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The structural analysis of the Dome of Discovery

Analyse structurale du " Dome of Discovery "

Die statische Berechnung des " Dome of Discovery "

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GENERAL DESCRIPTION OF THE METHOD OF ANALYSIS

The Dome of Discovery is a stiff triangulated space-frame in aluminium alloy shaped as a shallow spherical calotte with a heavy steel ring-girder supported on 48 tubular steel struts forming a system of 24 bipods articulated in the radial direction. The structure has 37 internal nodal points with 6 degrees of freedom each and 24 peripheral nodal points with 4 degrees of freedom each, and is therefore 318 times statically indeterminate.

The analysis was carried out by a combined relaxation and load transformation method specially devised for this purpose. The following is a very condensed outline of the method used and of some of the results obtained.*

The relaxation method is based on the principle of successive convergent approximations and consists of applying successive movements to imaginary constraints introduced at several points of a structure and placed so as to divide the structure into elementary structural units which can be readily analysed and may be statically determinate. The loads are imagined to act initially on the constraints which are considered "fixed." The movement of each constraint "relaxes" it, i.e. relieves it of part of the load, which is thereby transferred to the structure and simultaneously carries over forces and moments on to neighbouring constraints which are in turn relaxed.

The supports of the structure are treated as constraints; the sum totals of the actions carried over to them by neighbouring constraints give the actions exercised by the structure on its supports, equal and of opposite sign to the reactions of the supports.

The relaxation is complete when the residual actions on all the imaginary constraints are below predetermined negligible limits.

* A detailed description of the method of analysis and the full results are given in *The Structural Analysis of the Dome of Discovery*, by T. O. Lazarides, Crosby Lockwood & Son, London, 1952.

The combined method of analysis was based on the principle of decomposing unsymmetrical loads into a symmetrical and an antimetrical component and establishing for each component simple and exact group relaxation functions. By means of these functions the movements imparted to each nodal point or "spider" in turn in the course of the relaxation were automatically positively and negatively duplicated on spiders similarly situated with respect to planes of symmetry and of antimetry of the decomposed loading. In other words, this made it possible to restrict the analysis of the whole structure to that of an elementary wedge of symmetry and elementary wedge of antimetry; the boundaries of these wedges could be treated as structural boundaries with appropriate boundary conditions expressed in terms of relaxation coefficients. The wedges of symmetry and antimetry are shown on fig. 1.

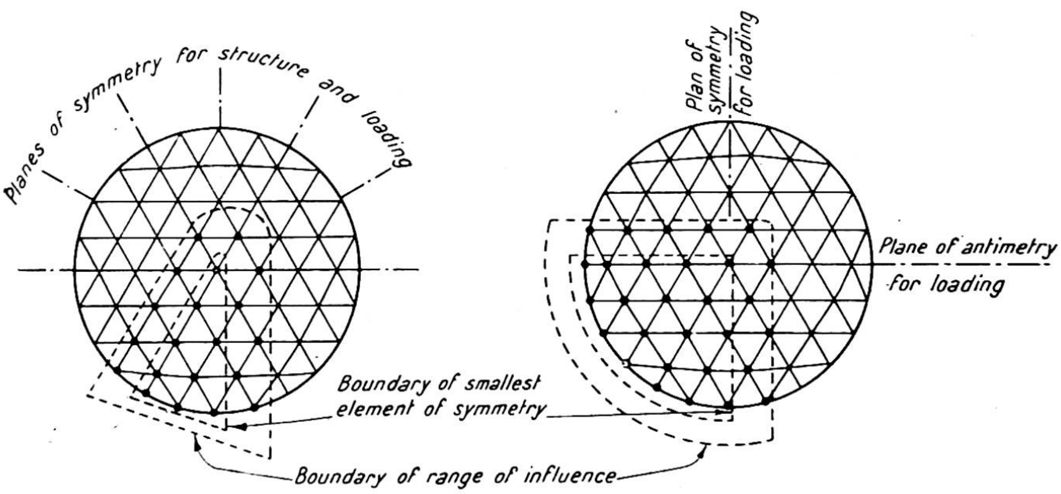


Fig. 1

The compounding of group relaxation operations was carried out as follows:

(1) Outline the element of symmetry to be analysed.

(2) Establish the range of influence for this element (i.e. the group of constraints adjacent to the internal boundaries).

(3) Establish influence coefficients for elementary movements on all constraints inside the range of influence. These are called the "elementary operations and influence coefficients for the general case" in order to distinguish them from the compounded operations for the symmetrical and antimetrical cases.

(4) Establish for all constraints situated on and within the boundaries of the element of symmetry the influence coefficients and relaxation operations compounded for the symmetrical case, as follows: perform in turn all possible operations on each of the inner constraints adjacent to the boundaries of the element and simultaneously the positive mirror image of these operations on each in turn of the outer constraints adjacent to the boundaries and symmetrical to the first group; sum the resulting effects on all the inner constraints affected by this duplication. For the unaffected constraints the coefficients are simply transcribed from the "general case."

(5) Do the same for the antimetrical case using negative mirror images, i.e. mirror images with all signs reversed.

This method of compounding is exact and the compounded operations differ from the elementary only with respect to the numerical values of the influence coefficients.

Three loading cases were analysed: own weight and uniformly distributed snow loads over half and over the whole Dome. All loads were assumed to be concentrated on the spiders.

The steel ring-girder was analysed both independently and together with the aluminium grid. The independent analysis gave valuable information on the elastic behaviour, under unsymmetrical loading, of a type of frame now coming into increasing use—a polygonal or round closed stiff frame supported on bipods allowing free radial but no tangential movement.

In the analysis of the Dome as a whole a choice had to be made between using one system of reference axes throughout or using several concordant systems of axes at different stages of the work. The following systems were finally adopted: (*a*) orthogonal parallel systems x, y, z at the two ends of a curved rib-element for determining the influence coefficients for each element, (*b*) orthogonal tangential systems of axes T, R, Q at each end of each curved rib-element, into which the previously obtained influence coefficients were converted in order to obtain interchangeable operation factors, (*c*) independent orthogonal reference systems R, S, T centred on each spider, into which the T, R, Q coefficients were converted in order to obtain operational factors for the relaxation of each spider. All systems of reference axes followed the same stereometric left-hand three-finger rule. The spider axes were determined as follows: axis R positive towards the centre of the Dome sphere, axis S tangential to the great circle passing through each spider and the summit of the Dome, positive towards the summit, axis T by the left-hand three-finger rule—tangential to the Dome sphere at right angles to axis S, positive from left to right viewed from above. A special convention was necessary for the summit spider. This choice of axes necessitated several conversion operations but decisively simplified the computations.

The sequence of operations used in the analysis can be summarised as follows:

(*a*) Draw a schematic plan view of the Dome showing ribs and ring-girder only (fig. 1).

(*b*) Choose plane of antimetry, i.e. decide, for the case of half snow load, which half should be loaded to give the most unfavourable results. (Note: the Dome can be divided into identical 60° or mutually symmetrical 30° wedges but not into identical quarters; turning the loading by 30° or by 90° therefore alters the loading conditions and considerably alters the distribution of tangential bipod thrusts. This alteration cannot be predicted exactly and it is therefore not certain that the case which was analysed was in fact the most unfavourable.)

(*c*) Outline on the plan view of the Dome the elementary wedges of symmetry and of antimetry (a 30° and a 90° wedge respectively) which will be used in the analysis, together with their ranges of influence.

(*d*) Mark all spiders with letter, number and upper and lower dash indices.

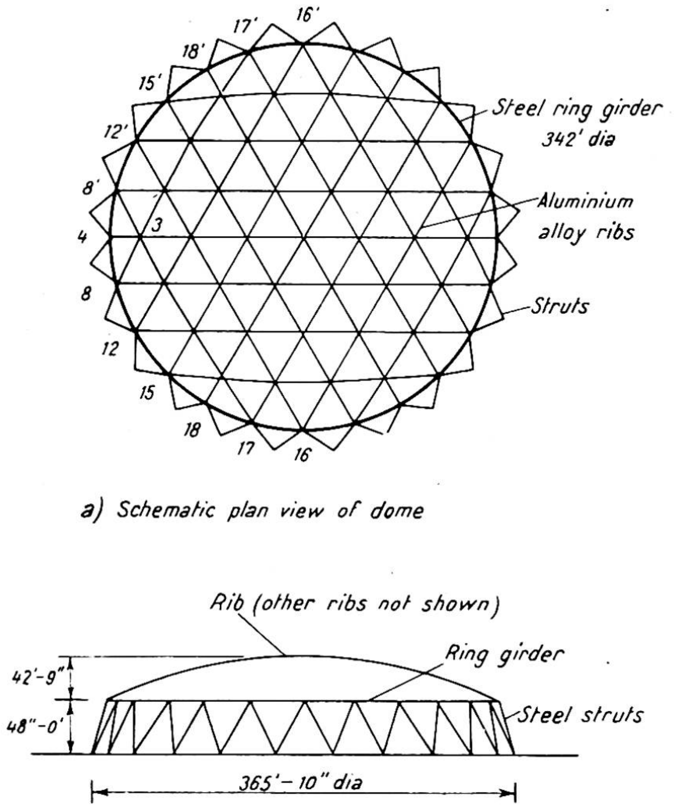
(*e*) Calculate exactly from basic structural data the position of each spider on the Dome-sphere, showing for each: the angle subtended by each of the elements converging on the spider; the position angles at both ends of each element.

(*f*) Calculate the influence coefficients for pure actions for all geometrically distinct rib-elements.

(*g*) Establish and solve in tabulated form sets of simultaneous equations giving the influence coefficients for pure movements for all geometrically distinct rib-elements.

(*h*) Enter these coefficients into conversion tables prepared for each geometrically distinct rib-element (letter indices only) and convert from x, y, z into T, R, Q reference system.

(*i*) Enter these T, R, Q coefficients into conversion tables prepared for each element in the influence range of the wedge of antimetry (number indices) and convert movements and actions from T, R, Q reference system into R, S, T reference system using position angle values from section (*e*).



a) Schematic plan view of dome

b) Schematic elevation of dome

Fig. 2

(*j*) Enter the R, S, T coefficients into the operational tables prepared for each spider in the range of influence of the wedge of antimetry. The fixed-end values corresponding to the spiders surrounding the reference spider are transcribed directly, the mobile-end values corresponding to the reference spider itself are added together.

(*k*) Compound these coefficients for the symmetrical case and enter into compounded operational tables prepared for each spider situated in the wedge of symmetry, using positive mirror images.

(*l*) Do the same for the antimetrical case using positive and negative mirror images as required.

(*m*) Prepare main relaxation tables and key plan (blanks); these are the same for both cases.

(*n*) Enter initial loading data for symmetrical case and relax to within predetermined negligible residues using appropriate compounded operational factors. Sum final total absolute movements for all spiders.

(*o*) Prepare re-compounding tables for individual rib-elements, enter total absolute movements of spiders, calculate forces and moments at each end of each rib-element.

(*p*) Prepare re-compounding tables for individual spiders, enter forces and moments in all rib-elements converging on each spider, check equilibrium of all spiders.

(*q*) Repeat from (*n*) to (*p*) for antimetrical case. Up to and including these operations all the forces and moments considered are those exercised *by* the the structure *on* the constraints. This convention has the advantage of being "natural" for analysis by relaxation because the movements applied then have the same sign as the residual actions which they are intended to liquidate.

(*r*) Compound the symmetrical and antimetrical cases to obtain the original unsymmetrical loading case (snow over half the Dome) by adding the movements for all spiders and the actions for both ends of each rib-element.

(*s*) Tabulate the final results for the five loading cases which can be obtained by simple combinations of the results of the analysis: own weight, snow over the whole Dome, snow over half the Dome, own weight plus snow over half the Dome, own weight plus snow over the whole Dome. In operations (*r*) and (*s*) the forces and moments are those exercised *by* the constraints *on* the structure, this convention being the more usual one in engineering practice.

Finally it should be mentioned that a complete and consistent system of checks was devised making it possible to discover numerical discrepancies, locate the errors by which they were caused, and follow through and eliminate all the consequences of these errors. Extensive use was made of elementary sketches, graphs and diagrams, particularly for the purpose of illustrating the sign conventions used at each succesive step.

Concerning the actual relaxation itself it was found that the successive operations tended to group themselves naturally into cycles, and the cycles occurred in natural sequences of two. Each successive attempt at reducing residual unbalanced forces constituting the first cycle of a pair, resulted in unbalanced residual bending and twisting moments, which were then in turn reduced in the operations of the second cycle. It was found advantageous to anticipate the effects of subsequent cycles and also of subsequent operations in each cycle by deliberate over- or under-relaxing.

A further essential feature was provided by the key plan, which made it possible to repeat previously carried out operations by direct transcription instead of referring back to the original operation sheets; when mutliplication or division was obviously required the only factors used were 1, 2, 3, 5 and powers of 10. This proved to be a decisive simplification.

Results of the analysis

The most striking aspect of the elastic behaviour of the Dome of Discovery under load is the very considerable difference in the distribution of the movements of spiders, of the internal forces and moments in the ribs and ring-girder, and of the bipod reactions for a symmetrical and for an unsymmetrical loading. Under a symmetrical loading the Dome only flattens out slightly, the distribution of spider movements and of the inner forces and moments is largely uniform, the bipod reactions are nearly identical and their tangential components are negligible. Under an unsymmetrical loading there is a marked tendency to sidesway despite the low rise-to-span ratio. This is due to the lack of rigidity of the ring-girder with respect to deformations which do not entail an overall lengthening and shortening of the perimeter and to the peculiar statical properties of the system of supports consisting of articulated bipods. The overall picture is that the grid tends to "slip away" from under the load; the rigidity of the ring-girder with respect to this type of deformation being practically nil, this tendency is checked by a transfer of tangential forces to some points only of the periphery, namely to the points where these tangential forces can be resisted by tangential bipod reactions. The result is that the supporting struts are loaded very unequally and that there are also some heavy thrusts in the ribs of the grid itself.

The final results cannot be given here in full, but a representative picture can be obtained by considering the behaviour of the ring-girder. Under full load conditions along the ring-girder are almost uniform, although there are small periodic variations giving the deformed ring-girder a slightly wavy appearance. The warping components are practically nil. These periodic variations have only a slight effect on the stresses in the ring-girder itself, but they appreciably affect the distribution of thrusts in the grid. The overall picture is that the grid pushes the bipods outwards and this movement is resisted by the ring-girder working in tension and in bending. While the stresses caused by direct tension and by bending are nearly equal the ring-girder is far more effective in tension than in bending, and a system of straight ties hinged at the rim spiders would therefore have been much more effective.

Under a half load conditions are radically different. The outward movements of the rim spiders and the tension in the ring-girder progressively increase in magnitude from the middle of the unloaded half to the middle of the loaded half. Rotations about the tangential axis are positive in the unloaded half (increasing the grid curvature) and negative in the loaded half (flattening the grid), but the greatest negative values are near the quarterpoints rather than in the middle as might have been expected. The most striking point, however, is the pattern of variation of the tangential bipod reactions; this pattern certainly appears strange at first glance, especially the solitary large positive thrust at the loading demarcation line flanked by negative values, and merits a more thorough examination.

Tangential thrusts in bipods can be caused in two entirely different ways: (*a*) by unequal thrusts in two grid ribs converging on a rim spider—the radial component of the resultant is taken entirely by the ring-girder, the tangential component is taken entirely by the bipod; (*b*) by unequal relative radial movements of rim spiders adjacent on either side to the spider supported by the bipod—these cause unequal thrusts in the spider, the resultant of which is taken entirely by the bipod. The distribution of these effects along the periphery follows quite independent laws.

Both these effects are present along the ring-girder when only half the Dome is loaded and it is their superposition which causes the pattern of variation of the tangential bipod reactions obtained in the analysis, and shown in the following Table.

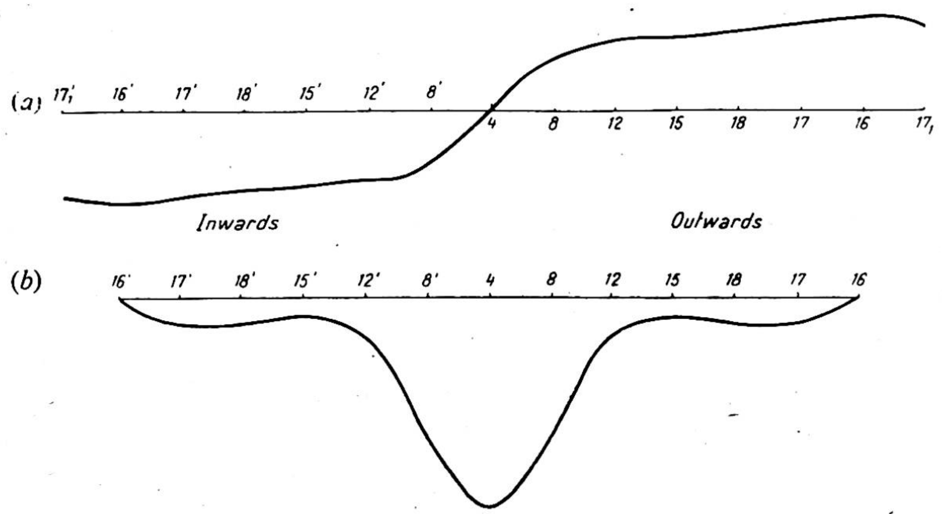
These effects were separated with the help of the influence coefficients used in the analysis to obtain the effects due to ring-girder action only; subtracting these from the total thrust we obtain the effects due to the unequal thrusts in the grid ribs.

It will immediately be appreciated that, taken separately, the two component effects do indeed follow a pattern the broad outlines of which could be deduced from the shape and general characteristics of the structure. It will also be appreciated that it would have been wellnigh impossible to predict from such general considera-

Tangential bipod reactions for load over half the Dome

	Spider No.	Total thrust	Thrust due to R.G. action only	Thrust due to grid action only
Middle of unloaded half . . .	16′	0	0	0
	17′	+ 7·831	+ 6·366	+ 1·465
	18′	+ 5·799	+ 6·116	− 317
	15′	−16·267	+ 4·454	−20·721
	12′	−29·952	+10·348	−40·300
	8′	−14·383	+34·885	−49·268
Demarcation line	4	+51·207	+52·618	− 1·411
	8	−14·807	+35·195	−50·002
	12	−29·952	+10·348	−40·300
	15	−15·843	+ 4·144	−19·987
	18	+ 5·799	+ 6·116	− 317
	17	+ 7·407	+ 6·676	+ 731
Middle of loaded half . . .	16	0	0	0

tions any details of the pattern of variation of either effect, let alone the exact relative proportions in which these two mutually antagonistic component effects are superimposed. On the other hand, the conditions along the ring-girder directly determine the reactions in the supporting struts and influence to a very large extent the distribution of forces and moments in the grid itself. It follows that a structure of this type can only be analysed exactly or not at all, as an analysis by approximate methods based on simplified and therefore necessarily incomplete structural analogies may easily lead to very large and unknown errors in unexpected parts of the structure, with the result that some of the structural parts and connections may inadvertently be grossly over or under-dimensioned.



Figs. 3

The absolute and differential radial movements for the rim spiders for the antimetrical loading case are shown schematically to an exaggerated scale on figs. 3(*a*) and 3(*b*). The deformation shown on fig. 3(*a*) is of a type one would naturally expect for an antimetrical load; thus it is obvious by inspection that h should be minimum at 16′, nil at 4 and maximum at 16 and that Δh should be nil at 16′ and 16 and maximum at 4, which is in fact a point of contraflexure. The variation of Δh

between these extremes is not quite what one would expect at first sight; in particular the transition between 8 and 12 is very abrupt and the secondary maxima at 17′ and 17 are unusual. These peculiarities are characteristic of the mode of support on bipods and of the manner in which the thrusts are distributed in the ribs of the grid. The general trend of the variation of the tangential bipod thrusts due to the ring-girder action only is now sufficiently clear.

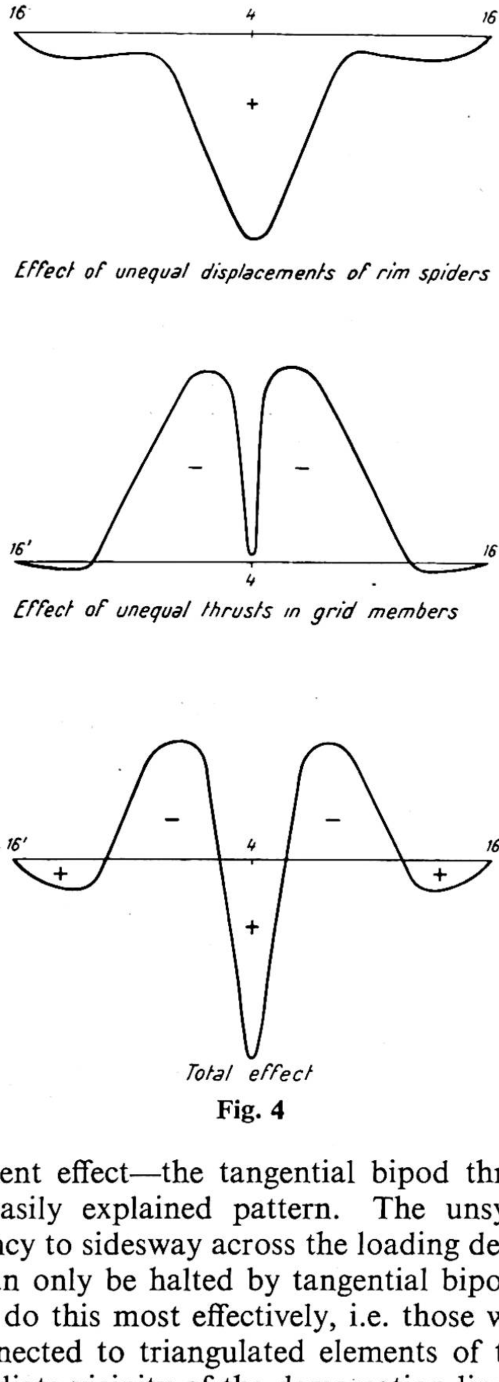


Fig. 4

The second component effect—the tangential bipod thrusts due to grid action only—also shows an easily explained pattern. The unsymmetrical loading will obviously cause a tendency to sidesway across the loading demarcation line and away from the load, which can only be halted by tangential bipod reactions exercised by those bipods which can do this most effectively, i.e. those which lie at a convenient inclination and are connected to triangulated elements of the grid. These are the rim spiders in the immediate vicinity of the demarcation line but *not* including those on the demarcation line itself owing to the almost complete lack of transversal rigidity of the grid elements connected to these spiders. The resulting interplay of tangential thrusts in the grid can be briefly summarised as follows. A severe cross-thrust in the loaded half of the Dome parallel to the loading demarcation line, two compensating pulls travelling across the Dome from the lower quarterpoint rim spiders, a cross-pull in the unloaded half of the Dome similar and parallel to the main cross-thrust but of smaller magnitude, and two compensating thrusts travelling across the Dome from the upper quarterpoint rim spiders.

The pattern of variation of the total tangential bipod reactions along the ring-girder (fig. 4) is thus adequately explained by the superposition of the two partial effects described above; in particular the presence of a large positive value at 4 flanked on either side by three consecutive large negative values without any transsition is explained by the superposition of the (positive) maximum ring-girder effect at the point of contraflexure 4 with a lacuna in the grid effect due to the lack of lateral stiffness of the rib 3–4 and the heavy concentration of the (negative) grid effect in the regions 15′–12′–18′ and 8–12–15, which are the only parts of the ring-girder where appreciable tangential components of the sidesway pull can be transmitted by a stiff triangulated grid—in other words, the only possible anchors against sidesway.

It is obvious that the extremely sharp variation of the tangential ring-girder reactions about the loading demarcation spiders shown on fig. 4 (from a large negative value to a very large positive value and back to a large negative value) cannot fail to produce heavy thrusts in the struts converging on these spiders. These struts are far more heavily loaded than all the others and owing to the peculiarities of the structure they cannot be relieved by shedding part of the load on to their neighbours. This condition could be radically altered either by triangulating all the terminal grid connections without any exceptions, or by converting some of the supporting bipods into tripods. In designing space-frames of great complexity particular care should be taken to avoid configurations in which the systematic or occasional overloading of some elements of the structure cannot be relieved by some form of load shedding.

Summary

The Dome of Discovery at the Festival of Britain, 1951, is a stiff triangulated space-frame in light alloy supported on radially articulated bipods and is 318 times statically indeterminate for arbitrary loading. A complete analysis for symmetrical and unsymmetrical snow load has been carried out by a combined relaxation and load transformation method specially developed for this purpose. This paper gives a brief summary of the method and of some of the results obtained.

Résumé

Le Dôme de la Découverte au Festival de Grande-Bretagne est une ossature rigide triangulée à trois dimensions en alliage léger prenant appuis sur un système de bipodes articulés dans le sens radial. La construction est 318 fois hyperstatique. Un calcul rigoureux pour surcharges symétriques et asymétriques dues à la neige a été fait à l'aide d'une méthode de relaxation et de transformation combinées spécialement établie. L'auteur donne un bref aperçu de la méthode et de certains des résultats obtenus.

Zusammenfassung

Die Tragkonstruktion des "Dome of Discovery" der Londoner Messe 1951 besteht aus einem steifen, 318fach statisch unbestimmten Raumtragwerk aus Leichtmetall, das getragen wird von stählernen in radialer Richtung gelenkig gelagerten Doppelstützen. Die genaue Berechnung für symmetrische und unsymmetrische Schneelast erfolgte mit Hilfe eines speziell für dieses Problem entwickelten Relaxationsverfahrens unter Benützung der Belastungsumordnung. Die Methode wird kurz beschrieben und einige Resultate werden angegeben.

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