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BII 2

Basic concepts of structural theory of aluminium alloys

Notions fondamentales concernant l'emploi des alliages légers dans la construction

Grundbegriffe einer Konstruktionstheorie für Aluminium-Legierungen

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HISTORICAL INTRODUCTION

The acceptance of a new material like aluminium for structural work represents a significant event in the history of engineering progress. The production and wide applications of ordinary and high-tensile steels in the entire gamut of the structural field ranging from screws to suspension bridges resulted in the neglect of development of other materials which possessed equally good physical and engineering properties. The development of aluminium alloys for structural purposes started in about 1905 when Conrad Claessen[1] obtained his patent for improving aluminium alloys by heat treatment. However, it was left to Alfred Wilm[2] to actually produce the first high-strength aluminium alloy termed "Duralumin." After several years of investigation, directed towards the production of an alloy for Zeppelin construction, Wilm, working at the Zentralstelle für wissenschaftliche und technische Forschungen in Germany, announced to the world of engineers and metallurgists in 1909 that the mechanical strength of some alloys could be substantially increased by a process of heat treatment. Not only did Wilm give the engineers a material of high specific tenacity (strength/weight ratio), but also laid in the hands of metallurgists a new principle of age hardening (or precipitation hardening) whose immense potentialities are as yet impossible to evaluate. The epochal character of Wilm's discovery can be appreciated from the fact that the advent of heat treatment introduced alloys which possessed the mechanical properties of mild steel with only one-third the weight. Prior to Wilm's discovery steel was the only known metal which could be hardened by heat treatment. The alloy composition worked out by Wilm had in addition to aluminium 4% copper, ½% magnesium, ½% manganese and slight traces of silicon and iron. It was actually produced by the Durener Metallwerke A.G. in Germany who coined the name Duralumin. Simultaneously with this development, Vickers,

[1] For references see end of paper.

Sons and Maxim introduced this alloy in England and France and acquired the patent rights for those countries.[3]

During the first world war it was realised that the pioneering work of the Wright brothers in evolving the airplane could only be put to a successful and practical use if airplanes could be made of a material having the strength of steel but the lightness of timber. The introduction of duralumin in the field in 1914 at once brought this metal into prominence because of its good mechanical properties, and it was extensively used in aircraft structures. By 1920 it had firmly established itself as a worthy and reliable constructional material for applications where high strength and light weight was essential. This was mainly due to the extensive and important work carried out at the National Physical Laboratory, England, the National Bureau of Standards, U.S.A., the aluminium research laboratories of the Aluminium Industrie-A.G., Neuhausen, Switzerland, and other large industrial concerns like the Aluminium Company of America.[4] Among its first uses as a major structural material was in the all-metal monoplane produced in 1920 by the Zeppelinwerke in Germany, followed by other large German dirigibles, the British R–34, and the American ZR–I airships.[5]

In spite of their large-scale use in aircraft structures, the field of application of aluminium alloys did not spread into other domains, especially in major stress-carrying components in structural engineering. This was mainly attributable to such factors as:

(*a*) Lack of knowledge of the principles of design and mechanical properties of aluminium on the part of designers outside the aircraft field.

(*b*) Difficulty of selecting alloys and sections due to general non-standardisation.

As it so often happens, it was only when both technical and commercial requirements combined to force the issue, as during and after the last war, that aluminium made some strides and established its mark as a useful material in structural engineering. However, at present the use of this light metal is limited in a very large degree to copying steel construction, which prevents full exploitation of its advantages. Aluminium has come into the structural engineering field *via* the aircraft industry, and as such should give a striking indication of its beneficial and progressive influence, provided it is not sidetracked into the familiar pattern of imitation of traditional materials. Besides direct substitution for steel, methods are available, and are outlined in this paper, by which an intelligent use can be made of the resources of accepted principles as adopted in the design of aircraft structures. The general principles of form strength and the basic concepts of strength of materials as developed through up-to-date engineering technique enable a truly rational approach to be made in formulating the theory of aluminium structures.

General comparison of aluminium and steel

Pure aluminium is very soft and ductile with a tensile strength of some 5 tons/in.^{2}, so that, except for pressing or deep drawing, it cannot be used for any structural work. By suitable admixture of other metals a variety of alloys are produced with strengths equal to and even above those of mild steel. For general structural purposes the ultimate tensile strength ranges from 16 to 32 tons/in.^{2} and the 0·1 % proof stress from 8 to 26 tons/in.^{2} By comparison with equivalent steel structures, the weights of those in aluminium are usually lower by 50 % in practice and nearly 90 % in the ideal case. In order to compare the two metals structurally the following typical values, as shown in Table I are assumed.

TABLE I

Properties	Steel	Duralumin
Ultimate tensile and compressive strength . .	27·5 tons/in.^{2}	25 tons/in.^{2}
Modulus of elasticity	30×10^6 lb./in.^{2}	10×10^6 lb./in.^{2}
Specific gravity	7·84	2·79

Based on the elementary principles of the theory of elasticity a comparison is made in Table II of the tensile, compressive and flexural properties of steel and aluminium having the stresses given in Table I.

TABLE II

Properties	Steel	Duralumin	Percentage economy
Equal tensile and compressive loading . .	W A	0·39 W 1·10 A	+61 W −10 A
Equal strength for beams	W A δ	0·38 W 1·07 A 2·64 δ	+62 W − 7 A —
Equal stiffness (deflection same) . . .	W A F	0·62 W 1·73 A 2·08 F	+38 W −73 A —

The factors considered herein are weight per unit length W, cross-sectional area A, relative deflection δ, and strength F. In terms of these factors the percentage economy (+*ve*) or excess (−*ve*) is given.[6] It will be observed from this table that by using an aluminium alloy member, there is throughout a saving in weight from 38% to 62%, although the volume of the metal is more than steel. For members in compression it was assumed that no buckling takes place. When such a failure is apprehended, the flexural rigidity has to be carefully considered, as it is this factor that actually measures the strength of struts. Detailed design of members in tension and compression based on the lines of steel design is not considered here, as it is given in several recent publications.[7–14]

In designing structures it often becomes necessary at times to determine which of the several available materials of construction when made into members of specified form have the least weight for a required strength or stiffness or will have maximum strength for a given weight, as in the fuselage and wings of airplanes, movable bridges and roofs, rolling stock and the like. A general idea of this concept can be readily had from a knowledge of the "specific tenacity" of a material. This term, first introduced in 1920 by Rosenhain[15] of England, is common in aeronautical parlance but not much known to structural engineers. The specific tenacity of a material is the ratio of the maximum stress in tons per square inch (ultimate tensile strength) to its weight in pounds per cubic inch. Table III gives the specific tenacity of some of the common representative structural materials.

It is quite evident that the specific tenacity does not give the complete criteria for design, as the questions of cost, durability, etc., are not covered; nevertheless it gives a clear picture of the relative strengths of various materials. It can be observed from

TABLE III

Material	Strength, tons/in.^{2}	Weight, lb./in.^{3}	Specific tenacity
Mild steel	28	0·286	98
Stainless steel (sheet material)	82	0·286	287
Duralumin	25	0·10	250
Aluminium 75–ST (sheet form)	35	0·101	347
Magnesium alloy	18	0·065	277
Laminated plastic (sheet form)	14	0·050	280
Sprucewood	4·2	0·0156	269

this table that the specific tenacity of aluminium alloys is nearly 2½ to 4 times that of steel.

In the case of direct tensile or compressive stresses, the governing criterion for design is the ratio of unit strength F, or modulus of elasticity E, to specific gravity or weight per unit volume W. For long and slender columns or for strength and stiffness in bending, ratios of these values with higher indices govern the criterion of design. This concept was first introduced by S. Livingstone Smith[16] in connection with the work on plastics, and was termed the "Criterion of Merit." This concept is of great use in aluminium design and requires to be carefully considered. Table IV gives the general values of the criterion of merit C, for various stress conditions for aluminium alloys. The member having the maximum value of C will be the most efficient from both the standpoints of strength and weight.

TABLE IV

Condition of stress	Criterion of merit, C		Ratio of C for aluminium to steel	
	Strength	Stiffness	Strength	Stiffness
Pure tension or compression	F/W	E/W	2·55	0·925
Bending—depth constant, width varied	F/W	E/W	2·55	0·925
Bending—depth varied, width constant	F/W^2	E/W^3	7·1	7·32
Bending—depth and width constant to give geometrically similar section	$F/\sqrt{W^3}$	E/W^2	4·24	2·6

It will be observed from this table that the value of the ratio of C for aluminium to steel (for stresses as given in Table I) is throughout more than unity, except for stiffness in the case of pure bending or compression. In the second and third cases for bending, the dimensions are varied to give minimum weight for required stiffness or strength, which is nearly seven times that of steel. However, when the sections are kept constant as in the fourth case of this table, the ratio of criterion of merit is nearly 60% more for strength than stiffness.

An analysis of structures in existence has revealed the fact that whenever aluminium is used to replace steel by directly copying from the latter the cost is invariably more, varying from two to seven times that of steel. For the purposes of economy, in structures where strength is of primary importance, high-cost high-strength heat-treatable alloys should be used, while low-cost low-strength alloys give economical results in the design of slender columns and also where deflection is the guiding factor. For both the above conditions it is technically sound and economical to increase the mass and dimensions of aluminium members.

A close study of the inherent characteristics of aluminium carried out by the author has indicated that in spite of the overall high cost of the light metal, its inherent properties can be turned to advantage even though some of them do not compare favourably with steel. A logical approach reveals that unlike the applications of steel, in which the metal is adapted to the preconceived structure, in aluminium the structure should be adapted to the material, by making a bold approach in design and in the evolution of sections suited for the particular work and design concepts governing the economics of the structure. Some of the salient features of this approach are discussed in the succeeding parts of the paper.

Basic concepts

A study of aircraft structural analysis reveals the close similarity between the airframe, its function and even the order of magnitude of loads, and the structural frame of a building. Such structures can be economically applied to structural engineering provided care is taken to see that they are not bodily copied, as this would be unpractical, for the extreme weight-saving required in aircraft is achieved at the expense of difficult fabrication and complex assembly not fully justifiable in structural engineering. However, the basic concepts typical in aircraft design can be adopted in the structural field when designing in aluminium alloys. Such principles are found in monocoque, semi-monocoque and sandwich construction, stiffened thin plates, tension field girders, corrugated sheets, tubular structures and space frames. Besides these, the general methods of application of non-dimensional column-curves for interpreting working stresses for compression design of thin plates, formed sections and stiffened panels in both the elastic and plastic zones offer a wide scope for creative thought and practice in aluminium design. These concepts, which have been used to a great extent by aircraft designers, are still foreign to the structural engineer and require to be carefully studied if a rational design procedure for aluminium is to be formulated.

Among the most important considerations are the very thin sections used in aircraft, in which shear and compression play a prominent part. Some members are designed so that they will not buckle locally, whereas others are permitted to buckle under their working loads so long as their design loads do not exceed their ultimate strengths. Such buckling, whether due to shear or compression, entails a redistribution of stress throughout the member in which it appears, and requires the use of methods and assumptions in the analysis of members which are new to the structural field. In light-weight construction the basis of efficiency lies in knowing the exact strength of materials used and in the accuracy of stresses imposed. In structures designed to withstand only static load, it is necessary to determine at the onset whether limiting loads will be based upon resultant stress or upon deformation.

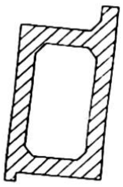
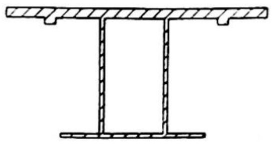
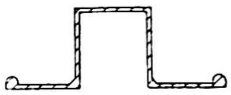
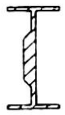
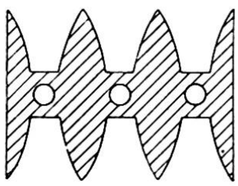
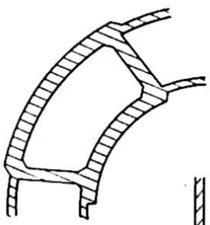
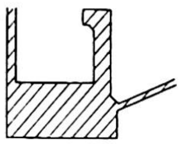
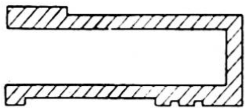
In light-weight design important considerations arise in two main types of structures, viz. complicated assemblies and lattice structures in which the maximum permissible stress rather than stiffness or deformation is the governing factor; and long and slender tension and compression members as well as thin-walled members wherein secondary stresses due to elastic instability and crumpling assume greater prominence than principal bending or compressive stresses.

In a correctly designed light-weight structure every component of the assembly must be so arranged and sectioned that it is utilised to the limit of maximum permissible stressing. Though easy to state, this is very difficult to achieve in actual practice. However, by avoiding the inefficient use of material in and around the neighbourhood of neutral zones, considerable economy can be achieved. Massive solid sections can be dispensed with, and in their place lattice structures and sheet-built columns employed. The ease with which aluminium can be extruded has resulted in the production and development of thinner structural sections as compared with steel. These thin sections when used in lattice and framed girders effect considerable economy in the material, because as a result of the correct distribution of their tension and compression elements, their component members exhibit no neutral zones. The resistance of I-sections against lateral loading, torque and overloading is low. Closed sections like rectangular and oval sections exhibit greater resistance, while buckling and cracking are less likely to occur, particularly in those sections having curved surfaces. The lattice structure assembled from tubes is in most cases the lightest and strongest. In fact, the shape and form of a stressed light-alloy member considerably influence its load-bearing capacity, as was shown by Thum[35] in his theory of form strength which required the member to have such a shape that the material was minimised at points of low stress and there was no stress concentration anywhere.

In view of the limitations of the rolling process the flanges of normal steel sections used in steel compression members are generally so thick that the stress at which failure by buckling of the flanges would occur is far above the normal calculated strength of the strut. Recently there has been a growing tendency in America to use stainless steel and light-gauge steel structural members for which special design specifications have been formulated.[36, 37] The production of very thin extruded sections in aluminium alloys necessitates a careful design of the relatively thin compression flanges, wherein buckling stresses usually fall below the permissible compressive strength of the strut. This particular aspect is being exhaustively dealt with in the Author's forthcoming publication[56] in which special consideration is given to deep and slender beams, lateral instability, web and flange buckling, shear-lag effects, torsion, and buckling of plates and shells. The low elastic modulus and the associated low shear modulus, influencing as they do to such a great extent in practice the bend and torque strength of the metal, as well as its tendency to crack, demand great attention being paid to loading capacity and moments of inertia, thereby directly influencing the structural form of elements built up from extruded sections. To give an indication of the possibilities, varieties and advantages of extruded sections, the author has given some of his suggestions in Table V. Such sections, which require to be judiciously used for the particular type of work, can be easily produced in aluminium, whereas their production in steel would present very great difficulties, if not be an impossibility. A further aspect of the reaction of materials to dynamic loading, specifically referred to in Germany as "Zeitfestigkeit," has recently become of importance, especially in light alloys. This aspect deals with the ability of the metal to sustain a given alternating load for a predetermined time, as against its fatigue or endurance strength which fixes the maximum alternating load which may be carried for an infinite period without destruction. This value is used in most parts of light-weight structures subjected to very severe stress and which after the lapse of the life period for which they were designed are intended to be replaced. The same value may also be used for the dimensioning of those parts of a structure which are so rarely subjected to peak stresses of the order assumed that they may be considered as serviceable for the entire working life of the structure.

The principle of continuity in design of beams and frames assumes considerable importance in view of the low modulus of elasticity of aluminium and of the fact

TABLE V

Form	Remarks
	Cavity-form independent of external shape.
	Main window-section frame, incorporating edging to take glass and rivets; and also having ample room for screws and beading.
	Top hat thin-walled sections having a high loading capacity in all directions, and beaded edges to prevent buckling or cracking.
	I-section with flanged edges in web to prevent buckling.
	Multi-cavity section having thin ribs with large surfaces.
	Tubular-shaped torsion and bending stress resistant window framing for roof-light, conforming to any curve and having edging for fixing.
	Form embodying a composite thin and thick section.
	Section having any desired decorative feature.

that load-bearing capacity is not necessarily a function of deflection. Thus a fixed beam supports 50% higher load (uniformly distributed) than a simply supported beam, while its deflection is only one-fifth of the latter. An intermediate state between these two appears more suitable in aluminium structures, for, in addition to reducing deflection, such partial fixity can produce positive and negative bending moments nearer to each other in magnitude of the order of $WL/16$ than with completely fixed ends wherein the negative moment of $WL/12$ is double the positive moment of $WL/24$ at the centre. The slightly greater deflection of aluminium beams gives a certain measure of springiness and thereby relieves fatigue; for it is a well-known fact that a human being experiences greater comfort while walking over resilient ground or suspended floors, which reduce the impact on the feet, than over solid floor. Halls for dance floors appear to be ideally suited for this material.

Having considered the general design aspects, the various specific types of basic formulations met with in aircraft are now considered and their methods of usefulness in the structural engineering field outlined.

Stressed-skin construction

The construction of wings of airplanes prior to 1930 was carried out in a simple manner. The main stress-carrying members comprised spars and bracings, which were covered all round by a fabric which in no way carried any load but only served to give aerodynamic smoothness to the wings to a certain extent. In fact, the whole design was like a simple braced girder. Realising the need for maintaining a highly smooth surface to reduce to the maximum the aerodynamic drag, metal covering slowly replaced doped fabric as a covering material. This sheet-metal skin not only acted as a mere covering, but also formed an integral part of the stressed system, carrying its share of stresses along with the spars and ribs. If the covering of thin sheets or webs is strong enough in carrying the loads without the necessity of internal stiffening members, the construction is termed "monocoque," from the French word meaning "single-shell." It is usually not possible for the skin to be thick enough to resist compression loads, and stiffeners are therefore necessary to form what is then termed a "semi-monocoque" structure. In such structures the thin webs resist torsional, shearing and tensile forces in the plane of the web, while the stiffeners resist compression forces in the plane of the web or small distributed loads normal to the plane of the web. Both these types are commonly termed stressed-skin structures.

In modern transport planes the fuselage is approximately a circular thin-walled aluminium-alloy cylinder, reinforced by circumferential and longitudinal stiffeners, termed stringers. The diameter of the cylinder is about 120 in., the thickness of sheet skin 0·025 to 0·072 in. and the depth of the ring about 2·5 in. It is obvious that a cylinder having a ratio of thickness to radius of the order of 1/1000 would buckle when subjected to small shear or tensile forces. The addition of a set of two stiffeners transforms the cylinder into a sturdy structure.

The principle of stressed-skin construction can be applied in structural engineering to a variety of members such as walls, floors and roof. Here the outer covering, especially in roofs, would be formed out of sheets of aluminium, which unlike ordinary construction would carry a part of the stresses normally carried by the truss. The whole roof unit comprising trusses, purlins and sheet covering, all in aluminium alloys, appears to be a very efficient way to preassemble the whole roof (or at least in suitable bays) and lay it on a structure as a finished product. When trusses are made of aluminium, the lightness of the unit enables it to be spaced closer than ordinary steel roof-trusses without any increase in load over the supporting walls or columns. The reduction in the distance between these trusses results in the deletion of purlins, because the stressed-skin of aluminium-sheet acting as roof covering can be fixed directly on the trusses, a procedure which cannot be followed in steel construction. It may be argued that the aluminium truss would deflect more than a steel one, because of its lower modulus of elasticity, and that this could only be avoided by using larger sections. This is not true, as the deflection and sections can be kept within normal working limits if only the designer can make an intelligent use of the variety of sections capable of being fabricated in this light metal. Thus by using a sort of benchlike section with offset connections as shown in fig. 1, a stable, strong and light truss can be produced. It will be observed from this figure, that members AC and BC are of a peculiar benchlike shape and are connected by pins at the points A, B and C. The member AB is a tensile piece and can be either a flat or a bar if the ends are pin-connected. When rigidity at supports A and B is available the member AB can be dispensed with. Members AC and BC have special offset connections to minimise deflection. The top compression chords of a conventional triangular roof truss deflect more than would a simply supported member due to axial compression in the chord which increases deflection once a small initial deflection is induced in it. With increase in loads, the deflection goes on increasing, which is naturally more pronounced in aluminium members if designed on conventional principles. However, by reversing the procedure as shown above, the eccentric connection of the chord causes an upward deflection which balances the downward deflection due to loads. Such a method of design not only compensates for the disadvantage of aluminium alloys having lower modulus of elasticity but also enables it to develop full strength with economy in weight.

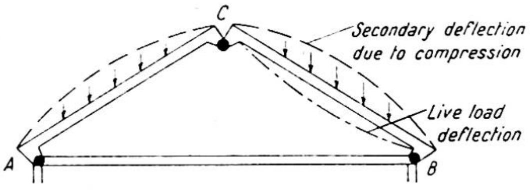


Fig. 1

In general the design of stressed-skin construction is in the main the design of flat and curved plates and sheets stiffened with extruded or rolled sections and subjected to bending, shear or torsion. The elastic buckling stress for thin rectangular plates is given by the equation $F=KE(t/b)^2$, where F is the critical stress, E is the modulus of elasticity, t and b are the thickness and breadth of the plate and K a constant depending on the linear dimensions of the plate. The value of the constant K governs the stress criterion, viz. whether it is the critical shear stress or the critical buckling stress or the critical compressive stress. Values of K are available from the principles of elastic stability as given by Timoshenko[17] and Perry.[18] After obtaining values of these stresses, they can be suitably combined for cases where the skin is subjected to a combination of two types of stresses. Thus initial buckling occurs when one of the following equations is satisfied:

(i) Compression and bending: $Z_b^{1\cdot 75}+Z_c=1$
(ii) Compression and shear: $Z_s^{1\cdot 5}+Z_c=1$
(iii) Bending and shear: $Z_b^2+Z_s^2=1$

where Z_b, Z_s and Z_c are ratios of stresses in plate to the critical stresses, viz. f_b/F_b, f_s/F_s and f_c/F_c.

When the plates are curved, as in the case of domes or arches, the equations for the critical stress remain the same, except that K has different values, being a function of the ratio of length to breadth of sheet and the ratio of the square of breadth to the product of radius of curvature and thickness of sheet.

The use of aluminium-alloy sheet for stressed-skin has a very great advantage. For the same weight it is roughly three times as thick as steel, and since buckling load increases as the cube of the sheet thickness, the strength contributed by aluminium is considerable. The principles of stressed-skin construction are applicable in floors and wall panels also. In the traditional type of structure the outside covering has no stress-carrying function. Since the skin extends over the entire surface of the panel, its cross-sectional area is sufficiently large to permit the use of a very thin skin, limited only by the criterion of corrosion, a feature not possible in steel. At times the loads to be resisted are so large that the stiffener spacing becomes so close as to make them impracticable for construction purposes. They are then replaced by a continuous corrugated sheet, riveted or spot-welded to the skin. The normally used corrugated sheet shown in fig. 2 does not develop its full strength, as the straight section between p and q buckles before the curved portions.

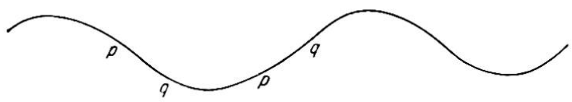


Fig. 2

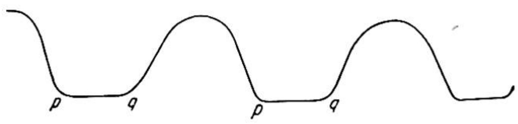


Fig. 3

A more efficient section would be one having flat-topped corrugations as in fig. 3. Here the flat section pq is adjacent to the strong curvature and is therefore better supported. The flat part also facilitates the attachment of other assemblies. Such sheets can be adopted in large built-up girders as compression flanges. This application appears to have been utilised only in the cantilever beams of aircraft wings, though they are quite useful in general structural engineering. Such stressed-skin structures lend themselves admirably to the design of roofs, both straight and curved, large domes, aircraft hangars (where the roof, sides and the main structure can be an integral unit), and floor and wall panels of ordinary and prefabricated structures.

Tension field beam

The tension field beam, not generally known outside the domain of aeronautical engineering, is an excellent example of the manner in which basic concepts of aluminium can be applied to efficient design in structural engineering. It is in the main similar to a steel built-up girder with web stiffeners as shown in fig. 4. The main difference is that the tension field beam is of very thin sheet-metal which is permitted to wrinkle or buckle under shear stress caused by the load. The wrinkled web acts like tension diagonals in ordinary open-web trusses, thus creating a "tension field" within the web. The theory of pure tension field beams was first developed and published by Wagner in 1929 and it is therefore also termed the Wagner beam.[19] In this theory it was assumed that the web was perfectly flexible and was not capable of resisting any diagonal compressive stress. In practice, however, these webs do resist to some extent diagonal compressive stress after buckling, and thus act in an intermediate range between shear-resistant webs and pure tension field webs. Beams with such webs are then termed semi-tension or partial-tension field beams.

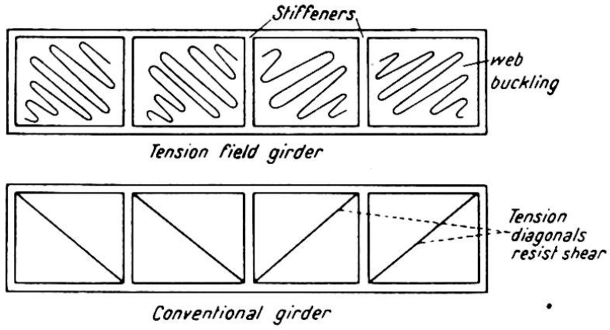


Fig. 4

The theory of semi-tension field beams demonstrates that a structural member does not necessarily fail under loads which cause large visible deformations in some of its elements such as the web of this beam. It follows therefore that such members can be made considerably lighter than what the more conventional or elementary design procedures permit. Unlike steel sheets which cannot be made less than 0·06 in. in thickness, aluminium webs of 0·025 in. can be successfully used in such beams without any danger of loss of strength due to corrosion. Since the development of the Wagner beam, many investigators have studied the problem of semi-tension field beams, the most extensive experimental work being undertaken by the National Advisory Committee for Aeronautics (N.A.C.A.) under the direction of Paul Kuhn in America,[38] and by Crowther and Hopkins in England.[39] In Kuhn's analysis it is assumed that a part of the shear load X is resisted by pure tension field action and the remaining load Y by shear-resistant beam action. Then the ratio of X/Y, termed the diagonal tension factor k, is given by the equation

$$k=\tan h(0{\cdot}5 \log_{10} f_s/F_s)$$

The stiffener compression forces P and the flange bending moments M are proportional to the vertical component of the web tensile stress, f_y. The values of these are given by

$$f_y=kf_s \tan\alpha$$

$$P=f_y td=kf_s td \tan\alpha$$

M, for stiffeners$=Pd/12$; and M, between stiffeners$=Pd/24$.

An effective width of web equal to $(1-k)d/2$ is assumed to act with the stiffener. The stiffener compression stress f_c is then given by:

$$f_c=P/A_e+(1-k)td/2$$

In the above equations,

f_s=shear stress,
F_s=buckling shear stress,
P=stiffener compression load,
M=flange bending moments,
f_y=vertical component of web tensile stress,
f_t=diagonal tension stress,
t=web thickness,
d=stiffener spacing,
E=modulus of elasticity.

The term A_e is used to denote the effective stiffener area. When there is a stiffener on each side of the web, A_e is equal to the true stiffener area A. However, when there is a stiffener on only one side, the value of A_e is given by $\frac{Ar^2}{e^2+r^2}$, where e is the eccentricity of compression load P, as measured from the centre of the web to the centroid of the stiffener area, and r is the radius of gyration of the stiffener. The angle of diagonal web tension is denoted by α and its value is given by:

$$\tan^2\alpha=\frac{S-S_x}{S-S_y}$$

where $S=f_t/E$ and is the unit strain along web diagonal; S_x is the unit strain in the beam flanges resulting from the compression caused by the web tension; and S_y is the unit strain in the vertical stiffeners caused by compression load P. For normal beam proportions where the flanges do not compress appreciably, S_y can be assumed to be zero. Then in terms of stiffener area, the value of α can be obtained as:

$$\cot^4\alpha=td/A_e+1$$

If the semi-tension field beam has equal stiffness in resisting the horizontal and vertical tension, the two tensions will be equal and the value of α will be 45°. In practice the flanges of such beams are more rigid in resisting compression loads than are the stiffeners, and as such α is less than 45° because the horizontal web stress is greater than the vertical tensile stress. Some indication of the values can be had from fig. 5, in which are plotted values of tan α against td/A_e for different values of k ranging from 0 to 1·0.

The analysis of straight beams can be extended to cover the special case of curved tension field web beams, commonly met with in aircraft structures, but necessarily limited in structural engineering. It can be used in the latter for curved built-up beams, and designed from the semi-empirical methods of analysis given by Kuhn and Griffith.[20]

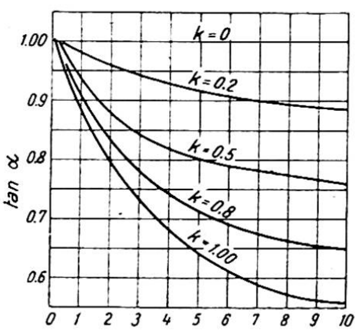


Fig. 5

Sandwich construction

A better known counterpart of the tension field beam is a sandwich panel, in which the flanges rather than the web are made of strong thin sheets. The advantages of using both low-density and high-density material can be obtained by using a relatively thick low-density material bonded between two faces of thin high-strength sheets. William Fairbairn adopted the idea of laminar construction when carrying out his notable investigations on fatigue and bridge design as far back as 1849.[21] Since then it has been used in aircraft structures to some extent. Since the last war its design and development have received considerable impetus mainly through the publication of Bruyne, Gough and Elam's classic paper and the active investigation by the Forest Products Laboratory, U.S.A., National Luchtvaartlaboratorium, Amsterdam, and the Royal Aircraft Establishment and College of Aeronautics, England.[22] The main purpose of this type of construction is to place the strong principal structural elements as far apart as possible to obtain a large moment of inertia of cross-section with the ensuing benefits of high flexural and torsional rigidity and low overall density. The core provides suitable shear connections, increases the relative moment of inertia and also acts in stabilising the facings so that they will not wrinkle until a high state of stress in the material is reached.

Aluminium alloys, with their high strength, low density, absence of corrosion and the availability of sheets of very thin size, offer a wide scope for use as facings of sandwich panels. Among the other materials available are stainless and mild steel, magnesium alloys, plastics, woven glass-fibre fabrics, paper fabrics and plywood for facings, and wood, honeycomb and pulp-base materials for cores. These panels when correctly designed can be used efficiently for structural purposes in floors, walls and roofs, and for light non-stress-bearing members in partitions, doors and refrigeration panels.[23]

The structural properties of sandwich panels depend upon the ratio of the core thickness to the thickness of the face materials. As the thickness of facings is increased, the panel becomes stronger, thicker and heavier. However, it cannot be indefinitely increased because there is one core/thickness ratio which gives the maximum flexural-strength/weight ratio and another which gives flexural-stiffness/weight ratio.

As in ordinary aluminium design, the strength/weight ratio also assumes importance in sandwich construction. From the general principles of strength of materials it is found that for a sandwich material resisting bending, the minimum weight is obtained when the weight of both the facings is approximately the same as the core material. To resist compression buckling loads, however, the total weight of both the facings should be approximately one-half the weight of the core material in order to obtain the minimum overall weight. Assuming the facings to be of a high-strength aluminium alloy 24 S–T, and a core material of density 0·01 lb./in.³, it can be worked out and shown that in order to resist the same bending moment, this type of sandwich panel would weigh 37 % less than a solid aluminium sheet. Similarly for equal compression buckling-load, it is found that the sandwich weighs only 21 % of the solid sheet of 24 S–T alloy. It is interesting to consider these values with other materials of construction given in Table VI.

TABLE VI

Sheet material	Ratio of weight of material to weight of 24 S–T alloy	
	Equal bending	Equal compression
Stainless steel	1·72	2·12
Magnesium alloy (of ultimate tensile strength 40,000 lb./in.²)	0·83	0·77
Laminated plastic	0·74	0·83
Sprucewood	0·42	0·31
Sandwich panel (as described above) . . .	0·37	0·21

It will be observed from this table that the sandwich panel with aluminium facings constitutes one of the lightest forms of modern construction procedure and is well adapted for floors and wall panels of structures. Instead of the ordinary smooth face sheets it is suggested that greater rigidity can be obtained by beading the faces or by providing integral ribs as shown in fig. 6. Early investigations revealed that bending

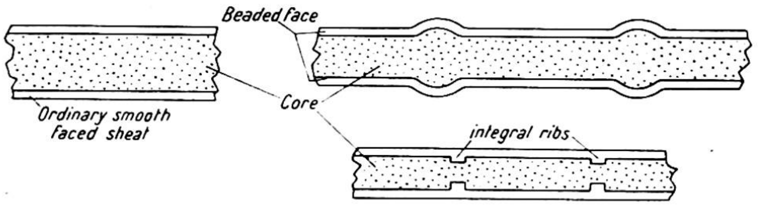


Fig. 6

rigidity and buckling loads of sandwich-type structural elements were considerably lower than those arrived at from the conventional bases of strength calculations, mainly because of the small shearing rigidity of the relatively thick core. Consequently the basic theory with their classic formulas could not be directly applied to calculate instability conditions. The theory had therefore to take into account (*a*) the shearing deformations, which as a rule are neglected in ordinary structural analysis, and (*b*) the anisotropic nature of face and/or core materials. Based on this concept various formulae were developed for compression, bending and buckling and general design features and evaluated by the author elsewhere.[22] A detailed analysis of the elastic and plastic stability of sandwich plates by the method of split rigidities has been recently made by Bijlaard and is worth studying.[27] Sandwich construction, with its high specific tenacity and ease of manufacture, resulting in the production of large sheets with uniform surfaces and absence of stringers and stiffeners or rivets, should commend itself as a good type of building material satisfying both architectural as well as structural standards.

Space frames

A logical development from aeronautics and one having very wide, but not yet fully explored, potentialities in structural engineering is the space frame. Most structures have up to now been designed as two-dimensional frames, without taking into consideration the favourable characteristic of the third dimension, mainly because a three-dimensional frame is difficult to visualise and also to analyse. The truss-type fuselage and aircraft wing structures have long realised the efficiency of space frames in which the strength of the torsional structure is utilised in carrying eccentric loads.

The general principles of this form of construction have long been known and the methods of calculating stresses given in many publications[24, 25, 26, 28, 32, 33]; yet its practical applications have not been made on a wide scale, except in some German and Swiss bridges described by Bowman,[28] Walther[29] and Haupt.[30] Triangular-section trusses have to a certain extent been used in other structural fields, such as radio and television towers, and in "diagrid" structures for floors and roofs evolved by Fowler and the late Dr. Pandya of India.[31] Very recently Professor Kavanagh illustrated the possibilities of this type of structure in the bridge field, by giving some pertinent figures for a welded two-lane deck highway bridge of 120 ft. span subject to specifications of A.A.S.H.O. for Highway Bridges.[34] According to him such a bridge of welded triangular section is 21% lighter than the conventional riveted structure. If, on the top of this, the item of flooring is omitted, because in both types they will be alike, then the remaining structure in triangular type is 49% lighter than its conventional counterpart. A similar analysis for a two-hinged spandrel arch bridge with braced decking of 22 ft. rise and 5 ft. depth at centre revealed a saving of 30% for the entire bridge, and 67% when the flooring was omitted. Another important factor is that in such space frames the actual space volume is reduced by 50% to 75% over conventional designs, with considerable improvement in the æsthetic value.

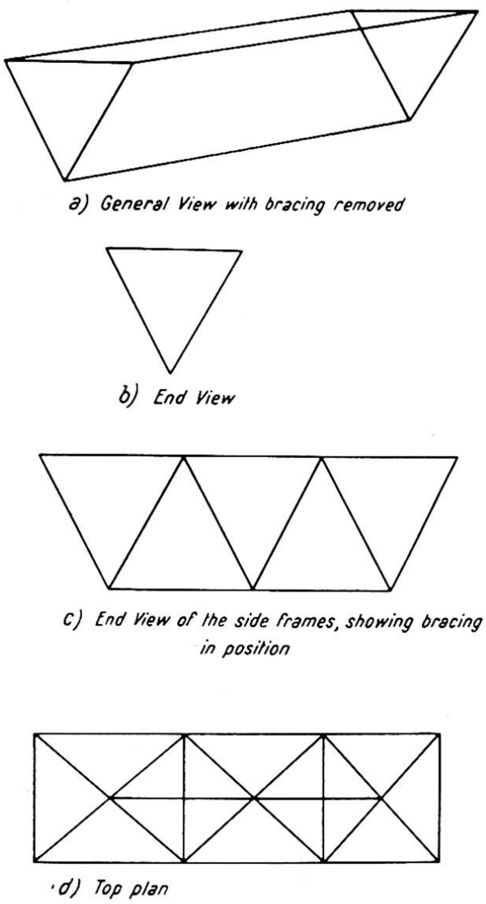


Fig. 7

A brief analysis of the triangular design given above strikingly reveals the vast economies that can be effected if in such structures steel is replaced by aluminium. Thus in ordinary bridges it is found that a saving of 50% in weight results from the use of the light alloys. Adding this to the extra saving in weight arising due to the triangular type construction of approximately 20%, it can be easily seen that an aluminium space frame correctly designed affords an overall weight-saving of some 70% over the conventional steel design. It may be of interest to point out that in the Arvida Bridge over Saguenay River in Canada (which incidentally forms the longest aluminium bridge of any kind in the world with a single fixed-arch span of 290 ft. and a rise of 47 ft. 6 in.) the saving in weight by using aluminium was 57%, while in the 100-ft. plate-girder-type bridge over the Grasse River at Massena, New York, the saving was 59% over the conventional steel structure.

The utility of triangular-type structures having been established, it is imperative to develop this form satisfactorily for major structural work, which is ideally suited for incorporation with aluminium alloys. The resulting framework is more or less on the lines given in fig. 7.

APPLICATIONS OF PLASTIC THEORY

The analysis of engineering structures is based on the theory of elasticity, the various concepts and applications of which were propounded by Hooke, Young, Navier, Poisson, Bernoulli, Coulomb, Clapeyron, Maxwell, Euler and Barré de Saint-Venant.[40, 41] According to the elastic analysis of a structure, each member is so proportioned that the most unfavourable combination of external loads, when multiplied by a suitable factor of safety, will just produce yield in that member. It is quite evident, and a long-standing fact, that a redundant structure is by no means on the verge of failure when yielding occurs in one of its members. If the external loads on a redundant structure are steadily increased, then the excess load on the member which has yielded is automatically taken up by other members which have not so far yielded and are capable of carrying a still greater load. The methods of applied elasticity also very conveniently ignore secondary stresses which are difficult to compute and stresses around rivet holes, around points of application of loads and reactions (stress-concentration) and around junction points such as in web and flanges of I-sections. It is therefore logical that the theory of elasticity, long considered a classic, should be replaced by a more correct theory to proportion structures which would collapse only when subjected to the maximum specified loads multiplied by the correct factor of safety. Realising this fact, the newer methods of design have come into prominence in which the inelastic behaviour of structures is given special consideration.[51] The inelastic behaviour indicates any type of general mechanical behaviour that is not elastic and covers the theory of anelasticity and the theory of plasticity (plastic flow and plastic deformation and limit design). The theory of anelasticity propounded by Zener[52] defines the inelastic characteristics of two-phase or polyphase materials in the range of small deformations and is at present of little interest in structural engineering. The theory of limit design has been dealt with fully by Van den Broek,[42] while the general theories of plasticity are being actively investigated by many workers in England, America, Germany and Russia. The original investigations of Kist,[43] Grunning,[44] and Maier-Leibnitz,[45] followed by the recent work of Baker[46] at Cambridge University and Prager in America,[47] have resulted in the development of methods of plastic design for several types of framed structures. Most of the applications of the plastic-design methods have been to steel structures, with little experimental or analytical work on light alloys. The author believes that the methods of limit design and the principles of shake-down theorems and plastic collapse[48] can be applied for designing frames in aluminium alloys also. The basic hypothesis of the value of the bending moment not exceeding a certain magnitude at which large changes in curvature occur at constant moments, though not strictly valid, appears to be accurate enough for some light alloys to allow useful results to be obtained. The fact that aluminium alloys do not exhibit the flat yield-characteristic of mild steel constitutes an advantage in that relatively much higher loads can be carried by redundant structures made of light alloys than can be carried by such structures in steel, because of the effects of continuity which contribute towards increasing the resisting moments. In view of the unsifted mass of literature existing on the theory of plasticity, from which the structural designer can find little of direct use, the author believes that the general principles of limit design furnish, in the present state of knowledge, the most reliable procedure to be adopted for structural design in aluminium alloys. The seeds of the theory of limit design, which presupposes ductile or semi-ductile stress distribution, and lays emphasis on permissible safe deformation rather than on permissible safe stresses, were first laid by Kist[49] of the University of Delft. These concepts were later expanded and modified and given a practical utilitarian value by Van Den Broek.[42] In fact, it is seldom realised that the classic formula of Euler used for compression design is a limit-design formula, because no symbol of stress appears in this function. In designing aluminium structures, especially continuous beams, the same procedure as followed for steel is adopted, without any modification, except that some value of the stress has to be decided upon to take the place of yield-point stress in steel. This stress should necessarily be low enough to avoid excessive deformations, but high enough so as to enable the material to be used to the maximum efficiency, the recommended being the 0·2% proof stress. A more practical method is to assume that the well-known stress function $M=fZ$ (M is the bending moment, f the maximum fibre stress and Z the section modulus) applies beyond the elastic limit, thereby utilising the fictitious stress as something analogous to the so-called modulus of rupture, without of course any existence of rupture. On this basis, experimental investigations were carried out by Panlilio [50] to chart out a possible future line of attack, and his findings are worthy of detailed study. The application of limit design to aluminium alloys has one disadvantage in that the comparative brittleness of these alloys does not allow a correct evaluation of exact figures to determine the minimum value of ductility consistent with safety for the requirements of different types of construction. Experimental investigation is still needed to correctly prognosticate the instability and rupture conditions, before an exact theory of plastic design can be formulated. In the analytical field the theory offers mathematical difficulties and forms an important field of research in the non-linear mechanics of deformable media.[53]

Safety factors

The application of the theory of limit design to aluminium structures offers a means whereby a constant factor of safety can be selected which is closer to the real safe margin than is provided by the conventional methods based on the elastic theory. In fact, the whole concept of safety factor requires to be re-analysed in the light of the recent development in the field of statistical analysis and the plastic theories. A comprehensive and probably the most modern treatment has been given by Pugsley [54] of Bristol University. As the author pointed out,[55] the various concepts in structural engineering adopted so far clearly reveal that what engineers usually designate as factor of safety is in reality a factor of uncertainty or a factor of ignorance. This is because allowance has to be made for such items as variations in the quality of the material, introduction of a new material for which test data are not available, exact interpretation of stresses and strains depending on a correct selection of the failure theory, and evaluation and estimation of exact loading arising in practice, all of which are extremely difficult to infer to the right degree of accuracy. As human observation is limited to a certain degree of accuracy attainable, the concept of safety can only be evaluated between maximum and minimum limits and not equated to an exact value. The relation between buckling resistance and slenderness ratio, in columns, is a typical example, in which functional, statistical and empirical laws exist in one and the same problem, and as such the margin of safety cannot be equated to any known quantity. Actually the safety factor is affected by two influences, viz. that which governs the stress induced in the structure or the load that produces the stresses, and those which govern the resistance of that structure or its carrying capacity. The laws of structural design, as far as they pertain to the margin of safety, have to be considered as a combination of functional and statistical relationships: functional so far as the laws of the theory of structure are concerned and statistical to the extent that real physical properties appear as parameters of the functional relations. Turning from these abstract considerations to practical engineering aspects, certain discrepancies can be observed if the safety factor for aluminium structures is unintelligently copied from steel construction. Thus mild structural steel with a yield-point strength of 15 tons has an ultimate strength about 80% higher at 26 tons and an elongation of over 20%; while a typical duralumin alloy with 0·1% proof strength of 26 tons has an ultimate strength only about 20% higher at 32 tons and an elongation of only 12%. It is quite evident, therefore, that if a design is based on proof strength or yield point, there is a proportionately less margin of safety between this and the ultimate strength in the case of aluminium alloys than in the case of steel. Further, to safeguard against rusting and corrosion, a minimum surface area has to be provided as a safe margin. In a fully stressed steel part of, say, 1 in. effective thickness, the loss of $\frac{1}{16}$ in. from each face by rusting, over a period of time, is alone sufficient to increase the original design stress by 1 ton/in.^{2} Such a consideration does not arise in aluminium alloys because of their very high corrosion resistance.

As the ultimate strength characteristics appear more pronounced, it can be easily realised that it is a function which cannot be left out of the picture of safety factor, if a rational approach to design is to be formulated. The importance of the theory of limit design, which is based upon the ultimate stress, can now be seen in its true perspective. As the structure becomes more complex and highly redundant, the gap between yield-point load, the theoretical load at which yield commences and the actual collapse load, also increases. In very extreme cases the collapse load was found to be as high as eight times the yield-point load. It therefore stands to reason that if a safety factor of 4 is employed for pin-connected structures, it can be easily reduced to 2 or 2·5 for redundant structures. Usually in aircraft structures the margin of safety is taken as $(F-f)/f$, where F is the allowable stress and f the calculated stress. In view of the meagre information, both analytical and experimental, available on this vexed question of safety factor for light-alloy design the author ventures to assert that a working stress of 1/2·5 to 1/3 of the ultimate stress should be adopted. A lower margin of safety than this, viz. 1/2, can be adopted for highly complex but fully static structures, while for bridges and other dynamic structures it should not be less than 1/3, especially where large impact forces are expected. The strength of the strongest aluminium alloys, and in fact all metals in general, is scarcely 1/30th to 1/40th of the value to be anticipated from the theory of perfect crystals as defined through the laws of metal physics.[57] A similar discrepancy is also observed for yield stress under shear. In general the region of perfect elasticity is very small, the metal does not fracture in a brittle manner but experiences a large plastic yield during which metals like aluminium work-harden, and at high temperatures and stresses exhibit the phenomena of creep. The concept of safety factor in design for high-temperature service, which arises in the case of the atomic pile for the production of nuclear energy, is considerably more complex. This is due to interrelation between time, temperature and strength, which introduces the duration of load at certain temperatures as a significant characteristic of this load. Since the effect of load fluctuations is to be combined with temperature fluctuations of varying durations, and since the former are not dependent on the fluctuations of resistance, even the statistical laws are not applicable in evaluating the safety factor in such problems.

Conclusion

In presenting this paper the author has tried to give a balanced appraisal of the utility and limitations of aluminium in structural engineering. A new outlook in designing light-alloy structures is stressed and avenues of approach for future developments indicated. The basic concepts of theory formulated here represent a unified attempt to establish fundamental principles necessary for a correct rational approach to the subject.

It is hoped that the paper will form a springboard for extending through experimental investigation and analytical research the existing meagre information in this uncharted domain of engineering.

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Summary

The paper gives a critical analysis of the fundamental principles of design governing the applications of aluminium alloys in structural engineering where up to a comparatively recent date the use of ferrous materials predominated. The inherent physical and mechanical properties of these light alloys enable them to be incorporated in major stress-carrying structures with great economy. In addition to their direct substitution for steel, methods are available and outlined in this paper by which an intelligent use can be made through the resources of accepted principles as adopted in the design of aircraft structures. The author believes that full exploitation of the potential advantages of light metal construction can only be made by an altogether original and rational approach formulated through basic concepts of strength of materials and principles of form strength, and not by imitating designs of steel structures as is done at present. In appraising the worth and limitations of aluminium structures, a digression on the general applications of the plastic theory and on the vexed question of the safety factor is also given to reveal the correct background of approach for design.

In presenting this balanced survey, it is hoped that the paper will form a springboard for extending through analytical investigations and experimental research the existing meagre information on the theory and design of aluminium structures.

Résumé

L'auteur expose une analyse critique des principes fondamentaux de l'emploi des alliages légers dans la construction, domaine dans lequel jusqu'à ces temps derniers les métaux ferreux étaient de beaucoup les plus employés. Les caractéristiques propres physiques et mécaniques des alliages légers permettent leur emploi dans des conditions économiques, dans la construction de nombreux ouvrages destinés à supporter des charges. Outre les possibilités de substitution directe à l'acier, on dispose actuellement de méthodes qui sont exposées dans le présent mémoire et qui permettent d'utiliser judicieusement les alliages légers suivant des principes tels que ceux qui sont adoptés dans la construction aéronautique.

L'auteur estime que l'utilisation intégrale des possibilités intéressantes des alliages légers, en matière de construction, n'est possible que sous une forme à la fois originale et rationnelle basée sur les notions essentielles de la résistance des matériaux et non pas par simple imitation des conceptions actuellement adoptées dans la construction en acier.

Après avoir mis en évidence les mérites des constructions en alliages légers et indiqué les limites qui leur sont imposées, l'auteur étudie les applications corrélatives générales de la théorie de la plasticité et la question controversée du coefficient de sécurité, afin de dégager les bases correctes de l'étude des ouvrages.

L'auteur espère que cet aperçu d'ensemble pourra fournir un tremplin aux recherches analytiques et expérimentales, en vue de compléter les informations restreintes dont nous disposons actuellement sur la théorie et le calcul des ouvrages en alliages légers.

Zusammenfassung

Der Verfasser gibt eine kritische Zusammenstellung der grundlegenden Entwurfsprinzipien für die Verwendung von Aluminium-Legierungen für Baukonstruktionen, wo bis in die jüngste Zeit der Gebrauch von Stählen vorherrschte. Die charakteristischen physikalischen und mechanischen Eigenschaften der Leichtmetall-Verbindungen erlauben deren äusserst wirtschaftliche Verwendung für zahlreiche Tragwerksarten. Neben ihrer direkten Anwendung als Ersatz von Stahl können sie nach den heute zur Verfügung stehenden und in diesem Aufsatz dargelegten Methoden zweckmässig entsprechend den für den Flugzeugbau entwickelten, bewährten Prinzipien verwendet werden. Der Verfasser ist der Auffassung, dass eine umfassende

Ausnützung der grossen Vorteile der Leichtmetallkonstruktionen nur auf eine zugleich originelle wie rationelle Weise möglich ist, die auf grundsätzlichen Ueberlegungen über Materialbeanspruchung und Formfestigkeit beruhen muss, und nicht durch einfache Nachahmung von Stahlkonstruktionen, wie es heute geschieht.

Nach einer Hervorhebung der Bedeutung und der Grenzen von Aluminiumkonstruktionen wird auch ein Hinweis auf die allgemeine Anwendung der Plastizitätstheorie und auf die umstrittene Frage des Sicherheitsfaktors gegeben, um damit das zweckmässige Vorgehen beim Entwerfen festzulegen. Der Verfasser hofft, dass die vorliegende allseitige Uebersicht den Anlass gebe zu einer Erweiterung der heute noch bescheidenen Kenntnisse über Theorie und Entwurf der Aluminiumkonstruktionen und zwar durch analytische Untersuchungen und experimentelle Forschung.

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BII 2

The structural analysis of the Dome of Discovery

Analyse structurale du " Dome of Discovery "

Die statische Berechnung des " Dome of Discovery "

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London London

General description of the method of analysis

The Dome of Discovery is a stiff triangulated space-frame in aluminium alloy shaped as a shallow spherical calotte with a heavy steel ring-girder supported on 48 tubular steel struts forming a system of 24 bipods articulated in the radial direction. The structure has 37 internal nodal points with 6 degrees of freedom each and 24 peripheral nodal points with 4 degrees of freedom each, and is therefore 318 times statically indeterminate.

The analysis was carried out by a combined relaxation and load transformation method specially devised for this purpose. The following is a very condensed outline of the method used and of some of the results obtained.*

The relaxation method is based on the principle of successive convergent approximations and consists of applying successive movements to imaginary constraints introduced at several points of a structure and placed so as to divide the structure into elementary structural units which can be readily analysed and may be statically determinate. The loads are imagined to act initially on the constraints which are considered "fixed." The movement of each constraint "relaxes" it, i.e. relieves it of part of the load, which is thereby transferred to the structure and simultaneously carries over forces and moments on to neighbouring constraints which are in turn relaxed.

The supports of the structure are treated as constraints; the sum totals of the actions carried over to them by neighbouring constraints give the actions exercised by the structure on its supports, equal and of opposite sign to the reactions of the supports.

The relaxation is complete when the residual actions on all the imaginary constraints are below predetermined negligible limits.

* A detailed description of the method of analysis and the full results are given in *The Structural Analysis of the Dome of Discovery*, by T. O. Lazarides, Crosby Lockwood & Son, London, 1952.

The combined method of analysis was based on the principle of decomposing unsymmetrical loads into a symmetrical and an antimetrical component and establishing for each component simple and exact group relaxation functions. By means of these functions the movements imparted to each nodal point or "spider" in turn in the course of the relaxation were automatically positively and negatively duplicated on spiders similarly situated with respect to planes of symmetry and of antimetry of the decomposed loading. In other words, this made it possible to restrict the analysis of the whole structure to that of an elementary wedge of symmetry and elementary wedge of antimetry; the boundaries of these wedges could be treated as structural boundaries with appropriate boundary conditions expressed in terms of relaxation coefficients. The wedges of symmetry and antimetry are shown on fig. 1.

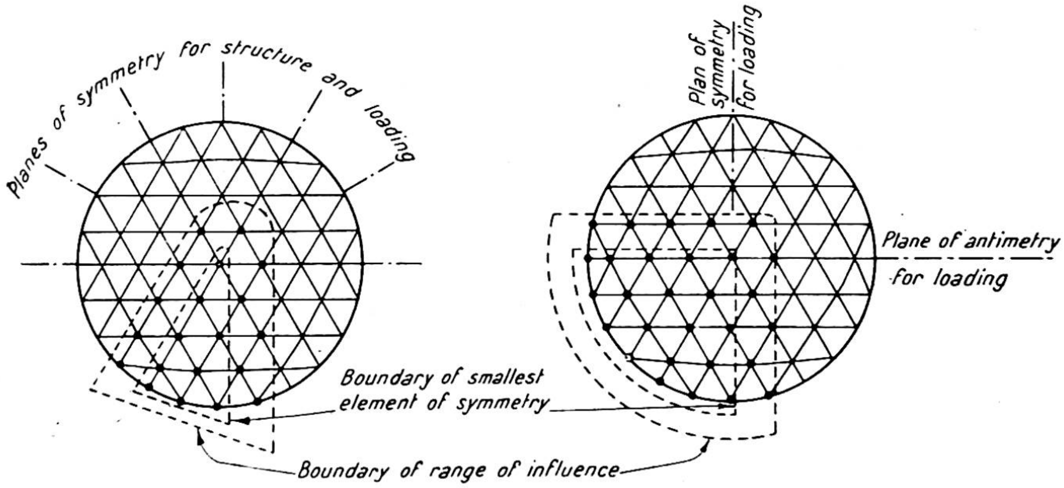


Fig. 1

The compounding of group relaxation operations was carried out as follows:

(1) Outline the element of symmetry to be analysed.
(2) Establish the range of influence for this element (i.e. the group of constraints adjacent to the internal boundaries).
(3) Establish influence coefficients for elementary movements on all constraints inside the range of influence. These are called the "elementary operations and influence coefficients for the general case" in order to distinguish them from the compounded operations for the symmetrical and antimetrical cases.
(4) Establish for all constraints situated on and within the boundaries of the element of symmetry the influence coefficients and relaxation operations compounded for the symmetrical case, as follows: perform in turn all possible operations on each of the inner constraints adjacent to the boundaries of the element and simultaneously the positive mirror image of these operations on each in turn of the outer constraints adjacent to the boundaries and symmetrical to the first group; sum the resulting effects on all the inner constraints affected by this duplication. For the unaffected constraints the coefficients are simply transcribed from the "general case."
(5) Do the same for the antimetrical case using negative mirror images, i.e. mirror images with all signs reversed.

This method of compounding is exact and the compounded operations differ from the elementary only with respect to the numerical values of the influence coefficients.

Three loading cases were analysed: own weight and uniformly distributed snow loads over half and over the whole Dome. All loads were assumed to be concentrated on the spiders.

The steel ring-girder was analysed both independently and together with the aluminium grid. The independent analysis gave valuable information on the elastic behaviour, under unsymmetrical loading, of a type of frame now coming into increasing use—a polygonal or round closed stiff frame supported on bipods allowing free radial but no tangential movement.

In the analysis of the Dome as a whole a choice had to be made between using one system of reference axes throughout or using several concordant systems of axes at different stages of the work. The following systems were finally adopted: (*a*) orthogonal parallel systems x, y, z at the two ends of a curved rib-element for determining the influence coefficients for each element, (*b*) orthogonal tangential systems of axes T, R, Q at each end of each curved rib-element, into which the previously obtained influence coefficients were converted in order to obtain interchangeable operation factors, (*c*) independent orthogonal reference systems R, S, T centred on each spider, into which the T, R, Q coefficients were converted in order to obtain operational factors for the relaxation of each spider. All systems of reference axes followed the same stereometric left-hand three-finger rule. The spider axes were determined as follows: axis R positive towards the centre of the Dome sphere, axis S tangential to the great circle passing through each spider and the summit of the Dome, positive towards the summit, axis T by the left-hand three-finger rule—tangential to the Dome sphere at right angles to axis S, positive from left to right viewed from above. A special convention was necessary for the summit spider. This choice of axes necessitated several conversion operations but decisively simplified the computations.

The sequence of operations used in the analysis can be summarised as follows:

(*a*) Draw a schematic plan view of the Dome showing ribs and ring-girder only (fig. 1).

(*b*) Choose plane of antimetry, i.e. decide, for the case of half snow load, which half should be loaded to give the most unfavourable results. (Note: the Dome can be divided into identical 60° or mutually symmetrical 30° wedges but not into identical quarters; turning the loading by 30° or by 90° therefore alters the loading conditions and considerably alters the distribution of tangential bipod thrusts. This alteration cannot be predicted exactly and it is therefore not certain that the case which was analysed was in fact the most unfavourable.)

(*c*) Outline on the plan view of the Dome the elementary wedges of symmetry and of antimetry (a 30° and a 90° wedge respectively) which will be used in the analysis, together with their ranges of influence.

(*d*) Mark all spiders with letter, number and upper and lower dash indices.

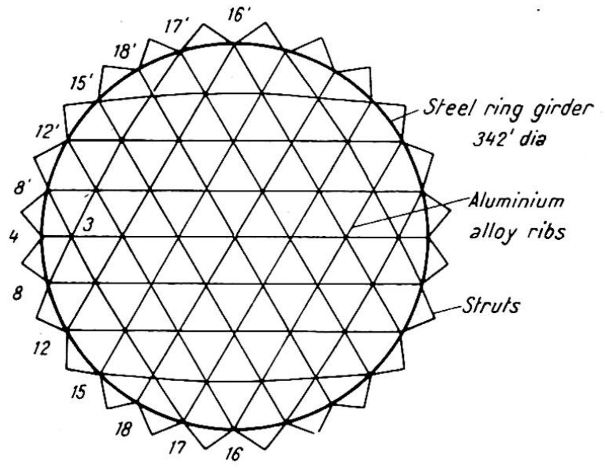
(*e*) Calculate exactly from basic structural data the position of each spider on the Dome-sphere, showing for each: the angle subtended by each of the elements converging on the spider; the position angles at both ends of each element.

(*f*) Calculate the influence coefficients for pure actions for all geometrically distinct rib-elements.

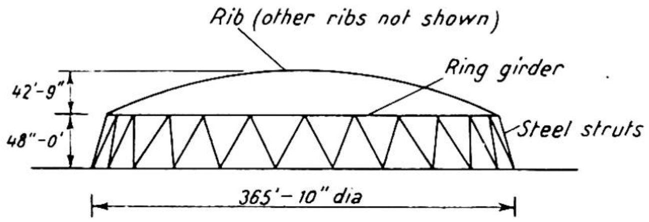
(*g*) Establish and solve in tabulated form sets of simultaneous equations giving the influence coefficients for pure movements for all geometrically distinct rib-elements.

(*h*) Enter these coefficients into conversion tables prepared for each geometrically distinct rib-element (letter indices only) and convert from x, y, z into T, R, Q reference system.

(*i*) Enter these T, R, Q coefficients into conversion tables prepared for each element in the influence range of the wedge of antimetry (number indices) and convert movements and actions from T, R, Q reference system into R, S, T reference system using position angle values from section (*e*).



a) Schematic plan view of dome



b) Schematic elevation of dome

Fig. 2

(*j*) Enter the R, S, T coefficients into the operational tables prepared for each spider in the range of influence of the wedge of antimetry. The fixed-end values corresponding to the spiders surrounding the reference spider are. transcribed directly, the mobile-end values corresponding to the reference spider itself are added together.

(*k*) Compound these coefficients for the symmetrical case and enter into compounded operational tables prepared for each spider situated in the wedge of symmetry, using positive mirror images.

(*l*) Do the same for the antimetrical case using positive and negative mirror images as required.

(*m*) Prepare main relaxation tables and key plan (blanks); these are the same for both cases.

(*n*) Enter initial loading data for symmetrical case and relax to within predetermined negligible residues using appropriate compounded operational factors. Sum final total absolute movements for all spiders.

(*o*) Prepare re-compounding tables for individual rib-elements, enter total absolute movements of spiders, calculate forces and moments at each end of each rib-element.

(*p*) Prepare re-compounding tables for individual spiders, enter forces and moments in all rib-elements converging on each spider, check equilibrium of all spiders.

(*q*) Repeat from (*n*) to (*p*) for antimetrical case. Up to and including these operations all the forces and moments considered are those exercised *by* the the structure *on* the constraints. This convention has the advantage of being "natural" for analysis by relaxation because the movements applied then have the same sign as the residual actions which they are intended to liquidate.

(*r*) Compound the symmetrical and antimetrical cases to obtain the original unsymmetrical loading case (snow over half the Dome) by adding the movements for all spiders and the actions for both ends of each rib-element.

(*s*) Tabulate the final results for the five loading cases which can be obtained by simple combinations of the results of the analysis: own weight, snow over the whole Dome, snow over half the Dome, own weight plus snow over half the Dome, own weight plus snow over the whole Dome. In operations (*r*) and (*s*) the forces and moments are those exercised *by* the constraints *on* the structure, this convention being the more usual one in engineering practice.

Finally it should be mentioned that a complete and consistent system of checks was devised making it possible to discover numerical discrepancies, locate the errors by which they were caused, and follow through and eliminate all the consequences of these errors. Extensive use was made of elementary sketches, graphs and diagrams, particularly for the purpose of illustrating the sign conventions used at each succesive step.

Concerning the actual relaxation itself it was found that the successive operations tended to group themselves naturally into cycles, and the cycles occurred in natural sequences of two. Each successive attempt at reducing residual unbalanced forces constituting the first cycle of a pair, resulted in unbalanced residual bending and twisting moments, which were then in turn reduced in the operations of the second cycle. It was found advantageous to anticipate the effects of subsequent cycles and also of subsequent operations in each cycle by deliberate over- or under-relaxing.

A further essential feature was provided by the key plan, which made it possible to repeat previously carried out operations by direct transcription instead of referring back to the original operation sheets; when mutliplication or division was obviously required the only factors used were 1, 2, 3, 5 and powers of 10. This proved to be a decisive simplification.

RESULTS OF THE ANALYSIS

The most striking aspect of the elastic behaviour of the Dome of Discovery under load is the very considerable difference in the distribution of the movements of spiders, of the internal forces and moments in the ribs and ring-girder, and of the bipod reactions for a symmetrical and for an unsymmetrical loading. Under a symmetrical loading the Dome only flattens out slightly, the distribution of spider movements and of the inner forces and moments is largely uniform, the bipod reactions are nearly identical and their tangential components are negligible. Under an unsymmetrical loading there is a marked tendency to sidesway despite the low rise-to-span ratio. This is due to the lack of rigidity of the ring-girder with respect to deformations which do not entail an overall lengthening and shortening of the perimeter and to the peculiar statical properties of the system of supports consisting of articulated bipods. The overall picture is that the grid tends to "slip away" from under the load; the rigidity of the ring-girder with respect to this type of deformation being practically nil, this tendency is checked by a transfer of tangential forces to some points only of the periphery, namely to the points where these tangential forces can be resisted by tangential bipod reactions. The result is that the supporting struts are loaded very unequally and that there are also some heavy thrusts in the ribs of the grid itself.

The final results cannot be given here in full, but a representative picture can be obtained by considering the behaviour of the ring-girder. Under full load conditions along the ring-girder are almost uniform, although there are small periodic variations giving the deformed ring-girder a slightly wavy appearance. The warping components are practically nil. These periodic variations have only a slight effect on the stresses in the ring-girder itself, but they appreciably affect the distribution of thrusts in the grid. The overall picture is that the grid pushes the bipods outwards and this movement is resisted by the ring-girder working in tension and in bending. While the stresses caused by direct tension and by bending are nearly equal the ring-girder is far more effective in tension than in bending, and a system of straight ties hinged at the rim spiders would therefore have been much more effective.

Under a half load conditions are radically different. The outward movements of the rim spiders and the tension in the ring-girder progressively increase in magnitude from the middle of the unloaded half to the middle of the loaded half. Rotations about the tangential axis are positive in the unloaded half (increasing the grid curvature) and negative in the loaded half (flattening the grid), but the greatest negative values are near the quarterpoints rather than in the middle as might have been expected. The most striking point, however, is the pattern of variation of the tangential bipod reactions; this pattern certainly appears strange at first glance, especially the solitary large positive thrust at the loading demarcation line flanked by negative values, and merits a more thorough examination.

Tangential thrusts in bipods can be caused in two entirely different ways: (*a*) by unequal thrusts in two grid ribs converging on a rim spider—the radial component of the resultant is taken entirely by the ring-girder, the tangential component is taken entirely by the bipod; (*b*) by unequal relative radial movements of rim spiders adjacent on either side to the spider supported by the bipod—these cause unequal thrusts in the spider, the resultant of which is taken entirely by the bipod. The distribution of these effects along the periphery follows quite independent laws.

Both these effects are present along the ring-girder when only half the Dome is loaded and it is their superposition which causes the pattern of variation of the tangential bipod reactions obtained in the analysis, and shown in the following Table.

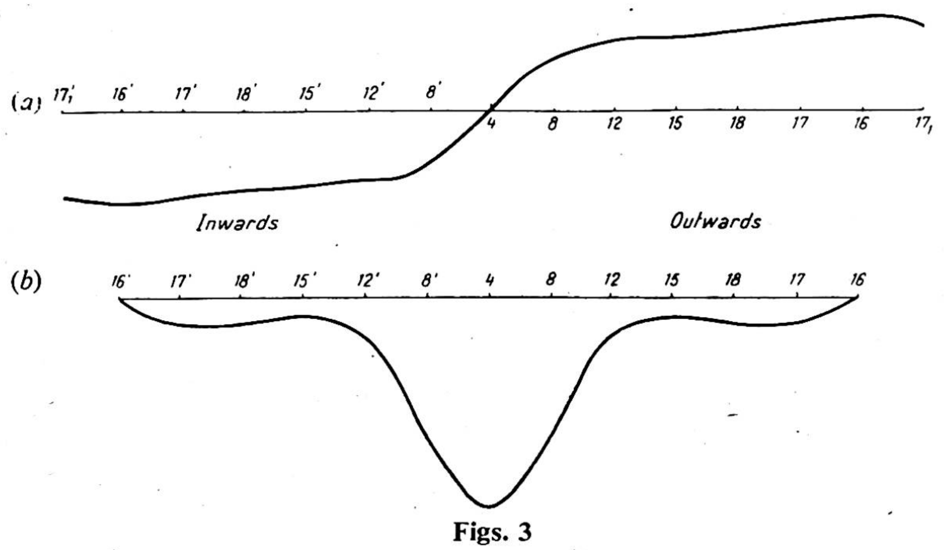
These effects were separated with the help of the influence coefficients used in the analysis to obtain the effects due to ring-girder action only; subtracting these from the total thrust we obtain the effects due to the unequal thrusts in the grid ribs.

It will immediately be appreciated that, taken separately, the two component effects do indeed follow a pattern the broad outlines of which could be deduced from the shape and general characteristics of the structure. It will also be appreciated that it would have been wellnigh impossible to predict from such general considera-

Tangential bipod reactions for load over half the Dome

	Spider No.	Total thrust	Thrust due to R.G. action only	Thrust due to grid action only
Middle of unloaded half . . .	16′	0	0	0
	17′	+ 7·831	+ 6·366	+ 1·465
	18′	+ 5·799	+ 6·116	− 317
	15′	−16·267	+ 4·454	−20·721
	12′	−29·952	+10·348	−40·300
	8′	−14·383	+34·885	−49·268
Demarcation line	4	+51·207	+52·618	− 1·411
	8	−14·807	+35·195	−50·002
	12	−29·952	+10·348	−40·300
	15	−15·843	+ 4·144	−19·987
	18	+ 5·799	+ 6·116	− 317
	17	+ 7·407	+ 6·676	+ 731
Middle of loaded half . . .	16	0	0	0

tions any details of the pattern of variation of either effect, let alone the exact relative proportions in which these two mutually antagonistic component effects are superimposed. On the other hand, the conditions along the ring-girder directly determine the reactions in the supporting struts and influence to a very large extent the distribution of forces and moments in the grid itself. It follows that a structure of this type can only be analysed exactly or not at all, as an analysis by approximate methods based on simplified and therefore necessarily incomplete structural analogies may easily lead to very large and unknown errors in unexpected parts of the structure, with the result that some of the structural parts and connections may inadvertently be grossly over or under-dimensioned.



Figs. 3

The absolute and differential radial movements for the rim spiders for the antimetrical loading case are shown schematically to an exaggerated scale on figs. 3(*a*) and 3(*b*). The deformation shown on fig. 3(*a*) is of a type one would naturally expect for an antimetrical load; thus it is obvious by inspection that h should be minimum at 16′, nil at 4 and maximum at 16 and that Δh should be nil at 16′ and 16 and maximum at 4, which is in fact a point of contraflexure. The variation of Δh

between these extremes is not quite what one would expect at first sight; in particular the transition between 8 and 12 is very abrupt and the secondary maxima at 17′ and 17 are unusual. These peculiarities are characteristic of the mode of support on bipods and of the manner in which the thrusts are distributed in the ribs of the grid. The general trend of the variation of the tangential bipod thrusts due to the ring-girder action only is now sufficiently clear.

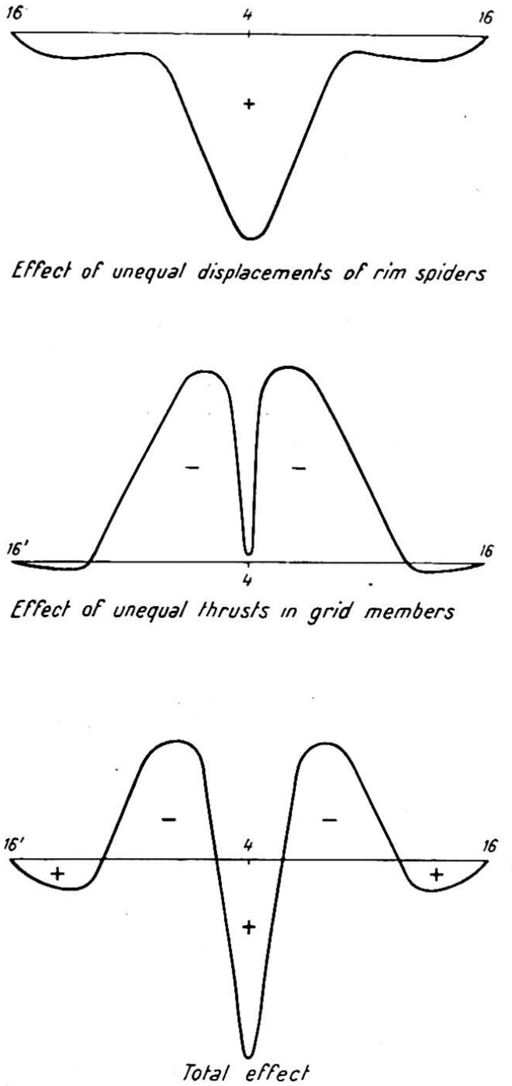


Fig. 4

The second component effect—the tangential bipod thrusts due to grid action only—also shows an easily explained pattern. The unsymmetrical loading will obviously cause a tendency to sidesway across the loading demarcation line and away from the load, which can only be halted by tangential bipod reactions exercised by those bipods which can do this most effectively, i.e. those which lie at a convenient inclination and are connected to triangulated elements of the grid. These are the rim spiders in the immediate vicinity of the demarcation line but *not* including those on the demarcation line itself owing to the almost complete lack of transversal rigidity of the grid elements connected to these spiders. The resulting interplay of tangential thrusts in the grid can be briefly summarised as follows. A severe cross-thrust in the loaded half of the Dome parallel to the loading demarcation line, two compensating pulls travelling across the Dome from the lower quarterpoint rim spiders, a cross-pull in the unloaded half of the Dome similar and parallel to the main cross-thrust but of smaller magnitude, and two compensating thrusts travelling across the Dome from the upper quarterpoint rim spiders.

The pattern of variation of the total tangential bipod reactions along the ring-girder (fig. 4) is thus adequately explained by the superposition of the two partial effects described above; in particular the presence of a large positive value at 4 flanked on either side by three consecutive large negative values without any transsition is explained by the superposition of the (positive) maximum ring-girder effect at the point of contraflexure 4 with a lacuna in the grid effect due to the lack of lateral stiffness of the rib 3–4 and the heavy concentration of the (negative) grid effect in the regions 15′–12′–18′ and 8–12–15, which are the only parts of the ring-girder where appreciable tangential components of the sidesway pull can be transmitted by a stiff triangulated grid—in other words, the only possible anchors against sidesway.

It is obvious that the extremely sharp variation of the tangential ring-girder reactions about the loading demarcation spiders shown on fig. 4 (from a large negative value to a very large positive value and back to a large negative value) cannot fail to produce heavy thrusts in the struts converging on these spiders. These struts are far more heavily loaded than all the others and owing to the peculiarities of the structure they cannot be relieved by shedding part of the load on to their neighbours. This condition could be radically altered either by triangulating all the terminal grid connections without any exceptions, or by converting some of the supporting bipods into tripods. In designing space-frames of great complexity particular care should be taken to avoid configurations in which the systematic or occasional overloading of some elements of the structure cannot be relieved by some form of load shedding.

Summary

The Dome of Discovery at the Festival of Britain, 1951, is a stiff triangulated space-frame in light alloy supported on radially articulated bipods and is 318 times statically indeterminate for arbitrary loading. A complete analysis for symmetrical and unsymmetrical snow load has been carried out by a combined relaxation and load transformation method specially developed for this purpose. This paper gives a brief summary of the method and of some of the results obtained.

Résumé

Le Dôme de la Découverte au Festival de Grande-Bretagne est une ossature rigide triangulée à trois dimensions en alliage léger prenant appuis sur un système de bipodes articulés dans le sens radial. La construction est 318 fois hyperstatique. Un calcul rigoureux pour surcharges symétriques et asymétriques dues à la neige a été fait à l'aide d'une méthode de relaxation et de transformation combinées spécialement établie. L'auteur donne un bref aperçu de la méthode et de certains des résultats obtenus.

Zusammenfassung

Die Tragkonstruktion des "Dome of Discovery" der Londoner Messe 1951 besteht aus einem steifen, 318fach statisch unbestimmten Raumtragwerk aus Leichtmetall, das getragen wird von stählernen in radialer Richtung gelenkig gelagerten Doppelstützen. Die genaue Berechnung für symmetrische und unsymmetrische Schneelast erfolgte mit Hilfe eines speziell für dieses Problem entwickelten Relaxationsverfahrens unter Benützung der Belastungsumordnung. Die Methode wird kurz beschrieben und einige Resultate werden angegeben.

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