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The limit of stress in the compression flanges of beams

Contraintes limites dans les membrures comprimées des poutres

Die Grenzspannung in den Druckgurten von Trägern

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Specifications for the design of structural metal beams usually limit the stress in the compression flange by consideration of its unsupported length, its width, and in some instances by its thickness and the depth of the beam. Most specifications do not consider the type of loading which produces the flange stress nor the end conditions which may affect the limit of that stress. A specification which provides one working formula for all conditions of loading, for all conditions of end restraint, and for flanges that may vary in section along their length, cannot provide constant factors of safety for all of the possible conditions.

The work of S. Timoshenko, as summarised in the *Theory of Elastic Stability*,† has been notable in the analysis of the elastic problem that is involved in the flanged beam subjected to bending. Karl De Vries' paper, "Strength of Beams as Determined by Lateral Buckling," with the several discussions,‡ has summarised the present status of the problem. Further consideration of the flange buckling problem seems justified with the objective of simplification and more general application to the varying conditions that may exist.

The following items are among the considerations that may affect solution of the problem:

(1) unsupported length of the compression flange,
(2) horizontal moment of inertia of the compression flange,
(3) torsional resistance of the beam,
(4) restraint to end rotation of the compression flange,
(5) thickness and width of the compression flange,
(6) variations in section of the flange,
(7) resistance of the tension flange, and
(8) point of application of load to the beam—whether at the top flange, bottom flange, or intermediate between the flanges.

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† S. Timoshenko, *Theory of Elastic Stability*, McGraw-Hill Book Co., 1936.
‡ *Trans. Amer. Soc. Civ. Engrs.*, **112**, 1245.

Some comparison has been made between the compression flange of a beam and a column, considering that the flange tends to buckle transverse to the web of the beam. The flange is considered to receive its load by shear transfer from the web. The manner in which this shear transfer is accomplished is a function of the manner in which the beam is loaded. For example, if a beam is subjected to pure bending the flanges receive full load at their ends; when the load is concentrated at the centre of the beam span the shear transfer is uniform per unit of length; and when the applied load is uniform the shear transfer is uniformly decreasing from the ends to the centre of the span. Thus the compression flanges may receive their load under conditions that vary from end loading to loading uniformly distributed along the length of the members.

The effect of the distribution of beam loading on the limit of stress may be demonstrated by comparison of similar loading conditions on a slender column. The classical Euler loading on a column of uniform section and having its ends free to rotate is expressed as $P=\pi^2EI/L^2=9{\cdot}87EI/L^2$. It may be shown that the same column having uniform increments of load per unit of length has a limiting load of $P=31{\cdot}6EI/L^2$, and when loaded with uniformly decreasing increments from the end to the centre, $P=20{\cdot}8EI/L^2$. Thus it would appear that the manner of loading is a major consideration affecting the limiting load by as much as 3·17 times.

Again, the effect of end restraint to rotation of the compression flange may be demonstrated by consideration of the free end and the fixed end Euler limits, which are in the ratio of 1 to 4. Degree of end restraint would affect values falling between these two.

Variation of the cross-section of a column along its length becomes an important consideration in establishing its limiting load. It is very difficult to assign an average value to the moment of inertia of a column which will fully account for the manner of variation. For example, a column may have a heavy mid-section or it may have heavy end-sections. In these cases the average moment of inertia may be the same but the limit of load would be different.

The torsional resistance of a beam to buckling of the compression flange might also be compared to a slender column having a spring placed to resist lateral deflection. Let fig. 1 illustrate a column with a spring which has zero load when the column is straight. When the column is bent toward the spring the restraining force is dependent upon the amount of deflection. Similarly, the simply supported beam illustrated in plan view in fig. 1(*b*) will have each cross-section throughout its length rotated through some angle β. The amount of the deflection a will determine the magnitude of the angles β along the length of the beam and consequently the amount of the torsional resistance. It would appear that the column of fig. 1(*a*) and the compression flange of the beam of fig. 1(*b*) would each have increasing loads required to maintain deflections of increasing magnitudes. However, in each case the restraining lateral force is zero when the member is straight and the critical load for the straight condition is the same whether or not the restraint is pending. In order to evaluate the effect of the torsional restraint of the beam for various amounts of lateral deflection of the compression flange it is necessary to assign values to the maximum angle of rotation of the beam and to define the law of variation of that angle along the length of the beam. The amount of torsional resistance must be small indeed when the flanges of the beam are straight or nearly straight. A condition of neutral equilibrium must exist while the beam flanges are straight. Higher values of load in the compression flange are likely, possibly because of torsional restraint that develops

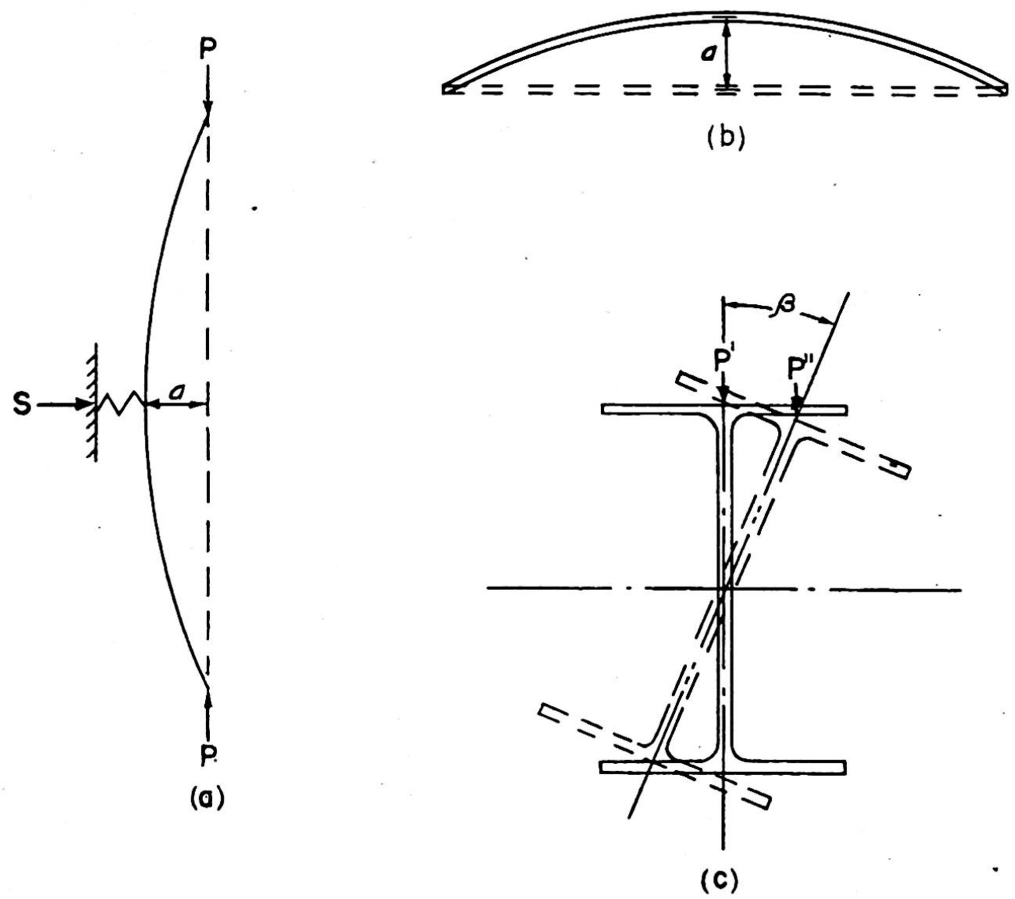

Fig. 1

with increasing angles of torsional rotation. The least value of load that will produce neutral equilibrium would seem to be that which occurs when the flanges are straight.

It has been assumed that the vertical load applied to the top flange of a beam tends to increase the torsional angle, resulting in a lowered limit of load. On this basis, a load applied to the bottom flange increases the limit of load. It follows that, if the flanges are straight, the vertical load would be in the plane of the web and consideration of top or bottom location would be eliminated.

If the designer is concerned with the load that will produce neutral equilibrium while the compression flange is straight, then a much simplified method may be used. In this case full consideration may be given to the effects of end restraint, variations in type of loading, and variations in the section of the compression flange.

It is not the intention of this paper to discuss buckling phenomena in the plastic range, that is, when the computed stress in the flange is greater than the proportional limit of the material. Also, it is assumed that the thickness of the compression flange is sufficient so that local crippling of the flange does not precede lateral buckling. For the purpose of this discussion it is considered that there are two limiting values of stress, either of which may control. One of these limits is the stress which compares with the yield point of the material and the other is the stress in the extreme fibres of the beam when a state of neutral equilibrium exists in a straight compression flange. It is acknowledged that higher stress values may be obtained before collapse of the beam, but it is believed that a factor of safety should be maintained with respect to the lower of these two defined critical stress values.

In order that the critical stress may be found for any given compression flange, it is assumed that the load will maintain a small lateral deflection of the flange. The amount of this deflection is immaterial so long as it does not produce an appreciable torsional resistance from the beam. The amount of the flange load is then such that any decrease would permit the flange to straighten and any increase would cause greater lateral deflection. The amount of the deflection that is assumed to be maintained is further assumed to be small enough so that it is immaterial whether the load is applied to the top flange or to the bottom flange of the beam. These assumptions are consistent with determination of the critical load for the straight flange.

The assumption of a small lateral deflection of the compression flange is a tool to be employed in evaluating the critical load in the compression flange. It is required that the load maintain the deflection in amount and the deflection curve in shape.

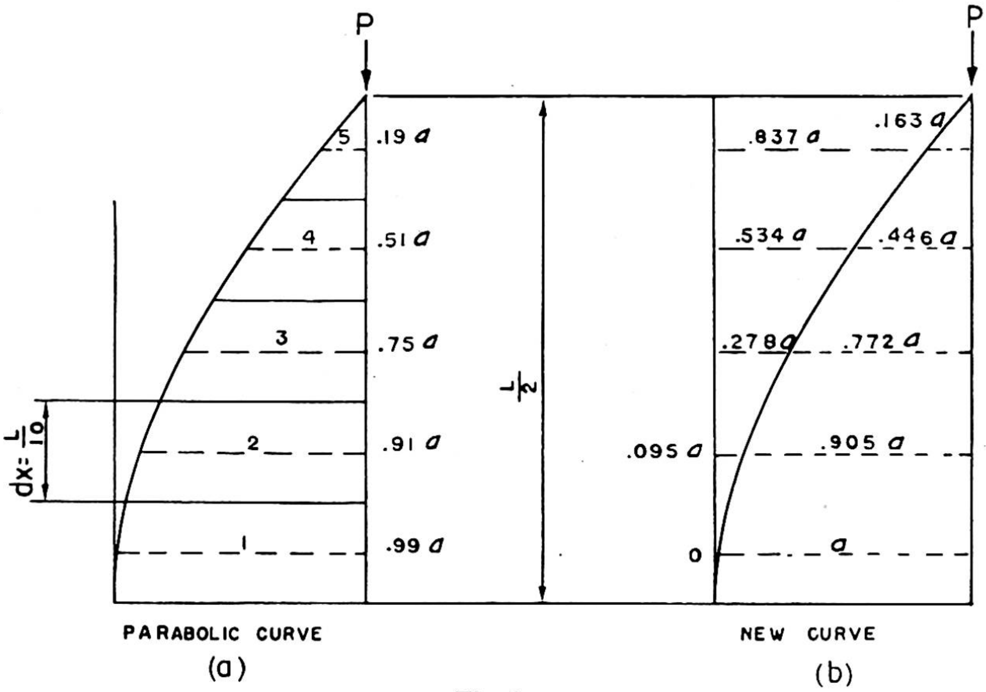

Fig. 2

Since the shape of the deflection curve is usually not known in advance, a process of iteration may be used to approach evaluation of the true curve. Fortunately, the series is rapidly converging so that the work is minimised. Again, the analogy of a column loaded at its ends may be used as an example. Assume that the deflection is a and that the shape of the curve is parabolic (while it is known that the curve is sinusoidal). Fig. 2(a) shows the ordinates to the parabolic curve for the centres of five equal divisions of the half length. The load P produces bending moments along the length of the column. The deflection at the centre may be computed from these bending moments and is expressed as $y=0{\cdot}1037PaL^2/EI$. Since $y=a$, then $P=9{\cdot}64EI/L^2$. If integrated continuously, the value of P would be found to be $9{\cdot}60EI/L^2$. These values are about 3 % less than the accepted value of $P=9{\cdot}87EI/L^2$, because of the assumption that the curve is parabolic. This approximation will normally be sufficiently accurate in view of the fact that the value of E will vary by more than 3 % from any assumed value. However, if deflections were computed at the centres of the five divisions, a new closer curve shape might be developed as shown in fig. 2(*b*). When the new curve is used in the same manner as the first approximation it is found that $y=0{\cdot}1025PaL^2/EI$, from which $P=9{\cdot}77EI/L^2$. This value is now about 1% below the accepted value. Continuation of the same process will yield results with an even greater degree of accuracy. If the sinusoidal ordinates of fig. 3 were used, the resultant value of $y=0{\cdot}1009PaL^2/EI$ produces $P=9{\cdot}91EI/L^2$. The only reason this value differs from the value of $9{\cdot}87EI/L^2$ is that the integration was performed in five finite parts rather than continuously.

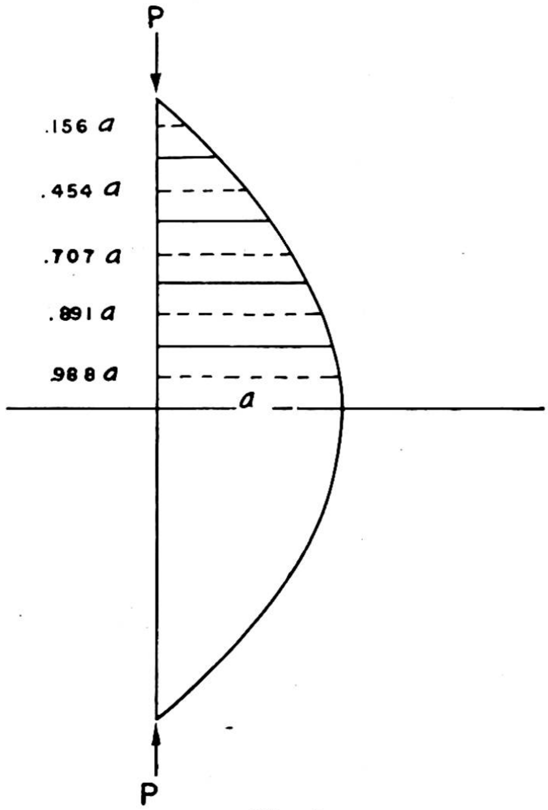

Fig. 3

In the case of pure bending in a flanged beam the flange stress is applied entirely at the ends of the beam. If the ends of the compression flange are free to rotate and the flange is of constant section, then the critical flange load is $F=9{\cdot}87EI/L^2$; the average stress in the flange is $F/A=9{\cdot}87EI/AL^2$, when A is the area of the flange and I is the moment of inertia of the compression flange about the axis along the web (for constant section rolled beams I is one-half of the $Iy-y$ value given in steel handbooks); the extreme fibre stress is $f=9{\cdot}87EIc/AL^2y$, when c is the distance from the neutral axis of the beam to the extreme fibres and y is the distance to the centre of the flange. Since $f=M/S$, in which S is the section modulus of the beam about its major axis, the critical value of $M=9{\cdot}87EIcS/AL^2y$.

Fig. 4(*a*) represents a flanged beam of uniform section simply supported and loaded with a concentrated load P placed at the centre. It is desired to find the load P which will induce a critical flange load F. If the half-span is divided into five equal divisions, the increment of load F that is applied to each division is $0{\cdot}2F$. Assuming that the compression flange deflects laterally in a parabolic shape with a maximum deflection of a, fig. 4(*c*) represents the column loading. Bending moments at the centre of the divisions are computed as follows:

	0	Point 5
$0{\cdot}2F\times 0{\cdot}32a=$	$0{\cdot}064Fa$	Point 4
$0{\cdot}2F$		
$0{\cdot}4F\times 0{\cdot}24a=$	$0{\cdot}096Fa$	
$0{\cdot}2F$	$0{\cdot}160Fa$	Point 3
$0{\cdot}6F\times 0{\cdot}16a=$	$0{\cdot}096Fa$	
$0{\cdot}2F$	$0{\cdot}256Fa$	Point 2
$0{\cdot}8F\times 0{\cdot}08a=$	$0{\cdot}064Fa$	
	$0{\cdot}320Fa$	Point 1

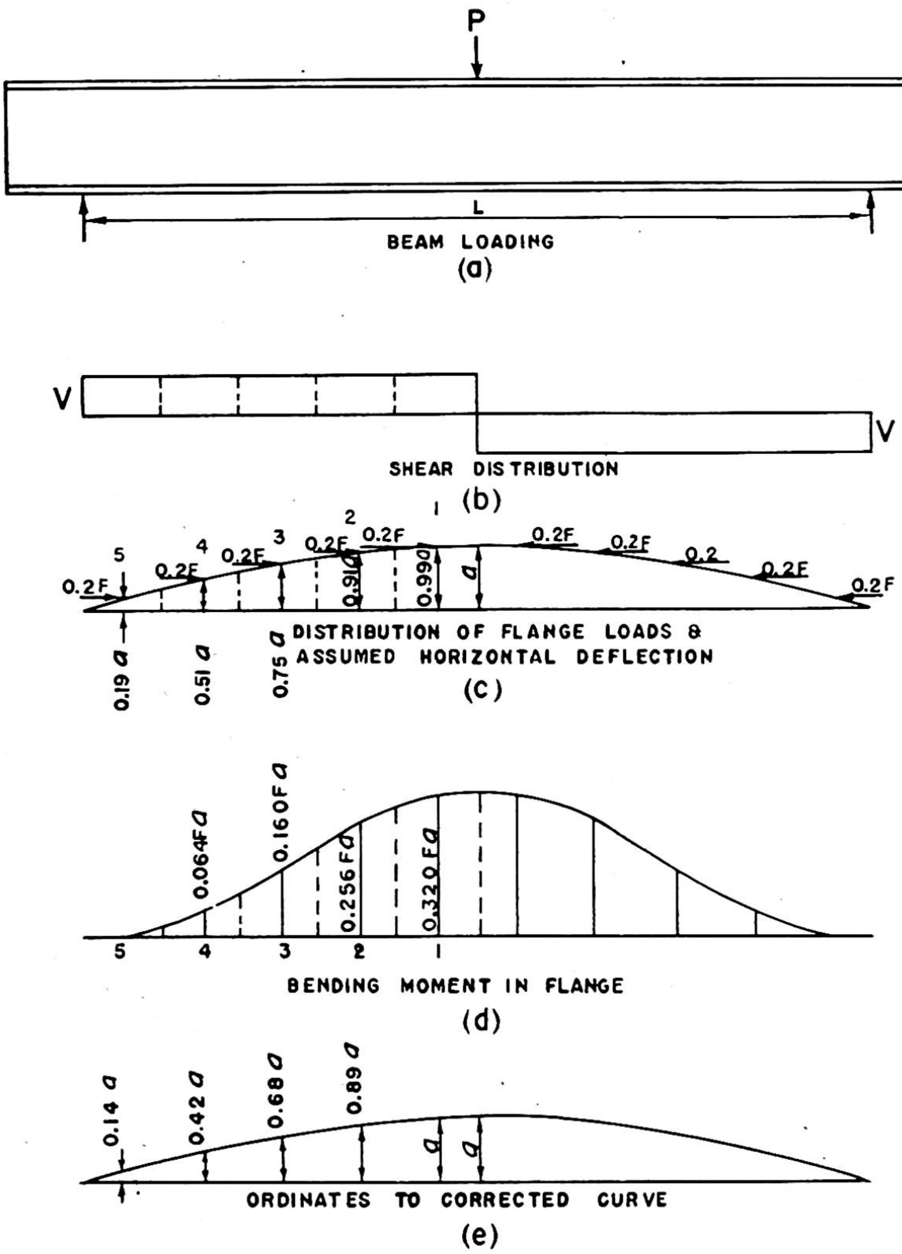

Fig. 4

These bending moments are plotted in fig. 4(*d*). The horizontal deflection of the centre of each division from the tangent to the elastic curve at the centre of the span may be computed by the use of the Moment-Area principles in the following manner:

$$
\begin{array}{llll}
0{\cdot}0320FaL/EI \times 0{\cdot}10L & = 0{\cdot}0032FaL^2/EI & & \text{Point 2} \\
0{\cdot}0256 & & & \\
\overline{0{\cdot}0576} & \times 0{\cdot}10 & = 0{\cdot}00576 & \\
0{\cdot}0160 & & \overline{0{\cdot}00896} & \text{Point 3} \\
\overline{0{\cdot}0736} & \times 0{\cdot}10 & = 0{\cdot}00736 & \\
0{\cdot}0064 & & \overline{0{\cdot}01632} & \text{Point 4} \\
\overline{0{\cdot}0800} & \times 0{\cdot}10 & = 0{\cdot}00800 & \\
 & & \overline{0{\cdot}02432} & \text{Point 5} \\
0{\cdot}0800 & \times 0{\cdot}05 & = 0{\cdot}00400 & \\
 & & \overline{0{\cdot}02832}FaL^2/EI & \text{End}
\end{array}
$$

Since the deflection of the end from the tangent to the elastic curve at the centre is $0{\cdot}02832FaL^2/EI$, the deflection of the centre will be $y=0{\cdot}02832FaL^2/EI$. The requirement is that $y=a$. Hence, $0{\cdot}02832FL^2/EI=1$, and $F=35{\cdot}3EI/L^2$, when I is the moment of inertia of the compression flange about its vertical axis. This value of the limiting load is approximate because it is based on an assumed shape of deflection curve. A closer value will result from a curve that is nearer the true shape of the deflection curve. Such a curve may be developed from the computed deflection at each point, when each such deflection is divided by $0{\cdot}02832FL^2/EI$ and the quotient is subtracted from $1{\cdot}0a$ as in the following computation:

$$1{\cdot}00a-\frac{0{\cdot}02832a}{0{\cdot}02832}=0 \qquad \text{End}$$

$$1{\cdot}00a-\frac{0{\cdot}02432a}{0{\cdot}02832}=0{\cdot}14a \qquad \text{Point 5}$$

$$1{\cdot}00a-\frac{0{\cdot}01632a}{0{\cdot}02832}=0{\cdot}42a \qquad \text{Point 4}$$

$$1{\cdot}00a-\frac{0{\cdot}00896a}{0{\cdot}02832}=0{\cdot}68a \qquad \text{Point 3}$$

$$1{\cdot}00a-\frac{0{\cdot}0032a}{0{\cdot}02832}=0{\cdot}89a \qquad \text{Point 2}$$

The new curve is plotted in fig. 4(*e*). A closer value for the limit of the force F may be found by repeating the calculations for bending moment and deflection, using this last curve:

$$
\begin{array}{lll}
0{\cdot}2F \times 0{\cdot}28a & = 0{\cdot}056Fa & \text{Point 4} \\
0{\cdot}2F & & \\
\overline{0{\cdot}4F} \times 0{\cdot}26a & = 0{\cdot}104Fa & \\
0{\cdot}2F & \overline{0{\cdot}160Fa} & \text{Point 3} \\
\overline{0{\cdot}6F} \times 0{\cdot}21a & = 0{\cdot}126Fa & \\
0{\cdot}2F & \overline{0{\cdot}286Fa} & \text{Point 2} \\
\overline{0{\cdot}8F} \times 0{\cdot}11a & = 0{\cdot}088Fa & \\
 & \overline{0{\cdot}374Fa} & \text{Point 1}
\end{array}
$$

$$
\begin{array}{lll}
0{\cdot}0374FaL/EI \times 0{\cdot}10L = 0{\cdot}00374FaL^2/EI & & \text{Point 2} \\
0{\cdot}0286 & & \\
\hline
0{\cdot}0660 & \times 0{\cdot}10L = 0{\cdot}00660 & \\
0{\cdot}0160 & 0{\cdot}01034 & \text{Point 3} \\
\hline
0{\cdot}0820 & \times 0{\cdot}10L = 0{\cdot}00820 & \\
0{\cdot}0056 & 0{\cdot}01854 & \text{Point 4} \\
\hline
0{\cdot}0876 & \times 0{\cdot}10L = 0{\cdot}00876 & \\
 & 0{\cdot}02730 & \text{Point 5} \\
0{\cdot}0876 & \times 0{\cdot}05L = 0{\cdot}00438 & \\
 & 0{\cdot}03168FaL^2/EI & \text{End}
\end{array}
$$

The new closer value for the limit of F is then found from the equation $y=a$. Thus, $y=0{\cdot}03168FaL^2/EI$, or $0{\cdot}03168FL^2/EI=1$, and $F=31{\cdot}6EI/L^2$.

The process might be continued, and it is found that a slight change in the value of F will occur, resulting in a final value of $F=31{\cdot}3EI/L^2$. Then $f=31{\cdot}3EIc/AL^2y$ and since $f=PL/4S$, $P=125{\cdot}2EIcS/AL^3y$.

It is noted that the critical load in the top flange is expressed as $F=KEI/L^2$, when K varies with the manner in which the loads are applied to the compression flange, or the continuity of the ends of the beam.

In the case of a uniformly loaded beam, the shear transfer from web to flange is uniformly decreasing from the end to the centre. Fig. 5(c) illustrates an assumed parabolic deflection curve with maximum ordinate a. The length of the beam is divided into ten equal divisions. The load applied to the flange per unit of length varies from a maximum value at the end to zero at the mid-span. The average value of VQ/I for division 5 is 9/10 of the value of VQ/I for the end of the beam; the average value is 7/10 for division 4, 5/10 for division 3, 3/10 for division 2, and 1/10 for division 1. Fig. 5(c) shows the distribution of the force F to the five divisions with $F/25$ at division 1, $3F/25$ at division 2, $5F/25$ at division 3, $7F/25$ at 4, and $9F/25$ at 5. The bending moment at point 4 will be $0{\cdot}36F\times(0{\cdot}51a-0{\cdot}19a)=0{\cdot}1152Fa$. The calculations for the bending moment at each point and the deflection of each point from the tangent to the elastic curve at the mid-span follow:

$$
\begin{array}{lll}
0{\cdot}36F \times 0{\cdot}32a = 0{\cdot}1152Fa & & \text{Point 4} \\
0{\cdot}28F & & \\
\hline
0{\cdot}64F \times 0{\cdot}24a = 0{\cdot}1536Fa & & \\
0{\cdot}20F & 0{\cdot}2688Fa & \text{Point 3} \\
\hline
0{\cdot}84F \times 0{\cdot}16a = 0{\cdot}1344Fa & & \\
0{\cdot}12F & 0{\cdot}4032Fa & \text{Point 2} \\
\hline
0{\cdot}96F \times 0{\cdot}08a = 0{\cdot}0768Fa & & \\
 & 0{\cdot}4800Fa & \text{Point 1}
\end{array}
$$

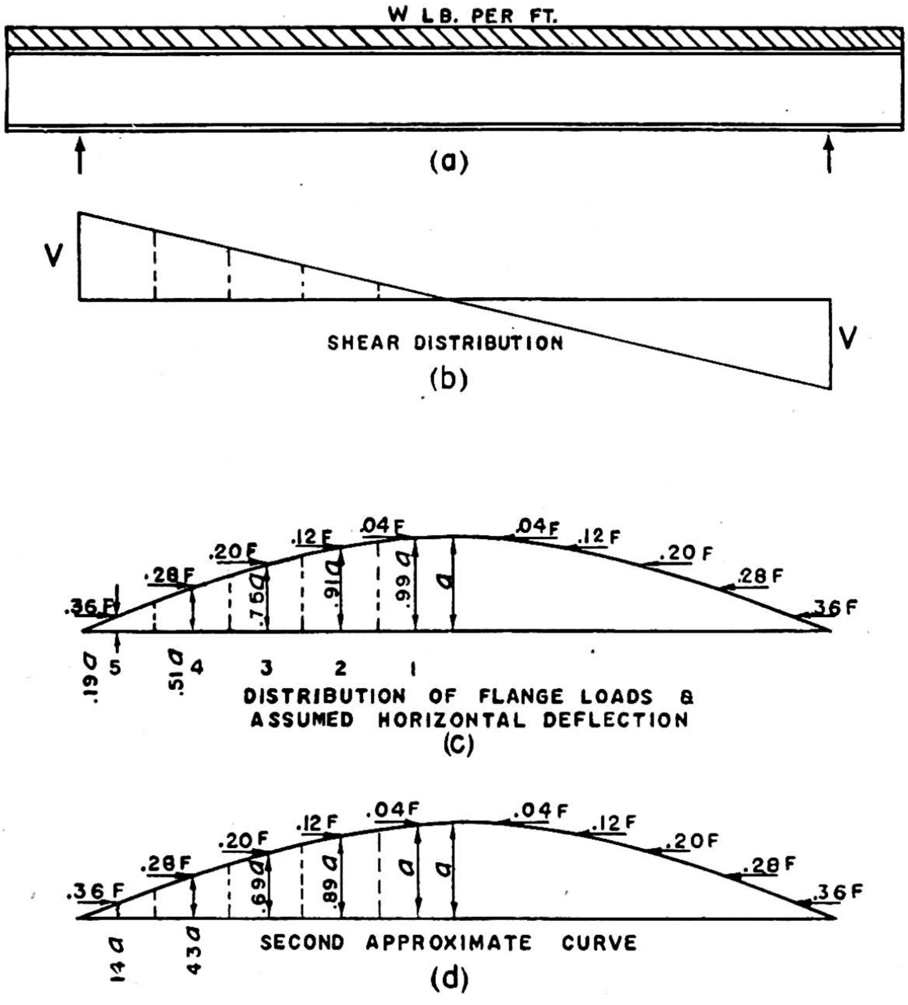

Fig. 5

			New ordinate
$0{\cdot}04800FaL/EI$	$\times 0{\cdot}10L = 0{\cdot}004800FaL^2/EI$	Point 2	$0{\cdot}89a$
$0{\cdot}04032$			
$0{\cdot}08832$	$\times 0{\cdot}10L = 0{\cdot}008832$		
$0{\cdot}02688$	$0{\cdot}013632$	Point 3	$0{\cdot}69a$
$0{\cdot}11520$	$\times 0{\cdot}10L = 0{\cdot}011520$		
$0{\cdot}01152$	$0{\cdot}025152$	Point 4	$0{\cdot}43a$
$0{\cdot}12672$	$\times 0{\cdot}10L = 0{\cdot}012672$		
	$0{\cdot}037824$	Point 5	$0{\cdot}14a$
$0{\cdot}12672$	$\times 0{\cdot}05L = 0{\cdot}006336$		
	$0{\cdot}044160FaL^2/EI$	End	0

Since the maximum deflection is found to be $y=0{\cdot}04416FaL^2/EI$ and $y=a$, then $F=22{\cdot}6EI/L^2$. This value of F is approximate, since a parabolic curve was assumed. The ordinates for a closer curve were found by dividing each deflection value by $0{\cdot}04416FaL^2/EI$ and subtracting these quotients from a. These new ordinates are shown in fig. 5(d).

By using the ordinates of (*d*), new values of bending moment, new deflections, and still another deflection curve are computed as follows:

$0{\cdot}36F \times 0{\cdot}29a = 0{\cdot}1044Fa$		Point 4
$0{\cdot}28F$		
$\overline{0{\cdot}64F} \times 0{\cdot}26a = 0{\cdot}1664Fa$		
$0{\cdot}20F$	$\overline{0{\cdot}2708Fa}$	Point 3
$\overline{0{\cdot}84F} \times 0{\cdot}20a = 0{\cdot}1680Fa$		
$0{\cdot}12F$	$\overline{0{\cdot}4388Fa}$	Point 2
$\overline{0{\cdot}96F} \times 0{\cdot}11a = 0{\cdot}1056Fa$		
	$\overline{0{\cdot}5444Fa}$	Point 1

				New ordinate
$0{\cdot}05444FaL/EI$	$\times 0{\cdot}10L = 0{\cdot}005444FaL^2/EI$		Point 2	$0{\cdot}89a$
$0{\cdot}04388$				
$\overline{0{\cdot}09832}$	$\times 0{\cdot}10L = 0{\cdot}009832$			
$0{\cdot}02708$		$\overline{0{\cdot}015276}$	Point 3	$0{\cdot}68a$
$\overline{0{\cdot}12540}$	$\times 0{\cdot}10L = 0{\cdot}012540$			
$0{\cdot}01044$		$\overline{0{\cdot}027816}$	Point 4	$0{\cdot}42a$
$\overline{0{\cdot}13584}$	$\times 0{\cdot}10L = 0{\cdot}013584$			
		$\overline{0{\cdot}041400}$	Point 5	$0{\cdot}14a$
$0{\cdot}13584$	$\times 0{\cdot}05L = 0{\cdot}006792$			
		$\overline{0{\cdot}048192FaL^2/EI}$	End	0

From this results the closer value of $F = 20{\cdot}8EI/L^2$, which is $2{\cdot}11\pi^2 EI/L^2$, when I is the moment of inertia of the compression flange about the vertical axis.

Table I gives values of K for simply supported beams of constant section which are supported against lateral movement only at their ends.

TABLE I

Values of K

Type of loading	K
Plane bending	9·87
Uniform load	20·8
Concentrated load at $0{\cdot}1L$	20·9
Concentrated load at $0{\cdot}2L$	23·8
Concentrated load at $0{\cdot}3L$	27·9
Concentrated load at $0{\cdot}4L$	30·4
Concentrated load at centre	31·3
Equal loads at $0{\cdot}1L$ and $0{\cdot}9L$	12·1
Equal loads at $0{\cdot}2L$ and $0{\cdot}8L$	15·1
Equal loads at $0{\cdot}3L$ and $0{\cdot}7L$	19·7
Equal loads at $0{\cdot}4L$ and $0{\cdot}6L$	25·3

The method that has been applied to the constant-section beam may be expanded to become applicable to the variable-section beam. Fig. 6(*a*) illustrates a welded beam with varying flange thickness, loaded with a single concentrated load at the centre of the span. The shear diagram is shown in fig. 6(*b*), and the moment of inertia of the entire beam about its horizontal axis is shown in fig. 6(*c*). The shear load between the web and the flange at any point is equal to VQ/I_x lb./in. Since the numerical value of V is constant throughout the length of the beam, the shear transferred to the flange of the beam from the web must be proportional to Q/I_x.

After the distribution of the flange loading from the web is determined, a

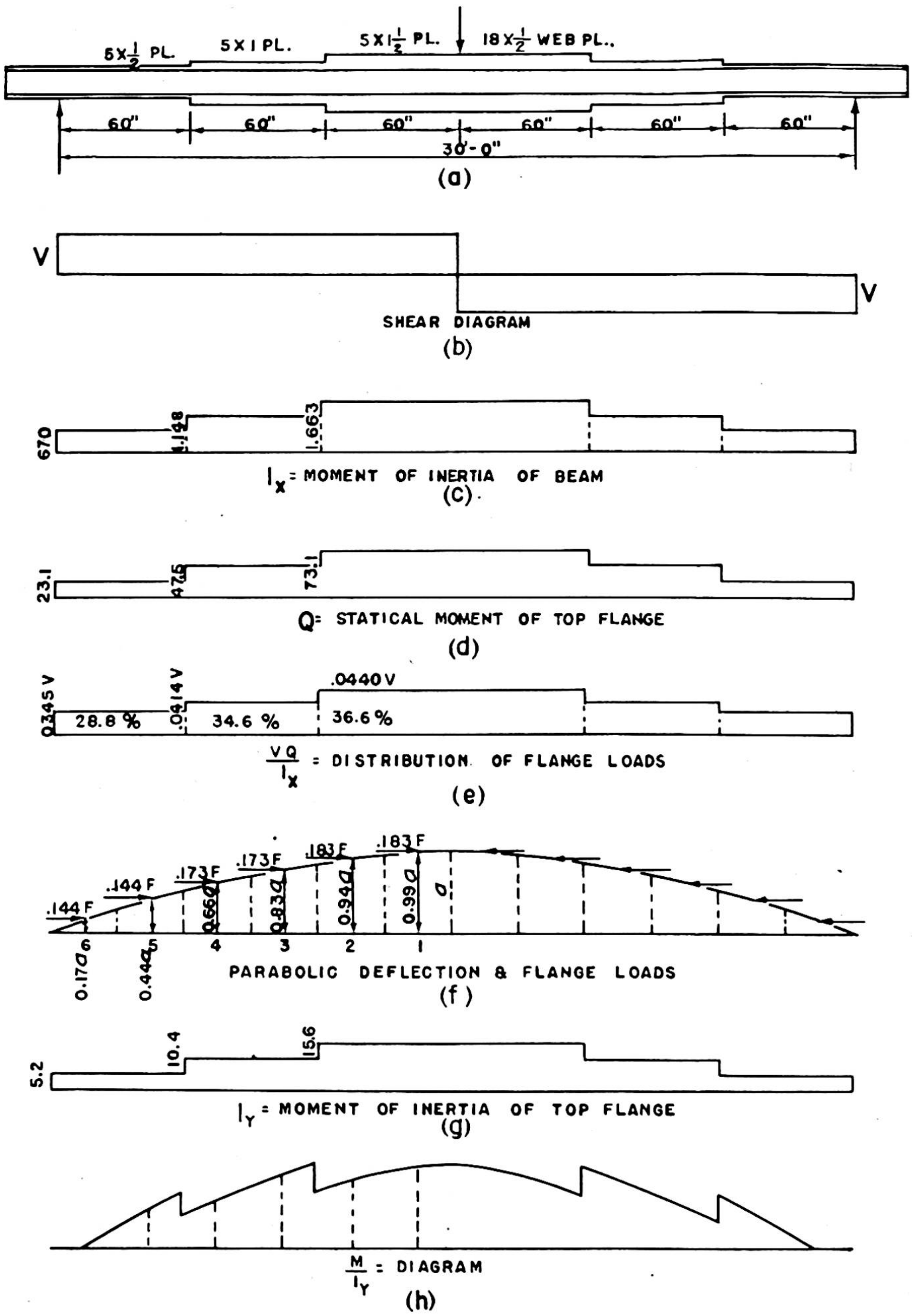

Fig. 6

parabolic horizontal deflection curve is assumed and corresponding bending moments are computed. The deflection curve may be computed by the Moment-Area method, areas of the M/EI diagram being used. The resulting deflection curve will be more nearly the true curve of deflection maintained by the flange loads. When a value of the maximum deflection is expressed in terms of the initial deflection and E, a value for F may be found.

Fig. 6(*e*) shows values of VQ/I_x and the percentage of the flange load that each 60-in. length of the web transfers to the flange. The half-length of the compression flange is divided into six sections of 30 in. each for the computation, and the centre of each length becomes a working point. These centres are numbered from 1 to 6 in fig. 6(*f*). The sum of the increments of F that are shown applied to the centres of these sections is equal to F, and these increments correspond with the VQ/I_x values in fig. 6(*e*). Ordinates to the assumed parabolic curve are shown in fig. 6(*f*) and are used to compute the bending moments at points 1 to 5 in the following manner:

Bending Moments

	0	Point 6
$0{\cdot}144F \times 0{\cdot}27a =$	$0{\cdot}03888Fa$	Point 5
$0{\cdot}144F$		
$0{\cdot}288F \times 0{\cdot}22a =$	$0{\cdot}06336Fa$	
$0{\cdot}173F$	$0{\cdot}10224Fa$	Point 4
$0{\cdot}461F \times 0{\cdot}17a =$	$0{\cdot}07837Fa$	
$0{\cdot}173F$	$0{\cdot}18061Fa$	Point 3
$0{\cdot}634F \times 0{\cdot}11a =$	$0{\cdot}06974Fa$	
$0{\cdot}183F$	$0{\cdot}25035Fa$	Point 2
$0{\cdot}817F \times 0{\cdot}05a =$	$0{\cdot}04085Fa$	
$0{\cdot}183F$	$0{\cdot}29120Fa$	Point 1
$1{\cdot}000F \times 0{\cdot}01a =$	$0{\cdot}01000Fa$	
	$0{\cdot}30120Fa$	Centre

Computation of Deflection

	M	dx/I	Mdx/I	x	$Mxdx/I$	Deflection at	New ordinate
(1)	$0{\cdot}29120Fa \times$	$30/15{\cdot}6 =$	$0{\cdot}5600Fa$	30	$16{\cdot}800Fa$	Point 2	$0{\cdot}93a$
(2)	$0{\cdot}25035Fa \times$	$30/15{\cdot}6 =$	$0{\cdot}4814Fa$				
			$1{\cdot}0414Fa$	30	$31{\cdot}242Fa$		
(3)	$0{\cdot}18061Fa \times$	$30/10{\cdot}4 =$	$0{\cdot}5210Fa$		$48{\cdot}042Fa$	Point 3	$0{\cdot}80a$
			$1{\cdot}5624Fa$	30	$46{\cdot}872Fa$		
(4)	$0{\cdot}10224Fa \times$	$30/10{\cdot}4 =$	$0{\cdot}2949Fa$		$94{\cdot}914Fa$	Point 4	$0{\cdot}61a$
			$1{\cdot}8573Fa$	30	$55{\cdot}719Fa$		
(5)	$0{\cdot}03888Fa \times$	$30/5{\cdot}2 =$	$0{\cdot}2243Fa$		$150{\cdot}633Fa$	Point 5	$0{\cdot}38a$
			$2{\cdot}0816Fa$	30	$62{\cdot}448Fa$		
					$213{\cdot}081Fa$	Point 6	$0{\cdot}13a$
			$2{\cdot}0816Fa$	15	$31{\cdot}224Fa$		
					$244{\cdot}305Fa$	End	0

In the foregoing computations it is found that the deflection of the end of the beam from the tangent to the elastic curve at the centre is $y=244{\cdot}305Fa/E$. By definition, the force F must just maintain the small deflection a. Hence, $y=a$ and $F=E/244{\cdot}305=122{,}800$ lb. The average flange stress at the centre of the beam will be F/A, or $122{,}800/7{\cdot}5=16{,}375$ lb./in.^{2}. The extreme fibre stress will be greater than the average flange stress, being equal to $16{,}375\times 10{\cdot}5/9{\cdot}75=17{,}635$ lb./in.^{2} The load P on the beam for which $Mc/I=17{,}635$ lb./in.^{2} will be such that $90P\times 10{\cdot}5/1{,}663=17{,}635$. Thus $P=31{,}030$ lb.

These values were computed on the assumption that the shape of the deflection curve that would be maintained by the force F is parabolic. If each of the deflections (times E) that were computed for points 2 to 6 is divided by $244{\cdot}305FA$ and the quotients are subtracted from a, a new shape of curve will be indicated which would be closer to the true curve.

The new ordinates in the foregoing computations are these values. The computations may now be repeated to obtain a closer value of F:

Bending Moment

$0{\cdot}144F\times 0{\cdot}25a=$	$0{\cdot}03600Fa$	Point 5
$0{\cdot}144F$		
$\overline{0{\cdot}288F}\times 0{\cdot}23a=$	$0{\cdot}06624Fa$	
$0{\cdot}173F$	$\overline{0{\cdot}10224Fa}$	Point 4
$\overline{0{\cdot}461F}\times 0{\cdot}19a=$	$0{\cdot}08759Fa$	
$0{\cdot}173F$	$\overline{0{\cdot}18983Fa}$	Point 3
$\overline{0{\cdot}634F}\times 0{\cdot}13a=$	$0{\cdot}08242Fa$	
$0{\cdot}183F$	$\overline{0{\cdot}27225Fa}$	Point 2
$\overline{0{\cdot}817F}\times 0{\cdot}07a=$	$0{\cdot}05719Fa$	
$0{\cdot}183F$	$\overline{0{\cdot}32944Fa}$	Point 1
$\overline{1{\cdot}000F}\times 0=$	0	
	$\overline{0{\cdot}32944Fa}$	Centre

Computation of Deflections

M	dx/I	Mdx/I	x	$Mxdx/I$	Deflection at	New ordinate
(1) $0{\cdot}32944Fa$	$\times 30/15{\cdot}6=$	$0{\cdot}6335Fa$	30	$19{\cdot}005Fa$	Point 2	$0{\cdot}93a$
(2) $0{\cdot}27225Fa$	$\times 30/15{\cdot}6=$	$0{\cdot}5236Fa$				
		$\overline{1{\cdot}1571Fa}$	30	$34{\cdot}713Fa$		
(3) $0{\cdot}18983Fa$	$\times 30/10{\cdot}4=$	$0{\cdot}5476Fa$		$\overline{53{\cdot}718Fa}$	Point 3	$0{\cdot}80a$
		$\overline{1{\cdot}7047Fa}$	30	$51{\cdot}141Fa$		
(4) $0{\cdot}10224Fa$	$\times 30/10{\cdot}4=$	$0{\cdot}2949Fa$		$\overline{104{\cdot}859Fa}$	Point 4	$0{\cdot}60a$
		$\overline{1{\cdot}9996Fa}$	30	$59{\cdot}988Fa$		
(5) $0{\cdot}03600Fa$	$\times 30/5{\cdot}2=$	$0{\cdot}2077Fa$		$\overline{164{\cdot}847Fa}$	Point 5	$0{\cdot}38a$
		$\overline{2{\cdot}2073Fa}$	30	$66{\cdot}219Fa$		
				$\overline{231{\cdot}066Fa}$	Point 6	$0{\cdot}13a$
		$2{\cdot}2073Fa$	15	$33{\cdot}109Fa$		
				$\overline{264{\cdot}175Fa}$	End	0

Since $y=264{\cdot}175Fa/E$ or $264{\cdot}175F/E=1$, then $F=113{,}561$ lb. Also, $F/A=113{,}561/7{\cdot}5=15{,}141$ lb./in.^{2}, and the maximum flange stress at the centre of the span is $10{\cdot}5/9{\cdot}75\times 15{,}141=16{,}306$ lb./in.^{2} Then $Mc/I=90P\times 10{\cdot}5/1{,}663=16{,}306$ lb./in.^{2} or $P=28{,}695$ lb.

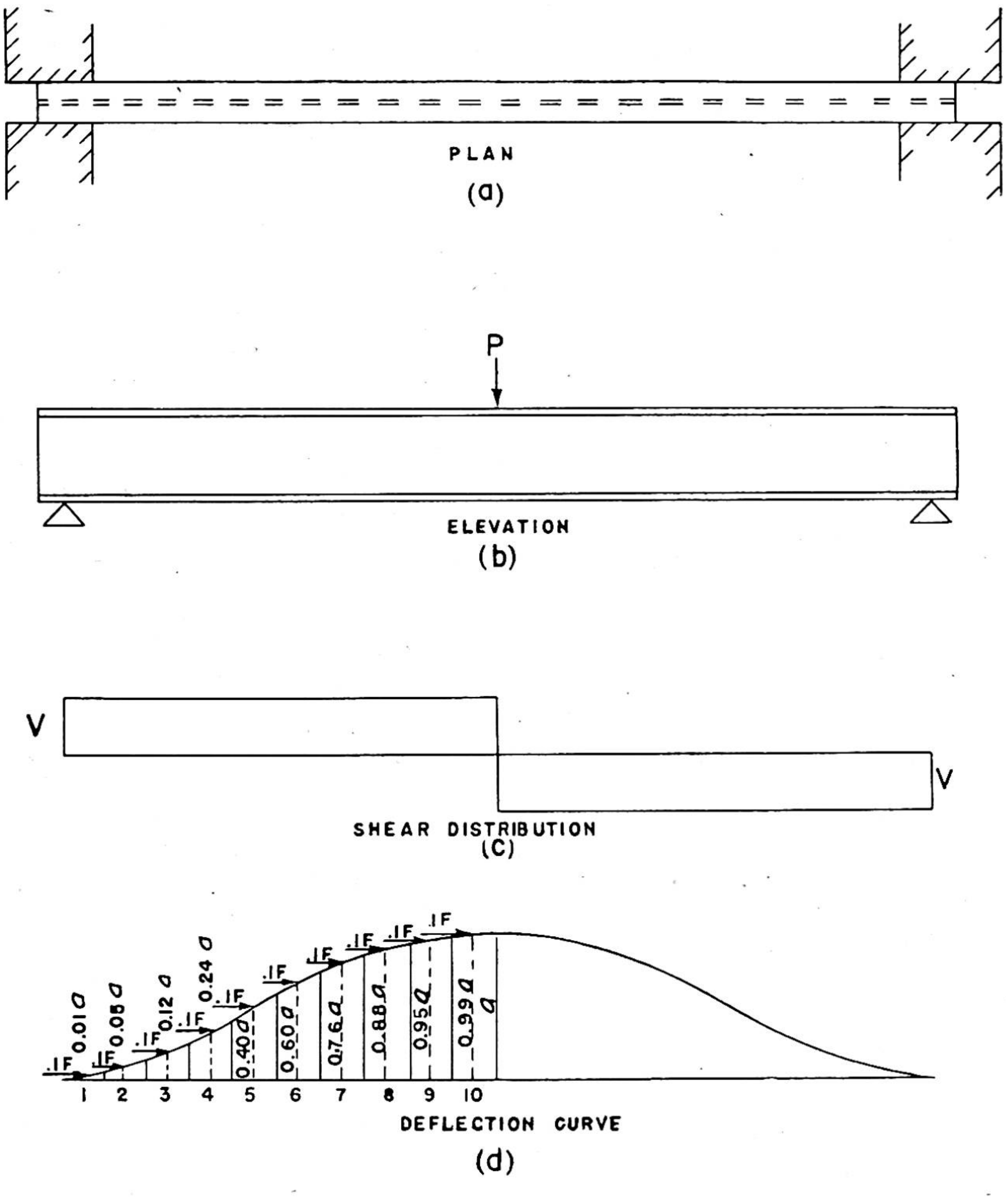

Fig. 7

It will be noted that the new values differ from the values first computed by less than 10%. Ordinates to a second new curve appear to be almost identical with those used for the second computation. Hence it would seem unnecessary to carry the computation further.

Fig. 7(*a*) illustrates the plan view of a simply supported beam of constant section. The top flange is assumed to be restrained from rotation in a horizontal plane. Fig. 7(*d*) shows the half-span divided into ten equal divisions and an assumed reverse parabolic deflection curve. The beam is loaded with a centrally placed concentrated load P as shown in fig. 7(*b*), hence an increment of $0{\cdot}1F$ will be applied to the compression flange at each division of the length.

Using the assumed curve shape, the simple bending moments are calculated in the usual manner. Since the end tangents to the elastic curve are prevented from rotating, the total M/EI area between the end and the centre of the span must be zero; hence, end moments must be of the magnitude that will accomplish this result. The sum of the simple moments at the ten divisions divided by 10 will then equal the end moment and the bending moment at any point will be the difference between the end moment and the simple moment at that point.

Bending Moments

			M
	0	Point 1	$-0{\cdot}207$
$0{\cdot}1F \times 0{\cdot}04a = 0{\cdot}004Fa$		Point 2	$-0{\cdot}202$
$0{\cdot}1F$			
$0{\cdot}2F \times 0{\cdot}07a = 0{\cdot}014Fa$			
$0{\cdot}1F$	$0{\cdot}018Fa$	Point 3	$-0{\cdot}189$
$0{\cdot}3F \times 0{\cdot}12a = 0{\cdot}036Fa$			
$0{\cdot}1F$	$0{\cdot}054Fa$	Point 4	$-0{\cdot}153$
$0{\cdot}4F \times 0{\cdot}16a = 0{\cdot}064Fa$			
$0{\cdot}1F$	$0{\cdot}118Fa$	Point 5	$-0{\cdot}089$
$0{\cdot}5F \times 0{\cdot}20a = 0{\cdot}100Fa$			
$0{\cdot}1F$	$0{\cdot}218Fa$	Point 6	$+0{\cdot}011$
$0{\cdot}6F \times 0{\cdot}16a = 0{\cdot}096Fa$			
$0{\cdot}1F$	$0{\cdot}314Fa$	Point 7	$+0{\cdot}107$
$0{\cdot}7F \times 0{\cdot}12a = 0{\cdot}084Fa$			
$0{\cdot}1F$	$0{\cdot}398Fa$	Point 8	$+0{\cdot}191$
$0{\cdot}8F \times 0{\cdot}07a = 0{\cdot}056Fa$			
$0{\cdot}1F$	$0{\cdot}454Fa$	Point 9	$+0{\cdot}247$
$0{\cdot}9F \times 0{\cdot}04a = 0{\cdot}036Fa$			
$0{\cdot}1F$	$0{\cdot}490Fa$	Point 10	$+0{\cdot}284$
$1{\cdot}0F$			

Deflections from the tangent to the elastic curve at the end may now be calculated as follows:

M	dx	x	Deflection		New curve
$-0\cdot207Fa$	$\times 0\cdot05L$	$\times 0\cdot05L=$	$-0\cdot0005FaL^2$	Point 2	$0\cdot038a$
$-0\cdot202Fa$					
$-0\cdot409Fa$			$-0\cdot0010FaL$		
$-0\cdot189Fa$			$-0\cdot0015FaL^2$	Point 3	$0\cdot116a$
$-0\cdot598Fa$			$-0\cdot0015FaL^2$		
$-0\cdot153Fa$			$-0\cdot0030FaL^2$	point 4	$0\cdot232a$
$-0\cdot751Fa$			$-0\cdot0019FaL^2$		
$-0\cdot089Fa$			$-0\cdot0049FaL^2$	Point 5	$0\cdot380a$
$-0\cdot840Fa$			$-0\cdot0021FaL^2$		
$+0\cdot011Fa$			$-0\cdot0070FaL^2$	Point 6	$0\cdot543a$
$-0\cdot829Fa$			$-0\cdot0021FaL^2$		
$+0\cdot107Fa$			$-0\cdot0091FaL^2$	Point 7	$0\cdot705a$
$-0\cdot722Fa$			$-0\cdot0018FaL^2$		
$+0\cdot191Fa$			$-0\cdot1009FaL^2$	Point 8	$0\cdot845a$
$-0\cdot531Fa$			$-0\cdot0013FaL^2$		
$+0\cdot247Fa$			$-0\cdot0122FaL^2$	Point 9	$0\cdot946a$
$-0\cdot284Fa$			$-0\cdot0007FaL^2$		
$+0\cdot284Fa$			$-0\cdot0129FaL^2$	Point 10	$1\cdot000a$
0					

Since $y=0\cdot0129FaL^2/EI$ and $y=a$, $F=77\cdot5EI/L^2$. The next approximation, using the curve developed from the first approximation, results in $F=75\cdot5EI/L^2$.

It will be seen by the illustrative examples that the procedure for finding the limit of stress in the compression flange of a beam follows a very definite plan. The step-by-step procedure may be outlined as follows:

(1) Identify the conditions of end restraint that affect the shape of the elastic curve for lateral buckling of the compression flange.

(2) Assume a nominal finite lateral deflection of the compression flange and a shape of curve that is in general agreement with the conditions of end restraint.

(3) Define the manner of loading of the compression flange consistent with the manner in which the beam is loaded.

(4) Compute bending moments along the length of the compression flange caused by the flange load and the assumed lateral deflections, and consistent with the conditions of end restraint.

(5) Compute the magnitude of the lateral deflection of the flange from the values of M, E, I, and the length of the beam, and expressed in terms of the magnitude of the assumed lateral deflection.

(6) A new deflection curve may be developed from the above step (5) and compared with the assumed shape of curve.
(7) When the assumed shape of deflection curve and the shape of the deflection curve found by use of the assumed curve agree, an equation between the computed maximum deflection and the assumed deflection will yield an expression for the limit of load in the straight compression flange of the beam.

Experimenters are familiar with certain phenomena in the testing of flanged beams. Load may be applied gradually to the beam with no apparent tendency for the compression flange to buckle sidewise until a certain load value has been reached. Once this critical value of flange stress has been reached, the compression flange may exhibit a tendency to bend principally in one lateral direction. Upon reaching a second critical value of flange stress, the compression flange may be easily moved from one deflected position to a deflected position in the opposite direction. Then, as increasing values of load are placed on the beam, the amount of lateral deflection that will remain placed in either direction increases also. The ultimate result occurs when the beam has been loaded so that lateral deflection in one direction continues to complete collapse.

It is noted by the experimenters that when a given load is suspended vertically from the bottom flange of the beam, the amount of lateral deflection of the compression flange is smaller than when the same load is placed on the top flange. This fact is consistent with principles developed by previous investigators pertaining to action after certain bending has taken place in a lateral direction.

It would seem that it should be possible experimentally to measure the angle of rotation of the central portion of the beam span that agrees with any value of superimposed load; then, with a sufficient number of measurements of such relations, the load at zero angle of rotation could be projected. Such measurements have been carried out successfully for several types of loading, but certain phenomena are troublesome to the experimenter.

The lateral deflection of the compression flange is sensitive to conditions of end restraint. It is not easy to obtain a truly simply supported beam with lateral support of the compression flange not restrained from end rotation. Also, it is found that the immediate past history of stress in the flange appears to affect the magnitudes of rotation angles of the beam cross-section that will be maintained by any given vertical load. The probable reason for this variation is that the experimenter is unable to control the maximum amount of rotation and the beam flange is subjected to stresses above the yield point in certain fibres. A different number of fibres have stress above the yield point with each value of rotation angle.

The following procedure has been found to produce satisfactory results experimentally. A load is placed upon the beam which does not cause general yielding but which is known to be well above that producing critical stress while the beam is straight. While the beam is under this load the compression flange may be moved in a lateral direction by a pressure of the hand, say to the left, and will stay in some such deflected position. Now the load may be gradually reduced and a record made of angles of rotation and corresponding loads. If the same procedure is repeated by rotating the beam to the right and recording the loads and angles, two sets of load-angle values will have been produced. Now, if these data are plotted, curves defining the two sets of data will intersect at a value of load checking very well the value of loading that produces critical flange load, while the beam is straight. A second set of data may produce a new set of different angle-load values, but the intersection of the two such curves produced continuously will usually give the same result for the critical point. Whether the load is applied to the top flange or to the bottom flange, and whether the load is vertical or inclined from some centre of loading, will affect the magnitudes of the angles maintained by any given loads on the beam. But it is of interest that any set of data produced from the same conditions of loading appear to project to the same critical value for the compression flange—while straight.

Summary

Determination of the limit of stress in the compression flanges of beams involves many considerations. Factors that are important in the literature on the subject include such items as the distribution of the load causing stress, the torsional resistance of the beam, the lateral stability of the compression flange, and others. Because of the complicated nature of a complete solution in the general case, specifications for design contain empirical formulae guiding the designer. The effects of the distribution of the loading, the type of end restraint, and variations in the section of the beam are known to have large effects but are not included as considerations in the design formulae.

It is herein presented that a revised definition of the neutral state of equilibrium will greatly simplify the considerations and provide the designer with a logical procedure for analysis. In this way he will not be dependent upon empirical formulae that must be conservative to a large degree. It is proposed that the neutral state of equilibrium for design purposes be defined as that having the smallest value; this value occurs while the flange is straight but buckling is imminent. Such a definition eliminates the necessity for consideration of the torsional resistance of the beam and of the loading position, that is, whether the load is on the top or bottom flange of the beam. The definition permits full attention to be given to the large factors affecting solution of the particular case considered. These large factors include the distribution of the loading on the beam, the condition of end restraint, and variations of section.

Special cases illustrate a general method of solution involving the use of common iteration processes and in some cases successive approximations.

Résumé

Le calcul des charges limites des membrures de compression des poutres fait intervenir plusieurs considérations. Les points importants traités dans la littérature spécialisée sont la distribution de la charge, la résistance à la torsion de la poutre, la stabilité latérale de la membrure de compression, etc. Par suite de la complexité d'une solution complète du cas général, les spécifications de détail font intervenir des formules empiriques destinées à guider le dessinateur. On sait que la distribution de la charge, le mode de fixation de l'extrémité de la poutre et les variations de son profil jouent ici un grand rôle, mais ne sont pas pris en considération dans les formules de dessin.

Nous montrons qu'une révision de la définition de l'état d'équilibre stable simplifiera sensiblement la question et fournira au dessinateur un processus logique d'analyse. Il n'aura ainsi pas à se fier à des formules empiriques qui sont nécessairement très conservatrices. Nous proposons de définir, pour le dessin, l'état d'équilibre stable comme celui qui a la moindre valeur; cette valeur se manifeste lorsque la membrure est droite, mais sur le point de se déformer. Une telle définition élimine la nécessité de considérer la résistance à la torsion de la poutre et de faire intervenir le mode d'application de la charge, suivant qu'elle est placée sur la membrure supérieure ou sur la membrure inférieure. Cette définition permet de concentrer toute l'attention sur les facteurs essentiels qui déterminent la solution dans le cas particulier étudié. Ces facteurs comprennent la distribution de la charge sur la poutre, le mode de fixation de l'extrémité de cette poutre et les variations de sa section.

Des cas particuliers illustrent une méthode générale de résolution qui entraîne le recours à des procédés d'itération courants et parfois à des approximations successives.

Zusammenfassung

Die Bestimmung der Grenzspannung in den Druckgurten von Trägern umfasst zahlreiche Ueberlegungen. Die in der Fachliteratur behandelten wichtigen Punkte sind die Lastverteilung, die Torsionssteifigkeit des Trägers, die seitliche Stabilität des Druckgurtes, u.a. Wegen der komplizierten Form der vollständigen Lösung im allgemeinen Fall finden sich in den Entwurfs-Normen empirische Formeln als Wegleitung für den Konstrukteur. Die grosse Bedeutung der Einflüsse der Lastverteilung, der Form der End-Festhaltung und der Veränderlichkeit des Querschnitts ist bekannt, doch sind diese Faktoren in den Entwurfsformeln nicht berücksichtigt.

Der Verfasser zeigt, dass eine verbesserte Definition des neutralen Gleichgewichtszustandes das Problem stark vereinfachen und dem Konstrukteur eine vernünftige Berechnungsmethode in die Hand geben kann. Er ist damit nicht mehr auf empirische Formeln angewiesen, die weitgehend veraltet sind. Der Verfasser schlägt vor, den neutralen Gleichgewichtszustand für den Entwurf dahin zu definieren, dass er den kleinsten Wert aufweisen soll; dieser Wert ergibt sich bei geradem Flansch unmittelbar vor dem Ausknicken. Die vorgeschlagene Definition macht die Notwendigkeit einer Berücksichtigung der Torsionssteifigkeit des Trägers und der Lage der Belastung, d.h. ob die Last am oberen oder unteren Flansch des Trägers wirkt, überflüssig. Die Definition erlaubt uns, unsere volle Aufmerksamkeit den entscheidenden Faktoren, die die Lösung des betrachteten, besonderen Falles beeinflussen, zuzuwenden. Diese entscheidenden Faktoren sind die Verteilung der Belastung über dem Träger, die Festhalte-Bedingungen an den Enden und die Veränderlichkeit des Querschnitts.

An Hand von Sonderfällen wird ein allgemeines Lösungsverfahren aufgezeigt, das die üblichen Iterationsvorgänge und in gewissen Fällen auch successive Approximationen umfasst.

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