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Experimental investigations into the behaviour of continuous and fixed-ended beams

Recherches expérimentales sur le comportement des poutres continues ou encastrées à leur extrémités

Experimentelle Untersuchungen über das Verhalten durchlaufender und eingespannter Balken

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1. Introduction

The behaviour beyond the elastic limit of mild-steel beams subjected to pure bending moments or bending moments combined with shear forces has been studied by Ewing (1903), Robertson and Cook (1913) and many others. The various theories suggested and the experimental evidence relating to them have been reviewed by Roderick and Phillipps (1949). It appears that, when considering annealed beams, the most satisfactory theory is that in which it is assumed that initially plane sections remain plane during bending, the longitudinal stress being related to the longitudinal strain as in a tension or compression test (see Roderick, 1948). Good correlation between bending and tension tests may be obtained if due regard is paid to the upper yield stress and to the rate of straining in the plastic range. The influence of shear forces has been investigated experimentally by Baker and Roderick (1940) and Hendry (1950) and theoretically by Horne (1951). It has been shown that, for practical purposes, shear forces have negligible effect on the behaviour of a beam. The stress distributions are also modified in the vicinity of concentrated loads, and this has been investigated experimentally by Roderick and Phillipps (1949) and theoretically by Heyman (unpublished). The simple plastic theory has also been found to apply approximately to rolled steel sections (Maier-Leibnitz, 1936), although correlation between bending and tension tests is here more difficult due to the variation in properties of the steel over any cross-section.

The simple plastic theory leads to important deductions regarding the behaviour of continuous and fixed-ended beams and rigid-jointed unbraced structures such as building frames. Due to the considerable pure plastic deformation which mild steel can undergo (of the order of 1 % strain, or ten times the strain at the commencement of yield), the curvature of the longitudinal centre line of an initially straight beam increases rapidly with practically no increase of bending moment as the section becomes fully plastic. The bending moment then approaches the "full plastic" value (see Roderick, 1948), and although at extremely high curvatures the beam may develop a higher moment of resistance due to strain hardening, the full plastic moment may be regarded as the highest moment to which the beam may be subjected and still retain its usefulness. When beams are continuous over a number of supports or *encastré* (i.e. fixed in position and direction at their ends), the high curvature which occurs in the vicinity of fully plastic sections enables the applied loads to be increased until the full plastic moment is reached at a sufficient number of sections for a "mechanism" to be formed, these sections being regarded as "hinges" with constant moments of resistance. Similar considerations apply to rigid-jointed unbraced frames as long as axial forces are small enough to have negligible effect on the bending moments in the members in which they occur. The application of such results to the calculation of collapse loads has been considered extensively by Bleich (1932), Baker (1949), Neal and Symonds (1950), Horne (1950) and others.

The above theoretical developments have been achieved by making certain extensions of the simple plastic theory as established by tests on simply supported members. The "plastic hinge" concept is only an approximation to the truth, corresponding as it does to infinite curvature at the assumed fully plastic sections. It is thus essential that these theoretical deductions should be tested experimentally. In the case of continuous beams, the simple plastic theory indicates that the order in which the spans are loaded, or the sinking of one support relative to the others, should have no effect on the value of the collapse load. In beams partially fixed against rotation at the ends, the degree of end restraint should similarly have no effect on the collapse loads as long as the moment of resistance of the end supports is at least equal to the full plastic moment of resistance of the beam. Moreover, the fact that full plasticity has been produced at some section or sections of a beam for one set of loads should not reduce the carrying capacity of that beam for any subsequent set.

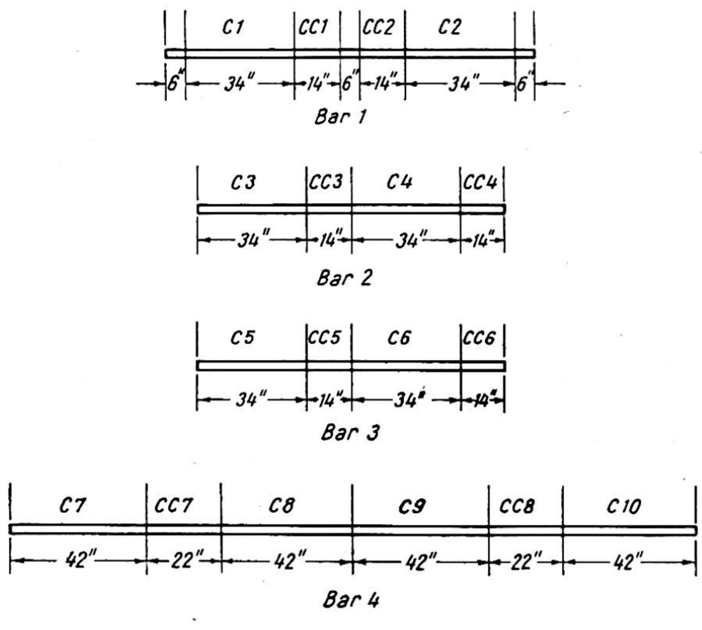

Fig. 1. Division of bars for continuous beam tests

While certain investigations on continuous beams have already been made by Maier-Leibnitz (1936) and Volterra (1943), no attempt to check these deductions systematically has yet been reported. It was for this reason that the investigations here described were undertaken.

2. Tests on continuous beams

(a) *Preparation of beams*

The beams were taken from 1-in. square bars of rolled mild steel in the "as received" condition, the bars being cut according to the scheme shown in fig. 1. All

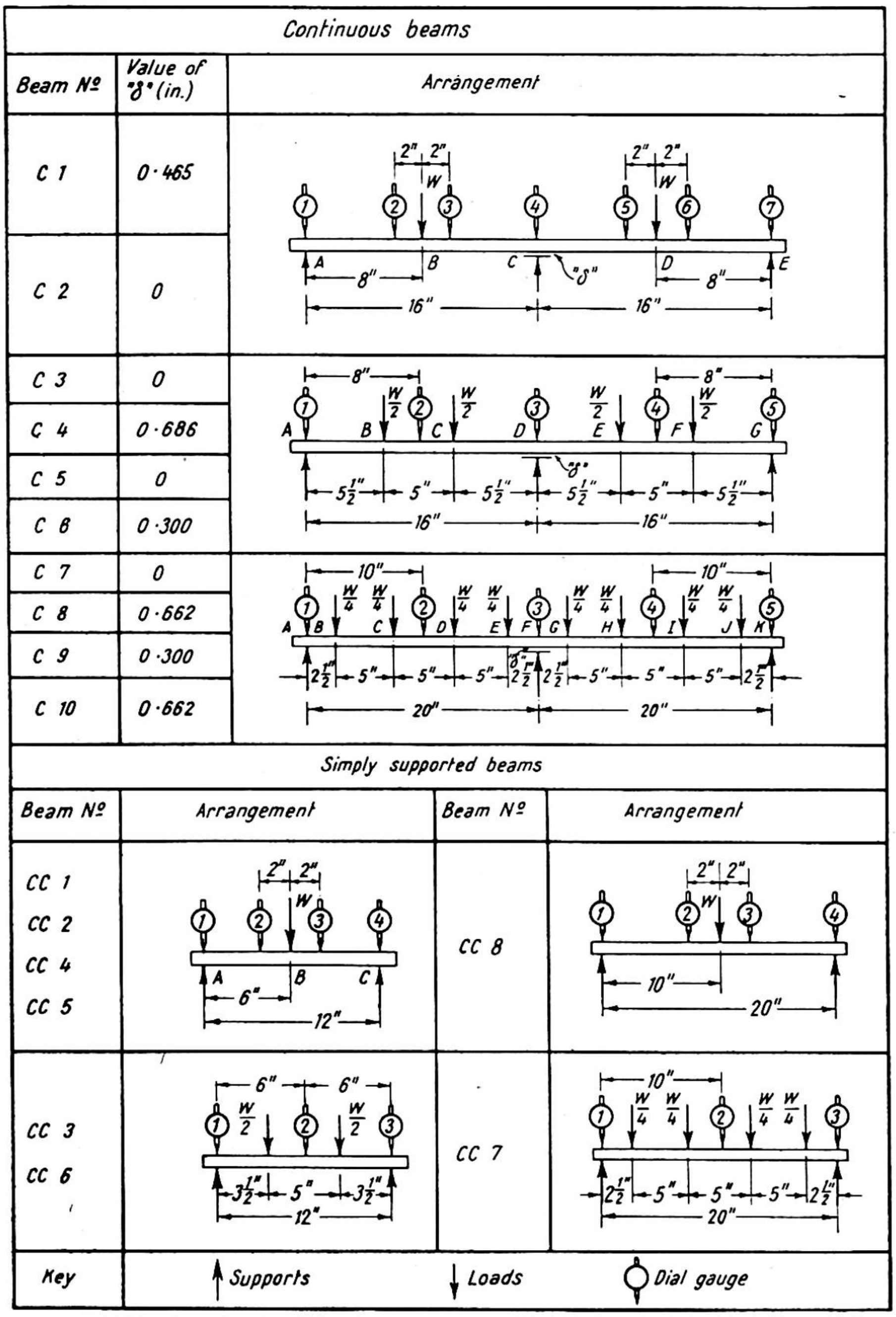

Fig. 2. Summary of continuous beam tests
(In beam CC8, for 2″ read $2\frac{1}{2}$″)

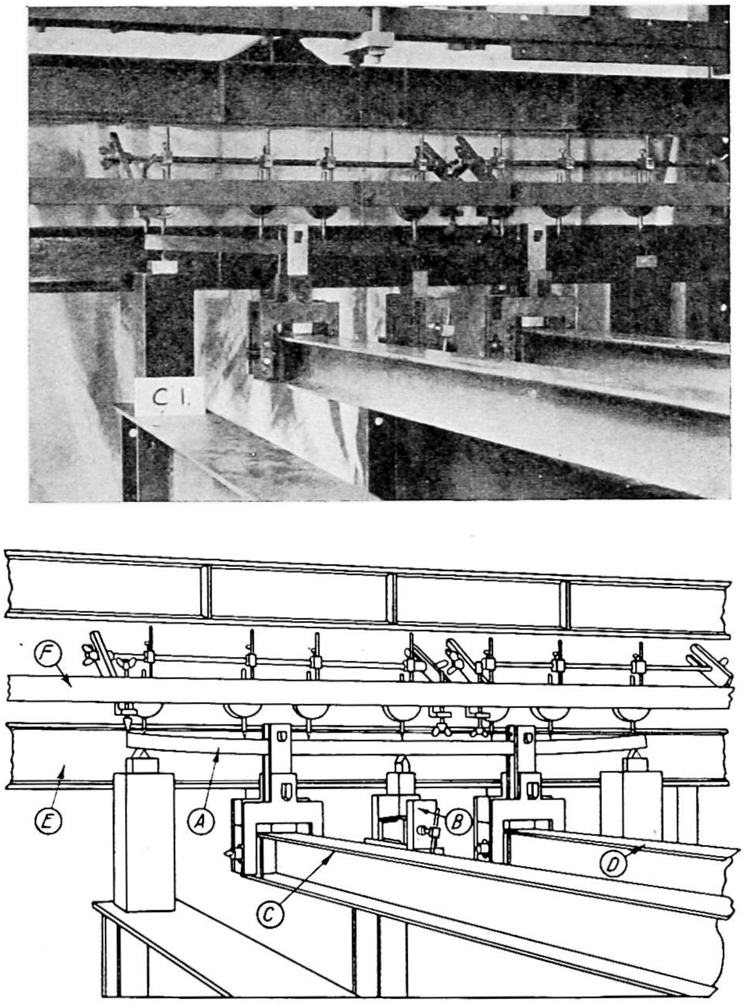

Fig. 3. Arrangement for testing continuous beams

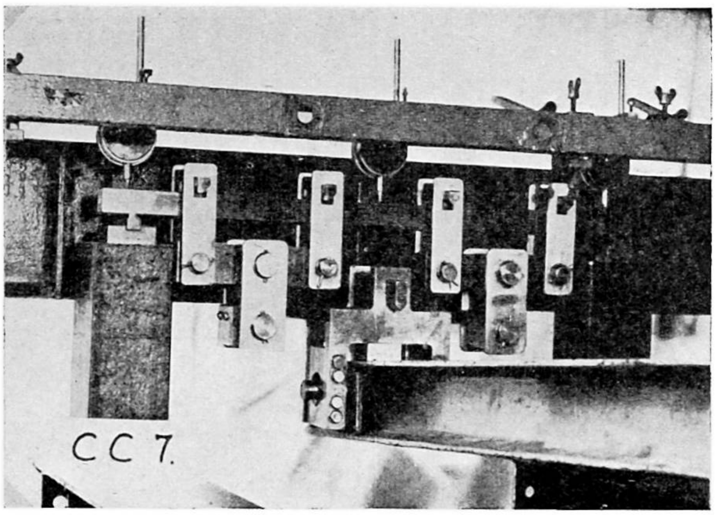

Fig. 4. Arrangement for testing simply supported beams

the beams were roughly planed to the required dimensions ($\frac{7}{8}$ in. square) and finished by surface grinding, thus imparting a polished surface which, as described below, enabled Lüders' wedges to be observed during the tests.

(b) *Description of tests*

The tests are summarised in fig. 2, which shows for each beam the positions of the supports, loads and dial gauges used to measure deflections. Some lengths were tested from each bar as continuous beams, while other lengths were tested as simply supported in order to obtain direct measurements of the full plastic moments. In some of the tests on the continuous beams, the central support was set a certain depth below the outer supports, and this is also indicated. Increasing loads were applied simultaneously to both spans of all the continuous beams except beam C5, in which span AD (see fig. 2) was loaded to collapse with only a small load on span DG. For all beams, when collapse had occurred in one span, the load on the other span was further increased until it also collapsed.

The tests were performed in a dead-load testing frame, a full description of which has been given by Baker and Roderick (1942). The arrangement for testing the continuous beam C1 is shown in fig. 3, in which A is the beam supported on knife-edges and B is a block by means of which it is possible to adjust the height of the central knife-edge. The load is applied by the levers C and D whose fulcra react against the member E, while the dial gauges for measuring deflections are supported on an independent frame of which F is a member. The simply supported beam CC7 was tested as shown in fig. 4, which also shows the linkage used to distribute the load from the lever equally to four knife-edges acting on the upper surface of the beam.

During the tests, as long as the beams remained elastic, finite increments of load were added at intervals of approximately two minutes, the dial gauges being read between each increment. After the first signs of creep had been observed, the addition of each load increment was delayed until no dial gauge showed a rate of increase greater than 10^{-4} in. per minute. Loading was continued until collapse occurred, this being characterised by a large increase of deflection for a small increase of load.

(c) *Test results*

The test results for all the beams are summarised in Table I, and are grouped according to the bar from which the beams were cut. The mean dimensions are given in columns 3 and 4. In the case of the simply supported beams, the values of the modulus of elasticity E calculated from the linear portions of the load deflection curves are given in column 5. The values of E quoted for the continuous beams are the mean of the values obtained for the simply supported beams cut from the same bar. Column 6 gives the collapse loads. In the case of the continuous beams, the mean of the values for the two spans is given; in no case did the difference between these values exceed 3·3%. Values of the full plastic moments may be deduced from the collapse loads by means of the simple plastic theory, giving the lower yield stresses quoted in column 7 of Table I. Assuming that each bar is of uniform material, the agreement between these stresses for beams cut from the same bar is a check on the accuracy of the simple plastic theory. The percentage variations of these yield stresses as compared with the average for the bar are given in column 9.

It has been shown by Heyman (to be published) that the assumption made in the simple plastic theory that there is no restraint in directions perpendicular to the longitudinal axis of a beam is invalidated in the vicinity of heavy load concentrations. This tends to increase the full plastic moment except where the maximum moment

TABLE I

	1	2	3	4	5	6	7	8	9	10	11	12
							Analysis by simple plastic theory ignoring the effect of load concentration			Analysis in which allowance is made for the effect of load concentration		
	Bar No.	Beam No.	Mean Width, in.	Mean Depth, in.	Estimated Modulus of Elasticity E, tons/in.^{2}	Load, tons	Lower Yield Stress, tons/in.^{2}	Mean Lower Yield Stress for bar, tons/in.^{2}	Per cent Difference	Lower Yield Stress, tons/in.^{2}	Mean Lower Yield Stress for bar, tons/in.^{2}	Per cent Difference
1	1	C1	0·875	0·876	13,360	1·125	17·83	18·02	−1·1	16·53	16·71	−1·1
2		C2	0·875	0·876	13,360	1·138	18·04		0·1	16·72		0·1
3		CC1	0·876	0·876	13,380	1·000	17·90		−0·7	16·59		−0·7
4		CC2	0·876	0·876	13,340	1·025	18·33		1·7	16·99		1·7
5	2	C3	0·875	0·875	13,120	1·560	19·03	18·34	3·8	17·96	17·48	2·7
6		C4	0·875	0·875	13,120	1·525	18·60		1·4	17·55		0·4
7		CC3	0·875	0·875	13,010	1·680	17·43		−5·0	17·43		−0·3
8		CC4	0·875	0·875	13,220	1·020	18·32		−0·1	16·98		−2·9
9	3	C5	0·875	0·875	12,930	1·225	14·93	14·91	0·1	14·09	14·21	−0·8
10		C6	0·875	0·874	12,930	1·230	15·01		0·7	14·16		−0·4
11		CC5	0·875	0·875	13,270	0·850	15·26		2·3	14·14		−0·5
12		CC6	0·875	0·875	12,590	1·380	14·44		−3·2	14·44		1·6
13	4	C7	0·875	0·876	12,945	1·670	18·13	17·84	1·6	16·46	16·60	−0·8
14		C8	0·876	0·875	12,945	1·670	18·12		1·6	16·45		−0·9
15		C9	0·876	0·876	12,945	1·650	17·92		0·4	16·27		−2·0
16		C10	0·876	0·876	12,945	1·700	18·38		3·0	16·69		0·5
17		CC7	0·875	0·875	13,060	1·150	17·16		−3·8	17·16		3·4
18		CC8	0·875	0·875	12,830	0·580	17·30		−3·0	16·54		−0·4

occurs uniformly over some length of the beam. This explains the lower than average yield stresses obtained for beams CC3, CC6 and CC7 (Table I, column 7). Roderick and Phillipps (1949) found that in their tests a satisfactory empirical allowance could be made for this effect by assuming that collapse was delayed until the full plastic moment had been reached at a section a distance away from the concentrated load equal to half the depth of the beam. The yield stresses for all the beams corresponding to this assumption are shown in column 10 of Table I, and the percentage variations from the mean values for separate bars are given in column 12.

There does not appear, from the figures given in columns 9 and 12 of Table I, to be any distinct advantage in accepting the complications introduced by Roderick and Phillipps. In either case the agreement is as good as could reasonably be expected, taking into account probable variations in yield stress in the bars. Ignoring signs, the mean values of the percentage variations given in columns 9 and 12 are 1·87 and 1·18 respectively. The application of the "t" test for the difference between means gives $t=1{\cdot}646$, corresponding to a probability of 0·12 that the difference between the means is due entirely to random causes. The improvement achieved with the second method of analysis, although discernible, is not therefore outstandingly significant. In their tests on simply supported beams, Roderick and Phillipps (1949) obtained much improved agreement by using this method, but it is to be noted that while these investigators tested carefully heat-treated beams, the tests here described were performed with the steel in the "as received" condition.

In a further attempt to decide between the two methods of analysis, tension tests were carried out on three specimens. Since the mean yield stresses given by the second method (column 11, Table I) are lower than those given by the first (column 8), it should thus be possible to reach some significant conclusion. The first two specimens (CT1 and CT2) were taken one from each end of beam C8, while the third (CT3) was taken from one end of beam C9. The specimens had a gauge length of 2·00 in. and a diameter of 0·282 in., and were tested in the Quinney Autographic Machine (see Quinney, 1938). The upper and lower yield stresses and the rates of strain in the plastic range are given in Table II. Calculations show that, during the

TABLE II

Tension Specimen	Upper Yield Stress, tons/in.^{2}	Lower Yield Stress, tons/in.^{2}	Rate of Strain in Plastic Range/sec. $10^{-6}\times$
CT1	22·48	17·99	18·20
CT2	19·53	17·32	0·767
CT3	22·31	18·16	18·20

beam tests, the mean rate of strain in the extreme fibres of the most highly stressed sections varied between $0{\cdot}7\times10^{-6}$ and $2{\cdot}0\times10^{-6}$ per sec. Hence the appropriate lower yield stress for bar 4 (see fig. 1) would be about 17·40 tons/in.^{2} Since the values obtained by the two methods of analysis were 17·84 and 16·60 tons/in.^{2} (columns 8 and 11 of Table I), the result is again inconclusive.

As an example of the load-deflection curves obtained, those for beams C1 and C2 are presented in figs. 5 and 6 respectively. In the case of beam C2, a theoretical load-deflection curve for dial gauges 3 and 5 has been calculated by means of the simple plastic theory, and is seen to be in good agreement with the observed values.

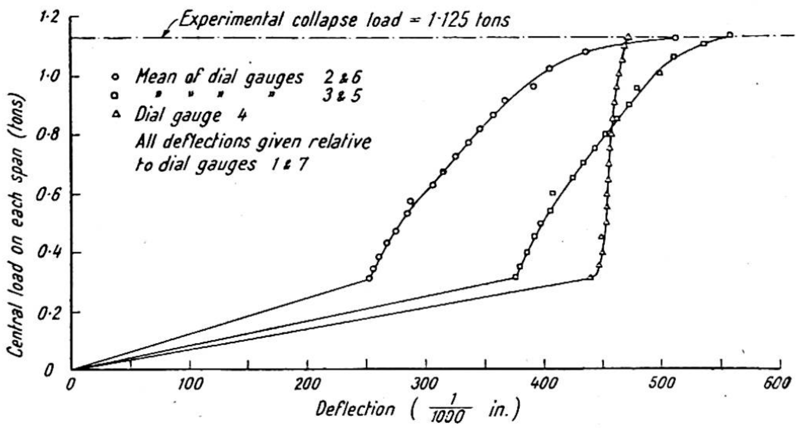

Fig. 5. Load-deflection curves for Beam C1

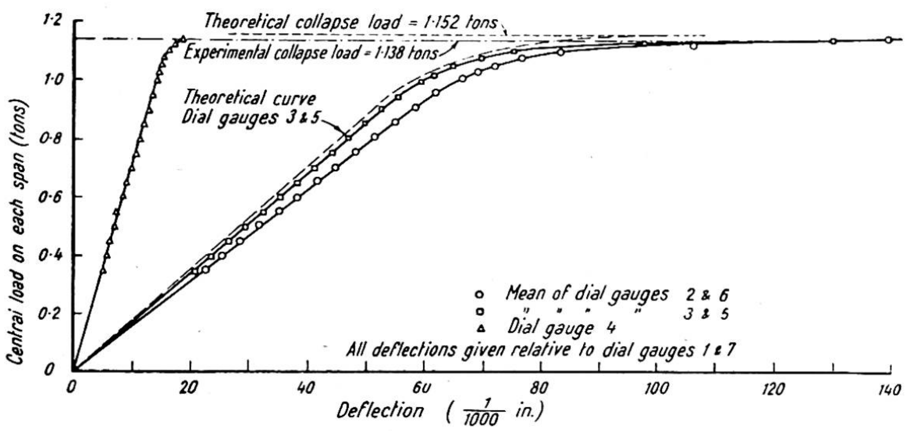

Fig. 6. Load-deflection curves for beam C2

In testing beams C1, C4, C8 and C10, the central support was set at such a distance below the outer supports that yield stress under a sagging bending moment was reached in the extreme fibres of the central section of the beam before contact occurred. As the loads on these beams were further increased, the central bending moment first decreased to zero and then increased until at collapse full plasticity under a hogging bending moment occurred over the central support. Simultaneously the position of the maximum sagging moment moved along the beam, as can be seen most clearly in the cases of beams C4 and C8. Thus in beam C4 (see fig. 2), a certain amount of yield under sagging moment occurred first at C and E, and finally at B and F, where full plastic moments developed at collapse. This may be traced in the appearance of Lüders' wedges on the side face of part of beam C4 after testing (fig. 7), in contrast to the absence of such wedges except at B and D on the face of beam C3, for which the supports were initially level. It will be observed from Table I that the sinking of the support and the occurrence of Lüders' wedges along the beam did not lead to any significant decrease in the carrying capacity of beam C4 as compared with beam C3. Similar remarks apply to beam C8, in which the maximum sagging moment moved first to sections E and G (see fig. 2), then to sections D and H, until finally full plastic moments were reached at collapse at

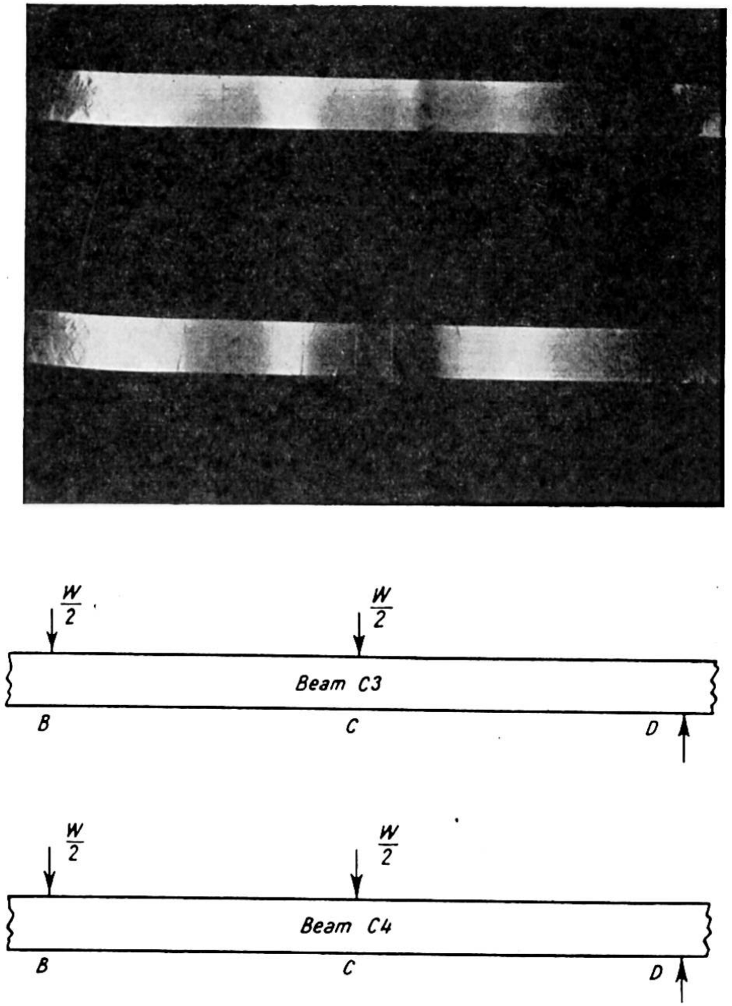

Fig. 7. Comparison of Lüders' wedges on beams C3 and C4

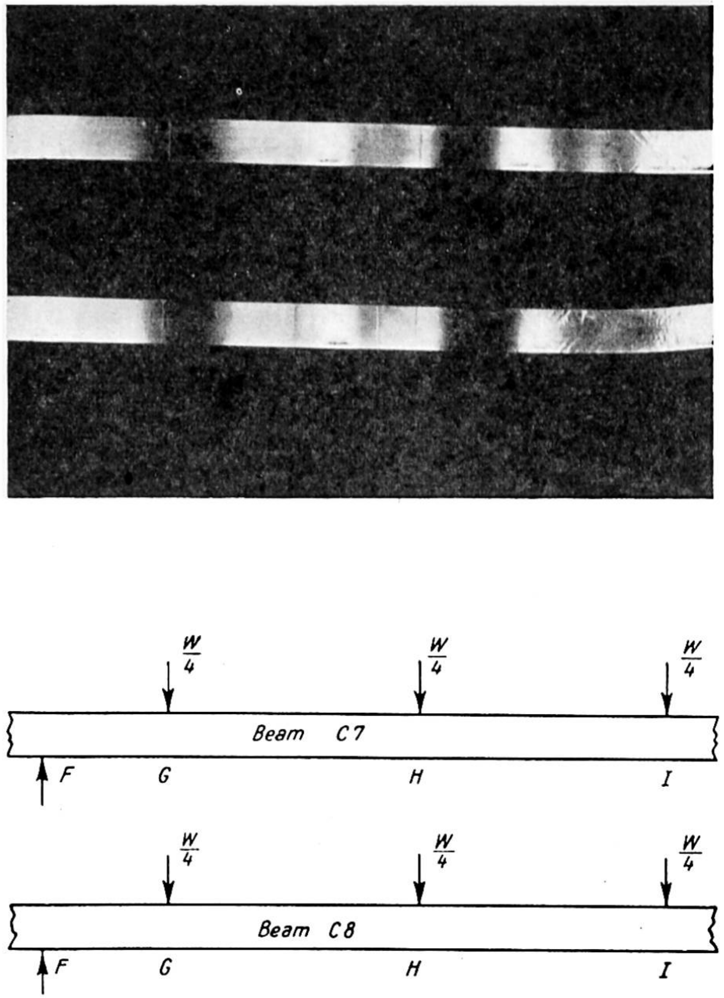

Fig. 8. Comparison of Lüders' wedges on beams C7 and C8

sections C and I. The side faces of parts of beams C7 and C8 after testing are compared in fig. 8. The theoretically deduced values of the moments at various sections of beam C8 at all stages of loading up to the collapse load are shown in fig. 9, and the progressive movement of the positions of the maximum sagging moments is apparent.

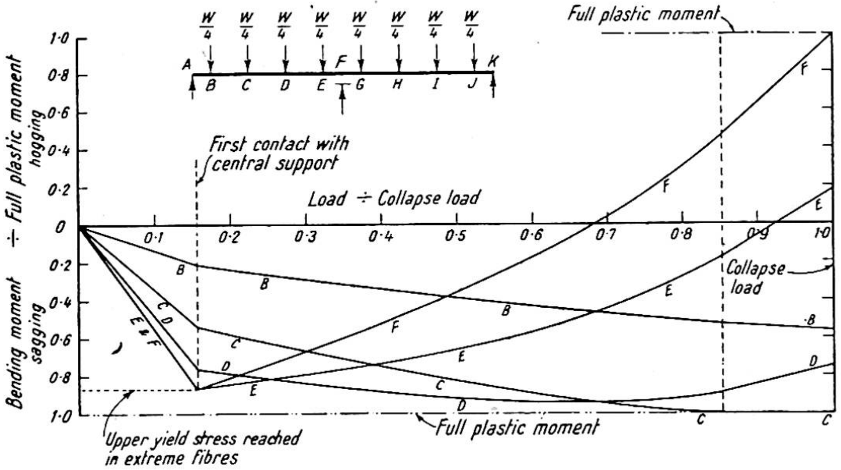

Fig. 9. Theoretical bending moment curves for beam C8

In test C5, the supports were at the same level, and equal loads of 0·50 tons were applied to each span. The load on span DG (fig. 2) was then kept constant while that on span AD was increased until collapse occurred at 1·20 tons. Finally the load on span DG was increased until this part of the beam also failed at a load of 1·25 tons.

3. Tests on fixed-ended beams

(a) *Preparation of beams*

The beams were all prepared from the same black mild-steel plate (dimensions 17 in. × 2 in. × $\frac{1}{4}$ in.) by cutting longitudinally (in the direction of rolling) into strips. The small size of the beams ($\frac{1}{4}$ in. × $\frac{1}{4}$ in. section) made it desirable to anneal at 900° C. and cool in air in order to reduce some of the effects of rolling and work-hardening. The beams were bent about axes perpendicular to the plane of the original plate.

(b) *Description of tests*

The tests are summarised in fig. 10. The beams E1–6 were tested over a span of 6·0 in. between end fittings which provided moments of resistance proportional to the rotations of the end sections of the beam. If a moment M lb. in. at the end of a beam corresponded to a rotation of θ radians, then $\theta = KM$ where K had the values for each beam given in the second column of fig. 10. The simply supported beams EC1 and EC2 had a span of 4·0 in. Fig. 10 shows the positions of the dial gauge used to measure deflections and of the mirror gauges used to measure rotations.

Tests E1, E2 and E3 were conducted to investigate the effect of various degrees of end fixity. Beams E4, E5 and E6 were subjected to loads at several sections (1, 2, 3 in fig. 10) in turn, each load being just sufficient according to the simple theory, to bring about collapse.

The arrangement for testing those beams which had the highest degree of end

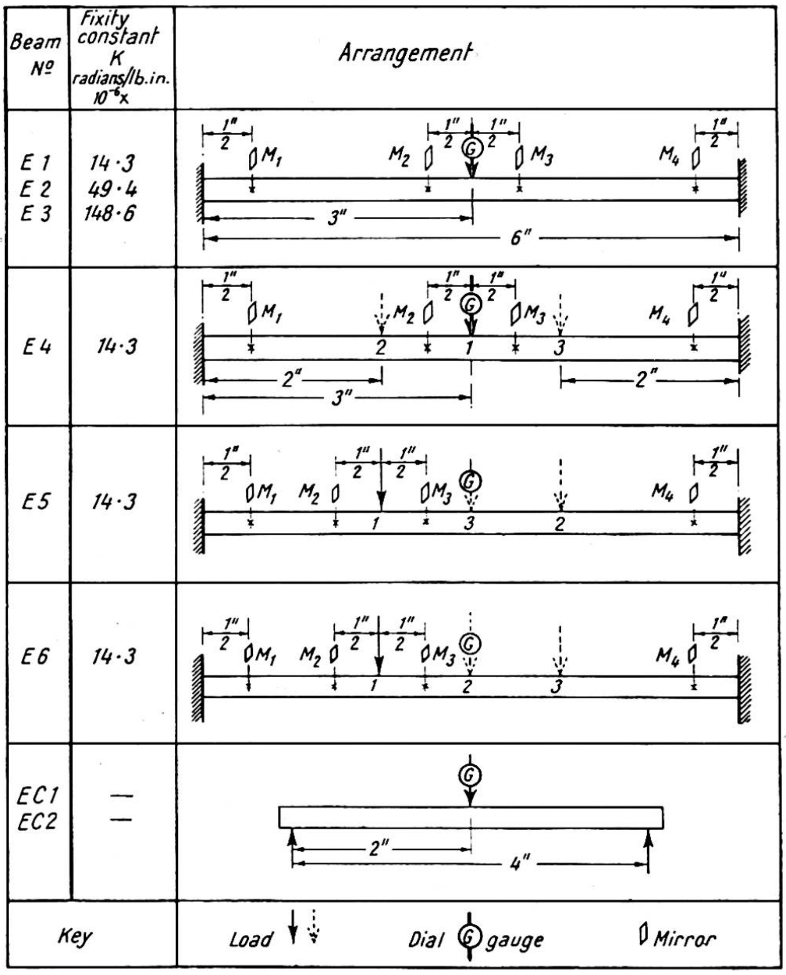

Fig. 10. Summary of tests on fixed-ended beams

fixity (beams E1, E4, E5 and E6) is shown in fig. 11, the load being applied by a chain acting through a yoke. The arrangement for testing beams E2 and E3 is shown in fig. 12, the clamping blocks on the end fittings having been removed for the sake of clarity.

During all the tests, load increments were made at approximately two-minute intervals until creep was first observed. Before each subsequent increment, the rate of creep on the dial gauge was allowed to drop to 10^{-4} in. during any two-minute interval.

(c) *Test results*

Beams E1, E2, E3, EC1 and EC2

The results are summarised in Table III, and columns 1 to 4 require no explanation. The end-fixity constants for the partially fixed-ended beams are given in column 5, from which it is possible to calculate the theoretical ratio of end to central moments for a central point load in the elastic range (column 6). The collapse loads are given in column 7, from which the lower yield stresses may be calculated by means of the simple plastic theory (see Table III, column 7). The percentage differences from the mean are given in column 10.

On the basis of the method suggested by Roderick and Phillipps for allowing for load concentration, these same collapse loads give the yield stresses shown in

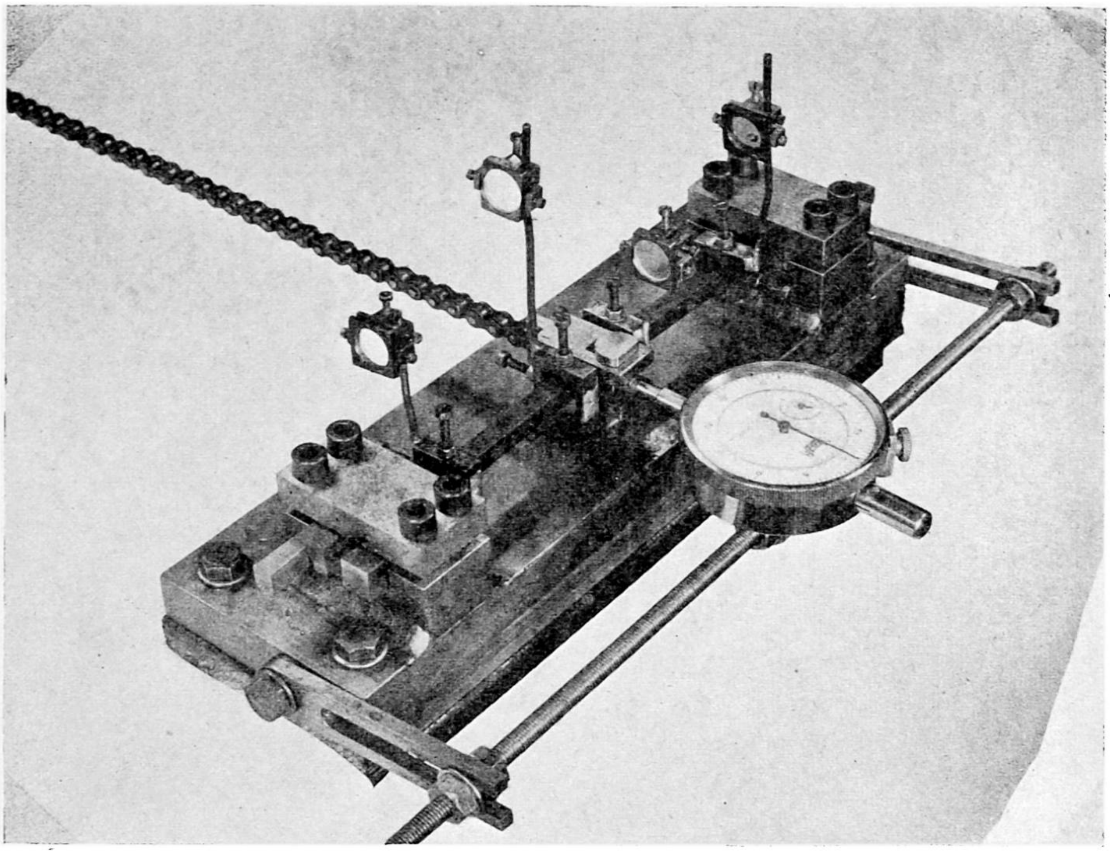

Fig. 11. Arrangement for testing fixed-ended beams

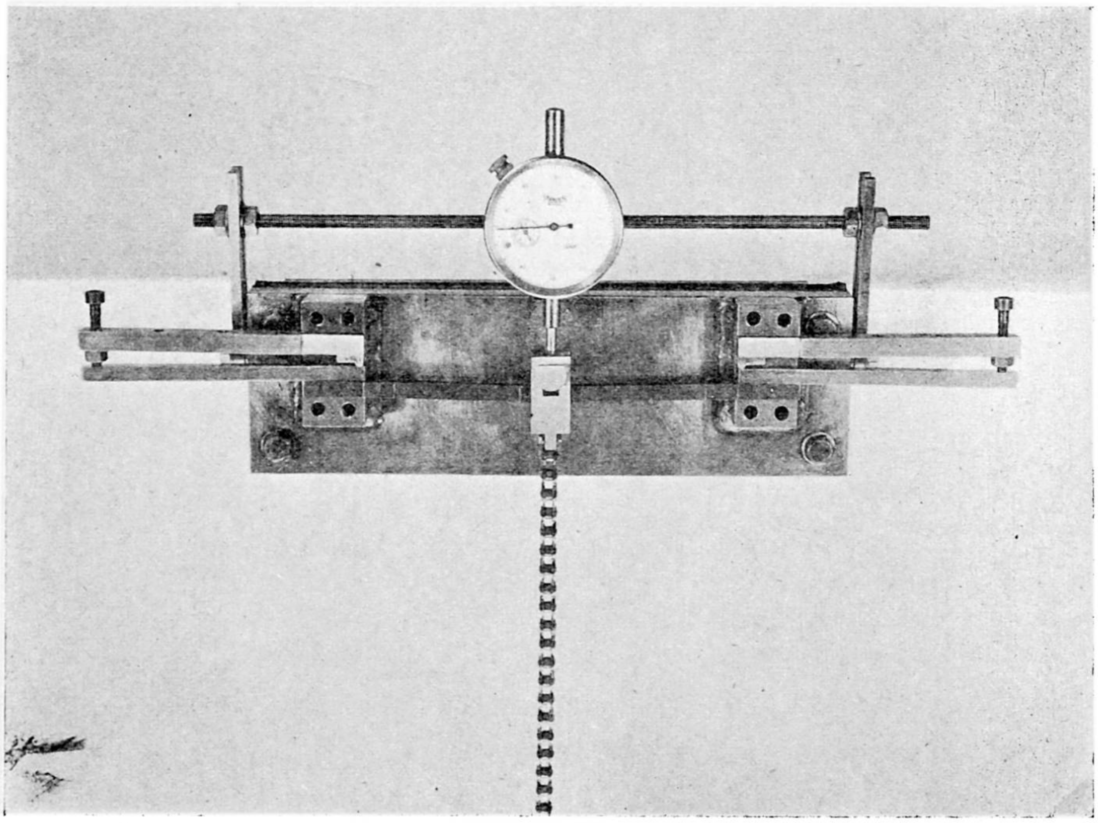

Fig. 12. Method of obtaining reduced end fixity

TABLE III

	1	2	3	4	5	6	7	8	9	10	11	12	13
								Analysis by simple plastic theory ignoring the effect of load concentration			Analysis in which allowance is made for the effect of load concentration		
	Beam No.	Mean Width, in.	Mean Depth, in.	Estimated Modulus of Elasticity E, tons/in.^{2}	End Fixity Constant K, radians/lb. in. $10^{-6}\times$	Ratio of End Moment to Central Moment in Elastic Range	Collapse Load, lb.	Lower Yield Stress, tons/in.^{2}	Mean Lower Yield Stress, tons/in.^{2}	Per cent Difference	Lower Yield Stress, tons/in.^{2}	Mean Lower Yield Stress, tons/in.^{2}	Per cent Difference
1	E1	0·254	0·253	13,440	14·3	0·912	258	21·3		−3·4	20·4		−2·6
2	E2	0·249	0·255	13,440	49·4	0·746	263	21·8		−1·2	20·9		−0·3
3	E3	0·250	0·256	13,440	148·6	0·490	269	22·0	22·1	−0·3	21·1	21·0	0·7
4	EC1	0·246	0·252	13,140	—	—	199	22·8		3·4	21·4		2·2
5	EC2	0·247	0·250	13,730	—	—	194	22·4		1·5	21·0		0

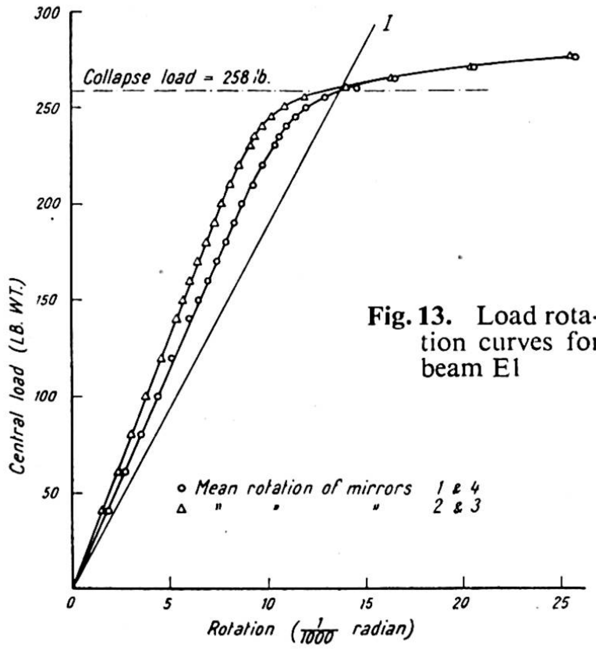

Fig. 13. Load rotation curves for beam E1

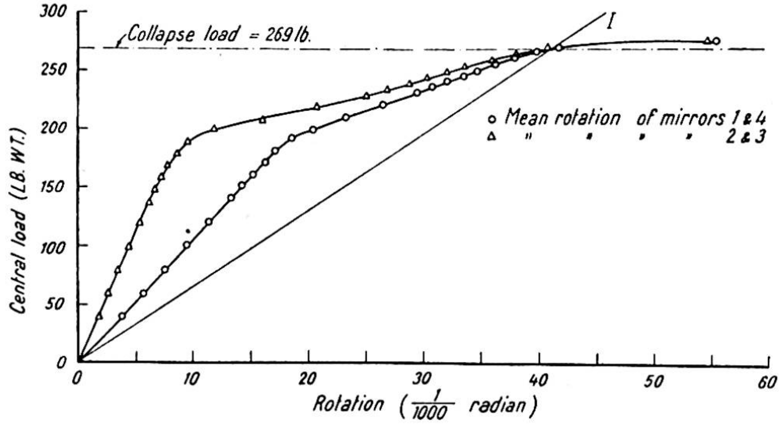

Fig. 14. Load rotation curves for beam E3

column 11, and the percentage differences from the mean are given in column 13. There is a certain improvement in the agreement between the yield stress values as compared with those given in column 8.

Load rotation curves are given for beams E1 and E3 in figs. 13 and 14 respectively. These curves do not indicate such definite collapse loads as obtained for the $\frac{7}{8}$-in. square beams described above. This may be due to strain hardening, and in order to obtain a consistent interpretation of test results, the collapse loads have been determined as follows.

Taking the value for the modulus of elasticity given in column 4 of Table III, and assuming some value for the collapse load, it is possible to calculate by means of the simple plastic theory of bending the rotation at any section of the beam when it is just about to collapse. Then the relationship between the assumed collapse load and the rotation is obtained as a straight line OI (figs. 13 and 14), and the collapse load is taken as the intersection of this line with the experimental load rotation curve. The figure quoted for any beam in column 7 of Table III is the mean of the three values obtained from the central deflection and the two pairs of mirrors (M_1, M_4 and M_2, M_3).

In the case of beam E1, the end moments were in the elastic range almost equal to the central moments (see column 6 of Table III), and the full plastic moment was reached at all three sections at practically the same load. With beam E3, however, the end moments were in the elastic range less than half the central moment, and the load rotation curves (fig. 14) indicate that full plastic moment was reached at the centre at a load of about 200 lb. Thereafter the rotations increased almost linearly with load up to 255 lb., soon after which full plastic moments developed at the ends and collapse occurred.

Beams E4, E5 and E6

The results for beams E4, E5 and E6 are summarised in Tables IV and V. Sufficient load was applied successively to the three loading positions (see fig. 10) to produce full plastic moments at the ends and under the load. Values of the lower yield stress calculated on the basis of the simple plastic theory are given in column 8 of Table IV.

TABLE IV

	1	2	3	4	5	6	7	8
	Beam No.	Mean Width, in.	Mean Depth, in.	Estimated Modulus of Elasticity, E, tons/in.^{2}	End Fixity Constant K, radians/lb. in. 10^{-6} ×	Order of Loading Positions	Maximum Load Actually Applied: Load, lb.	Maximum Load Actually Applied: Corresponding Lower Yield Stress, tons/in.^{2}
1	E4	0·249	0·255	13,440	14·3	C	260·0	21·5
2						D	292·5	21·5
3						E	292·5	21·5
4	E5	0·248	0·253	13,440	14·3	D	287·5	21·6
5						E	287·5	21·6
6						C	255·6	21·6
7	E6	0·247	0·254	13,440	14·3	D	285·0	21·3
8						C	253·3	21·3
9						E	285·0	21·3

TABLE V

	1	2	3	4	5	6	7	8	9
	Beam No.	Quantity	Unit	1st Load Position		2nd Load Position		3rd Load Position	
				Observed Maximum	Calculated Value at Collapse	Observed Maximum	Calculated Value at Collapse	Observed Maximum	Calculated Value at Collapse
1	E4	Central deflection	in. $\times 10^{-3}$	60·0	45·5	66·1	87·2	89·7	100·0
2		Rotation θ_1	radians $\times 10^{-3}$	18·1	13·6	30·9	46·4	28·7	31·6
3		,, θ_2	,,	18·8	13·6	22·7	39·4	10·1	15·0
4		,, θ_3	,,	17·5	13·6	4·0	15·0	29·7	39·4
5		,, θ_4	,,	18·4	13·6	22·0	22·9	46·5	53·5
6	E5	Central deflection	in. $\times 10^{-3}$	50·1	63·1	71·3	83·7	83·5	86·3
7		Rotation θ_1	radians $\times 10^{-3}$	29·2	39·6	26·8	29·7	32·8	32·5
8		,, θ_2	,,	31·4	39·6	7·5	15·1	11·3	13·7
9		,, θ_3	,,	9·3	15·1	27·1	39·6	12·5	13·7
10		,, θ_4	,,	12·2	15·1	35·6	44·1	33·9	35·2
11	E6	Central deflection	in. $\times 10^{-3}$	50·3	62·1	66·2	65·8	77·9	94·2
12		Rotation θ_1	radians $\times 10^{-3}$	28·1	39·0	27·4	27·6	27·2	31·5
13		,, θ_2	,,	31·4	39·0	14·3	13·5	7·8	14·9
14		,, θ_3	,,	10·0	14·9	11·0	13·5	25·4	39·0
15		,, θ_4	,,	11·7	14·9	20·5	17·8	36·5	49·1

It is possible by means of the simple plastic theory to calculate the theoretical deflections and rotations at collapse for the various positions of the load. These calculated values are compared with those observed in Table V. The rotations θ_1, θ_2, θ_3 and θ_4 refer respectively to mirrors M_1, M_2, M_3 and M_4. Except for the first loading position practically all the observed deflections and rotations are less than the calculated values. Hence the ability of a beam to sustain a given ultimate load is not adversely affected by the attainment of the full plastic moment at various sections due to other critical load distributions. This is true whatever the order in which the loads are applied.

Tension tests

Tension tests were performed on four specimens, of diameter 0·178 in. and gauge length 0·70 in., in a Hounsfield Tensometer. Specimens ET1 and ET2 were cut from the ends of beam E2 after testing, and specimens ET3 and ET4 were cut from the ends of beam E6. The upper and lower yield stresses obtained are given in Table VI.

TABLE VI

Tension Specimen	Upper Yield Stress, tons/in.^{2}	Lower Yield Stress, tons/in.^{2}
ET1	21·68	20·88
ET2	21·57	20·57
ET3	22·70	20·56
ET4	21·08	20·17

The lower yield stresses are in good agreement with each other, and have a mean value of 20·54 tons/in.^{2} Considering beams E1, E2, E3, EC1 and EC2 (see Table III), the method of analysis suggested by Roderick and Phillipps gives a mean yield stress in closer agreement with the yield stress from the tension tests than is obtained when the simple plastic theory is applied.

4. CONCLUSIONS

The general agreement between the values of the lower yield stress calculated from the collapse loads for both the continuous and the fixed-ended beams is satisfactory and shows that the simple plastic theory gives predictions of the collapse loads of such beams with sufficient accuracy for practical purposes. The method of allowing for stress concentration suggested by Roderick and Phillipps (1949) does not lead to any distinct improvement for the continuous beams, but does lead to slightly better agreement for the fixed-ended beams. The tension tests carried out in connexion with the continuous beams did not establish any conclusive results, but with the fixed-ended beams tension tests favoured the method of Roderick and Phillipps.

The tests on the continuous beams confirm that the predictions of the plastic theory are not upset by sinking of supports, even if sinking is sufficient to cause yield in the beam. The plastic theory is equally successful for all the load distributions investigated, and the failure of one span does not decrease the ultimate carrying capacity of an adjacent span.

The tests on the fixed-ended beams show that ultimate carrying capacity is independent of the degree of rigidity of the end connections as long as these are capable of resisting the full plastic moment. The carrying capacity is not adversely affected when full plastic moments are produced at a number of sections by different successive load distributions, and this is true whatever the order in which the loads are applied.

The work described in this paper was carried out at the Engineering Laboratory, Cambridge University, and forms part of a general investigation into the behaviour of rigid-frame structures under the direction of Professor J. F. Baker, Head of the Department of Engineering.

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Summary

According to the simple plastic theory, the collapse loads of mild-steel continuous and fixed-ended beams may be calculated by considering merely the requirements of equilibrium in relation to the external loads and the full plastic moments of resistance of the beams. It follows that sinking of supports, order of loading and degree of end fixity should have no influence on such collapse loads. In order to check these deductions, tests were performed on $\frac{7}{8}$-in. square beams continuous over two spans and on $\frac{1}{4}$-in. square single-span beams provided with varying degrees of end fixity. The influence of various types of loading and of varying orders of application of the loads were investigated. Control tests were performed on similar simply supported members, and tension tests carried out at controlled rates of strain on material taken from unyielded sections of the beams.

The results give consistent confirmation of the simple plastic theory, and show conclusively that the collapse loads may be calculated with sufficient accuracy for practical purposes by this means. During the loading of a continuous beam in which one support is initially lower than the others, there is, according to the simple plastic theory, a progressive movement of the sections of maximum sagging moments along the beam. This is demonstrated in the tests by the appearance of Lüders' wedges on the polished surfaces of the $\frac{7}{8}$-in. square beams.

Résumé

Suivant la théorie simple de la plasticité, les charges de rupture des poutres en acier doux, continues ou encastrées à leurs extrémités, peuvent être calculées par simple considération des exigences d'équilibre corrélativement aux charges extérieures et aux pleins moments plastiques de résistance des poutres. Il en résulte que l'affaissement des appuis, l'ordre de mise en charge et le degré de rigidité aux extrémités ne doivent exercer aucune influence sur ces charges de rupture. Pour vérifier ces déductions, des essais ont été effectués sur des poutres carrées de $\frac{7}{8}$ in. (22,2 mm.), continues sur deux portées, ainsi que sur des poutres carrées de $\frac{1}{4}$ in. (6,35 mm.) sur portée simple, avec différents degrés de rigidité aux extrémités. On a étudié l'influence de divers types de charges et de divers ordres de mise en charge. Des essais ont été effectués, à titre de contrôle, sur des éléments simplement posés sur leur appuis; on a également procédé à des essais de traction, sous des taux de tension contrôlés, sur des éprouvettes prélevées sur des sections n'ayant subi aucune déformation.

Les résultats obtenus fournissent une bonne confirmation de la théorie simple de la plasticité et montrent d'une manière concluante que les charges de rupture peuvent être calculées avec une précision suffisante pour les besoins de la pratique, d'après la méthode ci-dessus. Au cours de la mise en charge d'une poutre continue dont un appui est initialement plus bas que les autres, il se produit, suivant la théorie simple de la plasticité, un déplacement progressif des sections présentant les moments maxima d'affaissement, le long de la poutre. Ceci est mis en évidence, au cours des essais, par l'apparition de figures de Luders sur les surfaces polies des poutres carrées de $\frac{7}{8}$ in.

Zusammenfassung

Nach der einfachen Plastizitätstheorie können die Bruchlasten von durchlaufenden und eingespannten Balken aus Flusstahl allein aus der Betrachtung der Gleichgewichtsbedingungen bezüglich der äusseren Lasten und der vollen plastischen Widerstandsmomente der Balken berechnet werden. Es folgt daraus, dass Auflagersenkungen, Lastanordnung und Einspannungsgrad keinen Einfluss auf solche Bruchlasten haben sollten. Zur Ueberprüfung dieser Feststellungen wurden Versuche an über zwei Felder durchlaufenden, $\frac{7}{8}$ in. (22,2 mm.) starken und an einfeldrigen, verschieden stark eingespannten, $\frac{1}{4}$ in. (6,35 mm.) starken Rechteck-Balken durchgeführt. Die Einflüsse verschiedener Arten von Lasten und verschiedener Formen der Last-Aufbringung wurden untersucht. Zur Kontrolle wurden Untersuchungen an entsprechenden einfach gelagerten Balken gemacht und unter kontrollierten Spannungen Zugversuche an Material aus unverformten Trägerteilen ausgeführt.

Die ermittelten Resultate bedeuten eine gute Bestätigung der einfachen Plastizitätstheorie und zeigen überzeugend, dass die Bruchlasten mit für praktische Bedürfnisse genügender Genauigkeit nach dieser Methode berechnet werden können. Während der Belastung eines durchlaufenden Balkens, bei dem ein Auflager von Anfang an tiefer liegt als die anderen, ergibt sich, in Uebereinstimmung mit der einfachen Plastizitätstheorie, entlang dem Balken ein fortlaufendes Fliessen der Zonen grösster Momentenbeanspruchung infolge Einsenkung. Dies zeigt sich im Versuch durch das Auftreten von Fliessfiguren von Lüders auf den polierten Oberflächen der $\frac{7}{8}$ in. Rechteckbalken.