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IVa

Calcul des dalles champignons

Berechnung von Pilzdecken in Eisenbeton

Calculating flat slabs of reinforced concrete

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Designing a flat slab is, if the theory of elasticity be strictly adhered to, a comparatively difficult problem. It has been treated, among others by Dr. Lewe (*Pilzdecken und andere trägerlose Eisenbetonplatten*, Berlin, 1926). The question, however, usually concerns designing reinforced concrete, for which reason the problem does not necessarily have to be related to the theory of elasticity. The calculation of stresses according to the theory of elasticity always describes a system in equilibrium. It is of value from that viewpoint. Equilibrium can, however, be attained in an infinity of different ways and as regards reinforced concrete the actual materials qualities give great freedom for the shaping of a structure. When applying the foregoing, the theory of elasticity can either be entirely departed from, or also be modified in its traditional form, e.g., by basing the calculation on the assumption of aeolotropy to such a degree that the intended goal be attained. In spite of all this, the shaping must always be finally determined by actual observations on structures or models, representative of the building to be carried out, or of certain elements of the latter. This treatise will, from these general premises, attempt to give simple rules for determining designing moments in a flat slab and to state principles of shaping based on tests on models. Here it should be mentioned that available observations are deplorably scarce. Tests of ready flat slabs have certainly been carried out to a large extent. To find something essential in superficial accounts of isolated tests is, however, almost impossible.

However the problem be approached, it is obvious that for a flat slab with square panels and infinite extent the areas around the columns are characterised by nearly cylindrical symmetry, and that much of it will remain even with reasonable modifications in extent and shape. Since the areas around the columns are in ordinary cases of loading most affected, I have chosen them for special study and attempted to represent them in simple models, consisting of circular plates with cylindrically symmetrical load. The test series thus carried out aimed at giving rules for the reinforcement for moments. They will be described below. Concerning the sometimes still more important question of how shearing forces should be considered, the result is shown here of an investigation, published in *Betong*, No. 2, Stockholm 1946 by Forssell and Holmberg. It showed that at a circular capital with radius a the total shearing force was allowed to reach $P = 2.72\ (2a + h)h \cdot \tau_{beam}$ where h is the plate's total depth and τ_{beam} the shearing stress, allowable in a beam of the same concrete, and that in a corresponding manner at a square capital with the side $2a$ the total shearing force was allowed to reach $P = 0.87(\pi h + 8a)h \cdot \tau_{beam}$. Greater shearing forces than these demand reinforcement for shear with, in the present situation, a highly uncertain behaviour. An investigation by Graf published in 1938 by *Deutscher Ausschuss für Eisenbeton* (Heft 88) gives, after elimination of an error in calculation, some reason for the following rule : The reinforcement for shear shall be placed in a 45° direction. Its total area shall be $0.8\ P/\sigma_s$ where P is the shearing force and σ_s allowable stress in reinforcement. P may, however, not exceed by more than 50 % the value named by Forssell, Holmberg.

Tests with bent plates comprised six models in reinforced concrete. They were all shaped as circular plates with total diameter = 272 cm and total depth $h = 8$ cm. Effective depth (h_{eff}) for the both layers of reinforcement was 6.6 cm and 5.8 cm respectively. For reinforcement 8 mm plain bars with yield point 3 360 kg/cm^2 and ultimate stress 4 620 kg/cm^2 were used. Concrete used had in cubes (15 cm)^3 a compressive strength of 442 kg/cm^2. The tensile strength, found in prisms with a cross-section (15 cm)^2 was 27 kg/cm^2. The tensile strength, in bending tested on beams with cross-section 15 cm $\times$ 8 cm was $6\ M/15 \times 8^2 = 42$ kg/cm^2. The reinforced concrete's qualities were tested in six beams with total length = 250 cm, total depth, $h = 8$ cm, effective depth, $h_{eff} = 6.6$ cm and with varying breadth. Each of them was reinforced with three plain bars with equal spacing = 1/3 of beam's breadth. When tested they were placed with a span length of 231 cm and subjected to two symmetrical and equal concentrated forces with spacing 77 cm. Loading followed, as regards speed and magnitude of stresses, the same scheme as at testing of plates. Observations referred to rigidity and ultimate load. Rigidity D was determined by deflection measurements at three points between the loaded points. Values observed were set in relation to $D_{max} = 350\ 000\ h^3/12$. They are shown in fig. 1, where $k_b = 6\ m/h^2_{eff}$ and μ is the ratio of reinforcement in per cent. m is the intensity of moments. $k_{b\max}$ applies to maximum load. The figure's general shape and the result's dispersion are ordinary. With higher stresses, there appears the important circumstance that D increases at the same time as μ. In fig. 2 are shown the ultimate loads expressed in $k_{b\max}$ and set in relation to μ. There are also shown equivalent inter-relations when the ultimate loads are calculated on the assumption that the concrete compressive stress is constant = 442 kg/cm^2 and the steel stress = 3 360 kg/cm^2 and 4 620 kg/cm^2 respectively. The figure finally gives the supposed inter-relations taken as a basis for designing the plates. For the high values of the ultimate loads the explanation given by Jensen in *Bull.* No. 345 from Engineering Experiment Station, Illinois, is in general accepted, despite the fact that this is founded on the assumption of plain cross-sections, which is usually incorrect for reinforced concrete, where

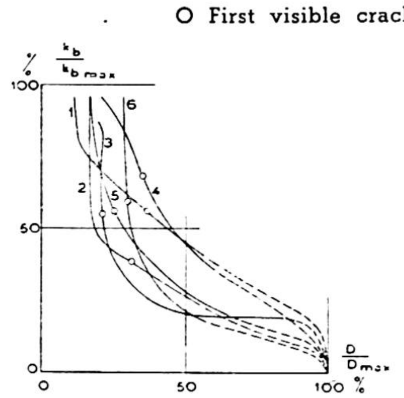

Fig. 1.

Plates I-III : 1 : $\mu = 0.384$ %
2 : $\mu = 0.771$ %
3 : $\mu = 1.149$ %.
Plates IV-VI : 4 : $\mu = 0.467$ %
5 : $\mu = 0.786$ %
6 : $\mu = 1.572$ %.

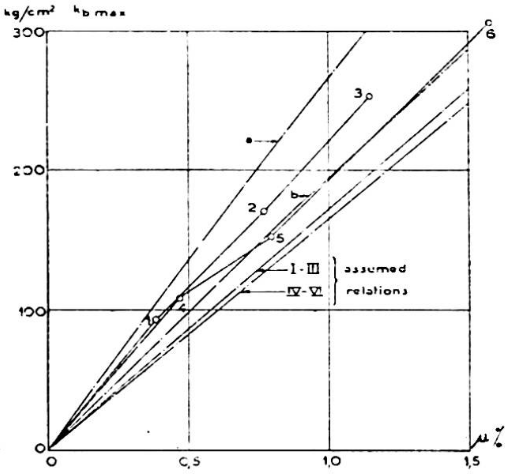

Fig. 2.

Curve a. Maximum value calculated from cube compressive strength of concrete and ultimate stress of reinforcement bars (σ = 442 kg/cm2 and σ = 4 620 kg/cm2 resp.).

Curve b. Maximum value calculated from cube compressive strength of concrete and yield point stress of reinforcement bars (σ = 442 kg/cm2 and σ = 3 360 kg/cm2 resp.).

the cross-section's steel part moves in relation to its concrete part. The dispersion observed for the beams must also be expected for the plates. The accuracy of the main tests is affected thereby.

The plates of the main tests were supported on a circle with a radius $a = 264$ cm and were centrally loaded on a circle with radius $b = 88$ cm. This does not entirely correspond to a flat slab, where the plate is monolithically joined to a capital, but was chosen for greater simplicity. The consequence should be that ultimate loads are correct but rigidities too small. On choosing the reinforcement arrangement consideration was given to the fact, shown by me in I. A. B. S. E., Publ. IX, that only conditions of equilibrium are satisfied, the amount of reinforcement needed being independent of material qualities. With very small differences in the total amount of reinforcement the plates I, III, V and VI, were designed as cylindrically aeolotropic plates with interrelation $n = D_r/D_\theta$ between the flexural rigidity in radial and circumferential directions = 1; 9/16; ∞ and 0, respectively; plate II was designed according to Johansen's method for determining ultimate loads (*Brudlinieteorier*, Copenhagen 1943) and plate IV was given square net reinforcement of bars with 10 cm spacing in the upper and lower layers. (The theory for plates with cylindrical aeolotropy has been developed by me in *Bygningsstatiske Meddelelser*, No. 3, Copenhagen 1948.) The models were designed for 1 100 kg dead load and for an effective load, P = 11 000 kg. With reference to observations on control beams, an effective load at the yield point of 12 500 to 14 500 could be expected. It was determined that the least number of radial bars, where these appear, should be eight. Ring bars were anchored by welding. Radial bars were welded to a steel plate in the centre. Ends of bars were anchored by plates 35 $\times$ 35 $\times$ 12 mm. Concerning plate IV a special investigation is needed of the probable yield point load. In general

$$P = \frac{2\pi b}{a-b}\left[(m_r)_b + \frac{1}{b}\int_b^a m_\theta\, dr\right].$$

$m_\theta = \xi (m_r)_b$ gives $P = 2\pi (m_r)_b\ (0.5 + \xi)$ where $0 \leqslant \xi \leqslant 1$ and ξ gives a measure for the exploitation of reinforcement between support and load. With this expression for P limiting values give 2 000 à 2 500 kg $< P <$ 8 500 à 10 000 kg in which due account is taken of the dead load. In the tests P was thus varied : 0⟶4 000⟶500⟶8 000⟶500⟶P_{max} at which the speed was such that P = 12 000 kg was reached after 280 min. Observations applied to cracks, deflections, yield point loads and maximum loads. Yield point loads in kilograms were for plates I-VI 13 000; 12 500; 12 500; 7 000; 18 000; and 10 500 respectively. From this plate IV receives $\xi \approx 0.7$. Concerning plate V it is noticed that, on the one hand, 18 000 is a value for P that is both high and difficult of explanation, and on the other hand that some welded joints broke at this load, with the result that the plate was never able to be finally tested. The ultimate loads (in kg) were for plates I-VI 18 200; 17 000; 16 800; 26 000; ? and 13 000 respectively. Ruptures occurred in plates I-III by shearing in cracks where radial bars ended, extending through the whole depth and caused by membrane stresses. Complete rupture did not on the whole occur in plate IV in spite of total deflection reaching 10.6 cm. Rupture in plate VI was caused by shearing round the loaded circle. Here from a structural viewpoint was an unsuitable region. The calculation postulated a discontinuity, which developed very suddenly, and gave rise to the low yield point load. If, as allowable load, there is chosen the lowest of 1/2 yield point load and 1/3 ultimate load there is obtained for plates I-VI 6 070; 5 670; 5 600; 3 500; 9 000 and 4 330, respectively. At the first loading to these values the total deflections for all plates were almost the same, namely 0.8 to 0.9 cm. The number of cracks was almost the same in all plates except in plate V where it was less. The first cracks were usually observed at an effective load of about 2 000 kg. Their width increased to a fairly similar degree for all plates except for IV and VI, which had the largest. Published details show that a crack width of 0.3 mm is the limit for the allowable in fairly heavily exposed structures. If it be assumed, as seems reasonable, that the final crack width is 50 % greater than that of the first loading 0.2 mm could be allowed for that case. With the allowable loads above proposed the average width for the five greatest cracks in plate V and VI was 0.35 and 0.23 respectively. This should not be allowed. Should the allowable loads with reference to the crack width be reduced there is obtained for plates V and VI 6 000 and 4 000 respectively. Determined in this manner the allowable load in kg divided by the weight of steel used in kg (kg/kg) is for plates I-VI 110; 105; 107; 60; 103 and 82, respectively. Thus compared, plates I; II; III and V are equivalent, and plates IV and VI inferior. The alternative shaping with rings + radial bars proved suitable. Radial bars only as in plate V were equivalent, but this involves considerable difficulty in manufacture. Plate IV seemed to show that reinforcement going from edge to edge gave great toughness. For the areas surrounding the columns in a flat slab these tests have thus given, as suitable reinforcement, rings + radial bars. Radial bars must be extended from the centre to the area's edge, by which means toughness can be expected. Too great a radial reinforcement must be avoided when taking into consideration manufacturing difficulties. Tests, with Professor Forssell's kind consent, have been carried

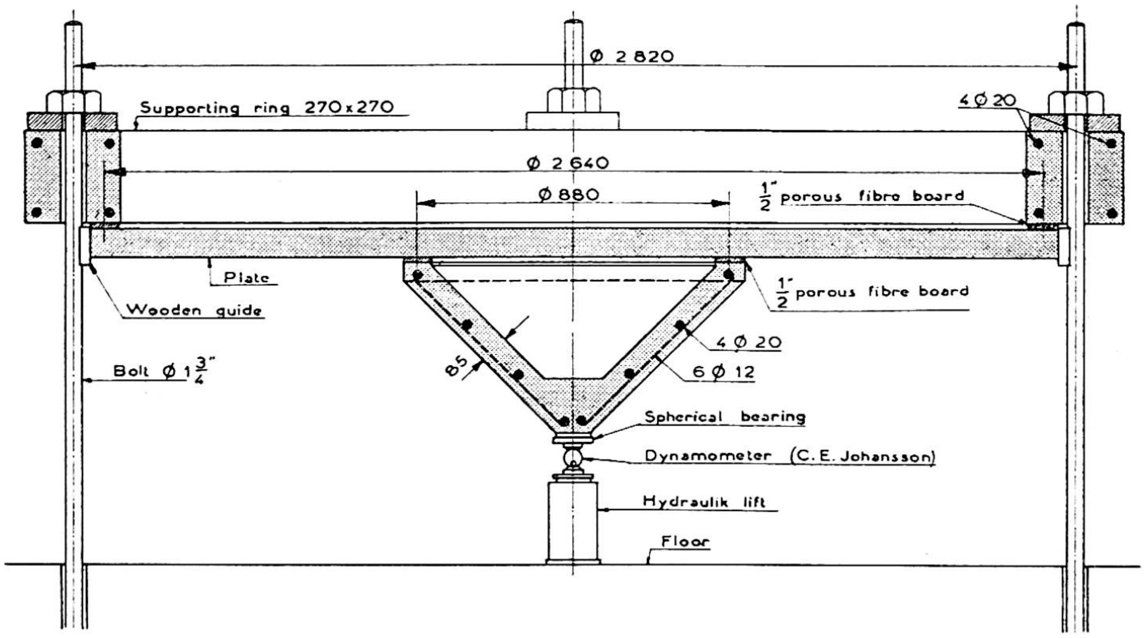

Fig. 3.

out at the Department for Structural Engineering at the Royal Technical University, Stockholm. Under my supervision plates I-III were tested by the then students K. Dierks and B. Pettersson as a graduation thesis, and in a corresponding manner plates IV-VI by the then students G. Orefelt and C. Sylvan. The test arrangement is shown in fig. 3.

Smulski published in *Engineering Record 1916* a report concerning an investigation similar in many respects. As far as the results can be found they are in agreement with mine. Attention is specially drawn to the fact that, in a series where radial reinforcement was varied from 36.2 % to 11.6 % of the total reinforcement, cracks were observed at the same part of the yield point load. This was a good illustration of reinforced concrete's great adaptibility to a chosen system for designing. Smulski's tests seem to show that, with reference to rigidity, radial reinforcement must not be less than 10 % of the total reinforcement. This is not contradicted by my investigation. When amounts of reinforcement are here compared, a correction is made for differences in effective depth.

Concerning flat slabs the question is posed as to how great an area with cylindrical symmetry should be chosen. With square equal panels with uniformly distributed load in all of them there arises, according to Lewe, $m_r = 0$ (polar system of coordinates with origo in the column's centre) roughly on a circle with radius $\beta \times l$ where l is the span-length and $\beta = 0.20$ to 0.25. From this is given the suitable order of magnitude. The capital is assumed circular with radius $\gamma_c l$. The intensity of the uniformly distributed load $= p$. The circumferential moment is written Cpl^2. The radial moment at the capital's edge is written $KCpl^2$. All radial bars are extended to $r = \beta l$. K is determined from the condition that radial reinforcement must not be less than 10 % of total reinforcement and that the reinforcement shall be given the highest possible exploitation. From that is derived as a reasonable value $K = 0.75$. $\beta = 0.250$ and $\gamma_c = 0.125$ then give an exploitation of abt. 99 %. In accordance with this, the thus calculated C is found in fig. 4. If the capital be not circular it is substituted with a circle with the same area. If the span-lengths is neighbouring panels are not equal the motive for the proposed arrangement ceases. 25 % deviation should however be acceptable. In such a case there is used in fig. 4 a value

$$l = 0{,}5\sqrt{(l_1')^2 + (l_1'')^2 + (l_2')^2 + (l_2'')^2}\,.$$

With reference to the fact that the next outermost line of columns takes up a greater part of the load on the outer panels than the outermost line of columns, the span-lengths of the outer panel in the calculations treated here are usually corrected with an increase of abt. 20 %. According to the given rule, the calculated radial reinforcement is enabled at load in every other panel to transfer to the column the moment $M = 2\ KC\gamma_c pl^3$ for which the column is designed. The average moment which is transferred to the neighbouring panel is as a result reduced by $m = M/l = 2\ KC\gamma_c pl^2$.

In continued designing the first requirement is a method for calculating the average moment in column strips and middle strips. In this connection it can be pointed out that a culculation founded on the theory for continuous beams with for each case sufficient accuracy gives the same average moment as Lewe's calculation. This circumstance can be used to advantage, whatever the method of designing may be. Lewe's moments are calculated for square capitals with edge $2\ \gamma_s l$. If the influence of the supports of the capitals' extent occurs according to the same rules with circular as well as with square capitals, there will apply in the calculation of the average moment $\gamma_s = 0.85\ \gamma_c$. In fig. 5 are shown for square, equal panels moments calculated on the one hand according to Lewe, on the other hand according to the theory for continuous beams. Curves indicating the numerical sum of the moments show the possibility of, for reasons of economy, exploiting reinforcement in one case of loading from another. This, with reasonable shaping, is most often scanty. In consideration of the risk of rupture, flat slabs are therefore as a rule excessively strong. (To abandon, therefore, as is the custom in U. S. A., the conditions of equilibrium, is nevertheless indefensible). Designing of exterior panels must be connected with designing of interior panels in the simplest manner.

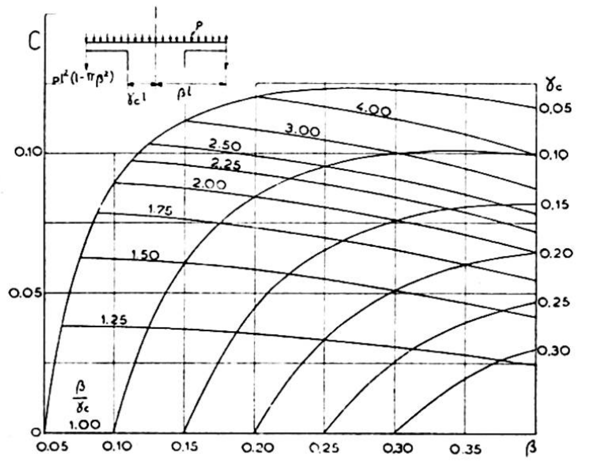

Fig. 4.

$$m_\theta = Cpl^3$$

$$C = \frac{(\beta - \gamma_c)(0.9540 - \beta^2 - \beta\gamma_c - \gamma_c^2)}{6\,[\beta - (1 - K)\,\gamma_c]}$$

$$K = 0{,}75$$

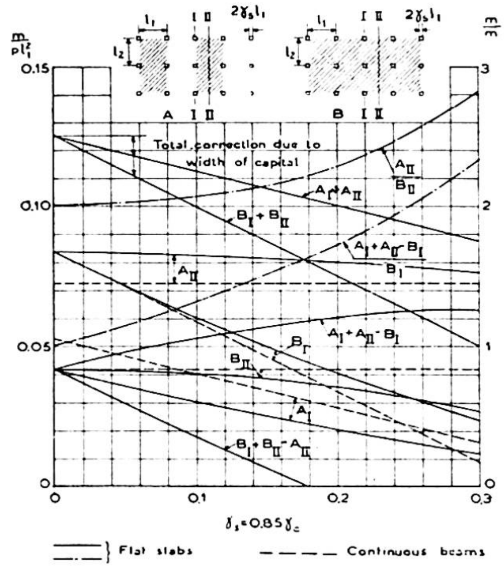

Fig. 5.

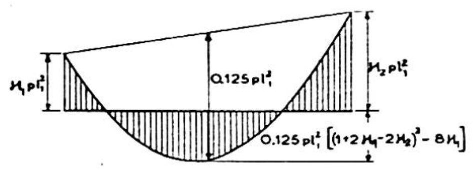

Fig. 6 (above).

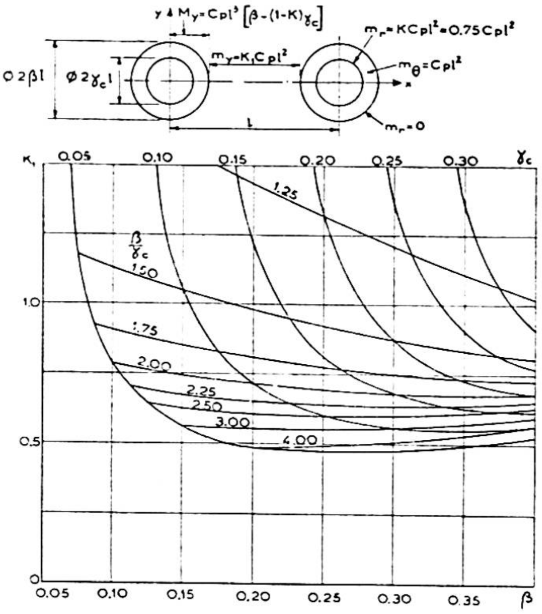

Fig. 7 (right).

In such cases the flat slab can be treated as infinitely extended up to the next outermost line of columns. The whole correction is placed on the moments of the exterior panels. It is carried out according to fig. 6. Moments of outer supports are determined by their moment-producing capacity. Careful economy usually requires that such exists. With different span-lengths for interior panels the moments can mostly be calculated according to fig. 5. For A_{II} no correction is needed. B_I is calculated with

$$l_1 = \sqrt{(l_1' - l_1'')^2 + l_1' l_1''}\ .$$

If B_{II} is determining, the figure's value is corrected with arithmetical means for the corrections at the support. After the average moment in the column strip has somehow been fixed there is obtained the moment between the columns as the difference between the total moment and the moments, which can be taken up in the areas at the columns. These latter, according to the method of consideration used, are at a column

$$M = 2\,Cpl^3[\beta - (l_1 - K)\gamma_c]\ .$$

The intensity of the remaining moments, calculated with reference to fig. 4, is shown in fig. 7. It would be unadvisable to accept greater values for K_1 than 0.75. As absolutely the greatest value, $K_1 = 1.00$ should apply.

The shaping of central parts of panels gives greater possibilities of variation. With an equally distributed load in equal square panels there arises cylindrical symmetry in the middle of the panel. It is then plausible to imagine a reinforcement of radial bars and rings here also. The need for this, however, has never been made clear by tests. Square net reinforcement is presumably no poor alternative. In such a case it is noticed that in the panel's centre no direction possesses special qualities; therefore the

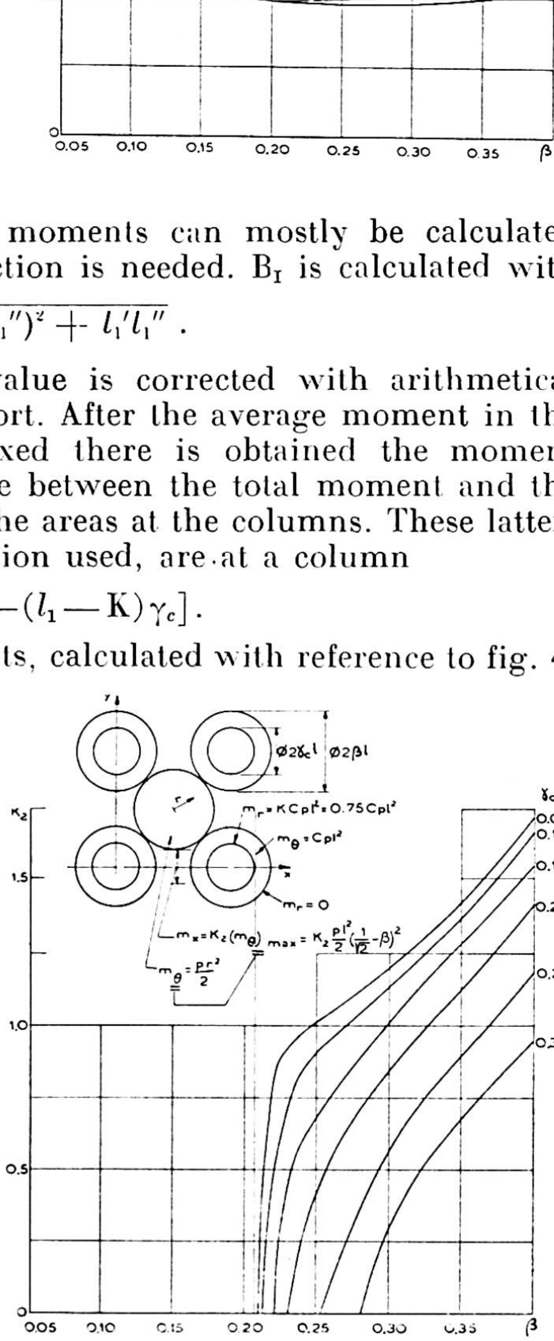

Fig. 8.

net can be turned in any direction whatsoever. If cylindrical symmetrical reinforcement be chosen, radial bars are properly excluded at an uniformly distributed load, which gives a very even stress distribution. Intensity of moments arising from this and calculated with reference to fig. 4 is shown in fig. 8. The demand for sufficient reinforcement in the strip between the columns should be fulfilled if the maximum intensity of moments there is considered to be at least 25 % larger than the average moment in the middle strip. It is very striking that cylindrical symmetrical reinforcement contains limited possibilities. There can scarcely be any motive for it at a load in any other line of panels.

As a guide for judging how great a part of a possible drop panel can be included in the capital, one is reminded of Lewe's work, according to which the radial moments at a circular drop panel's edge deviate with less than 5 % from moments at an absolutely rigid capital's edge, if for $\gamma_c = 0.08$; 0.15 and 0.25 respectively the drop panel's depth is 25 %; 50 % and 100 % respectively by the slab's depth.

The investigation has dealt with some simple cases. To treat all cases is unthinkable, since the shaping and loading can vary in an infinity of variations. The principles applied, which have been partly exploiting equivalents with continuous beams, partly extracting areas with cylindrical symmetry, can nevertheless be applied to other cases also, and give acceptable solutions for problems that arise.

Résumé

Suite à des essais décrits dans le présent mémoire, et à ceux exécutés par Smulski, une méthode de calcul des dalles champignons fut mise au point. L'auteur montre l'analogie avec les poutres continues ainsi qu'avec les dalles à symétrie axiale. Cette méthode peut être utilisée pour des cas simples et le résultat exprimé par des graphiques.

Zusammenfassung

Auf der Grundlage eigener Versuche, welche im vorliegenden Beitrag beschrieben werden und auf denjenigen von Smulski wurde eine Methode für die Berechnung der Pilzdecken ausgearbeitet. Grundsätzlich werden einerseits die Analogien mit dem durchlaufenden Balken, andererseits diejenigen der Platten mit Polarsymmetrie herangezogen. Diese Berechnungsmethode wird auf einfache Fälle angewendet, deren Ergebnisse graphisch ausgewertet werden.

Summary

On the basis of my own tests herein described, and of those of Smulski I put forward a method for designing flat slabs. Its principles are on the one hand to exploit equivalents with continuous beams, on the other to extract areas with cylindrical symmetry. The method is applied in simple cases and the results are given in nomograms.