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Autor: Mandes, Jerzy

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Application de l'équation du travail virtuel au calcul des poutres-parois

Die Anwendung des Prinzips der virtuellen Arbeit auf die Berechnung der wandartigen Träger

The application of the virtual work equation for calculating walls-beams

PROF. Dr JERZY MANDES

1. *Definition of Wall-Beam*

A wall-beam is a flat element, having its height greater than its span and a small thickness. We assume, that there does not exist any danger of buckling. They are used for building of : silos, shelters, coal-storages, etc. Walls-beams could be of single span, continuous, with brackets, having rigid or elastic support and constant or variable thickness.

2. *The Methods of Calculation for Walls-Beams*

The methods of calculation for walls-beams are based, until now, on the works of Bay, Bortsch, Cramer, Dischinger, L'Hermite, Hager and Hobel. All those methods could be brought to three fundamental types such as : general equilibrium equations, Airy's functions and differential equations. Since in this case the ordinary theory of bending is not applicable and previous ones have a number of faults like : limited adaptation, boundary or deformation conditions not fulfilled, disability to choose the right function of stresses etc., there was applied, therefore, Ritz-Timoshenko's method of virtual work. This method has following advantages : easy choice of stresses function, simple equations for stresses, generalizes the results for variable height, seizes the problem of buckling, applies to all kinds of loading by using continuous functions. An element supported continuously, at two points and a bracket were calculated by this method.

3. *Base of Virtual Work in Application to Walls-Beams*

The fundamental equation for a two-dimensional problem with the omission of the volumetric forces is :

$$\delta \int \int V \, dx \, dy = \int (\delta X_v u + \delta Y_v v) \, dx \, (dy) \, . \tag{1}$$

Strain energy per unit volume

$$V = \frac{1}{E} \left[\frac{1}{2} \sigma_x^2 + \frac{1}{2} \sigma_y^2 - \mu \cdot \sigma_x \cdot \sigma_y + (1 + \mu) \tau^2 \right] . \tag{2}$$

Surface forces

$$X_v = \sigma_x \cos (xv) + \tau \cos (yv) \qquad Y_v = \tau \cos (xv) + \sigma_y \cos (yv) \tag{3}$$

Constituent transfers

$$u = \frac{1}{E} \int (\sigma_x - \mu \sigma_y) \, dx \qquad v = \frac{1}{E} \int (\sigma_y - \mu \sigma_x) \, dy \, . \tag{4}$$

The angles of the normal to the edge of an element with axis x and y : xv, yv.

Let the function of stresses be of a type

$$F(x, y) = k_1 f_1 (x, y) + k_2 f_2 (x, y) + \ldots + k_n f_n (x, y) \tag{5}$$

with unknown parameters $k_1, k_2 \ldots k_n$, which are obtained from the equations

$$\frac{\partial}{\partial n} \int \int V dx dy = \int \left(\frac{\partial X_v}{\partial n} u + \frac{\partial y_v}{\partial n} v \right) dx \, (dy) \, . \tag{6}$$

Functions f_i must fulfil the boundary conditions. The stresses are determined from following equations :

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} \qquad \sigma_y = \frac{d^2 F}{\partial x^2} \qquad \tau = - \frac{\partial^2 F}{\partial x \partial y} \, . \tag{7}$$

4. *Calculation of a Single Element Supported at Two Points*

Let the stresses function be :

$$F = p \left[-\frac{1}{2} x^2 + \frac{1}{2 h^6} (a^2 - x^2) \cdot (-5.5 \, h^2 y^4 + 5 \, h y^5 - 0.5 \, y^6) + \frac{k}{a^2 h^4} \cdot (a^2 - x^2)^2 \cdot (h^2 y^2 - 2 \, h y^3 + y^4) \right] . \tag{8}$$

By means of equation (8) we obtain : stresses (7), strain energy (2), surface forces (3), constituent transfers (4). By substituting the obtained values from this expression into equation (6) we shall get the unknown parameter k :

$$k = - \left(2.73 \frac{h^2}{l^2} - 0.89 \right) : \left(\frac{l^2}{h^2} + 0.57 + \frac{h^2}{l^2} \right) . \tag{9}$$

expressed in the function of slenderness ratio $\frac{h}{l}$. Substituting now value of k, obtained from equation (9), into equation (7), we shall get three constituent stresses in a variable function x and y and also in the slenderness ratio $\frac{h}{l}$, having $\frac{h}{l} = 1$, $k = -0.72$ hence

$$\sigma_x = p\left[\frac{1}{2h^6}\cdot\left(\frac{h^2}{4} - x^2\right)\cdot(-66\,h^2y^2 + 100\,hy^3 - 15\,y^4) - \frac{2.88}{h^6}\cdot\left(\frac{h^2}{4} - x^2\right)^2\cdot(2\,h^2 - 12\,hy + 12\,y^2)\right]; \tag{10}$$

$$\sigma_y = p\left[-1 - \frac{1}{h^6}\cdot(-5.5\,h^2y^4 + 5\,hy^5 - 0.5\,y^6) + \frac{11.52}{h^6}\cdot\left(\frac{h^2}{4} - 3\,x^2\right)\cdot(h^2y^2 - 2\,hy^3 + y^4)\right]; \tag{11}$$

$$\tau = p\left[\frac{1}{h^6}\cdot x\cdot(-22\,h^2y^3 + 25\,hy^4 - 3\,y^5) - \frac{11.52}{h^6}\cdot x\cdot\left(\frac{h^2}{4} - x^2\right)\cdot(2\,h^2y - 6\,hy^2 + 4\,y^3)\right]. \tag{12}$$

The distribution of stresses is shown in figure 1.

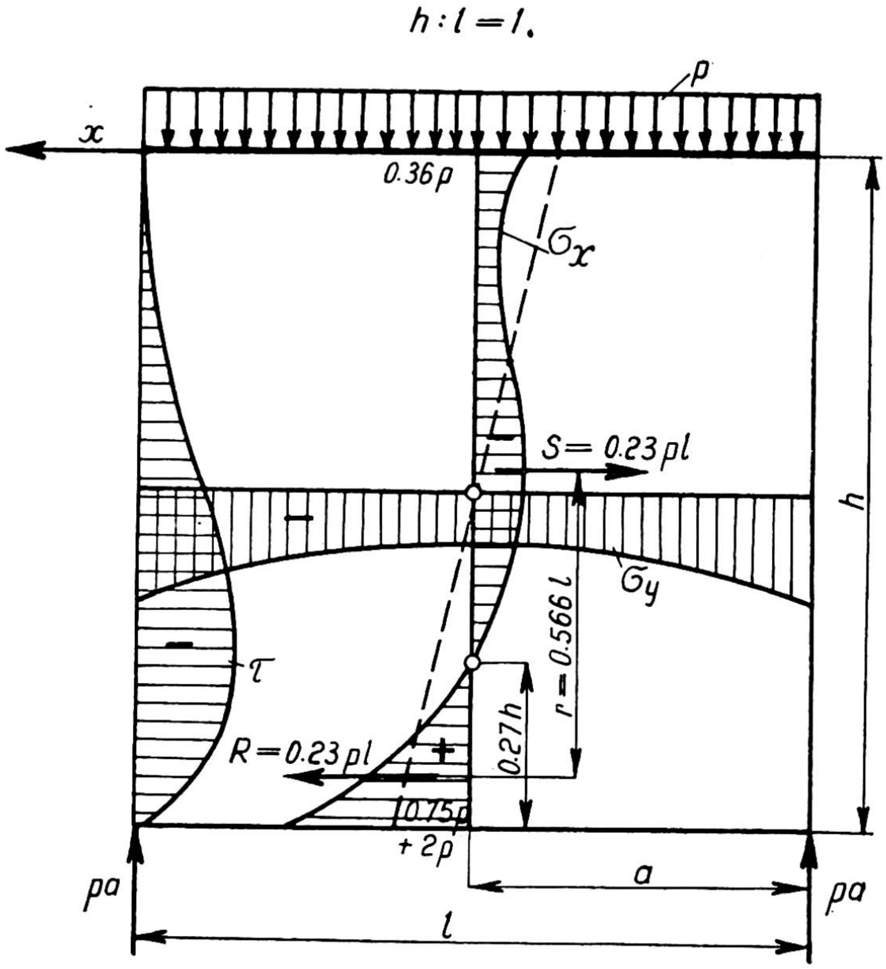

Fig. 1. Distribution of stresses.

The Curve σ_x introduces the normal horizontal stresses in the centre of the span of a beam. For comparison it is also given a curve of stresses obtained by means of Navier's theorem — dotted line. The numerical value, in the centre of the top edge of the beam, calculated by means of the equation for virtual work and also by Navier's theorem is — 0.36 p and — 0.75 p. In the centre of the bottom edge the values of above stresses are + 2.0 p and + 0.75 p.

The Curve τ introduces the tangential stresses in the section over the support but it is not a parabola as it happens in ordinary theory of bending. The greatest value of the tangential stresses on the neutral axis, with consideration of equation (12) is for $\frac{h}{l} = 1$ and $k = -0.72$, $y = 0.73\, h$

$$\max \tau = -\frac{3}{2} \cdot \frac{x}{h}\, p - 2\frac{x^3}{h^3}\, p\, .$$

With ordinary theory of bending

$$\max \tau = -\frac{3}{2}\frac{x}{h}\, p$$

hence by the theory of wall-beams, the maximum tangential stress gets a greater value.

The Curve σ_y introduces the normal vertical stresses in the middle of the height of a beam. Those stresses are usually neglected in the ordinary theory of bending. It is interesting to compare in both the methods the positions of the neutral axis, the values of compressive and tensile forces and also the arms of internal moments.

Assuming in equation (10) $\sigma_x = 0$ and $x = 0$ we shall get the position of the neutral axis at the centre of the span equal to 0.73 h. The value for the linear distribution of stresses is 0.5 h; hence the position of the neutral axis is much lower in walls-beams than in ordinary beams.

The values of the compressive and tensile forces are obtained by integrating the negative and positive areas enclosed by curve σ_x when $x = 0$,

$$\frac{h}{l} = 1 \text{ i.e. } S = \int_0^{0.73\,h} \sigma_x dy = -\int_{0.73\,h}^{h} \sigma_x dy = -R = -0.23\, pl\, .$$

By Navier's theorem we get a value

$$S_N = 0.5 \cdot \sigma_x \cdot 0.5\, h = -0.1875\, pl < S\, .$$

The arms of the internal moments, however, are respectively $m =$ 0.566 l and $m_N = 0.667\, l$.

A simple account shows, that the moment of the internal forces is practically equal to the moment of external forces, e.g.

$$M_W = R \cdot m = 0.23\, pl \cdot 0.566\, l = 0.129\, pl^2\, ;$$

$$M_Z = \frac{1}{8}\, pl^2 = 0.125\, pl^2\, .$$

The difference about 3 %.

The relationship between the stresses and the values of slenderness ratio

The investigation was done for the stresses in the centre of the top and bottom surface i.e.

$$(\sigma_x)_{\substack{x=0\\y=0}} = 0.5\, p \frac{l^2}{h^2}\, k \qquad (\sigma_x)_{\substack{x=0\\y=h}} = 0.5\, p \frac{l^2}{h^2} (4.75 + k)$$

for the stresses σ_y in the section over the supports i.e.

$$(\sigma_y)_{x=\pm a} = p \left[-1 - \frac{1}{h^6} (-5.5\, h^2 y^4 + 5\, h y^5 - 0.5\, y^6) + \frac{8\,k}{h^4} (h^2 y^2 - 2\, h y^3 + y^4) \right]$$

for tangential stresses τ, with $x = \frac{l}{3}$ i.e.

$$(\tau)_{x=\frac{l}{3}} = \frac{1}{3}\, p \frac{l}{h} \left[\frac{1}{h^5} (-22\, h^2 y^3 + 25\, h y^4 - 3\, y^5) + \frac{20\,k}{9\,h^3} (2\, h^2 y - 6\, h y^2 + 4\, y^3) \right]$$

and for tensile force in the centre of the span of a beam i.e. $x = 0$

$$R = -\int_0^{0.73\,h} \sigma_x dy = (0.26 + 0.0455\,k)\, p \frac{l^2}{h}\,.$$

Those stresses expressed by means of Navier's theorem will give :

$$(\sigma_x)_{\substack{x=0\\y=0}} = -0.75 \cdot p \frac{l^2}{h^2} \qquad (\sigma_x)_{\substack{x=0\\y=h}} = +0.75 \cdot p \frac{l^2}{h^2}$$

$$(\tau)_{x=\frac{l}{3}} = -2 \frac{pl}{h^3} \left(\frac{h^2}{4} - y^2 \right) \qquad R_N = 0.1875\, p \frac{l^2}{h}\,.$$

Having done the analysis of the given groups of the corresponding equation we come to the following conclusions : as the ratio $\frac{h}{l}$ grows bigger, the stresses σ_x are coming to zero, and by $\frac{h}{l} < 0.5$ the stresses are quickly increasing. The compressive stresses calculated by use of virtual work with respect to Navier's theorem are greater for $\frac{h}{l} < 0.65$ and smaller for $\frac{h}{l} > 0.65$. The tensile stresses are getting always greater values using the accurate method.

The stresses σ_y show that they are constant at $\frac{h}{l} > 1.5$. The stresses σ_x show nearly the same limits of stability. It means, that high beams are working like columns. This fact is also confirmed by the tangential stresses.

Further investigations show that for slenderness ratio $\frac{l}{h} \leqslant 0.5$, the values of stresses calculated by means of both methods come very fast closer to each other and the positions of the neutral axis are practically identical.

Conclusion

The beams having short height i.e. $\frac{h}{l} \leqslant 0.5$ gets the numerical values of stresses very close to that from Navier's theorem, and they are closer as the height of the beam is shorter. The beams with a big height i.e. $\frac{h}{l} > 0.5$ cannot be calculated by means of Navier's theorem because of big divergences and they are bigger as the beam gets higher. Those results are confirmed by the investigations of the deformations of beams. The solutions received by means of virtual work were checked by differential equations getting practically the same answers.

Résumé

Une poutre-paroi est constituée par un élément plat dont la hauteur est supérieure à sa portée et à sa petite épaisseur. On admet que le cas de flambage n'a pas lieu. Application : pour la construction des silos, blindages, abris, magasins à charbon, etc.

Les poutres-parois peuvent être à une travée, continues, à consoles, à un appui rigide ou élastique, à épaisseur constante ou variable.

Le principe de calcul est basé sur l'application de l'équation du travail virtuel dans la forme de Ritz-Timoshenko.

Cette méthode présente les avantages suivants : facilité du choix de la fonction des tensions; généralisation des résultats pour la hauteur variable et accomplissement du problème des déformations; enfin, elle est applicable à tous les cas de chargement en employant des fonctions continues.

On a calculé à l'aide de cette méthode : 1° un élément appuyé sur toute sa longueur; 2° un élément sur deux appuis; 3° un élément-console.

L'ordre du calcul est le suivant : 1° on écrit l'équation du travail virtuel; 2° on calcule le travail élastique, les forces superficielles et les composantes des déplacements; 3° on substitue ces valeurs dans l'équation fondamentale; 4° on détermine les tensions comme les dérivées partielles de la fonction des tensions.

Zusammenfassung

Ein wandartiger Träger ist ein scheibenförmiges Element, dessen Höhe grösser als seine Spannweite und dessen Dicke gering ist. Man setzt voraus, dass Beulgefahr nicht existiert. Solche Träger werden für Silos, Luftschutzkeller, Kohlenbunker, etc., angewendet.

Wandartige Träger werden einfeldrig, durchlaufend, mit Konsolen, auf festen oder senkbaren Stützen und mit konstanter oder veränderlicher Stärke ausgeführt.

Die Berechnung beruht auf der Anwendung des Prinzips der virtuellen Arbeit in der von Ritz-Timoshenko vorgeschlagenen Form. Diese Methode bietet folgende Vorteile : Erleichterung in der Wahl der Spannungsfunktion und einfache Gleichungen für die Spannungsberechnung. Ferner können die Ergebnisse auf veränderliche Höhe verallgemeinert werden, das Beulproblem mit einbezogen und alle Arten von Belastungen durch Einführung stetiger Funktionen erfasst werden.

Die Berechnung geschieht wie folgt : Aufstellung der Gleichung für die virtuelle Arbeit als Funktion der Formänderungsenergie, der Oberflächenkräfte und der Verschiebungskomponenten. Die Spannungen werden aus den partiellen Ableitungen der Spannungsfunktion bestimmt.

Summary

A wall-beam is a flat element, having its height greater than its span and a small thickness. We assume, that there does not exist any danger of buckling. They are used for building of : silos, shelters, coal-storages, etc. Walls-beams could be of single-span, continuous, with brackets, having rigid or elastic support and constant or variable thickness.

The principles of calculation are based on the application of the virtual work equation in form of Ritz-Timoshenko. This method has following advantages : easy choice of stresses function, simple equations for stresses, generalizes the results for variable height, seizes the problem of buckling, applies to all kinds of loading by using continuous functions. An element supported continuously, at two points and a bracket were calculated by this method.

The order of calculation consists on : establishment of the equation of virtual work, expressed by the elastic energy, surface forces and displacement components. The stresses are determined as partial derivatives of stress function.

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