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Calcul des dalles champignons

Comparaison des valeurs théoriques avec celles des moments spécifiées dans les formulaires

Die Berechnung von Pilzdecken

Vergleich der theoretischen Werte mit den in den Bestimmungen angegebenen Momentenkoeffizienten.

The calculation of flat slab floors

Comparing theoretical values with moment coefficients specified in flat slab codes

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Most countries now have a building code, in which rules are given for the calculation of flat slab floors. In these rules the moment coefficients always take an important place. They enable the designer without any theoretical analysis to construct these floors within certain limits. Mostly a numerical sum of positive and negative bending moments at in the codes defined critical sections shall be assumed as to be no less than a given amount. Furthermore the percentages of the bending moment at every section are defined for different cases; for example with and without the use of drop panels.

In the U. S. A. the flat slabs have and have had a large field of application. Apart from the regulations given by different cities, the American Concrete Institute issued her flat slab code in 1917 which was then considered as conservative. This code has frequently been revised and adapted to more modern practice; the regulations which are now in use are from the year 1947. In my opinion they are the best available set of flat slab regulations and superior to the ones formulated in other countries. In the comparison to be made they will be taken as the standard.

For the calculation of flat slabs there are many authors and consequently many methods of design. Almost all the authors derive their theory from the differential equation of a plate taking into account the boundary conditions. Thereby a plate of uniform thickness is assumed. The methods of Lewe, Navier, Lavoinne, Nadaï, Tölke and Hager employ

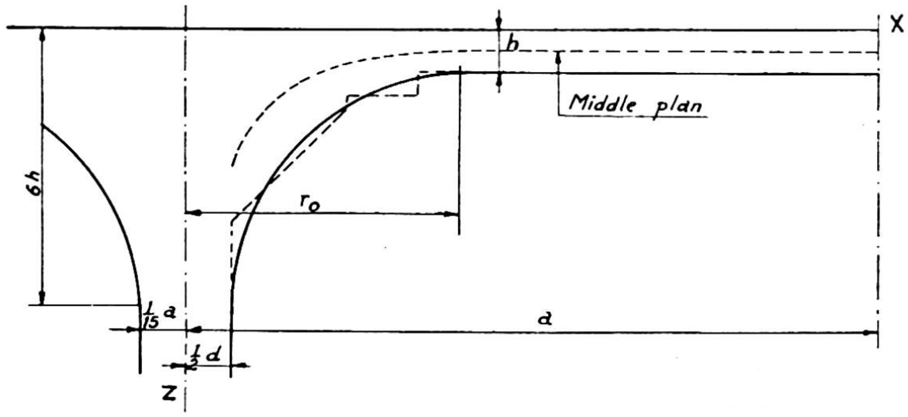

Fig. 1. Column head of hyperbolical shape.

a solution in the form of series. Grashoff and Eddy make use of an algebraic formula whereas Marcus replaces the differential equation of the plate by two equations of finite differences each of which has the form of the equation of a stretched membrane.

All these methods have the difficulty that afterwards the solution should be adjusted to the varying thickness of the column-head which has always been done rather roughly.

In the method of computation as developed by the author and published in 1948 ([1]) the column-head has been introduced from the beginning. Thus the problem is divided into two parts : the column-head-section and the slab proper. For the computation of the column-head it has been replaced by a hyperboloid (fig. 1). The solution of the differential equation is given in the form of series of which a limited number of terms are used. When the theoretical value for the bending moment in the plate-centre is compared for the different theories little divergency is found. For varying size of the area of column-head reaction, Lewe finds 0.117 to 0.122 pa^2, $(a = 1/2\ l)$, Tölke finds 0.114 pa^2, the author comes to 0.102 pa^2, if for the contraction-coefficient is taken $m = \frac{1}{\nu} = 6$. The small differences may be readily explained by the principle of de Venant. The assumption made in reference to the restraint in and the support by the column-head may vary greatly in the various theories, however they have little effect on bending moments and other stress functions in the plate-centre.

For the other bending moments and especially for the negative mo-

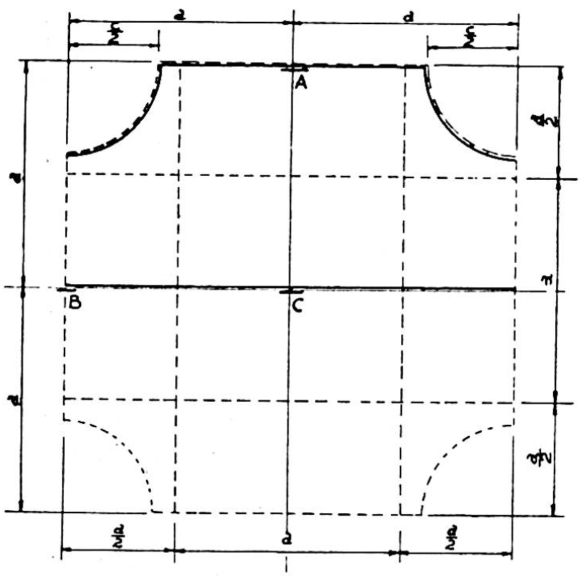

Fig. 2. Interior flat slab panel supported by columns arranged in a square.

([1]) A. M. Haas, *De berekening van paddestoelvloeren* (*The calculation of flat slab floors*), thesis 1948, Technical University of Delft, the Netherlands.

ment at the edge of or above the column-head there is little conformity. The different assumptions made in the theories are here of direct influence.

Large differencies do not only exist as a result of the theoretical analysis; there are also important discrepancies between the theoretical values as a whole and the corresponding moment-coefficients given in the code. As a result there is little confidence in either the theories or the codes values. In 1934 the English Code of Practice for flat slab floors still speaks of "the absence of a satisfactory and easily applied theory" and largely follows the requirements given in the United States of America. As things stand now there are two ways of approach to the problem. One can even spreak of two camps in which engineers who have to design flat slabs are divided. The ones using the code values do not look at the theory; on the other hand the engineers indulged in theoretical analysis do not as a rule bother themselves with the requirements stated in the regulations.

If the differencies can be explained this is of ultimate importance for the promotion of flat slab floors especially outside of the United States of America. This paper is an earnest effort in that direction.

Comparison will be made for an interior flat slab panel supported by columns arranged in a square. The bending moments in the points marked A, B, C and at the edge of the column-head will be considered (fig. 2 and 3). The results are shown in tables I and II.

Positive bending moment at the slab centre (Point C)

From the very beginning of the use of flat slab floors testloading revealed that the bending moments in this centrepoint were very small. It was one of the outstanding facts which made these floors so popular. On the other hand, as explained in the introduction, the theoretical value of the bending moment in this point known. In the older (American) flat slab codes this bending moment amounts to about 0.06 pa^2 ($a = 1/2\ l$) whereas an average from the theoretical analysis (average in regard to the

	Theoretical value	Value from the A. C. I. 1947	Difference
Negative moment at column-head	$-0.56\ qa^2$	$-0.52\ qa^2$	$-$ 7 %
Negative moment A.	$-0.065\ qa^2$	$-0.078\ qa^2$	$+20$ %
Positive moment B.	$+0.135\ qa^2$	$+0.104\ qa^2$	-23 %
Positive moment C.	$+0.102\ qa^2$	$+0.078\ qa^2$	-24 %

TABLE I. *Bending moments in critical points of a flat slab floor. Interior panel — Comparison between theory and code* various sizes of the columnhead) is 0.10 to 0.11 pa^2. The explanation for this discrepancy is that the theoretical value is derived for a homogeneous material whereas the moments incorporated in the codes are based upon experimental investigations (mainly measuring elongations during test-loading). These measurements besides measuring the deflections at several points consisted chiefly in determining the elongations of the steel reinforcement. From the results found the bending moments were established.

It is obvious that if the total bending moment is composed of different parts and one part is furnished by the elongations (= stresses) occurring in the reinforcing steel only a part of the total bending moment is found. This is the case here. The total bending moment is composed of two contributions viz. the moment due to the tensile stresses in the steel and that due to the tensile stresses in the concrete. In contradistinction to beams the relative contribution of the tensile stresses in concrete slabs is of importance.

For one who is familiar with the reinforced concrete theory and practice there arises the question whether the tensile stresses may be taken into account and if so, may be relied upon.

	Column-Head Negative moment		A. Negative moment	
	Formula used	Converted value	Formula used	Converted value
A. C. I. 1917	$-0.5 \times 0.09\,WL \left(L - \frac{2}{3} c\right)^2$	$-0.26\,qa^2$	$-0.12 \times 0.09\,WL \left(L - \frac{2}{3} c\right)^2$	$-0.062\,qa^2$
New York City regulations 1920	$-\frac{WL}{32}$	$-0.25\,qa^2$	$\frac{WL}{133}$	$-0.061\,qa^2$
A. C. I. 1947	$-0.5 \times 0.09\,WL \times \left(1 - \frac{2c}{3L}\right)^2$	$-0.26\,qa^2$ (*)	$-0.15 \times 0.09\,WL \times \left(1 - \frac{2c}{3L}\right)^2$	$-0{,}078\,qa^2$ (**)
English code 1938	$-0.046\,WL \left(L - \frac{2}{3} D\right)^2$	$-0.255\,qa^2$	$-0.016\,WL \left(L - \frac{2}{3} D\right)^2$	$-0.089\,qa^2$
Theoretical value $m = 6$		$-0.56\,qa^2$		$-0.065\,qa^2$
Lewe $m = 6$		$-0.3259\,qa^2$		$-0.0734\,qa^2$

(*) For the calculation of concrete in compression use $4/3 \times 3/2 \times 0.26\,qa^2 = 0.52\,qa^2$.

(**) For the calculation of concrete in compression use $4/3 \times 0.078\,qa^2 = 0.102\,qa^2$.

If the bending moment a slab can resist is determined by the conventional method of design the concrete in tension is supposed to be cracked. Thus only the reinforcement in tension is taken account of and a certain moment (M_3) will be found. If again this moment is established taking into account the tensile stresses in the steel (M_2) as well as in the concrete (M_1) a different amount will be found for the bending moment (fig. 4). For slabs with a low percentage of reinforcement the bending moment will be larger than when using the conventional method, even if the allowable tensile stress in the concrete is limited to a certain maximum. If for example the slab reinforcement is 0.4 %, n = the ratio of the modulus of elasticity of steel to that of concrete in compression = 10; m = the ratio of the modulus of elasticity of steel to that of concrete in tension = 40, the bending moment amounts to 5.09 bh^2 using the conventional method.

If the allowable tensile stress in the concrete is put at 20 kg/cm² the slab can resist a moment of 6.20 bh^2 per unit of width. Of this 6.20 bh^2 67 % or 4.18 bh^2 is due to the concrete in tension. The tensile stress

B. Positive moment		C. Positive moment	
Formula used	Converted value	Formula used	Converted value
$0{,}18 \times 0{,}09\,WL \left(L - \frac{2}{3}c\right)^2$	$+ 0.094\,qa^2$	$0.10 \times 0.09\,WL \times \left(L - \frac{2}{3}c\right)^2$	$+ 0.052\,qa^2$
$+\frac{WL}{80}$	$+ 0.100\,qa^2$	$\frac{WL}{133}$	$+ 0.061\,qa^2$
$+0{,}20 \times 0{,}09\,WL \times \left(1 - \frac{2c}{3L}\right)^2$	$+ 0.104\,qa^2$	$+0.15 \times 0.09WL \times \left(1 - \frac{2c}{3L}\right)^2$	$+ 0{,}078\,qa^2$
$+0{,}022\,WL \left(L - \frac{2}{3}D\right)^2$	$+ 0.122\,qa^2$	$+0.016\,WL \left(L - \frac{2}{3}D\right)^2$	$+ 0.089\,qa^2$
	$+ 0.135\,qa^2$		$+ 0.102\,qa^2$
	$+ 0.1818\,qa^2$		$+ 0.1176\,qa^2$

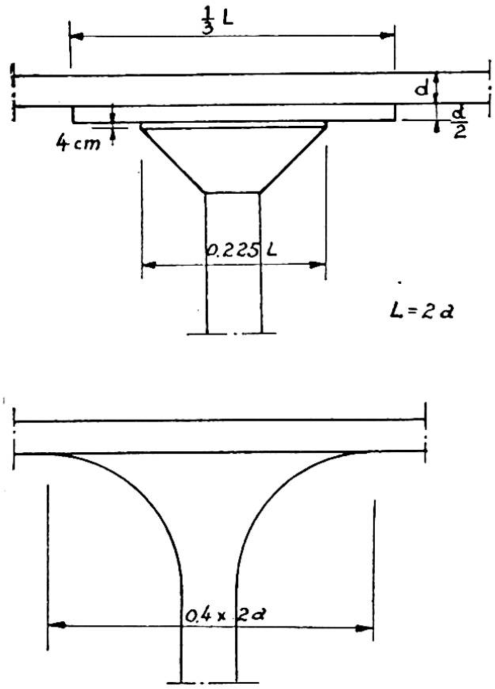

Fig. 3. Column heads used for the calculations of code-values (above) and for the theoretical values (right).

TABLE II. *Bending moments in critical points of a flat slab floor — Interior panel*

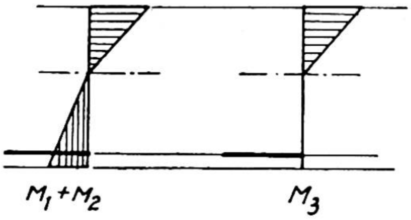

Fig. 4.

in the reinforcement corresponding to the 20 kg/cm² amounts to about 670 kg/cm².

If therefore the bending moment is established from elongation-measurements it will be derived from the value

$$6.20\ bh^2 - 4.18\ bh^2 = 2.02\ bh^2,$$

whereas the moment that the slab can take according to the conventional method is 5.09 bh^2 which is 2 1/2 as high. This shows clearly how small the apparent moment will be if the percentage of reinforcement is low. It also explains the large discrepancy between code and theory, which here in the centre-point amounts only to 24 % (see table I). For different ratio's of n and m, and for varying percentages of reinforcement one finds :

$$\sigma_c = 20\ \text{kg/cm}^2, \qquad n = 10, \qquad m = 40.$$

Percentage of reinforcement	x/h	M_1 Moment due to concrete in tension	M_2 Moment due to steel in tension	$M_1 + M_2$	$\frac{M_1}{M_1 + M_2}$	M_3 Moment according to conventional design
0.4	0.373	4.18 bh^2	2.02 bh^2	6.20 bh^2	0.67	5.09 bh^2
0.5	0.384	4.11 bh^2	2.61 bh^2	6.72 bh^2	0.61	6.30 bh^2
0.6	0.392	4.06 bh^2	3.08 bh^2	7.14 bh^2	0.57	7.50 bh^2

$$\sigma_c = 20\ \text{kg/cm}^2, \qquad n = 10, \qquad m = 10.$$

Percentage of reinforcement	x/h	M_1 Moment due to concrete in tension	M_2 Moment due to steel in tension	$M_1 + M_2$	$\frac{M_1}{M_1 + M_2}$	M_3 Moment according to conventional design
0.2	0 507	3.29 bh^2	0.21 bh^2	3.50 bh^2	0.94	2.61 bh^2
0.3	0.511	3.26 bh^2	0.33 bh^2	3.59 bh^2	0.91	3.87 bh^2
0.4	0.515	3.25 bh^2	0.45 bh^2	3.88 bh^2	0.88	5.09 bh^2

The next step is to find for which percentage of reinforcement the bending moments computed according to the conventional method equal the ones calculated when the concrete in tension is included.

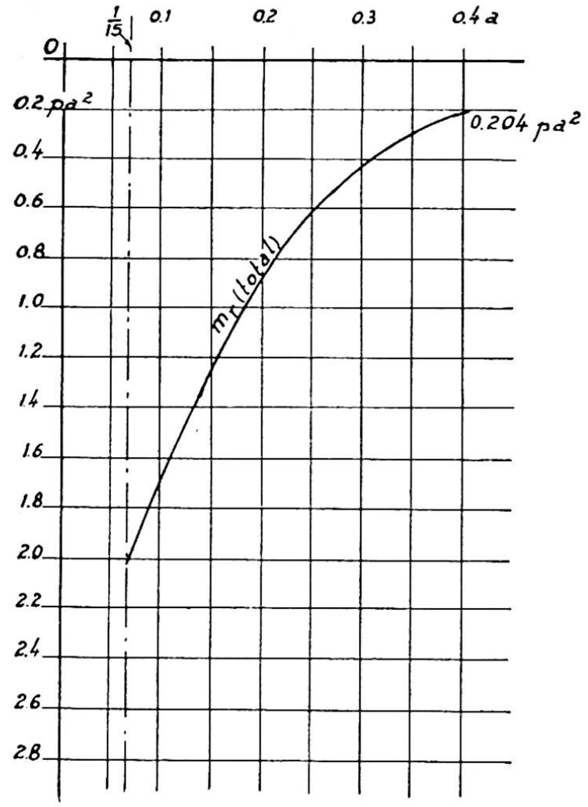

Fig. 5. Radial bending moment in hyperbolical column head.

For $\sigma_c = 15$ resp. 20 kg/cm² it is found for the relation

$$\frac{M_1}{M_1 + M_2}:$$

$\sigma_c = 15$ kg/cm²; $n = 10$

$m = 10$ 0.2 % reinforcement: 0.94
$= 20$ 0.3 % reinforcement: 0.83
$= 40$ 0.35 % reinforcement: 0.71

$\sigma_c = 20$ kg/cm²; $n = 10$

$m = 10$ 0.25 % reinforcement: 0.92
$= 20$ 0.35 % reinforcement: 0.81
$= 40$ 0.55 % reinforcement: 0.59

Above these percentages the bending moments calculated in the conventional way govern. These percentages will rank higher if the allowable maximum tensile stress in the concrete is taken higher. As a rule the percentage of reinforcement in and around the slab centre runs from 0.25 to 0.35. These under-reinforced parts come in the region here described and are below the maxima here given.

To which extent this also can be applied to the larger bending moments in the normal reinforced parts (such as the column-bands) greatly depends on the quality of the concrete. If the quality is high so that high tensile stresses can be expected in the concrete the same that is mentioned for the slab centre holds true for these bending moments. However one can no longer rely upon it.

Concluding for the slab centre-(under-reinforced) parts the small moment-coefficients of the codes can be readily explained. To these small bending moments always comes a complementary moment due to the tensile stresses in the concrete. One may here speak of a symbiose. In slabs with low percentage of reinforcement this complementary moment even exceeds the original one. Therefore the difference of 24 % (see table I) will in general be exceeded depending on the quality of the concrete.

Negative bending moment at the edge of the column-head

As can readily be understood the bending moment at or near the edge of the column-head is one of the critical moments in flat slab design. When moving from the edge of the column-head to the column axis the increase of the radial bending moment is more rapid than the increase of the moment of resistance of the column-head the critical section will be at the edge. When however due to the form of the column-head the moment of resistance does not — in the beginning — increase as swiftly as the bending moment the critical section (that is the section where the stresses are maximum) lies more inwardly. For a column-head without a drop-panel and with flares under 45 % the critical section is at the edge. For a column-head with a drop panel this section will be located somewhere between the edge of the column-head and the drop panel-edge depending on the thickness and the size of the drop.

In the building codes the bending moments are specified for bands which have mostly the width of half a panel. No distinction is made for bending moments differing over the width of the band. A clear comparison is therefore not possible. The best one can do is to compare the bending moment given in the code with the maximum bending moment resulting from the theory.

The theory of Lewe and others show its shortcomings when calculating the bending moments in this area. This is due to the assumption made, namely that a slab of uniform thickness extends above the column-head. Lewe finds — 0.326 pa^2 at the column axis and — 0.067 pa^2 at the edge. The theoretical analysis which takes into account a varying slab thickness finds larger bending moment viz. for

$$r = 0.4\ a : m = -\ 0.204\ pa^2$$

and for

$$r = 1/3\ a : m = -\ 0.3324\ pa^2;\ r = 0.225\ a : m = -\ 0.7\ pa^2.$$

These bending moments have been computed replacing the column-head by a hyperboloid which approximates the actual form as closely as possible (fig. 5).

When the bending moment at the edge of the column-head is taken as the critical one and 20 % is deducted for various reasons (plastic flow, etc.) one arrives at the high figure of 0.56 pa^2.

$$(0.8 \times 0.7\ pa^2 = 0.56\ pa^2)$$

In the rules given by the American Concrete Institute, for the first time in 1936, and also in the 1947-edition a clause has been introduced in which the calculation of the negative bending has been revised and made more severe. Instead of the whole width of the band only 3/4 of it should be used in determining the compressive stress in the concrete. Thus a value is found which is 4/3 higher than the one which follows when the whole section is introduced. The code even goes farther for drop panels which are often used. In that case the section to be introduced in the calculation should be restricted to 3/4 of the width of the drop panel. With a drop panel width of 1/3 of the panel this means that the compression in the concrete should be taken as $3/2 \times 4/3 = 2 \times$ the one which emanates from the calculation when the normal section is introduced. In the example the bending moment per unit of width will amount to $2 \times 0.26\ pa^2 = 0.52\ pa^2$. This comes very well in conformity with the theoretical value (0.56 pa^2).

The clause mentioned is restricted to the calculation of the compressive stresses in the concrete. In determining the amount of steel reinforcement no reduction of section is prescribed. This looks rather irrational but one should bear in mind that these excessive bending moments only occur in small areas. The steel is determined as a whole for a single band which has as a rule the width of half a panel. When excessive tensile stresses occur lateral distribution is very likely to take place as the quality of the concrete will be rather good in order to resist the high compressive stresses in the concrete. Concrete of good quality also means high resistance to bond which is essential to lateral distribution. When rupture occurs the reinforcing bars will be loaded to the yieldpoint and all will take a maximum load (plasticity-behaviour).

Positive and negative bending moment halfway between the columns (Points A and B)

These two moments are taken together as the phenomena by which they are influenced are partly the same. The difference is that the positive moment lies in an area of rather high percentage of reinforcement (columnbands) and the negative moment comes in an area of low percentage. Therefore only for the latter the tensile stresses in the concrete can be taken account of.

Already in 1925 a Hungarian civil engineer Dr. Nemenyi has drawn attention to the fact that a flat slab floor with its enlarged columnhead bears in itself a possibility of acting as a dome. When this really is the case large compressive stresses will occur and consequently a reduction in the elongation of the steel will be experienced in comparison with the case when only bending is considered. On this assumption Dr Nemenyi has made his calculations which resulted in a remarkable reduction of the bending moments, especially of the plate centre.

This assumption however in general does not hold true as the construction cannot take the large horizontal thrust which is essential for dome- (or arch-) action. In the case of uniform loading the thrusts per panel neutralize each other in horizontal direction at every column support. In the end panel a similar neutralization will not be at hand and the columns are in general by no means strong enough to take the heavy thrusts. Thus the theory advanced by Nemenyi can in general not be applied.

There is however an internal arch-action which though less effective may account for the discrepancies in the bending moments here mentioned. First of all when the flat slab is loaded there come into existence compressive stresses due to the form of the column-head. As a result of the loading the deflection of the edge of the column-head will not strictly be vertical but also a little horizontal. For a normal case it has been calculated that the horizontal deviation is about 1/14 of the vertical deflection. When two small bands of the slab are considered the compression in the plate can be determined when taking into account the horizontal deviation of the columnhead-edge. Thereby the assumption is made that the column-axis do not move and consequently the distances between the column-axis will not alter. When certain reasonable assumptions are made (slab thickness $= 1/17\ a = 1/34$ L; stiffness of the columnhead against compression twice as large as that of the plate) it has been found by the author that the compression in the point halfway between the column amounts to 9 % of the compressive stress due to bending. For the plate centre there is a relation of 8 % (fig. 6).

A small amount of internal arch-action comes into existence as a result of the type of construction. If a part of the slab is considered in which the column band and the column head lie, no appreciable arch-action can take place because the columns at the end cannot take the thrust. If however these columns are more or less hindered in their horizontal movement an arch comes into shape. The horizontal deviation is hindered because there

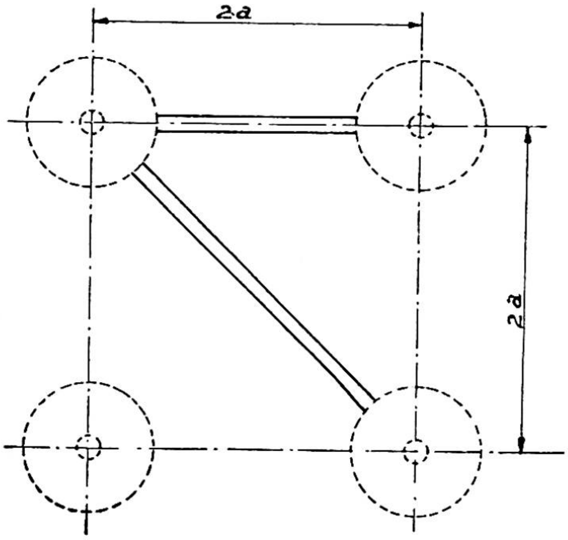

Fig. 6.

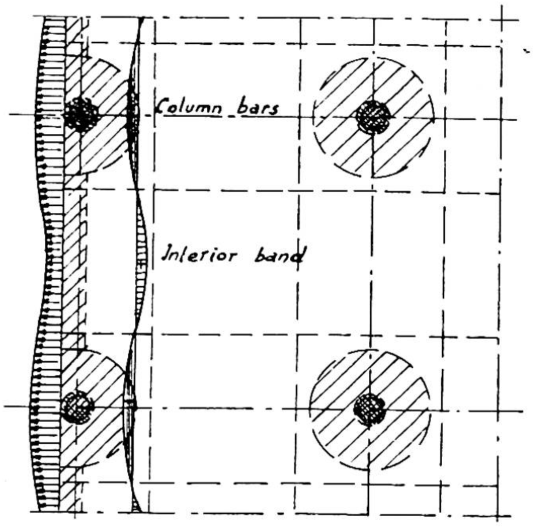

Fig. 7. Interior arch-action in an end panel.

is a continuous floor slab. On account of this, one part of the plate will experience compression (the column bands) and another part will be subject to tensile stresses (the bands running alongside of the columns). This being an internal action the sum of both the tension and compression must be zero. It is rather an intricate calculation to arrive to a reasonable distribution of these compressive and tensile stresses.

If a rough approximation is made and half of the panel-width is supposed to be in compression, the other half must be in tension. Thus the 8 to 9 % compression will correspond to 8 to 9 % tension. The assumed behaviour of an end panel is shown in figure 7.

When looking at the theoretical value for the positive bending moment this may be reduced with 9 % to be comparable with the moment-coefficient of the code : $0.91 \times 0.135\ pa^2 = 0.123\ pa^2$, whereas the code gives $0.104\ pa^2$ (a difference of 18 %).

For the negative bending moment the 9 % should be added to the theoretical value. It then amounts to $- 1.09 \times 0.065\ pa^2 = - 0.071\ pa^2$, whereas the code gives $- 0.078\ pa^2$ (a difference of 10 %). Thereby it should be considered that calculations are made for homogeneous material and the values in the codes are based upon elongation-measurements.

Concluding there shows itself to a good concurrence between theory and code-coefficients if all factors concerned are taking into account and the theory has been adjusted to the form of the construction. Of course there cannot be expected equality as the code-values are a result of a compromise of a number of considerations.

Also it should be borne in mind that the comparison finds place between values of unequal weight as the theory gives the stress-functions at each point. Whereas in the regulations one value is given for a section which extends over the width of half a panel generally.

Résumé

Lors de la conception des dalles champignons il faut faire choix entre les prescriptions réglementaires et les résultats théoriques. Ce mémoire compare les deux éventualités, en se basant d'une part sur les derniers règlements de l'American Concrete Institute datés de 1947, et d'autre part, sur les dernières théories faisant intervenir, dès le début, l'épaisseur variable de la tête de colonne. L'auteur attire l'attention sur la faible armature au centre de la dalle. Pour les parties faiblement armées, il faut ajouter un moment basé sur la traction dans le béton (béton tendu) au moment fléchissant correspondant aux armatures. D'autre part l'effet de voûte intérieur modifie les résultats obtenus par la théorie des plaques.

Une dernière ajoute au règlement américain, concernant les tensions de comparaison dans le domaine des moments fléchissants négatifs, donne des tensions de compression plus élevées en accord avec la théorie.

Ce mémoire compare les valeurs des moments fléchissants en quatre points critiques d'un cadre. Pour terminer l'auteur donne un exemple numérique.

Zusammenfassung

Beim Entwurf von Pilzdecken muss man sich entscheiden, ob man die Vorschriften anwenden oder die theoretischen Ergebnisse gebrauchen soll. In diesem Beitrag werden die beiden Möglichkeiten verglichen. Zu diesem Zweck werden die neuesten Vorschriften des American Concrete Institute vom Jahre 1947 als Grundlage angenommen. Sie wurden verglichen mit der neuesten Theorie, in der die veränderliche Dicke des Säulenkopfes schon zu Beginn in die Rechnung eingeführt wird. Es wird aufmerksam gemacht auf den kleinen Armierungsgehalt in der Plattenmitte. Für geringe Eisenprozent muss zu dem Biegemoment infolge der Armierung ein Biegemoment infolge der Betonzugspannungen gezählt werden. Ferner verändert eine innere Gewölbewirkung die Resultate der Plattentheorie.

Ein neuer Paragraph in den ACI-Vorschriften, der sich auf die Berechnung negativer Biegemomente bezieht, führt zu höheren Druckspannungen, was mit der Theorie übereinstimmt.

In vier kritischen Punkten, die in typischen Schnitten eines Innenfeldes liegen, werden die Werte der Biegungsmomente verglichen. Ein Beispiel ist beigefügt.

Summary

In designing flat slab floors a choice has to be made between two alternatives viz. following the regulations or using theoretical analysis. In this paper a comparison will be drawn between these two. For this purpose the latest regulations of the American Concrete Institute 1947 are taken as an example. They have been compared with the latest theory in which the varying thickness of the columnhead has been introduced from the beginning.

Attention is drawn to the low percentages of reinforcement in the slab center. For underreinforced parts of the slab a considerable bending moment due to tensile stresses in the concrete should be added to the bending moment due to the reinforcement. Furthermore an interior dome action modifies the results as given by the plate theory.

A comparatively new clause in the A. C. I. regulations in reference to the calculation of compression in the areas of negative bending leads to higher values of the compression stress which is in accordance with the theory.

In four critical points situated in typical sections of an interior panel the bending moment-values are compared. An example is given.

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