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Artikel: Stresses in gusset plates by use of an analogous grid

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Détermination des tensions dans les goussets des nœuds au moyen de grilles semblables
(Avec appendice concernant le problème analysé par Marvin Mass)

Die Bestimmung der Spannungen in Knotenblechen mit Hilfe eines analogen Netzes
(Mit Nachtrag über das von Marvin Mass analysierte Problem)

Stresses in gusset plates by use of an analogous grid (1)
(With appendix on problem analysis by Marvin Mass)

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The writer and some of his students have been engaged in investigating the stresses in plate structures by a grid analogy. The analogous grid is considered to be a physical substitute for the plate and the analysis of its stresses is undertaken by the procedures that the author developed for the analysis of wind stresses in tall building frames (2). To a structural engineer it will not seem strange that all of the important distortions of a plate can be reproduced in a substitute planar grid. This fact is almost obvious except for the introduction of Poisson's ratio which is not found to greatly complicate the analysis.

Of the planar structures so far studied one of the most important is the gusset plate of the truss or other structural joint. For study by the grid analogy the gusset plate often has two simplifying features : (1) It may have a centerline of symmetry and (2) since its forces intersect at a point its edges remain reasonably straight lines. It is upon these two observations that the method of analysis to be described has been based. This method is not sufficiently simplified as yet to be used effectively by a designer. It is still to be looked upon as a research tool. However, the writer feels

(1) For a thorough explanation of the procedures outlined here see *Numerical Methods of Analysis in Engineering*, Chapter 2, published January, 1949 by the Macmillan Co. (New York).

(2) L. E. Grinter, *Wind Stress Analysis Simplified* (*Transactions, American Society of Civil Engineers*, 1934, pp. 610-634). Also see the author's *Theory of Modern Steel Structures*, Vol. 2, The Macmillan Co. (New York) 1937 and 1949.

certain that further study will reveal means of standardizing certain analytical steps so that the procedure may become sufficiently simplified for convenient use in the design office.

The method to be described is a numerical procedure of relaxation and distribution following the writers' development of other applications of the Cross procedure of moment distribution. Here, alternate relaxation of force restraints in two directions along with distributions of moments will be needed. However, the joint-by-joint relaxation of force restraints is found to be too tedious for direct use and the necessity for such relaxations is therefore minimized by repeated use of stress estimates and moment distributions. By this procedure, termed " Strain Justification ", the problem has been reduced to practical limits of time consumption. Further simplification may depend upon the analysis of typical cases of force patterns which can be combined as needed to give first approximations of an analysis of almost any gusset plate. Final adjustments of grid stresses from an approximate to a nearly exact solution are not usually troublesome.

Summary of the analytical procedure

If a grid such as the one illustrated by figure 1 is acted upon by a single vertical load, the columns shorten unequal amounts as indicated. An estimate of the column stresses based upon experience with the method can usually be made with reasonable accuracy. Note however that in figure 1 we have as a simplifying feature a fixed base to the structural grid. A line of symmetry may be used with equal effectiveness. Also, it has been found that any boundary line of a typical rectangular gusset plate (where loads intersect at a point) remains sufficiently straight to serve as a base line from which we may calculate joint movements of the substitute grid structure after direct stresses have been estimated.

Once the direct stresses have been estimated and two perpendicular base lines of zero distortion chosen, we can locate readily the position of each joint of the substitute grid. Then based upon properties of the members, which will be derived to reproduce plate deformations, fixed-end shears in all members are calculated from relative displacements of the ends of the members. Now if our estimate of direct stresses had been exactly correct a simple distribution of moments would result in a set of shears in all members that would be in static equilibrium with the direct stresses and the loads. Instead, there will be some lack of equilibrium which will be expressed as a set of joint restraints acting in each of the two major directions.

If the joint restraints are relatively small, say 10 % to 20 % as significant as the original loading, they may be eliminated without difficulty by successive joint relaxation. But if they are more serious, it is far better to repeat the estimate of direct stresses one or several times until equilibrium is nearly achieved before relaxation of joint restraints is begun. With this brief explanation of the procedure to be used we will consider the proportioning of the gridwork to reproduce the deformations of the plate. We are here interested in stresses in the plate away from a rivet or a weld. Such stress concentrations can be superimposed on the general state of stress at any point desired since all stresses will be considered to be below the elastic limit.

Properties of the analogous grid

Reproducing direct stress deformations

The deformation of the plate under an uniaxial stress P/A is PL/AE for the length L. Since the area of the plate for a width L is Lt, this area must be replaced by each vertical member of the grid when the bar spacing of the grid is L.

$$A = Lt = bd \text{ (where } b \text{ and } d \text{ are dimensions of a grid member)} \quad (1)$$

For members of this area the longitudinal deformation of the grid under uniaxial stress corresponds exactly with the longitudinal deformation of the plate.

In equation (1), d is the depth of a bar in the plane of the plate and b is its dimension perpendicular to the plate so that $I_{bar} = 1/12\ bd^3$ for flexure in the plane of the plate.

Reproducing the shearing deformation of the plate

Consider the influence of a uniform shearing stress, P/A, across the top of the plate or of the grid. For the elastic Plate

$$G = \frac{s_s}{\Theta} = \frac{P/A}{\Theta}\ . \quad (2)$$

Also

$$G = \frac{E}{2\,(1+\mu)} = \frac{P/A}{\Theta}\ . \quad (3)$$

Hence

$$\Theta_{plate} = \frac{2\,P\,(1+\mu)}{AE}\ . \quad (4)$$

By considering the influence of shear upon joint rotations of the grid

$$\Theta_{grid} = \frac{PL^2}{6\,EI}\ . \quad (5)$$

By equating the θ values of the plate and the grid we obtain

$$\frac{2\,P\,(1+\mu)}{AE} = \frac{PL^2}{6\,EI}\ . \quad (6)$$

Substituting $A = Lt = bd$ and $I = 1/12\ bd^3$, and canceling terms, we have

$$\frac{L^2}{d^2} = (1+\mu) \text{ or } \frac{d}{L} = \frac{1}{\sqrt{1+\mu}} = \sqrt{1-\mu+\mu^2-\mu^3+\ldots} \quad (7)$$

For convenience, we may write

$$d = \frac{L}{\sqrt{1+\mu}} \text{ and } b = \frac{Lt}{d} = t\sqrt{1+\mu}\ . \quad (8)$$

When $\mu = 0$, $d = L$ and $b = t$.
When $\mu = 0.25$, $d = 0.89\ L$ and $b = 1.12\ t$.

Member sizes

Hence, the member sizes of the grid are established. The area bd of each member replaces the corresponding area Lt of the plate and the ratio b/d is the ratio t/L times the quantity $(1 + \mu)$. It is to be noted that the grid members in the horizontal direction are identical with those in the vertical direction. It may at first seem peculiar that the grid members in either direction replace the entire area of the plate. However, when we realize that plate material does have the ability to resist forces in both directions while grid members are uniaxial, we find this relationship reasonable.

Division of load between columns and beams

Although their positions may reverse with change in direction of the main loads, we will temporarily find it convenient to look upon vertical members as columns and horizontal members as beams or girders.

Length of column $= L$.

Breadth of column $=$ depth of beam $= d = \dfrac{L}{\sqrt{1+\mu}}$

Area of cross-section of column or beam $= bd$

$$= t\sqrt{1+\mu}\left(\frac{L}{\sqrt{1+\mu}}\right) = tL\,.$$

Force necessary to shorten column one unit

$$\Delta = 1.0 = \frac{PL}{AE}\,. \quad \text{Hence}\,, \quad P = \frac{AE}{L} = \frac{tLE}{L} = tE\,.$$

For unit plate thickness, $t = 1$ and therefore

$$P_{col} = E \text{ (for unit deformation and unit plate thickness)}\,. \qquad (9)$$

Force necessary to deflect beam one unit by flexure alone as a fixed end beam or as a double cantilever.

$$\Delta = 1.0 = 2\left(\frac{P\,(L/2)^3}{3\,EI}\right) = \frac{PL^3}{12\,EI}\,.$$

$$\text{But } I = \frac{1}{12}\,bd^3 = \frac{1}{12}\left(t\sqrt{1+\mu}\right)\left(\frac{L}{\sqrt{1+\mu}}\right)^3 = \frac{tL^3}{12\,(1+\mu)}\,.$$

$$\text{Hence } \Delta = 1.0 = \frac{PL^3}{12\,E}\left(\frac{12\,(1+\mu)}{tL^3}\right) = \frac{P\,(1+\mu)}{tE}\,.$$

For unit plate thickness, $t = 1$ and therefore

$$P_{beam} = \frac{E}{1+\mu} \text{ (for unit deflection and unit plate thickness)}\,. \qquad (10)$$

Relative stiffnesses for force distribution

$$\text{Column stiffness} = 1.0 \text{ (from } P = E \text{ when } \Delta = 1 \text{ and } t = 1)\,. \qquad (11)$$

$$\text{Girder stiffness} = \frac{1.0}{1+\mu}\left(\text{from } P = \frac{E}{1+\mu} \text{ when } \Delta = 1 \text{ and } t = 1\right)\,. \qquad (12)$$

These relations control the division of load or of joint force between the columns and girders meeting at any joint.

When $\mu = 0$, $d = L$, $b = t$ and the force distribution to columns and girders meeting at a joint is identical.

Relative stiffnesses for moment distribution

Since the cross-section of columns and girders are identical the resistances to joint rotation are the same for all members.

Grid analysis by strain justification

The grid analogy as applied to gusset-plate analysis will be outlined as a series of steps and then one very elementary example will be given so that the reader may have a better understanding of the numerical procedures involved. Of course, special devices are needed for more difficult problems, but space is not available for their consideration here.

(1) Study the gusset plate under consideration and select two perpendicular axes which will be assumed to remain straight lines. Although lines that distort under load have been used as axes, additional corrections are involved which will not be discussed here. A line of symmetry is one preferred axis. If no better choice appears obvious, a boundary line of the plate may be chosen as an axis when all applied forces intersect at a point.

(2) Using the lines chosen above as fixed bases, direct stresses in columns and girders of the grid are estimated. This procedure requires experience and judgment, but after a few elementary cases have been solved it is found that much more complex cases can be handled with assurance.

(3) Based upon estimated direct stresses and fixed axes or base lines the deformed coordinates of each joint become known, and the fixed-end shears in all members of the framework may be calculated therefrom. These shears give rise to fixed-end moments which may be balanced and distributed, and thus a first set of balanced moments are obtained.

(4) From the balanced moments a set of shears for all members are determined. These shears will be compared at each joint with the estimated direct stresses to see if the joint is in equilibrium. Since a perfect estimate of the direct stresses could not be hoped for, there will be some lack of equilibrium at each joint which can be expressed as two perpendicular joint restraints.

(5) At this point it is wise to compare the joint restraints in one direction with the loads and estimated direct stresses in the same direction. If the restraints considered individually or in groups represent an influence in excess of 20 % of the applied loads in that direction, the original estimate of direct stresses needs to be improved. Based upon preceeding work it is possible to improve the estimated direct stresses to reach this standard of accuracy without great difficulty.

(6) Once an estimate of direct stresses has been obtained which gives rise to joint restraints within the limitation set in step (5), a routine joint-by-joint relaxation of restraints, first in one direction and then in the other direction, will nearly complete the analysis. In this step, joints are not permitted to rotate.

(7) Of course, step (6) will change all girder shears with resultant changes in end moments, and the moment balance of step (3) will be disturbed. It is a simple matter to rebalance moments and determine the influence of the accompanying shear changes upon the balanced joint forces achieved in step (6). Now it becomes clear why a reestimate of direct stresses or even several reestimates may be preferred to a routine repetition of relaxation steps. When one is working in two directions the routine procedure becomes extremely tedious. An estimate of direct stresses can usually be made that eliminates much of this tedium.

(8) The steps of force relaxation and of moment distribution can be repeated as often as necessary to achieve any desired degree of accuracy of grid stresses. However, the accuracy attained in the analysis of the grid should be correlated with the fineness of the subdivision of the grid since the basic approximation of the method is found in the latter relationship.

(9) Subdivision of the grid need not be uniform. In fact it is clear that for equal accuracy throughout the grid the subdivision should vary in fineness being made finer in the regions of steep gradients of stress.

(10) The influence of Poisson's ratio μ is introduced into the grid merely through lengthening members in one direction by μ times the shortening of members perpendicular thereto or *vice versa*. However, since Poisson's ratio has not been found to influence stresses away from interior load concentrations, it will not be considered further here. As a matter of interest, when Poisson's ratio is introduced with loads applied at 45 degrees to the direction of the grid it is found that deformations in line with and perpendicular to the loads do bear the relationship predicted by Poisson. Hence the grid is more nearly isotropic than might be anticipated from its directional appearance.

Interpretation of grid forces as plate stresses

This interpretation is a study in statistics not unlike the interpretation of measured strains with electrical or mechanical strain gages. The grid subdivision is finite while a point stress in a plate is of a different character. However, there is considerable question as to the contribution of point stresses to the failure of a gusset plate. Most engineers would probably be satisfied to accept measured strains over one-inch gage lengths as indicative of stresses that might produce failure of a gusset plate. Such average or statistical strains or stresses can be obtained by the procedure of grid analysis, but point stresses can only be approximated. Of course, stresses around rivet holes, which are not obtained by the procedure of grid analysis, might be super-imposed upon the final stress pattern. Both experimental and mathematical results are available for this purpose.

We conclude then that the physical make-up of the grid gives it a different relationship to the structural plate than that borne by the idealized homogeneous, isotropic, continuous material considered by the mathematical theory of elasticity. In reality, internal grid forces represent statistical or average stresses over lengths controlled by the bar spacing of the grid. It is the author's belief that in many cases such statistical stresses when obtained from a grid of proper unit dimensions or bar spacing will give more useful relative numbers for picturing the remaining quasi-elastic range before structural damage results than will the point stresses determined by the mathematical theory of elasticity. The lattice or unit dimension of the grid for this purpose needs study since there should clearly be a finer subdivision of the grid where the stress gradient steepens. This approach to analysis and design might properly be termed the acceptance of a Statistical Theory of Elasticity.

Conclusions

Methods of numerical analysis when applied to plates and wall problems have in the past proved to be extremely tedious. Such methods required organized machine computations and therefore they could be of little help to a designer. The physical tool of grid analogy has been simplified by three devices (1) the concept of permitting initial relative movements of all joints in agreement with statics, (2) the concept of « strain justification » or checking the accuracy of this initial estimate of stresses and deflections and then revising that estimate as often as necessary to avoid tedious joint relaxations (3) the procedure of using a finer grid only in regions of steep stress gradients. The first two simplifications are merely refinements of the author's procedures of wind stress analysis of tall buildings. The third has been used in some form by most writers on numerical methods. These simplifications have reduced the time consumption to the point where a grid analysis for an important problem may soon become feasible within the time limitations of a design office. Further simplifications are in prospect.

The concept of a statistical approach to internal stresses is an important one. The fact that grid analogy automatically produces average stresses rather than point stresses is a favorable factor if used intelligently. It is particularly meaningful that the subdivision of the grid can be related roughly to the stress gradient. Thus it would be possible to control in a crude way the relationship between the statistical or average stress and

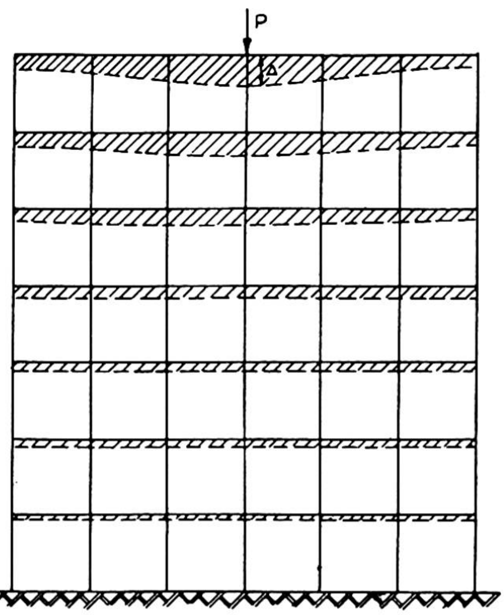

Fig. 1. Estimate of joint movements due to a vertical load on the grid.

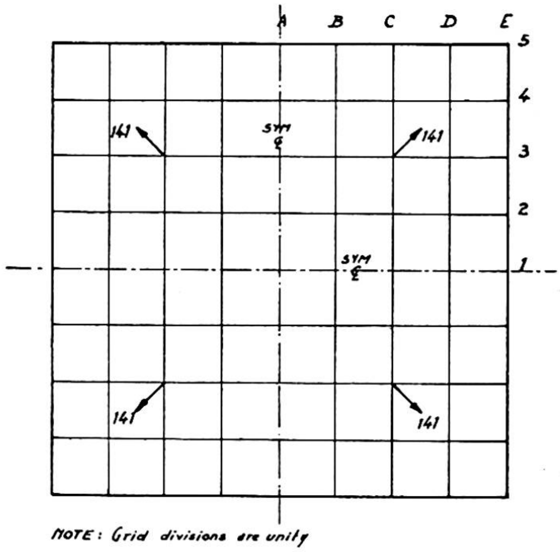

Fig. 2. Loaded gusset plate with grid superimposed.

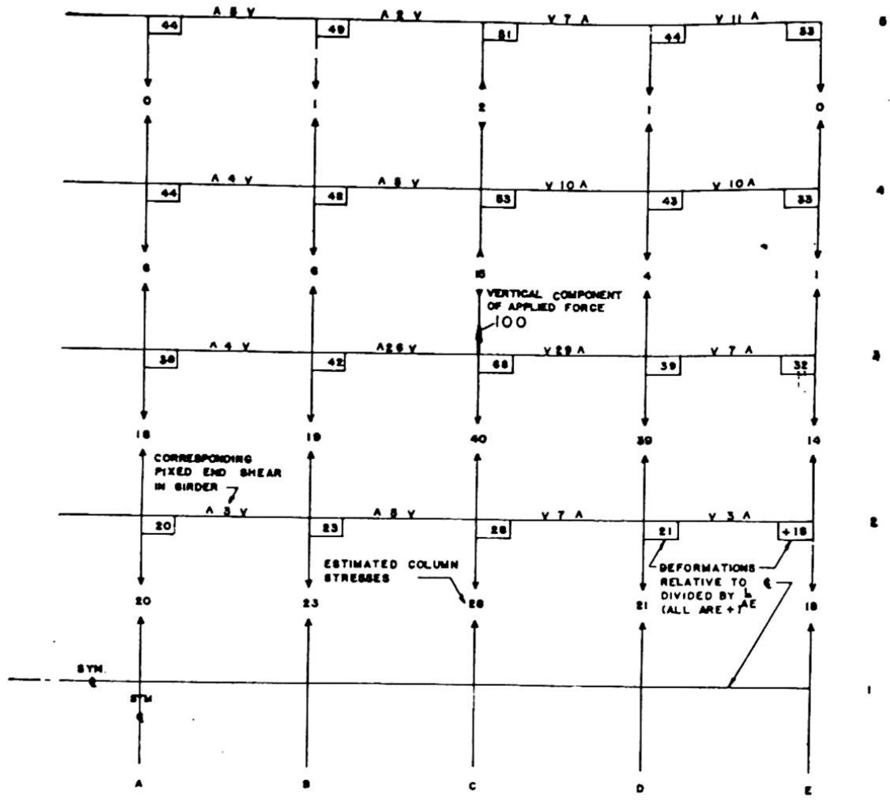

Fig. 3. Estimate of vertical column stresses.

When turned at 90° this sheet will also represent the estimate of horizontal girder stresses due to the horizontal force of 100.

the theoretical maximum stress. Finally grid analogy makes use of the physical concepts of column stresses, girder moments and shears, column shortening, joint rotation, moment distribution and joint relaxation that have become the standard working tools of the structural designer. They are merely put together here in an effective form for as rapid analysis as possible of the analogous grid.

Problem of gusset-plate analysis analyzed by Marvin Mass (3)

The simple problem illustrated by figure 2 was analyzed by the procedure of grid analogy. The subdivision into 8 by 8 grid spaces with two lines of symmetry reduces the study to that of a 4 by 4 grid. The initial estimate of stresses in the vertical direction due to the vertical component of the diagonal force is shown in figure 3. Based upon these estimated stresses in vertical members the joint movements (SL/AE) were computed and recorded in boxes at the joints. Since adjacent horizontal joints have unequal movements there are fixed-end shears developed in all horizontal members which are recorded on the member itself. If the sheet of figure 3 is turned at 90 degrees, it will also represent a proper estimate of horizontal girder stresses due to the horizontal component of the diagonal load shown in figure 2.

When fixed-end shears in horizontal and also vertical members are converted into fixed-end moments for members of unit length and recorded at the end of each member as in figure 4 the joints will not be in moment

(3) Assistent Professor, Department of Civil Engineering, University of Colorado.

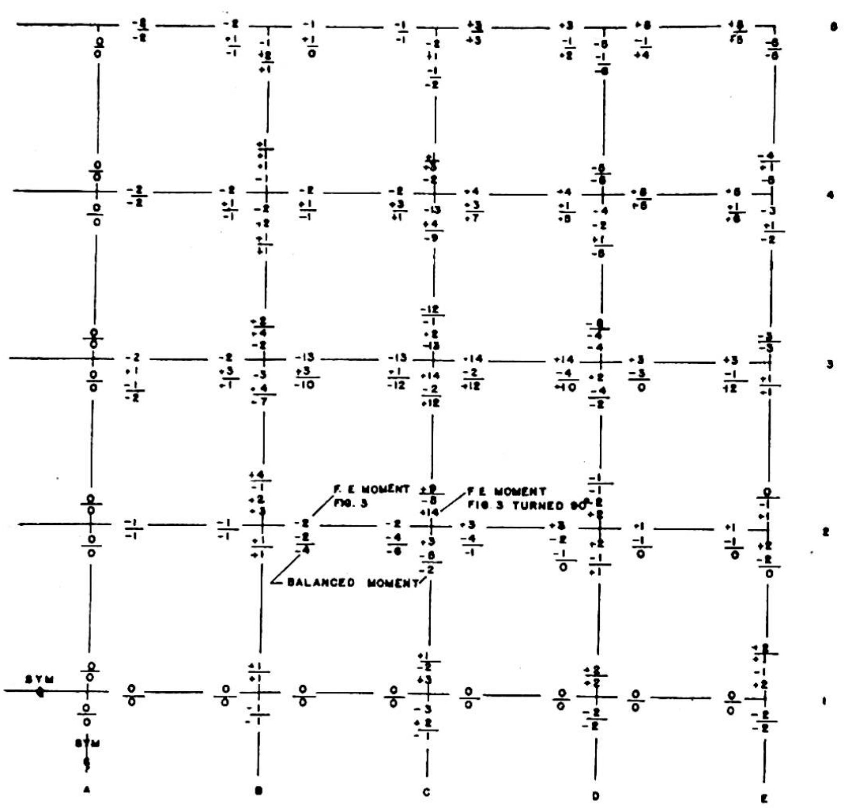

Fig. 4. First moment balance. Initial fixed-end moments computed from fixed-end shears in Fig. 3.

equilibrium. Hence on figure 4 is illustrated the distribution of moments to reach rotational equilibrium of the joints. The final shears in all horizontal members after moment distribution are computed from figure 4 and introduced along with the estimated stresses in vertical members on

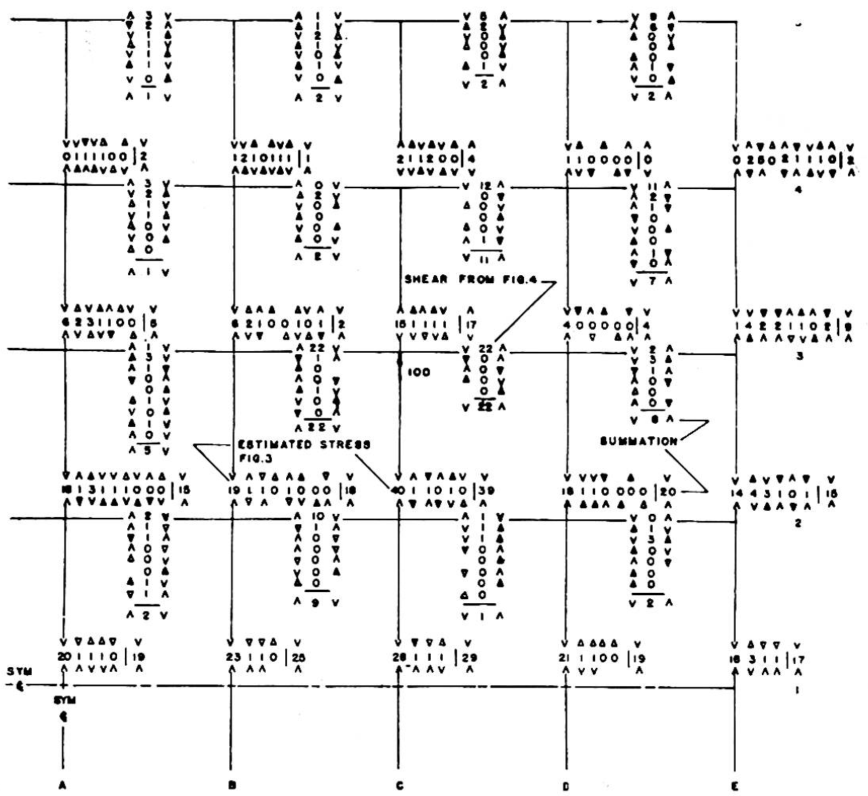

Fig. 5. First vertical force balance.

Arrow head indicates direction of action on adjacent joint. Arrow head is closed on side from which balancing force comes.

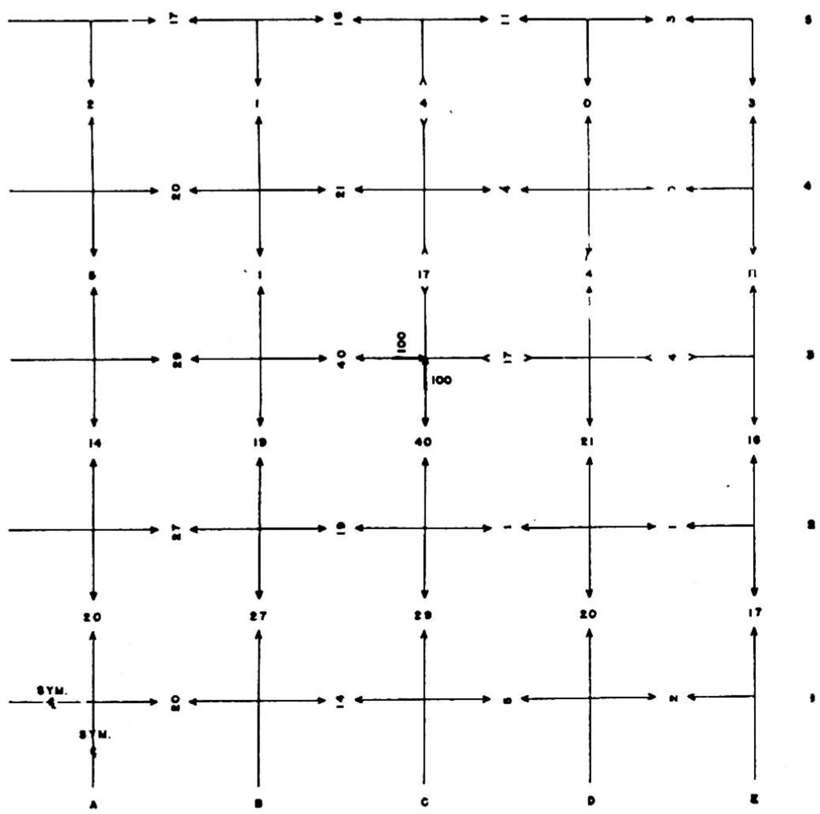

Fig. 6. Final direct stresses for one quarter of the gusset plate.

figure 5. Then the procedure of relaxing vertical joint restraints and permitting vertical translation without rotation joint-by-joint is carried out in figure 5. The first number at the top or to the left of a sequence of numbers represents the initial value. The final value occurs below or to

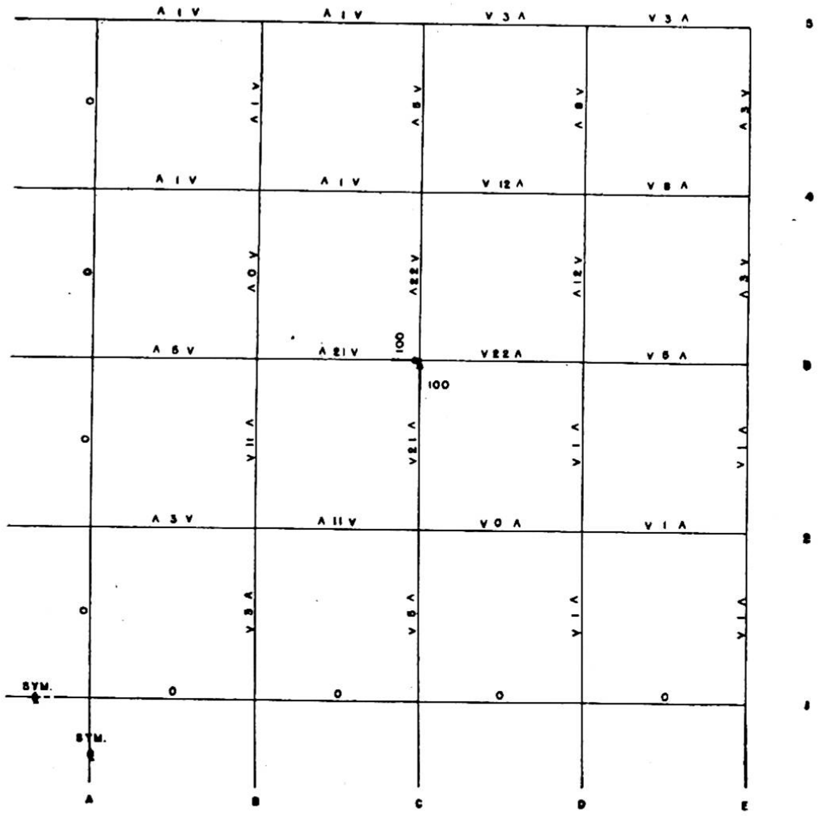

Fig. 7. Final shears for one quarter of the gusset plate.

the right of the sequence and is separated from the sequence by a short straight line. By turning the sheet of figure 5 at 90 degrees the effect of relaxation of restraints in the horizontal direction is obtained.

As in all instances of numerical procedures it is now necessary to reconsider the influence of the relaxation achieved in figure 5 upon the moment distribution of figure 4. Then any redistribution of moments found necessary will produce changes in shear with small accompanying joint restraints to be again relaxed. For the example chosen these corrections were all found to be minor and they are not illustrated here. However, the results in terms of the direct stresses in figure 6 and the shears in figure 7 do include these small corrections. If the subdivision of the grid had been somewhat finer it would have been possible to introduce average direct stresses at a joint and the average shear into Mohr's circle to compute a statistical or average value of the principal stress corresponding to the point on the plate represented by the joint of the grid. Finer subdivision would permit the computation of principal stresses closer to the application of the load but the ultimate stress-concentration would have to be superimposed upon the stress pattern obtained by grid analogy.

Résumé

Les déformations d'une tôle pleine, soumise à une sollicitation quelconque, peuvent être prédéterminées avec une approximation suffisante, en analysant, à la place de la tôle, un treillis constitué par des barres formant un réseau octogonal à nœuds rigides. La détermination des efforts dans un tel treillis se fait par la méthode préconisée par l'auteur pour la détermination des cadres à étage soumis à l'effort du vent. L'auteur montre que le cas spécial d'un gousset de nœud s'apparente au problème général par lequel les efforts s'appliquent dans la partie centrale. En annexe, il donne un exemple de calcul de cette méthode appliquée à un nœud de gousset du type classique.

Zusammenfassung

Die Deformation einer vollen Scheibe infolge einer beliebigen Belastung, können mit guter Genauigkeit näherungsweise bestimmt werden, indem statt der Scheibe ein zweckmässig ausgewähltes Netz von steif verbundenen, senkrecht aufeinander stehenden Stäben untersucht wird. Die Bestimmung der Kräfte in derartigen, steifen Stabwerken geschieht nach der vom Verfasser veröffentlichten Methode zur Untersuchung von durch Windkräfte beanspruchten mehrstöckigen Rahmen mittels Momenten- und Kräfteverteilung. Es wird gezeigt, dass das spezielle Problem des Knotenblechs denjenigen Platten- und Scheibenproblemen verwandt ist, bei denen die Kräfte ausschliesslich in der Mittelebene wirken. Ein Beispiel dieser Berechnungsmethode, angewandt auf ein typisches Knotenblech durch Marvin Mass, wird in einem Anhang vorgeführt.

Summary

By properly proportioning an analogous grid of rigidly connected members meeting at right angles the physical deformations of a continuous

plate due to any loading may be approximated with good accuracy. The analysis of the stresses in such a rigid gridwork follows from the author's previous publication of a method of analyzing for the wind stresses in a multistory building frame by moment and force distribution. The particular problem of a gusset plate is shown to be similar to any plate or wall problem where the forces exist entirely within the plane of the plate. An example of the method of analysis, as applied to a typical gusset plate by Marvin Mass, is presented in an appendix.