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Rubrik: V. Theory and research work on details for steel structures of welded and riveted construction

V

Theory and research work on details for steel structures of welded and riveted construction.

Theorie und Versuchsforschung der Einzelheiten der Stahlbauwerke für genietete und für geschweißte Konstruktionen.

Etude théorique et expérimentale des détails des constructions métalliques, rivées ou soudées.

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V

General Report.

Generalreferat.

Rapport Général.

L. Cambournac,
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Twelve papers have been submitted in reference to Question V. In what follows below it is proposed to analyse briefly each of these papers and to extract from them the propositions that can be regarded as conclusions of the Congress.

Paper by Dr. Grüning.

For the first time, in a bridge at Crefeld on the Rhine, contact joints have been utilised over the piers, with joint covers and rivets designed to carry only part of the total load. This arrangement was adopted after making two series of tests; in the first of these one half of the columns had no joints and the other half had joints with cover plates at mid-height representing 45 % of the cross section of the column and 52 % of its moment of inertia; in the second series one half of the columns were uncut while the other half were sawn across the middle and the two lengths simply butted against one another. The columns were subjected to compression extending over the whole cross section, either along the axis or eccentrically.

The tests showed that the columns formed with simple contact joints could carry the same loads as those without any joints, except for a reduction of 10 % in the case of those columns which had been sawn in two parts and were loaded eccentrically.

Dr. Grüning concludes that it is perfectly safe to use partially cover-plated contact joints in columns under compression provided that any peculiarities of the construction are taken into account when dimensioning the cover plates. Such a method of forming the joints may be extended to the compression members of bridges.

These indications are interesting, and the suggestion put forward by *Dr. Grüning* at the end of his paper on the subject of using contact joints in the compression members of steel structures deserves attention. It remains to be ascertained in practice, however, whether the economy realised in weight of metal and labour of fitting the cover plates will not be absorbed or outweighed by the cost of machining the contact surfaces.

Paper by M. Graf.

This paper deals with the testing of rivetted joints in steel St. 52 under alternating loads which change from tension to compression and back; or under repeated loads which do not change in sign. The tests showed:

1) that rivetted joints are capable of carrying a (30 %) greater amplitude of loading under alternating stress than under pulsating stress,
2) that the ratio between these two amplitudes falls off as the pressure around the edge of the holes is increased.

The author draws attention to those kinds of strain which produce a permanent effect. These are due to the play of the rivets in the holes and to the resistance of the members against slipping, the latter depending on the coefficient of friction of the surfaces in contact and on the adhesive force which results from rivetting. Usually fracture was found to begin at the edge of the rivet holes in the outer rows.

The striking feature of the interesting tests carried out by M. *Graf* is the relative magnitude of the forces applied to the joints. Even though we are concerned here with steel 52 which may have an elastic limit up to 35 or 40 kg/mm^2, the question arises whether, in practice, occasion is ever likely to arise for subjecting joints to such amplitudes of loading as were used in the tests (— 14 to + 14 kg/mm^2, or 0 to + 20 kg/mm^2). It would be valuable if experimental studies of joints under loads less intense than these could be carried out, especially with a view to ascertaining how great, under various conditions of test, are the alternating or pulsating loads at which the resistance to slipping of the portions joined is exactly reached (period of non-permanent strains).

Paper of M. Chwalla.

The author of this report makes a study of the buckling of the web plate in web plate girders. He deals with the case of a rectangular plate supported on all four edges without marginal restraint and subjected to pure bending on its own plane. He has found that where such a plate is provided with a horizontal stiffener placed at one quarter of its depth, as measured from the top, this stiffener will at first bear upon the plate; but as the rigidity of the stiffener increases the plate is reinforced after the moment is passed whereat the stiffener is no longer subject to buckling, and if the rigidity of the stiffener continues to increase, the plate may assume either of two different shapes under the same load: either a half wave in the direction of its length or a series of short ripples on either side of the stiffener. The author has found similar results in rectangular plates subject to compression and shear. He further shows that the approximate study of stiffened plates may be simplified by using the idea of a "substituted plate" which is assumed to exist in the compression portion represented by half the height of the plate under consideration.

The conclusion which the author draws from his experiments is that it is not possible to design horizontal stiffeners for thin plates simply from consideration of their own buckling tendency.

The report which has been briefly analysed above forms a theoretical contribution, of great interest, to the problem of stiffeners on plate web girders. One cannot but hope that the methods of calculation suggested by M. *Chwalla* may be applied in actual cases and may be followed by direct measurement of the actual deformations that occur in the webs and their stiffeners.

Paper by M. Ridet.

M. *Ridet* gives an account of experiments carried out for the purpose of measuring the secondary stresses in the verticals and diagonals of a single track N-truss steel railway bridge, with the decking below and the bracing above. He makes the following comparisons between the recorded results and the stresses calculated by the methods of *Pigeaud* and of *Fontviolant*:

Principal stresses.

Less by 28 % than the calculated stresses (owing to relief of the main girders by the longitudinal girders of the decking).

Secondary stresses.

1) In the diagonals the actual stresses are of the same order as those calculated.
2) In the verticals the actual stresses are at least twice those calculated.

This anomaly may be partially explained by unequal distribution of the stresses over the cross section of a vertical member due to the method of connecting the latter to the booms; also by the warping of the main girders, and by the influence of the gussets.

From his results the author deduces the following rules:

a) In triangulated trusses the use of vertical members should be avoided.
b) The connections of the truss should be designed in such a way as to afford a uniform distribution of the principal stresses.
c) The influence of the gussets should be studied.

The report by M. *Ridet* is a valuable contribution to the experimental study of triangulated structures. Before definitely adopting his preference fo the V-truss by comparison with N-truss it would appear desirable that new experiments should be carried out with a view to throwing more light on the question of why the secondary stresses, as measured in the verticals, agree less well with the results of calculation than those measured in the diagonals.

Paper by M. Krabbe.

This paper is a study of the rhomboidal type of truss, which is a double truss withhout verticals. Usually such trusses are designed by considering them to be made up of two V trusses. From his earlier investigations, in which the diagonals were free over their whole length, and account was taken of the rigidity of the booms, the author concludes that:

a) The rigidity of the booms is the preponderating factor.
b) The girder is stable even without verticals, and in any case the influence of these is purely local.
c) It is desirable to limit the depth of the booms in order to avoid excessive bending stresses.

In this new report on the subject the author puts forward a complete method of calculation in which account is taken not only of the rigidity of the booms but also of that of the diagonals and connections, and of the inequality of section of the booms. By considering the case of a girder provided with verticals, first assumed to be framed in and then as pin-jointed, he reaches that of the girder described at the beginning and he works out influence lines for the deformations in the boom members and diagonals and also for the moments existing at the ends of the members. Despite its initial complexity the problem is solved by reference to only three systems of equations of the *Clapeyron* type.

M. *Krabbe* has made an important theoretical contribution to the calculation of multiple truss girders without verticals. It is to be observed, however, that the author himself points out how good an approximation may be obtained by simpler methods of calculating such girders, and the question may be asked what are the cases wherein it is of practical importance to apply the complete method as given by him. This is a point on which engineers would like to obtain further information.

Paper by Prof. Campus.

The author begins by drawing attention to the importance of the intersections in the construction of vertically superimposed continuous steel frames. He recalls his earlier tests made on various uni-planar models of sheet metal, from which he deduced that the best form of joint consisted of two curved gussets, one below and one above the girder. He has proceeded to make tests on rivetted structures of this type and has been able to show that the presence of curved connections, fitted tangentially to the booms and vertical members, has the effect of relieving the principal elements of the construction; also that the transmission of stresses through them is effected gradually, and that the maximum stress occurs in the neighbourhood of the joint between the gusset and whatever member receives the greatest bending moment. These results are confirmed by tests on welded structures. Finally the author explains the general characteristics of rigid intersections and their method of calculation. He insists on the necessity of using a higher factor of safety than in the remainder of the structure.

This very clear and well documented treatment by Prof. *Campus* leads to the conclusion that the type of intersection which he describes ought to be used wherever it is not incompatible with constructional or architectural considerations.

Paper by Prof. Baker

The author draws attention to two of the results achieved in the course of experimental work in England by the Steel Structures Research Committee:

1) The first of these conclusions refers to the distribution of the bending moments transmitted by a beam to a stancheon and to different parts of the connection. Generally it is assumed that the amount of moment so transferred is proportional to the "rigidity" of the respective elements of the stancheon, but it has been found in the case of industrial structures, with only one exception, that the lower parts of the column receive a larger proportion of the moments than this implies. The anomaly may be explained as follows: the connection consists of a bracket below and a cleat above the beam, and while the former is able readily to participate in the deformation produced, the latter bears against the stancheon and engenders an axial compression in the latter. This tends to lessen the bending moment in that element of the stancheon which is above and to increase the moment in the element below.

The same observations were made on a steel framework of several stories with beams unequally loaded.

2) The second of the facts established has reference to the torsional stresses in double T-sections. In the case of beams these are due to eccentricity of loading, and in the case of stancheons to imperfections in the form of connection. Practically speaking, in the usual forms of construction the resistance to torsion of the various elements is adequate, but this torsion should be taken into account when the girder is unsymmetrically loaded.

The points made in Prof. *Baker's* contribution deserve all the more attention because the structures on which the observations were carried out appear to be of the usual industrial type. It would seem desirable to arrange similar tests on frameworks with other types of intersections in order to confirm whether the results are generally valid.

Paper by Mr. Andrews.

The tests already begun on steel frames were carried to destruction by the author, making measurements of the deflections. The tests were performed on a simply supported double T-beam, and also on frames built up from beams of that section with verticals dimensioned so as to attain their limit of resistance at the same time as the beams.

The author gives the diagrams and results obtained in these comparative tests. The cleats connecting the beams to the stancheons showed no deformation and the beams received much heavier stresses than the stancheons, a result which is at variance with the hypotheses used in the approximate calculation of this type of structure.

It would appear desirable to repeat these tests on small scale models, introducing different forms of joint at the intersections. This would, no doubt, enable more positive conclusions to be drawn than those derived from the tests of which Mr. *Andrews* has given an account.

Paper by M. Bleich.

The classical theory of bending in prism-shaped bars is based on the hypothesis of a linear distribution of stress over the whole of the cross section and on the absence of longitudinal forces in plain torsion. It has frequently been shown that this hypothesis is incorrect so far as bars built up of thin plates are concerned.

The authors propose to set up a general theory for such bars, whether of open or closed section. The hypotheses they adopt are the following:

a) The geometrical shape of the bar remains constant, the cross section not remaining plane but each element therein following *Navier*'s law.

b) Bending at right angles to the plane of the wall, and shear stresses due to bending, are neglected.

The differential equations of the problem are supplied by the equilibrium of the external and internal forces. The authors determine the strain energy in members of simple and multiple composition (the latter being the general case in actual application), and proceed to derive the differential equations for bending and torsion. These equations disclose the existence of an axis of torsion in the member, such torsion being zero when the resultant of the external forces passes through the centre of torsion of the section.

The authors then give indications as to how the bending and torsional stresses may be determined, and they further deal with the problem of unstable equilibrium:

a) buckling (member loaded concentrically), and

b) overturning (a member in which the median line has undergone deformation under the action of bending).

They show that the resistance to buckling of a member made up of thin sheets is less than the load calculated according to *Euler*, and does not attain that value unless the resistance to torsion is rather high. As regards overturning, the critical load is at a maximum when the normal force passes through the centre of torsion of the section.

There can be no question of discussion here the important contribution made by the authors to the study of the theory of bending and torsion in members formed of thin sheets, but the hope already expressed in regard to the theoretical studies of MM. *Chwalla* and *Krabbe* may here be repeated: namely that these results may form the object of practical applications and experimental confirmation.

Paper by M. Laffaille.

The problem involved in roofing over a building may be stated as follows: given the cubic volume and the external forces known to be acting, to construct a cover which will enclose the space and at the same time will carry to the supports the reactions derived from these external forces.

The author has solved this problem by forming the surface of thin sheeting. First of all he made semi-self-supporting roofs, with the sheeting carried on frames; then, after making tests on cardboard models, he abandoned the use of frames and stiffened the sheet itself transversely, so obtaining a roof that is entirely self-supporting. He gives several examples of such roofs which have been or might be carried out, and observes that in large spans it is necessary to take account of buckling.

He points out the novelty and pleasing appearance of these forms of roofing, and after explaining his theoretical and experimental investigations as regards the stiffeners he indicates the principle of the methods whereby stiffened sheet roofs may be designed.

The report of M. *Laffaille* would seem to offer some completely original solutions to the problem of roofing over buildings, and it may well be that this device, giving rise as it does to quite new forms, will be an enrichment of the art of architecture. It is too early, however, to pronounce on its economic importance, having regard both to first cost and to maintenance, and it is to be hoped that industrial applications will soon furnish an answer to this question.

Paper by M. Fava.

For the purpose of covering over the hall in the new station at Florence use was made of plate webbed girders of 30 m span of double T section with the axis bent at two points so that the ends make angles of 135° and 150° respectively with the central portion.

Tests were carried firstly on models made of transparent material, then on two steel girders constructed to a scale of $^1/_5$, finally on the girders themselves in position.

As a result of the preliminary tests the thickness of the flanges was made 20 to 30 mm at the ends, and the following observations were made on the finished structure:

1) In the straight portions of the girders the stress in the web obeys a law which is practically linear, but in the wings this stress falls off from the inside towards the outside face and from the middle towards the ends.
2) In the end portions the maximum stress differs by 250 % from that in the adjoining sections, but this stress agrees with the result of calculation when account is taken of the stresses in the wings.

These tests are of great interest and have been pursued very methodically. It may be hoped that a similar method may be applied in the study of constructional elements and may be repeated on a great many examples.

Paper by M. Kolm.

The author made tests on bridges in service in which measurements were made to ascertain how far a slab of reinforced concrete supported on steel girders contributes additional strength to the latter. It was found in the case of seven structures with slabs of very varying dimensions that the influence exerted by the slab is greater than can be attributed even to complete collaboration with the girders; the reason for this may lie in uncertainty as to the modulus of elasticity of the concrete, and also in the influence of the railings.

In an eighth structure tested the opposite result was obtained, the slab becoming detached. In the case of bridges of great width (in excess of 9 m) the slab exerts only a partial effect. In continuous girder bridges the slab is apt to crack in the zone of negative moment, and it is necessary, therefore, to design the reinforcement in this zone taking account of the collaboration between the slab and the steel girders.

Measurements of the kind discussed in this paper are extremely useful and it is to be hoped that they may be multiplied in all countries, so as to realise all possible economy in a type of bridge which is now universally accepted.

V 1

The Development of Steelwork Design and Details, shown in Heavy Movable Plant for Lignite Mining.[1]

Entwicklung des Stahlbaues und seiner baulichen Einzelheiten durch den Bau der fahrbaren Großgeräte des Braunkohlen-Bergbaues.[1]

Le développement de la construction métallique montré par la construction du grand outillage mobile des exploitations de lignite.[1]

Dr. Ing. K. Beyer,
Professor an der Technischen Hochschule Dresden.

It is the author's intention in the few minutes at his disposal to draw attention to a field of work which lies on the boundary of German structural steel engineering proper, and which owes its development to the last few years but has already attained considerable importance. The impulse for this development

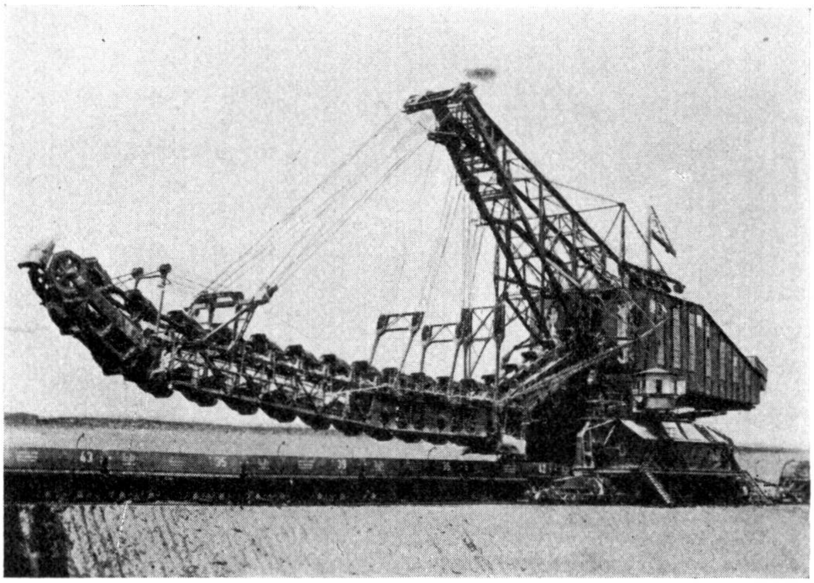

Fig. 1.

has been due to the German lignite mining. In Germany, lignite is obtained from large open casts, in seams which may be up to 60 m thick. The removal of the

[1] Extract from a longer publication to be made later.

overburden from the coal is effected by large moveable pieces of plant weighing as much as 5,000 tonnes, and the steel construction of these is in many ways similar to moveable steel bridge structures of the largest sizes. Their calculation and design has given rise to many new theoretical and constructional problems to which brief reference will be made here.

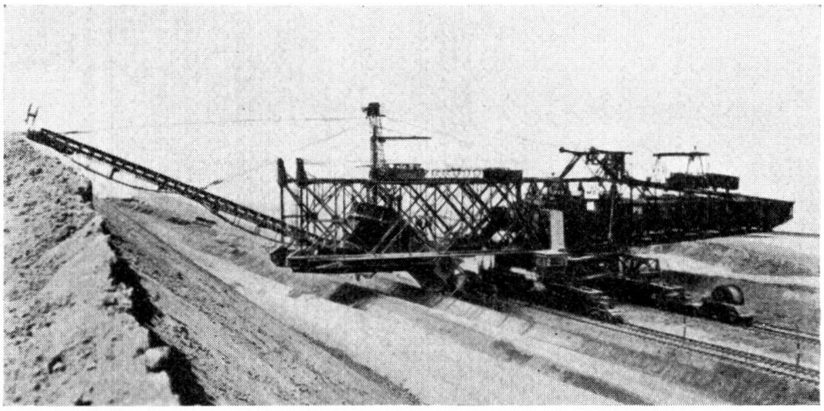

Fig. 2.

The steel structures in question serve to carry the plant for loosening and loading the coal (Fig. 1), dumping the spoil (Fig. 2) and refilling the emptied lignite pits by means of conveyor belts over the shortest routes (Fig. 3). These structures are subject to numerous external forces, some of which are scarcely known even as to their order of magnitude, and they move on rails layed

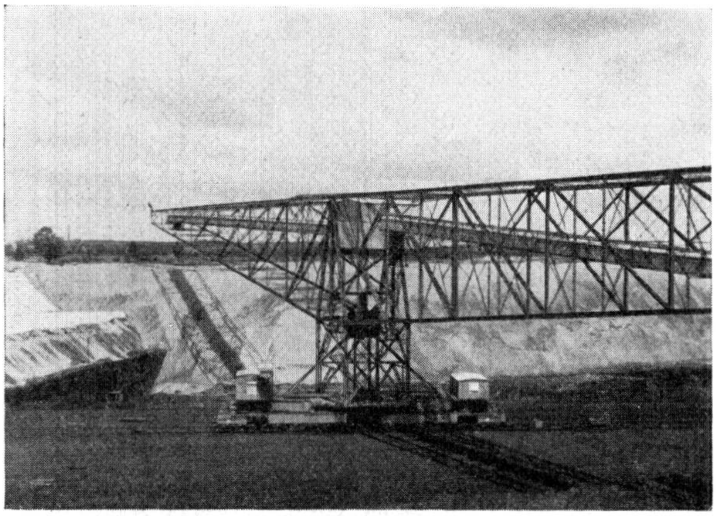

Fig. 3.

directly on the ground which are continuously being shifted about by machines, so that while the plant is operating its inclination to the horizontal is continually changing.

In this way a large elastically connected structure, subject to heavy loads, has to be supported over a large area at a large number of points which are moveable in all three dimensions so that only small supporting reactions are produced (Fig. 4). This is secured by the use of suitable balancing devices or hydraulically coupled cylinders, so arranged as to obtain a three-dimensional chain of elastic members with considerable freedom of displacement relatively to one another. The need for the supporting points to be free to move vertically

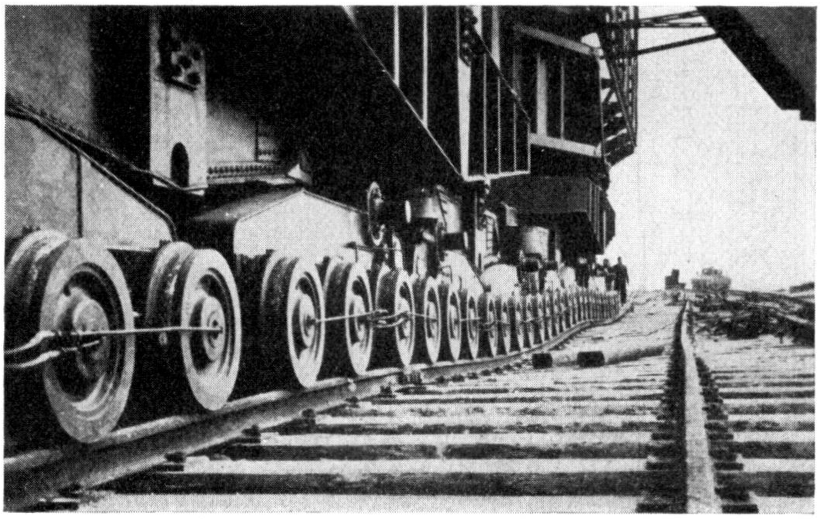

Fig. 4.

and horizontally (within certain limits) compels the adoption of a three-dimensional system which is kinematically determined and which, when the degree of freedom conferred upon it to allow for movement is removed, will remain kinematically rigid in all its positions, retaining no further power even of infinitesimal movement. In this way all reactions and intermediate forces between

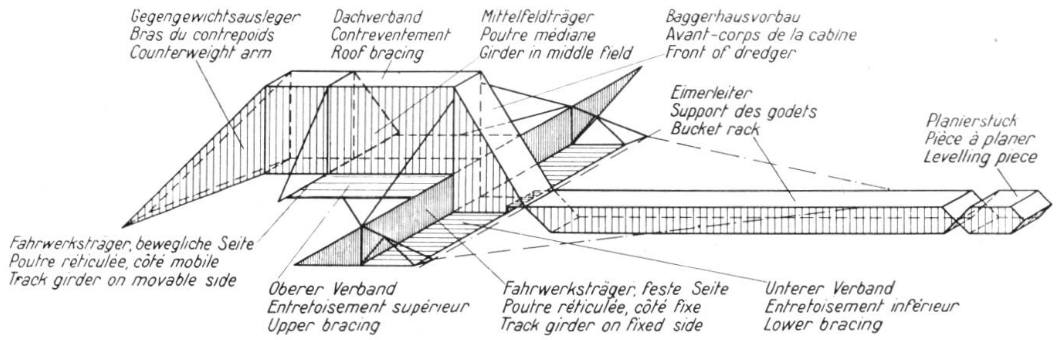

Fig. 5.

The winding gear and supporting mast of the bucket rack are omitted.

the members of the chain will be statically determinate for any system of loading in all three dimensions, regardless of how the machine as a whole is being moved about on caterpillar tracks or on wheels running on rails which may be curved in space relatively to one another. It follows, therefore, that the statically determinate system has to be such as will satisfy analysis carried down to each individual wheel, whatever system of external loading is assumed.

The members composing the three-dimensional chain are made up of lattice work, or space structures, connected by two, three or four bar links with journal, ball or roller bearings, or hinges. These members serve as supporting structures for mechanical and electrical plant; or for the balancing arms, motion gear and platforms of slewing apparatus. They have, therefore, to be either latticed or plated girders in three dimensional arrangement, rigid against bending or torsional effects in any direction.

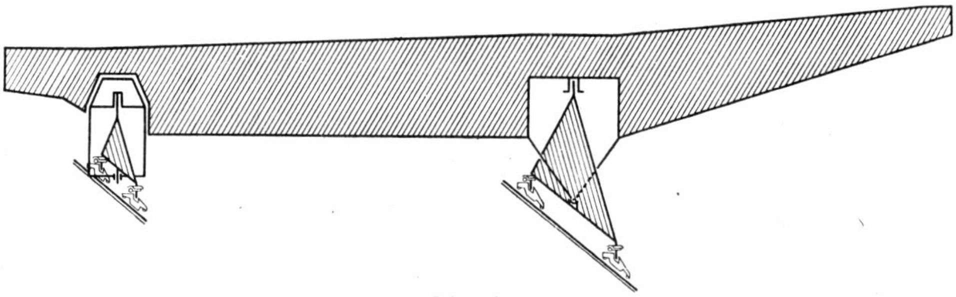

Fig. 6.

Examples may be seen in the outline sketch of the space frame work for a deep dredger with three-point support (Fig. 5), and in that of the structure moveable in three dimensions which serves to carry a spoil conveyor (Fig. 6). Here, under patents of the Mitteldeutschen Stahlwerke, the axis of the girder can be swung to 45 ⁰ from the direction of travel.

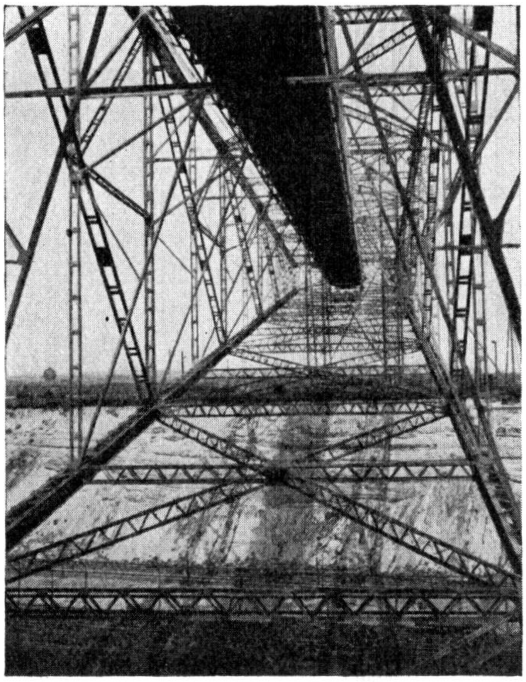

Fig. 7.

In all work of this kind the designer is compelled to concern himself with many three-dimensional problems foreign to the usual practice of structural engineering, and has continually to refer back to the fundamentals of mechanics and elastic theory in order to develop methods suitable for his needs, or to assess the validity of approximate solutions by making rigorous investigations of the statics of plates, shells or slabs.

Frequently these problems consist in the calculation of statically determinate or indeterminate lattice space structures under three-dimensional systems of loading (Fig. 7), and in the examination of surface structures for supporting a truck with a slewing motion and distributing the wheel loads (which are imposed on a relatively small portion of the platform) over as large an extent of the track as possible. For this purpose use is made of supporting structures which are rigid against bending and twisting, and which are carried on three points hydraulically compensated under heavy loads. Four pairs of plate web girders intersecting one another are gussetted on the upper and lower flanges in order to provide the necessary degree of rigidity against torsion.

The structure can also be developed from a circular girder with one or two concentric cylinders, having its flanges stiffened by horizontal discs. The reactions are then taken up either directly, or through two arms projecting from the ring (Fig. 8). The circular rail is carried on the outer girder and the remaining portion of the structure serves as a rule only to resist shear forces. The arrangement may also be adapted as in Fig. 9 (which shows a plan and cross section) whereby the transfer of forces is simplified and economy is realised. The conditions of stress and displacement of these elastic structures cannot be ascertained by the usual simple rules of design, but call for more general methods derived from elastic theory if a useful picture of the action is to be obtained and a useful basis for design is to be derived.

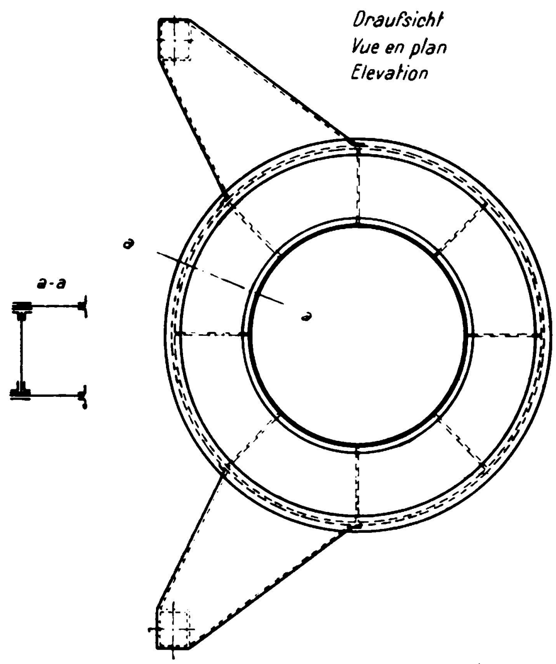

Fig. 8.

It will be seen, then, that the design of heavy plant for lignite mining compels a clearer elucidation of the action of space structures than is usually deemed necessary in bridge or building work. The solution of these structural

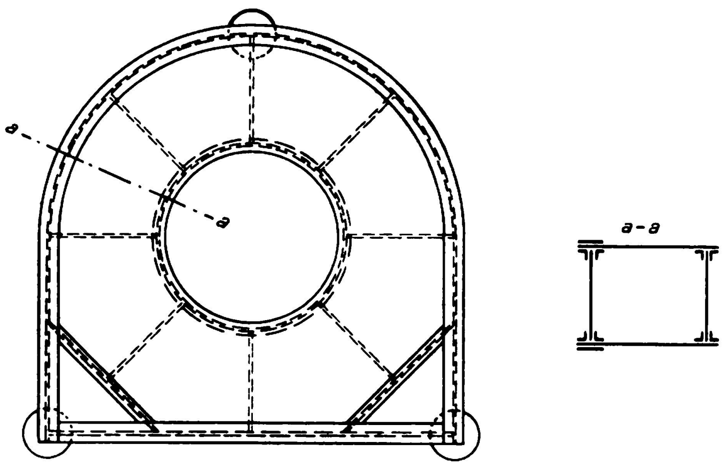

Fig. 9.

and mechanical problems is a service carried out by the Maschinenfabrik Magdeburg-Buckau, the Lübecker Maschinenbau-Gesellschaft, the Mitteldeutschen Stahlwerke, and the excavator department of Friedrich Krupp A.-G., Essen.

V 2

Semi-Experimental Method of Designing a Typical Structure.

Halb-experimentelle Berechnungsmethode eines grundlegenden Bauwerktypus.

Méthode de calcul semi-expérimentale d'un ouvrage classique.

R. Pascal,
Ingénieur conseil, Paris.

It is proposed to describe here a semi-experimental method of design which has been applied to two steel arch bridges, of 67 and 82 m span respectively, crossing the Seine between Neuilly and Courbevoie. These bridges are 35 m wide and are separated by a concrete arch of 32 m span between two massive abutments. The width of the roadway is 35 m but that of the central arch with its abutments is approximately 70 m. The work, occupying the site of the

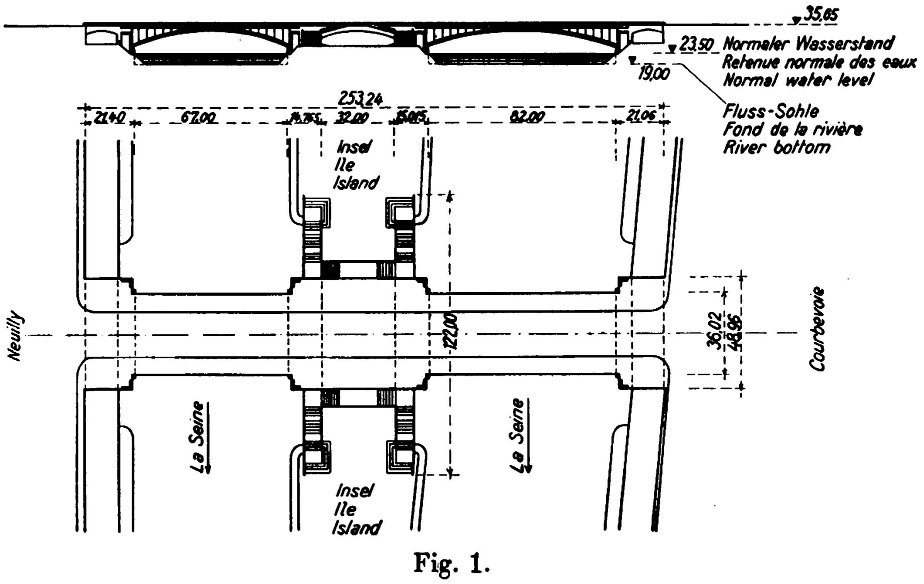

Fig. 1.

famous bridge built by Perronet in the reign of Louis XV, is at present under construction.

Fig. 1 is a key plan showing the general layout. The two steel bridges are of the same type. Each of the twelve arch ribs is hinged at its two ends, and between the springing and the quarter points nearest to the crown is rigidly connected to a cross frame composed of longitudinal members and verticals, the moment of inertia of these being much smaller than that of the arch. The frames so formed are connected by bracings below and cross girders above, so constituting an arcade in which the vertical members are also those of the main frames. In the central portion, adjoining to the crown, only the bracing members are provided. The sections of the arch ribs, verticals and bracings are hollow rectangular suitably stiffened. The cross members and longitudinals are rolled steel joists T. The work is to be carried out in steel 54, and all the connections are being made by welding. The bearings are of cast steel and the roadway is formed of reinforced concrete slabs.

The contractors for the masonry and foundations are the firm of *Leon Ballot,* and for the steelwork *Baudet, Donon & Roussel.* The work is being done for the Département de la Seine under the principal supervision of M. *Levaillant,* Ingénieur en chef des Ponts et Chaussées, and under the immediate charge of M. *Louis Alexandre Levy,* Ingénieur des Ponts et Chaussées, assisted by M. *Kienert,* Ingénieur T.P.E.[1]

Methods of calculation.

Each structure is a complex whole due to the main girders and cross bracings being inter-connected. The procedure followed in calculating the main girders will be explained first, and then the design of the cross bracings.

1) *Design of Main Girders.*

The type of structure under consideration, dictated as it is by the limited constructional depth available at the crown, is one that gives a pleasing effect to which the eye is well accustomed. The point to be noticed here is that in the central portion of the arch the moment of inertia increases outward from the crown as far as the first of the cross frames, which may be regarded as rigidly fixed, by the agency of its vertical member, to the solid central portion of the arch.

Structures similar to this have frequently been designed on the assumption that the frames were pin-jointed and, therefore, held at one end only. In the present case, however, the Administration des Ponts et Chaussées had specially asked the designers to consider each main frame as a rigid whole, and it was laid down that the roadway slab should not be taken into account for calculating the strength.

An attempt to work out the full continuity of the structure led, despite the simplicity of the method used, to impossibly complicated calculations and had to be abandoned. Advantage was accordingly taken of the work of Rieckhof and of the apparatus known under the name of Nu-Pu-Best. The steelworks were asked to build a metal model, to a scale of $1/25^{th}$, with its component elements so designed that their moments of inertia would be proportional to those of the

[1] Occurrences in France during 1936 led to many contracts being cancelled. In 1937 those for the steel bridges were awarded anew to contractors who are now engaged in carrying out the work on nearly the same lines, but in accordance with designs calculated in the offices of one of the firms concerned. The present paper is nevertheless being published in order to place on record a method of design believed not to have been previously described which is not affected by the circumstance that, for economic reasons, it was not carried into effect.

scantlings adopted in the preliminary design. By means of the Rieckhof bending apparatus it was then possible to determine the points of inflection, both real and virtual, for those members which had a small moment of inertia, but for reasons which will be evident this method could not be applied to the arches.[2]

By placing any given load in turn at every intersection point of the frame there was thus obtained the influence line for the shear force in the first vertical member to the left, and it was merely necessary to calculate the end frames in order to eliminate two hyperstatic quantities. Fig. 2 shows the shape of the influence line so obtained, and Fig. 2a the influence curve for the secondary thrust.

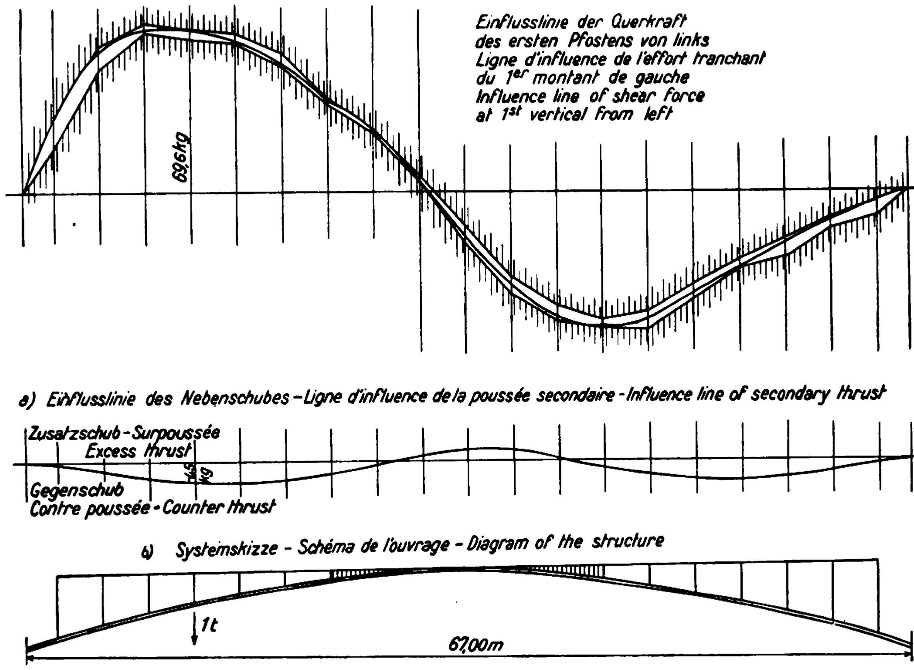

Fig. 2.

The arch had been investigated beforehand under the assumption that it was isolated, in order to determine the influence line for thrust —

1) under the action of unit vertical load;
2) under the action of unit horizontal load;
3) under the action of unit moment.

The calculations were made taking due account of the variations of the moment of inertia in the central portion. From the influence line described above, which is reproduced in Fig. 2, it was possible to work out influence tables for all the forces in all those sections of the structure which were to be welded, and at the same time a special investigation was made of thermal variations.

The engineers of Messrs. *Baudet, Donon & Roussel* suggested that it would be a great advantage if the joints in the model, which had at first been made cylindrical, were replaced by pieces corresponding in height to the sections of

[2] This method cannot properly be applied unless buckling effects have first been eliminated, but that is easily done.

the arches and the longitudinal beams respectively and a comparison of the model with the plan of the work made the utility of this suggestion apparent. Its importance was shown on examining a section of the model in polarised light, and at the same time this test furnished a useful justification for the method employed.

A calculation of the strains showed that the vertical and horizontal deflections of the monolithic frame were less than those which would occur in a pin-jointed frame.

Fig. 3 is a photograph of one of the models used. Special precautions were taken to eliminate the effect of friction due to the weight of the apparatus when arranged horizontally, and it was confirmed that the increase in the moment of

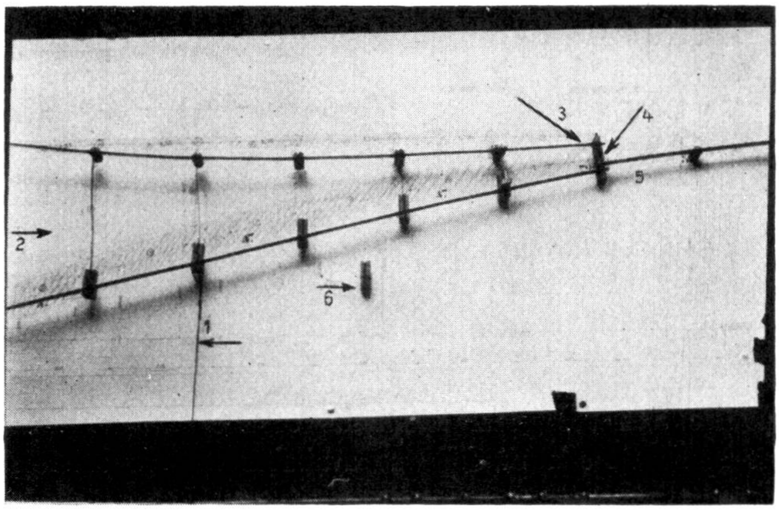

Fig. 3.

1 = A vertical load.
2 = Shear force x at the extreme left hand post.
3 = Total restraint of the last beam nearest to the crown.
4 = Unchangeable angle.
5 = Portion of variable sections (variability of moment of inertia).
6 = A single joint piece.

inertia due to the thickness of the joints had practically no effect on the results obtained.

Curve N° 2, together with curve 2A representing the influence line for secondary thrust, was plotted by comparing the results obtained in the calculation for the first frame to the left of the crown with those obtained by calculating the first frame to the right of the crown, these two series of calculations serving to check one another. In this way it was possible to delimit with approximate accuracy the region through which the curve would pass. Curve 2A was plotted at the same time as Curve 2.

2) *Calculation of the Cross Members and of the System as a Whole.*

By giving expression to the equality of the deflections occurring at each of the intersections between frames and cross members, a system of 228 equations with 228 unknowns is obtained; this can be considerably simplified, but remains quite insoluble. Could it be solved it would give the distribution of forces in the frames and the cross members for each case of loading under consideration.

What calculations could not give, it was decided to obtain by experiment. The firm of Baudet, Donon & Roussel were asked to build a model to scale of

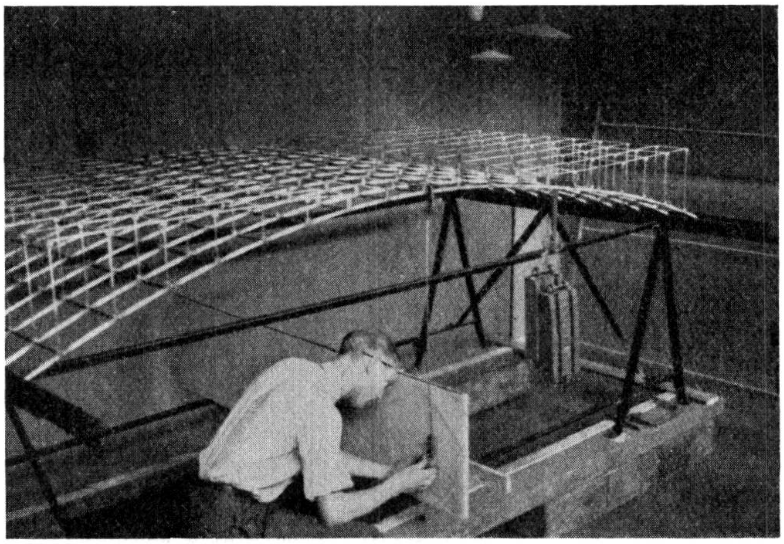

Fig. 4.
Three-dimensional model for measuring the vertical deformations.

$1/25^{th}$ of the system to be studied, conforming in its proportional dimensions to both the transverse and the polar moments of inertia so far as possible (see Fig. 4). At the same time a model was available of two arches identical with

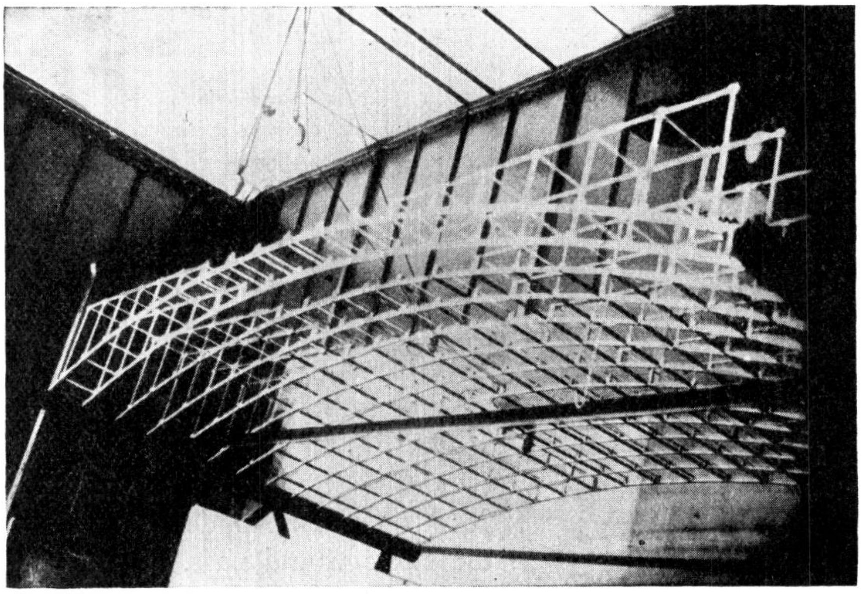

Fig. 5.
In the foreground the two-compound reference trusses;
below the tracing board.

the others, connected by bracings and cross members as in the main model. These two control arches may be seen in Fig. 5.

A load of 20 kg was applied to a large number of intersection points and the respective vertical deflections were measured. It was found that the horizontal deflections were very much smaller. When the elastic properties of the various cross members had been investigated a system of "compounding" or combined loading was established in the following way: each of the cross members whether simple or compound was replaced by a uniplanar model similar to that used for the

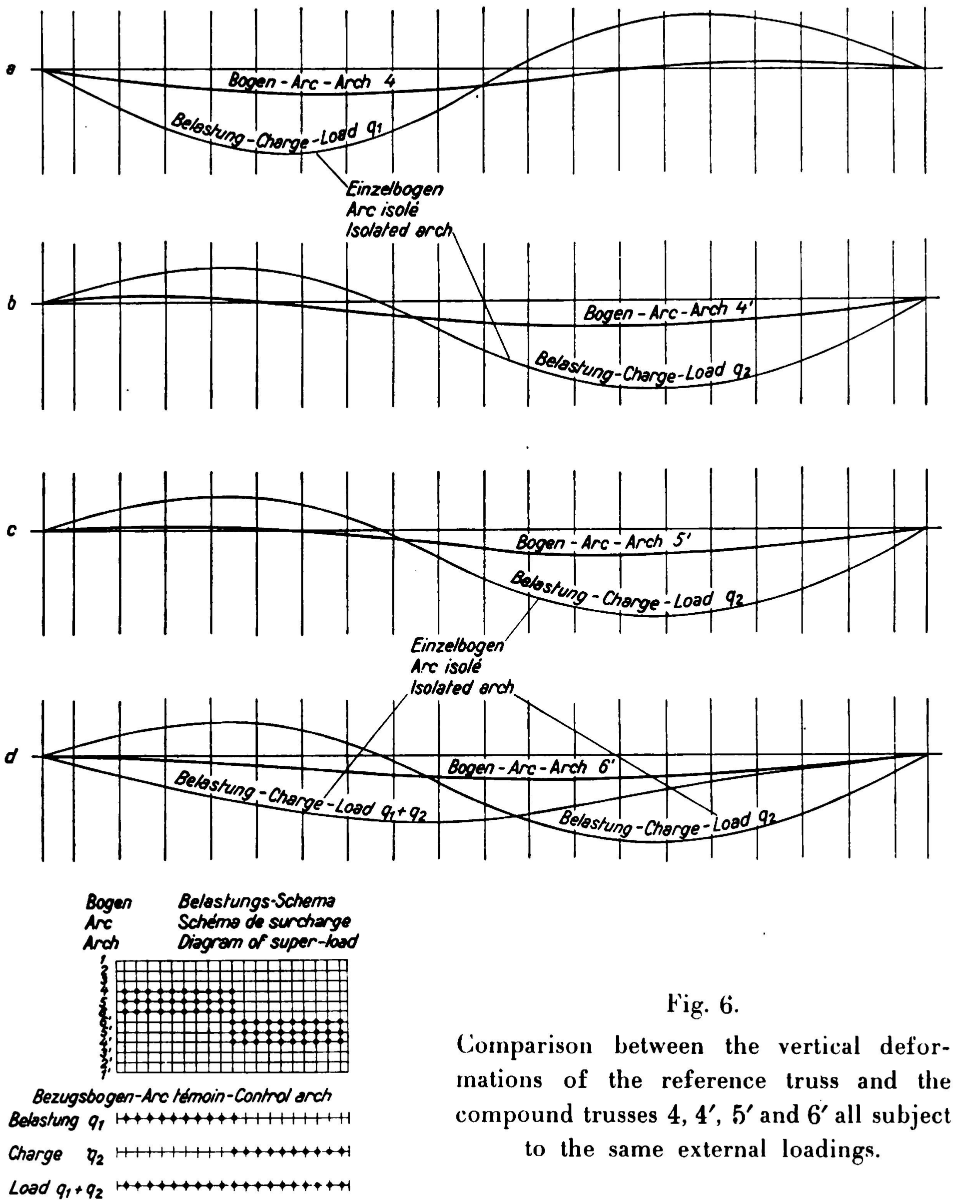

Fig. 6.
Comparison between the vertical deformations of the reference truss and the compound trusses 4, 4′, 5′ and 6′ all subject to the same external loadings.

determinations of curvature and points of inflections in the frames. The results thus obtained were not as accurate as in the case of the main frames, but it was possible to calculate the forces set up in the longitudinal, vertical, bracing and cross members to a sufficient degree of approximation. An influence table was compiled for each of the sections examined, and on comparing these tables with one another it emerged very clearly that the surcharge indicated in Fig. 6 was definitely the most unfavourable case; this applied to all of the sections considered.

It should be remarked incidentally that the cross members are not affected by dead load because the weight of the sidewalks is practically equal to that of the roadway, and the same is true in the case of live loads covering the full width of the structure.

A full investigation of the whole system would have involved "compounding" the cross frames, in exactly the same way as described above,[3] under loads assumed to be distributed in a chess-board pattern; but this would simply have furnished a check. From Fig. 6, which gives a comparison between the elastic properties of the control frame and those of each of the frames numbered 4, 5 and 6 under identical loads, it will be seen that there is a great difference in the vertical deflection which may amount to 60 to 70 %, and this shows that the corresponding actions in the various frames are considerably reduced by the presence of the cross members.

This result might indeed have been expected by analogy with the behaviour of a reinforced concrete slab supported on two edges and provided with distributing steel.

To summarise, it may be seen from this example that a structure can be designed by having recourse to experiment to provide what calculation is unable to furnish, without a disproportionate amount of labour. The essential point is to avoid empiricism and the doubtful degree of safety associated therewith. It must of course be understood that the experimental investigation should be followed up by calculation step by step. This method of using uniplanar and triplanar models can be applied to the study of a very large range of structures in two or three dimensions, composed of triangular lattice systems or frames. For instance, it would enable the study of secondary stresses in triangular lattice systems to be carried out with ease.[4]

The checks made by means of polarised light showed satisfactory agreement even in the case of very short members, and close agreement in the case of longer members.

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[3] We speak of imperfect "compounding" when the connections are merely situated at the places indicated by the three dimensional model, and of perfect "compounding" when the directions of the connections are the same also. It was established that within the limits of these experiments the difference between the two kinds of "compounding" was not negligible, but as a rule was not large.

[4] In the complete examination of a large structure over the Niger which is being undertaken it has been ascertained that the experimental method offers the only means of obtaining satisfactory results for the stresses in redundant members, and for secondary stresses.

V 3

Experiments on Rolled Sections Strengthened by Welding.

Versuche mit durch Schweißung verstärkten Walzträgern.

Essais sur poutres laminées renforcées par soudage.

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The beams tested were 30 cm deep and covered a span of 2.00 m. They were subjected to concentrated loads applied at the centre of each beam, and the following types were examined (Figs. 1 and 2):

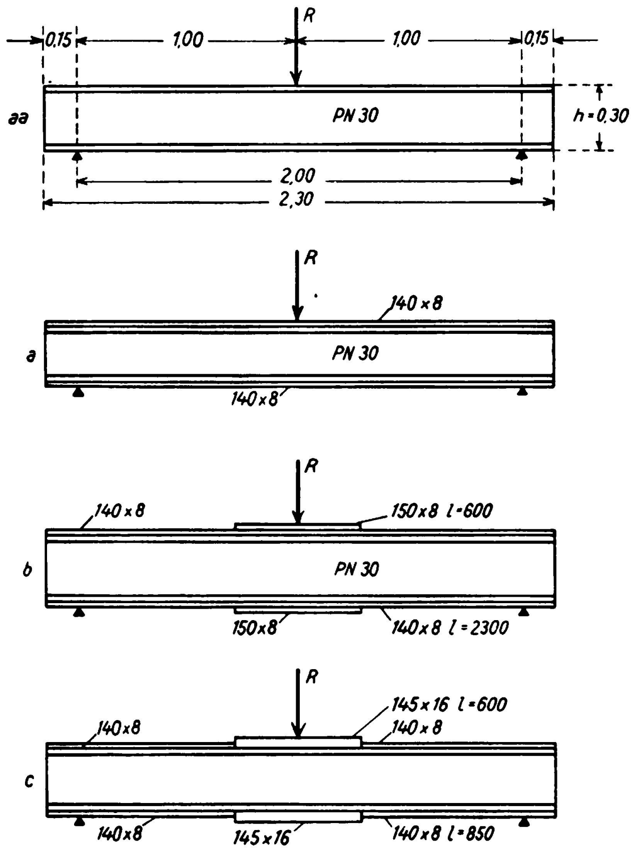

Fig. 1.

Type aa: Normal 30 cm joist.

Type a: Normal 30 cm joist reinforced with a 140 · 8 mm plate.

Type b: As type a, but reinforced with a 150 · 8 mm plate over a length of 60 cm at the middle of the span.

Type c: Same as type b, with the sole difference that instead of using two plates each 8 mm thick, a single plate measuring 145 · 16 mm was used at the centre (Fig. 3).

Each of these types was examined both without stiffeners (group I) and with stiffeners (group II) (Fig. 4). The latter group included two types of full webbed joists of compound section with h = 30 cm (Figs. 5 and 6).

Type d: with rivetted flange plates.

Type e: with welded flange plates.

Table I gives the weights of the beams examined (column 5) and the breaking loads R carried by them (column 4). In some cases a number of tests were carried out on beams of identical type in the same group, and R then denotes the arithmetic mean of the results so obtained. Column 6 in Table I gives the breaking load per unit weight (or specific breaking load) r = R/G and Table I has been used as a basis for calculating Tables II, III and IV which show the increase in absolute and in specific breaking load for one type of beam in relation to another. The addition of a flange plate increases the absolute load much more than the specific breaking load, and this increase is more marked in group II than in group I. On the other hand, the addition of a short flange plate (type b) is more advantageous in the case of group I. Type b is the most economical, giving values of R and of r approximately 10% higher, whatever the group under consideration. The welding of a plate to a joist is particularly advantageous when it is required to increase the strength of such a beam without greatly increasing its depth.

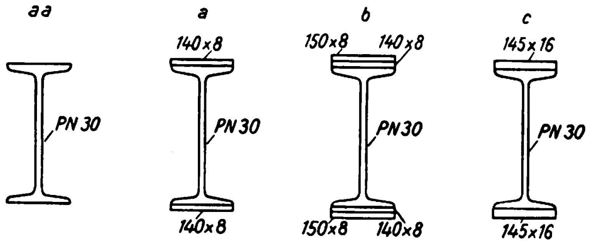

Fig. 2.

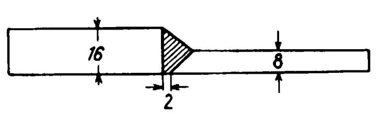

Fig. 3.

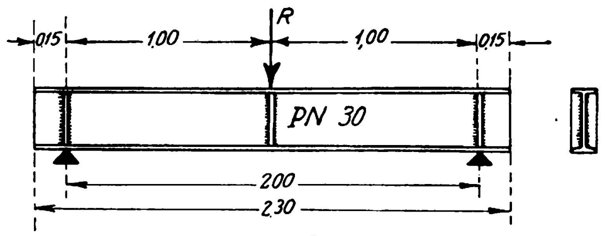

Fig. 4.

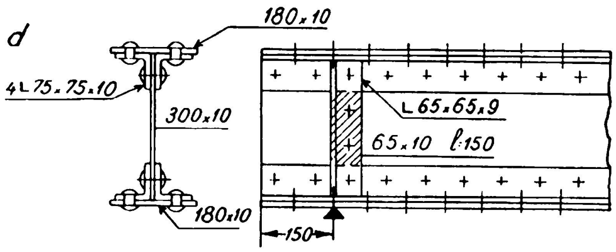

Fig. 5.

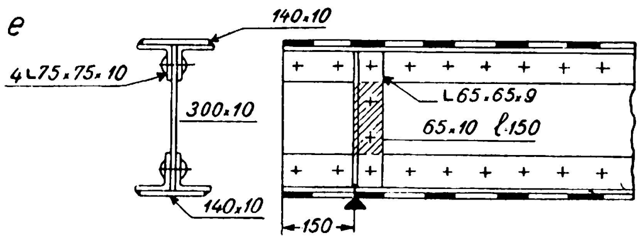

Fig. 6.

Table I.

1	2	3	4	5	6
Group	Type	Number of tests	R in t	G kg	r = R : G
I	aa	1	39.9	124.75	320
Beams	a	3	54.7	165.25	330
without stiffeners	b	1	62.5	176.55	354
	c	1	68.5	176.55	388
II	aa	3	48.4	133.58	362
Beams	a	9	71.3	174.08	409
with stiffeners	b	2	76.75	185.38	414
	c	1	84.5	185.38	455
	d	2	79.0	256.6	308
	e	2	74.9	242.2	309

Table II.

Group	I	II
$R_a - R_{aa}$	37.2 %	47.4 %
$R_b - R_a$	14.25 „	7.65 „
$R_c - R_a$	25.3 „	18.5 „
$R_c - R_b$	9.6 „	10.0 „

Table III.

Group	I	II
$r_a - r_{aa}$	3.12%	13.0%
$r_b - r_a$	7.3 „	1.5 „
$r_c - r_a$	17.5 „	11.2 „
$r_c - r_b$	9.6 „	9.9 „
$r_b - r_{aa}$	10.6 „	14.4 „
$r_c - r_{aa}$	21.2 „	26 „

Table IV.

Group	I	II
$r_{aa} - r_e$	2.8%	17 %
$r_a - r_e$	5.4 „	32.4 „
$r_b - r_e$	11.5 „	34 „
$r_c - r_e$	20.2 „	47 „

The two types d and e give almost the same value from the economic standpoint; r = 308 and 309, this being a little lower than is obtained with the rolled joists. It appears from Table IV that beams without stiffeners are from 2.8 to 20 % more economical than compound beams with stiffeners. Rolled joists with stiffeners show an economic advantage of between 17 and 47 %, but it is necessary to take account also of the amount of labour involved, which is much greater in compound beams than in others. In rivetted work compound beams are often preferred to rolled joists (for instance in the longitudinal and cross girders of bridges) since it is difficult to connect rolled joists to one another by riveting (as where the longitudinals have to be connected to the cross girders), but with welded work this difficulty disappears and there is every reason for preferring rolled joists to compound sections, whether reinforced or not; their construction is simpler, their weight lower and their strength greater. Herein lies one of the greatest advantages of welded over riveted work.

At the instant of failure of the beam the stress

$$\sigma = \frac{M}{J} \cdot v = \frac{M}{W}$$

should be equal to the breaking stress of the steel, assuming that failure is due to bending and that the steel exactly obeys *Hooke*'s law. But by reason of the horizontal break in the stress-strain curve the bending moment M, and with it the breaking stress, are increased by approximately 16 % in the case of double T sections, giving

$$M = \frac{R \cdot L}{4} = 1.15\, W \cdot \sigma \quad \text{or} \quad \sigma = \frac{R \cdot L}{4 \cdot 1.15\, W} = \frac{R}{B}$$

for

$$L = 200 \text{ cm}, \quad B = \frac{1.15\, W}{50 \text{ cm}}.$$

Table V gives the stresses σ as calculated by these fromulae. If the material were perfectly homogeneous and if all the beams were monolithic and all failed by bending the σ values in Table V would all be equal to the yield point of the metal. In monolithic beams of type aa and in semi-monolithic beams of type a, the material is better utilised than in beams built up of a number of elements, such as types d, e, b and c. This is the reason why the latter show less favourable results than the former. In type b the dangerous section is not necessarily at the centre of the beam; it occurs more probably at a distance of 250 mm from the centre, where the second flange plate is not yet effective. The values obtained for the beams without stiffeners are much lower than those obtained for the same beams with stiffeners (group II), and it may be concluded that the beams without stiffeners failed not by bending but by the collapse of the web. The concentrated load applied on the upper flange of the beam gives rise to transverse stresses, that is to say to a vertical compression under the load, and these stresses are at a maximum in the upper portion of the beam immediately below the flange; when they exceed the yield point the web buckles. For the purpose of calculating these transverse stresses σ_z Professor *M. T. Huber*[1] assumes that the compression flange behaves like a beam supported on an elastic base; where I_s is the moment of inertia of this beam, h_1 its height and δ the thickness of the web of the joist, we obtain

Table V.

Type	W cm^3	B cm^2	σ kg/mm^2 Group I	II
aa	653	15.07	26.4	32
a	958	22.05	24.8	32.4
b	1292	29.80	21	25.8
c	1292	29.80	23	28.4
d	1246	28.68		27.6
e	1154	26.50		28.2

$$\sigma_z = \frac{R\alpha}{2\delta}, \qquad \alpha^4 = \frac{0.4\,\delta}{I_s \cdot h_1}.$$

To take account of the rigid connection between the web and the flange, the transverse stress must be reduced by 8 % (by analogy with a uni-

[1] Prof. *M. T. Huber*: Etude des poutres en double té. Comptes-rendus des séances de la Société technique. Warsaw, 1923.

formly distributed load). In a joist PN 30, $\delta = 1.08$ cm, $h_1 = 26$ cm, and we have

$$\sigma_z = \frac{R}{A}, \qquad A = 6.55 \sqrt[4]{I_s}.$$

Table VI gives the values of σ_z calculated according to these formulae, which are much greater than the stresses caused by bending in the case of the beams without stiffeners (Table V, group I); this goes to prove that it is the transverse stresses which cause failure in the beams not provided with stiffeners.

Table VI.

Beams without stiffeners.

Type	J_s cm^4	A cm^2	σ_z kg/mm^2
aa	5.05	9.85	40.50
a	16.57	13.26	41.28
b	39	16.40	38.10
c	39	16.40	41.80

Table VII.

Beams with stiffeners.

Type	1,59 A cm^2	σ_z kg/mm^2
aa	15.70	32.5
a	21.14	33.8
b	26.0	29.55
c	26.0	32.75

It may surprise that the values of σ_z exceed the yield point, but this circumstance may be explained by the fact that the tests were not stopped at the precise instant when the stress σ_z reached the yield point; the load still continued to be increased, and as the result of the strains which occurred it was distributed over a strip of some considerable width, thereby reducing the stresses at the central point.

Assuming that the stiffeners placed immediately underneath the load R serve to distribute that load equally between the two flanges, we obtain transverse stresses amounting to only $\frac{1}{1.59}$ times as great,[2] hence

$$\sigma_z = \frac{R}{1.59\,A}.$$

Table VII shows the stresses as calculated by this formula in the beams without stiffeners; here again the values are higher than those in Table V, group II, but the differences are not so great as to exclude the possibility that the beams may have failed by bending. This may be seen in the illustrations. The lower flange of the beam without stiffeners (Fig. 7) is intact (the vertical stresses are here equal to zero), while the upper flange has bent as well as the web. In the beams with stiffeners (Figs. 8, 9 and 10) the two flanges have been visibly bent immediately under the point of application of the concentrated load, and this proves that the stresses due to bending have contributed to the failure of the beam. In the case shown in Figs. 8 and 9, the plate of the upper flange has become wavy, and in the case of Fig. 8 the weld seams have been torn away; this represents the effect of the transverse stresses, but it is also an effect of the

[2] *Bryla:* Influence des raidisseurs d'âme soudés aux poutrelles, sur leur résistance. Annales de l'Académie des Sciences techniques, Warsaw, 1935, I, p. 152.

buckling of the compression flange, which is free so to buckle intermediately between the weld seams. This explains why the flange plates have not bent in the beam with continuous weld seams (Fig. 10).

Fig. 7.

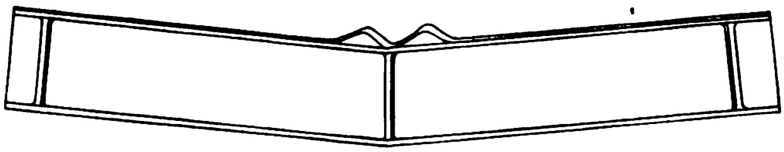

Fig. 8.

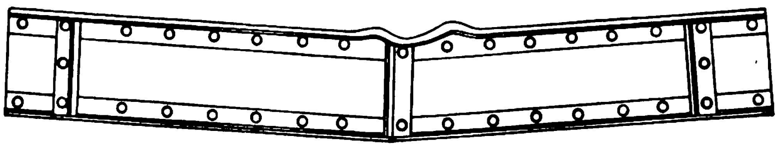

Fig. 9.

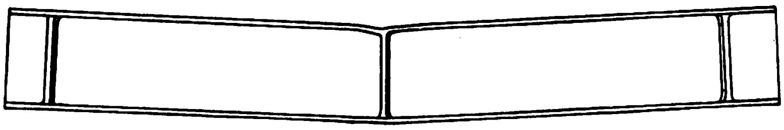

Fig. 10.

Conclusion.

The addition of a flange plate to a beam increases the value of the breaking load R (Table I). The specific breaking load (per kg weight of beam) $r = R/G$ increases in a smaller proportion. Beams of compound section are inferior to rolled joists, whether or not these are reinforced by flange plates. A single thick flange plate is to be preferred to two plates of smaller thickness. In beams without stiffeners, failure through collapse of the web is due to the vertical compression below the point of application of the concentrated load, and not to stresses caused by bending (see Tables V and VI and Fig. 7). In the case of beams provided with stiffeners the stresses due to bending have contributed to the failure (see Tables V and VII and Figs. 8, 9 and 10).

V 4

The Determination of Lines of Principal Stress in Riveted and Welded Structures.

Darstellung der Hauptspannungslinien an genieteten und geschweißten Konstruktionen.

Détermination des trajectoires des contraintes principales dans les constructions rivées et soudées.

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The technical development of structural engineering, the bridging of wide spans and the consequent need to make the fullest possible use of the materials while conserving adequate safety, requires the making of statical calculations which shall correspond as closely as possible with the stresses actually arising in the structure. Hitherto most calculation has been based upon far-reaching simplifications, and the condition of uno-axial stress which was easy to treat by calculation has formed the basis also of statical experiment. Yet it is only in the rarest cases that this assumption is in fact justified, and the true condition of stress is more often approximately duo-axial or tri-axial. As regards the cross sections of joints and intersections, corners of frames, and similar details of design in steel construction, at least a duo-axial condition of stress must be assumed if the normal and shear stresses are to be more accurately determined.

In view of the great number of marginal conditions it is difficult to determine the stresses by mathematical quantitative methods under this assumption, and consequently it is not possible to arrive in this way at satisfactory agreement between the values of stress as calculated and as arising in practice, in cases where no simple fields of stress can be distinguished. It appears expedient, therefore, to determine the fields of stress and the magnitudes of the stresses in certain sections of particular interest by means of experiments. In many practical cases an accurate knowledge of the direction and magnitude of the stress will probably enable a simple method of calculation, of sufficient accuracy, to be developed mathematically so as to yield results in satisfactory agreement with practice.

Experiments to determine the stresses of the components of duo-axial stress may be carried out by several methods. In one of the older of these the elongations at the corner points of a square net lying in the plane of stress are measured in several directions from which the magnitudes and directions of the principal stresses may be calculated and the lines of principal stress plotted.[1]

In the engineering laboratory of the Technische Hochschule at Darmstadt the determination of the lines of principal stress has been carried out with the aid of a lacquer susceptible to cracking which is coated over the experimental specimens to be examined.[2] When such a specimen is subjected to load the lacquer cracks at right angles to the direction of the maximum elongation and a family of lines of cracking results which corresponds to one system of the lines of principal stress; by making use of the known condition that the two systems of lines of principal stress must intersect one another at 90^0, it becomes possible to trace out the second system within the field of stress under examination. When the lines of principal stress have been determined, measurements of elongation are carried out at a number of points along suitable sections in the direction of the lines of cracking disclosed, and the magnitude of the principal stresses at these points is calculated.[3]

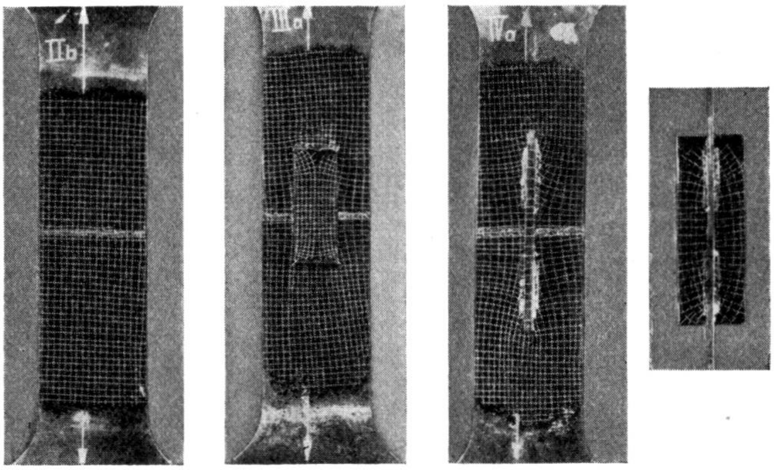
The field occupied by the lines of principal stress gives a useful picture of the distribution of stresses and strains in a specimen or structural member. In many cases important conclusions can be drawn directly from the trend and shape of the lines of cracking: thus a notably irregular distribution of the lines of principal stress indicates the position of the highly stressed places independently of any measurements and assists in the choice of cross sections for measurement. The field of stress, as indicated by the lines of principal stress, affords further exact information as to stress conditions. From a knowledge of this the proper assumptions to make as a basis for calculation can be ascertained.

Moreover the lacquer, through its liability to chipping, has the further property of giving a very early indication that phenomena of flow are about to take place, and this enables a good idea to be obtained of the plastic behaviour of the member under examination. Even within the region of the live load as calculated by the ordinary method small isolated zones may be found wherein plastic deformation is taking place, an occurrence which, but for this method, would escape observation. It is still easier to discern the widening of this region of flow that occurs when the load is increased, which can be followed clearly and intelligibly. In the same way, provided that the necessary experimental conditions can be satisfied, the lines of principal stress can be determined in uniplanar or three-dimensional forms even of large dimensions. Thus the starting point for new methods of calculation is afforded by experiments which have carried out on structures built either under practical conditions or (if necessary) in the form of a model to a reduced scale, and in either case the investigations can be made in the same material as is to be used in the actual structure.

In what follows below the lines of principal stress developed in a series of structural members under the experimental load will be described. Fig. 1 shows the system of such lines on simple tensile bars subjected to eccentric loading including a welded butt joint, a butt joint with a cover strap secured by end fillet welds, and finally by a butt joint with vertical plates connected to it by side fillet welds.

[1] See *Wyss:* Kraftfelder in festen elastischen Körpern.
[2] See Bautechnik, 1936, No. 23.
[3] Elongation line method of the Maybach firm.

In all these test bars the fields of stress were relatively simple. In the first of the bars, that welded with a simple joint, the cracks appeared at right angles to the axis of the bar, and these can easily be determined even assuming the condition of stress to be almost uni-axial. The second of the bars tested already shows, by the shape of the lines of force, the effect of the reinforcements welded on to it, this being recognisable in the deviation of the lines towards the straps. In this respect special attention should be drawn to the trend of the lines of principal stress in the cover strap itself. In the neighbourhood of the end weld the flow of the lines of force becomes irregular, concentrating at the corners of the end welds, while the middle is left free from stress. Moreover an arch shaped



Load applied

P = 11.40 t P = 15.20 t P = 15.55 t

Fig. 1.

Types of joint. Stress trajectories under working load.

plasticity figure occurs in the plate close to the seam, which implies very high stresses in this zone. This local yield, which has begun even within the range of the working load, may well be the reason for the low fatigue strength possessed by connections of this kind, their appearance on fracture under fatigue being similar to this.[4] Virtually the same result was obtained from the third experimental test bar shown in Fig. 1 where once again the strap, in spite of its cross section being partially separated from the bar, participates uniformly in the stress at its middle section — a fact which might also have been inferred from measurements of elongation. The end of this cover strap was tacked while being welded, and in the experiments chipping, originating at the tacking points, occurred in the lacquer, as may be clearly seen in Fig. 2, this again indicates zones in which yield has taken place even though the full working load is not developed. On this test bar being loaded to destruction, giving the result shown in Fig. 3, the yield zone is seen to have extended until fracture took place. The flow figure for the butt welded bar shows a much greater extension of the yield

[4] *Kommerell:* Experience obtained with Structures executed in Germany. IIId, Preliminary Publication of I.A.B.S.E. Congress, Berlin and Munich, 1936.

on one side than on the other, as the result of eccentric loading. The chipping of the laquer running at an angle of 45° is characteristic of the tensile yield point being exceeded, such chipping being caused by the formation of slip planes.

The trend of the lines of principal stress in compression members jointed in two ways is indicated in Fig. 4. In one of these specimens the plate was welded to an I-beam, in a second it was rivetted, and in a third example it was both

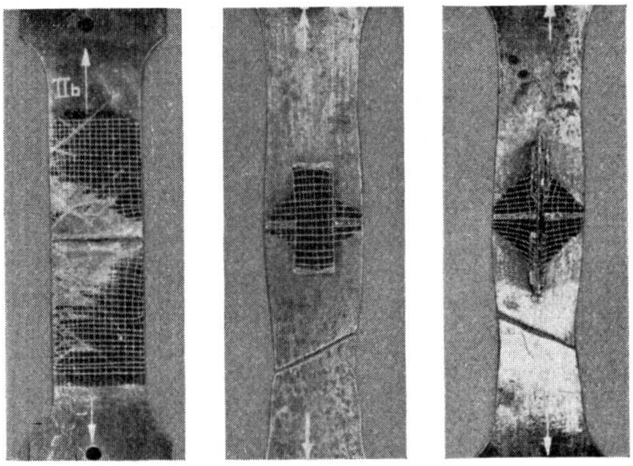
P = 15.20 t P = 15.55 t

Fig. 2.

Types of joint. Lines of flow in the lacquer under working load.

welded and rivetted. After the lines of principal stress had been developed the fields of stress in the plate were delimited as shown in Fig. 4. In the case of the first and third compression specimens the length of the connecting weld seam is indicated by points.

The welded form of connection gave a very irregular transfer of the forces into the seam and almost all the lines of stress are crowded in to the beginning of the



Rupture load

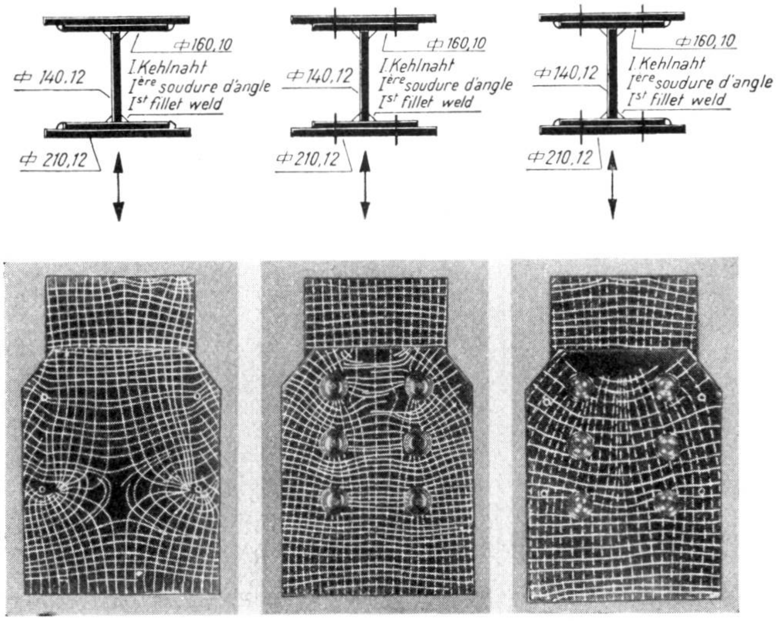
$P_B = 46.80$ t $P_B = 66.78$ t $P_B = 67.53$ t

Fig. 3.

Types of joint. Fracture under static load.

seam, which must, therefore, be subjected to higher stresses. The welded form of bar gave a considerably more equalised field of stress, but a uniform flow of the lines of force was obtained only by the combined use of rivetting and welding, implying that each of these methods of connection shared simultaneously in transferring the force.

From the available data it was possible to develop the trajectories in a girder with a proportion of h/l = 1/10 subjected to bending. Fig. 5 shows the results of this experiment. The girder was loaded with two isolated loads, 500 mm apart on either side of the centre, the span being 3000 mm. In the middle field there existed, therefore, a constant moment without shear, while in the end fields both these effects were present together. The shape of the lines so developed accurately reflects this statical condition. Whereas calculation gives a zero value of the



a) welded b) riveted c) welded and riveted

Fig. 4.

Compression test II.

Stress trajectories in welded and compound-jointed pieces. Front view.

shear at the middle height of the web plate in the central field, in the end fields the shear should be a maximum. This implies that in the first mentioned the lines should intersect with the axis of the girder at 90⁰, while in the end fields they should cut it at 45⁰. Intermediately between the two fields there is a transition zone, and apparently this must always occur in such cases where, statically speaking, a violent transition obtains from one to the other kind of stress. When the load was increased, chipping of the lacquer coating again occurred in the central field, indicating the yielding of the material. Despite the constant moment present throughout the length of this field, so that all cross sections therein should be equally stressed, the yield point was in fact reached earlier in some portions than in others (see Fig. 6). Another notable observation is that in the compression zone the development of the *signs of*

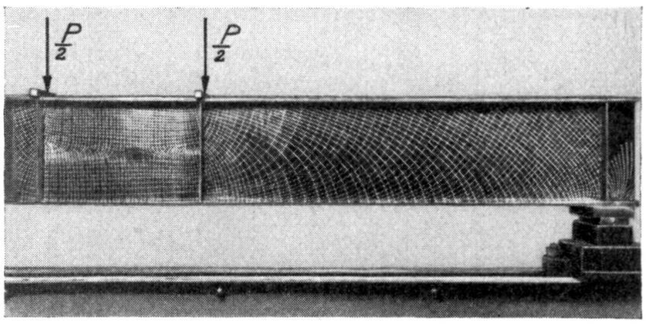
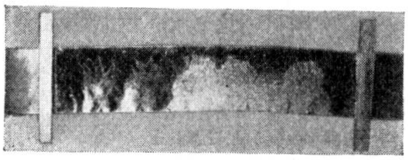


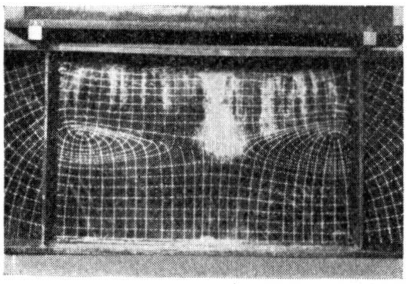
Fig. 5.

Stress trajectories in a girder subjected to bending. h : l = 1 : 10.

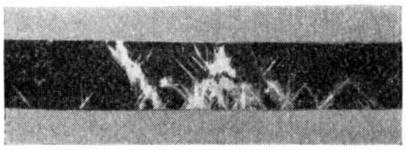
yield occurred at right angles to the direction of pressure, whereas in the tension zone it was at 45° to the direction of tension. Virtually the same observations were made in the compressive and tensile flanges of the middle field. In this instance it appears, therefore, that failure in the tension zone is initiated by the shear strength being exceeded, whereas in the compression zone it is another kind of failure that occurs, presumably through the compressive strength being exceeded. Hence the shear yield point is the determining factor as regards tension, and the compressive yield point as regards compression. It should be emphasised, however, that these observations have so far been carried out only on a small number of specimen parts, and that the results require confirmation by further experiments.



view from top



elevation



under side

Fig. 6.
Yield zones in the middle of a bent beam.

A further illustration, Fig. 7, shows the loss of carrying capacity by shear failure of the web plate in a short girder where the ratio of depth to height is 1 to 4. The lines of principal stress indicated in Fig. 7 were obtained at somewhat below the working load, but even under this loading yield zones appeared in the neighbourhood of the load points and of the end bearings. As the load increased this yielding spread from its origins outward, in the form of slip planes extending further and further into the end fields, these planes being inclined at 45° to direction of the principal stress lines, thus clearly indicating that the girder had failed through shear.

An exhaustive investigation of stress conditions was made in an experimental frame. In the horizontal members of the frame the lines of principal stress can be found in the way already described (see Fig. 8). In this experiment special attention was paid to the flow of lines in the neighbourhood of the corners, and a lack of symmetry of the lines was observed about the diagonal section. In a comparative specimen, made right angled in cross section and subjected to symmetrical loading, regular symmetrical curves were obtained as shown in Fig. 9 and these agree with the curves obtained by optical methods of stress measurement by Cardinal *von Widdern*.[5] At the corners of the frames no displacements of the curves towards the horizontal member are apparent. The reason for this is to be sought in the different methods of loading the vertical and horizontal members, shear and moments being operative in the former and longitudinal forces in the latter. The investigation was amplified by means of numerous measurements on elongation carried out in a series of cross sections, assuming a duo-axial condition of stress, and the results of this were recal-

[5] *Cardinal von Widdern:* Mitt. Mechan. Techn. Labor., Technische Hochschule, Munich.

culated for normal and shear stress conditions. The evaluation of the experimental results, and collation of the knowledge so obtained, is to be given in a separate publication. In order to afford a comparison of different methods of construction the frame was built with two different designs of corner construction, one side being provided with a strengthened angle gusset opposite to the web of the vertical member and the other side being stiffened. The change in flow of the lines of principal stress is shown in Fig. 10. At the end of the short stiffening there is seen a kink in the line, which suggests a distribution of the stresses over the depth of the web plate and implies a relief of stress in the region of the inner corner. At the same time the development of the lines of stress in the flange at the inner rounded portion shows that good support is afforded to the inner flange at the stiffened corner by contrast with the other corner.

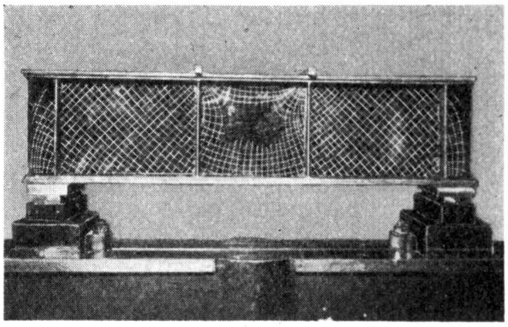


Fig. 7.
Stress trajectories in a girder subjected to bending. h : l = 1 : 4.

Fig. 11 shows the lines of principal stress on the inside of the inner flange, comparing an unstiffened and a stiffened corner.

To sum up, it may be stated that by tracing out the fields of stress, the yield zones and yield regions, an illuminating picture may be obtained of the

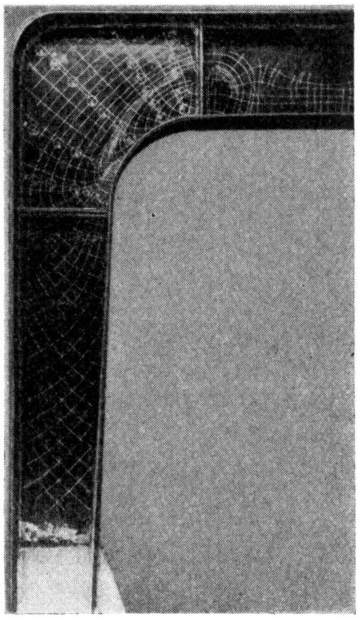


Fig. 8.
Stress trajectories in a frame of I-section.

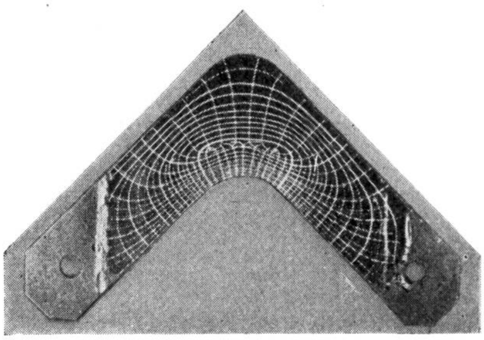
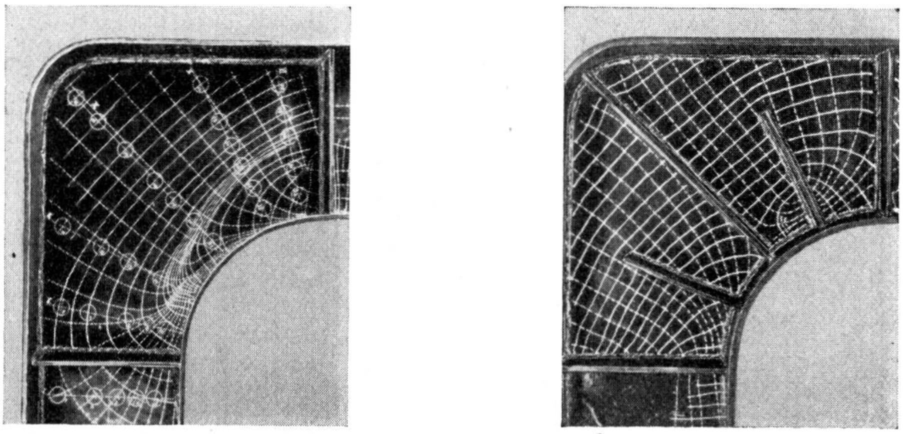


Fig. 9.
Stress trajectories in an angle piece of rectangular cross section.

behaviour of the structural members and details so examined. The delimitation of the fields of stress makes it possible to gain a further insight into the proportionate magnitudes and distribution of the stresses, especially under the condition of duo-axial stress. From these experimental results it may be presumed that opportunities will arise for correcting the existing methods of calculation to suit the duo-axial condition of stress, and so obtain agreement with the actual

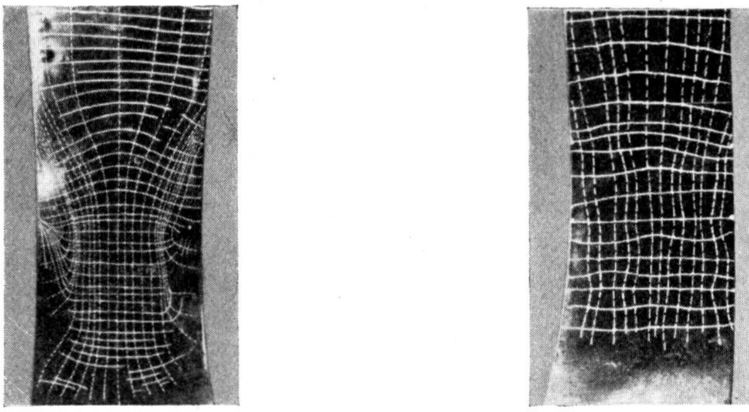


Corner a. Reinforced corner plate. Corner b. Non-reinforced corner plate but with stiffeners.

Fig. 10.

Comparison of stress trajectories in two corners of frames.

values. In the same way it will become possible for the fundamental assumptions necessary in pre-calculating the various important stress values to be correctly understood.



Corner a. Reinforced corner plate. Corner b. Non-reinforced corner plate but with stiffeners.

Fig. 11.

Stress trajectories in the inside flange of the corner of a portal frame.

The object of these experiments, however, must always be that of arriving at methods of calculation which can be generally applied in practice, show good agreement with the conditions actually arising, and enable the designer to build economical structures with adequate safety.

V 5

Experiments on Photo-Elasticity.

Spannungsoptische Untersuchungen.

Recherches photoélasticimétriques.

Dr. Ing. V. Tesař,
Paris.

At the present stage of the art of the engineer the solution of internal stresses in complicated structures is being considerably helped by experimental research. The most usual methods are based on the measurement of strains, the acoustical properties of vibrating cords, the variation in electric resistance of certain bodies, and the use of high frequency electric currents. All these methods yield excellent results, but they give information only in relation to those few points on which the measuring apparatus is mounted.

The case is altogether different as regards experiments carried out on models with the aid of polarised light, that is to say by the method of photo-elastic measurement. Under Question IVa the author has drawn attention to the value of such experiments in relation to problems of reinforced concrete construction, and what was said there applies, with little alteration, to steel work also.

Photo-elastic investigations facilitate the determination of rational forms of structure, of whatever nature, and allow the solution of internal stresses where other methods are ineffective. M. *Mesnager*, originator of the photo-elastic method, set up a laboratory for researches on small scale models by the aid of polarised light as early as 1900 at the Ecole Nationale des Ponts et Chaussées in Paris, and as an example of the experimental work recently carried out in this laboratory a brief description will now be given of investigations made in connection with the bridge at Neuilly. A description of this interesting work and of the methods of calculation applied to it has already been given by M. *Pascal*, and the present writer will confine himself to the experimental photo-elastic tests.

The subject of these was a central portion of the bridge (Fig. 1). The design of the model is shown in Fig. 2, and it should be noticed that the investigation had reference to the dead load of the bridge. It will be clear from these figures that the central portion of the bridge construction involves serious difficulties if attacked either by the usual methods of calculation or by experimental methods other than photo-elasticity, for it does not form a collection of elements that can be dealt with according to the ordinary rules of strength of materials; the necessary conditions for this are not fulfilled.

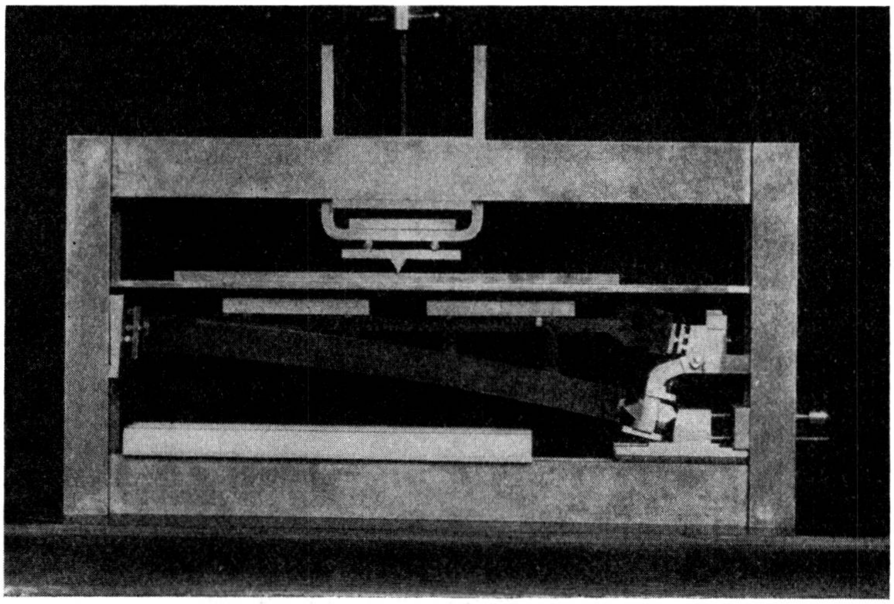


Fig. 1.

The model was made of xylonite to a scale of 2 cm to a metre and was subjected, apart from the external vertical loads, to the reactions of the missing

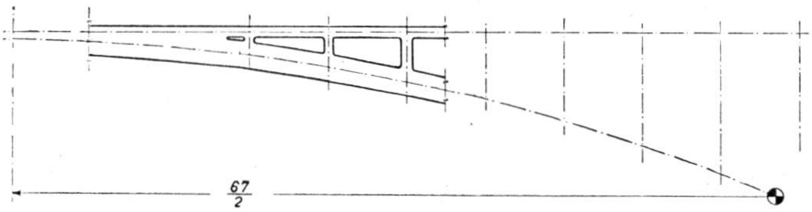


Fig. 2.

part of the bridge, such reactions having been determined by calculation and by other experimental methods to which M. *Pascal* has referred in his paper.

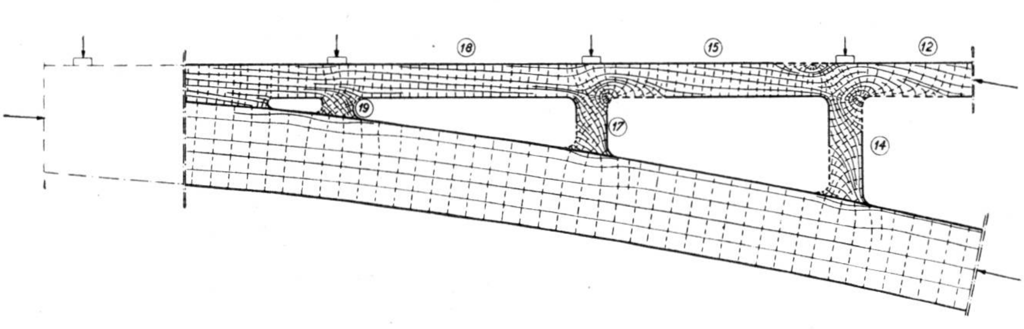


Fig. 3.

The photograph in Fig. 1 shows the experimental lay-out of the model with the system of loads, which were applied to a scale of 1 kg for 5 tonnes. The model was observed by polarised light in straight lines between two crossed nicols, thus obtaining the isoclinal lines from which in turn could be derived the isostatic lines (Fig. 3). By subsequent quantitative measurements it was

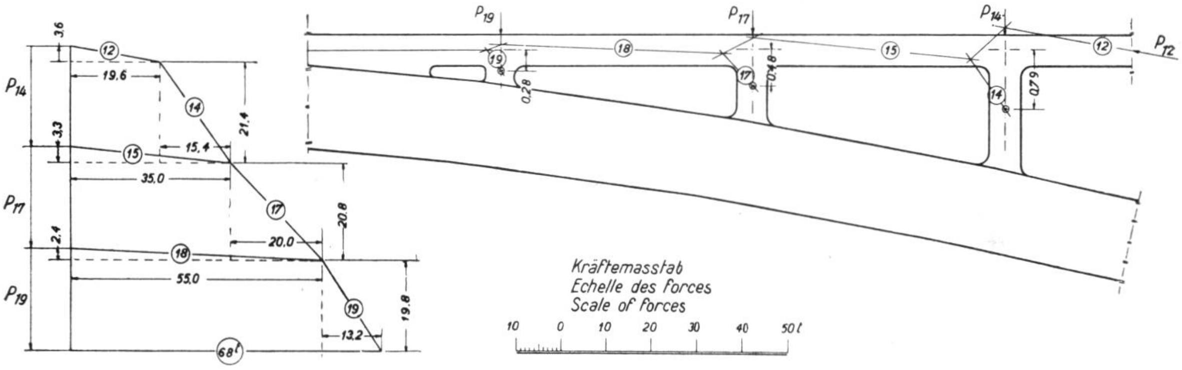


Fig. 4.

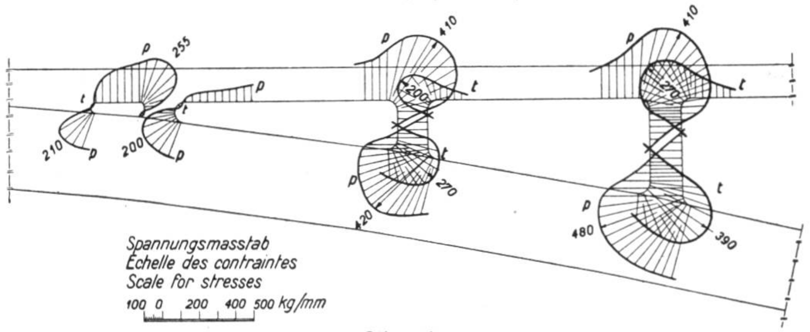


Fig. 5.

possible to fix the curve of pressure and plot the corresponding cremona diagram (Fig. 4).

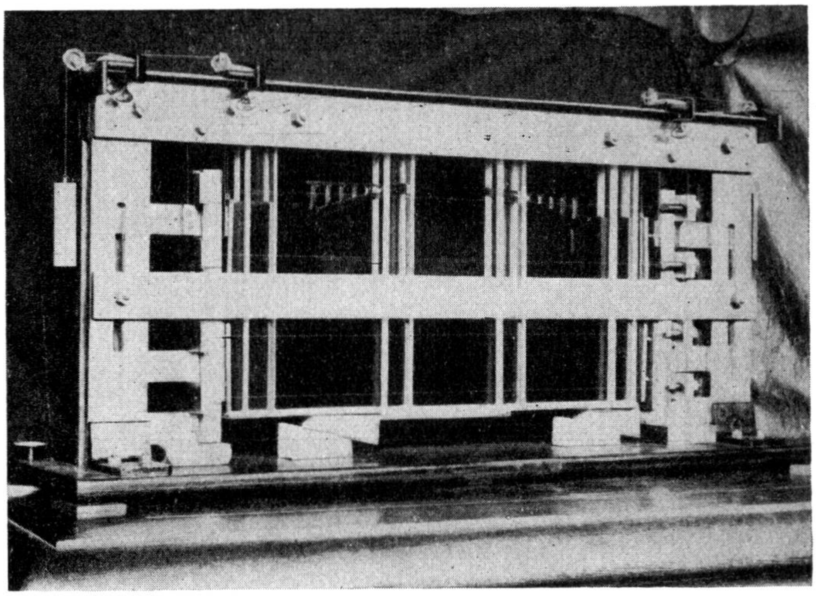


Fig. 6.

Fig. 5 is a diagram showing the stresses as measured along the edge of the verticals in the portion of the bridge under consideration.

In conclusion a few photographs will be given to indicate the experimental arrangement of the researches carried out in the Laboratoire des Ponts et

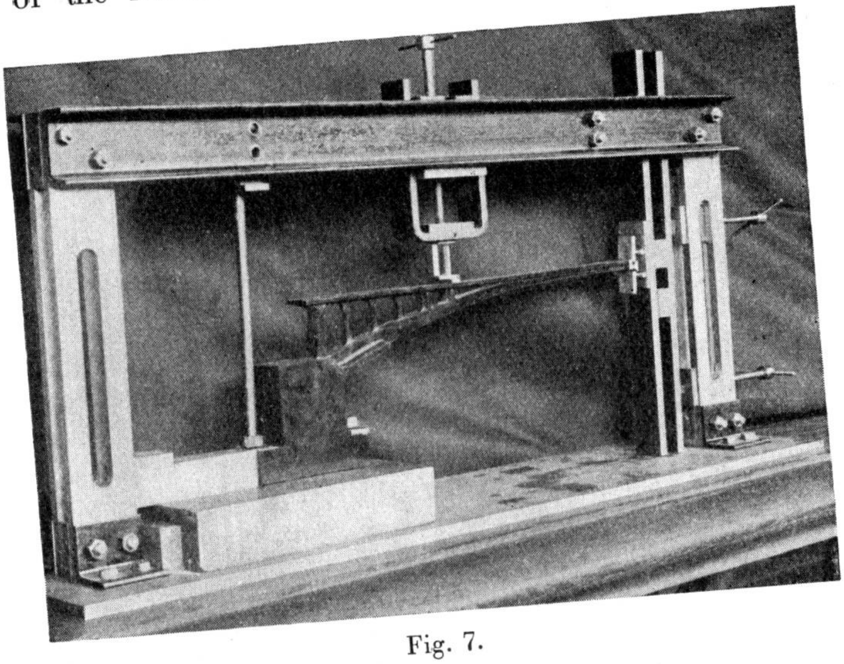


Fig. 7.

Chaussées at Paris for the Jirásek bridge at Praha. Fig. 6 represents the xylonite model of an arch of the bridge enclosed in a frame for the application of the

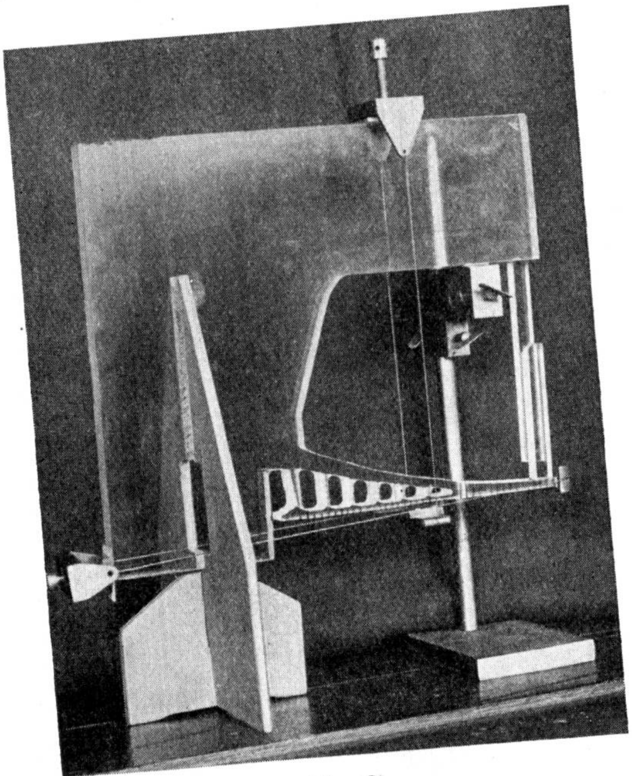


Fig. 8.

loads. Fig. 7 shows the three-dimensional model of one half of the same bridge set up for comparative investigation. Fig. 8 shows the glass model loaded by a vertical force and the horizontal thrust, and finally Fig. 9 gives photographs of the details of the end fixation of the arch, obtained directly by polarised light

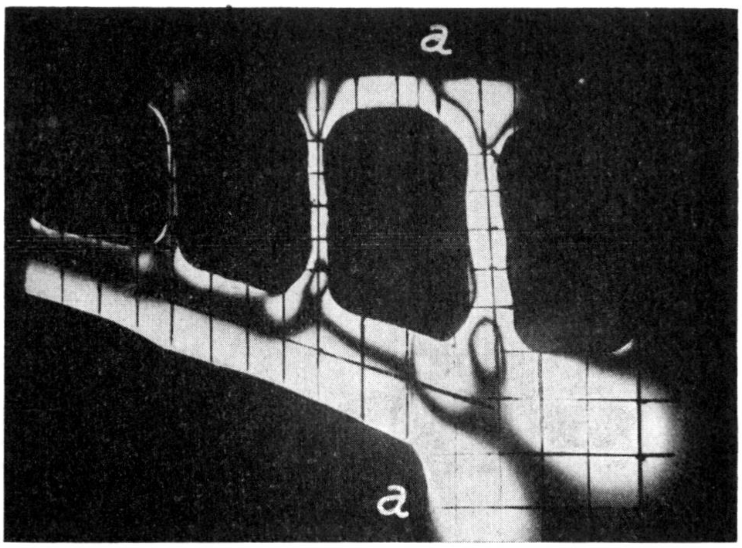


Fig. 9.

in straight lines, with the vertical and horizontal positions of the crossed planes of polarisation.

V 6

The Acoustical Measurement of Extension: its Application to the Determination of "Singular Points" in Structures.

Akustische Dehnungsmessung. Anwendung zur Bestimmung der singulären Punkte in den Bauwerken.

Mesure acoustique des allongements. Application à l'étude de points singuliers dans les constructions.

A. Coyne,
Ingénieur en Chef des Ponts et Chaussées, Paris.

One of the simplest, cheapest and most certain methods of auscultation available is that which makes use of acoustical principles, an idea which occurred to many workers a long time since and which has been brought into practice simultaneously in Germany and France during the last few years. The principle is as follows:

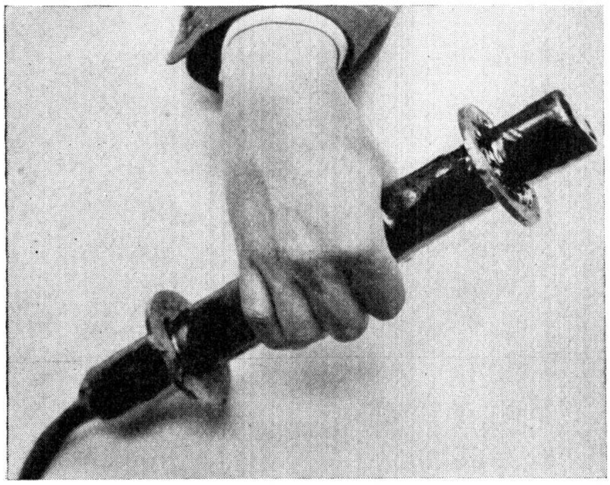


Fig. 1.
Outside appearance of the "acoustic telltale".

A vibrating cord having its ends fixed to the piece to be auscultated participates in the deformation of the latter, and its natural frequency varies in accordance with the elongations or compressions undergone. Where it is required to measure the stresses in concrete the cord is enclosed in a water tight corrugated tube which protects the apparatus without impairing its elasticity (Fig. 1 and 2).

The cord is excited from a

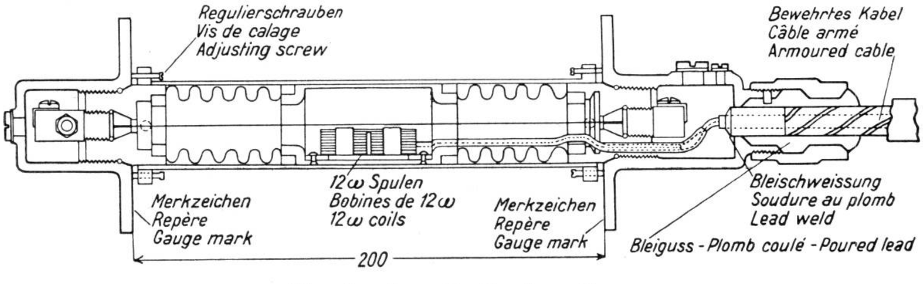


Fig. 2. Longitudinal section.

distance by means of an electro-magnet, receiving the discharge from a small condenser, and this has the effect of setting the cord into vibration. The vibrations

Fig. 3.
Valve amplifier for "acoustic telltale".

are detected in the same circuit by means of a valve amplifier (Fig. 3), the electro-magnet acting like a Bell telephone. At the central measuring station the

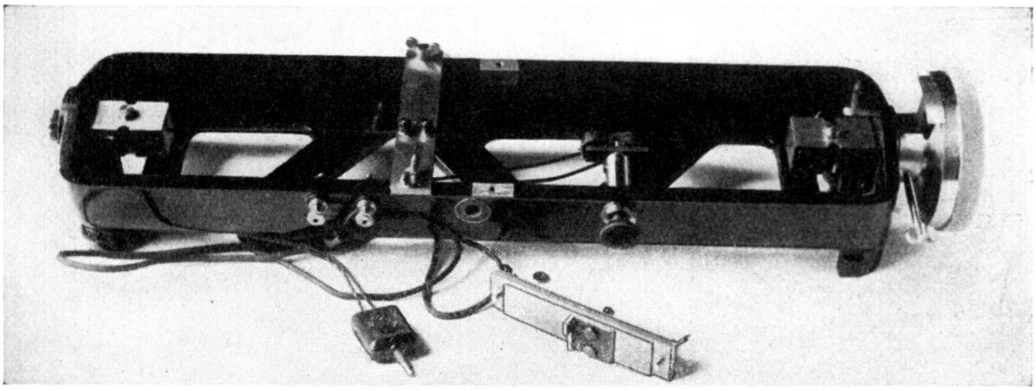


Fig. 4.
Standard frequency meter.

sound produced by the cord is compared with that of a standard frequency meter (Fig. 4), so affording an immediate indication as to the state of tension or compression in the concrete around the instrument. The amplifier and frequency meter may be housed in a case which is readily portable (Fig. 5).

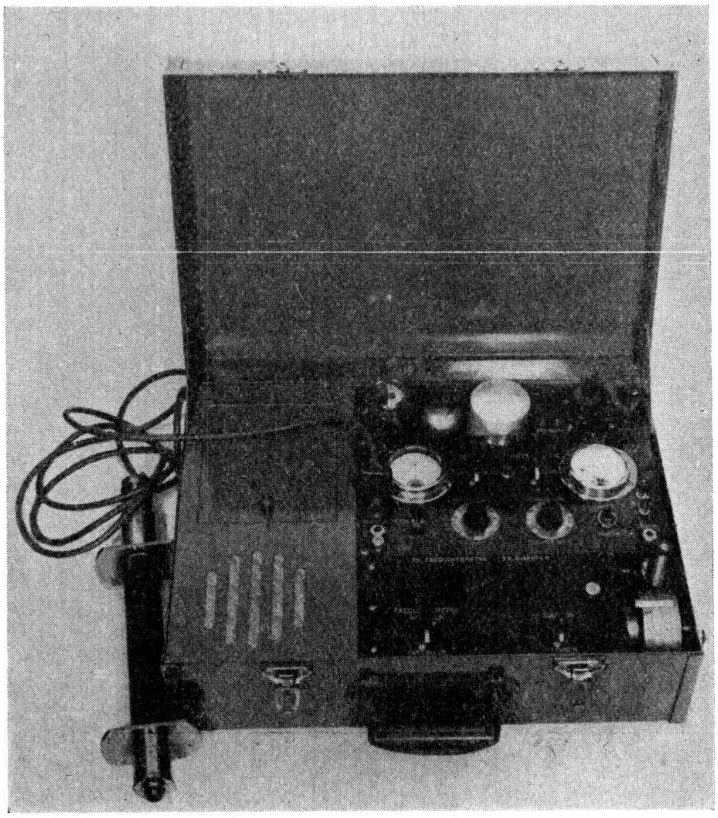


Fig. 5.

Acoustic set.

The first applications to be made by the author had reference to concrete and reinforced concrete, but the method is equally suitable for metal structures and has been of special use in studying the complex stresses existing at the intersections of frameworks and at the joints of pressure pipe lines. The steel wire is

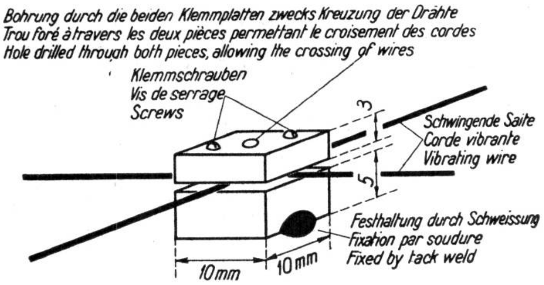


Fig. 6.

Attachment of acoustic cords to metal; diagrammatic.

attached to the metal structure in a very simple way by means of welded clamps (Fig. 6). The accompanying photographs (Figs. 7 and 8) show the application of the system to a thick sheet of metal.

Two examples of this kind of auscultation will now be given. One of them relates to an intersection point in the main girder of the Port de Pascau bridge over the Garonne, a structure which was auscultated at 51 points, by this acou-

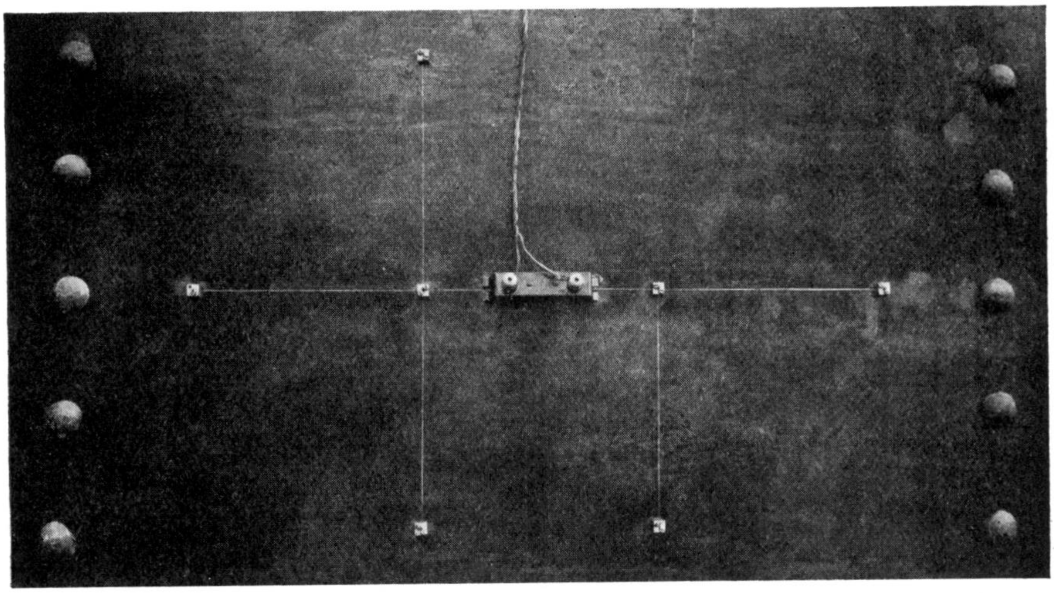


Fig. 7.

Attachment of acoustic cords to metal.

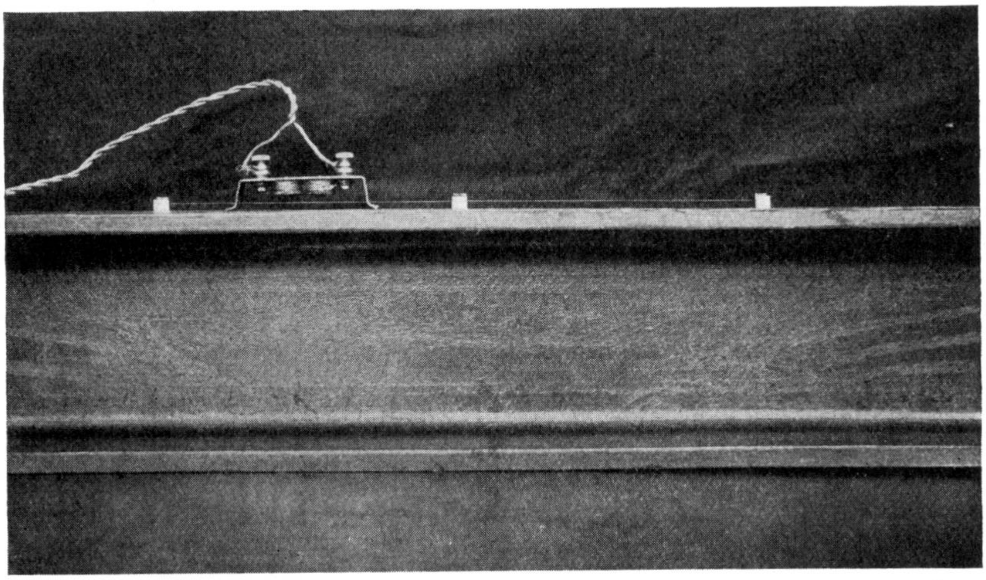


Fig. 8.

Auscultation of upper surface of an I-beam.

stical method in the course of the acceptance tests. The diagram (Fig. 9) shows the variations in elongation along the flange plates of one of the booms in the neighbourhood of the connecting gusset under a certain loading, the influence exerted by this gusset being clearly apparent.

The other example is that of a small scale model of a branch piece in pressure piping subjected to internal pressure. This was covered by a network of vibrating wires, as indicated in the drawing (Fig. 10), and the variations in length of these wires along two generatrices and two directrices are indicated for a particular pressure (Table I). It will be noticed that apart from the elongation of the plate itself the wires are affected by a considerable amount of elongation due to bending in consequence of their distance from the surface. These two elongations are distinguished by coupling each instrument with a second instrument fixed at the same point but further removed from the surface.

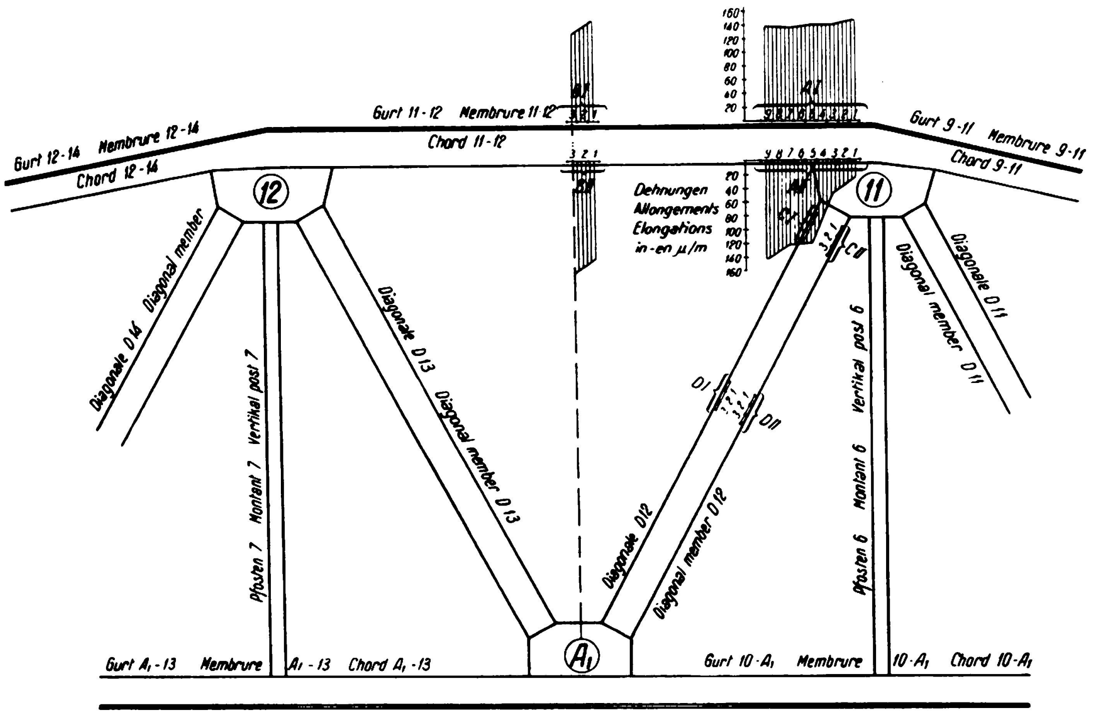


Fig. 9.

Auscultation of a steel girder.

The first advantage of this method lies in its economy, for it will be noticed that the number of measurement made on any given piece of steel can be multiplied very considerably at very small cost. A single wire, as shown in Fig. 7, might be used to indicate the elongations at a number of points, simply by dividing it into as many sections as are desired by means of clamps welded on intermediately between the two end points. In this way a whole network may be formed, from which no point is excluded. It will further be noticed that the auscultation is carried out by the operator from a central measuring post from which he has no need to move; this again greatly simplifies and cheapens the measurements and renders them almost simultaneous, since the adjusting operation adjustment when listening is very short. The method also allows of auscultating inaccessible points.

Finally it will be noticed that the causes of error are minimised by the use of sound as the medium of transmission. Indeed nothing is easier at the present time than to transmit a sound, either over a wire or even without wires, *without* *altering its frequency.* The sensitiveness of the apparatus is very high, and in the laboratory it may reach one millionth. In practice, it need hardly be stated, so

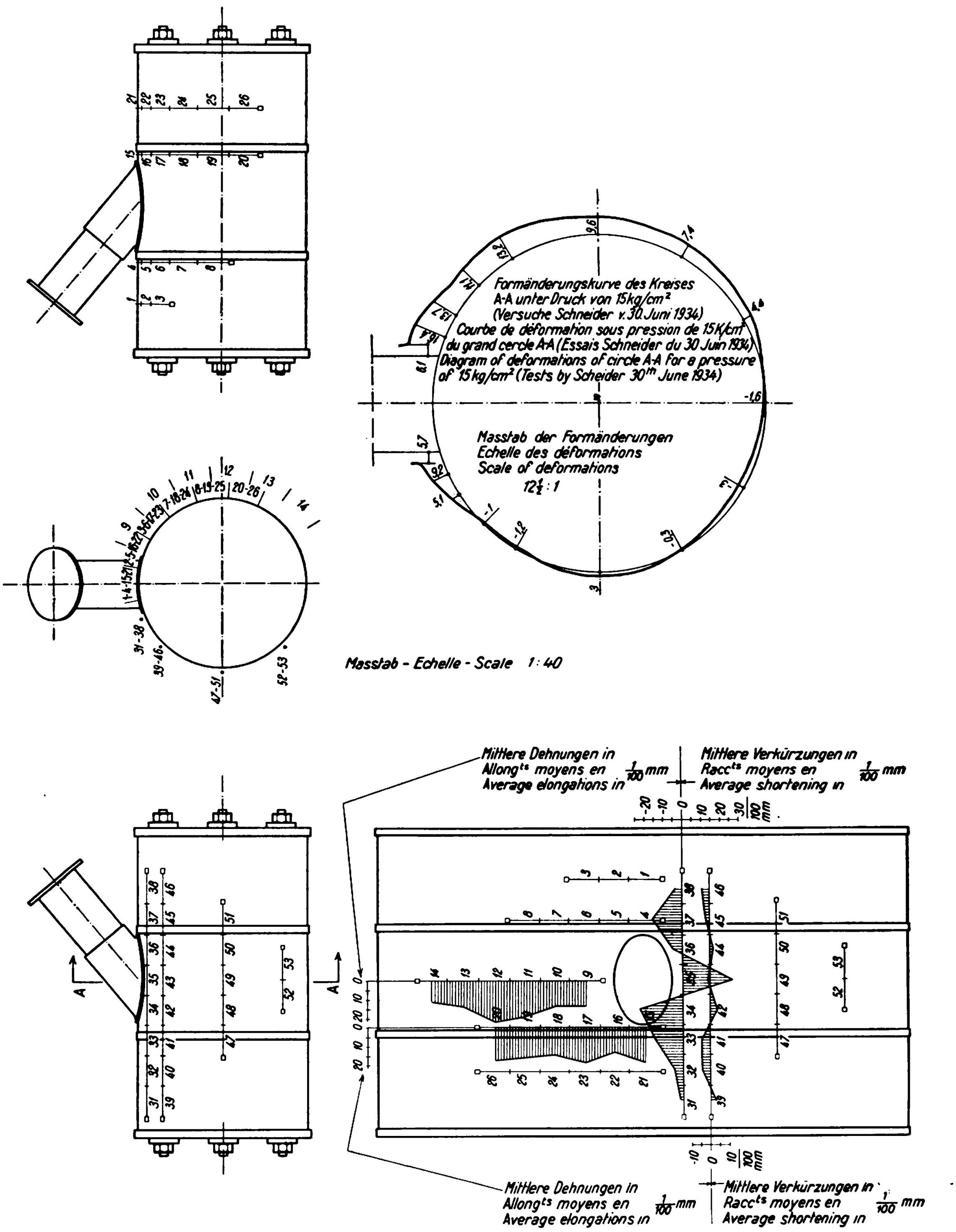


Fig. 10.

Auscultation of a small scale model of a branch in pressure pipe.

high a precision is unnecessary and in any case is unattainable on account of the inevitable differences in temperature as between the wire and the member under auscultation, but this difficulty — which is easy to circumvent — applies equally to all kinds of extensometer.

The method is also applicable for the measurement of dynamic stresses, by means of oscillographic recording.

Table I.

Elongation of strings in $^1/_{100}$th of mm in relation to test pressure.

Strings on parallel circles					Strings parallel to axis of tube				
Nrs.	Test pressure			Observations	Nrs.	Test pressure			Observations
	5 kg/cm^2	10 kg/cm^2	15 kg/cm^2			5 kg/cm^2	10 kg/cm^2	15 kg/cm^2	
1	18.6	36.6	55.0		31	— 1.4	— 5.0	— 9.0	
2	14.2	28.1	43.4		32	— 4.3	— 8.1	— 12.5	
3	17.7	34.6	54.6		33	— 14.5	— 29.2	— 49.0	
4	11.1	20.3	29.1		34	— 22.6	— 47.4	— 79.0	
5	10.8	24.0	36.4		35	+ 26.3	+ 57.1	+ 86.0	
6	13.4	27.0	41.6		36	— 4,9	— 12.0	— 25.5	
7	15.4	32.9	46.2		37	— 15.6	— 34.6	— 60.0	
8	16.2	31.0	49.3		38	— 4.0	— 11.6	— 17.0	
9	13.4	24.6	35.2		39	+ 2.6	+ 1.2	+ 0.3	
10	13.4	25.9	42.6		40	— 4.6	— 10.1	— 21.2	
11	18.9	39.4	58.8		41	— 4.6	— 10.4	— 19.9	
12	21.8	43.3	51.6		42	+ 3.8	+ 6.1	+ 2.8	
13	13.7	29.1	48.1		43	— 0.8	— 3.2	— 8.0	
14	11.1	23.4	37.6	Length of string: 12.2 cm	44	+ 2.1	+ 2.0	— 0.9	
15	18.4	38.2	61.5		45	— 1.7	— 6.0	— 13.6	
16	12.9	25.3	36.6		46	— 4.0	— 11.2	— 20.8	
17	18.5	37.0	57.0		47	— 2.5	— 4.9	— 11.2	
18	14.5	29.0	44.9		48	0.0	— 1.3	— 4.9	
19	15.9	32.4	51.0		49	— 1.8	— 3.2	— 9.2	
20	17.2	35.6	54.5		50	+ 2.0	+ 1.7	0.0	
21	18.8	34.8	54.3		51	— 2.9	— 1.4	+ 1.8	
22	13.4	27.6	42.6		52	+ 8.4	+ 3.5	— 5.0	
23	12.7	26.4	41.3		53	0.0	— 2.7	— 10.3	
24	21.1	43.6	65.0						
25	16.6	33.6	52.3						
26	18.6	37.0	59.3						

V 7

Experiments on Welded Frame Intersections, with Special Reference to Vierendeel Girders Subject to Heavy Dynamic Stresses.

Versuche mit geschweißten Rahmenecken, insonderheit für dynamisch hoch beanspruchte Vierendeelträger.

Essais sur noeuds rigides soudés, spécialement de poutres Vierendeel soumises à de fortes sollicitations dynamiques.

Dr. Ing. A. Dörnen,
Dortmund-Derne.

The two comparisons between riveted and welded designs of the same steel frame components, which are represented respectively in Fig. 1 (corresponding to Fig. 3 of the paper by *Schaper* in the Preliminary Publication, p. 1370) and in Fig. 2, go to indicate the superiority of welding both from an aesthetic and a technical point of view. The frame shown in Fig. 2 can, indeed, scarcely be made at all by riveting without objectionable features: a number of the rivets have to be excessively long; many of them cannot be hammered effectively if at all; a still larger number are incapable of replacement. It is clear, in fact, that we are here at the limit of what is feasible with riveting.

In both comparisons the welded design is also definitely the more economical, as the cost of construction is approximately 17 % greater in the riveted design for both Fig. 1 and Fig. 2. So far as the example shown in Fig. 1 is concerned this difference has been confirmed in the construction of 27 riveted and 25 welded frames. In the case of full webbed frames the conditions which favour welding are indeed particularly marked, and it was only to be expected that, as Professor *Campus* shows in his paper, the Vierendeel girder which consists of practically nothing but frame corners would lend itself especially well to welded construction.

In view of this circumstance *Dr. Schaper*, Director of the Reichsbahn, called for fatigue tests on welded frame corners of Vierendeel girders in 1932, and such experiments were carried out by *Dr. Krabbe* and the author in the works of the latter from 1933 to 1936 with the object of constructing welded Vierendeel corners for use in railway bridges subject to heavy dynamic stresses, the criterion to be satisfied being that such corners made in St. 37 should be able to withstand two million changes of load under an altenating stress of $\pm$ 1400 kg/cm^2 without damage. In order that useful results might be obtained it was necessary that the specimens should not be too small, and they were made roughly one third of the full size used in railway bridges of class "N" of 50 m span. Fig. 3 and 4 below show the testing machine and its method of working, the frequency of alternation being approximately 25 per minute. Altogether 27 specimens were tested.

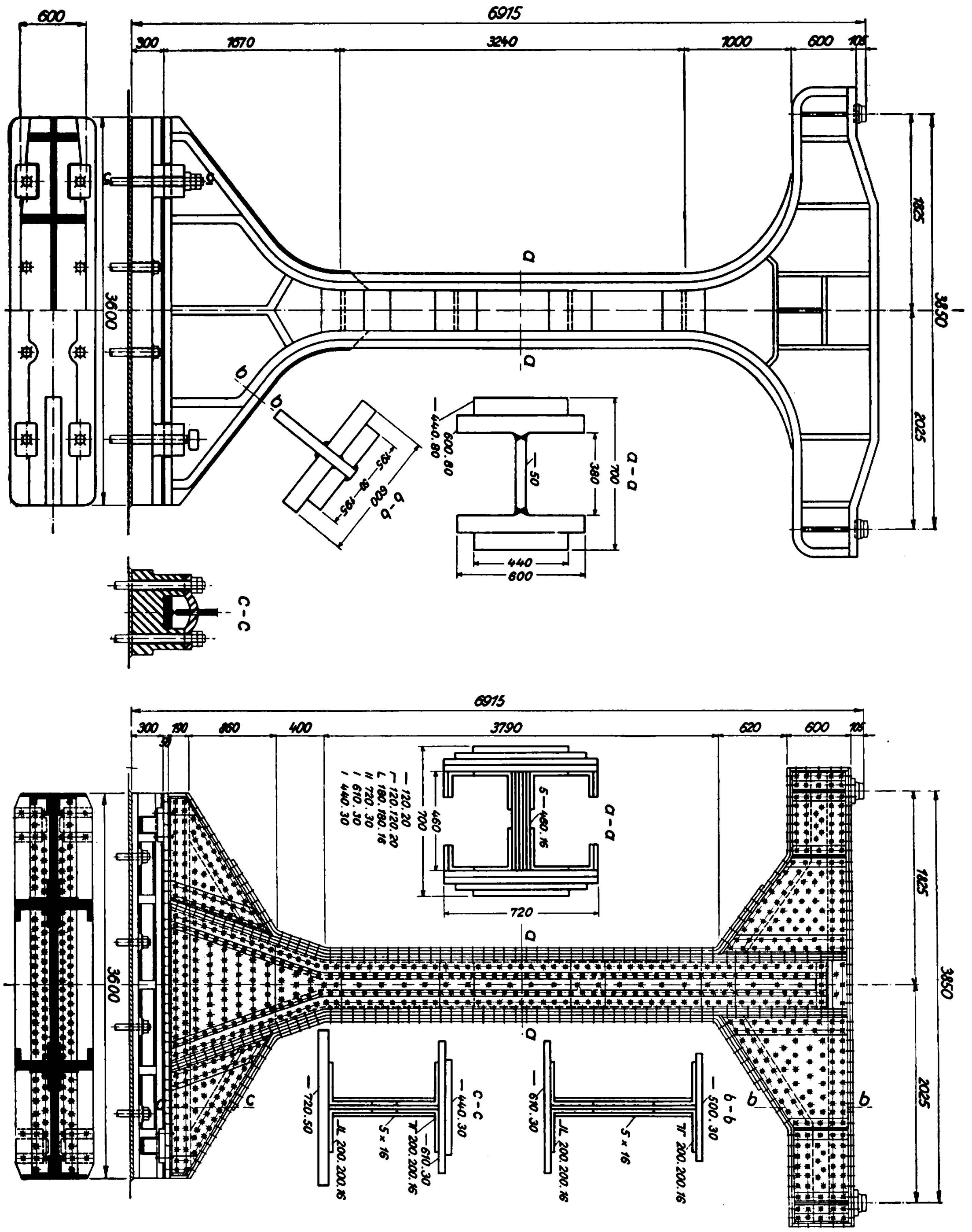


Fig. 2.

Welded Vierendeel girders had already been used in Germany for statically stressed structures. Thus Fig. 5 shows a portion of the main girder for a signal gantry at Stendal,[1] in the design of which endeavours have been made to ensure

[1] Dr.-Ing. *Schröder:* Zustandsänderungen und Spannungen während der Schweißung des Stahlbaues für das Reiterstellwerk in Stendal. Der Bauingenieur, 1932, No. 19/20.

a properly graduated transmission of stress by thickening the web plate at the corners and welding ribs onto it. Here, as in the next example, no special welding considerations arise as the structures were subject mainly to statical stresses. As

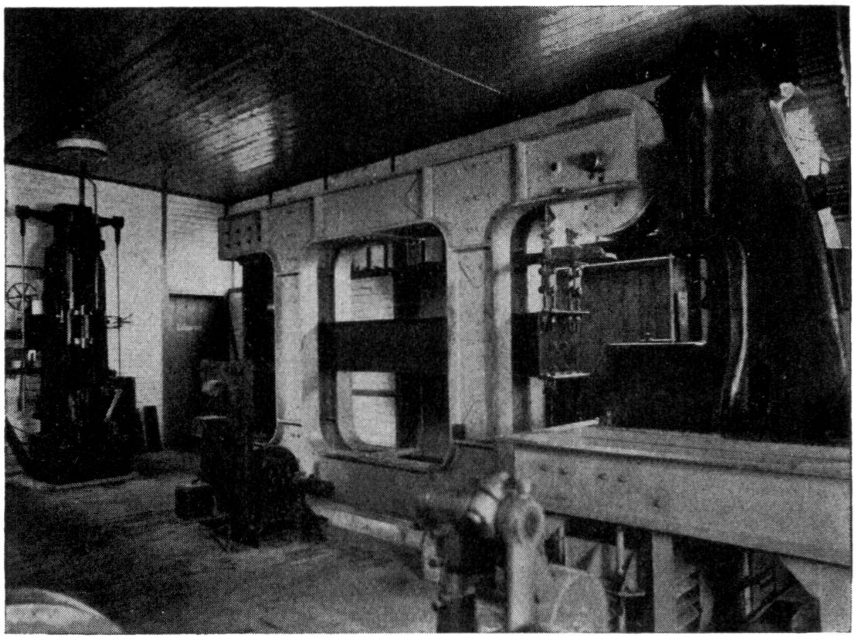


Fig. 3.

a further example, Fig. 6 shows one of the completed girders of 25 m span for the main station at Düsseldorf, and Fig. 7 the detail of the intersections. This

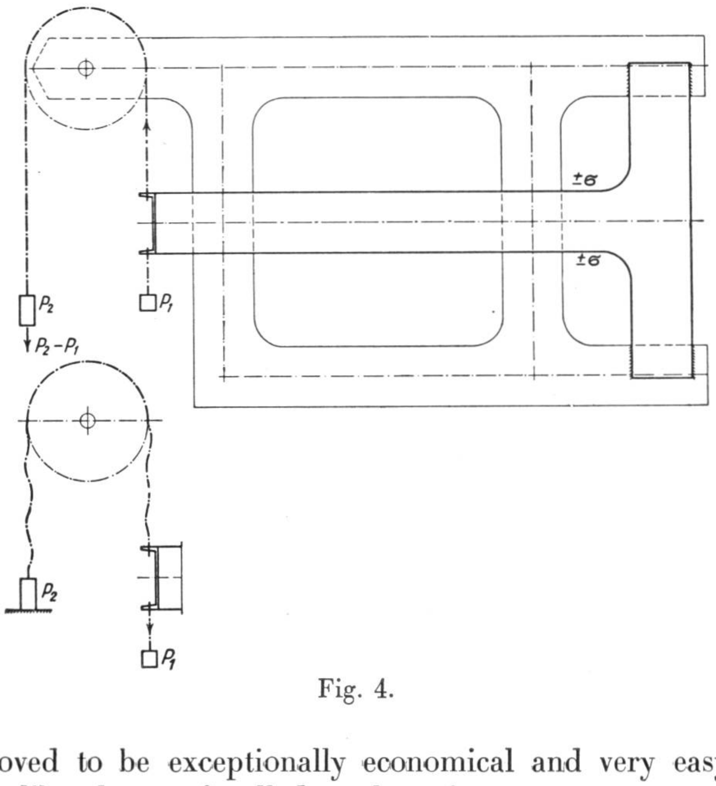


Fig. 4.

construction proved to be exceptionally economical and very easy to carry out, despite the very liberal use of rolled steel sections.

The fatigue tests undertaken had reference, however, to corner pieces subject to dynamic stress. The various designs of corners tried are indicated in Figs. 8 and 9, and fall into three groups:

In the first group the flanges of the vertical members are made continuous with the flanges of the booms and no special structural elements are provided to transmit the forces between them (Figs. 8a, 8b, 9e).

In the second group the flanges are stiffened against deformation by welding plates onto or below them (Figs. 8c and 8d).

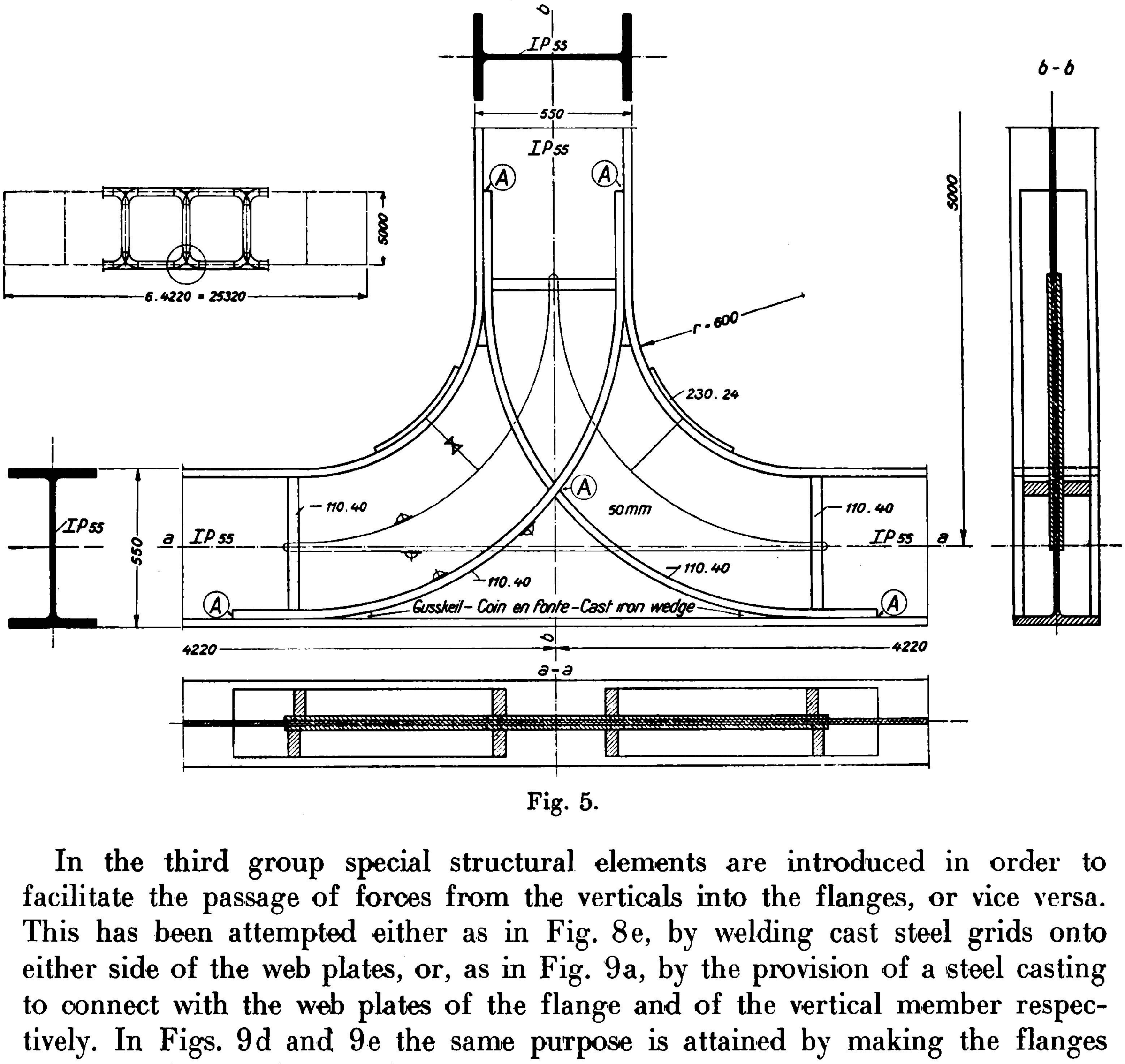


Fig. 5.

In the third group special structural elements are introduced in order to facilitate the passage of forces from the verticals into the flanges, or vice versa. This has been attempted either as in Fig. 8e, by welding cast steel grids onto either side of the web plates, or, as in Fig. 9a, by the provision of a steel casting to connect with the web plates of the flange and of the vertical member respectively. In Figs. 9d and 9e the same purpose is attained by making the flanges of the verticals and of the booms intersect one another.

The corner piece designed as in Fig. 10 was the first to satisfy requirements by withstanding two million alternations of load $\pm$ 1400 kg/cm^2 and subsequently 1.5 million alternations at $\pm$ 1800 kg/cm^2 without manifesting any defects. It is extremely simple and differs from that shown in Fig. 9e only in the substitution of rolled T sections for the flat flanges. The period that the corner shown in Fig. 9e, with flat flanges, withstood the test was considerably shorter — presumably because in Fig. 9e the "neck seams" are directly on the flange whereas in Fig. 10 they are 30 mm away.

While the tests were in progress a special requirement arose, namely the construction of a girder for a rail-car travelling at about 200 km per hour, to

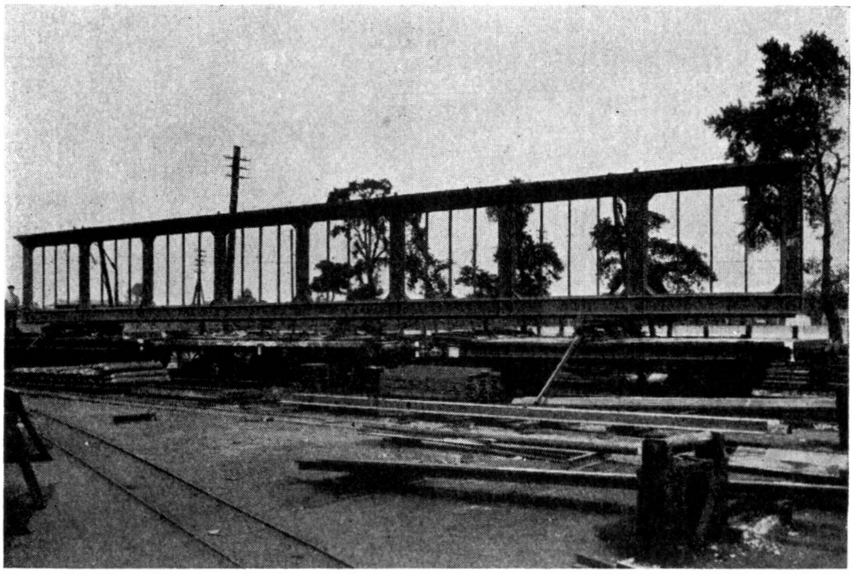


Fig. 6.

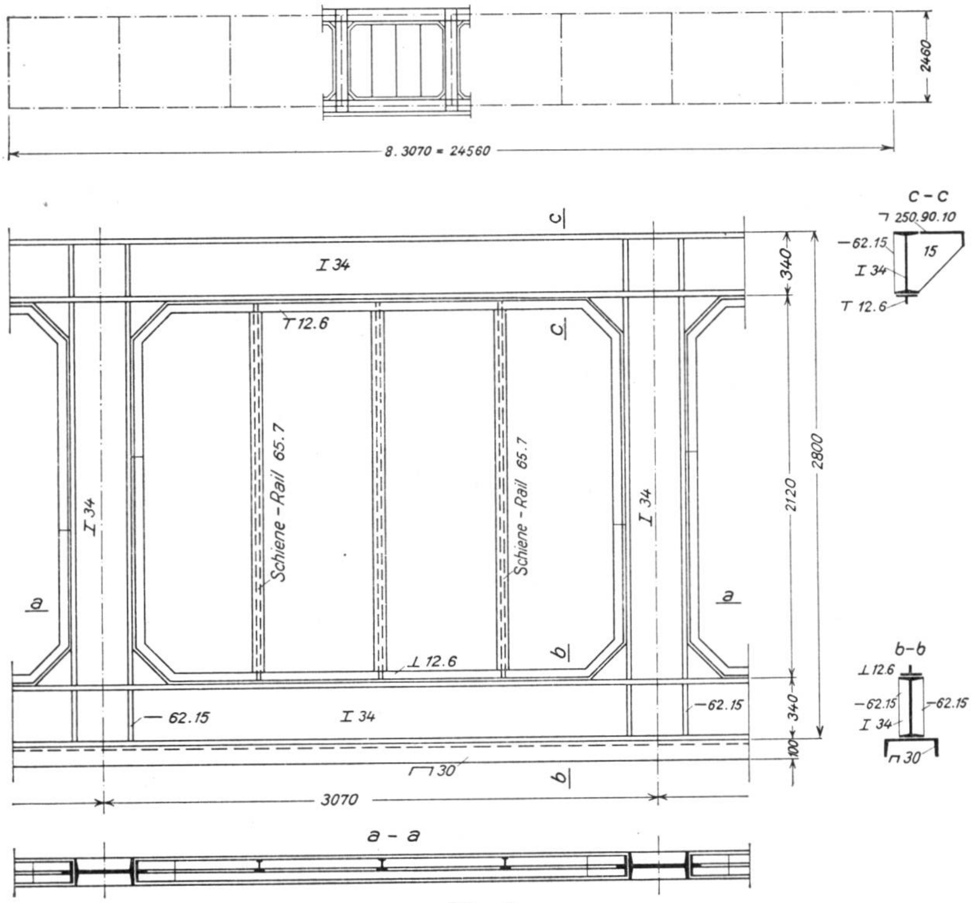


Fig. 7.

resist very heavy dynamic stresses and to be in the form of a Vierendeel girder up to 24.2 m long. Fig. 11 shows the cross section based on the experiments, and the design of the girder. If the web plate is assembled by welding in advance of the flanges such a girder can be kept practically free from welding stresses

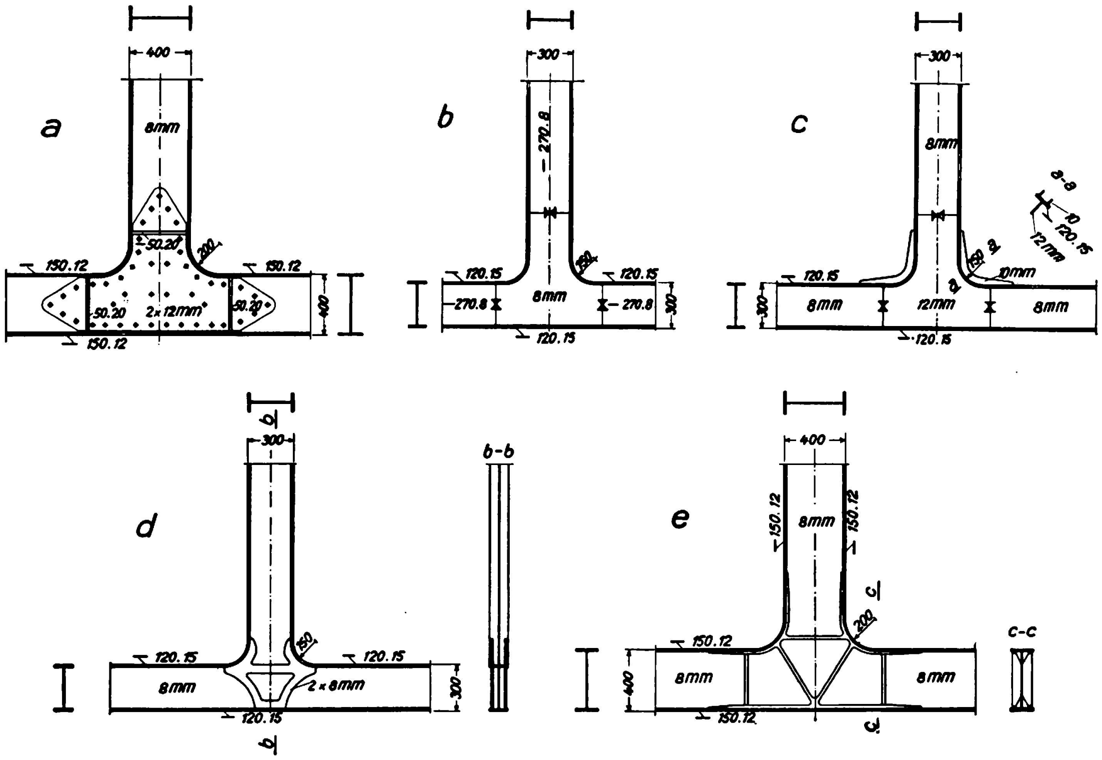


Fig. 8.

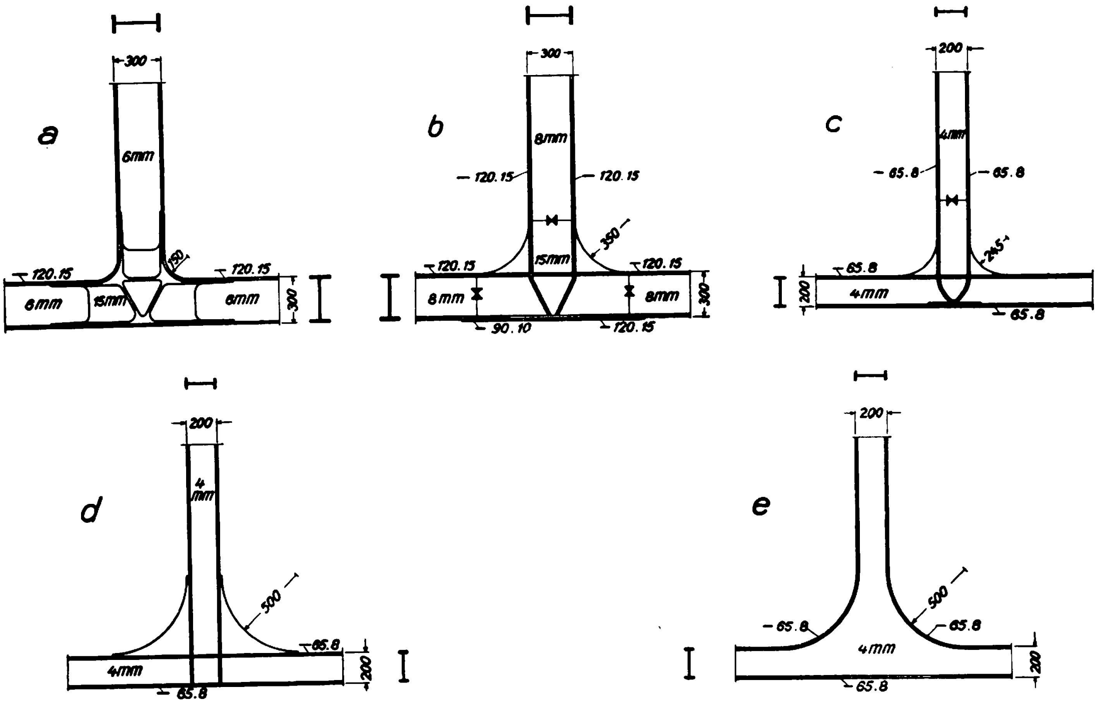


Fig. 9.

due to external effects. Fig. 12 is an isometric view of part of the car frame. The connection between the flange and the web is made jointless by the use of specially rolled section on the *Dörnen* system, and the cross girders are riveted on. The welded construction for the train 61 m in length weighs only about 17 tonnes.

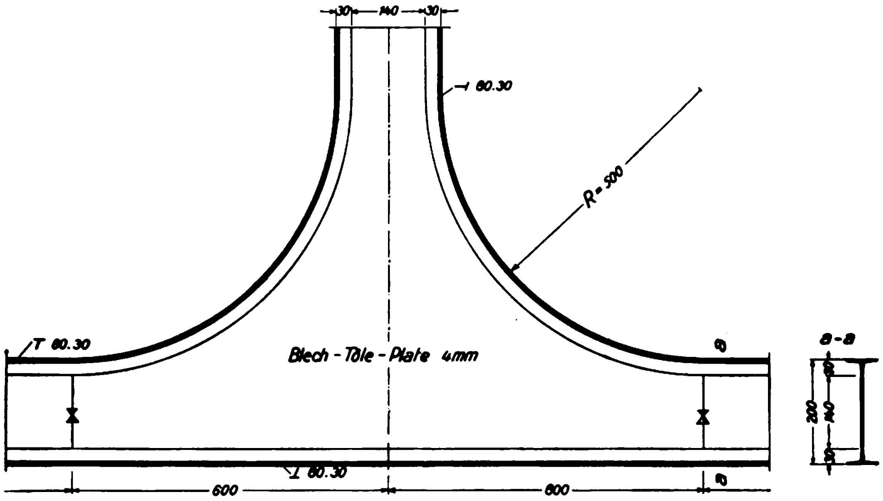


Fig. 10.

As is well known, welded structures behave quite differently from riveted structures as regards dynamic stresses, and the author desires specially to emphasise this before comparing the results of his fatigue tests with those mentioned by Professor *Campus* who used statical tests when measuring his stresses.

1) Under dynamic stresses the most desirable type of corner design is that in which the flanges of the booms are continuous with those of the verticals. In this respect any discontinuity is to be avoided as far as possible.

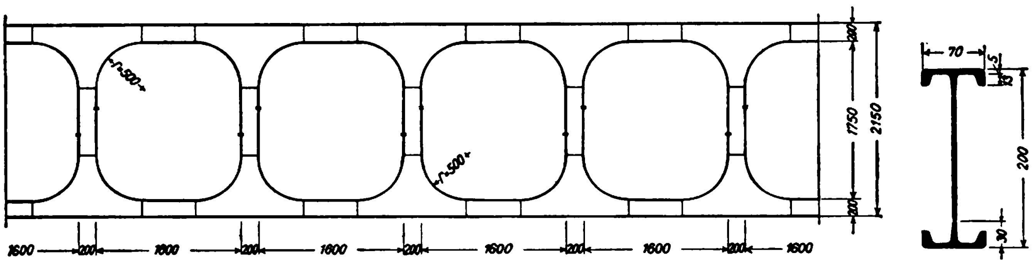


Fig. 11.

2) With a view to avoiding excessive radial forces in the seams connecting the web to the flanges the radius of the inside of the corner must not be made too small. The author is in agreement with Professor *Campus* that an elliptical or hyperbolic shape with a very gradual transition to the straight line, is the best form.

3) The smaller the radius of curvature the greater will be the difference in stress in the curved portion between the edge and the middle of the flange, because the greater is the extent that the edges of the flanges relieve themselves of their stresses, by buckling if they are in compression or by stretching across the curve if they are in tension. Under the low frequency of 25 alternations per minute this effect could be observed and the working of the specimens (which gave the impression of live organisms) could be followed with the naked eye.

4) Under present conditions of welding the presence of transverse seams and ends of fillet seams, and crowding together of seams, is particularly to be avoided

at the points of transition between the curves and the straight lines. In this matter the experiments entirely confirm the results of the investigation which served as

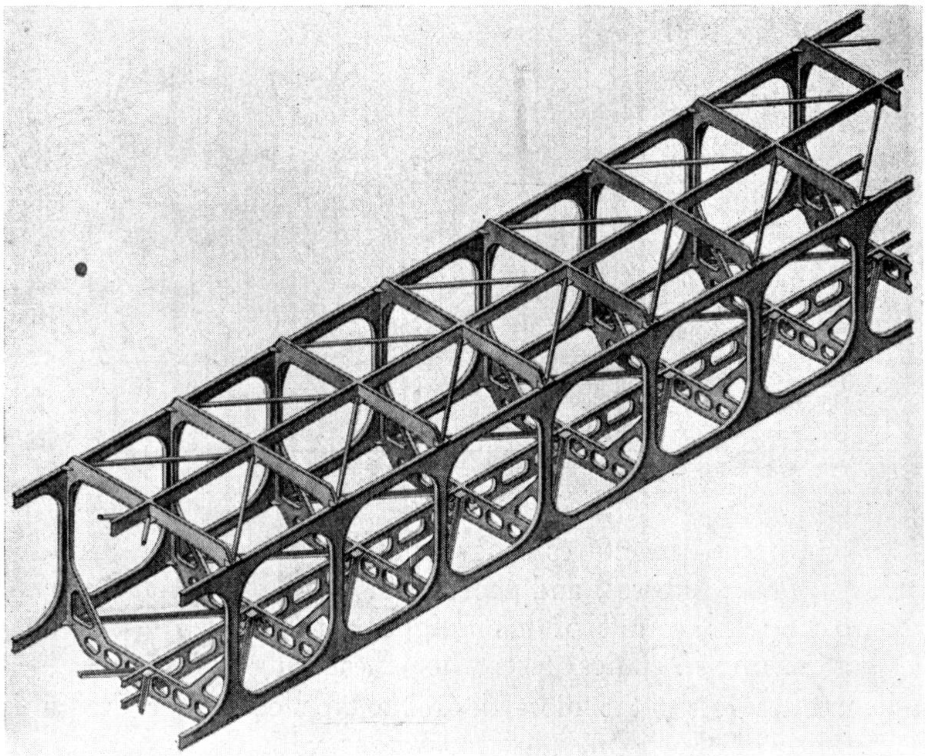


Fig. 12.

the basis for the welding regulations for bridges on the German Reichsbahn. The unfavourable behaviour of these seams is particularly marked at the corners of frames, because there the notch effect is augmented by the change in cross section. On this account the writer feels less inclined than *Campus* to approve the welded intersections used in the road bridge at Lanaye.

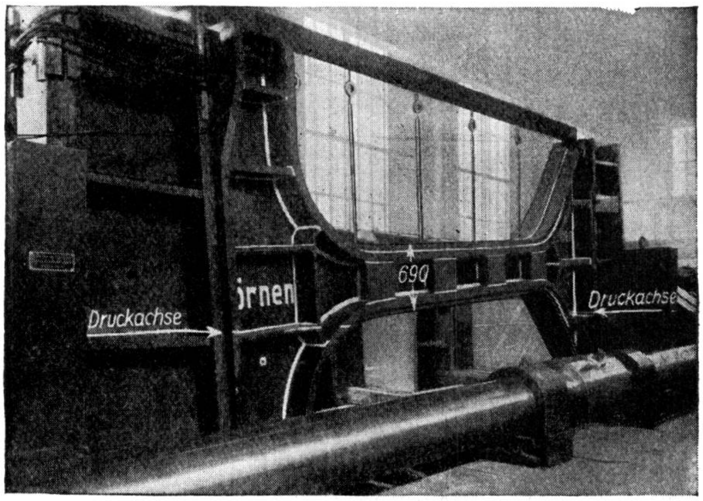


Fig. 13.

5) The web plate should be suitably strengthened at the intersection, which is best accomplished by using a thicker plate. The connection between this plate and the plate of normal thickness must be arranged outside the curved portion and the X-seams forming it must be kept free from internal or external stress. The author would further recommend that important seams be made thicker than the cross section to be connected and that the weld and its neighbouring portions be brought up to a red heat and the weld metal hammered out to the thickness of the plate, and ground smooth.

6) Corners with special structural elements to ensure the transmission of the flange forces were adopted as early as 1930 by *Dr. Schröder* in the signal gantry at Stendal, and as shown in the paper by *Campus,* have not confirmed expectations. It is wrong, however, to conclude from this that the idea underlying the design was faulty, the reason being that an unfavourable effect was exerted by the transverse seams which it was impossible to avoid.

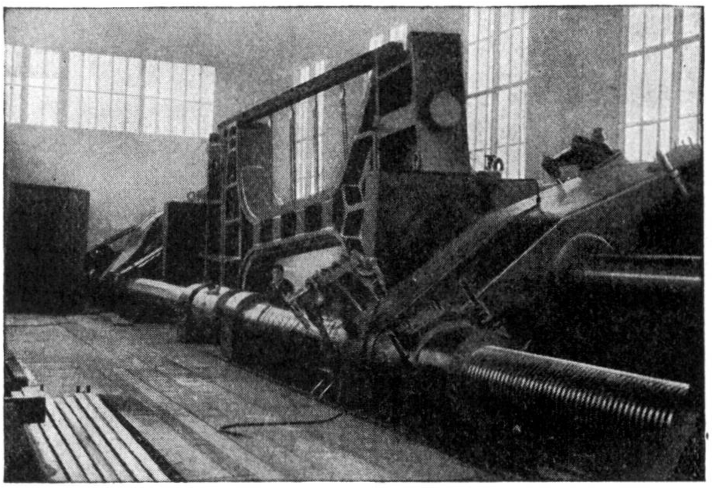


Fig. 14.

7) Having regard to the complicated conditions of stress which arise at the corners of the frame, cross girders, bracing members and the like are best attached by means of riveting.

The possibility of so constructing full webbed framed designs that the carrying capacity suffers no diminution through welding stresses is indicated by an experiment on the full sized welded columns represented in Fig. 2 which was carried out for the German Reichsbahn in the Staatliches Materialprüfungsamt at Berlin-Dahlem using the 3000-tonne testing machine of the Deutsche Stahlbau-Verband, Berlin. Figs. 13 and 14 show the column loaded eccentrically to correspond with the conditions arising in service. Fig. 15 shows the column greatly deformed after receiving an eccentric load of 1300 tonnes. In regular service it will carry approximately 330 tonnes. Large deformations first occurred under a pressure of 1300 tonnes, without any cracks being observed in the main seams or in the parent material itself, and only a few of the caulking seams connecting the stiffeners of the web plates with the flanges being loose. No defects which might have impaired the carrying capacity of the column had occurred. This experiment should go far to refute any apprehensions that may be felt as regards welding stresses in such structures.

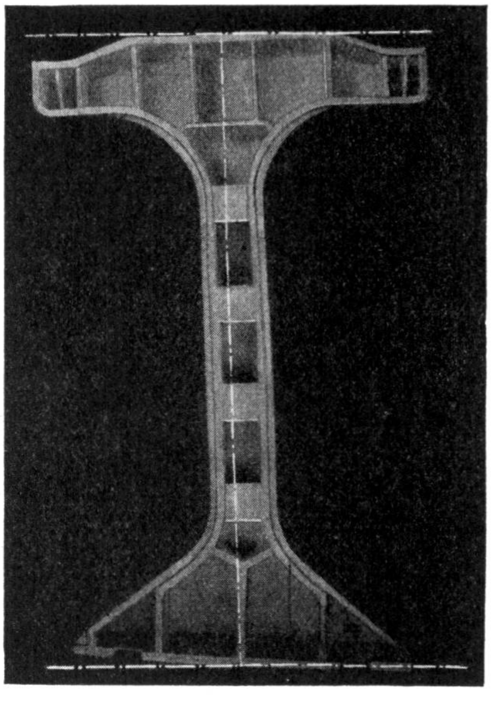


Fig. 15.

(Figs. 13 to 15 have kindly been placed at the author's disposal by the publishers of the journal "Elektroschweissung", No. 7, 1937).

V 8

Strengthening of the Ill Bridge near Strasbourg.

Die Verstärkung der Illbrücke bei Straßburg.

Le renforcement du pont sur l'Ill près de Strasbourg.

H. Lang,

Ingénieur en Chef de la Voie et des Bâtiments, Chemins de Fer d'Alsace et de Lorraine, Strasbourg.

In a paper presented before the Congress M. *Bastien* has referred to bridge works recently carried out by the French railways. The present writer wishes to give some details of one of the works there mentioned, namely, the strengthening of the bridge over the River Ill near Strasbourg, by means of a new method of great interest. He feels the more free to do this as the work in question was the idea not of himself but of M. *Goelzer,* whose name will already be familiar to readers.

The bridge over the Ill comprises two identical independent spans of 52 m each, and was erected some sixty years ago to carry the double line of railway from Strasbourg towards Germany. In each of the spans there are two main girders

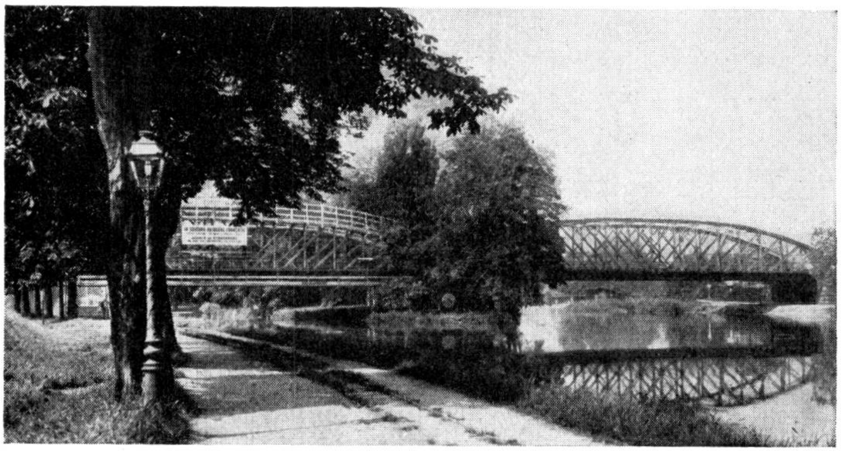


Fig. 1.

of the Howe truss type with the verticals in compression and the diagonals in tension (Fig. 1). The existing bridge was inadequate to carry the heavy locomotives now in use, and its weakness may be judged from the stresses as calculated in accordance with the current rules, which work out as follows:

Upper booms in compression	15.85 kg/mm²
Lower booms in tension	16.40 kg/mm²
Verticals in compression	18.27 kg/mm²
Diagonals in tension	18.29 kg/mm²

whereas a limit of 11 kg/mm^2 ought not reasonably to be exceeded in an iron structure.

One solution would have been simply to replace the bridge, but this, while psychologically simple, was in fact complicated by the need to keep traffic open, and it would moreover have been extremely costly, and also regrettable because the decking of the bridge, although weak, was in a very good state of preservation. For strengthening the bridge, recourse might have been had to direct reinforcement of the girders by the addition of cover straps to the booms, and angle bars to the verticals and diagonals. This again appeared to offer a simple solution but was found to be impracticable in view of the extreme weakness of the connections in the existing structure, consisting as it did of girders wherein the quest after lightness at the time of construction had been carried to extremes and far beyond what was reasonable, especially in the shaping of the gussets which left no room even for the addition of even a single rivet. The difficulty was especially apparent in the neighbourhood of the connections of the cross girders, and neither the engineers consulted on the matter nor the designing staff of the administration concerned were able to put forward any satisfactory proposal. Moreover it was an essential condition for the execution of the work that no portion of the existing construction should be dismantled under load, even under dead load, so that no new strains should arise on account of adaptation when the member in question was being replaced — strains such as might normally be acceptable in a new construction, but not in one which was already old.

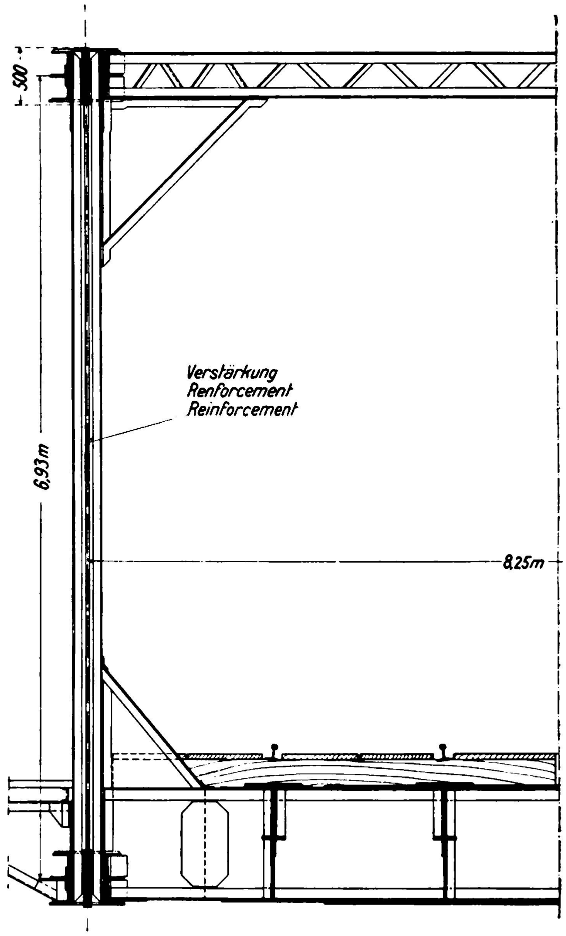


Fig. 2.

Yet a third solution was considered and abandoned: namely, that of erecting new girders of similar construction alongside the existing ones. The result would have been both ugly and heavy, and as the new and old members would have been separate from one another the conditions under which they would have had to act would not have been very satisfactory.

It remains to describe the solution worked out by M. *Goelzer* and now in course of being carried into effect, one of the spans having already been completed. The old girders consisted of two booms of open box section with projecting flanges, connected together by plate-webbed verticals and by flats forming the diagonals (Fig. 2). Within each of these girders a new flat reinforcing girder of the same geometrical form is being built up, inserted between the walls of the old girders, and connected to these by means of flat welded plates.

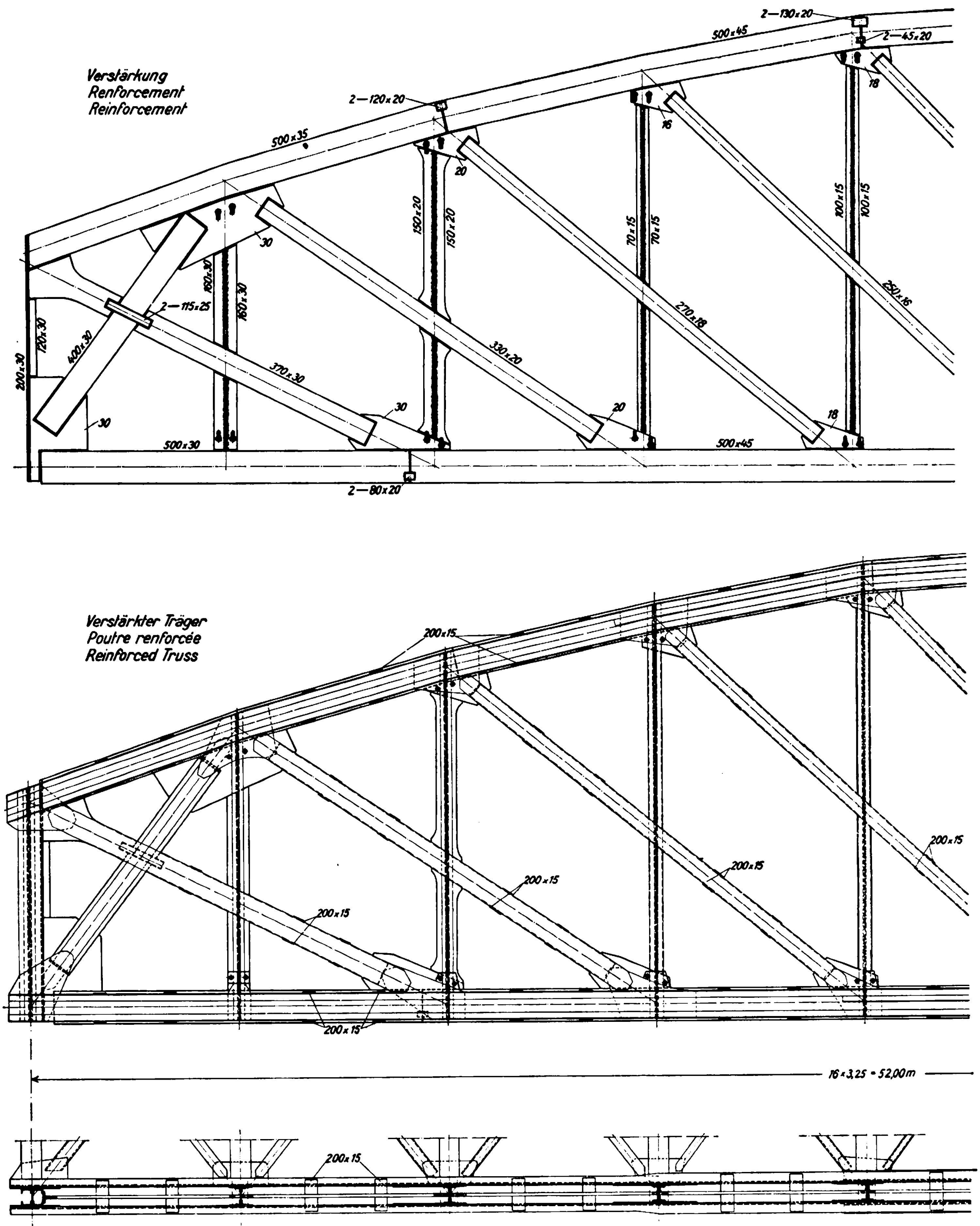


Fig. 3.

The new girder is formed of broad steel plates 40 to 50 mm thick (Fig. 3). It is entirely of welded construction using butt welds wherever possible, especially in the booms and diagonals, with welded fish plates where necessary. All the new pieces are of Martin steel. The booms have largely been welded in the shop, and the remainder of the welding carried out on the site before any part was erected (Fig. 4).

All that was necessary to enable the new members to be placed in position was to form a few slots beforehand in the web plates of the verticals so as to accommodate the boom members and the diagonals (Fig. 2).

It was desirable to load the new girder before connecting it to the corresponding parts of the old in order that the new dead load might not be carried on to the old girders (as frequently happens in reinforcing work), and so that on the contrary, part of the old dead load might be picked up by the stronger new girder. With this object the old girder was lifted on jacks before completing the connections, and was caused to bear upon the new one. The relief afforded

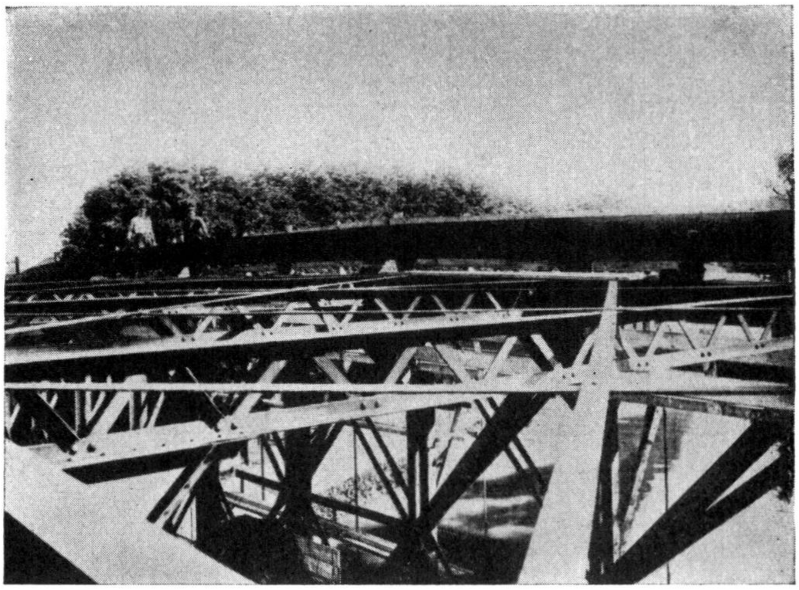


Fig. 4.

to the old girder in this way amounted to approximately one third of its weight, so that the stresses in the old members will not in future exceed 11 kg/mm^2. The amount of strengthening afforded is considerable, since the weight of the new girders is approximately 50 % that of the old.

It should be observed that the strengthening effect is rendered completely effective through the close union that has been obtained between the corresponding old and new elements (Fig. 5).

The operations were, of course, carried out without interrupting the railway traffic, the latter merely being confined to a single track under a reduced speed. The strengthening of the rail bearers, cross girders and bracings involved no special features.

The writer it not unaware of the criticisms which might, in principle, be advanced against a scheme of this kind, especially one in which iron and steel are associated with one another; but the number of essential welds between steel and iron is in fact limited, the three elements that make up each of the girders being connected mainly at their intersections. Moreover practice has shown that such work is successful as in the bridge at Brest and in the case of the Pont d'Austerlitz in Paris — and to the engineer that is what matters.

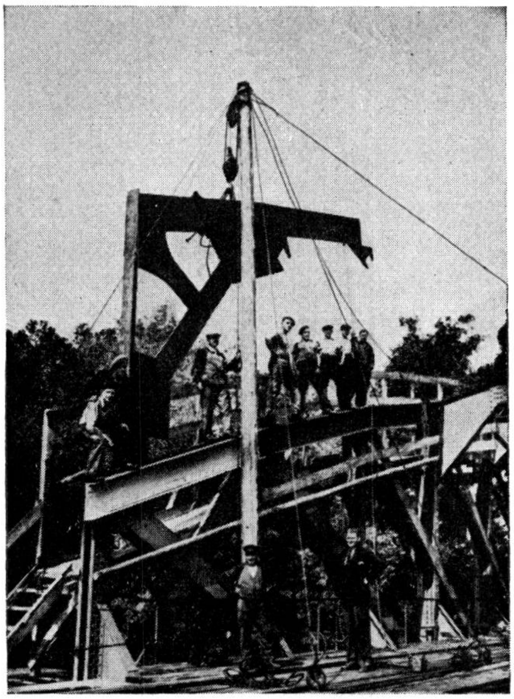


Fig. 5.

V 9

Systematic Tests on Floor Systems Comprising Reinforced Concrete Slabs on Steel Girders.

Planmäßige Versuche an Decken aus Stahlträgern mit Eisenbetonplatten.

Essais systématiques sur planchers constitués de poutrelles métalliques surmontées de dalles en béton armé.

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Ingénieur des Arts et Manufactures, Paris.

A brief explanation is given of the results obtained in the course of systematic tests carried out by the Bureau Securitas, in collaboration with the Office Technique pour l'Utilisation de l'Acier, on the strength of floors composed of steel joists carrying reinforced concrete slabs. These tests were made during the first few months of 1935 in the new Laboratories for Building and Public Works. (Laboratoires du Bâtiment et des Travaux Publics.)

At first sight the action of floors composed of joists embedded in concrete would appear to be similar to that of reinforced concrete members, assuming that adequate bond exists between the rolled sections and the concrete. But the mechanism of the phenomena of bond are so little understood that any theoretical investigation of the problem was deemed to be uncertain, and only systematic experiments, carried to the point of breakdown, were thought to be acceptable as a source of useful information.

It was sought to determine, for each such type of floor, the limits within which a simultaneous development of the strength of the concrete and of the metal might be relied upon as in reinforced concrete construction. With a view to studying separately the various factors usually taken into account which make up the resistance, two series of tests were carried out for each type of floor, the first being intended mainly to arrive at the factor of safety and the conditions of resistance to bending moments; the second more particularly for studying the effect of shear forces.

The mode of operation and results obtained in these tests have been described in greater detail in the Compte-Rendus du Centre d'Etudes Supérieures de l'Institut Technique (1934—35, 22nd Session), of which the following is a summary:

I. Types of floor studied.

It was sought to reproduce conditions corresponding to those likely to be encountered in the floors of the usual types of residential buildings. The cross section of the members considered are shown in the accompanying illustration (Fig. 1), each element consisting of two joists (I PN 10 in the first case and I PN 12 in the second) spaced at 0.80 m centres and supporting a slab 4.50 cm thick, the total depth being the same in the case of all the floors, namely 14.5 cm.

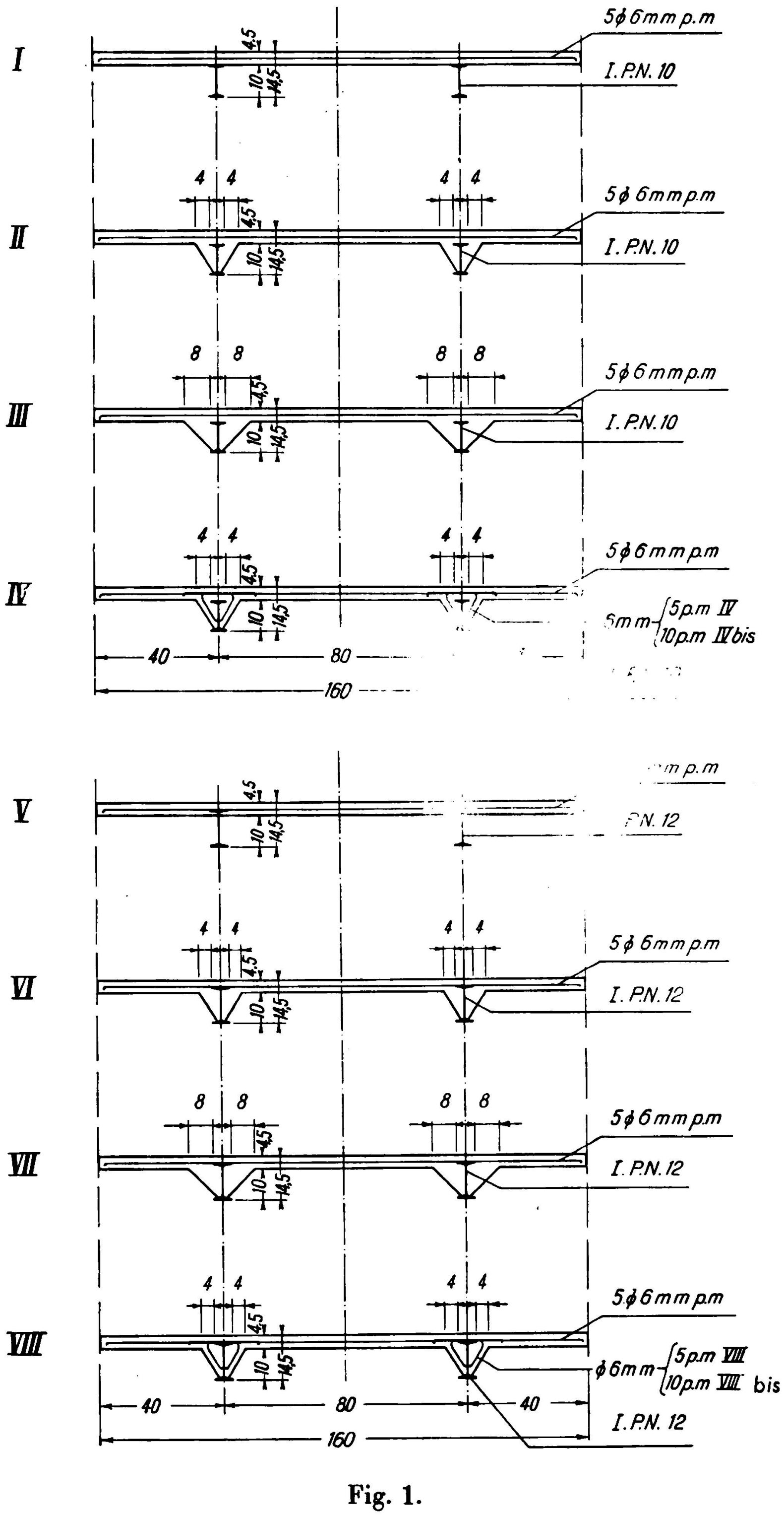

Fig. 1.

Cross sections of the types examined.

Item N° I comprises a concrete slab resting directly on the upper flange of the joists, a type which it was deemed desirable to examine in order to determine, in some degree, the lower limit of the phenomena producing bond between the concrete and the metal. Item N° V consists of two joists of 12 PN in which only the upper portion is embedded 2 cm deep in the concrete slab. All the other types involve concrete which encloses the web of the joists as far down as the fillet of the lower flange. In elements IV and VIII there are binding elements of 6 mm dia. which pass through holes punched in the web and are bent over into the slabs.

The steel used for the joists was of the usual commercial quality, having an elastic limit of 30 kg/cm² and a breaking stress of 40 kg/mm². The quality of the concrete was ascertained from a large number of samples which were tested in compression and in tension.

II. Arrangements made for the test.

A) *Bending Tests.*

These were carried out on specimens having a free span of 4.00 m, the loads being applied as shown in the attached illustration (Fig. 2) by means of a loading

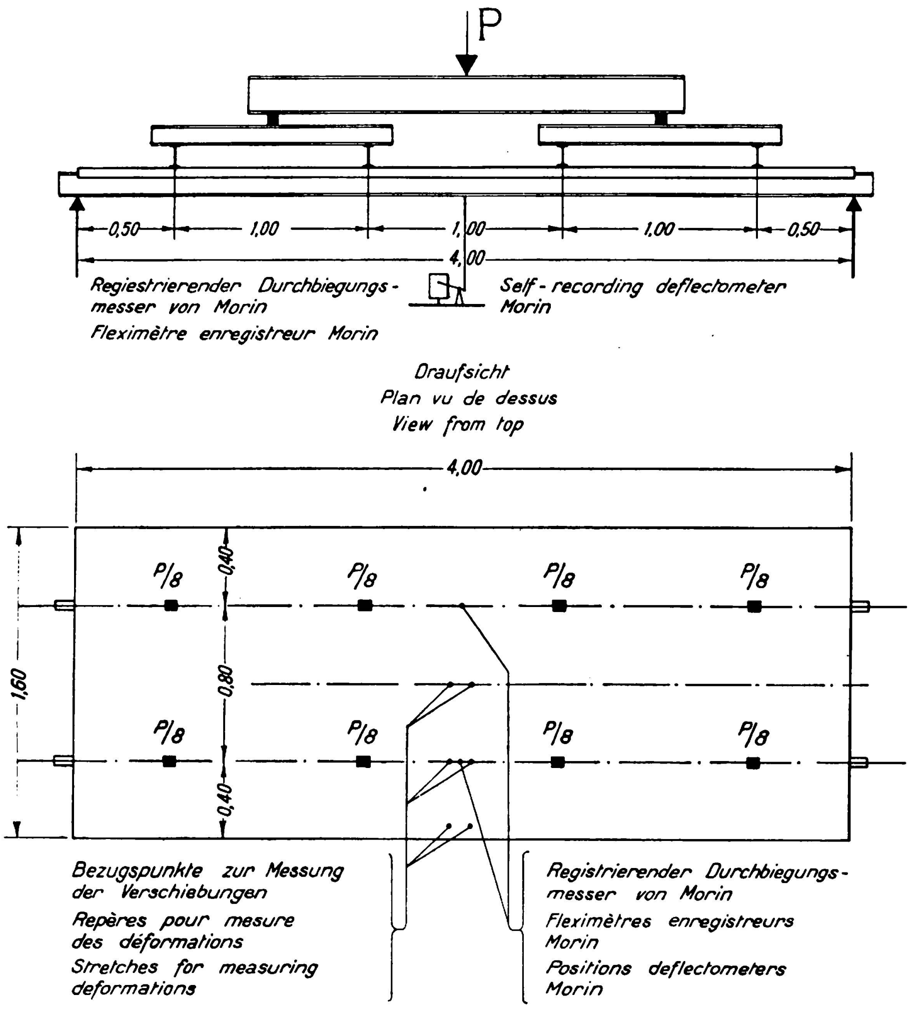

Fig. 2.

Bending test.

lever specially arranged for tests of this kind, in the Laboratory for Building and Public Works.

During the tests the elastic deflections were read by means of recording apparatus arranged underneath the axes of the joists. It was also sought to determine, by means of a *Huggenberger* extensometer, the elastic contraction of the concrete on the upper surface of the slab, and the elongation of the metal at the lower flanges of the joists.

B) *Tests of resistance to shear forces.*

These were made with the object of determining the conditions of breakage of the different types of floor under consideration when subjected to the action of concentrated loads applied in the neighbourhood of the supports, the intention being to determine whether such failure would take the form of a slipping of the rolled joist within its surround of concrete owing to breakage of the bond, or that of the slabs becoming detached from the joists.

The tests were made on elements 2.00 m in length which were subjected to concentrated loads successively applied in the neighbourhood of the two ends, by the agency of screw jacks forming part of the powerful loading machine which has recently been constructed at the Laboratory for Building and Public Works.

III. Results obtained.

A) *Bending tests.*

The results obtained in the bending tests on elements 4.00 m span are shown in table I.

Table I.

No. of specimen	Concrete			Bending moment causing breakage kgm	Limit of elastic behaviour kgm	Deflection in mm under total load P			Stress under load P = 4000 kg			
	age in days	compression kg/cm²	tension kg/cm²			2000 kg	4000 kg	6000 kg	R_b calculated kg/cm²	R_b measured kg/cm²	R_a calculated kg/mm²	R_a measured kg/mm
				each beam								
I	29	270	23	*1350*	800	15						
II	28	280	23.4	*2915*	1550	5.05	9.8	16				
III	40	315	26.4	*2680*	1550	5.4	10.4	16	58.5 50	69 46.5	16.1 16.8	14.3
IV	20	255	20	*2340*	1550	6.2	12.9	19.5	58.5 50	58.5 39	16.1 16.8	15.8
V	31	325	27	*3330*	2050	5.2	10.4	15.5	51 42.5	52 34.7	13.3 13.8	11.3
VI	23	280	22,5	*3162*	2050	4.8	9.6	15.1	51 42.5	52 34.7	13.3 13.8	11.3
VII	23	225	20	*2910*	2050	4.75	10	15.5	51 42.5	50 33.3	13.3 13.8	11.5
VIII	20	275	18	*3020*	2050	5	10.2	15.6	51 42.5	52 34.7	13.3 13.8	15

Note: R_b = stress in concrete; R_a = stress in steel.

In all the types studied, with the exception of N° I, the bond between the concrete and the joists held good until failure occurred as the result of excessive tension in the steel, which agrees with the classical theory of failure of reinforced concrete members. That is to say, when the elongation of the metal becomes very great the axis is displaced towards the upper surface of the concrete between the joists, and the concrete is cracked by excessive compression.

Item N° I was the only one in which the slab was observed to become detached from the rolled joist before failure occurred, this detachment being observed at the centre of the span under a load of P = 1000 kg. In all the other types the simultaneous resistance of the steel and of the concrete slab certainly held good up to the point of failure under the same conditions as in reinforced concrete floors provided with round bars.

According to the permissible moments determined by the usual methods of calculation with m = 10 and assuming a limiting stress of 12 kg/mm², the factor of safety against breakdown works out at between 3.15 and 3.9.

By examination of the diagrams recorded on the deflectometers during the test it was possible to determine the limits of elastic behaviour for the different types of floor under definite conditions of application of the load. The bending moment so found corresponded to a stress in the metal calculated by the usual methods of the order of 37 kg/mm², and therefore close to the elastic limit of the metal.

The measurements made of the shortening experienced by the upper face of the concrete slab, and of the extension undergone by the lower flange of the rolled joists, enabled the stress in the steel to be determined, and hence, by assuming a modulus of elasticity equal to 22×10^3 kg/mm², to determine the stress in the concrete, putting m = 10 or m = 15. Further calculations were made of the stresses existing in the concrete and in the steel under different loads, again taking m successively as 10 and 15. The results obtained are given in the last columns of the table above, and these show satisfactory agreement between the stresses as calculated from the deformations and the stresses as calculated directly from the hypothesis m = 10. This agreement being at least as good as in concrete slabs reinforced with round bars, the methods usually adopted in the calculation of ordinary reinforced concrete members may properly be applied in the design of floors consisting of embedded joists.

B) *Resistance to Shear Force.*

The results obtained in the test are shown in the table below.

In Specimen N° I the joists became detached from the slab under a load of the order of 4 tonnes, but it was not possible to observe any longitudinal slipping in the neighbourhood of the supports. It would appear, therefore, that from the point of view of increasing the safety of systems of this kind any devices intended solely to eliminate such slipping, such as the use of flat bars welded to the upper flange, are less effective than arrangements for anchoring the rolled sections to the concrete slabs by means either of binding wires passed through holes punched in the web or by means of a spiral welded to the upper flange of the joists.

In all the other specimens breakage occurred after the appearance of tensile cracks in the concrete, through an excess of compression which followed upon the excessive elongation of the steel. No failures of bond of the joists close to the supports appeared until fracture actually occurred, with very large deformation. The shear force at breakage is of the order of at least 7 tonnes for specimens containing joists 10 PN and 9 tonnes for those comprising joists 12 PN.

C) *Distribution of compressive forces in the slabs.*

Advantage was taken of the tests in an attempt to determine how the compressive forces are distributed within the slabs. Most of the official regulations in force lay down limits to be allowed for the width of the slab that may be

Table II.

No. of specimen	Concrete			Shear force causing breakage kg	Corresponding couple kgm
	Age in days	Compression kg/cm²	Tension kg/cm²		
I	29	340	27	4 675	1170
II	28	340	27	8 450	2175
III	40	305	24	8 300 6 900	2080 2080
IV	23	260	19.4	8 300 7 900	2080 2350
IV bis	20	225	20	8 050 8 800	2160 2640
V	30	340	27	10 500 9 000	2620 2700
VI	34	275	26	9 650 8 600	2420 2580
VII	30	230	19	10 300 11 900	2570 3570
VIII	20	280	20	12 500 12 800	3120 3850
VIII bis	20	225	20	12 600 13 200	3150 3950

assumed to act in compression at the centre of the span of the joists, but they give no precise rule applicable to the case where heavy concentrated loads produce maximum moments in sections close to the supports.

It was confirmed, in the first place, that in the specimens 4.00 m long which were examined the shortening of the concrete as measured at the upper face of the slab at the middle of the span (Fig. 2) was the same over the whole width of the slab. Compressive stresses, therefore, are uniformly distributed.

Extensometers placed close to the supports showed that the shortening was greater at the centre of the joists than at the edges; and the measurements made, which it would take too long to describe in detail here, indicated that the following simple rule is justified: if the effective width of slab acting in compression is taken as bounded by two straight lines making angles of 30 to 35° with the joists, the compressive stress so calculated for each section, by applying the usual method, differs only slightly from the maximum stress as measured immediately over the joist.

Conclusion.

To sum up, it may be said that in the case of *all types of floor* examined, except N° I, the results of the tests carried out indicate the propriety of calculating the normal stresses by the methods usually adopted in the case of reinforced concrete floors, the agreement between stresses so calculated and stresses which actually exist being at least as good as in the case of reinforced concrete members. These methods give a factor of safety of the order of 3.5.

As regards tangential stresses in the sections examined, the experiments show that with the exception of N° I the strength of the floor is in practice not limited by any excess in the tangential forces but by the value of the normal stresses.

It is proper to observe that these results were obtained with concrete of good quality. If it is desired to make use of the resistance afforded by the concrete it is necessary that the workmanship applied to this material should be sufficiently careful to ensure at least as high a resistance as in reinforced concrete work.

It is considered that the conclusions stated above apply only to sections similar to those actually studied, wherein, in particular, the neutral axis lies close to the lower surface of the slab and to the upper flange of the rolled joist. It is hoped to amplify these results by tests shortly to be undertaken on sections different from the above.

V 10

Design of the Ends of a Bridge with Parallel Booms.

Ausbildung der Enden einer Brücke mit Parallelträger.

Disposition des extrémités d'un pont à membrures parallèles

Geh. Regierungsrat Dr. Ing. A. Hertwig,
Professor an der Technischen Hochschule, Berlin.

The design of the ends of bridges with parallel booms is a matter in regard to which marked difference of opinion exists between engineers and architects. Some of the latter prefer, in particular, the arrangement shown in the accompanying sketch for the end features of bridges having parallel booms and rhomboidal web members. In addition to this it is very often demanded that the end portal frame shall be left completely open between the upper and lower boom — in other words that there shall be no intermediate transverse stays. In the design for one of the new bridges recently constructed over the Rhine it was originally proposed to carry down the upper wind bracing as far as the point marked a, but the idea was subsequently discarded and the necessity arose, therefore, to investigate the stability of the end portal frame. It was thereupon discovered that both in practice and in the literature the scope of such calculations is confined to symmetrical deformations (see, for instance, *Bleich*).

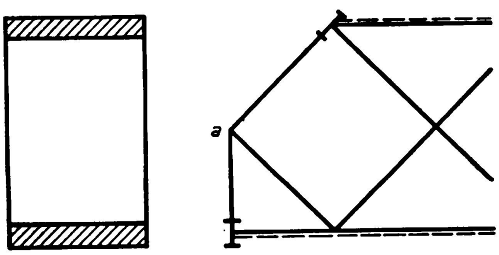

Fig. 1.

It is usual to consider only the most unfavourable assumption, namely that the uprights are hinged at both top and bottom, in which case the buckling load is given by the expression

$$P_K = \frac{\pi^2 E J_V}{h^2}.$$

Assuming, now, that for one sided deformation the moments of inertia J_o J_u are very large, so that the full restraint of the uprights can be taken into account, we find that the buckling load is the same as before:

$$P_K = \frac{\pi^2 E J_V}{h^2}.$$

As a rule, however, the moments of inertia J_o and J_u are not large enough to justify the use of this approximate formula resting on the hypothesis of one-sided deformation.

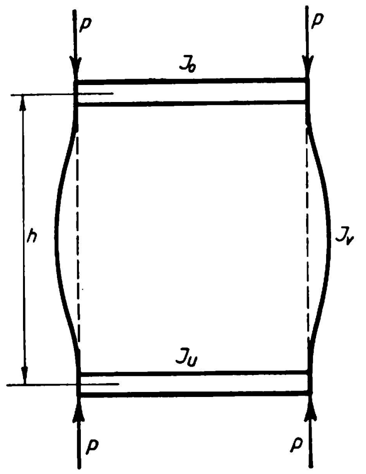

Fig. 2.

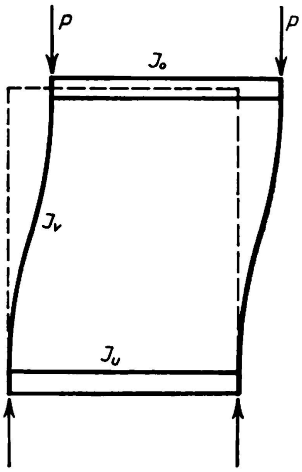

Fig. 3.

For the purpose of the Rhine bridge aforementioned, it was found necessary to provide a very heavy cross brace at the level of the upper boom, and the kink occurring in the line of the uprights at the end portal frame had a further unfavourable effect.

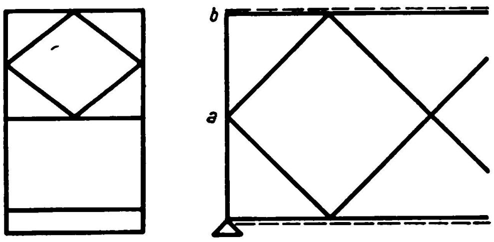

Fig. 4.

It has to be admitted, therefore, that the solution indicated in the sketch detracts from the quality of the structure at two important points solely for the sake of appearance.

If it is essential for the girder to have its corners cut off it is preferable that the wind bracing should be carried down to point a, and it is better still to design the end of the bridge after the manner of Fig. 4 with stays and braces between a and b.

A precise investigation of the buckling conditions in end frames under one sided deformation is given in the paper by *Hertwig* and *Pohl* which appeared in Stahlbau, 1936.

V 11

Stability of the Webs of Plate Girders Taking Account of Concentrated Loads.

Die Stabilität der Stegbleche vollwandiger Träger bei Berücksichtigung örtlicher Lastangriffe.

La stabilité des âmes de poutres pleines, calculée en tenant compte des charges locales.

Dr. Ing. K. Girkmann,
Privatdozent an der Technischen Hochschule Wien.

Plate girders are occasionally called upon to carry loads on the compression flange intermediately between stiffeners, as for instance in crane girders, in the rail bearers of railway bridges, and also in the main girders of such bridges when the sleepers rest directly over them.

An examination of the stability of the web-plates of plate girders loaded in this way has been made by the present author firstly in his paper „*Stegblechbeulung unter örtlichem Lastangriff*"[1] and the basis of the treatment there adopted will be outlined here before proceeding to explain a simplified method.

To determine the limit of stability of the web plate accurately in each particular case would involve lengthy calculations. In order to reduce these and to arrive at results possessing general validity a number of approximations had necessarily to be introduced. In the first place the distribution of load along the edge of the web plate owing to the loaded flange being sufficiently rigid to resist bending with the aid of the cross stiffeners is treated separately, and is calculated under simplified assumptions. Such distributions of load p (x) are represented in Figs. 1a and 1b. The former shows an arrangement in which the stiffeners are practically inoperative, representing the case of a plate girder in which the loaded flange has little stiffness to resist bending, or where the stiffeners are placed far apart. Fig. 1b, on the contrary, shows the distribution in a girder with a thick flange and closely spaced stiffeners, the latter considerably reducing the load along the edge of the web plate by transferring a portion of it directly from the flange to parts of the web plate in which there is little risk of buckling. This cooperation of the stiffeners has been dealt with in an approximate way by considering the

[1] Sitzungsberichte der Akademie der Wissenschaften in Wien, math.-nat. Kl., Abt. IIa, Vol. 145, Nos. 1 and 2, 1936.

flange as a bar and the web plate as a separate element and by making use of the following expression for the edge loading p (x) of the latter:

$$p(x) = \sum p_n \cos \frac{n\pi x}{3a} \qquad (n = 1, 3, 5 \ldots) \tag{1}$$

with the coefficient

$$p_n = \frac{2P}{3a} \cdot \frac{1}{1 + \frac{2J_o}{t}\left(\frac{\pi}{3a}\right)^3 n^3} \cdot \left\{ \frac{3a}{n\pi c} \sin \frac{n\pi c}{3a} - \cos \frac{n\pi}{6} \cdot \frac{\sum\limits_{m=1,3..} \frac{3a}{m\pi c} \sin \frac{m\pi c}{3a} \cdot \cos \frac{m\pi}{6} \cdot \frac{1}{1 + \frac{2J_o}{t}\left(\frac{\pi}{3a}\right)^3 m^3}}{\sum\limits_{m=1,3..} \cos^2 \frac{m\pi}{6} \cdot \frac{1}{1 + \frac{2J_o}{t}\left(\frac{\pi}{3a}\right)^3 m^3}} \right\} \tag{2}$$

Here a denotes the distance between the stiffeners, t the thickness of the web plate, c the half width of the loaded area and J_o the moment of inertia of the loaded flange referred to its horizontal axis. The second bracketted term in Equation (2) represents the effect of the stiffeners, which is often negligibly small. (Equation (2) has been so written that the transition to c = 0 can be immediately effected.) Relations similar to those represented by Equation (2) may also be developed for the case where several loads symmetrical to the y-axis are present in the field under consideration.

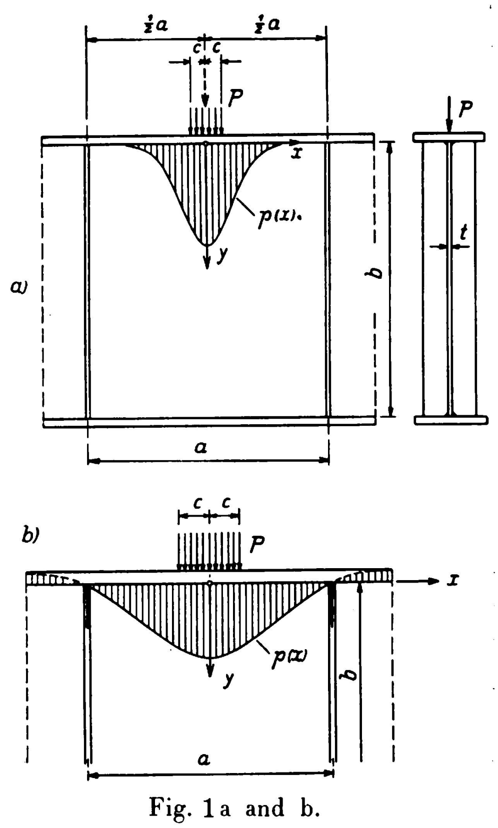

Fig. 1a and b.

With the aid of the distribution p (x) the stresses in the part of the web plate in question may now be calculated. In the earlier paper by the present author, already cited, these stresses have been computed from two constituent parts: namely from the elementary stresses σ_{x1} and τ_1 on the one hand, and from the panel stresses σ_{x2}, σ_y and τ_2 which are a consequence of the edge loading p (x), on the other. The shear stresses τ_1 are left out of consideration in what follows below, and the examination of stability is based upon *a condition of stress in the web plate which is symmetrical with regard to the y axis.* The value of the critical stress and load have been determined by reference to the "energy criterion of safety against buckling".

A calculation of the potential energy of the bent web plate showed that part of the energy resulting from the stresses σ_{x2} is relieved by the relevant portion of the stresses τ_2. As a first approach it was deemed permissible, therefore, to ignore the stresses σ_{x2} and τ_2 in the examination of stability, merely estimating the stress σ_x by reference to ordinary bending theory and then calculating the stresses σ_y which are not obtainable by elementary means, as a function of the stress in the web plate. It has been found from comparative calculations that the critical stresses determined in this way differ only slightly from those found by taking account of all stresses arising from local effects by using the stricter method.[1]

In what follows below the web stresses will be calculated by this simplified method. Where b represents the depth of the web plate (measured between the rivet lines of the flange angles in the case of rivetted girders) and σ_o represents the extreme fibre stress (estimated by elementary means) due to all the loads at the centre (x = 0) of the portion of the web plate EF to be examined (Fig. 2a), then the bending stress at this section will be given by

$$\sigma_x = -\sigma_0 \left(1 - \frac{2\,y}{b}\right). \tag{3}$$

The same expression will also be used for the bending stress of all other sections of the web within EF.

For determining the stresses σ_y only the portion of the girder CD (Fig. 2b) with the load P is taken into account. The length of this portion of the girder is equal to three times the spacing of the stiffeners a and is therefore large enough to allow of the stresses σ_y within EF being determined with sufficient accuracy. Applying Airy's stress function for the parallel strips developed with the edge load p (x) for the length CD = 3a as a half period, we obtain

$$\sigma_y = -\sum_{n=1,3\ldots} \left\{ c_{1,n} \cdot e^{\frac{n\pi y}{3a}} + c_{2,n} \cdot e^{-\frac{n\pi y}{3a}} + c_{3,n} \cdot \frac{n\pi y}{3a}\, e^{\frac{n\pi y}{3a}} + c_{4,n} \cdot \frac{n\pi y}{3a}\, e^{-\frac{n\pi y}{3a}} \right\} \cos\frac{n\pi x}{3a}. \tag{4}$$

The constants $c_{1,n} \ldots c_{4,n}$ are to be determined from the edge conditions of the web plate. Since only the stresses σ_y are calculated from the stress function, the strict fulfilment of the conditions of transfer (compound action of flange and web) may be ignored, thereby not only shortening the calculations but enabling the results to be expressed in a more general form.

The examination of stability is made over again in relation to panel spaces, and the equations for buckling are introduced under the assumption that the edges of the web plates bounded by the flanges and the stiffeners are "freely supported". Flanges and stiffeners are regarded as stiff so far as bending at right angles to the plane of the plate is concerned, and in examining the field EF (Fig. 2) a bent shape is assumed which forms only a half wave in the direction of the length of the girder. The results are then safely applicable for $a \leqq 0.9\,b$, for if there were no local loading of the web plate the latter would tend to bend into a half wave under the influence of the bending stresses σ_x for to long as $a < 0.9b$, of a local effect enters into play, which tends to create symmetrical buckling, then the buckling will continue to be in the form of a half wave even for greater values of $\frac{a}{b}$ though the half wave may be differently formed. A more accurate examination of these conditions was rendered impossible by excessive amount of development it would have involved, and the limit of $a = 0.9b$ has, therefore, been adopted here.

In order to reduce the number of terms to a minimum the following expression has been applied for the buckling $w(x, y)$ of the panel EF:

$$w = \left(A \cdot \sin\frac{\pi y}{b} + B \cdot \sin\frac{2\pi y}{b} + C \cdot \sin\frac{3\pi y}{b}\right)\cos\frac{\pi x}{a} \tag{5}$$

which satisfies the marginal conditions in the form of "Navier's conditions". By means of this equation of three terms the amount of buckling of the plate can be ascertained with approximate accuracy, particularly when the effect of the stresses σ_y by comparison with σ_x becomes less pronounced (that is to say when the place of greatest buckling of the plate is further removed from the loaded flange). This is in fact true of the more important applications arising in practice.

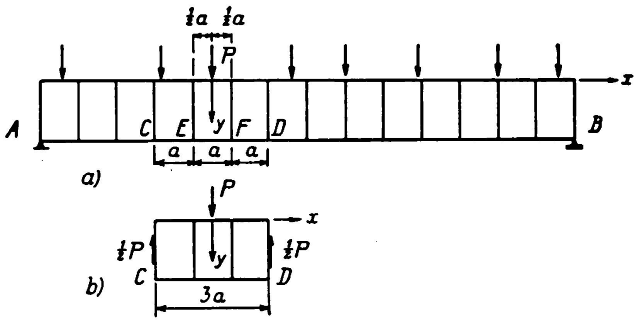

Fig. 2a and b.

The potential energy e of the stretched and bent web plate EF is now calculated[2] with the aid of Equations (3), (4) and (5). For each variation δ_w in the condition of strain, δ_e must be equal to zero. If the series of constants A, B and C in Equation (5) is varied in turn, and the corresponding $\delta_e = 0$ is worked out, the following equations for buckling are obtained:

$$A\left[\frac{1}{k}\cdot\frac{\pi^2}{12\beta}(1+9\beta^2)^2 - 2\varkappa r_1\right] - B\left[\frac{8}{3}\beta + \varkappa r_4\right] - C\varkappa r_5 = 0,$$

$$-A\left[\frac{8}{3}\beta + \varkappa r_4\right] + B\left[\frac{1}{k}\cdot\frac{\pi^2}{12\beta}(4+9\beta^2)^2 - 2\varkappa r_2\right] - C\left[\frac{72}{25}\beta + \varkappa r_6\right] = 0, \tag{6}$$

$$-A\varkappa r_5 - B\left[\frac{72}{25}\beta + \varkappa r_6\right] + C\left[\frac{1}{k}\cdot\frac{\pi^2}{\beta}\cdot\frac{27}{4}(1+\beta^2)^2 - 2\varkappa r_3\right] = 0.$$

Here $\beta = \frac{b}{3a}$, $k = \frac{\sigma_0}{\sigma_e}$, σ_e (the Euler stress) $= \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$, $\varkappa = \frac{P}{at}\cdot\frac{1}{\sigma_0}$

and $r_1 \ldots r_6$ denote the summations of the following series:

$$r_1 = \sum_{n=1,3..}\frac{ap_n}{P}\left[\frac{1}{n^2\beta^2} + \frac{n^2\beta^2}{(4+n^2\beta^2)^2}\right]\varphi_n,$$

$$r_2 = 4\sum_{n=1,3..}\frac{ap_n}{P}\left[\frac{1}{n^2\beta^2} + \frac{n^2\beta^2}{(16+n^2\beta^2)^2}\right]\varphi_n,$$

$$r_3 = 9\sum_{n=1,3..}\frac{ap_n}{P}\left[\frac{1}{n^2\beta^2} + \frac{n^2\beta^2}{(36+n^2\beta^2)^2}\right]\varphi_n,$$

[2] *A. Nadai:* Elastische Platten, Berlin 1925.

$$r_4 = 4 \sum_{n=1,3..} \frac{a p_n}{P} \left[\frac{1}{(1+n^2\beta^2)^2} + \frac{1}{(9+n^2\beta^2)^2} \right] n^2 \beta^2 \psi_n,$$

$$r_5 = 6 \sum_{n=1,3..} \frac{a p_n}{P} \left[\frac{1}{(4+n^2\beta^2)^2} + \frac{1}{(16+n^2\beta^2)^2} \right] n^2 \beta^2 \varphi_n,$$

$$r_6 = 12 \sum_{n=1,3..} \frac{a p_n}{P} \left[\frac{1}{(1+n^2\beta^2)^2} + \frac{1}{(25+n^2\beta^2)^2} \right] n^2 \beta^2 \psi_n,$$

with the auxiliary terms

$$\varphi_n = \frac{18}{36 - n^2} \sin \frac{n\pi}{6} \cdot \frac{1 - e^{-n\pi\beta}}{(1 + e^{-n\pi\beta}) + \frac{2 n\pi\beta e^{-n\pi\beta}}{1 - e^{-n\pi\beta}}}$$

and

$$\psi_n = \frac{18}{36 - n^2} \sin \frac{n\pi}{6} \cdot \frac{1 + e^{-n\pi\beta}}{(1 - e^{-n\pi\beta}) - \frac{2 n\pi\beta e^{-n\pi\beta}}{1 + e^{-n\pi\beta}}}$$

As the constants A, B and C must be different from zero the divisor determinant of Equation (6) must be zero. From this Equation the minimum root $k = k_{min}$ differing from zero can be calculated. The critical stress at the edge of the web plate in the field EF is then $\sigma_{o,\,kr} = k_{min} \cdot \sigma_e$ and the corresponding critical value of the panel load P is $P_{kr} = \varkappa$ at $\sigma_{o,\,kr}$.

To allow of evaluating the critical stresses and loads for the case of inelastic buckling also, the results already obtained for materials possessing unlimited elasticity are recalculated with the aid of the buckling stresses in a bar assumed as a standard of comparison. In determining the maximum stress imposed on the plate, account must now be taken also of that portion of the stress which arises from local effects with regard to σ_x and the somewhat larger amount of stress $(\sigma_y)_{x=0,\ y=0}$ may be substituted for this.

In considering these results it must be remembered that only an approximate equation was used for determining the amount of buckling w (x, y), a circumstance which implies excessively high critical stresses. On the other hand various assumptions made in the calculation are too stringent; for instance, it has been assumed that the edges of the plate are capable of rotation, whereas the fixation stresses are always elastic, and the fixation into the flange may considerably increase the stability of the web plate. As regards the inelastic buckling, it is to be observed that the maximum stress is produced in the plate along an edge which is stiffened, and moreover that the incidence of the latter is purely local, so that plastic deformation may bring about a partial compensation of the stress.

It is possible that the *true* buckling load may be considerably higher than the critical value. The resistance to deformation offered by the adjacent panels of the web will tend to hinder its further buckling, and it is possible, moreover that the resistance to bending of the loaded flange may enable further increases in load beyond the stability of the web plate. On the other hand a slender flange will itself require the support afforded by the web plate in order that it may not buckle in the plane of the latter.

The problem here arising will be treated more exhaustively in a later paper which will also include tables for the practical application of the method of calculation developed.

V 12

The Phenomena of Buckling.

Über Kipperscheinungen.

Sur les phénomènes de déversement.

Privatdozent Dr. F. Stüssi,
Beratender Ingenieur, Zürich.

The problem of stability against buckling and bulging has now been the subject of extensive researches, but the theory of collapse by torsion (tilting) has received little attention. In design practice, up to the present, the lack of a simple formula for calculation to be applied in determining stability of members subject to bending has usually led to the compressive flange being considered as separated from the remaining portion of the girder. In what follows below the relationship between this view of the matter and the complete solution of the problem of the collapse (tilting) of girders of **I** section[1] will be pointed out. For this purpose we shall take as an example the simplest case of all, where the bending moment is constant and the girder so loaded is of constant **I** cross section.

If, now, the bending of the flange is neglected, that is to say the critical moment is determined as for a beam of rectangular cross section, we obtain the value:

$$M_{o,\,kr} = \pi \cdot \frac{\sqrt{B_2 \cdot C}}{l}, \tag{1}$$

wherein $B_2 = E.J_y$ represents the lateral resistance to bending and $C = G.J_d$ represents the resistance to torsion. The effect of vertical bending, which as a rule is small, is here neglected.

If now we consider the compressive flange as detached, then the product of *Euler*'s buckling load P_E multiplied by the distance h of the flanges determines the critical moment of the girder in the usual way:

$$M_{Fl,\,kr} = P_E \cdot h = \frac{\pi^2 \cdot B_2 \cdot h}{a\, l^2}, \tag{2}$$

since the resistance of a flange to bending may be equated to one half of the resistance of the girder to lateral bending.

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[1] *S. Timoshenko:* Sur la stabilité des systèmes élastiques. Annales des Ponts et Chaussées, 1913. — *S. Timoshenko:* Stability of plate girders subjected to bending. Preliminary Report, I.A.B.S.E., Paris, Congress 1932. — *F. Stüssi:* Stability of a girder subject to bending. Publications, I.A.B.S.E. Vol. 3, 1935. — *F. Stüssi:* Exzentrisches Kippen. Schweizerische Bauzeitung. Vol. 105, 1935.

According to *Timoshenko*, the critical moment of the **I** beam is

$$M_{kr} = \pi \frac{\sqrt{B_2 \cdot C}}{l} \cdot \sqrt{1 + \frac{\pi^2}{a^2}} \tag{3}$$

wherein a^2 is a simplification for the expression

$$a^2 = \frac{4\,Cl^2}{B_2\,h^2}$$

If, now, the value obtained by this simplification is introduced into Equation (3), a right-angled triangle as in Fig. 1 will serve to indicate the simple connection that exists between the three values of the critical moment which are under consideration:

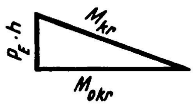

Fig. 1.

$$\underline{M_{kr} = \sqrt{M^2_{o,\,kr} + (P_E \cdot h)^2}.} \tag{4}$$

In order to picture the numerical significance of the two components the critical extreme fibres stresses for an **I** NP 16 beam are represented in Fig. 2. Throughout the elastic region M_{kr} is only slightly larger than $M_{o,\,kr}$; hence the buckling load on the compression flange plays only a very subordinate part in the determination of the critical moment. The assumption usually made in design that the carrying capacity of the girder is governed only by the buckling load on the compression flange is unsatisfactory, attributing, as it does, a decisive influence to what is only a subordinate partial influence. The correctness of the complete solution of the problem should be evident from the experimental points plotted in

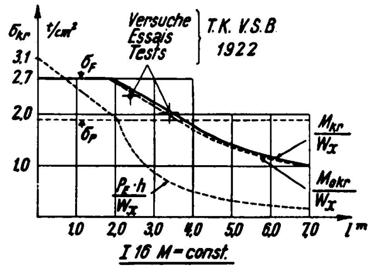

Fig. 2.

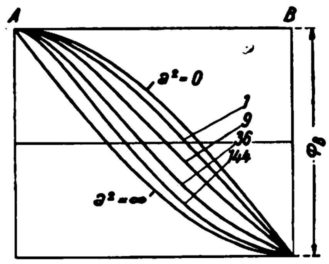

Fig. 3.

Fig. 2 which correspond to the condition of loading here considered, and which are reproduced from an unpublished report of the Technische Kommission des Vereines Schweizerischer Brückenbau und Eisenhochbau Fabriken.[2]

The simple connection between the critical moment of the rectangular beam and the buckling load of the compression flange, as represented in the right-angled triangle, is as consequence of the fact that with the loading here under consideration the deformation curves for buckling and for tilting are both sine curves, and therefore are both of the same form. The author has investigated,

[2] T.K.V.S.B.: Bending tests at the Swiss Federal Testing Institute, Zürich, May 1932. Lateral buckling of the compression flange of an I-beam. Experiments Nos. 1 and 5.

as an example, the case where this agreement in shape cannot be obtained, namely the case of a cantilever loaded with a constant bending moment. The two deformations which correspond to the beginning of instability due to torsion and to buckling, react upon one another, so that the curve of distortion φ changes its form according to the dimensions. The shape of this curve is represented in Fig. 3 for different values of the abbreviation a^2, wherein $a^2 = 0$ corresponds to the buckling problem (resistance to torsion $C = 0$) and $a^2 = \infty$ corresponds to resistance to torsion (stiffness of flange $= 0$). Since the mutual effect of the two limiting curves is the same for every case, assuming an arbitrary amount of fixation, and is the same, therefore, if the stiffness is increased, the critical moment of the **I** girder must be greater here than is measured by the hypotenuse of the triangle which has for its other two sides

$$M_{o,\,kr} = \frac{\pi}{2} \cdot \frac{\sqrt{B_2 \cdot C}}{l}$$

and

$$P_E \cdot h = \frac{\pi^2}{4} \cdot \frac{B_2 \cdot h}{2 \cdot l^2} = \frac{\pi^2}{4\,a} \cdot \frac{\sqrt{B_2 \cdot C}}{l}.$$

Here again, however, the effect of the buckling load for the compression flange is only of subordinate importance under practical conditions (Fig. 4).

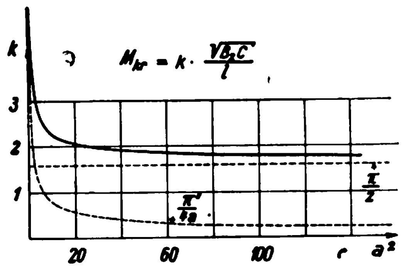

Fig. 4.

The accuracy of the investigations on buckling of beams, now available in a simple form, should make it possible to pay attention to this important group of problems of stability in the structural practice.

V 13

Results of experiments on compression members composed of two unequal angle bars.

Ergebnisse von Versuchen mit Druckstäben aus zwei ungleichschenkligen Winkelstählen.

Résultats des essais de compression sur des cornières à ailes inégales.

Dr. Ing. H. Maier-Leibnitz,
Professor an der Technischen Hochschule, Stuttgart.

Bars composed of two angles with unequal legs are frequently used in trusses as, for instance, in those of roofs, both for the chords and diagonals. In the case of the chords, the angles are usually made continuous over the gussets. With compressed diagonals the forces are transmitted as a rule by means of gusset plates. The clearance between the angles is either constant throughout, or is wider at the middle.

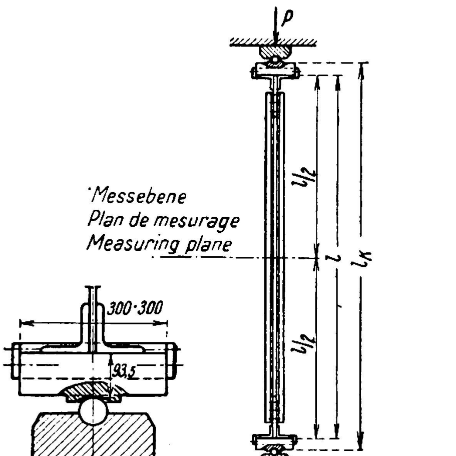

Fig. 1.
Disposition of test.

In the case of compressed members composed of two parts, according to DIN 1050 (Version of July 1937), the "ideal ratio of slenderness" λ_{yi} has to be used for checking the buckling tendency in the phase x — x (see fig. 4).

In developing the formula for λ_{yi} it is assumed that the axis x — x of the bar consisting of two parts is an axis of symmetry of the total cross section, but in the case of bars composed of two angles connected to one another by packing pieces this assumption is not correct. Theoretical investigations relating to such bars have apparently not been carried out up to the present time; nor have any values been made available of the buckling resistance of such bars as determined by experiment.

In 1936 the author accordingly carried out experiments[1] on nine double bars and also on two single bars serving for comparison, and a summary and supplementary discussion of these experiments will be given below.

[1] See the journal „Der Stahlbau". Vol. 9 (1936), p. 166 foll.

The tests were made with the bars arranged in a vertical position, as shown in Fig. 1, wherein it should be noted that the load was imposed through hinges and pressure plates. The tested bars were arranged in such a way as to make the geometrical axis through the centre of gravity coincide as nearly as possible with the axis of the testing machine. After more or less heavy initial loads had been imposed these were removed from the bar and the latter was shifted parallel to the axis of the machine a sufficient number of times to ensure that on again being loaded the deflection at the middle of the bar would be reduced to a minimum. The tested maximum load P_k was usually accompanied by a sudden bending out of the middle of the bar. Fig. 2 represents the „slender" test bars; Fig. 3 the „forced" bars and test bar ⑧ which consists of one angle.

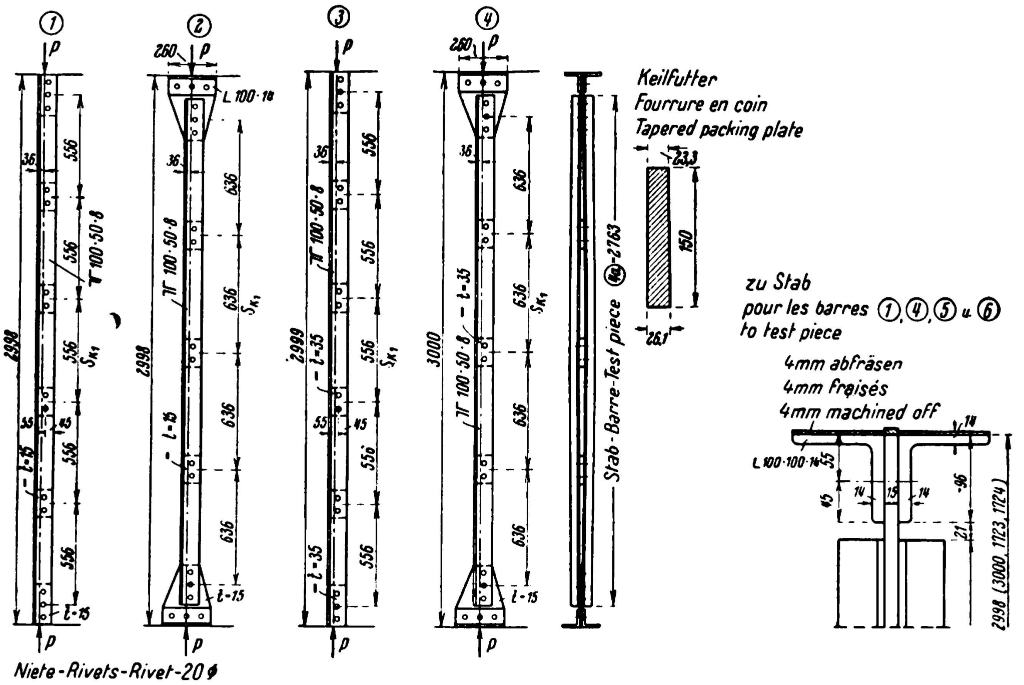

Fig. 2.
Slender bars.

The *buckling load* (in german tons = t) may be seen from the diagram in Fig. 4 which shows the bar ④a which was formed from the bar ④ after testing, and also the second specimen ⑩ consisting of one part. The end surfaces of this bar bore against the horizontal pressure plate of the testing machine which has been secured against tilting.

In the case of bars ① to ④, the calculated buckling load P_k is determined by reference to the tabulated values of the cross sections of the bars with the aid of the relationship $\sigma_k = \frac{20\,726}{\lambda^2}$ t/cm² and in the case of bar ⑤ by the relationship

$$\sigma_k = 2.8905 - 0.008175\,\lambda \text{ t/cm}^2$$

always taking account of the ratio of slenderness λ_{yi} In the case of the single bar ⑧ the value of P_k is determined by reference to the moment of inertia J_η.

Conclusions.

a) In the bars without gusset plates, (1), (3) and (4a), the experimental buckling loads are larger than the buckling loads determined by calculations from λ_{yi}.

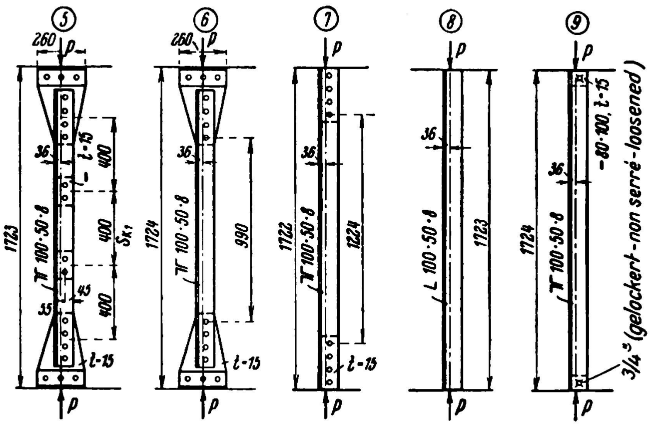

Fig. 3.
Stout bars.

b) Bars with gusset plates, Nos. (2), (4) and (5): the slender bars marked (2) and (4) each showed a buckling load 10 % greater as found by experiment than as calculated in reference to λ_{yi}. The stout bar marked (5) shows a smaller value than that calculated from λ_{yi} despite the fact that its yield point stress 3.27 t/cm² is considerably greater than $\sigma_s = 2.4$ t/cm² which was used as a basis for determining σ_k.

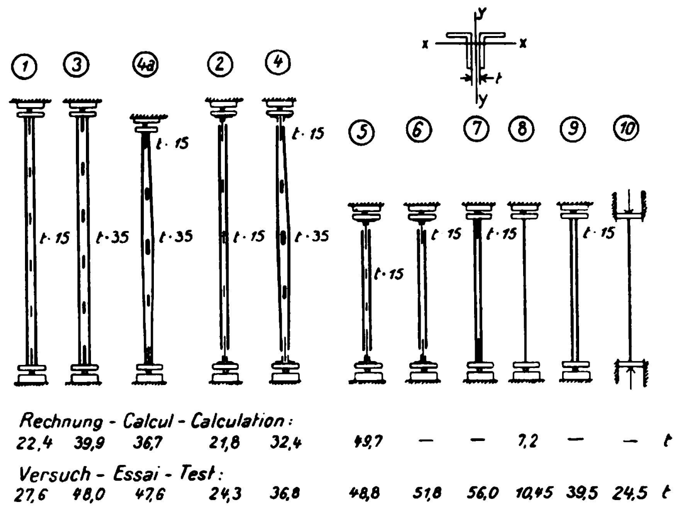

Fig. 4.
The most important results of calculation and test.

c) Bars (6) and (7).

α) In bar (5) the two packings at the points in one third of the total length as required by DIN 1050, have been provided, but in bar (6) these packings have been omitted. In spite of this the maximum buckling load for bar (6) was greater than that for bar (5). It must be concluded from this that the restraint imposed on the ends of the bars by the gusset plates exerts a great influence over the magnitude of the buckling load.

β) Bar (7) showed a still greater buckling load than bars (6) and (5). It may be supposed that the packings provided on bar (5) in accordance with the regulations has not increased the buckling load.

d) Bars (8), (9) and (10). The results obtained in these experiments are bound up with questions previously discussed by the author in reference to experiments on timber struts composed of several parts. In the case of the latter the experimental results cannot be applied directly to the conditions of a strut in the actual structure, for the pressure plates generally used at the ends of the bars in compression tests obscure the effect due to the packings intended to ensure the co-operation of the single bars in the making up of a compound bar.[2] Conditions are similar in the case of steel bars made up from two parts.

Bar (8) gave an actual buckling load of $P_k = 10.45$ t and the double bar (9), which had *no connection of any kind* in the whole length between end plates, gave not merely twice the buckling load of the single bar but 3.8 times as much. The action of this unconnected double bar may be eplained from the circumstance that the projecting legs exert a certain end-fixing effect on the separate bars at the pressure plates. Actually the experiments carried out on the control bar (10), the ends of which rested directly against the pressure plates in the machine 1724 mm apart, showed a buckling load of 24.5 t, or rather more than half the buckling load for bar (9).

It may be suppposed that in the case of test bars arranged like (1), (3) and (4a), the buckling load depends partly on the nature of the bearing of the compound membeı against the pressure plates, as explained above. One should beware, therefore, of assuming that the surplus of actual buckling load (found in testing a double bar in the usual way) over the buckling load of two single bars pinned at the end must be attributed to the operation of the connections (packings).

In the case of bars which in the actual structure have their ends connected by gusset plates, the action of the connections can be estimated only by means of experiments arranged as in the case of bars (2), (4) and (5).

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[2] See. for instance, the journal „Der Bauingenieur" 17 (1936), p. 1.

V 14

Strengthening of the Austerlitz Viaduct in Paris by Electric Arc Welding.

Verstärkung der Austerlitzbrücke der Pariser Stadtbahn durch elektrische Licht=Bogenschweißung.

Renforcement du Viaduc d'Austerlitz par soudure à l'arc électrique.

M. Fauconnier,
Directeur des Travaux Neufs de la Compagnie du Métropolitain de Paris.

Line No. 5 of the Metropolitan Railway of Paris crosses the Seine close to the Gare d'Austerlitz by a steel bridge covering a single span of 140 m between the banks, which is a greater span than that of any other bridge in Paris.

The structure consists of a three-hinged arch, with the side hinges carried not on the abutments but on cantilevers which project 14 m. The result is an

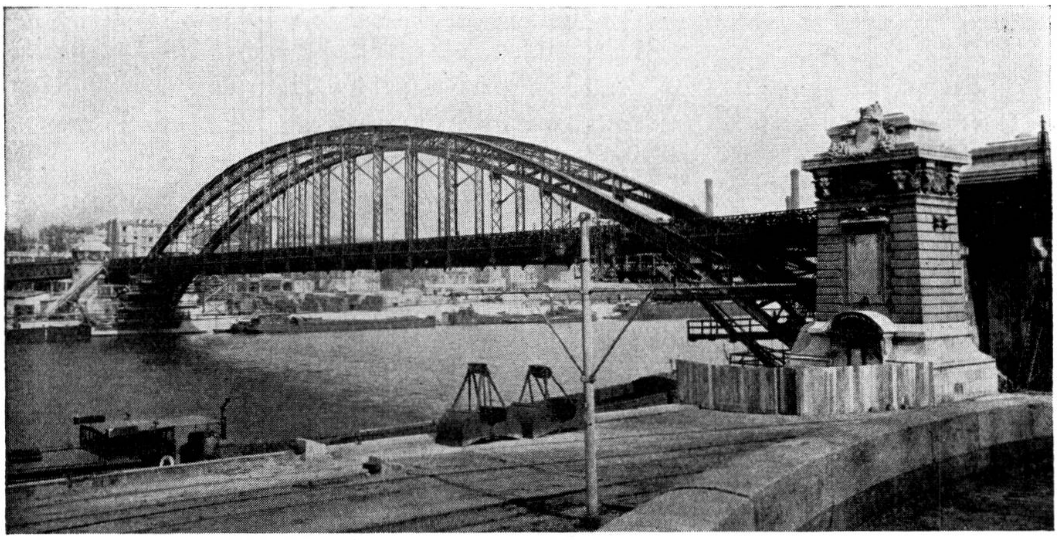

Fig. 1.
Austerlitz bridge.

appearance of lightness which has deservedly won the admiration of engineers (Fig. 1). The weight is 820 tonnes. Unfortunately the design was carried out on the assumption of a light train load of 121 tonnes with a length of 50 m, whereas to-day, thirty years after the construction of the bridge, the typical train for the Metropolitan Railway measures 105 m in length and weighs 420 tonnes. The live load is consequently more than trebled.

The bridge is now being strengthened by the Company and the conditions governing this work are, to the author's knowledge, without precedent anywhere in the world. The effective cross sections of the arch are being increased by as much as 60 %, and the weight of the structure is being increased from 800 to 1000 tonnes. The arch is of the three hinged type and its span is considerable. The equilibrium of the cantilevers must be re-established in such a way that the line of pressure continues to pass through the middle third, and all this work must be done under a traffic of 700 trains a day across the bridge.

The reinforcing members, which are 70 × 140 mm flats and various rolled sections, are being placed between the lines of rivets, and the strength elements of the original structure are being maintained complete.

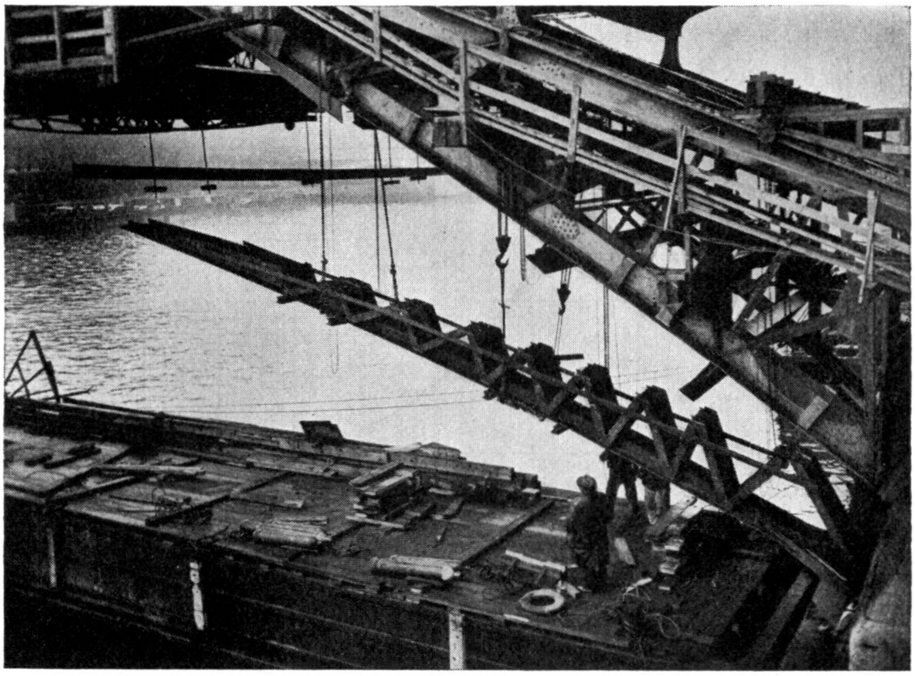

Fig. 2.
Strengthening of springings.

The condition of stability for the cantilevers (namely that the line of pressure should remain within the middle third) is being satisfied by adopting the expedient of welding into place under the lower flange a supplementary construction which is fabricated beforehand and is erected in a single operation.

The calculation has been based on the following simple assumption: namely that the added steel receives no stress except from live load, while the old steel carries the whole of the dead load and at the same time participates in resisting live load. The validity of this assumption has been proved by direct experiment.

It might be thought that it would be essential to take account of internal stresses due to welding, but actually this is a mistake, for such stresses are local in their incidence, and in proportion as they tend to become dangerous they also tend to be compensated by the high ductility which is characteristic of Steel 37.

The original metal is an extra mild Martin steel containing less than 0.10 % of carbon and is, therefore, perfectly suited for welding. Its breaking stress is 40 kg/mm² and the elongation 32 %. The steel used for the reinforcement has purposely been chosen milder still so as to ensure perfect welding qualities, its breaking stress being 37 kg/mm² and elongation 34 %, and the chemical composition being required to comply strictly with the following analysis:

$$C \leq 0.10\ \%$$
$$Mn \leq 0.40\ \%$$
$$Si \leq 0.20\ \%$$
$$P \leq 0.04\ \%$$
$$S \leq 0.04\ \%$$

The resilience is required to be not less than 10 kgm/cm² as measured on a *Mesnager* specimen.

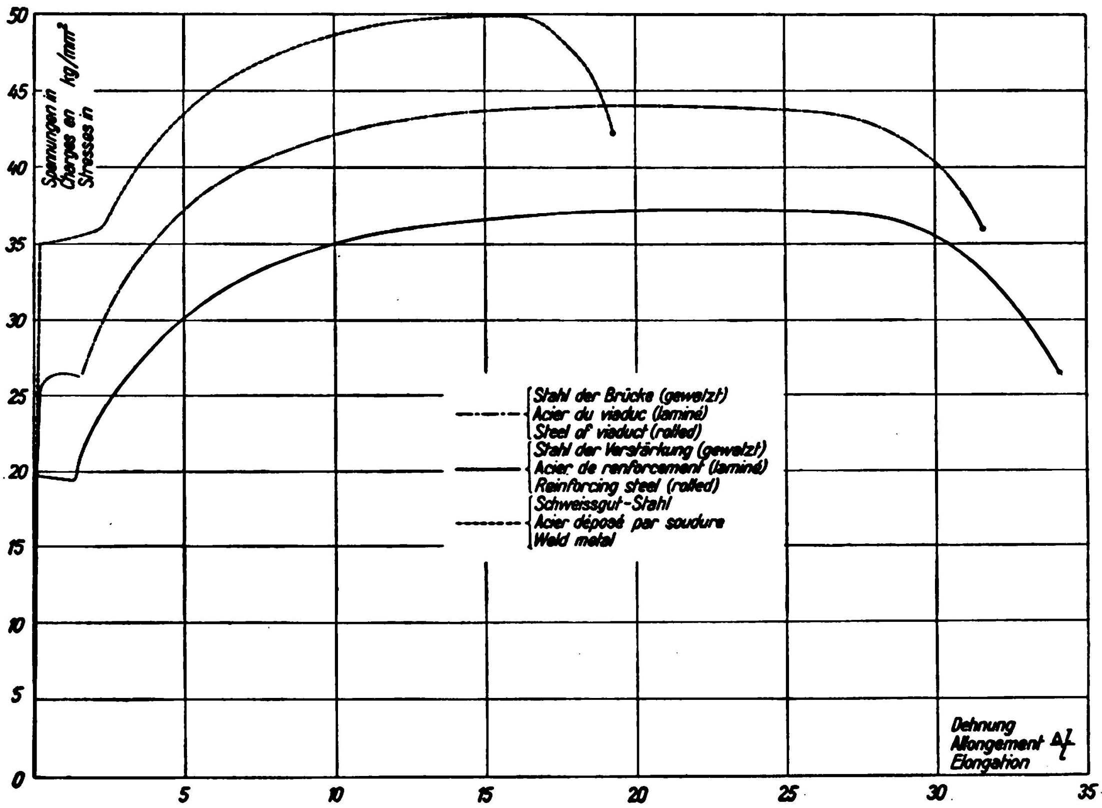

Fig. 3.

Stress-deformation diagrams of steel.

The upper curve in Fig. 3 represents the cast metal deposited from the electrodes, giving 48 kg/mm² ultimate strength and 20 % elongation, with a minimum resilience of 8 kgm/cm² on the *Mesnager* bar. All three curves in Fig. 3 are of similar shape.

An essential condition to be satisfied is that any elongation of the metal between the limits of 1 and 15 per 1000 should be unaccompanied by increase in stress. Now, in arc welding, it is the case that the molten metal contracts by 10—15 per thousand in the process of changing from a semi-plastic state, wherein it offers practically no strength, to its final condition with the normal ultimate strength of 45—50 kg/mm^2; hence it is important, when working on tightened members, that the parent material close to the weld seam should be capable of following the deformation imposed, by itself extending.

By virtue of this property, which the diagrams indicate to be a characteristic of really mild steel, the final equilibrium is obtained without the internal stresses exceeding the elastic limit at any point or at any time.

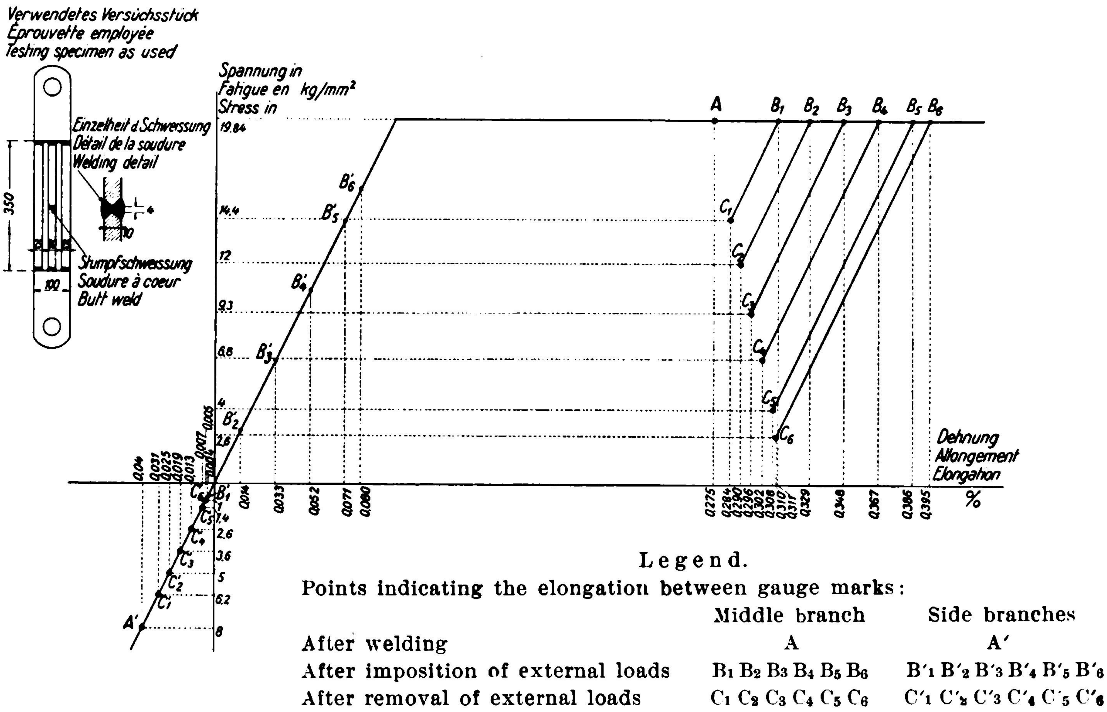

Fig. 4.

Diagram of internal stresses under the influence of external forces. Deformation diagram.

A large number of measurements of internal stresses were carried out systematically, and the results so obtained — usually of the order of 14 kg/cm^2 — were below the elastic limit. This still leaves open the question whether, under the effects of external loads (dead and live) added to the internal stresses, the total stress might not rise to an excessive value, but it has been shown, notably by *Dr. Kommerell* and Mr. *G. Fish* by reference to the stress strain diagram, that such apprehension is unjustified, and the author has confirmed experimentally as regards simple compressive and tensile strains that the internal stresses are relieved under the action of the external loads. The following essential fact was established: if any external load is applied momentarily to a welded system in such a way as to cause *permanent* elongation of any kind in any element of that sytem, then, when the force is removed, an internal effect occurs which reduces the residual internal tension within the element concerned.

The specimen used had three branches, as shown in Fig. 4, the central piece being jointed. At the centre the shrinkage due to welding causes simple deformations which can easily be measured with a *Huggenberger* deformeter; these are limited to an elongation of the central portion which is thus placed under tensile stress, and to a shortening of the side bars which are placed in compression. The application of external load to this specimen is effected in a tensile testing machine, fitted with arrangements to avoid the introduction of bending stresses. The different stages of the test are represented in the diagram, Fig. 4, and its accuracy is remarkable; it will be seen that in the central piece the action of the external load has enabled the tensile stress to be reduced from 19.8 to 2.6 kg/mm^2

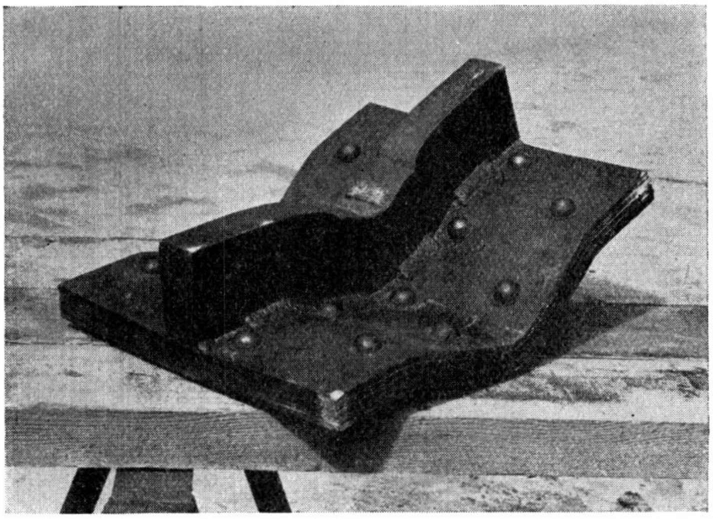

Fig. 5.

1st Homogeneity test.

(Deformation without rupture, under a pressure of 700 tons.)

and the permanent elongation is increased from 0.275 % to 0.310 %. By carrying the test further it would be possible entirely to eliminate the internal stresses. No sign of cracking was observed.

In order that this relief of internal stresses may be ensured in practice it is necessary that the ductility of the parent metal, the weld metal and the cast metal should be as high as possible, and should be of the same order of magnitude for all three. Homogeneity is the quality required.

The existence of this homogeneity has been shown by a curious kind of test. To a member composed of five riveted plates there was affixed a bar measuring 60×130 mm by means of weld seams 16×16 mm in cross section, carried out in several runs. These large seams were made discontinuous, and immediately close to the rivets there was nothing but a small seam of 5×5 mm.

This connection withstood, without fracture, a load of 700 tonnes. Fig. 5 shows the unbelievable amount of deformation sustained by the piece in question, and affords an answer to all possible criticism; homogeneity has in fact been obtained.

For the purpose of comparison the same test was carried out with a continuous seam of 8 × 8 mm (reduced to 5 × 5 mm close to the rivets). Instead of 700 tonnes being applied without breakage as in the preceding test, the failure of the seam then took place under a load of 170 tonnes (Fig. 6).

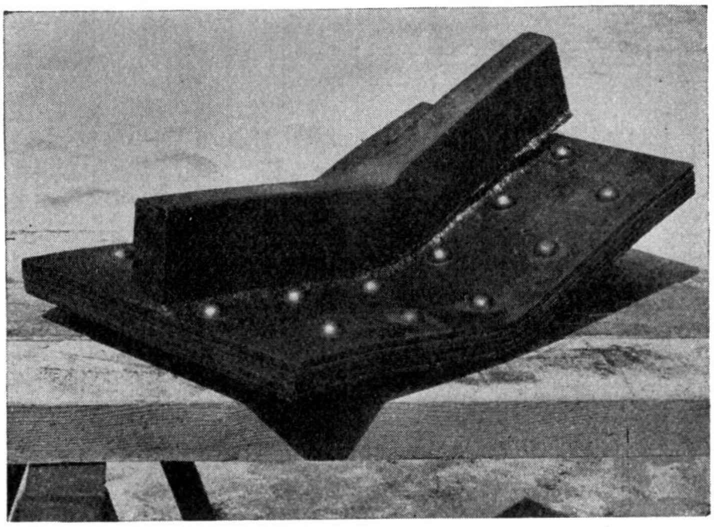

Fig. 6.

2nd Homogeneity test.

(Ruptured under a load of 170 tons.)

To sum up, the work now being carried out on the Métropolitain in Paris is the most difficult kind of welding job that can arise, because applying to structure parts not only tightened but under traffic the whole time. The operation is already well advanced; its success is directly dependent on the remarkably high quality of the electrode used (type L 40 as supplied by the Soudure Autogène Française) and, above all, on the choice of a very ductile reinforcing metal, namely Steel 37 of *prescribed chemical composition.*

V 15

A New System of Suspension Bridges.

Neues System für Hängebrücken.

Un nouveau système de ponts suspendus.

Prof. G. Krivochéine,
Ingenieur, General-Major, Prag.

The object of the system of suspension bridge construction now to be described is to reduce the horizontal tension in the cables and consequently the weight of the bridge. The system consists of a cable of which the horizontal tension is taken up not by stiffening girders, but by a polygonal arch (Figs. 1, 2 and 3), the stiffening girders serving merely to resist bending moments. The proposed system can be carried out in a statically determinate form (Fig. 4a), in which case there are two hinges in the side opening and one hinge capable of longitudinal displacement in the central opening; after the bridge is erected all three hinges, or the central hinge alone, can be eliminated, in which case the central hinge should be constructed in such a way as to be able to carry only bending moments without transmitting longitudinal forces (Fig. 4d). The system would then be statically indeterminate in one or three degrees.

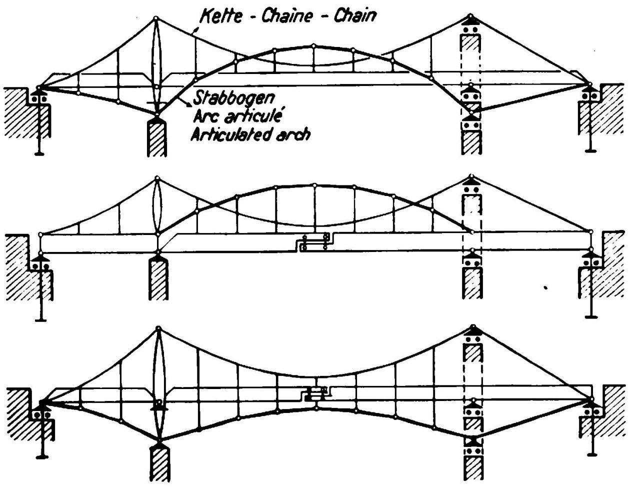

Fig 1—3.

For the statically determinate system (Fig. 4a) the horizontal tension of the cable can be calculated from the conditions of equilibrium of the left hand portion as indicated in Fig. 4b, and of the left half of the main girder (Fig. 4c):

$$1.\quad H_1 = H_2 = H,$$

$$2.\quad +C\,(l_o - c) + Hz_1 = 0,$$

$$3.\quad +C\left(l_o + \frac{1}{2}l\right) + A \cdot \frac{1}{2}l - Hz_2 = 0,$$

$$4.\quad C + A = \frac{1}{2}P.$$

If the hinge G lies on the line CA, we have

$$H = P \frac{l}{4(f_1 + f_2)}.$$

The simple suspension bridge with the horizontal tension eliminated gives

$$H_1 = P \frac{l}{4 f_1}$$

$H = \frac{1}{2} H_1$ and if $f_1 = f_2$.

In other words, the horizontal tension in the cable of the proposed type of suspension bridge combined with an arch is only half as great as the correspon-

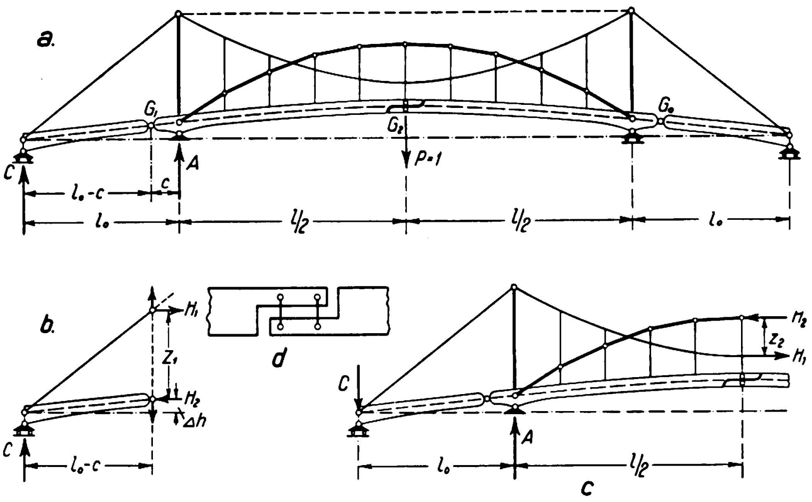

Fig. 4.

ding tension in a suspension bridge with a stiffening girder alone. In this fact lies the principal characteristic of the proposed system, which results in great economy.

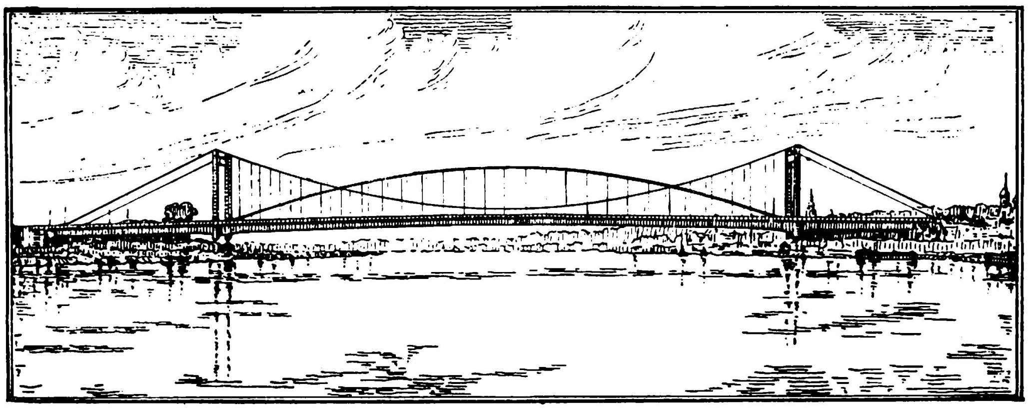

Fig. 5.

Design for a suspension bridge with unstiffened arch, at Cologne-Mülheim, on the system of Professor G. G. Krivochéine.

For instance, for the Rhine bridge at Cologne-Mülheim, having a central span of 315 m as shown in Fig. 5, this system would realise an economy of one million RM, that is 20 %.

The system has been suggested by the author:

1) In the competition for the design for the road bridge over the Elbe at Bodenbach-Tetschen in Czechoslovakia (l = 118 m).
2) For the road bridge over the Elbe in Aussig, Czechoslovakia (l = 124 m).
3) In the competition for the design of the railway and road bridge in Porto-Novo, Dahomey, Africa (1 = 169 m) for the French cable works of Leinekugel-le-Cocq. (Etablissements Metallurgiques, Corrèze.)

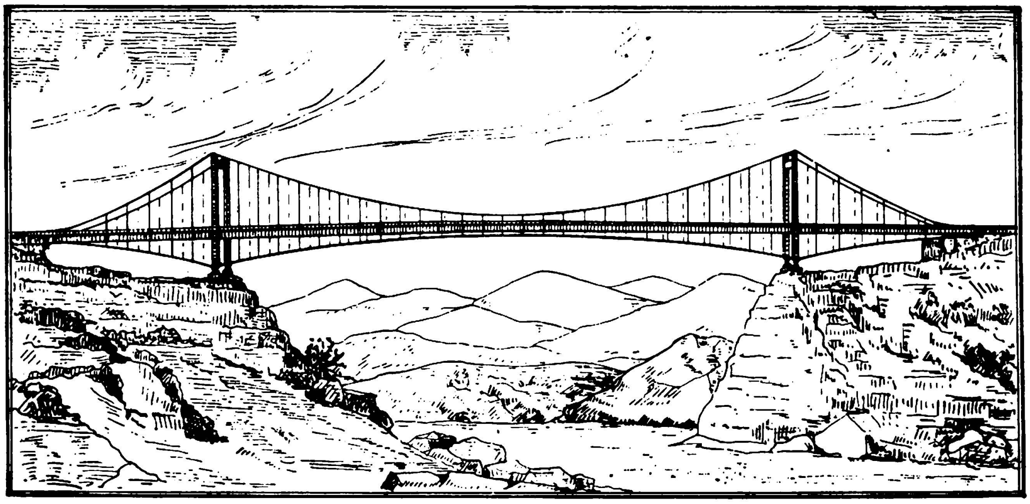

Fig. 6.

Suspension bridge with unstiffened arch.
System of Professor G. G. Krivochéine.

The opinion was expressed by a large German firm that the preliminary design was not satisfactory from an aesthetic point of view because the arch introduces a foreign element into the otherwise clear and pleasing appearance of the suspension bridge but Figs. 3 and 6 show sketches of suspension bridges in which the arch is placed completely below the frame, so that this critisism loses its force.

V 16

Stability of Rectangular Plates Under Shear and Bending Forces.

Die Stabilität rechteckiger Platten unter Schub- und Biegebeanspruchung.

La stabilité des plaques rectangulaires soumises au cisaillement et à la flexion.

Dr. S. Way,
East Pittsburgh, Pa., U.S.A.

1) *Introduction.*

In the design bridge, ship and aircraft structures, problems arise having to do with the stability of rectangular plates with various types of edge loading.[1] With loading higher than a certain critical value lateral deflection from the initial plane of the plate takes place. It may sometimes be permissable for a structure to carry a load higher than the critical value but a knowledge of the critical load is always desirable.

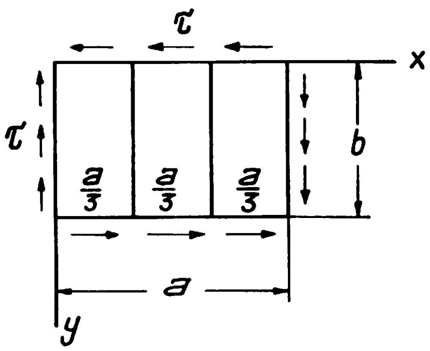

Fig. 1.

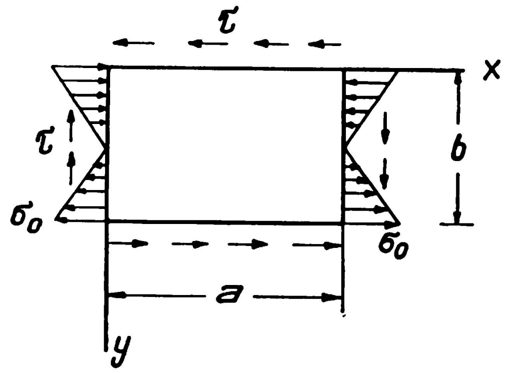

Fig. 2.

Two problems in the buckling of rectangular plates are discussed in this paper. The first, Fig. 1, is that of a plate having two stiffeners. The loading consists of uniformly distributed shearing forces on the edges. The second problem, Fig. 2, is that of a plate loaded by uniform edge shear and linearly distributed tension and compression at the ends. In both cases we assume all four edges are simply supported.

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[1] An extensive bibliography on the stability of plates has been given by *O. S. Heck* and *H. Ebner*, Luftfahrtforschung, Vol. 11, 1935, p. 211.

The method employed is the energy method, in which the critical load is calculated from the condition that the work of the edge forces during buckling is equal to the stored elastic energy. The form of the deflection must be that which makes the critical load a minimum.

2) *The Plate in Shear with Two Stiffeners.*

The problem of a rectangular plate in shear having no stiffeners and that of a plate with one stiffener have been solved by *Timoshenko*,[2] while *Southwell*[3] and *Skan* have treated the case of an infinitely long strip with edge shearing forces. Transverse stiffeners serve to increase the critical load value for a plate, and the greater the rigidity of the stiffeners the greater is this increase, in general. It is found, however, that beyond a certain point increasing the rigidity of the stiffeners in a plate with edge shear loading is useless, as the stiffeners will remain straight and only the panels of the plate will deflect.

To solve the problem of the plate shown in Fig. 1, a general expression is assumed for the deflection surface in the form of the double trigonometric series

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{\pi m x}{a} \sin \frac{\pi n y}{b} \tag{1}$$

each term of which satisfies the boundary condition of simply supported edges. Using this expression three energy quantities may be calculated, the elastic energy of the plate V, the elastic energy of the stiffeners $V_{s1} + V_{s2}$, and the work done by the edge shearing forces during buckling, V_1. The torsional rigidity of the stiffeners will be neglected. These three energy quantities in terms of the derivatives of w are:

$$V = \frac{D}{2} \int_0^a \int_0^b \{(w_{xx} + w_{yy})^2 - 2(1-\mu)(w_{xx} w_{yy} - w_{xy}^2)\} \, dx \, dy \tag{2}$$

$$V_{s1} + V_{s2} = \frac{B}{2} \int_0^b \left\{ \left(w_{yy}\right)^2_{x=\frac{a}{3}} + \left(w_{yy}\right)^2_{x=\frac{2a}{3}} \right\} dy \tag{3}$$

$$V_1 = -\tau h \int_0^a \int_0^b w_x w_y \, dx \, dy \tag{4}$$

where h is the thickness of the plate, B the flexural rigidity of the stiffeners and D the flexural rigidity of the plate,

$$D = \frac{E h^3}{12(1-\mu^2)} \tag{5}$$

[2] *S. Timoshenko:* Eisenbau, Vol. 12 (1921), p. 147.

[3] *R. V. Southwell* and *S. W. Skan*, Proc. Royal Society, London, Series A, Vol. 105 (1924), p. 582.

The condition for buckling is that

$$V + V_{s1} + V_{s2} = V_1. \qquad (6)$$

Equation (6) leads to an expression for τ_{cr}, the critical shearing stress, in terms of the constants A_{mn}. The next step is to minimize τ_{cr} by differentiation with respect to the constants. The equations $\delta \tau_{cr}/\delta A_{mn} = 0$ lead to a set of linear equations for the constants A_{mn}. The value of τ_{cr} is finally obtained by equating to zero the determinant of this system. The greater the number of terms considered in the infinite order determinant, the greater will be the accuracy of the τ_{cr} value calculated.

It happens that the linear equations obtained by minimizing τ_{cr} with respect to the constants A_{mn} consist of two groups, one group containing terms with $m + n$ odd and the other group containing terms with $m + n$ even. That group of equations should be used which leads to the lowest value of τ_{cr}.

It is convenient to measure the critical load by the ratio τ_{cr}/σ_e, where σ_e is the Euler stress $D\pi^2/hb^2$. In Table I values of τ_{cr}/σ_e are given as found by calculation from sixth order determinants for the two groups of equations, for the particular case when $B = 0$[4]. The ratio $\frac{a}{b}$ is designated by β.

Table I.

τ_{cr}/σ_e Values for Various β Values. $B = 0$.

β	1	1.2	1.5	2	2.5	3
τ_{cr} / σ_e $m + n$ even	9.42	8.06	7.14	6.59	6.32	6.14
τ_{cr} / σ_e $m + n$ odd	11.55		8.09	6.74	6.21	6.04

Values of τ_{cr}/σ_e for β less than unity can easily be derived from the above values and are given in Table II. The lowest τ_{cr}/σ_e value of the two is given.

Table II.

τ_{cr}/σ_e for $\beta < 1$. $B = 0$.

β	1	0.833	0.667	0.500	0.400	0.333
τ_{cr} / σ_e	9.42	11.60	16.06	26.40	38.80	54.40

For any appreciable amount of stiffening the system with $m + n$ odd leads to the least values of τ_{cr}. We equate to zero the determinant of the coefficients of the constants

$$A_{21},\ A_{12},\ A_{32},\ A_{23},\ A_{41},\ A_{14}$$

[4] The values for $m + n$ even agree with those obtained by *Timoshenko* (note 2). The terms used for $m + n$ even were A_{11}, A_{22}, A_{33}, A_{13}, A_{31} and A_{42}, and for $m + n$ odd A_{21}, A_{12}, A_{32}, A_{23}, A_{41}, A_{14}.

and the resulting equation enables us to calculate τ_{cr}/σ_e directly when β and the stiffener rigidity are given. The stiffener rigidity is, for convenience, measured by the ratio $\gamma = B/aD$. Values of τ_{cr}/σ_e are given in Table III. It will be noted that increasing the stiffener rigidity increases the load necessary to buckle the plate.

Table III.

τ_{cr}/σ_e Values for Various β and γ Values.

$\beta = 1$		$\beta = 1{,}2$		$\beta = 1{,}5$		$\beta = 2$		$\beta = 2{,}5$		$\beta = 3$	
γ	τ_{cr}/σ_e	γ	τ_{cr}/σ_e	γ	τ_{cr}/σ_e	γ	τ_{cr}/σ_e	γ	τ_{cr}/σ_e	γ	τ_{cr}/σ_e
0	11.55	20	36.7	0	8.09	0	6.74	0	6.21	0	6.04
10	32.75	25	40.1	5	19.43	2	13.07	1	10.3	0.2	7.29
20	41.6	30	43.2	10	25.2	5	18.2	2	13.1	0.4	8.32
30	48.5			15	29.5	10	23.8	3	15.1	0.6	9.21
40	54.4									0.7	9.59

If the stiffeners are made very rigid the calculated critical load for the plate will be greater than the critical load for each of the three panels. The situation then is such that the stiffeners will remain straight and only the panels of the plate will buckle. The condition that the stiffeners remain straight is that their rigidity be greater than that necessary to make the critical load for the plate equal that for one panel. The critical load for one panel is calculated by making the assumption that all edges are simply supported. Actually, each panel is partially constrained by the adjacent panel or panels.

We let γ_{min} be the minimum value of γ for which the stiffeners will remain straight. To illustrate the method of calculation we shall consider the case of $\beta = 1.2$. The β value for one panel of this plate is 0,400 and the corresponding critical load ratio is 38.8 as given in Table II. By plotting graphically the values given for τ_{cr}/σ_e with $\beta = 1.2$ in Table III we find that $\tau_{cr}/\sigma_e = 38.8$ when γ is 23. Hence γ_{min} is 23 for $\beta = 1.2$. In the same manner values of γ_{min} for other values of β may be found.

It is useful to present the results in terms of the dimensions of one panel instead of the dimensions of the plate. Let c be the distance between the stiffeners. We introduce the symbols β' and γ' defined by

$$\beta' = \frac{c}{b} \qquad \gamma' = \frac{B}{cD}.$$

In Table IV, values of τ_{cr}/σ_e and γ'_{min} are given for various panel ratios. These values have been plotted in the curves in Fig. 3. We note that the required stiffener rigidity for two stiffeners is not very much greater than the required rigidity for one stiffener. For three or more stiffeners, the γ'_{min} value would probably be only very slightly larger than the value for two stiffeners.

Table IV.

One Stiffener			Two Stiffeners		
β'	γ'_{min}	τ_{cr} / σ_e	β'	γ'_{min}	τ_{cr} / σ_e
0.500	30.4	26.4	0.333	120	54.4
0.625	12.6	17.9	0.400	69	38.8
0.750	5.8	13.3	0.500	34	26.4
1.000	1.66	9.42	0.667	10.8	16.06
			0.833	4.2	11.61
			1.000	2.0	9.42

B) *Plates Loaded by Combined Shear and Bending Forces.*

If, as shown in Fig. 2, the plate is loaded by uniformly distributed edge shearing stress, τ, and linearly distributed tension and compression at the ends, $\sigma = \sigma_o\ (1-2\,y/b)$, the critical bending stress $\sigma_{o\,cr}$ will depend upon the magnitude of the shearing stress. Similarly, the critical shearing stress may be said to depend upon the magnitude of σ_o. For convenience we introduce the parameters $\varkappa$ and ρ defined as follows:

$$\varkappa = \frac{\sigma_{o\,cr}}{\sigma_e} \qquad \rho = \frac{\tau}{\sigma_e}.$$

To solve the problem, the expression (1) which satisfies the boundary conditions for simply supported edges may again be assumed for the buckling deflection. The elastic energy of the plate after buckling will be given, as before, by Equation (2). The work V_1 of the edge forces during buckling will be, in this case,

$$V_1 = \frac{h}{2}\int_0^b dy \int_0^a \sigma_o\left(1 - \frac{2\,y}{b}\right) w^2{}_x\, dx - \tau h \int_0^a \int_0^b w_x w_y\, dx\, dy \tag{7}$$

The condition for buckling is that $V = V_1$. This leads to an expression for $\sigma_{o\,cr}$, which is then minimized with respect to the constants A_{mn}. The equations $\delta\varkappa/\delta A_{mn} = 0$ became a linear system in A_{mn}. The magnitude of $\varkappa$ is obtained by equating to zero the determinant of this system. The order of the determinant used determines the accuracy of the result. We here use the determinant of the coefficients of the eight[5] terms A_{11}, A_{12}, A_{13}, A_{21}, A_{22}, A_{23}, A_{31}, A_{33}, and the calculated values of $\varkappa$ for various values of β and ρ are as given in Table V.

These values have been plotted in the curves in Fig. 4. When $\varkappa = 0$ we have the condition for the buckling of a plate in pure shear, while for $\rho = 0$ we have

[5] *Stein* made calculations with four terms. Considerably lower $\varkappa$ values are obtained above by using eight terms. *O. Stein*: Der Stahlbau, Berlin, Vol. 7 (1934), p. 57.

Table V.

Values of $\varkappa$ and ρ For Various β Values.

$\beta=1$		$\beta={}^4/_5$		$\beta={}^2/_3$		$\beta={}^1/_2$	
ρ	$\varkappa$	ρ	$\varkappa$	ρ	$\varkappa$	ρ	$\varkappa$
0	25.6	0	24.5	0	23.9	0	25.6
2	24.6	4	22.8	4	23.05	4	25.4
4	22.2	8	17.7	8	20.35	8	24.3
6	18.4	10	13.25	12	15.23	12	22.55
8	12.4	11	10.01	14	11.04	16	19.94
9	6.85	12	4.61	15	8.0	20	16.13
9.42	0	12.26	0	16.09	0	24	10.26
						26	5.44
						26.9	0

the condition for the buckling of a plate with bending forces at the ends. The values of ρ for $\varkappa = 0$ agree very closely with those obtained by *Timoshenko* for the case of pure shear. The slight disagreement for small values of β is due to the fact that among the eight terms used in these calculations there are only five $m + n$ even terms, while *Timoshenko* uses six terms. For pure shear and $\beta = {}^1/_2$ the difference in the values of τ_{cr}/σ_e for five and six term calculations is only 2 %. For pure bending ($\rho = 0$) the values of $\varkappa$ agree with those obtained by *Timoshenko* using three terms.

4) *Numerical Examples.*

Suppose we have a plate with edge shear loading which we wish to reinforce with two stiffeners. Let a = 2000 mm, b = 1000 mm, h = 7 mm, E = 21,000 kg/mm², $\mu = 0.3$. Let it be required to find the load causing the plate to buckle and the proper rigidity for the stiffeners.

$$D = \frac{21{,}000 \cdot 7^3}{12\,(1 - 0.09)} = 660{,}000 \text{ kg mm},$$

$$\sigma_e = \frac{660{,}000 \cdot \pi^2}{1{,}000{,}000 \cdot 7} = 0.93 \text{ kg/mm}^2.$$

For $\beta = 2$ we have $\beta' = 0.667$, and we have from Table IV, $\gamma'_{min} = 10.8$ and $\tau_{cr}/\sigma_e = 16.06$ from which $\tau_{cr} = 14.94$ kg/mm² and $B = 10.8 \cdot 0.667 \cdot 660{,}000 = 4750 \cdot 10^6$ kg/mm². If one stiffener were used, the critical shearing stress would be 8.77 kg/mm², and the required stiffener rigidity $1096 \cdot 10^6$ kg · mm².

As a second example, take the case of the end portion of the web of a plate girder. Let the depth b, be 2000 mm and the thickness h = 8 mm. Let it be required to find the proper stiffener spacing for the end of the girder and also the stiffener rigidity so that buckling will occur when $\tau = 10$ kg/mm².

$$D = \frac{21{,}000 \cdot 8^3}{12 \cdot 0.91} = 985{,}000 \text{ kg mm}$$

$$\sigma_e = \frac{985{,}000 \cdot \pi^2}{4 \cdot 10^6 \cdot 8} = 0.3035 \text{ kg/mm}^2$$

$$\tau_{cr} = 10 \text{ kg/mm}^2$$

$$\frac{\tau_{cr}}{\sigma_e} = 32.9.$$

By Fig. 3 we see that $\beta' = 0.44$ so that the proper stiffener spacing is given by

$$c = 2000 \cdot 0.44 = 880 \text{ mm}.$$

To find the stiffener rigidity we assume that every third stiffener in the girder is perfectly rigid. We then use the curve for γ'_{min} in Fig. 3 for two stiffeners and find $\gamma'_{min} = 50$. The required stiffener rigidity is therefore

$$B = 50 \cdot 880 \cdot 985{,}000 = 43{,}300 \cdot 10^6 \text{ kg} \cdot \text{mm}^2.$$

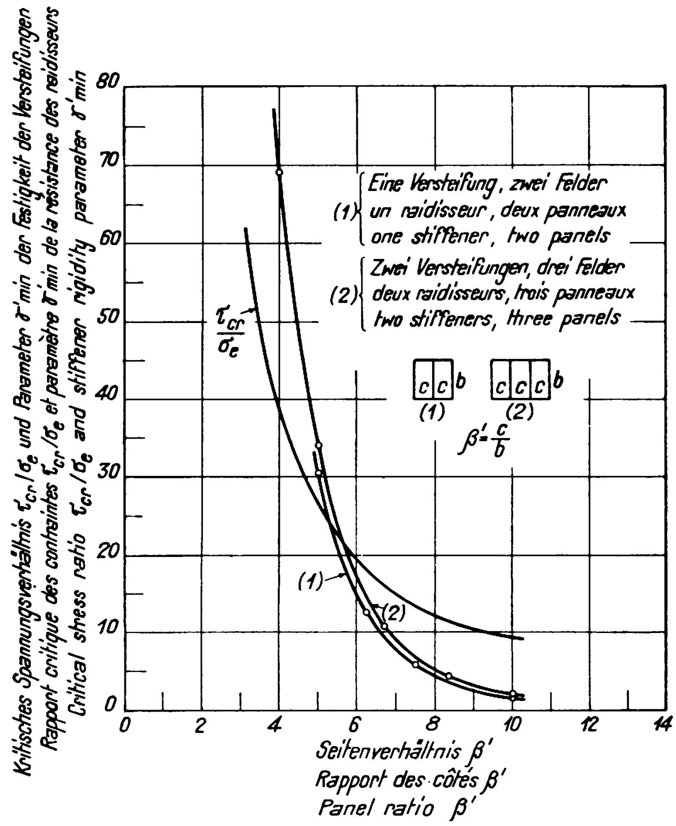

Fig. 3.

Required stiffener rigidity for given panel proportions.

As a third example, suppose we have a rectangular panel with $a = 1000$ mm, $b = 2000$ mm, $h = 10$ mm which is loaded by bending and shearing forces as shown in Fig. 2. Let the bending stress, σ_o, be 10 kg/mm² and let it be required to find the shearing stress which will cause buckling. In this case $\beta = {}^1/_2$ and

$$D = \frac{21{,}000 \cdot 10^3}{12 \cdot 0.91} = 1{,}923{,}000 \text{ kg mm}$$

$$\sigma_e = \frac{1{,}923{,}000 \cdot \pi^2}{4 \cdot 10^6 \cdot 10} = 0.474 \text{ kg/mm}^2$$

$$\varkappa = \frac{10}{0.474} = 21.1.$$

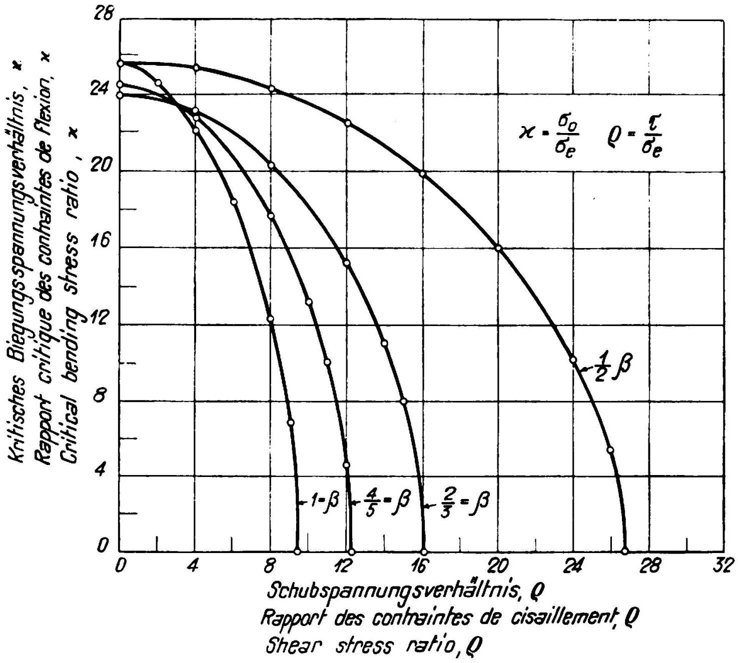

Fig. 4.

Critical loads combined shear and bending.

From Fig. 4 the value of ρ for $\beta = {}^1/_2$ and $\varkappa = 21.1$ is 14.3. The corresponding value of τ is $14.3 \cdot 0.474 = 6.78$ kg/mm².

V 17

Investigation of the Buckling of a Parabolic Arch in a Vierendeel Girder under Compression.

Untersuchung über das Ausknicken des parabelförmigen Druckgurtes eines Vierendeel-Trägers.

Etude du flambage d'ensemble de l'arc parabolique comprimé d'une poutre Vierendeel.

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The parabolic arch of a Vierendeel girder, like that of a bowstring girder, is subject to conditions as regards buckling which are difficult to define exactly, for not only is it built in at its ends which form part of the lower (or tensile) boom and of the roadway structure, but at the same time it is rigidly fixed to the verticals which are themselves very stiff and which connect with the roadway structure in the form of inverted portals.

It may nevertheless be assumed, at any rate in the central portion of the arch where the height of the verticals varies only slightly, that the arch acts like a straight boom carried on verticals of constant height. By treating the arch within this zone as if it were an ordinary straight boom, and by calculating the load and the buckling length according to the well known methods of *Engesser*, *Timoshenko*, *Pigeaud*, etc., it will thus be possible to arrive at a limiting stress.

With a view, however, to more exact understanding of the problem, attempts have been made to apply *Timoshenko*'s method taking due account of the end fixations and assuming a shape of the deformed axis which agrees well enough with the truth but is also simple enough to avoid unduly complicating the calculations. Moreover, since the phenomenon of buckling will tend to make its appearance in the central zone where the verticals are highest, it has been assumed as a further simplification that the vertical members in question are of constant height.

The numerical results show that the lengths of waves caculated in this way justify the assumptions made. In studying ordinary buckling effects in a straight piece, intermediately between joints, the deformed axis is compared to a simple sinusoid; here, in order to take account of the conditions of end fixation, the shape of the deformed axis was chosen in accordance with the equation

$$y = f\left(\sin \pi \frac{x}{L} - K \sin 3\pi \frac{x}{L}\right)$$

wherein the arch, supposed to be straightened along the median plane of the beam, is taken as the x-axis; the origin is taken at one of the ends, and the y-axis (parallel to which the deflections are measured) is taken at right angles to the plane of the girder. It is further assumed that the thrust is constant troughout the arch between the two supports.

The principle underlying the approximate method of *Timoshenko* is that of equating the external work performed by the thrust with the work performed by the ends of the verticals and by the internal stresses.

Work due to thrust.

The displacement of the thrust is equal to the difference in length between the deformed arch and its projection:

$$\Delta x = \int_0^L (ds - dx), \quad ds = \left(1 + y'^2\right)^{\frac{1}{2}} dx$$

where, approximately, $ds = \left(1 + \frac{1}{2} y'^2\right) dx$

$$ds - dx = \frac{1}{2} y'^2 \, dx$$

$$\int_0^L (ds - dx) = \Delta x = \frac{1}{2} \int_0^L y'^2 \, dx$$

$$T_Q = Q \cdot \Delta x.$$

Resisting work performed by the ends of the uprights.

It will be assumed that this thrust is continuous along the arch (*Engesser*). It should be noted that this approximation has much better justification in the Vierendeel girder than in the lattice girder, for in the former such continuity is largely realised by the enlargement of the verticals at top and bottom, and the assumption made is more likely to be true than that of isolated reactions. The thrust at the end of a vertical is conditioned by two terms which depend on the stiffness of the vertical member itself and on the stiffness of the cross girder into which it frames.

If no account is taken of the reinforcing effect of the connecting gusset and h is the height of the vertical member; p the span of the cross girder; I_h and I_p the moments of inertia of these respective members ($\varepsilon = EI$), and Σf is the deflection of the head of the vertical member under a load of one tonne, we obtain

$$\Sigma f = \frac{h^2 \cdot p}{2 \varepsilon_p} + \frac{h^3}{3 \varepsilon_h}.$$

Moreover, the unit reaction continuously distributed is Cy where y is the deformation at a given point and λ is the width of the panel, whence

$$\lambda C \Sigma f = 1 \quad \text{and} \quad C = \frac{1}{\lambda \Sigma f}.$$

The work done by the sides will be

$$\int_0^y \int_0^L C\, y\, dy\, dx = \frac{C}{2} \int_0^L y^2 dx.$$

Work due to internal stresses.

When account is taken only of the elastic energy of bending, we obtain the expression

$$T_1 = \frac{1}{2} \int_0^L \frac{M^2}{\varepsilon}\, dx$$

or expressing M in terms of y′,

$$T_1 = \frac{1}{2} \int_0^L \varepsilon y''^2 dx = \frac{\varepsilon}{2} \int_0^L y''^2 dx,$$

I being assumed constant.

General equation.

As indicated above, the deformed shape of the neutral axis was taken to be in accordance with the equation

$$y = f\left(\sin \pi \frac{x}{L} - K \sin 3\pi \frac{x}{L}\right)$$

where $y = 0$ at the two ends and $y = f\ (1 + K)$ at the centre

$$y' = f \frac{\pi}{L}\left(\cos \pi \frac{x}{L} - 3K \cos 3\pi \frac{x}{L}\right)$$

$$\int_0^L y'^2 dx = f^2 \frac{\pi^2}{L^2} \cdot \frac{L}{2} (1 + 9K^2)$$

$$\int_0^L y^2 dx = f^2 \frac{L}{2} (1 + K^2)$$

$$y'' = -f \frac{\pi^2}{L^2}\left(\sin \pi \frac{x}{L} - 9K \sin 3\pi \frac{x}{L}\right)$$

$$\int_0^L y''^2 dx = f^2 \frac{\pi^4}{L^4} \cdot \frac{L}{2} (1 + 81K^2).$$

The fundamental relationship may be written

$$P \cdot \frac{1}{4} \frac{f^2 \pi^2}{L} (1 + 9K^2) = \frac{C f^2 L}{4} (1 + K^2) + \frac{1}{4} f^2 \frac{\pi^4}{L^3} \varepsilon (1 + 81K^2)$$

And finally:

$$\underline{P = \frac{\varepsilon \pi^2}{L^2} \frac{1 + 81K^2}{1 + 9K^2} + C \frac{L^2}{\pi^2} \frac{1 + K^2}{1 + 9K^2}.}$$

For the case of simple buckling this takes the form

$$P = \frac{\varepsilon \pi^2}{L^2} + C \frac{L^2}{\pi^2}.$$

including a first general term of *Euler*'s formula and a complementary term obtained from the ends of the vertical members. On determining the minimum value of P in accordance with the rule

$$\frac{dP}{dL} = 0$$

the following results are obtained, L being the buckling length:

$$C \cdot \frac{L_1^2}{\pi^2} = \frac{\varepsilon \pi^2}{L_1^2}$$

The buckling load

$$P_1 = 2 \frac{\varepsilon \pi^2}{L_1^2} = 2 \frac{C \cdot L_1^2}{\pi^2}; \quad P_1 = 2 \sqrt{C \cdot \varepsilon}; \quad L_1 = \pi \sqrt[4]{\frac{\varepsilon}{C}}; \quad \varepsilon = E \cdot I.$$

To fix the minimum value for P according to the new hypothesis we may write

$$P = \frac{\varepsilon \pi^2}{L^2} \cdot A + \frac{CL^2}{\pi^2} \cdot B.$$

The condition $\frac{dP}{dL} = 0$ is expressed by

$$B \cdot \frac{CL_2^2}{\pi^2} = A \cdot \frac{\varepsilon \pi^2}{L_2^2}$$

$$L_2^4 = \pi^4 \cdot \frac{A}{B} \cdot \frac{\varepsilon}{C}$$

$$L_2 = \pi \cdot \sqrt[4]{\frac{A}{B}} \cdot \sqrt[4]{\frac{\varepsilon}{C}}$$

$$P_2 = \frac{\varepsilon \pi^2}{L_2^2} \left(1 + \frac{A}{B}\right)$$

$$A = \frac{1 + 81 K^2}{1 + 9 K^2}$$

$$B = \frac{1 + K^2}{1 + 9 K^2}$$

$$\frac{A}{B} = \frac{1 + 81 K^2}{1 + K^2}.$$

Juxtaposing these results with those above it is seen that

$$L_2 = L_1 \sqrt[4]{\frac{A}{B}}$$

$$P_2 = \frac{\varepsilon \pi^2}{L_1^2} \sqrt{\frac{B}{A}} \left(1 + \frac{A}{B}\right) = P_1 \cdot \frac{1}{2} \sqrt{\frac{B}{A}} \left(1 + \frac{A}{B}\right)$$

To evaluate these expressions we must choose a value of K.
If the condition $y' = 0$ is strictly observed at the ends we obtain

$$f \frac{\pi}{L}(1 - 3K) = 0; \qquad K = \frac{1}{3}.$$

This value of K will give the value 5 to the coefficient A in the term $\frac{\varepsilon \pi^2}{L^2}$ for P. It appears that in the most favourable circumstances the maximum value of A cannot exceed 4, which would correspond to perfect end fixation. This would imply $K^2 = \frac{1}{15}$ or $K = \frac{1}{4}$ approximately. On applying these values of K we obtain the following results:

(1) $K^2 = \frac{1}{9}, \quad \frac{A}{B} = 9, \quad \sqrt[2]{\frac{B}{A}} = \frac{1}{3}$

$$P_2 = \frac{\varepsilon \pi^2}{L_1^2} \cdot \sqrt[2]{\frac{B}{A}} \left(1 + \frac{A}{B}\right) = \frac{\varepsilon \pi^2}{L_1^2} \cdot \frac{10}{3}$$

$$L_2 = L_1 \cdot \sqrt[4]{\frac{A}{B}} = L_1 \sqrt[2]{3} = L_1 \cdot 1.73$$

(2) $K^2 = \frac{1}{15}, \quad \frac{A}{B} = 6, \quad \sqrt[2]{\frac{B}{A}} = \sqrt{\frac{1}{6}} = \frac{1}{2.45}$

$$P_2 = \frac{\varepsilon \pi^2}{L_1^2} \cdot \frac{7}{2.45} = 2.85 \cdot \frac{\varepsilon \pi^2}{L_1^2}$$

$$L_2 = L_1 \cdot \sqrt{2.45} = L_1 \cdot 1.565$$

(3) If $K^2 = 0, \quad L_1 = L_2, \quad P_2 = 2 \cdot \varepsilon \frac{\pi^2}{L_1^2}.$

Numerical application.

A numerical application of these results was made for the case of a parabolic Vierendeel girder for a single track railway bridge of 100.10 m span and 14.30 m rise ($^1/_7$th), divided into eleven panels of 9.10 m width.

Here L_1 = approximately 34 m.
$P_1 = 2 \times 1500 = 3000$ tonnes.

Assuming a built-in arch with $K^2 = \frac{1}{15}$

$$L_2 = L_1 \cdot 1.565 = 53 \text{ m} = L_2$$

$$P_2 = \frac{\varepsilon \pi^2}{L_1^2} \cdot 2.85 = 4250 \text{ t}$$

If $K^2 = \frac{1}{9}, \quad L_2 = L_1 \cdot 1.73 = 58.5 \text{ m} = L_2$

$$P_2 = \frac{\varepsilon \pi^2}{L_1^2} \cdot \frac{10}{3} = 1500 \cdot \frac{10}{3} = 5000 \text{ t}.$$

For the arch under consideration the axial thrust due to full live load amounts to 1.035 tonnes at the crown and 1.230 tonnes at the springings, taking due account of dynamic augment.

These calculations show that in the most unfavourable case, which is the limiting condition for a girder without end fixation and with constant height, the critical buckling load of the whole structure represents a factor of safety of nearly 3. Making the more correct assumption that the ends are to a considerable extent built-in, and are firmly held by the ends of the vertical members and cross girders, with $K^2 = \frac{1}{15}$ the factor of safety will be at least 4.

It is found that the arch is of approximately the same strength whether considered as bearing only on the verticals or as being fixed at the ends so that its whole length acts as a single piece. It may also be concluded from these results that the rigidity of the vertical members confers upon the bridge a considerable margin of lateral strength; a margin which increases with the rigidity of the verticals themselves and with the continuity of their connection to the arch. These conditions are fulfilled best of all in the Vierendeel type of girder here considered.

V 18

Measurement of Transverse Accelerations Arising in Bridges.

Messung der an Brücken auftretenden Querbeschleunigungen.

Mesures des accélérations transversales auxquelles peuvent être soumis les ponts.

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Arising out of researches carried through in recent years on the French railways endeavours have been made to perfect a form of accelerometer in which the natural period of vibration is susceptible to measurement and in which the sensitivity is confined to a particular direction.

Generally speaking all existing forms of accelerometer are based on the same principle, namely the inertia of a mass, the displacement of which relatively to the structure is measured and is taken as proportional to the force represented by the mass of the structure multiplied by the acceleration which the latter undergoes. A simple calculation is enough to show that if the natural frequency of vibration of the apparatus is to be high (several thousands per second) the displacement on the mass must be extremely small (being of the order of $^1/_{10}{}^{th}$ to $^1/_{100}{}^{th}$ of a mm).

In these circumstances use has been made of the properties of piezo-electric quartz for the purpose of constructing an alternative form of apparatus. By means of a spring a certain mass is kept in contact with a piece of quartz, and variations in the pressure exerted on the latter, due to the mass being under acceleration, release in the quartz quantities of electricity, which are converted by means of a triode valve into a current which is recorded by means of an oscillograph. The natural period of vibration of such an apparatus is in excess of what can be recorded by the oscillograph employed (1000 per second).

Moreover, certain measurement arise in railway practice (for instance, the recording of very small longitudinal accelerations in a body subejcted to vertical accelerations of relatively high value) which make it necessary to devise some arrangement which will render the apparatus practically insensitive to accelerations perpendicular to a particular direction. This requirement has been met by special attention to the manner in which pressure is exerted on the quartz.

The idea arose of using this apparatus for measuring the transverse accelerations in bridges — a problem of evident importance, since in the course of time such vibrations may give rise to appreciable stresses. The apparatus here described was one which seemed eminently adapted to this purpose, as its high natural frequency of vibration and its sensitivity blased in one particular direction allow of measuring these transverse accelerations even though the vertical accelerations may be relatively much greater. A few examples of recards obtained in this way on railway bridges are given below.

Position of testing apparatus.

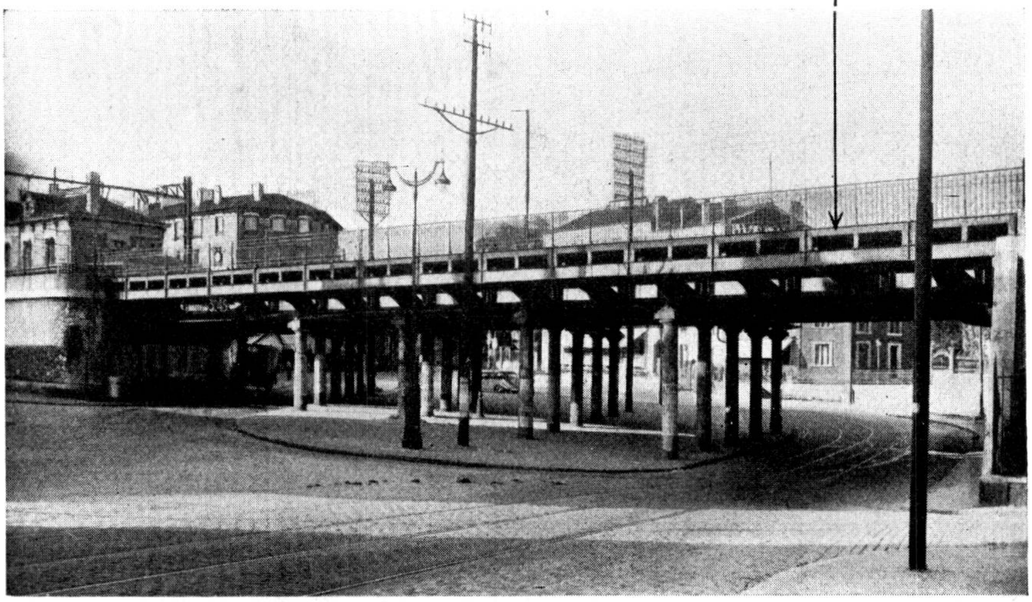

Fig. 1.

Skew deck-type bridge at Vitry-sur-Seine. Weight of bridge about 300 tons.

The graphs Nos. 1 and 2 show the vertical and lateral accelerations as measured in a skew bridge having a weight of 300 tonnes with the decking carried above the girders. Graphs Nos. 3 and 4 refer to a skew bridge weighing 120 tonnes, again with the decking above the girders. Graphs Nos. 5 and 8

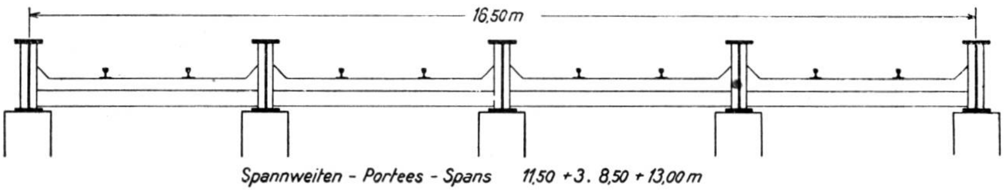

Fig. 2.

Cross section.

relate to a square bridge weighing 600 tonnes with the decking below the girders, the measurements having been made at the upper and lower booms respectively. The following are the values of the accelerations.

For the 300-tonne skew bridge (Figs. 1 and 2).

Vertical accelerations: graph 1 — at high frequencies g/2; at the natural period of the vibration of the bridge g/10.

Lateral accelerations: graph 2 — at high frequencies g/5; at the natural period of vibration of the bridge g/13.

For the 120-tonne skew bridge (Figs. 3 and 4).

Vertical accelerations: graph 3 — at high frequencies g/1.2; at the natural period of vibration of the bridge g/5.

Lateral accelerations: graph 4 — at high frequencies g/2; at the natural period of vibration of the bridge g/7.

Position of testing apparatus.

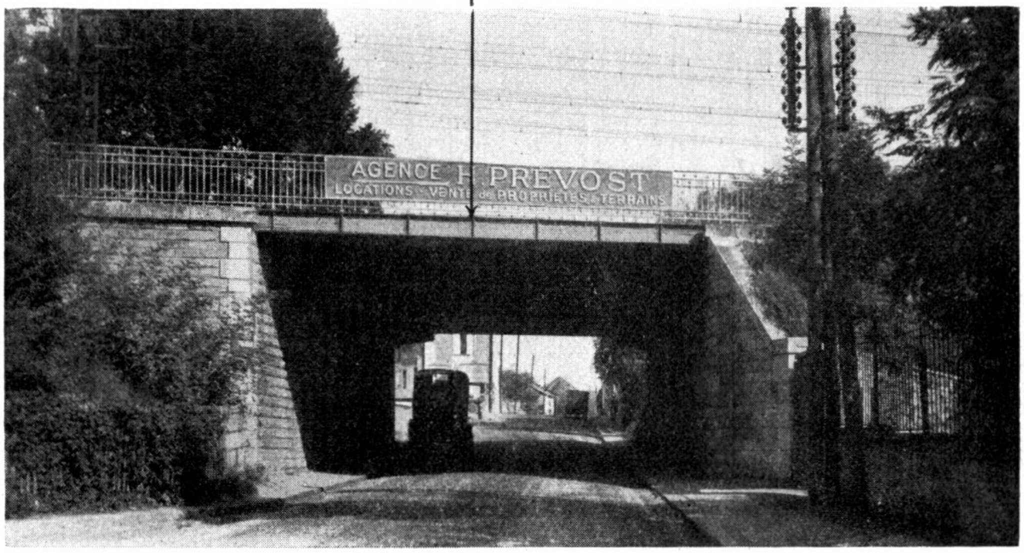

Fig. 3.

Skew deck-type bridge at Epinay-sur-Orge. Weight of bridge about 120 tons.

For the 600-tonne straight bridge (Figs. 5 and 6).

1) Measurements carried out on the upper portion of the girder. Vertical accelerations: graph 5 — at high frequencies g/0.8. No apparent natural period of vibration.

Lateral accelerations: graph 6 — at high frequencies g/1.25. No apparent natural period of vibration.

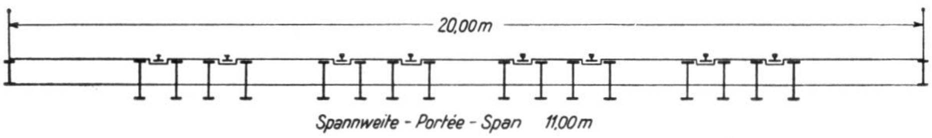

Fig. 4.

Cross section (diagramatic).

2) Measurements carried out on the lower portion of the girder.

Vertical accelerations: graph 7.

At high frequencies g/0.66.

At the natural period of the vibration of the bridge g/2.6.

Lateral accelerations: graph 8.

At high frequencies g/1.6.

At the natural period of vibration of the bridge g/7.

It will be seen that according to these graphs the greatest lateral accelerations were those obtained in the 600-tonne straight bridge (g/1.25 at the upper portion of the girder and g/1.6 at the lower portion).

In the other bridges the acceleration did not exceed g/2 laterally.

Position of testing apparatus.

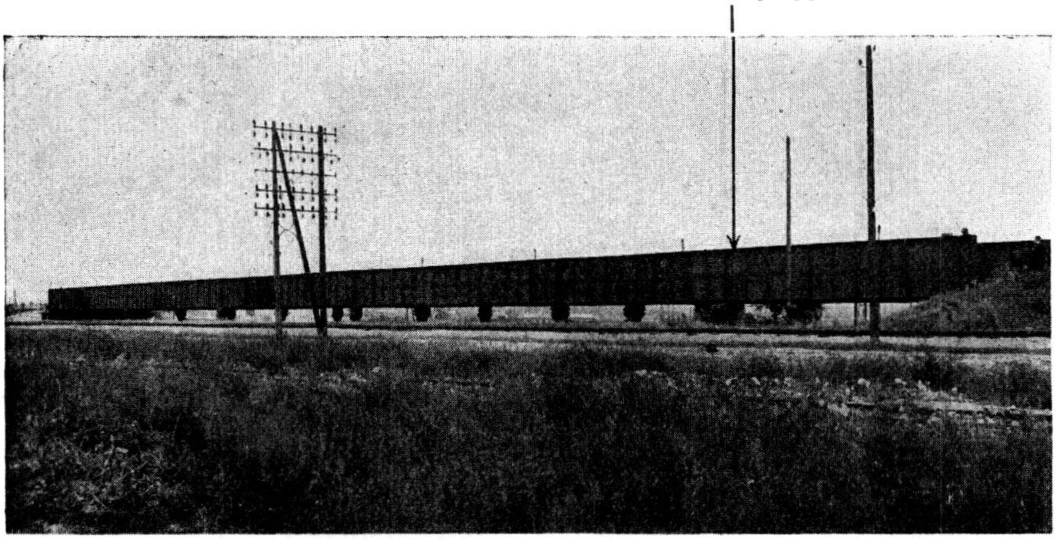

Fig. 5.

Straight through-type bridge at Maisons-Alfort.

Weight of bridge about 600 tons.

In addition, measurements of the acceleration have recently been effected on masonry bridges carrying railways; the values obtained were slightly lower than those recorded from the steel bridges.

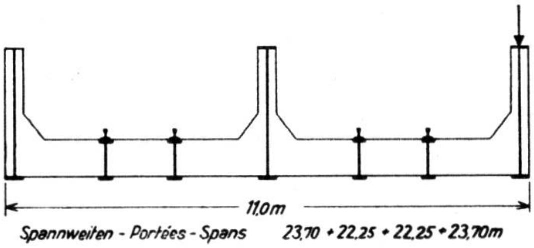

Fig. 6.

Cross section.

It should be noted that these figures are given only as examples, the object of the present note not being to study the acceleration to which bridge are in fact exposed and the consequent stresses arising in them under traffic, but merely to indicate the possibilities of use of this new apparatus.

Diagram 1
g = 32 mm

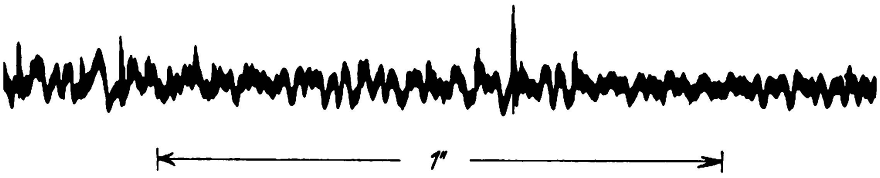

Diagram 2
g = 64 mm

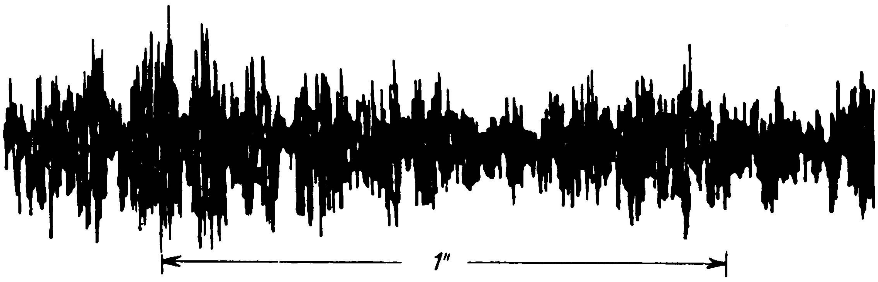

Diagram 3
g = 32 mm

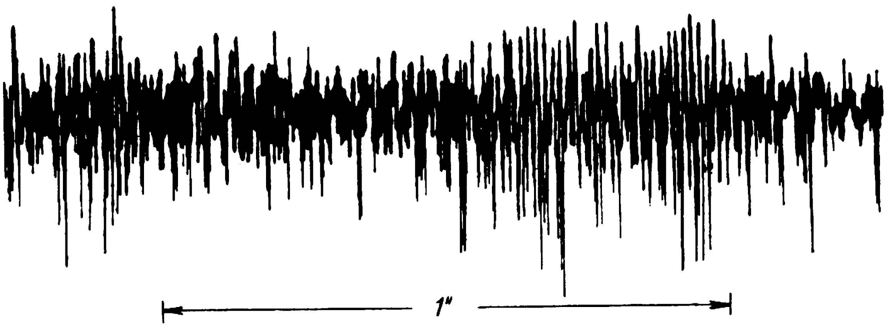

Diagram 4
g = 64 mm

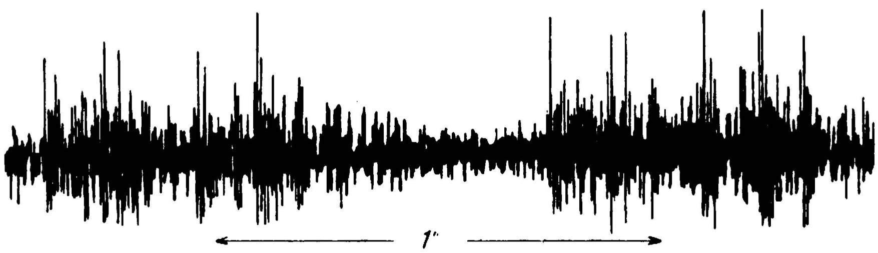

Diagram 5
g = 16 mm

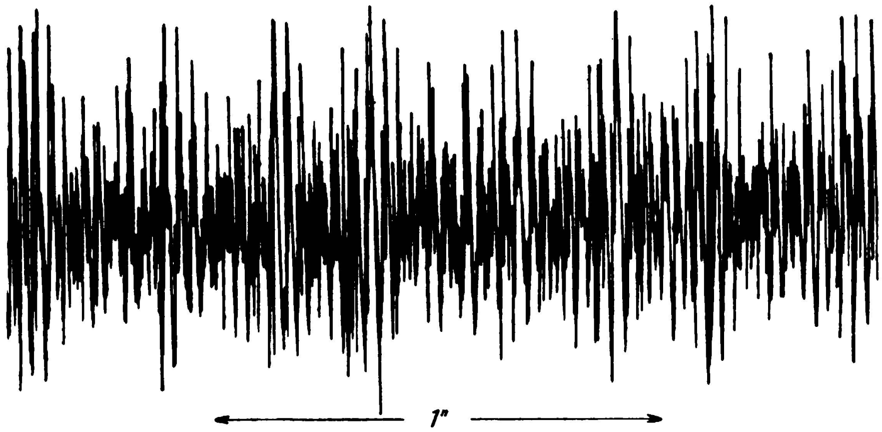

Diagram 6
g = 32 mm

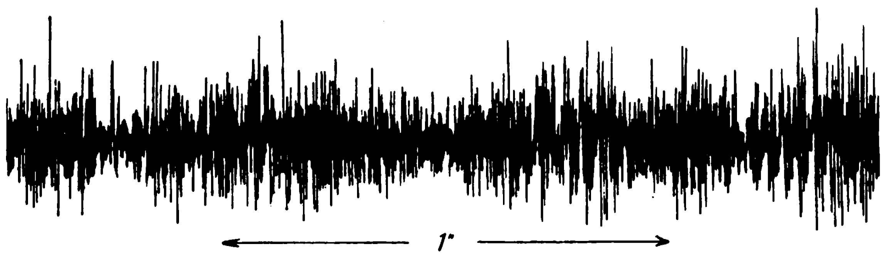

Diagram 7
g = 16 mm

Diagram 8
g = 32 mm

These records were obtained from the passage of different trains.