IV b

Wide-span bridges.

Weitgespannte Brücken.

Ponts de grande portée.

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IVb 1

The Shuttering and Concreting of Long Span Reinforced Concrete Girder Bridges.

Rüstung und Betonierung bei weitgespannten Eisenbetonbalkenbrücken.

L'échafaudage et le bétonnage des grands ponts en poutres de béton armé.

Dr. Ing. L. Pistor,
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Engineering work in the last few years has proved the possibility of building reinforced concrete beam bridges on piers of medium height across spans of 30 to 65 metres, without difficulty either from a constructional or an economic point of view, which are able to compete with steel bridges. By adopting the special measures discussed in the Preliminary Report[1] a further considerable increase in their scope is to be anticipated. This increase in span has hitherto been attributed by many to the adoption of higher stresses, and it is true that by adopting higher stresses in the concrete and in the steel the limiting span can easily be forced up to a high value. To a much greater extent, however, the problem is one of taking suitable statical measures to eliminate excessive bending stresses, or, ultimately, to eliminate cracks; at the same time, it is a problem which affects the whole constructional operation.

In work of this kind the question of scaffolding and concreting has already become important and will be more so as the size of the structure is increased so as to involve greater masses in proportion to area, greater depth of beams, and correspondingly greater cross sections. The proportions hitherto in use have varied between 0.6 and 1 m^3/m^2 of area, and portions of the structure which were statically and constructionally independent have been concreted in a continuous process covering 400 to 900 m^2 of ground area, the maximum values being in the case of continuous girders (for instance, the Saubachtal bridge and the Denkendorf viaduct for the Reichsautobahn, and also the bridge over the Oder near Oppeln).[2] In constructions of this kind the rate of concreting is limited by technical and economic factors, and difficulties may arise through lack of space, complication of girder design and overcrowding of the reinforcement. An output of about 15 m^3 of solid concrete per hour might be regarded as the upper limit. In beam bridges it is indeed not impossible, as in arch construction,

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[1] *Dischinger:* Preliminary Report, page 775 foll.

[2] See *Pistor:* Die neuere Entwicklung des Baues weitgespannter Eisenbetonbalkenbrücken in Deutschland. Die Bautechnik 1936, N°. 43, page 630 foll.

to subdivide monolithic parts into layers, but in the case of beams which are stressed mainly by bending and shear the practice is highly undesirable if not altogether inadvisable in view of the heavy reinforcement against shear, and of the narrow neck of the beam which increases the practical difficulties; hence concreting in one integral piece which includes also the roadway slabs is the desideratum to be sought.

The precondition for continuous concreting is immobility of the falsework. This, however, is practically unattainable on account of the variations in load along the length of the girder, deformations will arise which differ both in magnitude and in the time of their incidence, and these are combined with questions of ground supports, timber connections and other factors. The falsework for beam bridges is in itself heavily loaded, and with present practice as regards bridge design, favouring a limited number of large main girders, the load on a line of falsework may be between 5 and 8 tons per metre. In large bridges the deformations which arise as the loading is increased by the deposition of fresh concrete upwards from below, on top of concrete which has already set and hardened, cause an undesirable distribution of the stresses and may cause cracking. The measures to be taken for the avoidance of harmful effects have been known for a long time, and are:

a) Concreting in strips.
b) Continuity of the falsework with a view to avoiding local irregularities of the supports.
c) Pre-loading of the whole of the falsework and elimination of the deformations before concreting.

All these methods are in use. Concreting in strips has mostly been carried out by closing with shuttering those portions of the beams which are over the supports and filling in these directly after the girder is finished, or by closing the gap after an interval of a few days. This method was used in some of the new bridges — for instance, in the Denkendorf viaduct[3] for the Reichsautobahn and in the cantilever beams for the Saale bridge at Bernburg[4] — and a similar method was adopted for making the suspended girders. In this way the influence of the varying amount of stiffness as between the supporting piers and the spans themselves is eliminated — though at the cost of the homogeneity of the girder, since the resulting construction joints, when they extend into the tension zone, must be regarded as cracks produced beforehand. Within the span, if the falsework has been properly designed and is not too heavily loaded, it is true that fairly uniform settlement may be counted upon, but there can be no guarantee against local deformations due to yielding of timber connections, etc.

The same difficulties arise when continuous scaffolding is adopted, an expedient which indeed avoids the presence of working joints in the concrete but which does not exclude the possibility of local settlements due, in particular, to concentrations of load at the supporting trestles. Every irregularity in loading, as, for instance, that which occurs in the neighbourhood of openings left for navigation and the like, is a possible source of danger. Hitherto this method has

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[3] *Schächterle:* Beton und Eisen 1936, N°. 1, page 1.

[4] *Nakonz:* Bautechnik 1936, N°. 15, page 216.

been applied only in one instance, namely, the Sophia bridge at Bamberg[5]. (Fig. 1).

The third method, that of pre-loading the falsework, enables the greater part of the deformations to be made to occur before the introduction of concrete, so that the concreting can be carried out quite continuously and there is nothing to prevent the formation of a monolithic structure. There is a further advantage in the fact that local settlement is detected at an early stage in the application of the preliminary loading, so that suitable precautions can be taken, and the desired positions of the soffits of the beam can be regulated with great accuracy after the pre-loading is completed.

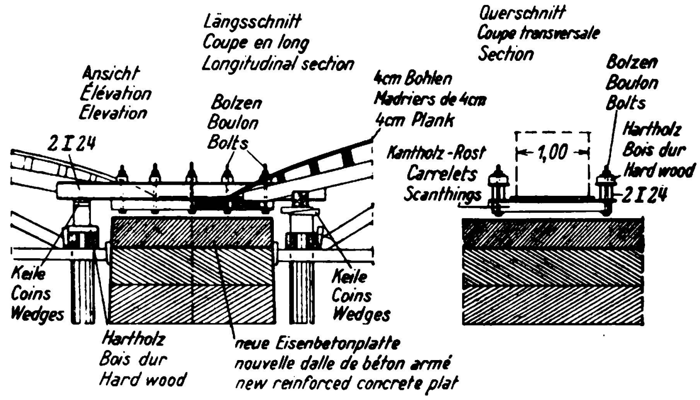

Fig. 1.

Pre-loading in this way — the first instance of its use on so large a scale and with such accuracy — was applied by the author, at the instance of the autorities concerned, in his design for the Reichsautobahn bridge over the Inn, in which a total of mare than 1000 linear metres of girders were built in this way (Fig. 2). Boxes were provided on the inside face of the shuttering of the girders, and these were filled with the gravel to constitute the loads, a weight equal to two-thirds that of the concrete being supplied. To attain the full amount of loading it remained necessary to add to the girder 0.6 m³ of concrete per metre of length, and this was possible within the time required for the setting of the concrete; not until the latter had been placed in position was the gravel removed, keeping pace with the progress of the concreting. Further relevant details will be found in the paper cited below[6] and also in the section of the Congress concerned with reinforced concrete construction.

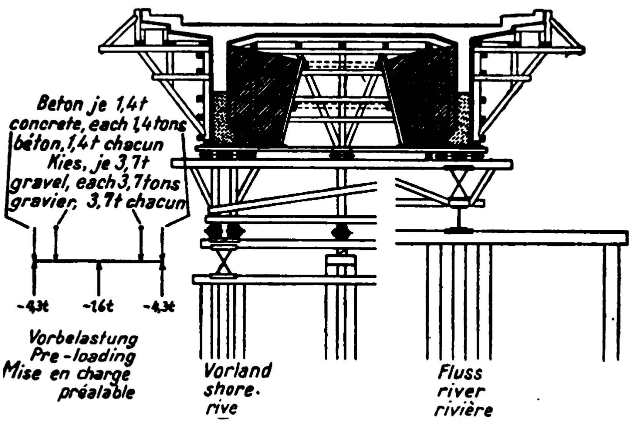

Fig. 2.

Pre-loading on a very simple system was applied in the construction of the Sophia bridge at Bamberg,[5] making use of crane rails. Another application, similar in form to that employed at the Inn bridge, was made with a view to eliminating the effects of the deflection of the *Melan* girder built as a suspended span, 27.0 m long, in the bridge over the Saale at Bernburg[4] (Fig. 3). The work

[5] *Berger:* Bauingenieur 1932, N°. 21/24, page 305 foll. — *Berger:* Final Report of First Congress, Paris 1932, page 359.

[6] *Endrös:* Beton und Eisen 1935, N°. 3, page 27 foll.

at the Inn bridge showed that the costs of pre-loading are low and are out of all proportion to the increase in safety during construction so obtained. The complete freedom of the bridge from cracking must be attributed largely to the elimination of the deflections in the falsework.

Of the thirteen long span reinforced beam bridges[2] which have been built since 1933, it appears, so far as can be established, that two were built by pre-loading, four by temporary interruption of concreting over the supports, and seven were built by continuous concreting without special precautions. It must be recorded that even in the last of these categories it was found possible to build the bridges completely free from surface cracks, despite the fact that not inconsiderable settlements of the falsework were observed. It appears, however, that the latter occurred at the right time, before the concrete had began to set, and the more so because in continuous concreting at first a kind of division into strips was practised. The success obtained in these particular bridge works cannot, however, be held to rebut criticisms of the method.

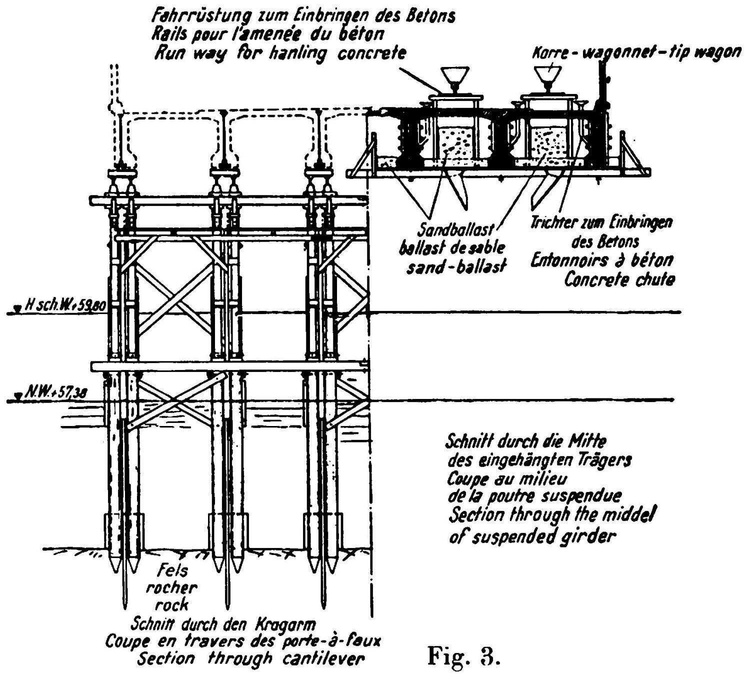

Fig. 3.

Where the work is notably larger in size, and especially where it consists of continuous girders, it will be impossible to avoid the necessity for subdivision into zones even with the precautions mentioned above. However, any large increase in size will probably require the adoption of pre-stressing on technical and economic grounds, and since, if this is the case, bending tensile stresses are completely or largely eliminated, there will be no objection to working joints if suitably situated. The advantages of pre-stressing apply here again in regard to the execution of the work.

Conclusion.

In the consruction of large reinforced concrete beam bridges which are mainly stressed by bending, continuous concreting should be practised on those portions which form statical units.

The harmful but unavoidable deformations of the falsework (or of stiff reinforcement) must be compensated. This can be properly ensured only by pre-loading the supporting scaffolding work as was practised in the case of the whole of the Inn bridge, and in the case of the suspended span in the Saale bridge at Bernburg. The method is simple and relatively cheap. Its application as a matter of principle would mean a further step towards the elimination of uncontrollable influences, and would increase the likelihood of successful construction.

IVb 2

Long Span Bridges.

Weitgespannte massive Brücken.

Ponts de grande portée.

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Doubtless the most satisfactory way of determining the maximum span of bridge capable of being built in a particular material would be actually to build it. This, of course, is not always possible. An alternative is to prepare a design, if possible a design for a project ready to be carried out. Either of these methods is subject to the disadvantage that it holds good only in reference to a particular conjunction of permissible stress, load, ratio of rise to span, length of span, etc., and does not allow conclusions to be directly drawn in reference to other sets of conditions.

For this reason it appeared desirable to develop formulae that would be generally applicable.

1. Reinforced concrete arch bridges.

The author has arrived at definite formulae of this kind for concrete *arch* bridges. In order to do so it was necessary, of course, to simplify by idealising certain of the usual mathematical treatments, and this must be borne in mind when estimating the reliability of the result.

The following assumptions were made, using the notation indicated in Fig. 1.

a) An arch, based on the "pressure line" due to the normal loading condition: i. e. dead load plus half the live load uniformly distributed over the span.

b) Distribution of the total load q over the length of the bridge in accordance with the relationship

$$q = q_S \left[1 + (m - 1)\, y/f\right] \tag{1}$$

$$m = \frac{q_K}{q_S} \tag{2}$$

c) Variation of section of the arch according to

$$F = \frac{F_S}{\cos\varphi} \tag{3}$$

d) Stress at centre of gravity, under normal load condition a), given by

$$\sigma_m = \mu\sigma_{perm} \qquad 0 < \mu < 1 \tag{4}$$

Here μ serves to indicate to what extent under the assumed condition of loading, the permissible stress σ_{perm} is utilised. μ may therefore be designated the coefficient of utilisation. From Fig. 1 we obtain:

$$H = \frac{(Q+G)\cdot\beta l}{f} \tag{5}$$

and from (4) it follows that

$$H = \mu\cdot\sigma_{perm}\cdot F_S \tag{6}$$

From (1) we obtain the distance $\beta\cdot l$ from the springing for the load $G+Q$ (where G represents the weight of the arch itself and Q that of the superstructure and roadway with one half of the live load):

$$\beta = \frac{m-1}{2c\sqrt{m^2-1}} \tag{7}$$

$$c = \operatorname{Ar\,cosh} m = \operatorname{Ar\,cosh}\sqrt{\left(\frac{q_K}{q_S}\right)} \tag{8}$$

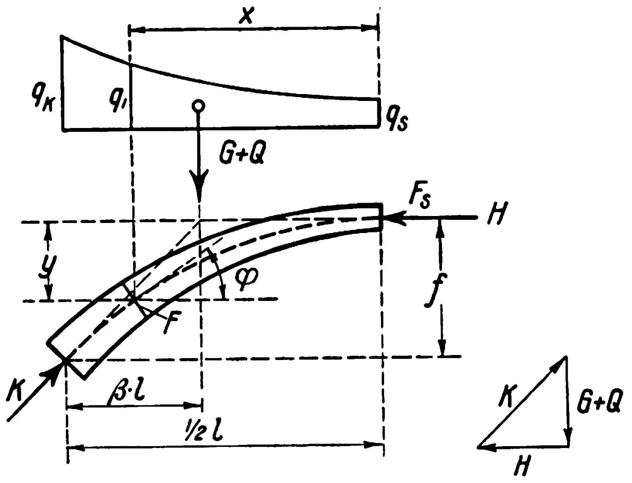

Fig. 1.

Equations (1) and (3) enable the cross section of the arch to be calculated and hence the weight G of one half of the arch.

$$G = \frac{1}{2}\gamma F_S\cdot l\,[1+\alpha(f/l)^2] \tag{9}$$

$$\alpha = \frac{c}{(m-1)^2}(\operatorname{Sin} 2c - 2c) \tag{10}$$

Combining (5), (6) and (9) we obtain

$$\varkappa = \frac{G}{Q} = 1 : \left[\frac{2\mu\sigma\cdot f/l}{\gamma\cdot l\cdot\beta\,[1+\alpha(f/l)^2]} - 1\right]$$
$$\varkappa = 1 : \left[\frac{\mu\sigma}{\gamma\cdot l}\cdot\frac{2n}{\beta(1+\alpha n^2)} - 1\right] = 1 : \left[\frac{\mu\sigma}{\gamma\cdot l}\cdot\delta - 1\right] \tag{11}$$

wherein, for abbreviation, there is written

$\sigma = \sigma_{perm}$, $n = f/l$, $\gamma = \gamma_B$ = specific weight of the arch by volume. (12)

In view of (7) and (10) the new coefficient δ becomes:

$$\delta = \frac{2n}{\beta(1+\alpha n^2)} = \frac{4nc\cdot\sqrt{m^2-1}\cdot(m-1)}{(m-1)^2 - cn^2(\operatorname{Sin} 2c - 2c)} \tag{13}$$

The figure $\varkappa$ obtained from equations 11 and 13 gives the proportion between the weight of the arch itself and that of the super-structure including the roadway and half the live load; in other words, the *relative amount of material* required for the arch. $\varkappa$ provides a convenient means of assessing the result of various influences on the amount of material required and on the possible span. In particular, the condition:

$$2\mu\sigma n = \beta\gamma l(1+\alpha n^2) \tag{14}$$

applied to the case when $\varkappa = \infty$, determines the theoretical limit of feasibility for the arch. It will be seen that this is governed only by the geometrical conditions and by the mechanical properties of the material, not by the intensity of the loading.

In the case of arch bridges of "open superstructure", wherein suspension bars, spandrel walls, or columns transfer the weight of the roadway onto the arch, the approximate assumption may be made that the loading of the arch under normal conditions of loading is distributed in approximately the same way as its own weight:

$$m = \frac{q_K}{q_S} = \frac{g_K}{g_S} = \frac{1}{\cos^2 \varphi_K} = 1 + \left(\frac{dy}{dx}\right)_K^2 \tag{15}$$

Hence m is determined as a function of the ratio of rise to span $n = f/l$ in accordance with the formula:

$$\frac{1}{2c}\sqrt{\frac{(m-1)^3}{m^2-1}} = \frac{1}{2 \operatorname{Ar} \cosh m}\sqrt{\frac{(m-1)^3}{m^2-1}} = n \tag{16}$$

Here the coefficients α and β determined from equations (7) and (10) as being dependent on m, and also δ according to equation (13), have been reduced to functions of the ratio of rise n alone. In the following table the coefficients m, α, β, δ are given for various ratios of rise to span:

$f/l = n$	m	α	β	δ
$1:\infty = 0$	1.00	5.33	0.25	0
$1:10 = 0.1$	1.18	5.47	0.242	0.784
$1:7 = 0.143$	1.38	5.60	0.234	1.068
$1:5 = 0.200$	1.83	5.70	0.223	1.460
$1:3.5 = 0.286$	3.00	5.87	0.201	1.920
$1:2.5 = 0.400$	7.50	6.86	0.165	2.310
$1:1.78 = 0.562$	20.00	8.13	0.129	2.450
$1:1 = 1.000$	100.00	10.67	0.095	1.805

The only item now outstanding in order to make use of the fundamental formula (11) is the coefficient of utilisation μ (4). The value of this increases in proportion as the dead load becomes the predominant factor, and it also increases with the permissible stress, so that as a general rule it is higher in the case of long span bridges than in that of smaller spans. To a large extent, however, it can be varied in accordance with the design and method of construction of the bridge. Indeed, it may be stated that the improvement of the coefficient of utilisation constitutes the chief problem before the designer of a long span arch bridge, and it follows from this that no universally valid statement can be made as to the magnitude of μ. It is the business of the designing engineer to regulate this figure according to the conditions of each particular case.

For the purpose of the preliminary calculations, the author has made use of the following relationship based upon considerations which are here omitted for lack of space:

$$\mu = a \cdot \sqrt[4]{\sigma_{perm}} \qquad (\sigma_{perm} \text{ in t/m}^2) \tag{17}$$

Here a is a figure amounting to between 0.1 and 0.12 for the ratio of rise to span liable to arise in practice. The highest values for a are applicable where $f/l = \frac{1}{5}$ to $\frac{1}{4}$, but the value of a decreases in either steeper or flatter arches:

Hence (11) becomes:

$$\varkappa = 1 : \left(\frac{a \cdot \sigma^{5/4} \cdot \delta}{\gamma \cdot l} - 1\right) \tag{18}$$

With the aid of this equation all the figures and graphs given below have been calculated. These latter are correct only under the special assumptions (15) and (17) while equation (10) is of more general validity. It may be remarked that the curves do, in principle, represent actual conditions.

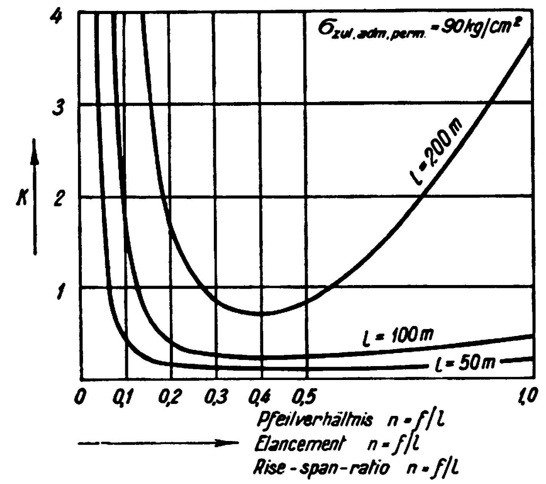

Fig. 2.
Material requirements in relation to rise-span ratio.

$$\varkappa = \frac{\text{weight of arch}}{\text{weight of superstructure} + \frac{1}{2}\text{ live load}}$$

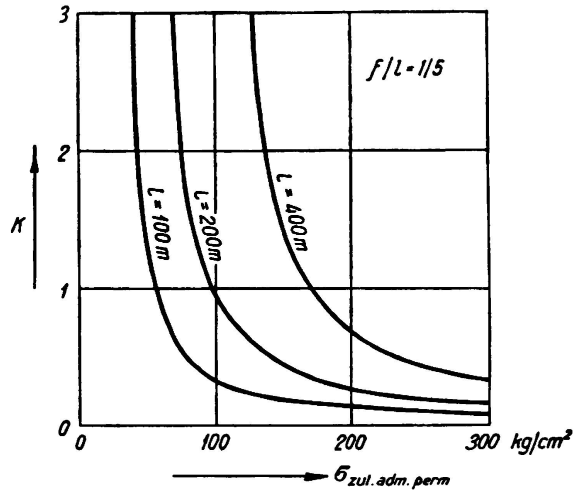

Fig. 3.
Material requirements in relation to the permissible stresses.

In Fig. 2 the requirement of material $\varkappa$ is shown relatively to the ratio of rise to span n. All the curves reach their minimum for a ratio of rise to span of approximately $^1/_3$. In practice a flatter arch is more usual, and the curves rise towards the left, but for moderate spans up to 50 m they to so only slowly, so that the excess quantity of material if a flatter arch is chosen is not great. If, however, the span becomes large the curve of $\varkappa/n$ rises very steeply, and such spans are practicable only if the ratio of rise to span is kept near the minimum indicated by this curve. Actually the optimum value is not exactly as indicated here, but a little further left, somewhere between $^1/_4$ and $^1/_6$, because, among other reasons, the super-structure over a flatter arch is cheaper and easier to build, and a non-uniform distribution of the live load produces in such cases smaller stresses.

Fig. 3 shows the relative requirement of material as a function of the permissible stress. The curves indicate that the saving in material that can be realised by increasing the permissible stress is much smaller in small spans than in large spans. For instance, if the stress is increased from 100 to 150 kg/cm² the reduction in $\varkappa$ for a span of 100 m is from 0.32 to 0.20, equal to 37 %, but with 200 m span it is from 0.93 to 0.43, equal to 54 %. The principal point to be noticed is that there is a lower limit of permissible stress for every span, below which the construction of that span becomes impracticable.

Assuming a particular ratio of rise to span, it is possible to calculate the theoretical maximum span to correspond with any given permissible stress. In practice, of course, such spans are not attained. The question of how far practice falls short of the theoretical limit is not solely a technical one, but is determined by various considerations, especially those of an economic nature. It might be approximately correct to assume that in practice about two thirds of the theoretical maximum span is attainable. Fig. 4 has been calculated on this basis, showing the practically attainable spans for reinforced concrete arch bridges as

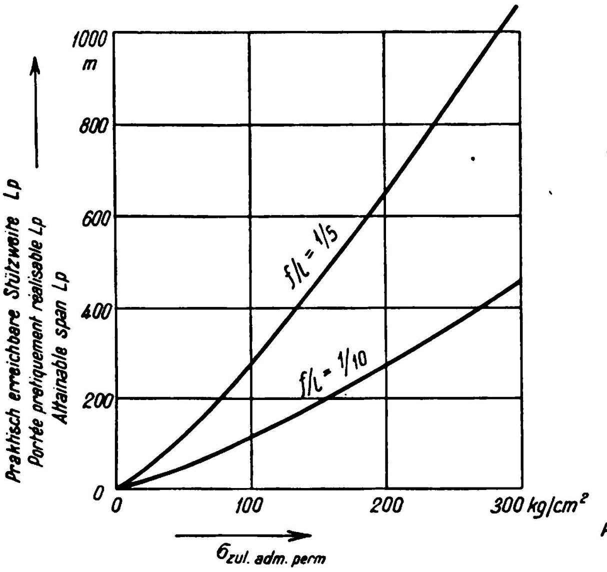

Fig. 4.

Spans attainable in practice with reinforced concrete arch bridges.

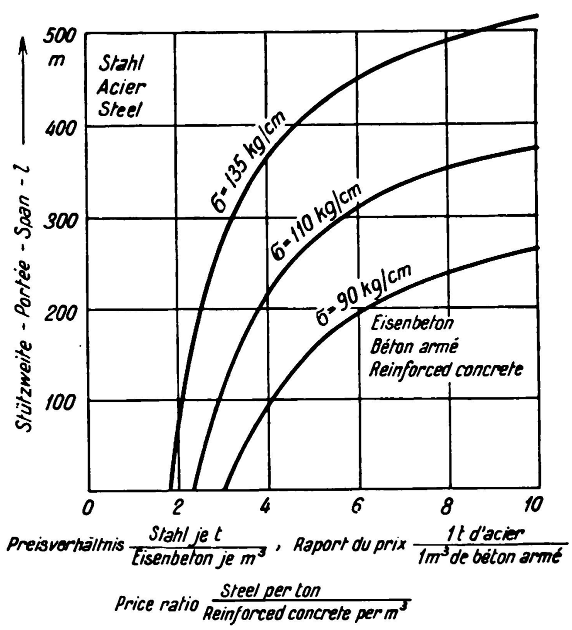

Fig. 5.

Cost comparison between steel and reinforced concrete arch bridges.

functions of the permissible stress σ in the form of two curves plotted to correspond with the ratios of rise to span $^1/_{10}$ and $^1/_5$ respectively. It will be seen that in the latter case approximately twice the span can be obtained as with the flatter arch which has a ratio of $^1/_{10}$. Special attention may here be drawn to a few important points in the $^1/_5$ curve: using a concrete with a permissible stress of 100 kg/cm² an arch of about 270 m span can be built; with 200 kg/cm² the possible span is 600 m, and for 300 kg/cm² it is about 1000 m.

To a certain extent it is possible to exceed these spans, but only at the cost of a greatly increased consumption of material in the arch and to the detriment of the competitive power of reinforced concrete. The most important of its competitors is the steel arch, and since the formulae here developed are applicable also to the latter a comparison of cost between concrete and steel arches can be made. For small spans, as a rule, the reinforced concrete is the more economical; then a certain critical span is reached at which the cost of the two materials is equal, and beyond that point the steel arch is the more economical. The critical span depends on the proportionate cost of the two materials. Fig. 5 shows the limiting spans in relation to the proportionate cost of 1 ton of steel and 1 m^3 of reinforced concrete. The permissible stresses are taken at 2100 kg/cm^2 for the steel and at 90, 110 and 135 kg/cm^2 for the reinforced concrete. The region in which the reinforced concrete is economically more advantageous lies to the right and below the curves, and the corresponding region for steel lies to the left above.

What is particularly notable in this diagram is the large increase in the reinforced concrete zone which accompanies the by no means immoderate increase assumed in its permissible stress. For instance, taking a cost ratio of 4 to 1, the limit of competability of the reinforced concrete with $\sigma_b = 90$ kg/cm^2 is about 100 m span, whereas with 110 kg/cm^2 it becomes 220 m, and with 135 kg/cm^2 as much as 360 m.

It would scarcely be possible to express in a more striking manner the paramount importance of improving the quality of concrete and thereby increasing its permissible stress. In this connection it should be noted that everything which can be done towards reducing those additional extreme-fibre stresses which are a consequence of irregular distribution of the live load, temperature variations, shrinkage, etc. will have the same effect as an increase in the permissible stress. The great efforts that are being made with this object are, therefore, fully justified.

II. Reinforced concrete beam bridges.

Apart from arch bridges, which type alone comes into question for the largest spans, reinforced concrete construction in the form of beams has a large field of application for medium spans. In these structures the distribution of the moments over the supports and within the spans plays a similarly decisive part in determining the amount of material required as is played by the choice of the ratio of rise to span in arch bridges. The following considerations will serve to indicate how far this is true.

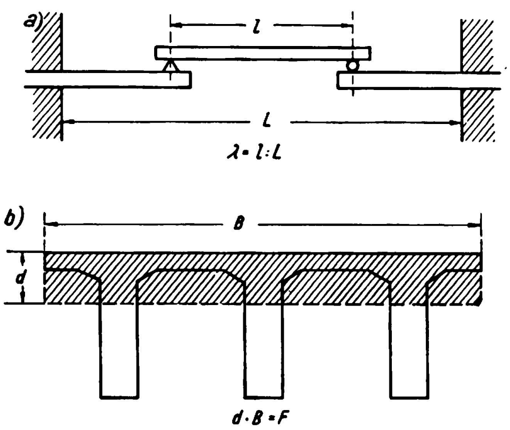

Fig. 6.

An opening of width L may be bridged by a structure consisting of a freely supported girder of span l carried on two symmetrically arranged cantilevers (see Fig. 6a). The proportion between the spans l/L will be denoted by λ, the value of which lies between 0 and 1. $\lambda = 1$ corresponds to the case of a simple beam on two supports and $\lambda = 0$ to that of two cantilevers each of length L/2. Between these limits there may be any intermediate forms similar to that sketched.

It was a matter of difficulty[1] to work out generally valid formulae similar to those for arch bridges. As a beginning, therefore, a series of designs were prepared for road bridge superstructures in reinforced concrete of different spans and with different values of λ and the amount of material required for each of

[1] See *Gaede:* „Balkenträger von gleichem Widerstande gegen Biegung.“ Die Bautechnik 1937, Heft 10, S. 120/122.

these was expressed in terms of thickness d of a slab of the same volume, and the area of the bridge floor (see Fig. 6b). The calculations were based upon the reinforced concrete stresses of 60 and 70 kg/cm^2 prescribed in the German regulations, the latter of these values being used for the increased stresses in the region of negative moments.

In Fig. 7 the average quantity of material d, in m^3/m^2 of floor area, is shown as a function of the span ratio λ for several different spans L. According to this, the minimum quantity of material is obtained with $\lambda = 0$ for all spans; that is to say, in a structure consisting of two cantilevers alone. The quantity of material increases as the length of the simply supported girder increases, and reaches its maximum value when $\lambda = 1$ which corresponds to the case of a beam between two supports. The curves become more and more steep as the total span L in-

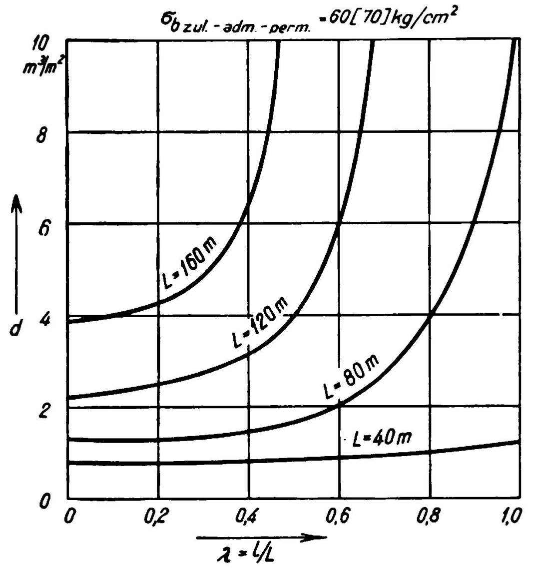

Fig. 7.

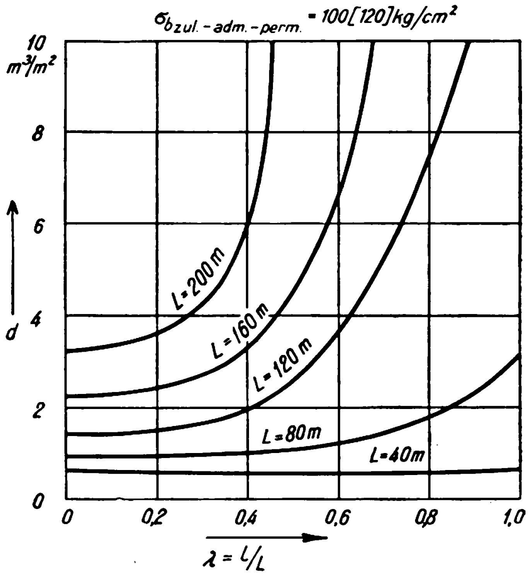

Fig. 8.

Material requirement d for reinforced concrete girder bridges in m^3/m^2.

creases, which shows that for very large spans a simple beam, or even cantilevers with proportionately long beams between them, would be uneconomical or even impossible to build. Large spans can be bridged, with an economically acceptable amount of material, only by the use of cantilever girders acting in the same way as those with small intermediate girders. It is not essential in such a case to introduce hinges, for the same effect may be obtained by means of hingeless continuous girders provided there is a suitable distribution of the moments of inertia and artificial pre-stressing (by dropping or lifting the supports) is applied. If side openings are not otherwise available for the purpose of imposing the heavy restraint moments, it may be expedient to introduce special measures for this purpose, such as counterweight arms, or specially provided openings at the sides, etc.

Fig. 8 shows a corresponding result to that in the preceding graph, assuming a permissible stress of 100 or 120 kg/cm^2 such as may conceivably be realised in the future.

For practical reasons λ will not be reduced to zero, but the span of the intermediate beam will be made say 0.2 to 0.4 of the total span. The values obtained from Figs. 7 and 8 to correspond with this condition are incorporated in the next diagram, Fig. 9, which shows the quantity of material for suitably arranged cantilever beams in relation to the spans, using either of the stresses already contemplated.

As in the case of the arch bridges, a comparison between the quantities of materials in these cases and those in steel girder bridges allows the limits of competability of reinforced concrete by comparison with steel to be determined. These limits are represented in Fig. 10 once again in relation to the cost ratio between 1 ton of steel and 1 m³ of reinforced concrete. Here again emphasis should be laid down on the great increase of the region in which reinforced concrete may be enabled to compete by raising the permissible stress. Measures designed to reduce the extreme fibre stress of the concrete, such as those suggested by Professor *Dischinger*, play the same part as a corresponding increase

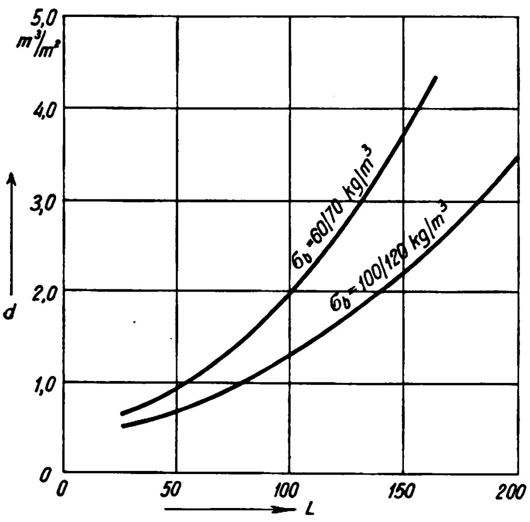

Fig. 9.
Material required for reinforced concrete girder bridges in m³/m².

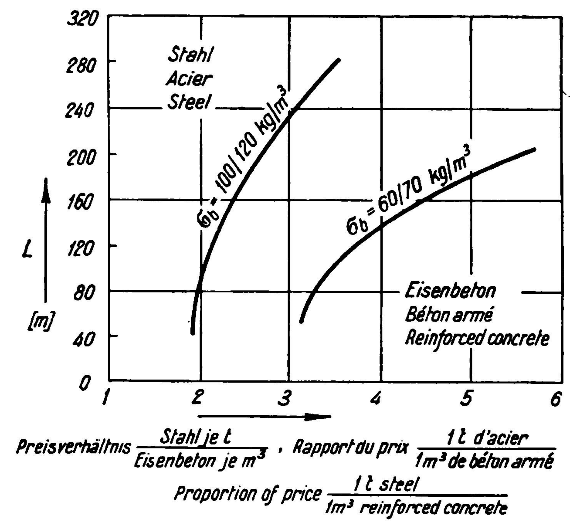

Fig. 10.
Cost comparison between steel and reinforced concrete beam bridges.

in the permissible stress, and offer like the latter, a suitable means of increasing the competability of reinforced concrete in this important field of application.

Finally the author would draw special attention to the fact that these comparisons of cost, whether for arch or beam bridges, must not be regarded as more than rough approximations and in particular that no account has been taken of the greater quantity of material usually necessary in the piers and abutments in consequence of the great weight of the reinforced concrete superstructure, these quantities being so greatly dependent on special local circumstances as not to admit of generalised treatment. It must not be overlooked that this circumstance will tend to swing the balance of the comparison more or less strongly to the disadvantages of reinforced concrete.

An attempt to take into account the supports in a comparison of cost for arch bridges has been made by *Dr. Glaser* in Zeitschrift des Oesterreichischen Ingenieur- und Architektenvereins, 1934, N° 39/40, pages 233 and foll., and similar solutions might possibly be made for beam bridges. This work by *Dr. Glaser* followed upon an earlier one by the present author in Bauingenieur, 1934, Nos 13/14 and 17/18.

IVb 3

The Bridge at Prato.

Die Brücke von Prato.

Le pont de Prato.

G. Krall,
Professor der Universitäten Rom und Neapel, Rom.

The arch is, of all forms of construction, that which makes the best use of the material, but for the purpose of bridging wide spans there are many cases in which it is not the best form to use. Where the conditions of the foundation are such as to preclude any large horizontal thrust, while at the same time the use of an arch with a tie is out of the question, the natural tendency is to consider some system of girder or frame, and to exemplify this a description will now be given of the foot bridge over the Bisenzio near Prato in Tuscany which has a span of 60 m and was carried out under contract for the municipality of Prato by the Società Ferrobeton of Rome.

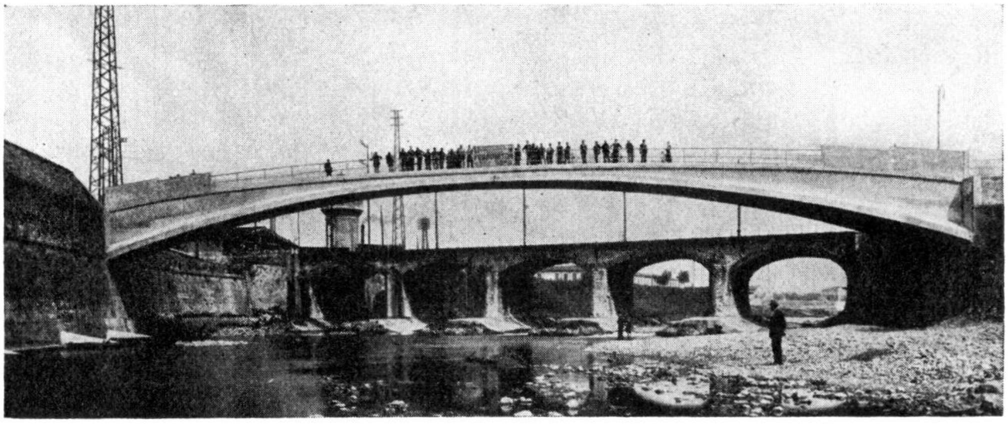

Fig. 1.
Bridge over the Bisenzio at Prato. (View of completed work.)

Fig. 1 shows the completed work, while Figs. 2 and 4 are plans. The structure was required to satisfy the following conditions: in view of possible floods the river was to be bridged by a single span of statically determinate construction, and the soffit under the crown was to correspond in height with the rail level of a railway bridge existing upstream.

Since the suitable foundation lay more than 10 m below the level of the abutments the adoption of an arch design was not feasible, and at the same time

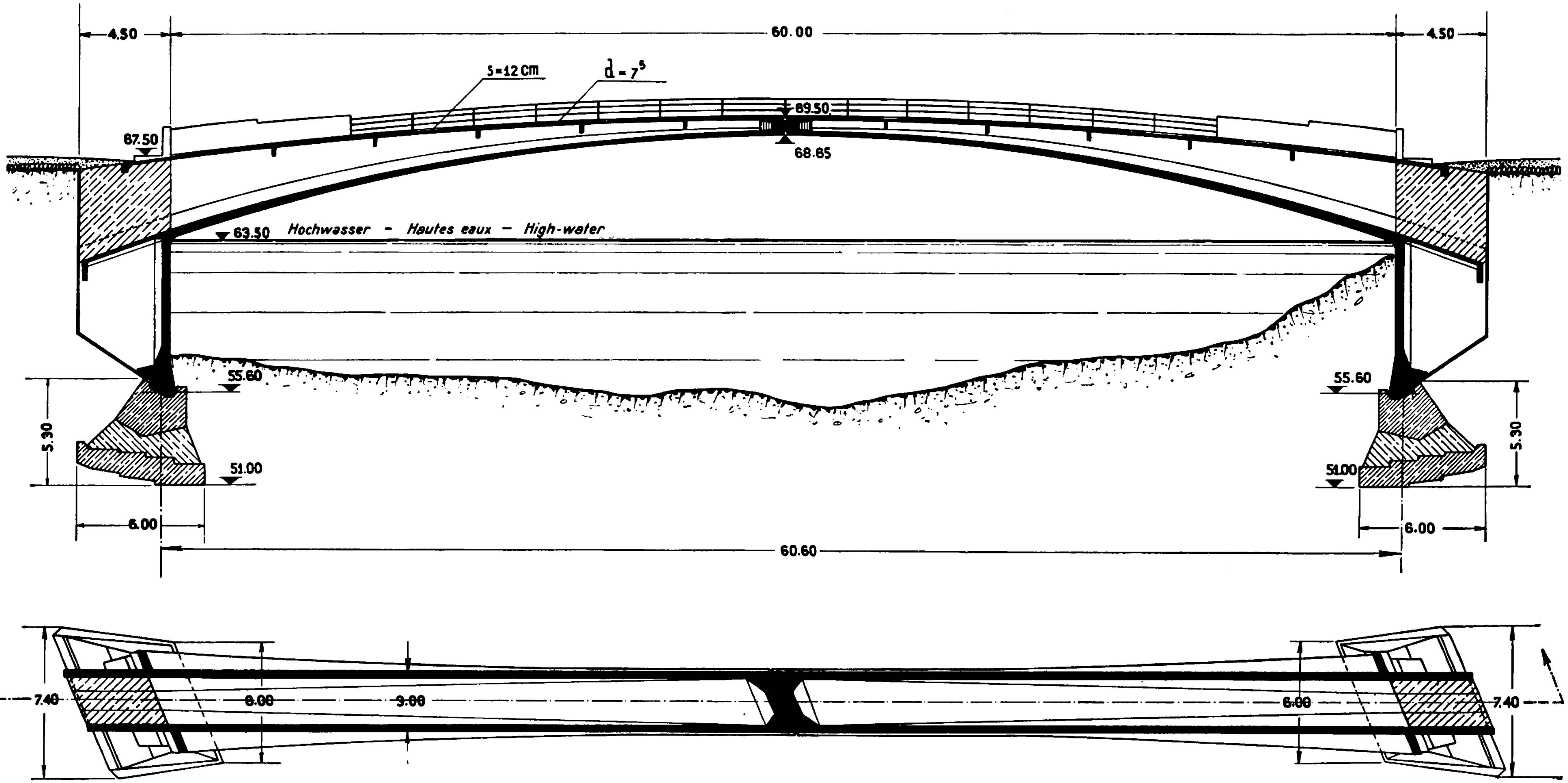

Fig. 2.

Plan and elevation.

a built-in beam construction was ruled out straight away by the fact that it would have been statically indeterminate. Neither would an arch with a tie have been practicable, and in the present case this would not have been a suitable design having regard to the limited width of the bridge and to aesthetic considerations. In these circumstances it was decided to adopt a three-hinged frame design, which allowed of very small foundations while, at the same time, the permissible foundation pressure of 1.5 kg/cm² would not be exceeded.

The effective width of the roadway is 2.50 m and the constructional width 3.00 m, so that the width of the bridge amounts to $^1/_{20}{}^{th}$ of the span. For various

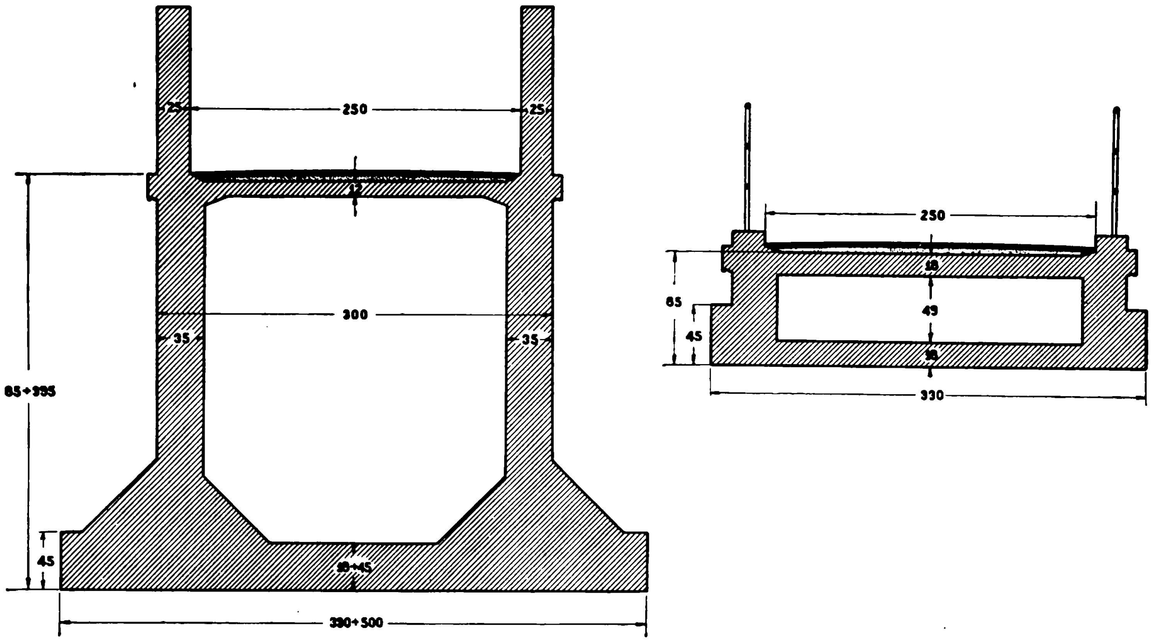

Fig. 3.

Cross section.

reasons, and particularly in order to prevent the development of lateral or overturning forces, the lower slab was widened to 5.50 m at the springings; this had the effect of strengthening the supports, increasing the lateral resistance and improving the appearance of the structure. The skew position of the bridge, as laid down in the specification, involved difficulties in so far as it was proposed to construct the hinges on the Considère system using flat steel reinforcement, and on this account it was decided to adopt a type of hinge which would permit a sliding motion along an axis at right angles to the plane of loading in addition to rotation about that axis. As, however, the construction of such a roller bearing in steel would have been too costly, and its construction in reinforced concrete too difficult, the idea of introducing roller hinges was abandoned and the erection was carried out with the aid of hydraulic jacks supported on well greased plates in such a way as to allow at any rate some degree of sliding to take place. In this way the effect of the permanent load and of the settlement of the supports could be taken up by mutual adjustment of the two portions of the bridge before the hinges were built in.

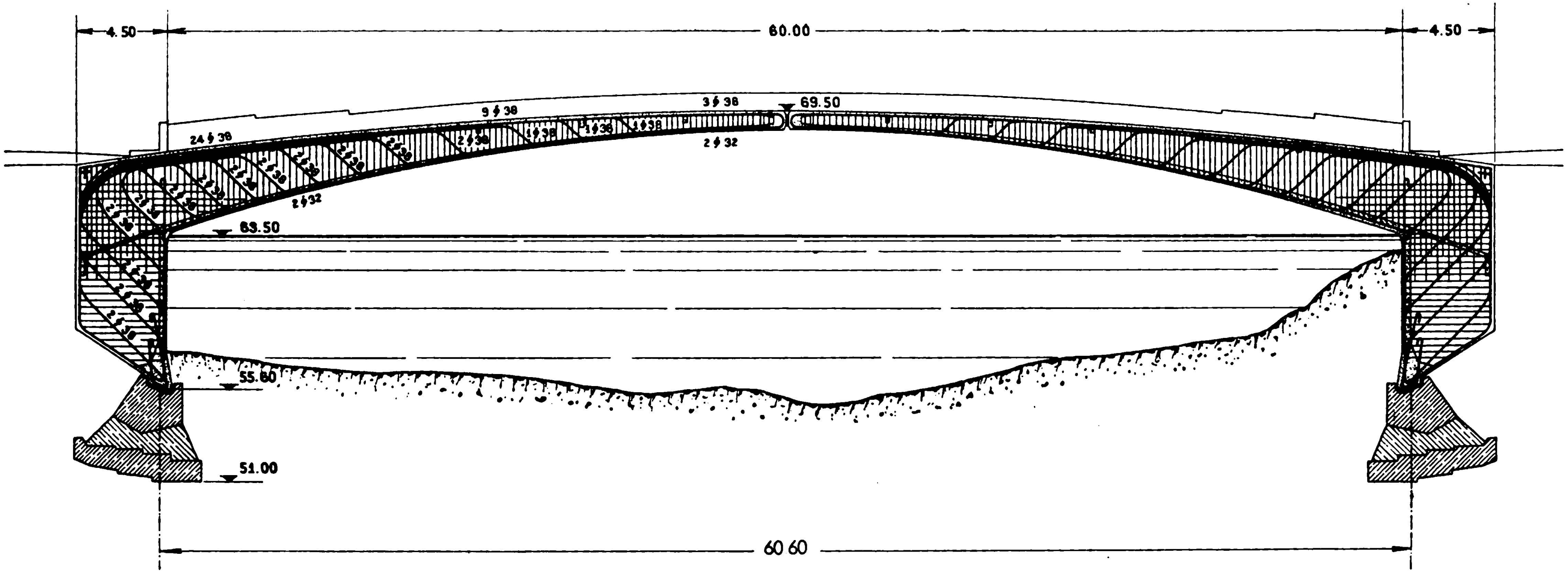

Fig. 4.

Details of reinforcement.

The jacks adopted (which were constructed in the Monteverde works of "Ferro-beton") have the advantage that they can be blocked in any position by the use of set screws, so that when the reinforcement is completed a constant control can be exercised over the moments. In the case of the bridge here described there was no necessity to measure the lateral pressure as the design was a three-hinged one and any such lateral thrust would become independent of the movement once the bridge has been separated from its centreing; such, at any rate, is theoretically the case where small displacements are concerned, assuming ideal hinges at the springing and free supports.

The concreting of the hinges had to be undertaken with great care after the scaffolding had been struck since in this case — by contrast with a built-in

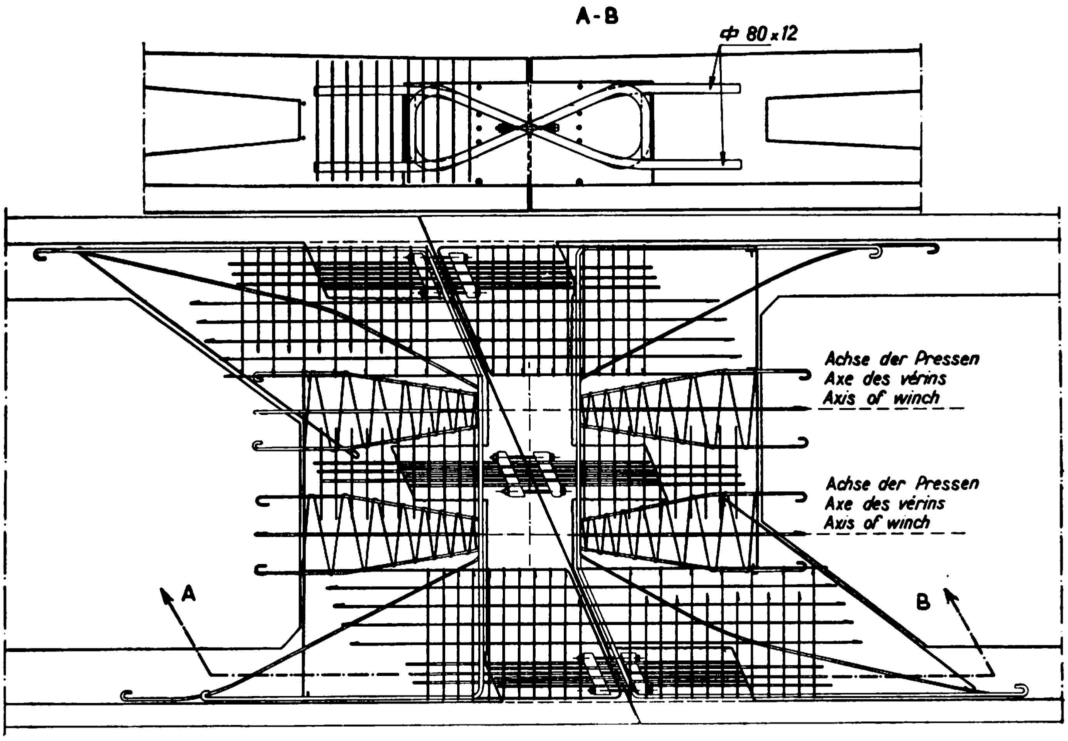

Fig. 5.

Crown hinge.

arch — the hinges must be allowed the necessary freedom of movement. Fig. 5 shows the construction of the crossed hinge members from flat steel bars 80 × 12 mm, arranged in three groups each containing six such bars; after concreting the hinge each bar remained fixed on either side. Finally the two cavities at the jacks were concreted (see Figs. 5 and 6). These lightly stressed portions of the cross section either co-operate, or else the hinges which are only partially under compression enter into full operation.

By suitable calculation the positions for the jacks can be decided in such a way as to avoid any fixing effect, or in other words so as to ensure that the jacks are properly centred.

The loading test of the bridge was carried out with the live load of 650 kg/cm^2 in the most unfavourable position, with the load covering nearly the whole of the span, and the amount of elastic deflection at the crown was found to measure 6 mm. Dynamical measurements (which were not carried out in great detail and are not, therefore, worthy of description here) indicate that the structure attains some degree of stiffness through the concreting of the crown sections close to the hinges.

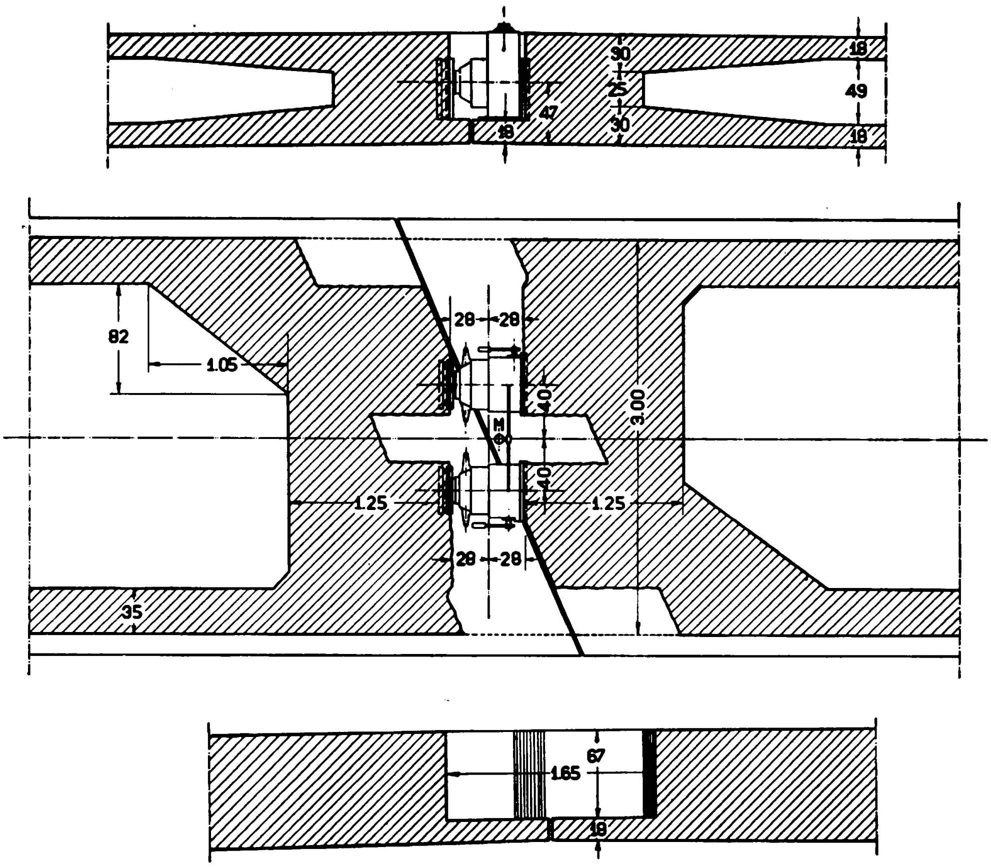

Fig. 6.

Jacks at crown.

It would have been desirable to erect the piers in masonry seatings in order to allow their free movement, but this precaution was excluded by considerations of cost. The total cost of the work amounted to 200000 lire.

From test results on the finished structure it may be concluded that the system of construction adopted was the best one for its purpose of covering a large span with a bridge of good appearance, carried on small foundations, at limited cost.

IVb 4

Bridges in the New Maritime Station at Naples.

Brücken im neuen Hafenbahnhof in Neapel.

Les ponts dans la nouvelle gare maritime de Naples.

G. Krall,

Professor der Universitäten Rom und Neapel, Rom.

In the new maritime station at Naples, shown in Figs. 1 and 2, which was constructed by the Società Anonima Italiana Ferro-Beton, the chief point of interest is the two bridges which connect the two portions of the construction at the landward and seaward ends. The building is a two storey one and its

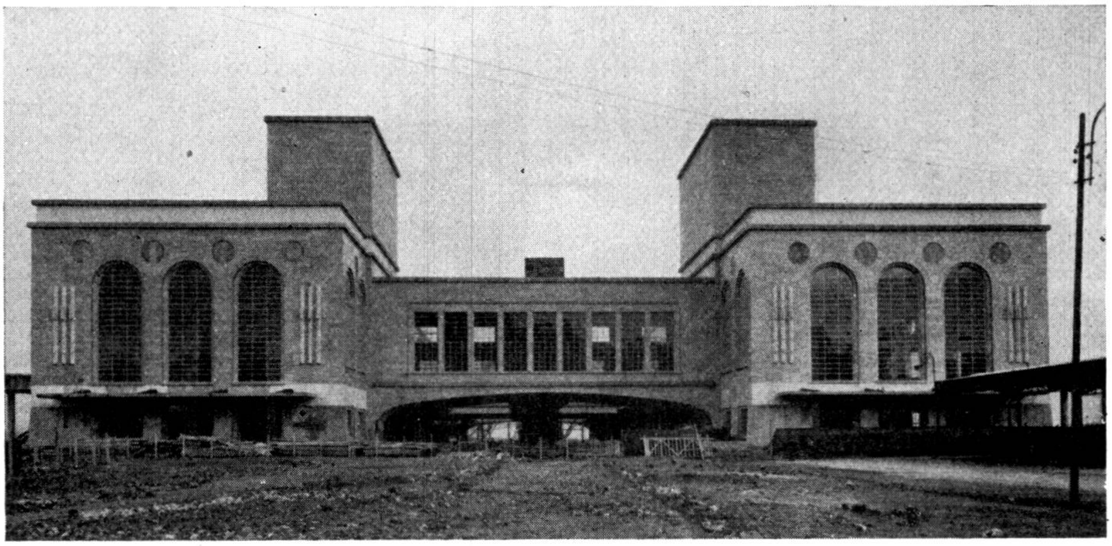

Fig. 1.

supports are the two side walls. One of these walls forms a closed frame in which the hangers are sufficiently slender to allow of its bending resistance being neglected; the other is a double Vierendeel girder as shown in Fig. 4. The two storeys are respectively at 10.15 and 19.10 m above sea level, and span across the railway tracks which are situated at R.L. 3.0 with a clear span of 35 m. The floor at R.L. 10.15 is of coffered construction, the coffers being rectangular with their sides 3.80 and 3.30 m long. The longitudinal ribs are built into the two transverse box-girders at the ends which are thus made particularly resistant to torsion. The box cross section of the end cross girders is enclosed between the two outermost cross ribs and the corresponding portions of the slab, and the

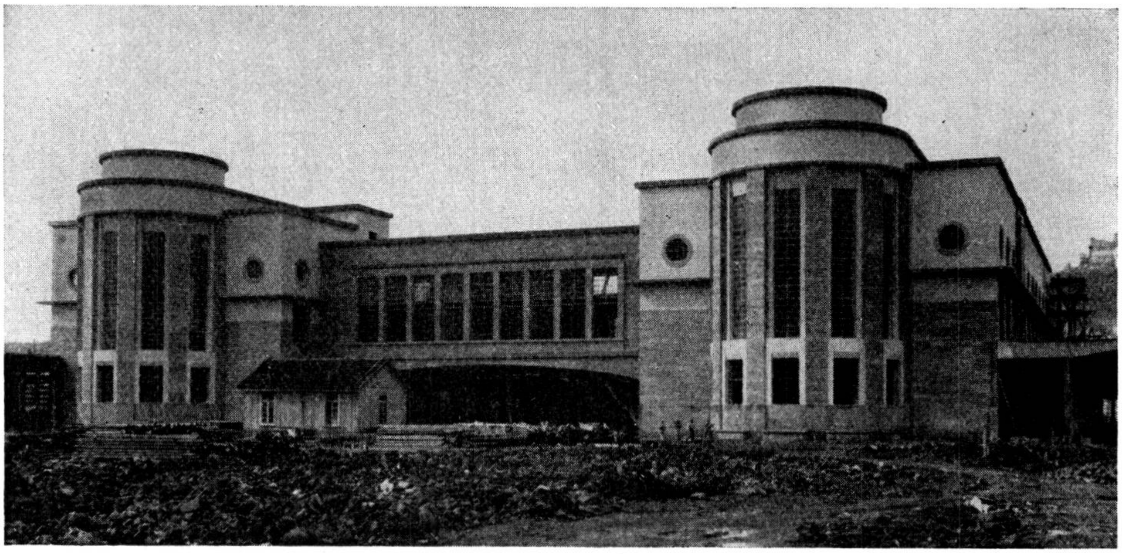

Fig. 2.

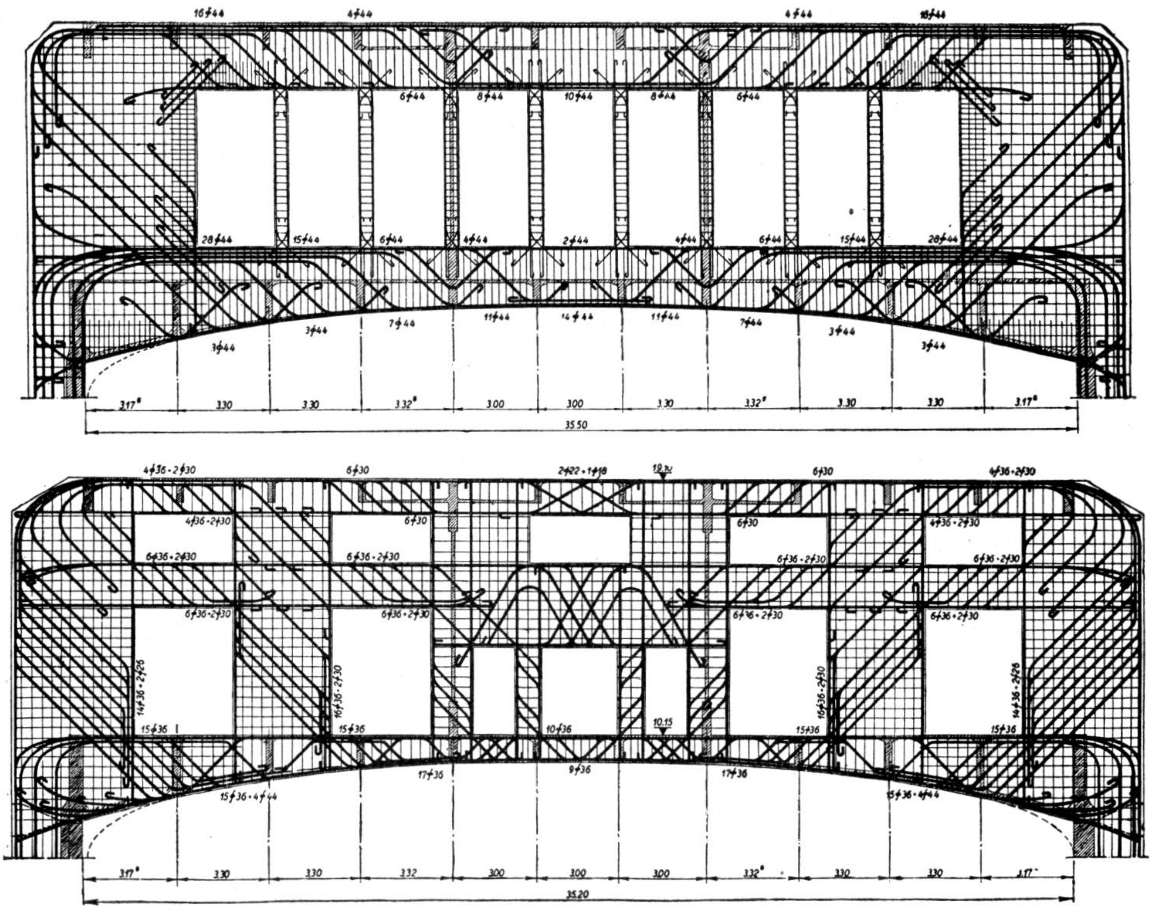

Fig. 3 and 4.

remaining cross ribs may be regarded as suspended from the two main supporting walls. The cross ribs along the axes of the cross walls, which run through the whole height of the storey, may therefore be regarded as infinitely stiff by comparison with the remainder.

IVb 5

The Saalach Bridge on the German Alpine Road.

Die Saalachbrücke an der Deutschen Alpenstraße.

Le pont de Saalach de la Route allemande des Alpes.

Dr. Ing. H. Olsen,
München.

The discussions on solid walled reinforced concrete structures offer an occasion to describe a remarkable bridge on the German Alpine road, which illustrates the possibility of applying the available knowledge to skew arches also.

Fig. 1 shows the general layout of the Saalach bridge recently completed near Bad Reichenhall, consisting of three completely built-in reinforced concrete arches. The respective spans are of 23.8, 24.7 and 28.2 m with a thickness of the arch barrel of 0.60 m at the crown and 1.0 m at the springings. The two intermediate piers are 3.5 m wide and rest upon strong reinforced concrete foundations which were built inside sheet pile cofferdams.

All three arches are skew, the piers and abutments being parallel to the direction of the stream and making an angle of 60^0 with the axis of the road. The latter, which is 9.0 m wide, is enclosed between massive side walls and has a gradient of 5 % in the direction of the length of the bridge, in addition to which, since the bridge is situated at the end of a curve, the road surface has a super elevation of 3 %. The footway 0.80 m wide on the upstream side, and the kerb 0.25 m on the downstream side, are lined with granite. The parapets are 0.60 m high by 0.45 m wide and are covered over with coping stone slabs. The external faces of the bridge are lined with masonry.

Statical features.

As is well known, the statical examination of skew arches has hitherto been conducted from several radically different points of view. One method of calculation commonly adopted for such arches is to assume them to be divided up into a number of independent arched strips spanning between the abutments in a direction parallel to the side walls, but this assumption is unsatisfactory because the loads on the arch are in fact carried to the abutment along the direction of the skew span. Moreover, Navier's law of stress distribution holds good only for cross sections normal to the gravitation axis of the structure.

In the present work the statical investigation was made by considering the two side walls each 1.5 m thick as supporting the masonry lining over a thickness of 1.0 m and a radial height of 1.9 m and enclosing the sides to the arch, the

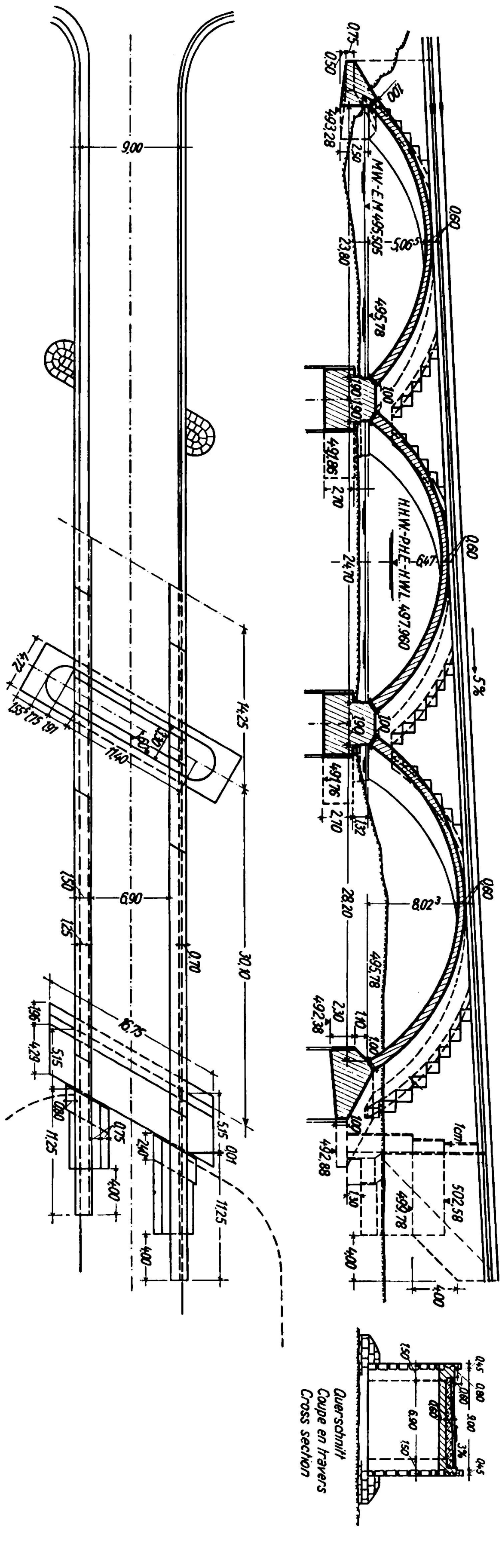

Fig. 1. Saalach Bridge, structural features.

upper parts of the construction being stepped to follow the coursing of the masonry. In this way a boxed section is constituted wherein the play of forces is three-dimensional, the arch acting as a circular cylindrical shell between the two lateral walls. In this way the combined loads acting on the arch barrel are transferred for the most part to these two facing walls. This action is due to the fact that the elements of beams, which exist in the direction of the generatrices of the shell, wedge against one another when loaded, so that if the arch has a sufficient curvature the condition of stress in these elements is almost free from bending effects.[1]

Over a certain width, known as the cooperating width, the shell participates in the deformation undergone by the above mentioned arched side walls. According to *Finsterwalder*[2] and *Craemer*[1] this width depends, among other factors, on the degree of fixation. It was determined at the crown as 1.2 m and at the springings as 1.8 m measured at right angles to the inner face of the aforementioned arched facing walls.

The loading which appertains to each such facing wall includes apart from its dead weight the loads transferred to it from the shell. These last were determined and include the live load according to the Class 1 of the German bridge standard, enabling the walls to be

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[1] *Craemer:* Zusammenwirken von Scheibe und Schale bei Bogenscheibenbrücken. Der Bauingenieur, 1936, p. 99.

[2] *Finsterwalder:* Die querversteiften, zylindrischen Schalengewölbe. Ing.-Archiv, 1933, p, 43.

designed at once in the usual way as built-in arches taking account of temperature and shrinkage stresses, and hence enabling the thrust and moments at various cross sections to be deduced. The three statical unknowns could then be

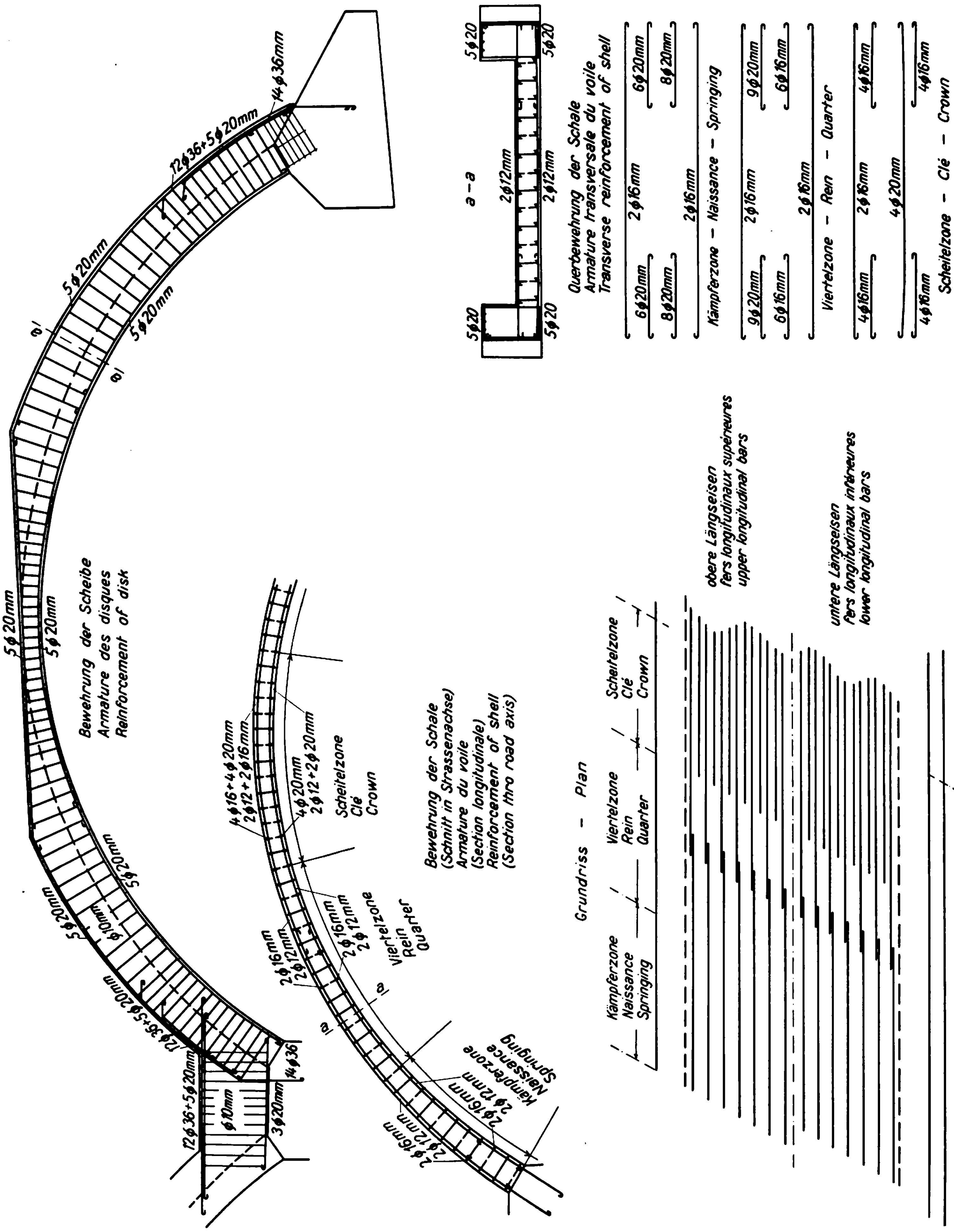

Fig. 2. Reinforcement of the arch.

solved from the deformations of the arched wall, displacements and rotations resulting from all loads and temperature changes being brought directly into the calculations. In the same way the effect of live loads was considered, on the assumption, firstly, that such loading covered one half of the span, and secondly that it covered the whole arch. The most unfavourable stresses, determined from the moments at the nuclear points and referred to the cross section at the crown of the largest of the arches made up of the "walls" and cooperating portions of the shell, were 42.6 kg/cm^2 (compression) and -16.4 kg/cm^2 (tension). The corresponding stresses at the quarter points were 4.2 and -8.1 kg/cm^2, and at the springings 52.6 and -55.4 kg/cm^2.

The reinforcement required may be seen from Fig. 2. Assuming a permissible stress of 1200 kg/cm^2 in the steel, it amounts to five round bars of 20 mm diameter in the crown and to twelve round bars of 36 mm diameter plus five

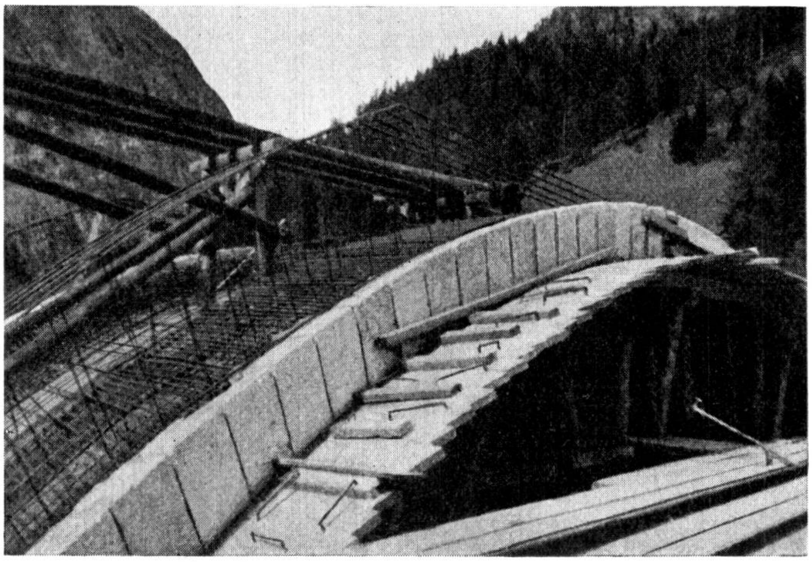

Fig. 3.

Reinforcement of the side walls.

bars of 20 mm diameter at the springings. The reinforcement at the crown was inserted on top and soffit and made continuous to the springings. The heavy reinforcements and the heavy stresses existing in the concrete under full load show that the walls, as explained above, do the greater part of the work. Fig. 3 shows the completed reinforcements in the facing walls.

In the shell (arch barrel), on the contrary, the reinforcement is only lightly stressed. At the middle it has to transmit normal forces so long until equilibrium is attained between the arched facing walls and the barrel arch; the magnitude of this depends, among other factors, from the load on the arch and therefore varies from one section to another. Shear stresses are produced which give rise to normal stresses in the direction of the generatrices of the shell. Differences in the respective stresses where the shell is bonded into the "walls" necessitate transverse reinforcement at the crown, quarter points and springings, as shown in Fig. 2, but in the middle portion of the arch barrel which is free from bending effects the light reinforcement there indicated is sufficient. Fig. 4 shows the completed reinforcement of the shell.

The transfer of the arch loads to the abutments and intermediate piers is effected mainly by the "facing walls" so that the skewness of the arch is of little statical significance. The maximum pressure on the foundation amounts to 4.9 kg/cm^2 on the right abutment and 4.4 kg/cm^2 on the piers. The left abutment is founded on rock and exerts a maximum pressure of 6.9 kg/cm^2.

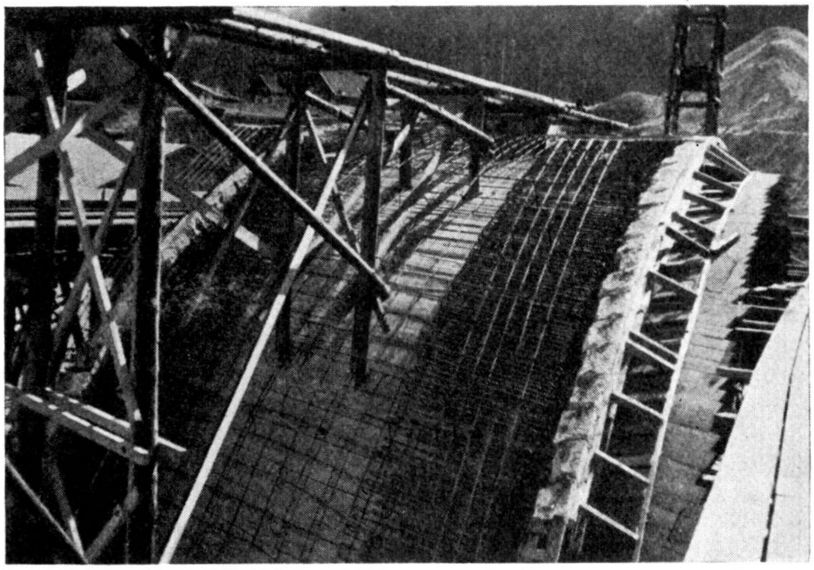

Fig. 4.
Reinforcement of the shell.

Construction.

The concreting mixing plant was placed on the right bank. The concrete was made with an admixture of 250 kg of trass Portland cement per m^3 in the

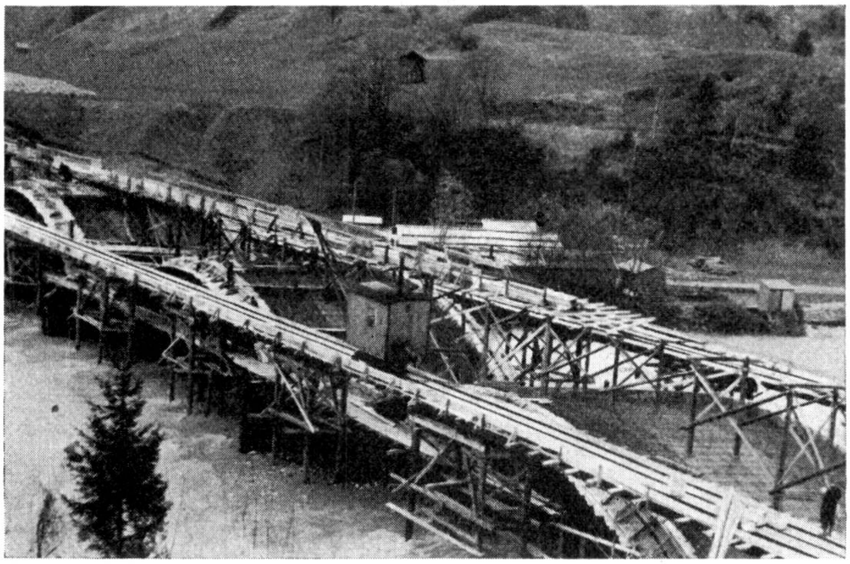

Fig. 5.
General view of work on site.

case of the foundations and abutments, and 300 kg/m^3 cement for the arch barrel and facing walls. The construction of the cofferdams for the foundations of the piers and right abutment was begun in October 1935, and when excavation had been completed the concreting of the foundations and abutments was undertaken.

Once the piles to carry the falsework had been driven the latter was erected simultaneously in all three openings, consisting of eight frames with a system of bracing. Screw jacks were provided to allow of lowering the falsework without shock. A service bridge was constructed, both upstream and downstream of the falsework, with rails for the travelling cranes used in placing the masonry lining. Fig. 5 is a general view of the site.

The concreting of the arches, by chuting through flexible pipes from a high level scaffold, was carried out in strips in such a way that those portions of the

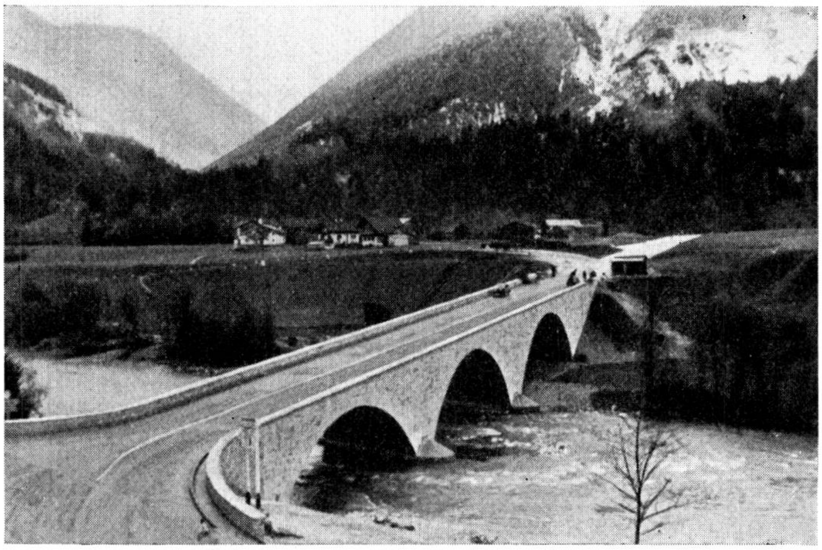

Fig. 6.

View of the finished bridge.

arch which had already hardened would not be stressed by movements of the falsework. Cube tests at 28 days gave compressive strengths of approximately 250 kg/cm². After allowing a period of four to six weeks for the concrete in the arches to harden the scaffolding was dropped from below all three arches, the maximum amount of sinking which then occurred at the crown being 1.4 mm. After removing the shuttering from the barrel arches the masonry lining of the end walls was carried up simultaneously with the concrete. The provision of continuous expansion joints was not necessary.

Fig. 6 shows the finished bridge. The 120 m length of the structure makes it visible from afar, and it blends pleasingly in to the surrounding mountain landscape.

The design of the bridge was entrusted by the Bavarian Staatsbauverwaltung to the author who also had to deal with all statical and constructional details and was responsible for supervising of the work. The plans for issue to the contractors, and the statical calculations, were carried out by *Dr. Craemer*, to whom is due also the utilisation of the "facing walls" as part of the carrying system.

IVb 6

Note on the Paper by Boussiron.[1]

Bemerkung zum Referat Boussiron.[1]

Note concernant le rapport Boussiron.[1]

H. Lossier,
Ingénieur Conseil, Argenteuil.

Variation in moments of inertia.

The principle of varying the moments of inertia of different elements in a hyperstatic structure, so as to satisfy certain technical or economic conditions, is one which has undoubted advantages. It confers greater flexibility than the use of hinges and as a rule is simpler than the expedient of bringing initial stresses into play by artificial means.

In the general case of a structure covering several spans, wherein all the elements are monolithic, the distribution of the stresses depends severally on the characteristics of the arches, the piers, the abutments and the ground bed. By ringing the changes in these characteristics it is possible to modify the action of the hyperstatic system in question even without altering the arrangements of the spans, thus obtaining several solutions which give equal strength, but differ in appearance, deformations and cost. For instance, if the piers are relatively very rigid the section of the arches may be reduced to a minimum, because, in the limit, these tend to act as independent elements built into fixed supports. On the other hand if the arches are made stiffer it is permissible to adopt more slender piers, and in the limit the conditions of stress imposed upon the latter tend to approximate to those which would exist in posts carrying and rigidly embedded into a beam.

In two instances of structures with very high piers the author has been able in this way to realise advantages amounting to between 15 and 23 %, by comparison with the arbitrary method which consists of designing the piers as if they had to resist the difference in thrust between the adjacent spans, while the piers are assumed to be rigidly fixed. The comparative calculations for different sections can very rapidly be carried out, especially by using the rigorous graphical method already published by the author in the Bulletin Technique de la Suisse Romande in 1903 in an article entitled «Théorie Générale de l'arc élastique continu sur appuis rigides». As well as in Le Génie Civil, 1908, under the title «Calcul des ponts en maçonnerie». He has made use of similar principles in

[1] See Preliminary Publication of the Berlin Congress 1936, page 729.

his article entitled «Le réglage du fonctionnement des poutres continues» in Le Génie Civil, 1935.

The study by *M. Boussiron* is confined to the special case of a simple arch resting on supports which are assumed to be indeformable. It is based on the principle of greatly reducing certain of the sections, and it differs from the solution using semi-hinges in hoop-reinforced concrete, which was devised for the same purpose by Considère, in the avoidance of any sudden interruption of continuity. It has not been proved that an equalisation of the maximum bending moments at the springings and at $^1/_4$ span is the optimum condition from an economic standpoint, but the method of *M. Boussiron,* it cannot be denied, certainly enables a considerable reduction in the volume of reinforced concrete to be effected by comparison with other methods. Generally speaking, as stated by the present writer at the Liége Congress, the action of a reinforced concrete structure is far from being invariable in character. It changes in the course of time under the effect of diverse and complicated causes which are not yet sufficiently understood, and which operate especially on the linear dimensions, the elasticity and the plasticity of the concrete.

The author has observed, in the case of built-in arches of the ordinary type, that under the same external loading the strains grow less at the springings and greater at the crown and of $^1/_3$ span with the passage of time, the differences being sometimes of the order of 20 % after a period of some ten years. Moreover, if calculations for hyperstatic structures are based only on considerations of elastic strains their results can be true only relatively and at a particular instant of time. The degree of uncertainty which attends them, should, therefore, logically lead to the adoption of a margin of safety greater in hyperstatic structures than in those types of structures which do not depend for their action on the amount of deformation. Again, the several experiments on small scale models which the writer carried out a few years ago went to show that, as a general rule, arches containing semi-hinges offer a smaller margin of safety against frequently repeated stresses than is the case with elements of practically uniform sections throughout. From this point of view it would appear, *a priori*, that arches of the type considered by *M. Boussiron* must occupy an intermediate place between semi-hinged and ordinary fixed arches. But the question is practically of academic interest only, for in long span bridges fatigue, properly so-called, scarcely enters into play.

As regards structures of exceptional size such as are anticipated in the future, the expedient of regulating the action of the arches would appear a priori, to offer fewer advantages in these than in bridges of smaller span, on account of the reduced ratio of live to dead load in the former. Nevertheless the suggestions made by *M. Boussiron* are so original in conception and so thoroughly worked out that their interest is a very real one, and they serve to illustrate the remarkable applications which he has made.

Limiting spans for road bridges.

If consideration is given only to the limits imposed by the mechanical strength of the material used, the approximate spans which admit of being obtained with the concretes now available, using the most perfect methods of manufacture and placing in the job, are approximately as follows:

1400 m for arch bridges, with rise-span ratio 1 : 5,
500 m for continuous girder bridges.

From the study entitled «L'avenir du béton armé et du métal pour les ponts de très grande portée», presented by the author before the Societé des Ingénieurs Civils de France and also in London in 1934, it would appear, from the economic point of view, that the cost of such works tends to increase very rapidly when something like the following spans are reached:

400 m for arches in lightly reinforced concrete,
800 m for arches in heavily reinforced concrete,
1000 m for steel arches,
1500 m for suspension bridges.

From a comparative point of view the economic advantage would appear to favour:

Lightly reinforced concrete arches for spans up to 250 m.
Heavily reinforced concrete arches up to 700 m.
Suspension bridges from 700 m upwards.

It need hardly be stated that these figures are based on hypotheses which may differ very appreciably from the truth in each particular case, and for this reason their value is entirely academic.

The principal factor which places reinforced concrete at an economic disadvantage by comparison with suspension bridges once a certain span is exceeded is its low "coefficient of utilisation", meaning the ratio of strength to the weight per volume unit. In order to increase this ratio it would be necessary to increase the former of these terms or to diminish the second. The author's researches now in hand appear to indicate that one method of simultaneously realising both these conditions would be to make use of concrete that has been mixed from light aggregates and is pre-compressed in a transverse direction by means of suitable binding. Approximately one half of the weight of a reinforced concrete member is represented by the gravel, so that by substituting natural or artificial materials of light weight the density of the concrete may be reduced considerably. These special aggregates, however, usually have a lower strength than the customary kinds of gravel, and it is essential to make use of them in elements which are subject to compression in all directions. Members made in this way are subject to larger deformations than those formed from ordinary aggregates, but this fact is not a disadvantage in very large structures where the dead load represents the largest part of the total.

In a later communication the author proposes to publish the results of French experiments now in hand regarding the use of cements which neither contract nor expand and which would enable reinforcing bars to be pre-tensioned *in all directions* on the job without the use of any special apparatus, thus subjecting the concrete to compression or integral interlocking. (This is dealt with in his article of 29th February 1936 in Le Génie Civil.)

IVb 7

Long Span Reinforced Concrete Arch Bridges.

Weitgespannte Eisenbeton-Bogenbrücken.

Ponts en arc de béton armé à grande portée.

Dr. Ing. A. Hawranek,
ord. Professor an der Deutschen Technischen Hochschule, Brünn.

The author's paper on "Long Span Reinforced Concrete Arch Bridges" had reference firstly to new and more exact calculations making use of different moduli of elasticity at different points in the arch; secondly to the theory of deformations arising under variable values of E and I, and thirdly to the case of an arch in which the line of thrust is prescribed and of which only the final results are stated. Apart from this, suggestions relating to materials were considered, followed by others as to the construction of large arches, the necessary falsework and the constructional details, which may promote the development of large reinforced concrete bridges. Finally a proposal was made for a new method of constructing large arched bridges. In the contribution to the discussion which follows below some amplifications of this last question will be put forward; also numerical calculations to show the application of the theoretical treatment, and misprints in the Preliminary Report will be pointed out.

I. Theoretical investigation. Results of numerical calculations made in accordance with the theory explained in the preliminary report.

1) Application of varying moduli of elasticity to a hollow cross section. Elastic theory.

The treatment of varying moduli of elasticity in arch bridges of hollow cross section (Point 3, page 789 in the Preliminary Report) can be stated in a more general way than was done in the Preliminary Report. A difference was assumed between the modulus of elasticity E_1 and E_2 for the concrete in the lower and upper slabs of the hollow cross section respectively, but these moduli were assumed to be constant through the whole length of the arch. It is possible, however, to assume varying values of E_1 and E_2 at different cross sections of the arch, somewhat in accordance with the progress of the constructional work which causes the concrete to be of differing age in each section. This closer analysis is also called for by the higher moments that arise in the process of striking the centreing with the aid of hydraulic jacks. (See *Hawranek,* Schweizerische Bauzeitung, 1937, Vol. 110, p. 153.)

Using the symbols indicated in Fig. 1 the moments, normal and shear forces are given by:

$$M_x = \mathfrak{M}_x + M - Hy - Vx \qquad \text{and} \qquad k_1 = \frac{E_1}{E_2} + 1 \qquad k = \frac{k_2}{k_1}$$

$$N_x = H \cos\varphi + Q_x \sin\varphi$$

$$Q_x = V + \mathfrak{Q}_x = V - \sum_x^{l/2} G \qquad k_2 = \frac{E_1}{E_2} - 1$$

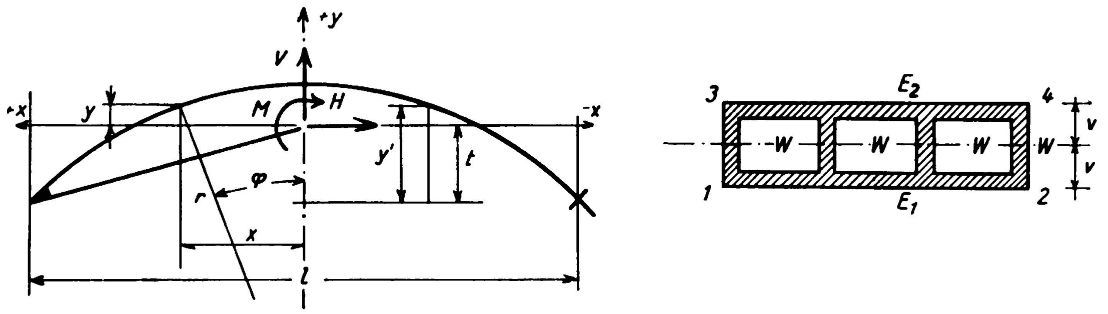

Fig. 1.

Hence we can obtain the three statically indeterminate values H, V, M and the distance t to the elastic centre of gravity:

$$H = \frac{\int \frac{\mathfrak{M}_x y k_1 ds}{E_1 J} - \int \frac{\mathfrak{M}_x k_2 v \cdot ds \cdot \cos\varphi}{E_1 J} + \int \frac{\mathfrak{Q}_x y k_2 ds \sin\varphi}{v E_1 F} - \int \frac{\mathfrak{Q}_x k_1 ds \sin\varphi \cdot \cos\varphi}{E_1 F} + 2\varepsilon t l}{\int \frac{y^2 k_1 ds}{E_1 J} - \int \frac{y k_2 v ds \cos\varphi}{E_1 J} - \int \frac{y k_2 ds \cos\varphi}{v E_1 F} + \int \frac{k_1 \cos^2\varphi \, ds}{E_1 F}}$$

$$V = \frac{\int \frac{\mathfrak{M}_x x k_1 ds}{E_1 J} - \int \frac{\mathfrak{M}_x k_2 v \cdot ds \cdot \sin\varphi}{E_1 J} + \int \frac{\mathfrak{Q}_x x k_2 ds \cdot \sin\varphi}{v E_1 F} - \int \frac{\mathfrak{Q}_x k_1 ds \cdot \sin^2\varphi}{E_1 F}}{\int \frac{x^2 k_1 ds}{E_1 J} + \int \frac{k_1 ds \cdot \sin^2\varphi}{E_1 F}}$$

$$M = \frac{\int \frac{\mathfrak{M}_x k_1 ds}{E_1 J} + \int \frac{\mathfrak{Q}_x k_2 ds \sin\varphi}{v E_1 F}}{\int \frac{k_1 ds}{E_1 J}} + H \frac{\int \frac{y k_1 ds}{E_1 J} - \int \frac{k_2 ds \cos\varphi}{v E_1 J}}{\int \frac{k_1 ds}{E_1 J}}$$

$$t = \frac{\int \frac{y' k_1 ds}{E_1 J} - \int \frac{k_2 v ds \cdot \cos\varphi}{E_1 J}}{\int \frac{k_1 ds}{E_1 J}}.$$

According to this general theory an arch with $l = 400$ m and $f = 100$ m was worked out numerically (Preliminary Report, Fig. 5, p. 802), taking the moduli of elasticity in the soffit barrel as being $E_1 = 470\,000$ kg/cm² at the springing and 350 000 kg/cm² at the crown, and the corresponding figures for the upper barrel as $E_2 = 350\,000$ kg/cm² in the springing and 230 000 kg/cm² at the crown, with a gradual transition in between (elastic theory). These variations in value for the elasticity would correspond to the necessary period of time taken in constructing so large an arch. With loading over *one half of the arch* on the right hand side, equivalent to p = 1 tonne per metre run, the statically indeterminate values worked out at

H = 102.11 tonnes (as against 101.304 tonnes when E = constant; Δ = + 1.0 %).
V = 35.544 tonnes (as against 38.597 tonnes when E = constant; Δ = — 7.9 %).
M = 3473.593 tonnes · m (as against 3707.348 tonnes · m when E = constant; Δ = — 6.3 %).

The *moments* at specified points are:

At the left hand springing + 3180.150 tonne · m (as against + 2467.348 tonne · m when E = constant; Δ = + 28.9 %).

At the crown + 77.791 tonne · m (as against + 67.348 tonne · m when E = constant; Δ = + 15.3 %).

At the right hand springing — 2603.314 tonne · m (as against — 2092.652 tonne · m when E = const.; Δ = + 24.4 %).

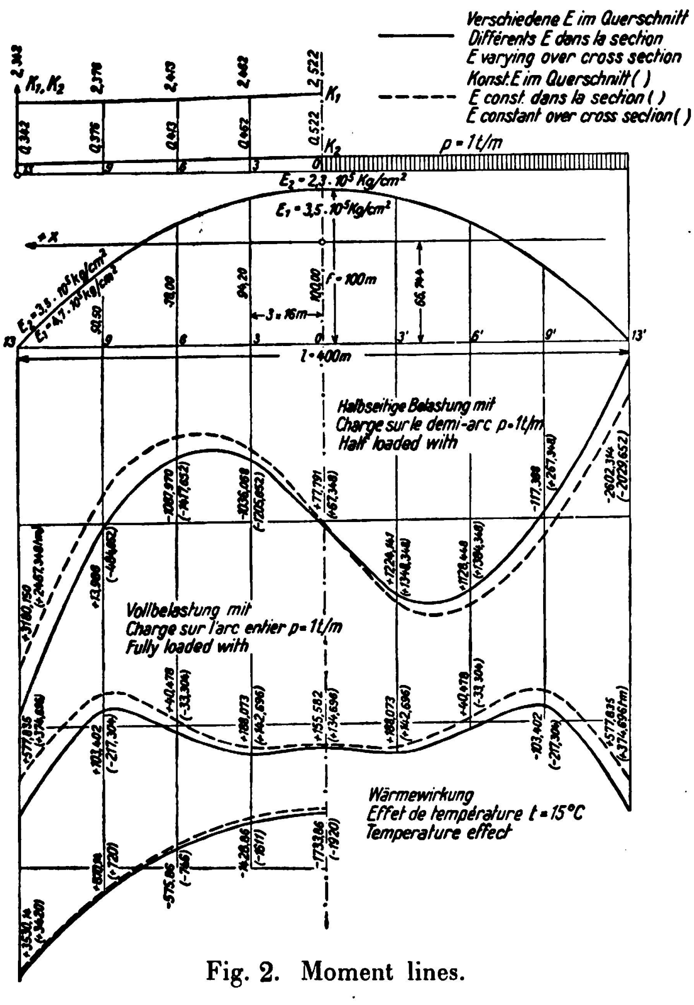

Fig. 2. Moment lines.

With loading at 1 tonne per metre run over the *whole span*, the moment at the springing becomes + 577.835 tonne · m (as against 374.696 tonne · m with constant E); Δ = 54 %. The relevant moment curves are reproduced in Fig. 2, those for a varying E being shown as full lines and those for a constant E as dotted lines. The varying values of k_1 and k_2 are also given in the figure. The moment due to thermal effect with t = 15° C have been determined and are shown in tonne-metres; as regards these the differences are smaller.

It will be seen that the differences between the moments in the springings and at the crown respectively are considerable, and are always greater with E varying than with E constant, but as regards the thermal effect they are greater only in the neighbourhood of the springing.

2) *Deflection theory for the Built-in Arch with its axis along line of thrust.*

In this case also the theory has been further developed, and the origin of co-ordinates 0 has been displaced to the centre between springings. The loading curve g_x is assumed to vary in accordance with cosh αx corresponding to the distribution of weight in an actual arch, and account has also been taken of thermal effect and of shrinkage. In the theoretical development the value d′ corresponding to the normal forces has been assumed more accurately than in the Preliminary Report.

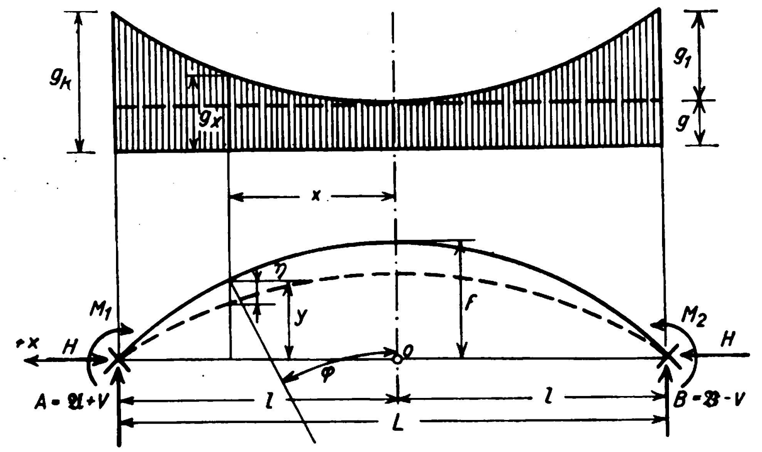

Fig. 3.

Equation for axis of arch:

$$y = f(1 + v) - f v \cosh \alpha x$$

$$v = \frac{1}{m-1}; \quad m = \frac{g_k}{g} = \cosh k$$

$$k = \text{arc} \cosh m; \quad \alpha = \frac{k}{l}$$

Specific loading:

$$g_x = g \cosh \alpha x; \quad g_1 = g_k - g$$

Differential equation for line of flexure:

$$\eta'' = -\frac{M_x}{EJ' \cos\varphi} + \frac{N_x}{EF'} \cdot \frac{d^2y}{dx^2}(1 + \cos^2\varphi) + \frac{1}{EF'} \cdot \frac{dN_x}{dx} \cdot \frac{dy}{dx} \mp \varepsilon t \cdot \frac{d^2y}{dx^2}$$

with $c^2 = \dfrac{H}{EJ' \cos\varphi}$

$$\eta'' = -c^2 \left\{\frac{\mathfrak{M}_x}{H} - y\left(1 \mp \frac{\varepsilon t \alpha^2}{c^2}\right) + \frac{V_1}{H}(l - x) + \frac{M_1}{H}\right\} - c^2\eta - c^2\left[\frac{d' \cdot J}{F} + \frac{\varepsilon t f \alpha^2 (1 + v)}{c^2}\right]$$

so that $d' = f v \alpha^2 \cosh \alpha x \left[(1 + \cos^2\varphi) + \left(1 - \frac{\dot{y}^2}{2} + \frac{3}{8}\dot{y}^4\right) f^2 v^2 \alpha^2 \sinh^2 \alpha x \cos\varphi\right]$

$$\eta'' + c^2\eta + c^2 F(x) = 0$$

$$\mathfrak{M}_x = \frac{g_1 v}{\alpha^2}(\cosh k - \cosh \alpha x)$$

Solution of the differential equation:

$$\eta = A \sin cx + B \cos cx - \left\{\frac{\mathfrak{M}_x}{H} - f \cdot w\,[1 - v(\cosh \alpha x - 1)] + \frac{V_1}{H}(l - x) + \frac{M_1}{H} + \frac{d'J}{F} \mp \frac{\varepsilon t f \alpha^2 (1 + v)}{c^2}\right\} + \left(-\frac{g_1 v}{c^2 H} + \frac{f v w \alpha^2}{c^2} - R\right) \cosh \alpha x$$

where $w = 1 \mp \dfrac{\varepsilon t \alpha^2}{c^2}$,

$$R = -\frac{\alpha^3 v}{c^2(\alpha^2 + c^2)}\left(\frac{g_1}{\alpha H} - f w \alpha\right)$$

Under a loading g_x we obtain:

$$M_x = H\left[\frac{z}{\alpha c}\,\frac{\sinh k}{\sin cl} \cos cx - \frac{d'J}{F} + \left(f v - \frac{g_1 v}{\alpha^2 H} + \frac{z}{\alpha^2}\right) \cosh \alpha x\right]$$

$$M_1 = H\left[\frac{z}{\alpha c} \sinh k \,\text{cotg}\, cl - \frac{d'J}{F} - \frac{1}{\alpha}\left(\frac{g_1 v}{H} - z\right) \cosh k\right]$$

The horizontal thrust H may be calculated by equating the sum of the horizontal compressions in the elements of the arch to zero, with $\triangle l = 0$; i. e. from the following equation:

$$0 = -\left(\frac{H}{EF' \cos\varphi} \mp \varepsilon t\right) l - \frac{f v}{2}\left[z + \left(\frac{H}{EF' \cos\varphi} \mp \varepsilon t\right) \cdot f v \alpha^2\right] \cdot \left(-l + \frac{1}{\alpha} \sinh k \cosh k\right)$$
$$- \frac{f v \alpha c B}{c^2 + \alpha^2}(c \sinh k \cos cl - \alpha \cosh k \cdot \sin cl).$$

The moments and the deflection were determined according to this theory for the same arch of 400 m span due to the *dead load* of the bridge itself. The curve of loading is shown in Fig. 4. It indicates 189.5 tonne/m at the springing and 99.0 tonne/m at the crown.

The moment in the springing amounts to — 11041.553 tonne · m (as against — 7080.942 tonne · m by the elastic theory; $\Delta = 56.0\,\%$) and the moment at the crown works out at + 4357.149 tonne · m (as against 3889.062 tonne · m by the elastic theory; $\Delta = 12.1\,\%$).

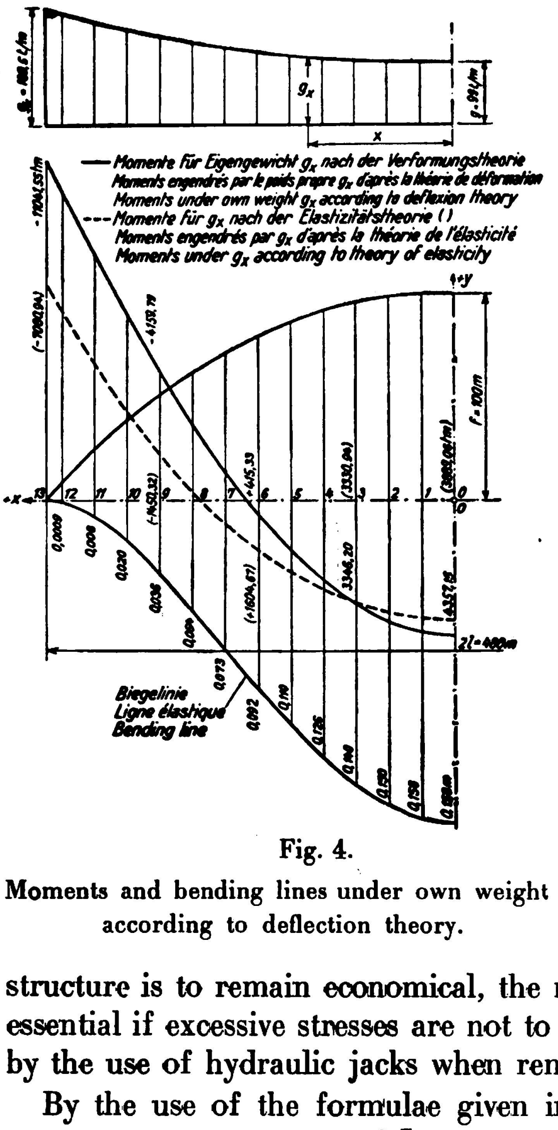

Fig. 4.

Moments and bending lines under own weight according to deflection theory.

Here again considerably greater differences in the moments are obtained than those between the results of the deflection theory and the elastic theory when respectively applied to an arch with a parabolic axis, under permanent full loading. In the crown and in the springing they are larger and only in fields 3 and 6 of the span are they smaller, these fields being each one eighth of the span.

The deflection of the arch was likewise determined under the dead load of the bridge itself; and it amounted to 158 mm at the crown, which corresponds to $^{1}/_{2530}\,$l. The deflection line is incorporated in Fig. 4.

These results also show the necessity for more exact investigation in the case of an arch with its axis along the line of thrust, since a parabolic axis does not correspond with the true conditions, especially if the rise be large.

The fixing moments at the springing of a built-in arch are always the heaviest, and, since for large spans the permissible stresses require to be increased if the structure is to remain economical, the more exact determination of the moment is essential if excessive stresses are not to occur in practice. It is also made necessary by the use of hydraulic jacks when removing the falsework.

By the use of the formulae given in the Preliminary Report which made use of a fixed value of $d = \frac{2\,J}{r\,F_m}$, it is also possible to take account of the *varying* value of d, and this is discussed elsewhere. Numerical calculations indicate that the true value is larger and that the influence on the moments is not negligible. (*Hawranek,* Der Bauingenieur 1937, p. 719.)

As regards the more exact method of calculation for arch bridges (whether in steel or in reinforced concrete) by the deflection theory, it may be stated in general that nothing but numerical solutions for different forms of arch will correctly indicate the importance of the influence exerted by the various assumptions or the various terms in the formulae. It is necessary, therefore, that such calculations should be carried out and made available, enabling comparisons between the results of different methods of calculations. If measurements made on completed bridges are available these should be compared with theory, and in making the comparison reliance should be placed not on the elasticity theory but on the deflection theory, taking due account of all relevant circumstances in interpreting the experimental results, for otherwise mistakes are unavoidable. This applies particularly to reinforced concrete arch bridges.

The value of making exact calculations should not be underestimated and one should not be deterred by the amount of work involved, for a clarification of the conditions is important both for the safety of these structures and for their economy. When everything has been cleared up by research a further increase in the permissible stresses may be contemplated. Further investigations might then be made as to the influence of deformation in very flat arches, as to the effect of varying relationships of live to dead load on the moment, and as to other conditions governing variations in the moments of inertia.

Among the problems, manifest to everyone, on which experimental research is still needed, is that of elucidating how the load is transferred from the roadway to the arch through the columns or by hangers; in other words the nature of the local stresses at these places when the supports are placed relatively close together or far apart. Such investigations are particularly required in relation to arches of hollow cross section.

The aspects of arch design which have been referred to above or in the original paper, and the further theoretical researches on problems of arches that are required as a step to the adoption of higher permissible stresses, are dealt with by *Boussiron.* The author's choice of an arch of *constant* cross section for a design for 400 m span is arrived at from the same considerations as are advanced by *Boussiron* in his paper.

II. Properties of materials.

The increase in the permissible compressive stress for concrete to 150—200 kg/cm², as proposed in the paper for long span bridges, should not be too difficult to justify on the basis of cube strengths. Both the German and the French engineers have expressed this opinion.

The latter already contemplate permissible stresses in the concrete of 150 kg/cm² with 1 % reinforcement, or even of 240 kg/cm² with 3.6 % of hooping, where the ultimate strength of the concrete is 450 kg/cm². As regards the use of hooping further experiments are, however, still required on specimens with a flat rectangular cross section, reinforced by spirals *close together* after the manner frequently adopted in the case of hollow sections.

As regards the effect of these arrangements of spirals as compared with spirally bound columns, opinions differ, and the advocacy of particularly high permissible stresses in the concrete in hooped hollow sections of this kind is therefore to be received with caution, quite apart from the difficulties in concreting that are inherent in this type of reinforcement. If the ultimate compressive strength of concrete is brought to up to about 700 kg/cm² then hooping may not be required.

As regards the *physical* properties of the concrete, further investigation is desirable, as these exert a particularly marked effect on all reinforced concrete arch bridges. Here the coefficient of *thermal expansion* of the concrete plays a part. Usually this is taken to be of the same value as for the steel, but since the true value for pure cement is considerably higher than for the aggregate (and since, moreover, it differs according to the source of the cement) the value for the concrete must depend on the mixture. According to the experiments of *S. L. Meyers*[1] on cement, the coefficient of thermal expansion also increases with age. In the case of high silicate cements the increase is quite considerable, but in that of high lime cements it is only small, even after several months. In the first mentioned case, after nine months, it amounts to almost double the initial value. If it were found that the same phenomenon held good for European cements it would be necessary to take this into account. In the actual concrete the effect will, of course, be considerably less, but it is nevertheless present. So far as is known the coefficient of thermal expansion of concrete is lower than that of the steel, varying around 0.000009 for 1^0 C in the former compared with 0.00001234 in the latter. This difference gives rise to stresses in both materials, and therefore to moments in the arch.

Another question is that of the *heat of setting* which reaches its maximum after 15 to 20 hours and causes increases in temperature of 40 to 60^0 C. In test cubes the temperature again becomes equal to that of the air after about two days have elapsed, but according to experiments on bridges by *C. R. Whyte*[2] the fall in temperature may take 12 to 20 days according to the position in the cross section, during which time the concrete will long ago have hardened. If the coefficients for thermal expansion differ for the concrete and the steel this will entail transfers of stress between the two materials.

Neither the differing coefficients of thermal expansion as between the concrete and the steel, nor the heat of setting will, indeed, exert much effect on arch bridges, but they do play a part in the great complex of questions relating to materials.

As regards *shrinkage* and *plastic deformation* in the concrete, a great deal has been said in the paper. The critical phenomenon of plastic deformation has not yet been fully explained, and in particular it has not yet been ascertained whether the two phenomena may influence one another; moreover the question arises whether apart from any other relationships between them heat may not also play a part. There is here a great gap in our knowledge of the properties of concrete. As regards the effect of these phenomena on reinforced concrete arch bridges, the author has elsewhere[3] advanced some speculations, and has put forward a new method of calculation.

It is important in this context also to take account of *plastic* compression in the *abutments*, which undergo a permanent contraction in the direction of the load and thereby increase the span. This contraction of the abutments will be

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[1] Eng. News Record, 1935/I, p. 424.

[2] Eng. News Record, 1936/I, p. 693.

[3] *A. Hawranek:* Zukunftsfragen des Baues weitgespannter Eisenbeton-Bogenbrücken mit besonderer Berücksichtigung der Plastizität des Betons. Beton und Eisen 1937, N°. 2.

appreciable only if they are not surrounded by subsoil water, and if the foundation blocks are of large dimensions. When hinges used in the arch are placed in a forward position the total length from the hinges to the base of the foundation should be taken into account when calculating the plastic contraction, besides that of the arch itself.

III. Pre-stressing of the reinforced bars in reinforced concrete arches.

On account of the shrinkage effect, and of that portion of the plastic strain which does not operate in a purely plastic manner, tensile stresses are set up in the concrete, and additional compressive stresses arise in the reinforcement even with purely plastic contraction. Unless these changes are neutralised by the use of hydraulic jacks in the process of striking the falsework, or in the case of small spans where this procedure is not adopted, these additional stresses can be reduced or eliminated by *pre-stressing* the reinforcing steel as already indicated in the report.

In such a case the reinforcing bars should be made continuous through the whole length of the arch, their component lengths being welded together and provided with screw turnbuckles, somewhat staggered in position along the length, for the purpose of tensioning. The round bars, embedded in the abutments over a sufficient length, must be provided with supports at intermediate points consisting either of reinforced concrete separators which have already been hardened and placed against the shuttering, or of steel frames adapted to the whole or the hollow cross sections of the arch and are supported by the shuttering, to allow of accurate and uniform placing of the steel. These intermediate members placed at intervals of 10 to 15 m are intended to transfer the compressive stresses arising from the tensioning of the steel, on to the falsework. In the case of arches of hollow cross section the intermediate diaphragm walls may be made to serve this purpose. The turnbuckle may be placed in the shrinkage gap left open in concreting, thus making it possible to adjust the stress in the section about to be concreted, after the main stressing for the whole length of the arch has been already carried out.

If it were desired to introduce a tensile stress of 1000 kg/cm^2 in the steel over a length of 100 m it would be necessary to stretch the length of the latter by 4.76 cm. Such an adjustment would tend to upset the continuous curvature of the steel bars by changing it towards a polygonal shape, and would thereby reduce the moment of inertia at the kink points and result in a varying thickness of cover, unless the intrados and extrados were made polygonal in shape. The stability of the intermediate pieces for adjusting the position of the steel bars placed near the extrados can be attained by concreting those steel frames into position.

Using this procedure it would be possible, in most cases, to adopt ordinary commercial steel.

Another method is to make use of a *rigid* form of reinforcement with provision for driving in wedges between the steel arch and the falsework so as to obtain prestressing in the steel, but these might differ as between the upper and lower booms.

In the case of *hinged arches* the steel might be temporarily built in to the abutments, and if necessary at the crown, in order to allow of stressing, and after the concrete had been placed the steel at the hinges might be cut through.

The question whether to adopt this new method or to make use of a high quality steel without eliminating the compressive stresses must depend on the relative economy of the two proposals.

IV. Design for an arch of 400 metres span.

In the paper a new idea was put forward for constructing large span reinforced concrete arch bridges (Fig. 5, p. 802).

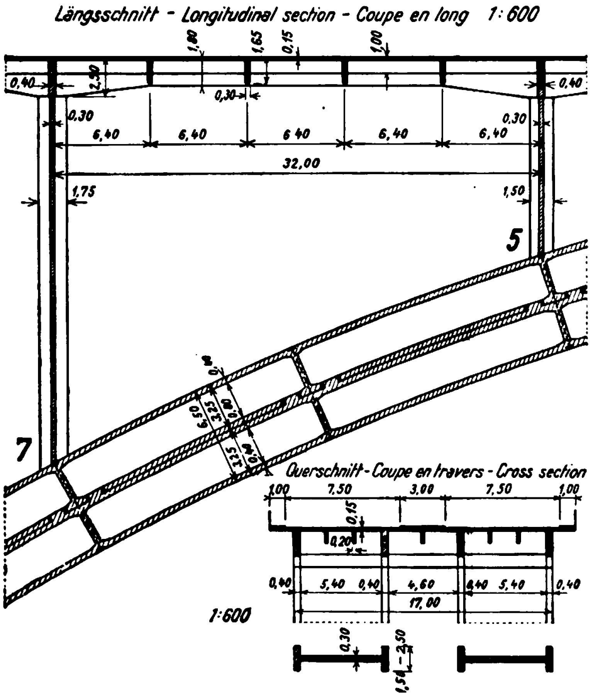

Fig. 5. Roadway construction.

The proposal consists in making the main arch double, consisting of two similar superimposed but separate arches whereof the lower one would be built on centreing and would, when finished, be released by suitable procedure in striking the falsework and adjusted at this stage by means of hydraulic jacks, using the most effective means possible to eliminate shrinkage stresses and settlement due to plastic strain. Using this first arch as centreing, the second arch would then be concreted and released in the same way. The two arches would then be connected together so as to co-operate.

A longitudinal and cross section of the roadway and of the supports are shown in Fig. 5. In what follows below some amplifications and improvements of this proposal will be explained.

1) *The connection between the two superimposed separate arches.*

The main feature of the connection between the two arches as shown in Fig. 6 is the use of *dowels* which are dovetailed not only in elevation but also in cross section. This latter characteristic absolutely prevents the separation of the two arches as the grout filling in the gaps is wedge shaped and is reinforced. The wedges serve to prevent the upper arch from rising. Since these dowels are placed close to the transverse and longitudinal strengthening provided by the ribs on the concrete slabs, there is no danger that the slabs may be torn out of the arch. As a further precaution round steel anchorages passing right through both arches are provided; these are concreted into the lower arch and project into steel pipes which are concreted into the upper arch. The play of the anchoring bar within the pipes does not interfere with the adjustment of the arches by hydraulic jacks, and when the final condition has been obtained the cavities left in the pipes can be filled with cement grout and the nuts on the end of the pipe can be tightened up.

At a later stage in the construction, when the roadway is in position, there will be no tendency for the two arches to separate from one another, and on the contrary the increasing shrinkage effect and plastic strain being larger in the

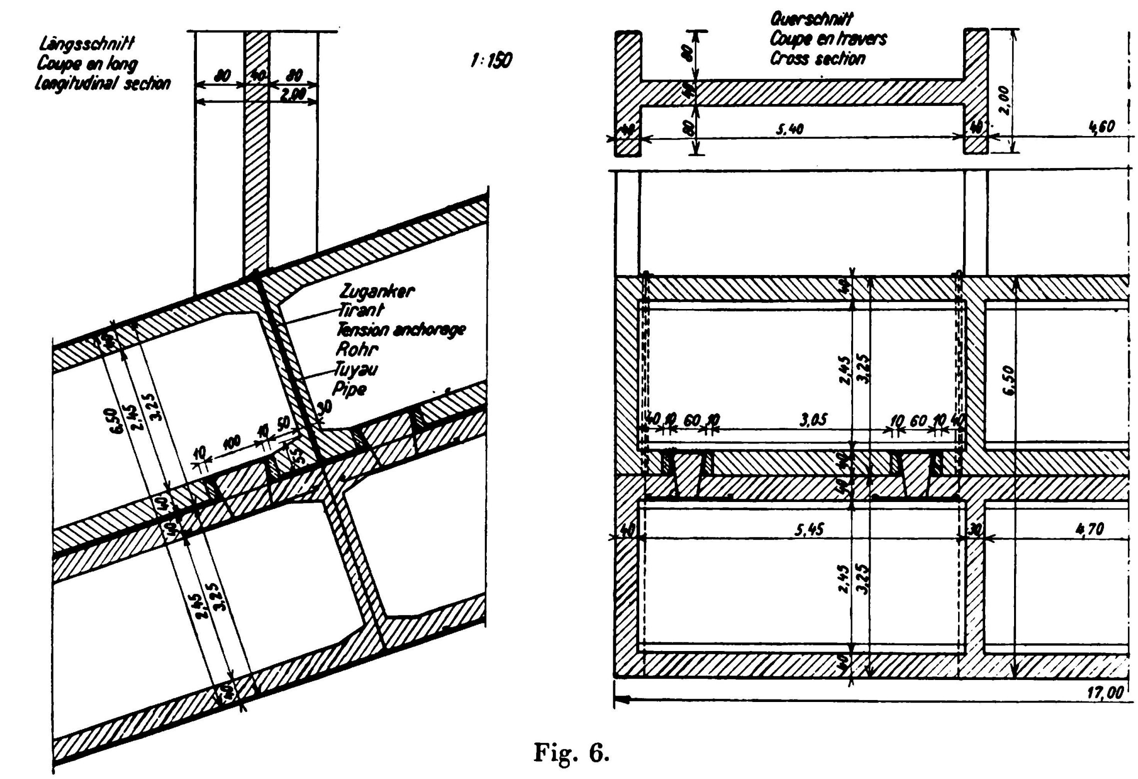

Fig. 6.
Connection of the two arches.

upper arch than in the lower arch will counteract any such tendency or may even reverse it.

This would be subject to confirmation by the fact that when load is added by constructing the roadway, or when a larger amount of shrinkage takes place in the upper arch than in the lower, the upper arch will not only be supported by the abutments but, apart from this, it can be considered as being *elastically supported* over its whole length by the lower arch. Such mutual reactions, and the additional moments produced in either arch, are susceptible to calculation.

This idea of a *compound arch* is one which admits of further elaboration, particularly as regards obtaining an increased moment of inertia in the lower arch by adding upward ribs or vertical walls which can be built into the upper arch.

Finally there is yet another possibility: the load carrying arch might be divided into two principal ribs connected by intermediate walls (provided these offered the requisite amount of lateral resistance to buckling) and in this way a portion or the whole of the falsework could be used over again. With a moderate ratio of rise to span further advantages might be secured, mainly a saving in the cost of the falsework.

In the Preliminary Report only the principle of this solution was explained.

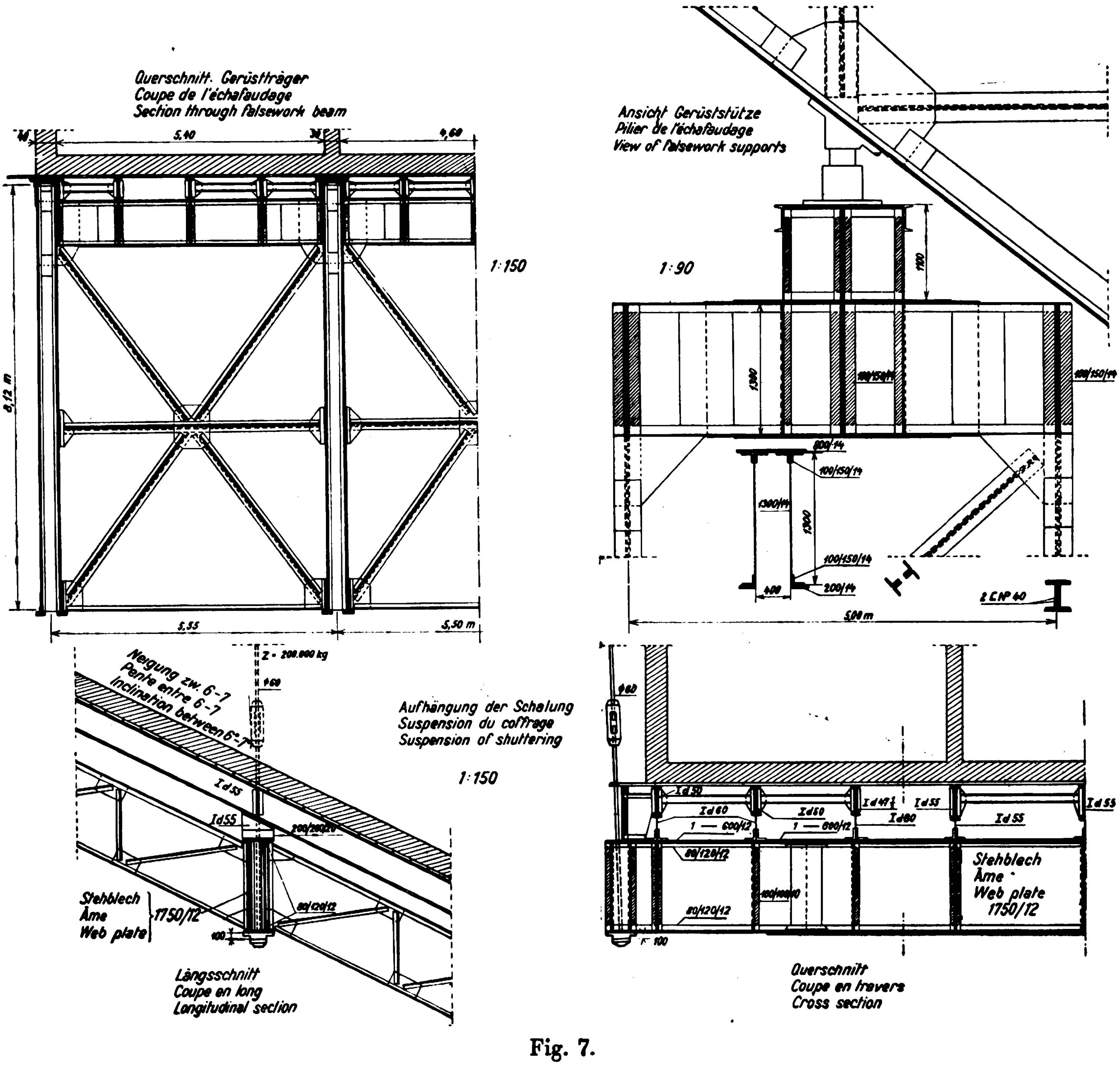

Fig. 7.

Shuttering and bracing frame.

2) *Centreing and Falsework.*

The falsework for the arch has been calculated and designed in detail (Fig. 7). The fixed portion consists of four main lattice trusses which are arranged underneath the vertical spandrel walls of the arch and are supported on towers through the medium of hydraulic jacks. The steel beams which serve to carry the shuttering for the arch itself are supported on cross girders which rest in turn on the four main lattice girders, in the fixed portion of the falsework, while the cross girders in the suspended portion at their ends are carried by suspension rods from the cables. Owing to the great width of the bridge double webbed plate girders are necessary, which at their ends carry vertical lattice girders for stiffening the falsework in that direction, and which are connected by wind bracings (Fig. 8).

It would, however, be advantageous to use not two main cables but three or four of smaller size, thus enabling the cross girders and the wind bracings in the central portion of the falsework to be made much lighter on account of the

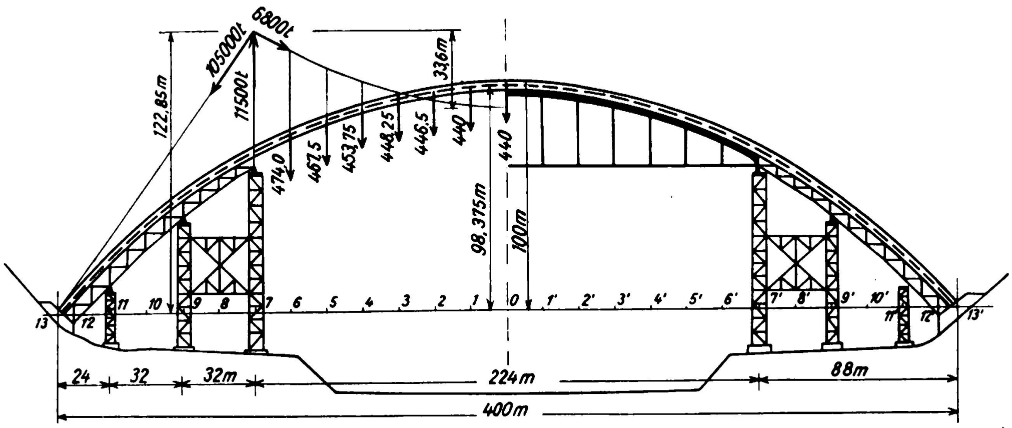

Fig. 8.

Forces in a carrying cable.

intermediate support afforded; at the same time this would be more economical since weight would also be saved in the suspension cables. The pylons could also be used to carry a cableway.

The loading of two cables for a single reinforced concrete arch with shuttering, cross girders, longitudinal girders, cable and suspension bars amounts to $g' = 55$ tonnes per metre run, which would correspond to a total pressure on the pylon of $P = 11{,}500$ tonnes and to a tension in the anchoring cables of $R = 10{,}500$ tonnes; the maximum tension in the cable over the central opening would be $S = 6{,}800$ tonnes. Fig. 8 shows these loads and reactions arising in one cable. With permissible stress of 7,000 kg/cm^2 in the cables, their cross section if only two cables are used becomes

$f_r = 1{,}500$ cm^2 for the anchorage cable and

$f_s = 972$ cm^2 over the central opening.

The total weight of steel in the falsework will be:

Weight of pylons	2 × 575 tonnes =	1,150 tonnes
Cables and suspenders		1,100 tonnes
Fixed falsework	2 × 600 tonnes =	1,200 tonnes
		3,450 tonnes.

In relation to the space filled, which is $^2/_3$ rds. $f \cdot l \cdot b = 480{,}000\ m^3$, the total weight of the falsework will be 3450 tonnes / 480,000 m^3 = 0.0072 tonnes per m^3.

In the case of the Traneberg bridge with a span of 181 m the weight of the falsework amounted to 948 tonnes over the total width of the bridge, which was 18 m, and the total space filled was

$$^2/_3 \times 181 \times 26.2 \times 18 = 56{,}000\ m^3,$$

corresponding to 0.0167 tonnes per m^3. In the present case, therefore, although the shuttering has been designed to cover the whole width of the bridge, the specific weight of steel per sq. metre is less than one half (actually 0.43) of the corresponding amount in the Traneberg bridge, and this despite the fact that in the latter case the falsework was used twice over. This is evidence of the economy of the proposal.

The total cost of the falsework may be estimated at:

2,150 tonnes a 3,000 Czech crowns =	6.45 million crowns
1,100 tonnes a 5,000 Czech crowns =	5.50 million crowns
K_1 =	11.95 million crowns

which compares with the following for the Traneberg bridge:

744,000 Swedish crowns $\times$ 6.55 = K_2 = 4.83 million Czech crowns. The ratio of cost $K_2 : K_1 = 0.405$ practically corresponds with the ratio of weight which was 0.43. If the falsework girder and supports were to be used twice over a still greater saving could be realised. Re-use in this way, the sequence of work being planned accordingly, is feasible — especially if the towers are to remain in situ — but has not here been considered.

With a flat arch, instead of using *vertical* towers for the cables as here proposed, it would be possible to use pylone inclined outwards, with the advantage that the tension in the anchoring cable and in the connecting portion over the central opening could be made equal, and therefore the same cross section could be used throughout the length of the cable. Apart from this the cable saddle would be easier to design as no additional anchoring cable would be necessary and the single cable of uniform cross section would merely have to be bent over the saddle. An inclined pylon of the kind envisaged would, however, complicate the towers of the falsework, as there would have to be an inclined strut reaching to the ground below.

Finally, it would be possible instead of using the suspended construction of falsework as here proposed, to make use of a steel arch with a tie spanning across the reduced central field of 224 m, having its bearings on the towers (Fig. 8). Such an arch, to be used twice over for constructing a twin reinforced concrete arch, could be shifted from one position to the other. In a moderately small span such a lateral displacement of the steel arch with its supporting structure would be possible, and in this way considerable saving in the cost of falsework could be realised.

3) *Quantities of Material.*

In regard to the quantities required, the following volumes of concrete may be estimated:

Arch	15,650 m³
Cross walls of arch	610 m³
Roadway and supports	5,690 m³
	21,950 m³

which amounts to 0.0456 m³ of material per m³. of the space filled, not counting the abutments. The total in the design by *Dischinger* is 20,800 m³ corresponding to 0.0434 m³ of concrete per m³ of space filled.

Assuming a daily output of 100 m³ the construction of the arch would require 156 working days, or about three months for each separate arch.

For the shuttering 4,040 m³ of timber are necessary.

4) *Moments, Thrusts and Stresses.*

The statical calculations have resulted in the moments and normal forces given in Tables 1 and 2, the live load assumed on the bridge for the purpose of designing the arch being taken as 6 tonnes per metre run, and the range of temperature $\pm$ 15° C, with shrinkage reckoned for — 15° C. The wind pressure is taken as 250 kg/m².

Table 1.

Moments in tonne-m.

	Point 0 (crown)	3	6	9	13 (springing)
Dead load of roadway and columns (without arch)	— 13260	— 240	+ 4520	+ 10860	— 21960
Live load on one half p = 6 tonnes per m	+ 8090	— 7234	+ 8306 — 8506	+ 1604 — 2908	— 12554 left + 14804 right
Thermal effect + 15° C E = 210,000 kg per cm²	$\mp$ 1920		$\mp$ 746		$\pm$ 3420
Shrinkage — 15° C	+ 1920		+ 746		— 3420
Σ 1—4	— 14776				— 41354
Wind moment: bending	$\pm$ 10780		$\pm$ 937		+ 53723
Wind moment: torsion	0		$\pm$ 3522		$\pm$ 3295

Table 2.

Normal forces in tonnes (compression —).

	Point 0 (crown)	3	6	9	13 (springing)
Dead load of a single arch alone	— 8760	— 9000	— 9780	— 10730	— 12800
Dead load of roadway and columns	— 5367	— 5544	— 5818	— 7050	— 8400
Live load on one half $p = 6$ tonnes per m	— 608	— 604 — 644	— 694 — 649	— 862 — 629	— 1125 — 600
Thermal effect $\pm 15^\circ$ C	$\mp$ 53.3		$\mp$ 48.7		$\mp$ 35.9
Shrinkage -15° C	+ 53.3		+ 48.7		+ 35.9

With these moments and normal forces the maximum stresses work out as shown in Table 3 for points 0, 6 and 13 in the arch; — the dead load, live load over half the span, temperature drop of — 15^0 C and shrinkage for 15^0 C being taken into account.

Table 3.

Maximum concrete stresses in the arch (kg per sq. cm).

Point	Dead load Loading on one half Thermal stress for -15° C Shrinkage for — 15^0 C	Wind 250 kg/m²	Total unadjusted	Total adjusted by pressure $H_z = 280$ tonnes
0 (crown)	86.7 compression 30.5	$\mp$ 9.6	96.3 compression	97.1 compression
6	92.9 39.9 compression	$\mp$ 0.84	93.7 compression	106.0 compression
13 (springing)	166.1 13.1 compression	$\pm$ 48.0	214.1 compression	163.4 compression

To reduce the total stress of 214.1 kg/cm² in the springing an additional force H_z must be provided at the crown, which can be effected by the use of hydraulic jacks, and the resulting moment at the springings will be

$$M_1 = H_z \cdot f = 100\ H_z$$

The normal force in the springing becomes

$$N_z = H_z \cos \varphi_K = 0{,}672 \cdot H_z.$$

If the condition is laid down that the maximum stresses in either springing due to dead load, live load on one half of the bridge, thermal effect for — 15^0 C and shrinkage — 15° C must be equal, the following equation must be satisfied, assuming concentric pressure at the crown:

$$\frac{N_1 + N_z}{F} + \frac{M_1 - H_z f}{W} = \frac{N_2 + N_z}{F} + \frac{M_2 - H_z f}{W}$$

and

$$H_z = \frac{W}{2f}\left(\frac{N_1 - N_2}{F} + \frac{M_1 - M_2}{W}\right)$$

Here N_1, M_1 relate to the left hand springing and N_2, M_2 to the right, on inserting the absolute values of the two springing moments as follows:

$M_1 = 41{,}354$ tonne-m $N_1 = 35{,}053.2$ tonnes
$M_2 = 13{,}996$ tonne-m $N_2 = 34{,}528.2$ tonnes
$F = 39.1\ m^2$ $W = 54\ m^4$

and assuming $H_z = 280$ tonnes, the maximum concrete stresses that can ever arise are those at the springings which work out at 115.4 kg/cm², or including wind 115.4 + 48 = 163.4 kg/cm², the reinforcement in the arch being only 1 %. All other stresses in the arch will be lower, amounting, inclusive of wind, to 97.1 kg/cm² at the crown and 106 kg/cm² at the quarter point. Thus the effect of the pressure imposed by jacks is to reduce the maximum stress at the springings by about 30.5 % without wind or by about 23.6 % including wind.

A further reduction in the compressive stress of the concrete could easily be obtained by building the arch with a small lateral batter in the neighbourhood of the springings alone, this would allow the maximum stress inclusive of wind to be as now as 150 kg/cm². The same effect, however, is obtainable with a thrust of $H_z = 340$ tonnes without such lateral haunching. By this means the extreme fibre stresses at the springings and at the quarter points would become equal.

This shows that a relatively slender arch of constant cross section with a maximum stress in the concrete of approximately 150 kg/cm² is practicable, and would necessitate only the very moderate pressure of 340 tonnes at the crown when the centreing is struck.

The new proposals for the construction of long span bridges put forward here and fully supported by calculation may, therefore, be looked upon as an acceptable method for promoting the further development of reinforced concrete arch design.

IVb 8

Bridge Problems in Albania.

Über Brückenprobleme in Albanien.

Le calcul des ponts en Albanie.

G. Giadri, Ingenieur,
Generalsekretär des Arbeitsministeriums Tirana.

The problem under discussion here has reference to special statical investigations carried out on a hingeless reinforced concrete arch bridge of 55 m span, built at Gomsice in Northern Albania and opened to public traffic in 1933 (Fig. 10). Normally, in Albania, the more important structures are designed by engineers of the Ministry of Public Works, but in the present case the Statical Office had the task of checking the plans put forward by the Italian-Albanian firm who were contractors for the work, and of confirming its stability by reference to first principes. An account will be given here of the methods adopted by the structural department in accomplishing this object, which were partly of their own devising.

To determine the three supernumerary unknowns for the built-in arch reference was made simultaneously to two statically indeterminate systems, based respectively on the assumption of a built-in beam and on that of a two-hinged arch.

A knowledge of the influence lines for the moments M_{oa} and M_{ob} of the built-in beam, together with that for the horizontal thrust H_o in the two-hinged arch, made it possible to determine the influence lines for the moments at the springing M_a and M_b and, also for the horizontal thrust H of the built-in arch, directly and definitely, without making use of the centre of gravity of the elastic weights.

The first step in determining M_{oa} and M_{ob} was to determine the moment M_β in a beam freely supported on one end and built-in at the other end, after which the fixation was assumed to be cut through in the usual way and the *Maxwell-Mohr* deflection line was determined from the load ordinates $\frac{x}{l} \cdot \frac{1}{J}$ due to an auxiliary moment $M_\beta = 1$, subsequently dividing by the elastic reaction at the support where the end fixation had been cut through (Fig. 4).

In this way x_s represented the length of the arch, corresponding to the distance measured between the left-hand support and the load ordinate (Fig. 2). From M_α which was the lateral inversion of M_β, and from M_β itself, the Statical Office were now able by a simple method to derive the influence lines of M_{oa} and M_{ob} for the built-in beam.

Here the governing factor was the end inclination τ of the influence lines for M_α and M_β (Fig. 4). On applying the moments M_{oa} and M_{ob} to this angle τ for the built-in beam, M_{oa} and M_{ob} were made to operate to the right and left of the points of fixation of the arch.

Reference to the condition of equilibrium $\Sigma M = 0$ served to determine the relationship $M_{ob} = M_\beta - M_{oa} \cdot \tau$ on the right-hand side, and the relationship $M_{oa} = M_\alpha - M_{ob} \cdot \tau$ on the left. On solving these two equations for M_{oa} and M_{ob}, and on substituting $M_{oc} = \frac{M_{oa} + M_{ob}}{2}$ and $M_{od} = \frac{M_{oa} - M_{ob}}{2}$, the result is $M_{oc} = \frac{M_\alpha + M_\beta}{2(1+\tau)}$ and $M_{od} = \frac{M_\alpha - M_\beta}{2(1-\tau)}$. These simple expressions enabled the influence lines for M_a and M_b of the built-in arch to be calculated without reference to other loading schemes except that required for determining the moments in the beam due to the horizontal thrust H_o of the two-hinged arch. The expression $M_{od} = M_d$ could be regarded as a final result, enabling the effects of one-sided loading of the built-in arch to be taken into account. $M_d = \frac{M_a - M_b}{2}$ acts quite independently and causes no horizontal thrust when the two-hinged arch is loaded in either of its hinges. Hence M_d could be separated from $M_c = \frac{M_a + M_b}{2}$ which gives rise to a horizontal thrust (Fig. 5).

The remaining problem was then limited to the examination of a doubly indeterminate statical system, the available basis of reference being the two-hinged arch. Here the end slope τ_o of the influence line for horizontal thrust H_o played a similar part to the end

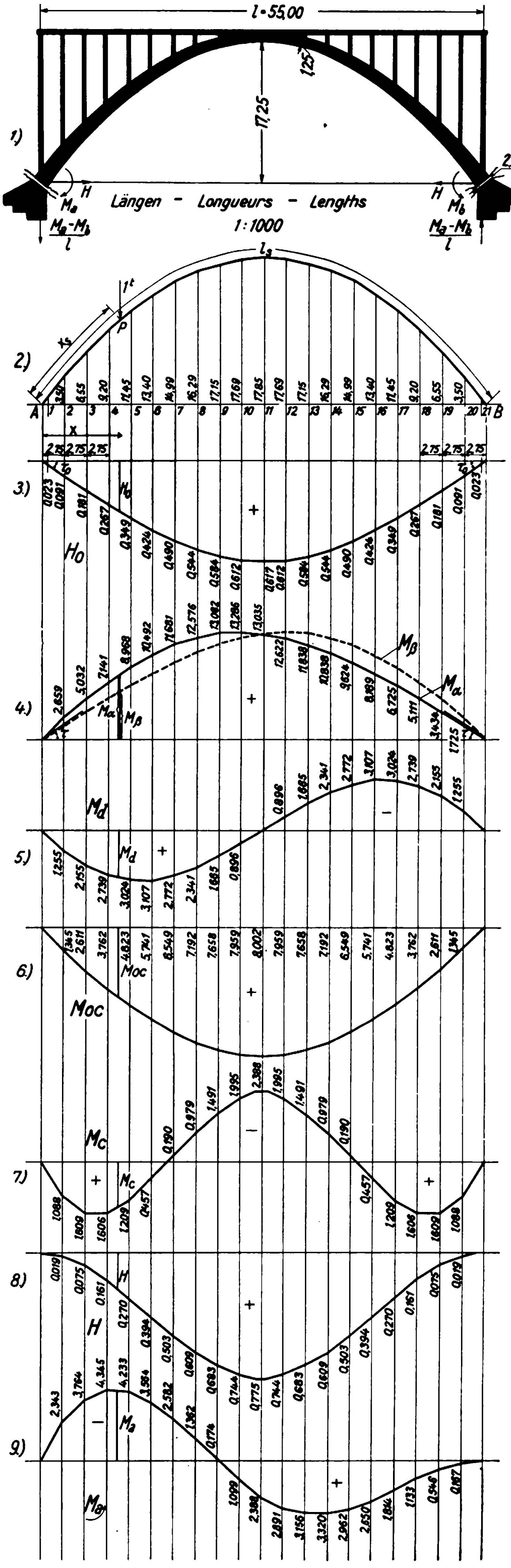

Fig. 1—9. Influence lines for the indeterminate quantities H, H_a and M_b.

slope angle τ in the influence line for M_β in Fig. 3. On loading the hinges of the two-hinged arch with the redundant H and M_c and, equating the horizontal forces at one hinge, the relationship $H = H_o - M_c \cdot 2\,\tau_o$ is reached, from which the influence line of horizontal thrust H may be determined. The influence line of M_c could then rapidly be determined.

Adopting the notation $X_c = M_a + M_b$ and introducing at both hinges $M_c = -\frac{1}{2}$ due to an auxiliary force $X_c = -1$, there was obtained firstly the equation

$$M_c = +\frac{1}{2} \cdot \frac{\int \left(\frac{1}{2} - \tau_o \cdot y\right) \frac{M_o\,ds}{J}}{\int \left(\frac{1}{2} - \tau_o \cdot y\right)^2 \frac{ds}{J}}$$

wherein $\int \left(\frac{1}{2} - \tau_o \cdot y\right)^2 \frac{ds}{y}$ is identified with the constant value δ_{cc}.

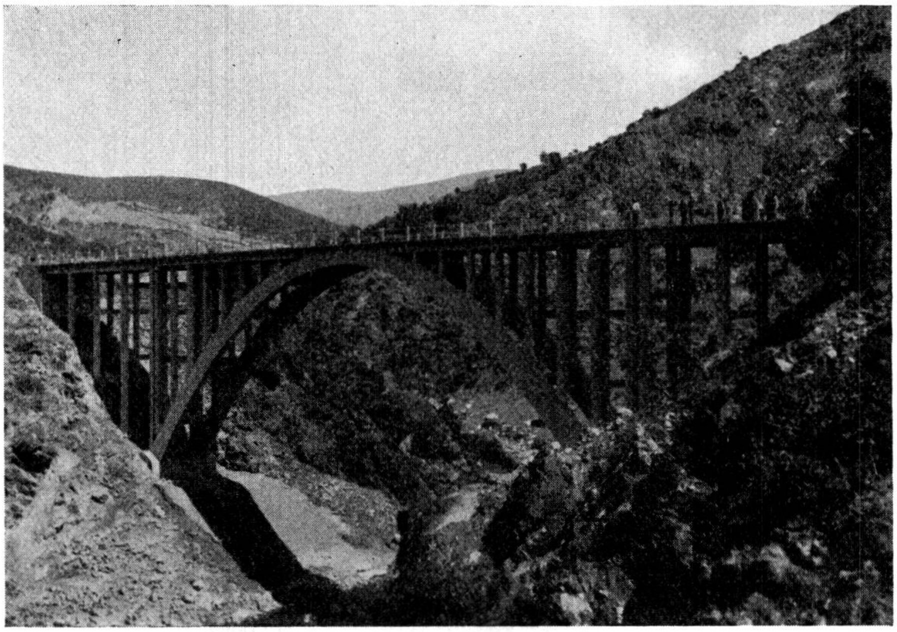

Fig. 10.
Gomsice-Bridge.

On separating the dividend term into two factors and introducing $M_{oc} \cdot \delta_{zz} = \int \frac{M_o \cdot ds}{J}$ and $H_o \cdot \delta_{hh} = \int \frac{y \cdot M_o \cdot ds}{J}$ the value of M_c appears as a function of known magnitudes and there is obtained

$$M_c = \frac{M_{oc} \cdot \delta_{zz}}{4\,\delta_{cc}} - \frac{H_o \cdot \tau_o \cdot \delta_{hh}}{2\,\delta_{cc}}$$

wherein the two constants δ_{zz} and δ_{hh} were found from $\int \frac{ds}{J}$ and $\int y^2 \frac{ds}{J} + \int \frac{ds}{F}$ respectively (Fig. 7). By the value $\int \frac{ds}{F}$ account was taken of the influence of normal forces. In the case of the dividend terms representing redundant quantities the effect of the normal forces was neglected, since the bridge had a rise of 17.85 m. Finally from $M_c + M_a$ the influence line for the redundant moment M_a could be determined. The influence line for the unknown quantity M_b was found by analogy from M_a (Fig. 9).

A temperature effect of $t = \pm 20^0$ C could easily be calculated from δ_{hh}, δ_{cc} and $E \omega tl$.

On the basis of these considerations it was desired to reach an immediate verdict as to the statical performance of the arch. The usual method of calculating built-in arches is simple enough, but it requires the calculation of beam moments for three loading systems. The treatment of the three supernumerary quantities independently of one another is bound up with the condition that the displacements represented by terms of opposite sign cancel out, and this involves a fourth operation, namely, the determination of the centre of gravity of the elastic weights. The supernumerary quantities must themselves be regarded as deriving from general equations which cannot be independently evolved. In confining the calculation of the beam moment to only two systems of loading, and in eliminating reference to the centre of gravity of the elastic weights, the Statical Office of the Albanian Ministry of Public Works believe they have discovered a simplified process for the calculation of influence lines of the supernumerary quantities for built-in arches.

Finally it should be mentioned that the Statical Office have likewise arrived at their own formulae for the calculation of continuous girders. By means of the formulae the influence lines for the moments over the supports in girders which are continuous over three or four spans can be very easily determined, provided that the influence line of the moments over the supports in a girder continuous over two spans is already known. The method has been applied to the solution of various problems in reinforced concrete. It offers the advantage that an alteration in the moment of inertia can very easily be taken into account, and also that it dispenses with the unilluminating tables which are otherwise necessary.

IVb 9

Disadvantages of Thin Construction in Reinforced Concrete.

Nachteile der dünnen Eisenbetonkonstruktionen.

Inconvénients des constructions minces en béton armé.

J. Killer,
Baden (Schweiz).

During the last few years a number of reinforced concrete bridges, even of considerable spans, have been built wherein some of the elements are dimensioned very lightly: for instance load-carrying walls only 10 cm thick, and arches 20 cm or less. Most of this work is either in "beams reinforced with polygonal arches" or in the walls of the box sections of true arch bridges.

Desirable as it may be to build structures with a minimum of material, the adoption of excessively light cross sections in bridge work must be looked upon as a mistake in such cases as these where not only have statical conditions to be fulfilled but external influences such as weathering and frost play an equally important part in the design of the structure.

Just as a natural stone decays in course of time, so also concrete, a material usually inferior in quality to good stone, is liable to suffer from exposure to these external effects. It must be remembered that completely frost-proof concrete has not yet been produced, for even the densest concrete contains a small percentage of pores, wherein water may penetrate and frost cause damage. It is further to be remembered that many bridges cross high above valleys in positions where they are particularly exposed to wind, weather and snow, and these are influences to which special attention ought, therefore, to be paid when dimensioning the various parts. It may be true that bridges have been built without, up to the present, showing signs of damage from frost, but it must not be concluded from this that no risk of such damage exists. We have as yet no basis for estimating how concrete will behave when old and in what form the effects of fatigue may eventually be manifested, for the practice of concrete construction is still too young to afford such a basis.

It is, therefore, desirable to use the utmost caution in dimensioning, and to err on the side of liberality where heavily stressed reinforced concrete structures are concerned. Once frost damage begins to affect structural members which have been too thinly constructed a few centimetres loss of material may endanger the carrying capacity of the bridge, and especially in the case of the main supporting members of arches, it would be difficult, if not impossible, to remedy such damage; hence arches, especially, should not be made too light. Apart from the possibility that the bridge may later be subject to increased static loads, for which it may be inadequate, there is the further consideration that bridges are monumental works which with proper materials ought to last for centuries.

It may also be recalled that in the case of heavily stressed bridge members the temporary work such as the scaffolding, shuttering, etc. costs much more than the concrete itself and this should be an additional reason for not attempting to save on the concrete. Moreover, the unit cost of concrete is higher in thin structures than in thick ones, the expenditure on such plant as service bridges, cranes, mixing machines, etc. being the same in either case, and the labour required for concreting in light constructions being much greater. Again, it is now known that very fluid concrete, such as is used in thin construction, tends to reduce the resistance to frost and this is an additional reason for avoiding excessively thin design.

The increase in mechanisation on constructional works makes it possible nowadays to produce the actual concrete at very low cost, while on the other hand the erection of the scaffolding and shuttering continues to be purely handwork. As wages increase the percentage proportion of the total cost which is attributable to the scaffold and shuttering increases, and the proportion attributable to the concrete becomes less: hence the remarkable fact emerges that in the case of thin, highly stressed bridge work the proportion of cost of the permanently stationary items is small, while that of the plant necessary during the construction of the bridge is large for the reason already discussed, and is even increasing. It follows that in bridge construction heavier designs are to be preferred to lighter. The former, moreover, are less susceptible to shock and vibration. In this matter there is much to be learnt from the old masters of bridge building, who paid special attention to making the principal part of their structures strong.

It may be true that the use of light heavily stressed structures promotes development in structural design; but in bridge work they are out of place, and such practice may lead to damage through frost greatly to the prejudice of reinforced concrete as a system of construction.

IVb 10

Progress in the Architecture of Reinforced Concrete Structures.

Fortschritte der Architektur der Kunstbauten in Eisenbeton.

Progrès de l'architecture des ouvrages d'art en béton armé.

S. Boussiron,
Paris.

Several speakers on steel construction have very properly referred to the attempts that are being made to establish the architecture which may be regarded as proper to this material, and they have been wise to emphasise the importance, from this point of view, of collaboration between the engineer and the architect.

In steelwork — a form of construction now able to celebrate its centenary — plentiful examples dating from the earliest times may be found of fine achievements born of just such collaboration. The recommendation continues, however, to be apposite: indeed becomes all the more so when, after almost a century devoid of architectural mark, endeavours to fix the style of the age in which we live are beginning to meet with success, and when nothing should be left undone to promote that appreciation of natural beauty which the modern trend of travel for pleasure has done so much to stimulate.

Reinforced concrete should be the object of similar endeavours, and may perhaps be a more promising one by reason of the ease with which it enables a combination of the forms and sections considered best by the engineer and the architect in any given circumstances. All the works mentioned by various contributors as being representative of present trends in major reinforced concrete construction furnish evidence of efforts being made in the same direction.

It will be found, however that in the great bulk of examples the arch is placed below the roadway. This preference need occasion no surprise, for it is very natural; indeed one should always endeavour to place the roadway above the massive portion of the structure, in order that no obstacle may interfere with the view of the surrounding country. Ultimately these structure owe their lasting charm to their resemblance to the beautiful masonry bridges which have come down to us from past centuries. The largest of these concrete arches are also the most beautiful; the Elorn bridge at Plougastel by M. *Freyssinet* and the Traneberg bridge at Stockholm by M. *Kasarnowsky* owe most of their beauty to being like amplifications of masonry arches.

Conditions are less favourable where the engineer is forced by considerations of head room to adopt the type of design in which the arch comes above the roadway, but cases where circumstances compel the use of a bow-string design are fortunately rare, being instances where aesthetics have played no part in the general layout of the crossing. Some freedom can almost always be left to the engineer and the architect in the option of slightly lifting the roadway above the springings of the arch, so as not to emphasise the connection between the arch and the abutments.

The construction of a bridge of 161 m span over the Seine at La Roche-Guyon gave the author an opportunity of pursuing his endeavours to improve the aesthetics of this type of construction. It is from this point of view that the matter is mentioned here, theoretical and constructional developments being fully discussed in the general paper.

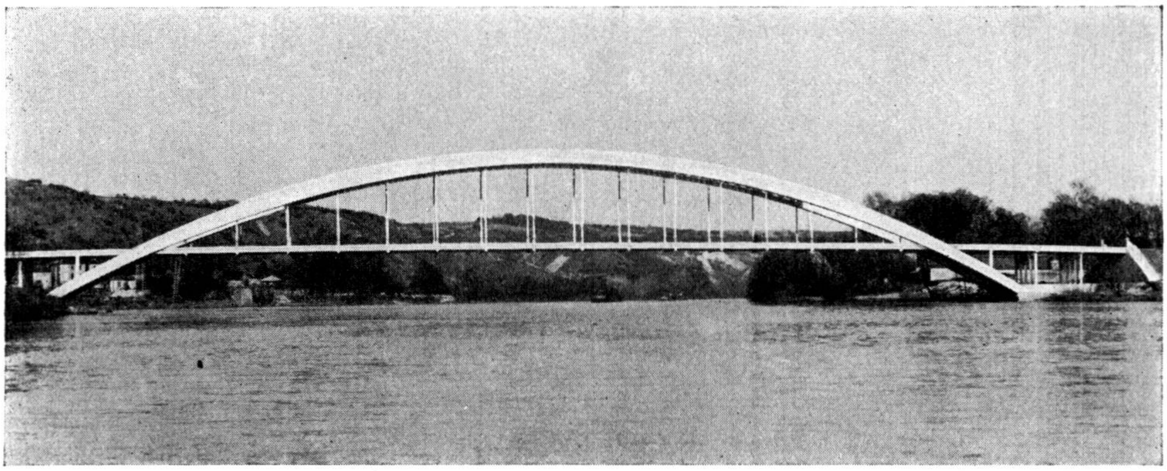

Fig. 1.

Bridge over the River Seine at La Roche-Guyon. Span: 162 m.

First as regards the type of bridge (Fig. 1). In the author's opinion the best position for a roadway which intersects the arch is such that two-thirds of the rise occurs above and one-thrid below the roadway. It will be noticed how little obstruction is caused to the wind by this solution, and, as we have just remarked, the cases where this amount of obstruction is inadmissible must be very rare. The arch has purposely been made very thin; its degree of slenderness being, it is believed, the largest that has hitherto been attained, for the mean depth of the section is only $^1/_{80}{}^{th}$ of the span, the depth being two metres in a span of 161 m. In a structure of this type, more akin to steelwork than to masonry, lightness is the quality which best emphasises the properties of reinforced concrete. It is a quality which is desirable also in order to minimise the obstruction to visibility occasioned by the intersections of the arch with the roadway. Over the whole of the length between these intersections the visibility is in fact almost as good as if the arch were below the roadway, because the light suspension bars 8.50 m apart form only negligible obstacles. The lightness thus obtained may be appreciated very well from Fig. 1.

Another question which has long occupied the author's thoughts, in reference to bridges where the arch comes above the roadway is that of bracing. In his opinion large cross bars, or any kind of triangulation, are unsuitable; he considers that they interfere with the general sense of a free invitation to passage, which is characteristic of this type of structure. In long span bridges, especially, every component ought to flow in the direction of the span.

The solution adopted in the La Roche-Guyon bridge is shown in Fig. 2. Considerations of rigidity undoubtedly called for some kind of lattice work, but here the necessary triangulation has been provided in the form of members of multiple

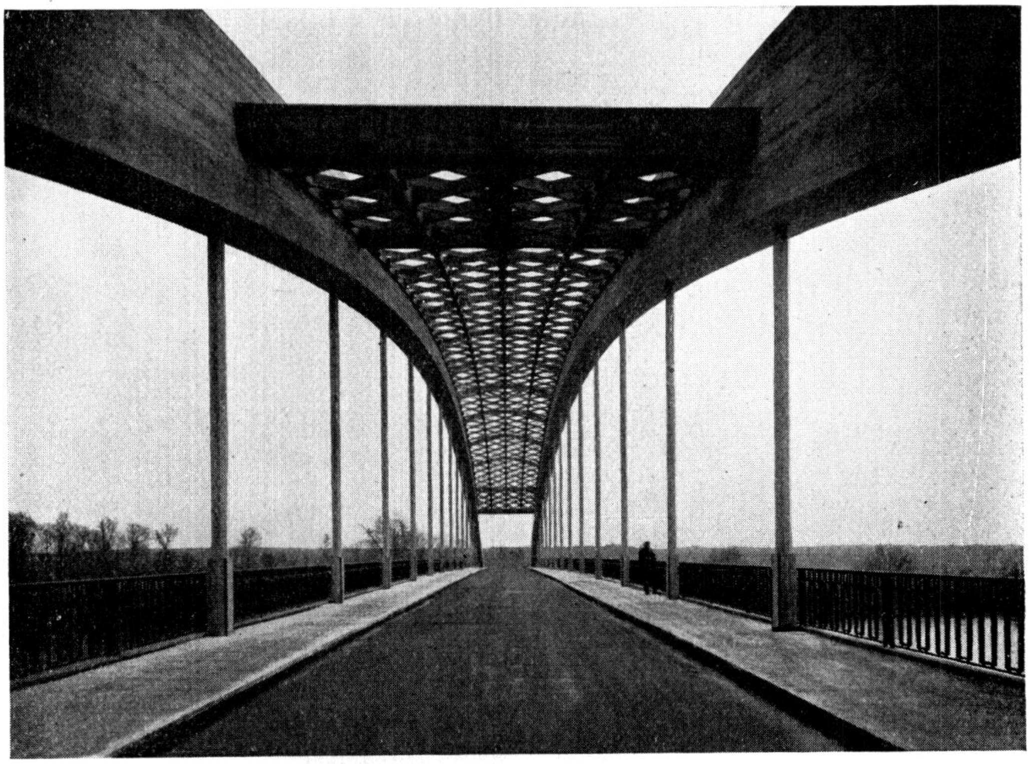

Fig. 2.

lattice construction, more suggestive of the open mesh work of a ceiling than of lattice girders proper. Moreover (apart from the two first struts which form the entrance portal) no other cross piece is to be seen throughout the length of the bridge; the lattice work is carried by arches which spring successively without interruption, along the whole extent of approximately 100 m. It is true that the erection of these lattice members is more expensive than that of a smaller number of bars of heavier section, but by comparison with the total cost of work of this magnitude the difference is small.

The author would hesitate to suggest that equal merit might not be claimed by other solutions of the aesthetic problem; he feels, nevertheless, that the I.A.B.S.E. may be interested in this example of endeavours to make headway as regards this difficult problem appertaining to that type of reinforced concrete bridge in which the arch comes above the roadway. Fig. 3 shows the setting of the bridge in the surrounding country.

It will be granted that in a structure possessing a degree of slenderness greatly in excess of any hitherto obtained some form of overhead bracing was essential.

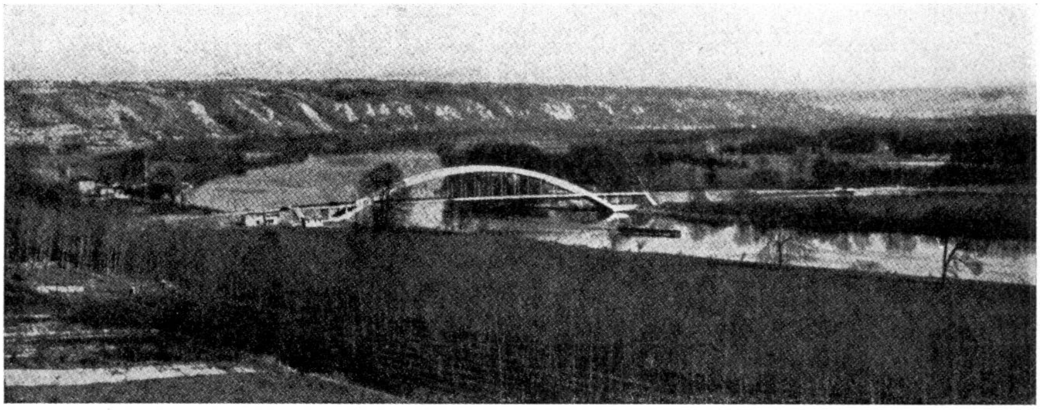

Fig. 3.

The author would gladly have dispensed with this if it had been possible and has made every effort so to arrange it as not to contradict the impression of slenderness.

IVb 11

On Arch Bridges with Inclined Hangers.

Über Bogenträger mit schräg gestellten Hängestangen.

Sur les ponts en arc avec suspentes obliques.

A. E. Bretting,
Oberingenieur i. Fa. Christiani & Nielsen, Kopenhagen.

In his interesting paper on new considerations arising in the construction of large reinforced concrete structures M. *Boussiron* has drawn a number of conclusions which, in the opinion of the present writer, are not capable of justification.

He makes a comparison between the self-stable arch at La Roche Guyon and the arch with inclined suspension bars in the Castelmoron bridge, and suggests that, for various reasons, the latter system of construction is unsuitable for spans in excess of about 150 m. It would appear from this that the true working principle of the system with inclined suspenders has not been quite understood by M. Boussiron, for the question is not one of a parabolic lattice girder with triangular mesh in which the web members must not carry compression, but of an arched construction wherein the effect of the alternating inclined arrangement of the suspenders is greatly to diminish the moments. This working principle holds good even if all the suspenders sloping in one direction cease to operate, and even in such a case the moments are very considerably less than in the corresponding arch of the ordinary type.

The firm of Christiani and Nielsen have constructed such arches in large numbers, including three structures with spans of over 100 m, the largest example being the Castelmoron bridge mentioned by M. *Boussiron* in his paper. In the course of designing these bridges it has in fact been found that the type of arch with inclined suspension bars is not so well adapted for small spans and that its full advantages are realised only in really large spans. When it is mentioned that in the main arches of the Castelmoron bridge, which is a road bridge for heavy traffic, with a span of 143 m and a distance of 8.5 m between centres of main girders, the dimensions at the crown are only 100 by 120 cm, it is difficult to suppose that even here the system has reached its limit of possible application. Designs for larger spans up to more than 200 m have been worked out on several occasions and there appears absolutely no obstacle to the construction of such bridges in still larger spans.

The effectiveness of the system depends not only on the inclination of the suspension bars but also, to an important extent, on the proportion of live to dead load. Its advantage increases with the relative importance of the dead load, and since it is the latter, not the live load, that grows with the span, the most favourable conditions are realised only in large spans.

The inclination of the suspenders will not, of course, be reduced progressively with the increase in span as M. *Boussiron* assumes. On the contrary, there is a certain optimum inclination from which one should not deviate more than can be helped, and in the very large spans it may be necessary for the suspenders to cross one another — an arrangement which would be perfectly feasible, though it has not been necessary in the bridges actually constructed up to the present time.

For further details reference may be made to the paper by *Dr. O. F. Nielsen*, the inventor of the system, printed in the first and fourth volumes of the "Publications" of the I.A.B.S.E.

IVb 12

Application of the Theory of „Internal Elastic Bond" to the Equilibrium of Arches.

Anwendung der „Bindungs-Elastizitäts-Theorie" für das Studium der Bogen.

Application de la théorie de „l'élasticité à liaisons internes" à l'équilibre des arcs.

Prof. Ing. E. Volterra,
Rome.

Deformations in elastic solids may be simply and systematically studied if, instead of relying on special hypotheses as to stresses (Saint-Venant), the study is based on a special hypotheses regarding displacements.

In many cases the displacements of elastic bodies may be supposed to occur very approximately as if there existed special internal hinges. By the introduction of such hinges some degree of freedom is removed from the system and from the analytical point of view a notable simplification results, the problems of elasticity being reduced to ordinary total differential equations (instead of partial differential equations). The greater simplicity of the new method consists in this device, and the author proposes to name it the Method of Internal Elastic Bond. It lends itself very well to the study of deformations in elongated elastic solids, whether their axis be straight or curved.

The hinges thus introduced within the elastic solid are represented by the assumption that transverse sections (that is to say sections at right angles to the axis) remain plane; in other words any plane section undergoes only pure deformation in its own plane, with rigid displacement.

Thus, in the simplest case of all, that of a plane arch, the analytical study of deformations reduces itself to the problem of solving a system of four ordinary total differential equations having four unknowns.

The solution of these equations provides us with the components of the elastic displacement.

Consider the case of a beam of constant thickness with a straight axis. Here the differential equations can be integrated at once, and the shear force and bending moment will at once be obtained. Now let us suppose that the beam, while still of constant cross section, has a curved axis. It may then be shown that the displacements can be calculated by the addition of correcting terms to the displacement obtained in the first case, these correcting terms being made to take account of the curvature of the arch. By the introduction of new correcting factors it is also possible to extend the solution to the case where the thickness varies.

It is impossible to explain the details of the calculation in a few words but these may be found in the notes already published by the present author in the „Rendiconti dell'Accademia dei Lincei" of Rome and in the Comptes Rendus de l'Académie des Sciences, Paris.[1]

The different types of arches which are important in engineering — namely those having circular, elliptical, parabolic, etc., axes — can all be related to the theory briefly summarised above.

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[1] See *E. Volterra:* 1° Elasticità vincolata e sua schematizzazione matematica. Rend. Acc. R. dei Lincei vol. XVI — serie 6°—2° semestre fascicolo 5 e 6 — settembre 1932. — 2° Questioni di elasticità vincolata: 1° Componenti di deformazione e potenziale elastico in coordinate qualsivogliono id., id., vol. XX id., id. fascicolo 11 dicembre 1934. — 3° Id., id., id.; II° Forma appropriata del ds^2 e conseguenze del vincolo geometrico. Id., id., id., vol. XX id., id., fascicolo 12 dicembre 1934. — 4° Id., id., id.: III° Espressione della ϑ e della ψ nel caso generale. Le equazioni dell' elasticità vincolata pei solidi la cui fibra baricentrica è piana. Id., id., id. vol. XXI, id., id., fascicolo 1° — gennaio 1935. — 5° Id., id., id.: IV° Significato del vincolo geometrico. Id., id., id. — Seduta del 1° marzo 1936. — 6° Sugli archi elastici piani: 1. Le equazioni differenziali delle deformazioni. Id., id., id. Seduta del 15 marzo 1936. — 7° Id., id., id.: 2. Direttrice rettilinea Id., id., id. Seduta del aprile 1936. — 8° Id., id., id.: 3. Direttrice, qualsiasi. Id., id., id. Seduta del 19 aprile 1936. — 9° Sur la déformation des arcs élastiques. Comptes rendus des séances de l'Académie des Sciences, t. 202, p. 1558.

See also *E. Volterra:* 10° Elasticità libera ed elasticità vincolata. Applicazioni del concetto di elasticità vincolata. "Atti del Congresso Internazionale di Matematica" di Zurigo, settembre 1932. — 11° Ricerche sugli archi elastici: 1° Metodo generale ed applicazione alle travi ad asse rettilineo. "Annali dei Lavori Pubblici" Anno 1936 — XIV°.

IVb 13

The Theoretical Maximum Spans of Reinforced Concrete Arch Bridges.

Die theoretisch größtmöglichen Spannweiten von Eisenbetonbogenbrücken.

Les portées théoriquement maxima des ponts en arc de béton armé.

Dr. techn. F. Baravalle,

Ingenieur im Stadtbauamt Wien.

M. *Boussiron,* in his exhaustive and interesting paper as printed in the Preliminary Report, has included a calculation and diagram of the average concrete section of reinforced concrete arch bridges of different spans in relation to the concrete compressive stress where the ratio of rise to span remains constant at $\frac{f}{l} = \frac{1}{5}$ (Preliminary Report, page 739, Fig. 11). The basis for his calculations is the theoretical principle which he explains $\left(l = \frac{\varepsilon \cdot R^n}{e^\alpha} \ldots\right)$ and the assumption that the arch has to carry a live load of 2 tonnes per linear m (corresponding to $\sim \frac{2}{4} = 0.5$ tonnes per m^2) in addition to its own weight and a dead load of 4,6 tonnes per m, representing the roadway, suspension bars, etc. The variation in temperature is assumed at $\pm 25^0$ C.

From the curves given it will be seen that for $\frac{f}{l} = \frac{1}{5}$ and $\sigma_{bperm} = 100$ kg/cm^2 the maximum possible span is approximately 600 m, or with $\sigma_{bperm} = 150$ kg/cm^2 approximately 900 m.

In amplification of this work and of the contribution to the discussion made by Professor *K. Gaede,* the present writer proposes to give an account of his own investigations which lead to a generalised determination of the maximum possible spans of reinforced concrete arch bridges.

Basic assumptions and principles.

According to this study, the form of arch which allows the longest span is of the hingeless type, built in at either end and supporting the roadway above.

Using the method of calculation given by *Dr. A. Straßner*,[1] the thicknesses at the crown and springing corresponding to different amounts of rise with $\sigma_{b\,perm} = 100$ and $150\ \mathrm{kg/cm^2}$ will be determined on the assumptions stated below:

I. *Nature of arch.*

Fixed hingeless arch of full cross section with the roadway above.

II. *Calculation.*

(The basic idea is that subject to a particular law of change of loading the axis of the arch may be represented as a geometrical function of the line of thrust for dead load, and that the statically unknown values may then be obtained from the equations of elasticity. In the same way, the variation in thickness of the arch may be calculated from a law of change. The notation adopted by *Dr. A. Straßner* is retained here throughout.)

Further: —

1) The planes of action of the forces coincide with the principal longitudinal planes of symmetry.

2) The system of axes in a vertical direction is determined through the choice of values m_a and m_b such that

$$\gamma_a + \varepsilon_a m_a = 0$$
$$\gamma_b + \varepsilon_b m_b = 0$$

In other words, the angle of the abutment at the springing under a loading $H = 1$ and the angle of the built-in cross section are in agreement for equal or opposite loading.

3) The system of axes in a horizontal direction is determined by a suitable choice of values z_a and z_b such that

$$z_a(\alpha_a + \beta + \varepsilon_a) = z_b(\alpha_b + \beta + \varepsilon_b).$$

4) Equilibrium exists between internal and external forces.

5) The modulus of elasticity E remains the same for the whole of the arch.

6 The distribution of stress follows *Navier*'s straight line law.

7) The proportion between stress and strain is constant (Hooke's law) $\sigma = \varepsilon \cdot E$

8) $Z = \int z^2 \cdot \frac{r}{r+z} \cdot dF \approx J$, or accurately: $J = \int z^2 \cdot dF$

$$\frac{r}{r+z} = 1 - \frac{z}{r} + \left(\frac{z}{r}\right)^2 - \left(\frac{z}{r}\right)^3 \ldots\ldots\ldots$$

In the case of a rectangular cross section this gives

$$Z = J\left[1 + \frac{3}{5}\left(\frac{d}{2r}\right)^2 + \frac{3}{7}\left(\frac{d}{2r}\right)^4 \ldots\ldots\ldots\right]$$

which in turn gives $Z = 1.0015\ J$ when $r = 10\ d$.

[1] *Dr. A. Straßner:* Neuere Methoden zur Statik der Rahmentragwerke. Berlin 1927.

9) No account is taken of the following:

a) The value $\frac{M}{r}$ in relation to N in the expression $\varepsilon = \frac{1}{E \cdot F} \cdot \left(N + \frac{M}{r}\right)$;

hence $\varepsilon = \frac{N}{E \cdot F}$.

b) The value $\frac{\varepsilon}{r}$ in relation to $\frac{M}{EJ}$ in the expression for $\frac{\Delta d\varphi}{ds} = \frac{M}{EJ} + \frac{\varepsilon}{r}$;

hence $\frac{\Delta d\varphi}{ds} = \frac{M}{EJ}$.

10) The arch is symmetrical and is firmly fixed on each side, so that $z_a = z_b = \frac{l}{z}$

$$m_o = \frac{\int y_o \cdot dw}{\int dw}; \qquad \psi = 0.$$

11) A geometrical law of loading holds good: $g_z = g_s \left(1 + \frac{y'}{f}(m-1)\right)$.

12) The axis of the arch coincides with the line of thrust under its own weight:

$$y' = \frac{f}{m-1}(\text{Cos}\, \zeta k - 1).$$

13) The moment of inertia of any given cross section of concrete varies in accordance with a geometrical law:

$$\frac{Js}{J_z \cos\varphi} = 1 - (1-n) \cdot \zeta^1.$$

14) The calculation of thickness of the arch is governed solely by the compressive stress in the concrete, all tensile stresses being taken up by the steel reinforcement.

III. *Loading.*

1) From own weight:

The change in cross section from the crown to the springing follows in accordance with the above-mentioned law from the equation

$$\frac{Js}{J_z \cos\varphi} = 1 - (1-n)\, \zeta^1, \text{ wherein } n = \frac{Js}{J_k \cos\varphi_k}.$$

2) Due to the weight of the floor construction and superstructures of the bridge.

A load of 2 tonnes per m^2 will be assumed to include the average weight of the roadway paving, the slab and the longitudinal and cross girders. In addition the following load in tonnes per m^2 will be allowed for the necessary superstructure:

1.9 tonnes per m^2 for spans up to . . . . $l = 250$ m
4.0 tonnes per m^2 for spans . . . . $l = 500$ m
8.0 tonnes per m^2 for spans . . . . $l = 750$ m

This simplification involves errors, which do not, however, appreciably affect the final result.

3) Due to live load.

Here a uniform moving load equal to $p = 1.0$ tonnes per m^2 is assumed, corresponding approximately with the loading for an (Austrian) first-class highway (Oenorm B 6201, Case 1). Since we are dealing mainly with spans of over 100 m the value p is made amply high enough to cover the possibility of a future increase in loading. With very large spans the live load has so small an effect that a reduction in p would not alter the final result, consequently the value of $p = 1.0$ tonnes per m^2 is here retained. For the subsequent calculation of M_p and N_p the ordinates of the influence lines calculated by *Dr. A. Straßner* are utilised.

4) The variation in temperature is taken at $\pm$ 15° C and allowance for shrinkage is made by assuming a drop in temperature of — 15° C. These assumptions appear to be entirely justified on the basis of experiments carried out on the Langwieser viaduct and on the Hundwilertobel bridge in Switzerland. By the adoption of special methods of construction the shrinkage effects can be considerably reduced, but in the present investigation this possibility has been neglected.

5) No account is taken of stresses due to wind loading, braking loads and movement of the supports.

Results.

a) *Investigation with* $\sigma_{b\,perm} = 100\ kg/cm^2$.

Firstly only the arch of 250 m span will be considered. By plotting the thicknesses of the arch calculated in accordance with the various amounts of rise,

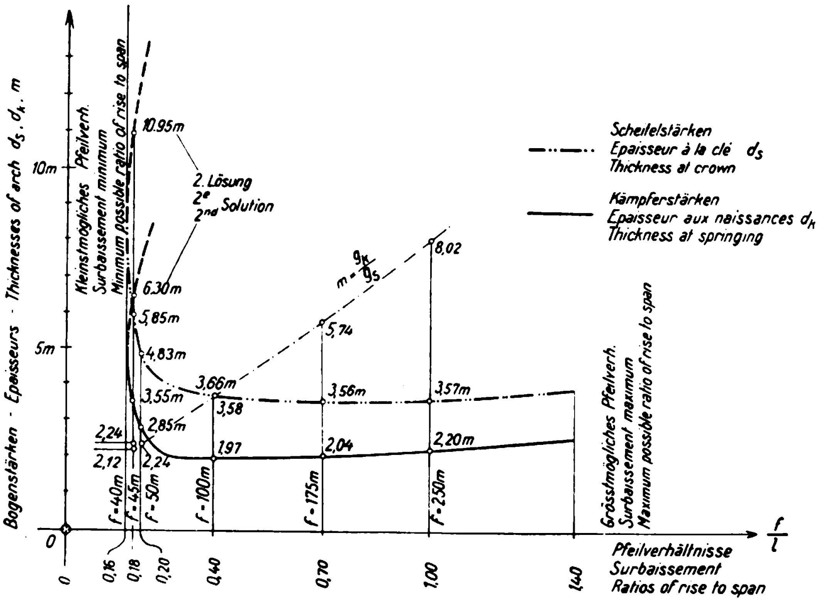

Fig. 1.

Thicknesses at crown and springings in arch bridges of 250 m span with varying amounts of rise. $\sigma_{b\,perm} = 100$ kg/cm².

a clear picture of their magnitude to correspond with varying amounts of rise f is obtained (Fig. 1).

In the resulting curves the ordinates for the smallest possible ratio of rise become tangents, but then the curves assume their maximum curvature and as the rise of the arch increases they rapidly flatten out. The thicknesses of arch after touching their minimum value again increase slightly when the amount of rise f continues to increase, but subsequently the curves straighten very rapidly, still with small increments. The curves finally terminate at that ratio of rise which corresponds to the calculated maximum f possible. The broken line represents the relationship

$$m = \frac{g_k}{g_s}.$$

The relation of the crown thickness to changes in the rise alone is further represented in Fig. 2, which will be readily understood from Fig. 1.

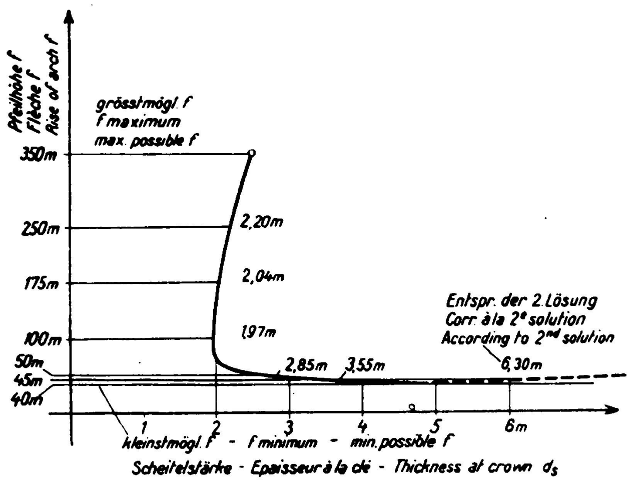

Fig. 2.

Crown thicknesses in arch bridges of 250 m span with varying amounts of rise.
$\sigma_{b\ perm} = 100\ kg/cm^2$.

For the sake of conciseness the further investigations will be discussed only from the point of view of the thickness at the crown. Since the crown and the

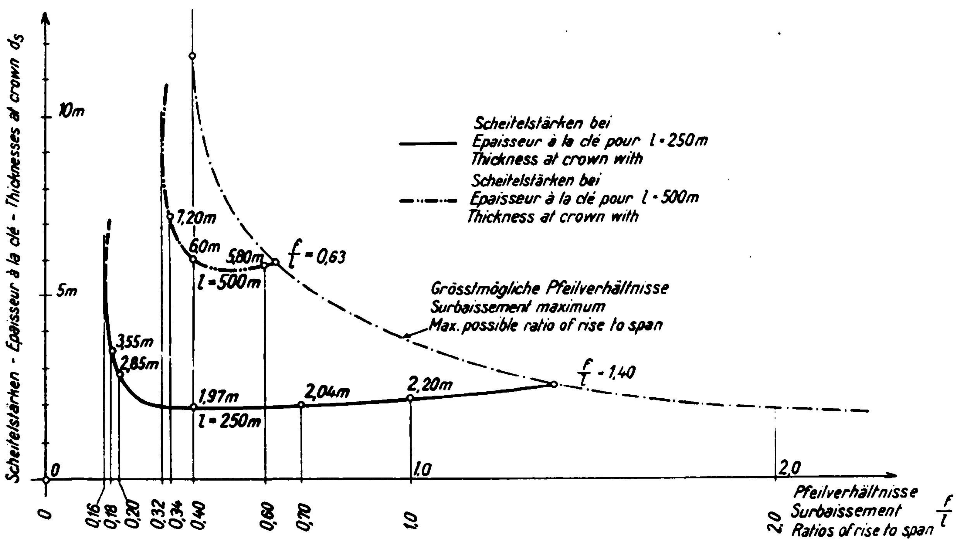

Fig. 3.

Crown thicknesses in arch bridges of varying spans and varying amounts of rise.
$\sigma_{b\ perm} = 100\ kg/cm^2$.

springings of an arch are subject to the same laws the results obtained in relation to the former will apply also to the latter.

If, now, the calculated crown thicknesses of arches of different spans are plotted as in Fig. 3, it will be seen that the end points of the several curves indicating the maximum possible amounts of rise of arch admit of being con-

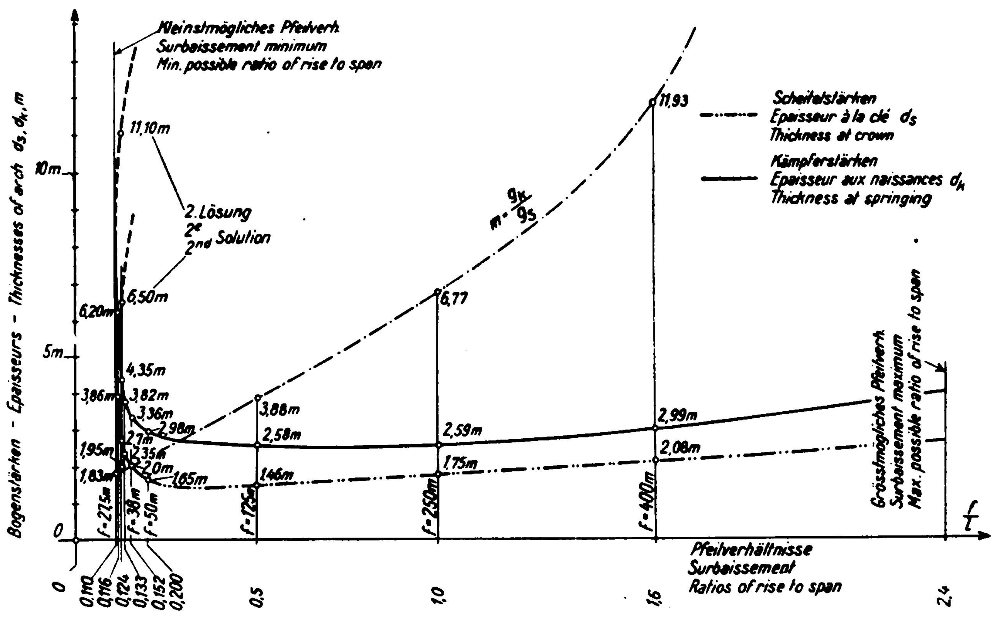

Fig. 4.

Thicknesses of crown and springings in arch bridges of 250 m span with varying amounts of rise.

$\sigma_{b\,perm} = 150$ kg/cm^2.

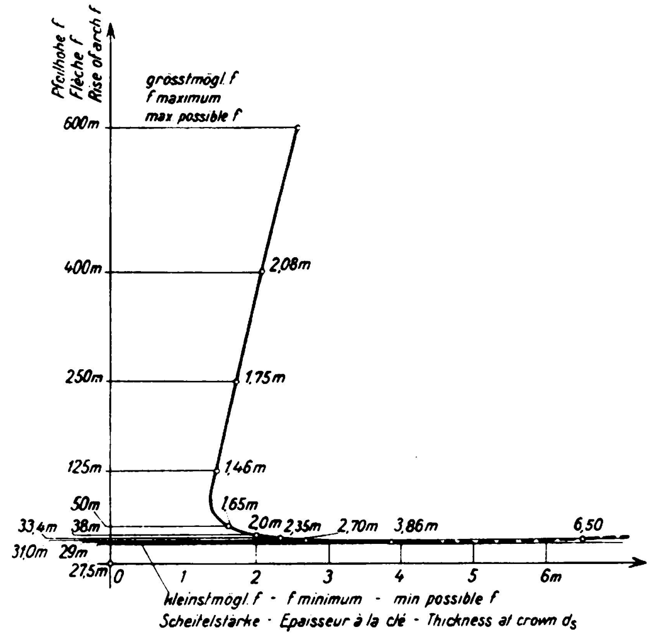

Fig. 5.

Crown thicknesses in arch bridges of 250 m span with varying amounts of rise.

$\sigma_{b\,perm} = 150$ kg/cm^2.

nected by a smooth curve, which is shown as a broken line in the figure. *If such points for different spans, and also for the ratios corresponding to minimum amounts of rise, are plotted as in Fig. 7, the intersections indicate the maximum obtainable lengths of span.*

The area enclosed between the upper and lower bounding lines covers all the possible arches. It will be seen that as the span increases this area is rapidly reduced until finally it becomes a point. *Thus the maximum possible span, in this case $l = 650$ m, is possible only with one particular ratio of rise, namely* $\left(\frac{f}{l} = 0.40\right)$.

b) *Investigations with* $\sigma_{bperm} = 150\ kg/cm^2$.

Here again reference will at first be made only to the case of the arch of 250 m span represented in Fig. 4. The dependence of the thickness at the crown on the varying amount of rise is represented in Fig. 5 and the lines so obtained show a similar trend to those found under a). Hence what has been said above is also valid here, but in accordance with the greater permissible stresses for the concrete the limiting values are now different as shown in Figs. 4 to 7. *The maximum span is seen to be 1000 m, which once again is only obtainable with a ratio of rise equal to* $\frac{f}{l} = 0.40$.

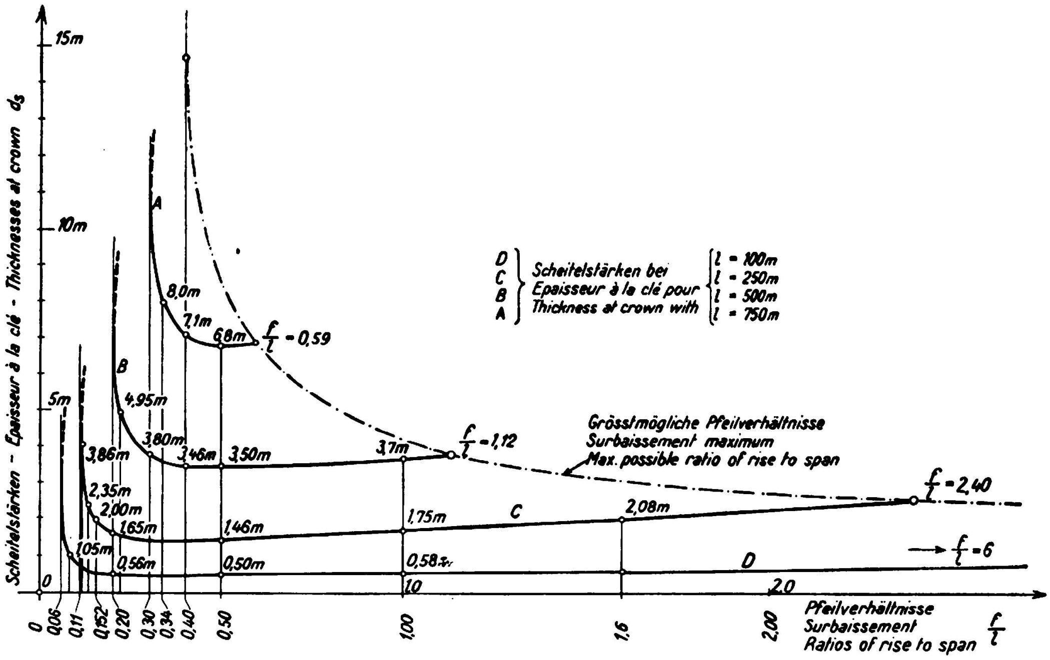

Fig. 6.

Crown thicknesses in arch bridges of varying spans with varying amounts of riso. $\sigma_{b\ perm} = 150$ kg/cm².

The range of possible ratios of rise $\frac{f}{l}$ for different spans and different permissible stresses in the concrete is shown in Fig. 7.

The possibility of further progress in the construction of reinforced concrete arch bridges rests on the fact that it is being seriously contemplated, even today, that compressive stresses in the concrete of 200 to 300 kg/cm² might be used in exceptional bridges of this type. Preliminary designs have been made for reinforced concrete arches of 400 m free span (*Hawranek*, with $\sigma_{bperm} = 160$ kg/cm²) and even 1000 m free span (*Freyssinet*, with $\sigma_{bperm} = 280$ kg/cm²).

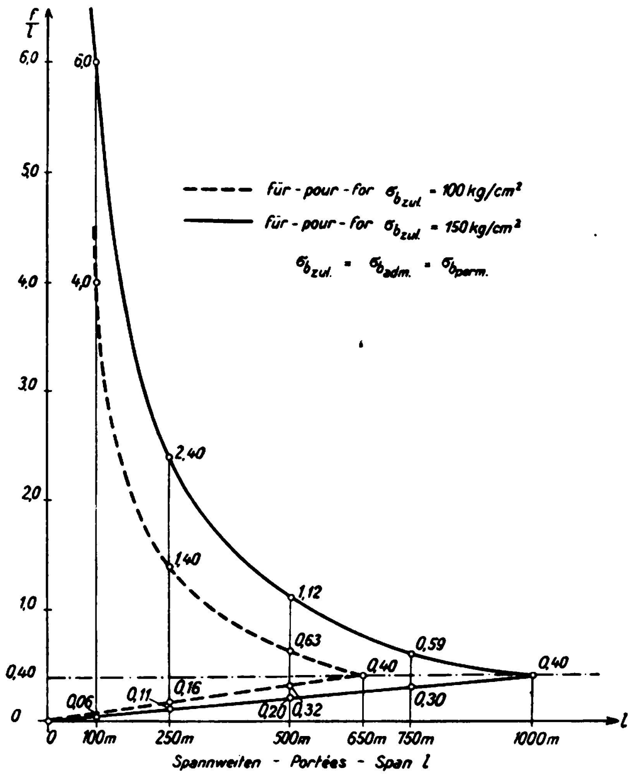

Fig. 7.

Range of possibility of arch bridges, and limiting values of spans, under the assumptions made.

The development of technique in the preparation and placing of concrete, together with the theoretical investigations that are being made into the statical conditions which govern structures of this character, render it likely that such designs may actually be realised.

IVb 14

The Bridge over the Esla in Spain.

Die Brücke über den Esla in Spanien.

Le pont sur l'Esla en Espagne.

C. Villalba Granda,

Ingénieur des Ponts et Chaussées, Madrid.

The bridge at Plougastel and the one at Traneberg built five years later are splendid achievements in the construction of long span arch bridges. The Esla bridge now being built in Spain will have a span exceeding either of the two just mentioned. At the site of the bridge the Esla is an artificial lake more than 40 m deep, and the bridge is intended to carry the railway connecting Zamora with Coruña, which is a double track standard gauge line (Fig. 1). The author was responsible for the definite planning of the work after a preliminary design by M. *Gil*.

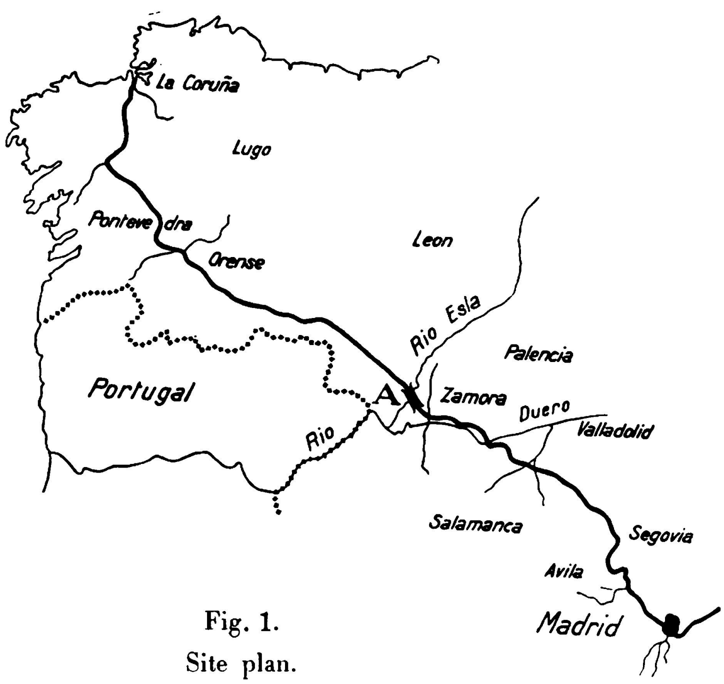

Fig. 1.
Site plan.

The following is a comparison of the principal dimensions of the three bridges just mentioned:

	Plougastel	Traneberg	Esla
Clear span	172.60 m	178.50 m	192.40 m
Theoretical span	186.40 m	181.00 m	172.00 m
Rise	35.30 m	27.00 m	38.80 m
Working stress in concrete	75.0 kg/cm²	98.5 kg/cm²	86.0 kg/cm²

The bridge consists of a main central arch and a series of side arches forming access viaducts on either bank (Figs. 2 and 3).

The central arch is hollow, containing three longitudinal cells, and the side faces are battered at 1.5 % uniformly for all elements of the central portion.

At the crown the width is 7.90 m and the thickness 4.52 m; at the springing the corresponding dimensions are 9.063 m and 5.50 m. The decking bears on the arch through the medium of columns. The access viaducts consist of five arches each of 22 m span on the Zamora side and three arches also of that span on the Coruña side.

I. Description of the Work.

Access viaducts. The arches forming the access viaducts are semi-circular with an intrados of 11 m radius; they are 1.10 m thick at the crown and are reinforced by rolled sections and round bars of steel (Fig. 4). The longitudinal deck girders are rigidly connected to the uprights which are 9.50 m high and situated vertically above the piers; intermediately they are carried by rockers on the uprights which are 2.10 m high, and at the crown they rest upon the extrados of the arch itself through the medium of sliding bearings. The piers of the viaducts are hollow, their longitudinal walls being 0.90 m thick at the top and the buttresses 0.10 m thick; the transverse walls are 1.50 m thick with buttresses 0.25 m thick. The height of the piers, measured from the top of the foundations to the springing of the arches, varies between 9.70 m and 38.70 m; the depth of the foundations between 1.22 to 6.77 m these being carried down to rock which is perfectly sound and compact.

Main arch. The height and principal dimensions of the main arch have already been stated, and it may be added that the sections consist of two rings of equal thickness varying from 0.70 m at the crown to 1.05 m at the springing, connected together by four walls with a constant thickness of 0.40 m (Fig. 5). The reinforcement of the arch is provided solely with the object of resisting secondary stresses. The neutral axis is given by the equation

$$y = 206.7\ (x - 2x^2 + 2x^3 - x^4)$$

where y represents the absolute value and x is the ratio between the abscissa of the point under consideration and the span, the origin being taken at one of the springings. Under dead load this neutral axis has practically the shape of a catenary curve, and the total cross section of the arch, as well as its moment of inertia, are designed to ensure that their vertical projection shall remain constant.

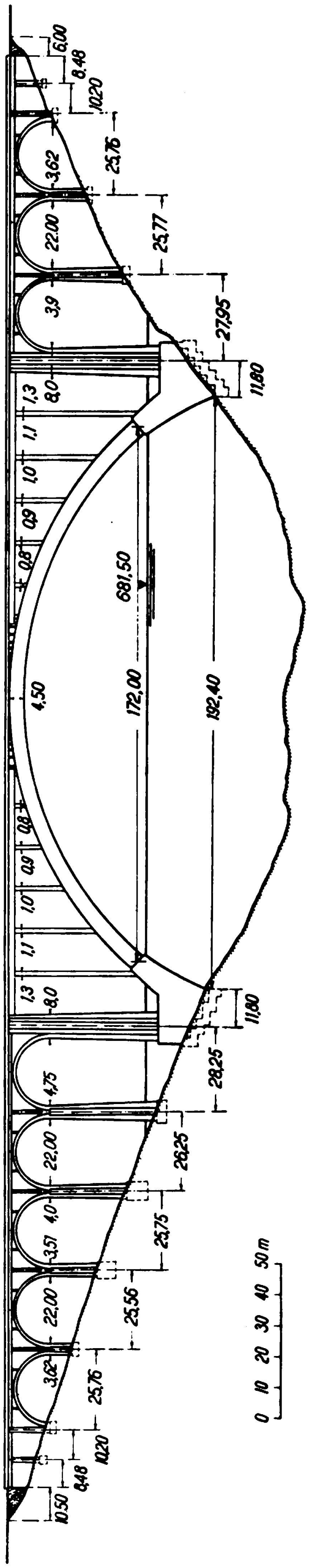

Fig. 2. Elevation of the bridge.

The superstructure of the main arch are in three distinct portions. The central portion, which is 20 m long, has small longitudinal walls which serve to retain a filling road metal. Intermediately, over a length of 12 m, the floor slab (which is continuous over five openings) is carried on cross walls built into the decking

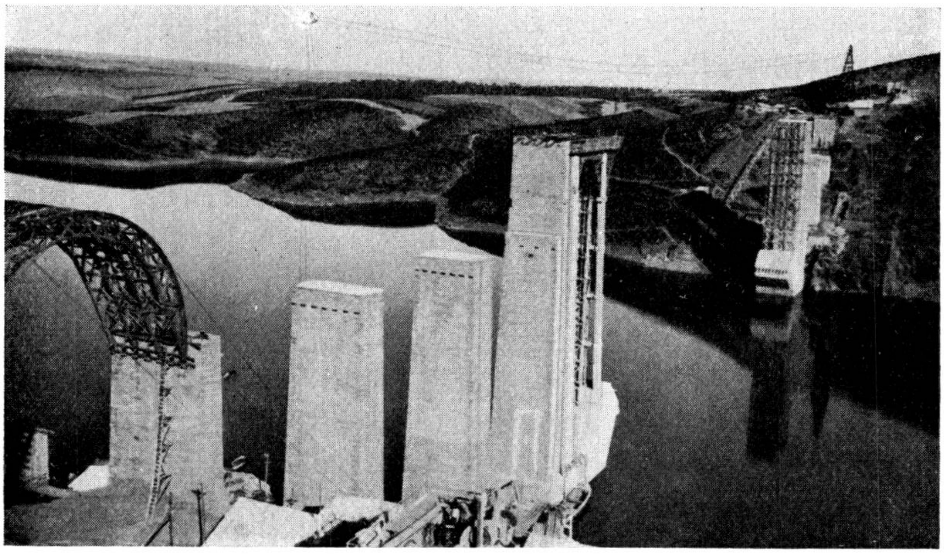

Fig. 3.
General view.

at the top and hinged at the bottom on the arch. The outermost part of the construction consists of 12.50 m spans carried onto the arch through columns. The floor itself consists of a slab 0.20 m thick supported on four beams measuring 1.80×0.60 m, the maximum height of the columns being 38.72 m.

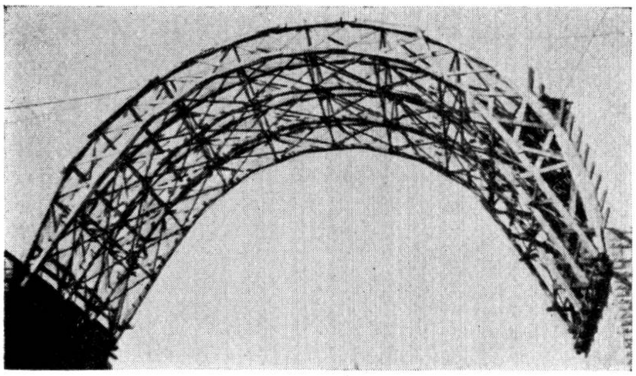

Fig. 4.
Centreing for 22 m arch.

The massive pier-abutments (Figs. 6 and 7) have a cross batter of 2 %. The abutments proper (springings) are reinforced with rolled sections and round bars, the former surrounded by circular hooping. At the springing the piers are 10.41 m wide by 6.70 m high.

As regards foundations, slate and quartzite lie exposed in the ground; hence all that was necessary was to reach down to a sound portion of the rock.

Materials. The end abutments of the access viaducts were built in ashlar masonry and the foundations of cyclopean concrete containing 150 kg of cement per cubic metre. The facings were formed of concrete blocks with 250 kg/m^3; the filling is of concrete with 200 kg/m^3; the small arches and decking of concrete with 350 kg/m^3. Under the main span the foundations were built of cyclopean concrete containing 200 kg of cement per cubic metre; the abutments of concrete containing 325 kg/m^3; the arch itself of concrete containing 400 kg/m^3 and the superstructure of concrete containing 350 kg/m^3. Artificial Portland cement was used throughout.

The total cost of the work has been estimated at 6.5 millions of pesetas.

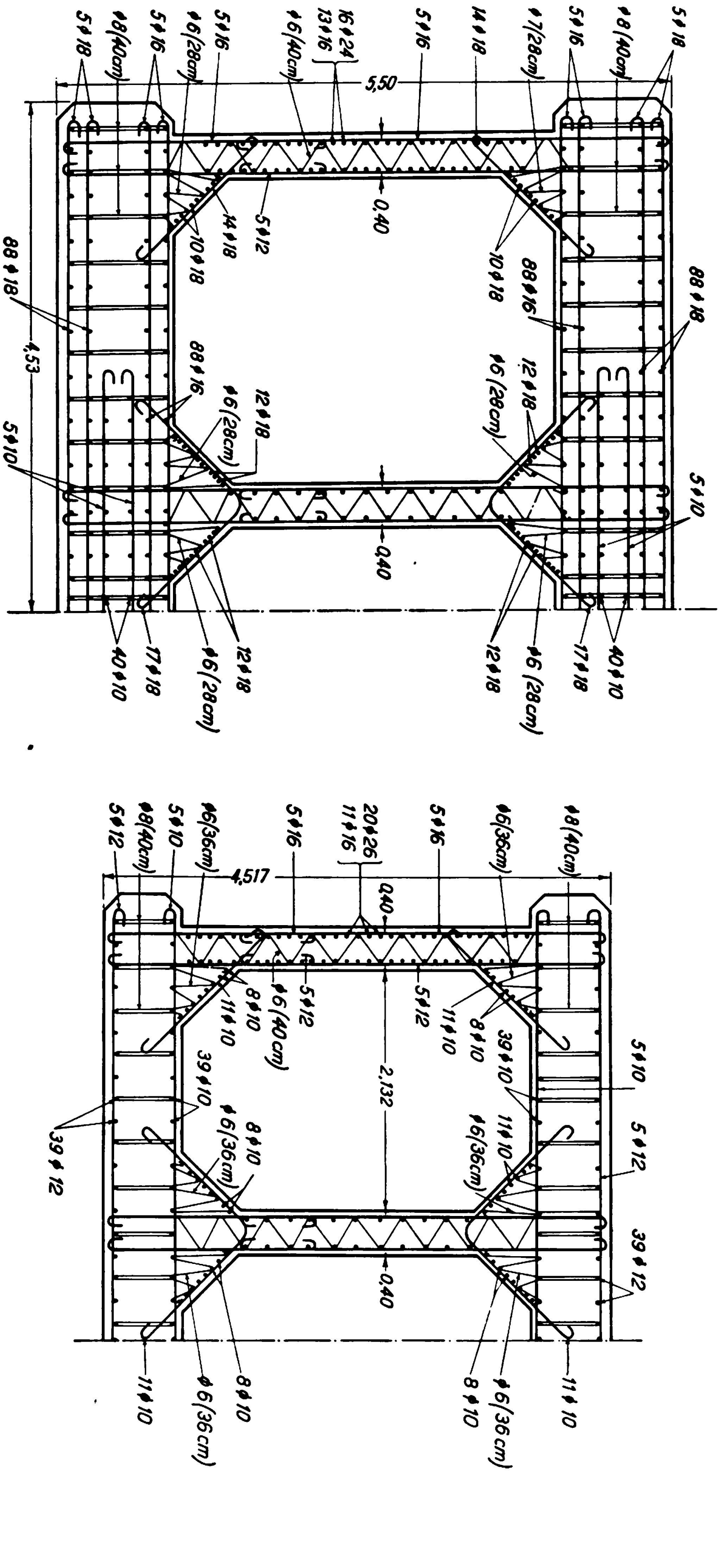

Fig. 5.
Sections at springing and crown.

II. Statical Calculations.

a) *Access viaducts.* The circular arches have a theoretical diameter of 23.40 m. They were designed with an allowance for a variation in temperature of $\pm 20^0$ C, including shrinkage. The maximum stresses, taking account of braking loads and wind, amount to 46.5 kg/cm² for the compression of the concrete at the springing and 1010 kg/cm² for tension in the steel. The shear stresses due to torsion in the arch may reach 5 kg/cm². Flexure of the piers and strains in adjacent arches were taken into account when designing each arch.

The frames working the superstructure were very exhaustively calculated, taking due account of all possible effects.

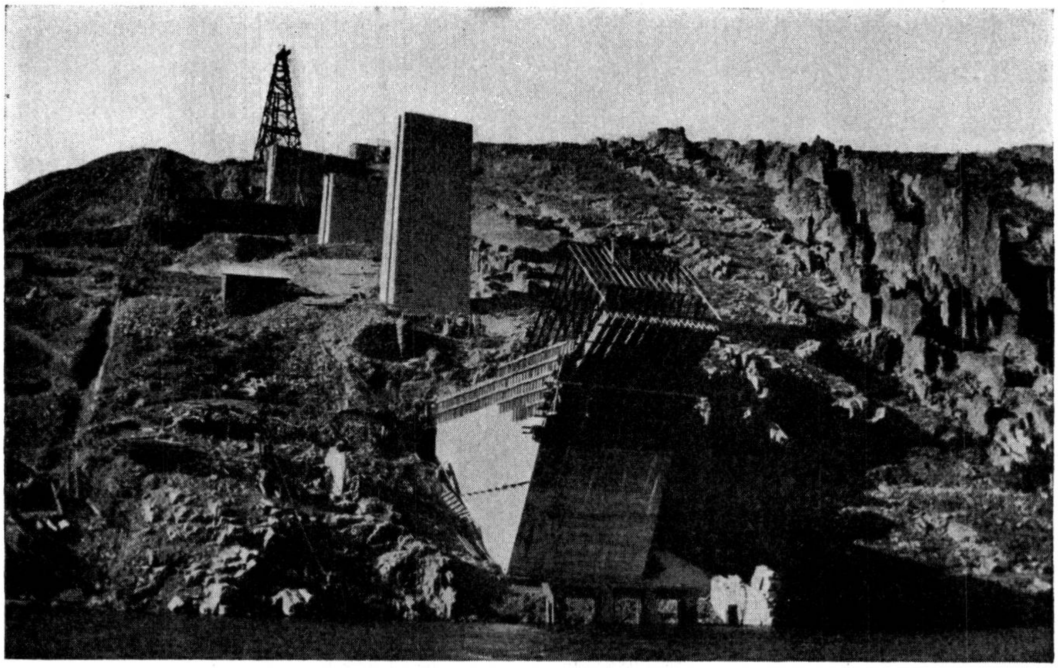

Fig. 6.

Abutment (on Zamora side) showing reinforcement; in the background a mast of the cableway crane.

b) *Main Arch.* The axis of the arch is a parabola of the fourth degree, this being the shape which conforms most closely to the dead load pressure line out of dead weight. The load assumed in the calculation consists of two typical trains as prescribed in the Spanish regulations, and this gives rise to stresses which are less than one half of the dead load stresses.

In long span arches of this type secondary stresses become very important. For the purpose of calculation account was taken of variations in temperature, collaboration between the arch and its superstructure, and buckling of the arch. In view of the great size of the structure it was assumed that the wind would act only on one part of the bridge at a time.

The method of calculating the polar moment of inertia for the solid and hollow rectangular sections was that given by *Mesnager* and *Föppl. Lorenz* and

Pigeaud have worked out the general case of a section of any shape, but have stated only approximate solutions. For the case where the sections consist, as here, of several cells there is a choice of several methods:

1) The section may be assumed to be solid, the distribution of stresses on that assumption may be explored, and the stresses which ought to be carried by the non-existent hollow portions may be transferred to the top, bottom and the vertical elements. This solution is positive and simple, but gives too heavy stresses.

2) The central vertical diaphragm wall is neglected, so that the section has in effect only one internal cavity. On this supposition a great deal of the actual rigidity against torsion is neglected.

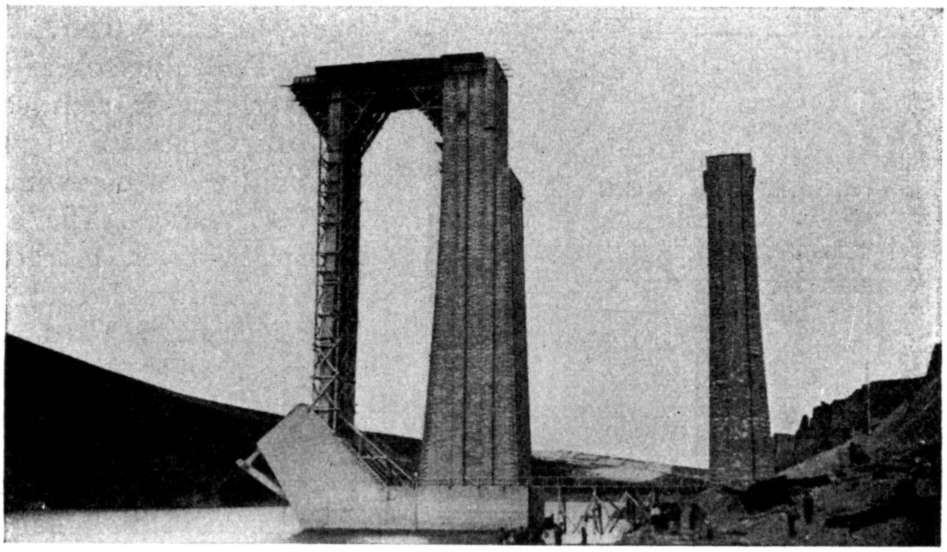

Fig. 7.

Abutment pier with a portion of the superstructure for erecting the centreing of the main arch.

3) It may be assumed that the twisting moment is distributed among the three cells and that the three portions of the section have the same angle of deformation. It was this last method that was in fact adopted.

The influence lines for all the hyperstatic reactions were determined by the method of virtual loads and superposition of forces. A variation of temperature of $\pm$ 150° C was assumed. In designing the columns account was taken of braking and acceleration, also of the effect of deformation of the arch and columns, etc.

Large arches need to have very solid abutments in order to ensure the fixation of the arch. In order to check the elasticity possessed by these abutments influence lines for the deformation were calculated, by assuming the arch to be comparable to a beam of varying moment of inertia which was rigidly built in. These influence lines were calculated by *Müller*'s method, and it was found that the displacement of the extreme section of the arch at the abutments amounted to 0.0044 mm, which value is very low.

III. Construction.

The volume of concrete required for the construction of this work amounts to 32000 m³, which represents 28000 m³ of broken stone and 15000 m³ of sand. The available plant provided has a capacity of 100 m³ of stone and 30 m³ of sand in eight hours. The site is served by a cable-way 500 m long carried on two wooden pylons 28 m high, the speed of travel being 1 m per second vertically and 4 m per second horizontally. It is operated by a 46 H.P. motor.

The falsework as designed is a timber arch consisting of frames 3.50 m high, built up of pieces measuring 23 × 7.5 cm. The arch will spring from brackets made of reinforced concrete, and on these brackets hydraulic jacks will be provided for the adjustment of the centreing.

The falsework will be erected with the aid of a suspension bridge consisting of three sets of carrying cables (Fig. 8), and will be braced by a network of cables to ensure stability (Fig. 9). The suspenders will be rigid cables of 8.1 mm dia., and the 15 carrying cables will be arranged in groups of five.

The falsework is relatively very light in design as a special sequence of operations is to be followed in concreting: first the lower third of the arch resting on three rollers, then the corresponding portion on two rollers, finally the remaining two-thirds of the arch at the

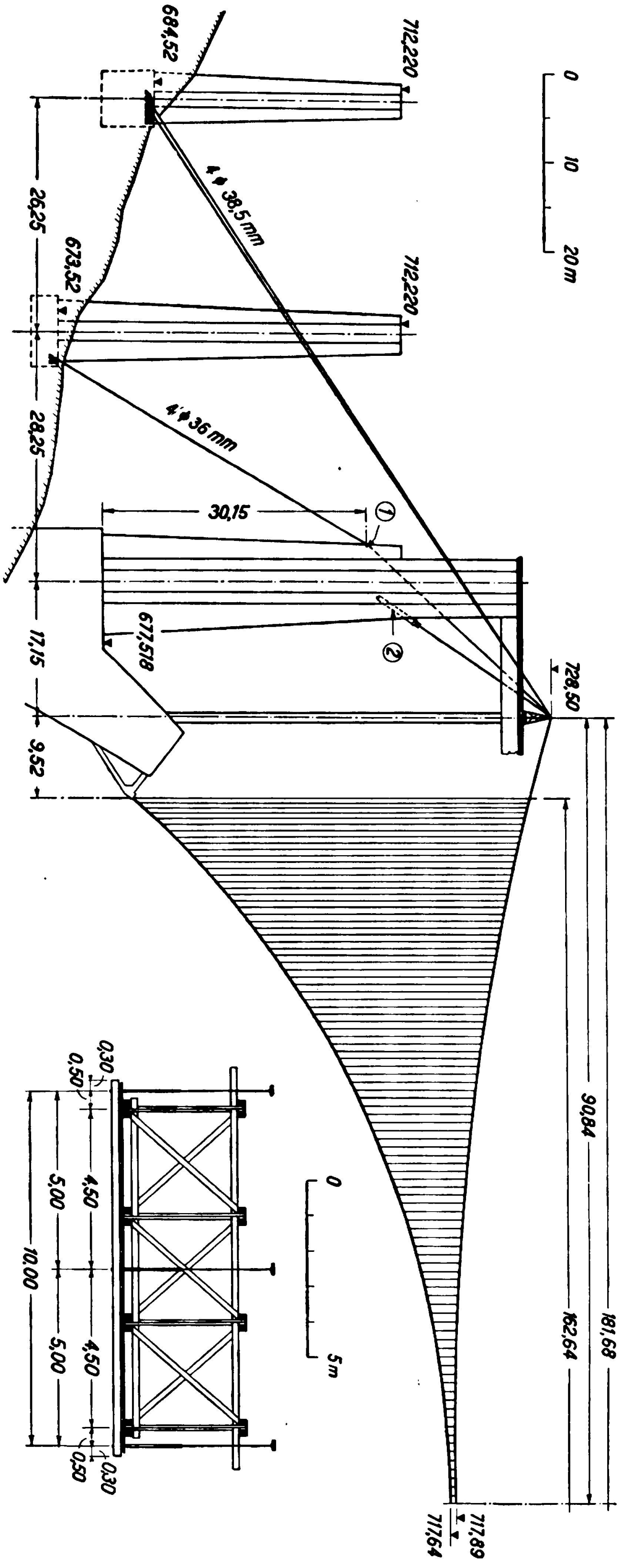

Fig. 8. Suspension bridge for erecting the centreing:
① Notched timber member. ② Anchorage of cable.

crown. The maximum stress imposed on the timber of the falsework will amount to 78 kg/cm^2.

No special arrangements for releasing the falsework are being made, as adjustments will be carried out with the aid of 36 hydraulic jacks placed at the crown.

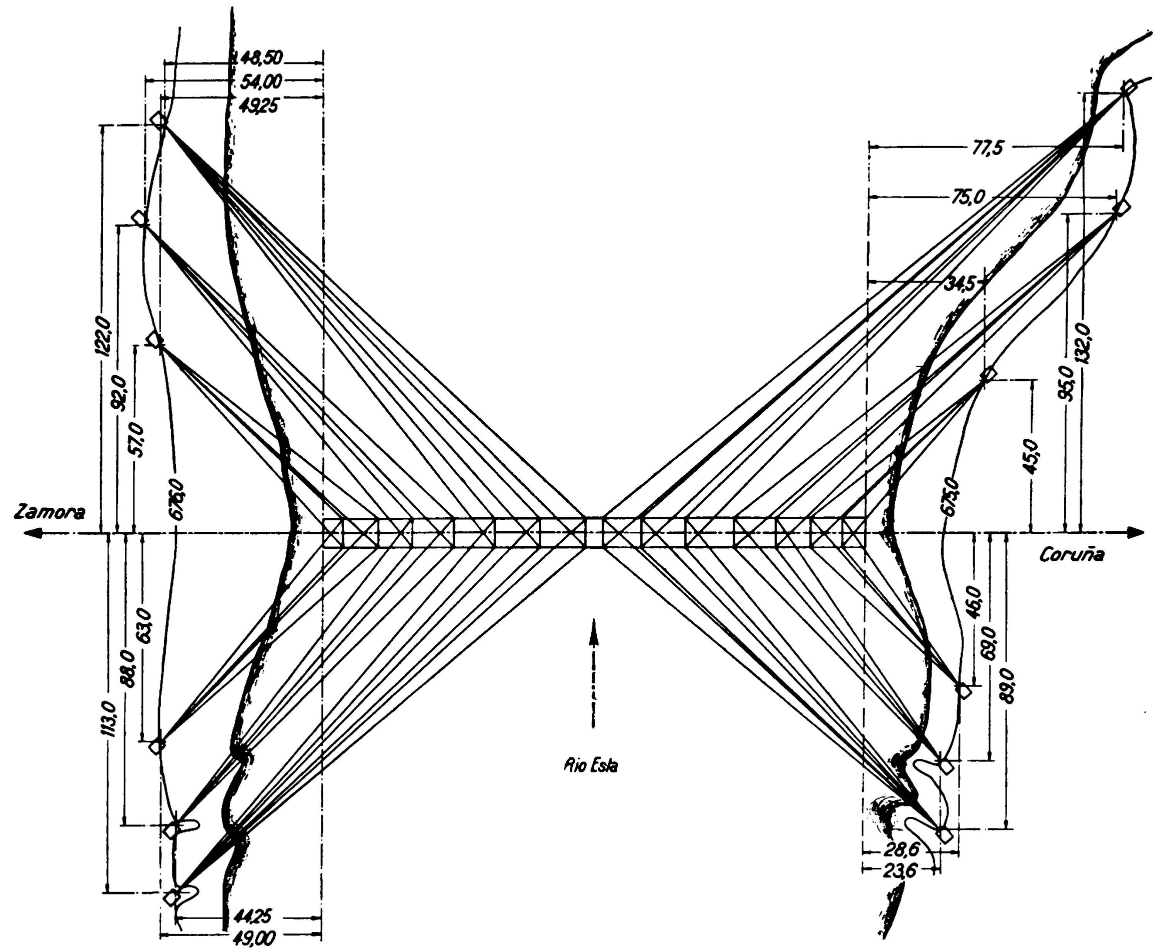

Fig. 9.

Cable system to ensure stability.

Provision has been made for 86 auscultators to be embedded in the concrete mass, thus, enabling the accuracy of the calculations and hypotheses to be checked at any time. The site is in a district where the climate is hard and dry, and these observations may be expected to furnish important knowledge on the phenomena of shrinkage and creep, especially when compared with the results already obtained at the Plougastel and Traneberg bridges which are situated on northern coasts where the climate is damp and cold.

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