Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 2 (1936)

Rubrik: IIIa. Influence of dynamic and frequently alternating loading on welded structures: research work and its practical application

III a

Influence of dynamic and frequently alternating loading on welded structures
(Research work and its practical application).

Einfluß dynamischer und häufig wechselnder Lastwirkungen auf geschweißte Konstruktionen
(Versuchsforschungen und Auswirkung auf die praktische Ausführung).

Influence des actions dynamiques sur les constructions soudées
(études expérimentales et résultats pratiques).

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IIIa 1

General Considerations on Welding.

Allgemeine Betrachtungen über das Schweißen.

Considérations générales sur le soudage.

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The results obtained between 1928 and 1933 by Professor *O. Graf* in his experiments on fatigue in the Materials Testing Laboratory of the Technical University of Stuttgart have been summarised in a report by the present writer entitled "On fatigue strength of rivetted and welded joints and the design of dynamically stressed structural members based on conclusions drawn from fatigue tests" (I.A.B.S.E. Zurich, 1934). Since that time our knowledge of the effects of dynamic and frequently repeated loading on rivetted and welded members has been extended and deepened by a great number of researches.

It has been found, in agreement with parallel investigations in other research laboratoires, that the fatigue resistance of structural steels of different compositions and origins is affected by irregularities in the material, by surface conditions and by the treatment received in the workshop. It has likewise been found that unfavourable effects in this respect may be associated with the presence of contractions or enlargements of the cross section, as well as holes and weld seams, and that when fracture occurs under steadily increasing static loads, it is preceded by a considerable amount of plastic deformation, which in turn is connected with the phenomenon of cold working. Under a condition of pulsating or alternating stress, many times repeated, the plastic deformation and the hardening brought about by the cold working effect are much reduced, and the fatigue stress value may lie a good deal lower than the normal breaking stress value. Under static loading, local increases and concentrations of stress may be mitigated by the equalising action which occurs in the plastic region but under dynamic loading such local concentrations of stress tend to manifest themselves in a reduction of the fatigue resistance. This effect is more marked in hard than in soft steels, and those in which an increase in the breaking strength or yield point is accompanied by a large increase in the susceptibility to notching are unsuitable for use in bridge construction.

Test bars of St. 37 and St. 52 respectively gave tensile strengths of $\sigma_B = 40$ and 57 kg/mm², and surge load limits of stress $\sigma_U = 27$ and 31 kg/mm². The ratio of the surge load limit of stress to tensile breaking stress is 0.68 for St. 37 and 0.55 for St. 52, the corresponding value as obtained on bars with holes in them being 0.50 for St. 37 and 0.36 for St. 52. The reduction is greater in the case of St. 52 than in that of St. 37, a fact which may be attributed to the former being more susceptible to notching.

The usual grade of structural steel, St. 37, is equally adapted for rivetting and welding, but high tensile steels of St. 52 grade show considerable differences as regards weldability according to their compositions and origins. High carbon or alloy steels which have been incorrectly treated either mechanically or thermally have been known to result in failures. Thus a steel St. 52 containing 0.25 % C, 1.20 % Mn, 0.76 % Si, 0.5 % Cu, 0.023 % S and P, which gave the high tensile strength value of $\sigma_B = 58\ \mathrm{kg/mm^2}$ and a yield point $\sigma_F = 44\ \mathrm{kg/mm^2}$ with 20 % elongation, nevertheless showed very fine cracklike notches radiating from the edge and these developed into dangerous cracks when the usual unavoidable shrinkage stresses occurred on the steel being welded. In steels of grade St. 52 having smooth surfaces free from notches and containing less than 0.18 % C and less than 0.5 % Si these effects have not hitherto been observed.

It follows, therefore, that if St. 52 is used for welded structures its optimum composition must be confirmed by experiment. Another matter calling for consideration is the choice of the correct welding rod to give the best welded joint on the parent material in question, and this is best decided by using the commonest forms of electrodes to make butt welds in test bars wich can be subjected to fatigue tests to determine the surge load limit of stress. The composition and crystal structure, both of the parent metal and the weld metal, should be ascertained, and also their liability to cracking when welded.

I. The first German Regulations for Welded Steel Structures (DIN 4000 for bridges and structural work) appeared in 1931. These were based on statical tests carried out in the material testing laboratory at Dresden. At that time a connection made with side fillet welds was considered more reliable than a butt joint, and the joints were, therefore, covered with fish plates after the manner of rivetted work; in other respects also welded bridges were treated in the same way as the accepted forms of rivetted bridges, the same formulae being used in each case to determine the cross sections. For instance under alternating stress with St. 37 the stress was calculated as $\sigma = \dfrac{M_{max} - 0{,}3\, M_{min}}{W_n} \leqq \sigma_{zul} \leqq 14\ \mathrm{kg/mm^2}$. For weld seams in the region of pulsating stress the American formula $M = \max M + \dfrac{1}{2}\,(\max M - \min M)$ was introduced.

The earliest fatigue tests to be carried out on welded structural elements showed surprising results by comparison with the static tests. It was found that the efficiency of a welded connection depended not only on the mechanical properties of the weld, which were influenced by the thermal effects of the welding process on the properties of the parent material and transition zone, but also, to a still greater extent, on the flow of stress, and that the principal criterion governing breakage through fatigue was in fact the internal distribution of stress.

Hence the butt joint which allows an undisturbed flow of stress shows considerably better values of "pulsating" strength than the strapped joint with side fillets which involves deviations in the flow and concentrations of stress. Consequently, if butt connections were reinforced by straps with fillet seams the fatigue strength was found to be considerably reduced, whether against alternating or pulsating loads, and the originating crack for fatigue breakage was found to start from the end fillets. Moreover, it was found that light concave fillet welds, causing a gradual transfer of stress from the weld to the plate, gave better results than full fillet welds, contrary to what had hitherto been thought.

On the basis of these experimental results it was prescribed in DIN 4100, issued in 1931, that a butt weld might be used with four-fifths of the permissible stress of the parent metal. This decision led to the development of oblique seams, but it was found in later experiments that the lengthening of the weld seam in this way did not enable the fatigue strength of the connection to be increased by comparison with a square butt seam. On the other hand, where cover straps to the connections were provided it was found possible to increase the fatigue strength by taking suitable constructional precautions such as by bevelling the ends of the straps in order to ensure that the change in the cross section should be gradual, and by careful after-treatment of the fillet seams by milling, particularly at the beginning of the side fillets. In accordance with these facts the regulations had to be again amended and redrafted, and the occasion was seized to introduce a fundamental distinction between structures carrying static stresses and those carrying mainly dynamic stresses.

If the fatigue strength of either an ordinary or a high tensile structural steel is plotted in a diagram the result is the σ_o line shown in Fig. 1, referred to lower stress value σ_u. The amplitude $\sigma_S = \sigma_O - \sigma_U$ is determined from the distance between σ_O line from a line inclined at 45^0. The shaded area represents the "pulsating" zone, those portions of the diagram which extend beyond this limit being ruled out for practical purposes, because the deformations imply that the yield point or crushing stress is exceeded. In the remaining region of the diagram the curve of fatigue strength may be approximately represented by a straight line. The amplitude decreases somewhat in the region nearing the yield point. According to the experiments of Professor *Graf* it is amply accurate enough to assume that the amplitude σ_S is of uniform amount throughout the tensile and compression zone for welded constructions; in other words it may be assumed that $\sigma_S = \sigma_U = 2\,\sigma_W$.

The factors governing the dimensions of bridge members subject to traffic loads, which cause pulsating stresses, are the respective maximum values of the normal forces, the shear forces and the bending moments under the influence of the stationary load and of the live load multiplied by the impact coefficient φ. The stresses calculated from these maximum values must fall within the safe limits as indicated in the region $\sigma_{D\,zul}$.

On the basis of experiments recorded in *Dauerfestigkeitsversuche mit Schweiß-verbindungen* (Berlin, 1925, VDI-Verlag) the limits of stress as adopted by the Drafting Committee for Regulations for Welded Railway Bridges (on which both scientific research and practical interests are represented) are those indicated in Figs. 2a and 2b.

There is an unmistakeable tendency to force the permissible stresses to a maximum, and the values of $\sigma_{D\,zul}$ are in fact very close to the alternating stress values as determined by experiment (average values for 2000000 alternations of stress), with the result that the available margin of stress is at places only

1 kg/mm² even when no account is taken of the unavoidable shrinkage stresses. Retaining the methods of calculation prescribed by the "BE" (Rules for calculating steel bridges) a variable "shape coefficient" α is introduced in addition to the vibration factor γ appertaining to an unjointed member, in order to allow

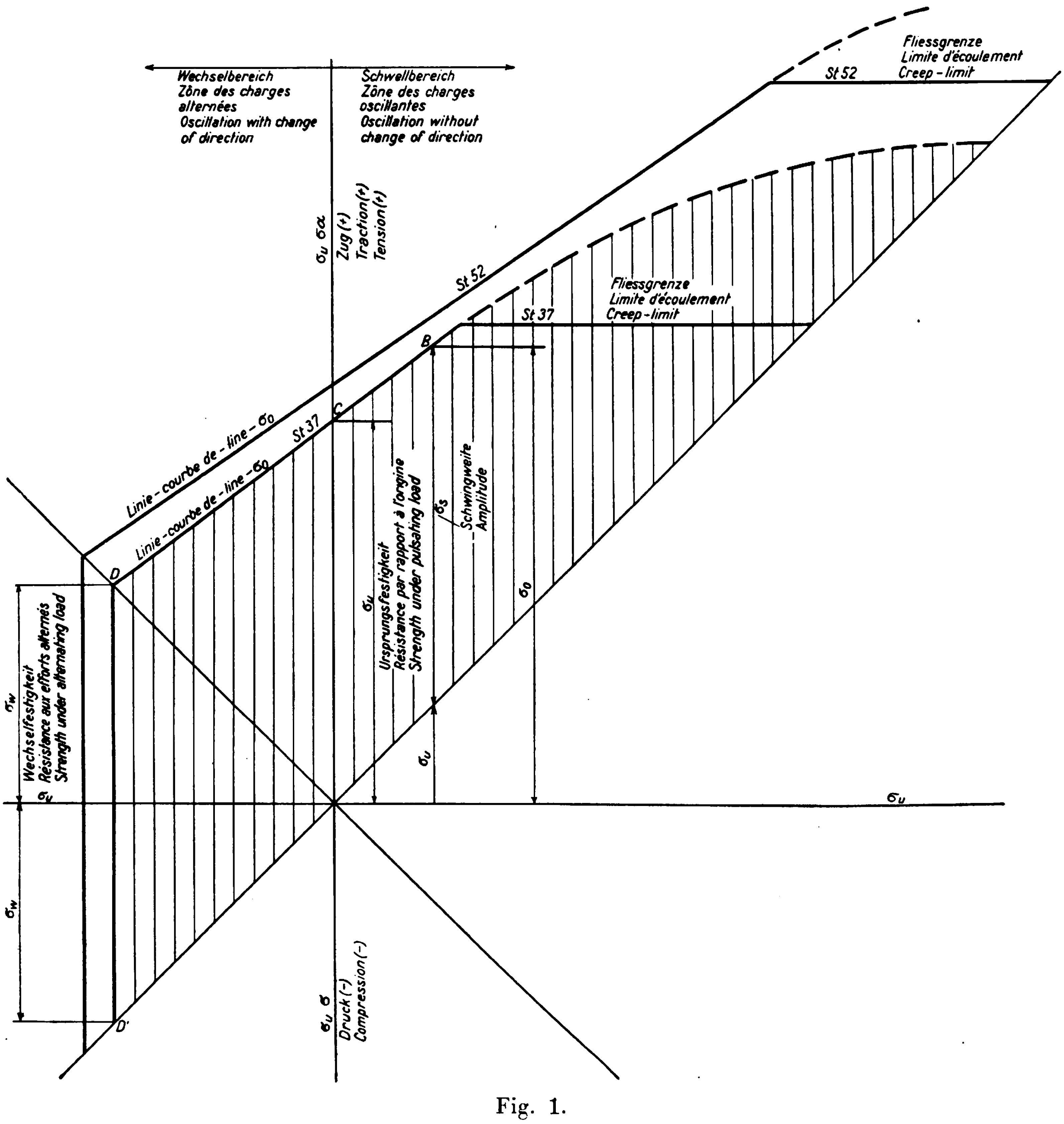

Fig. 1.

Curve of fatigue strengths σ_0 for St. 37 and St. 52 in tension.

of treating a dynamically loaded member in the same way as a member subjected to stationary loading. According to $\sigma_1 = \frac{\gamma}{\alpha} \cdot \frac{\max M_I}{W_n} \leqq \sigma_{zul}$ the maximum values of bending moments ($M_I = M_g + \varphi \cdot M_p$) for dead load and live load respectively are to be multiplied by the coefficient γ and again multiplied by the coefficient α according to the nature of the construction and that of welding. Thus the calculation is performed on the basis of imaginary stresses σ_I, whereas the stresses which actually arise are given by $\sigma_{max} = \frac{\alpha}{\gamma} \cdot \sigma_I \left(= \frac{\max M_I}{W_n}\right)$.

The values of γ and α applicable to railway bridges are shown in Tables 1, 2 and 3 of the "Provisional Regulations for Welded Solid-Webbed Railway

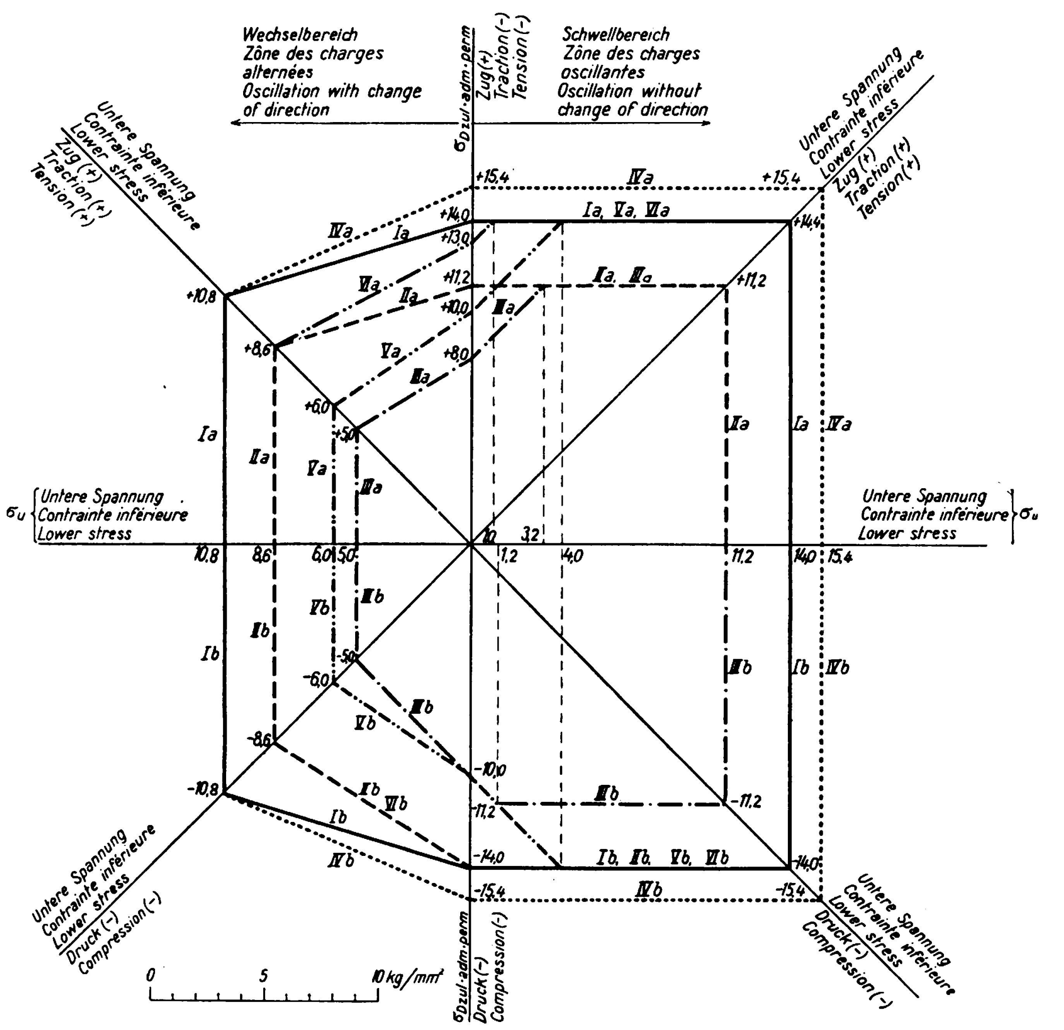

Fig. 2a.

Diagram of permissible stresses σ_{Dzul} in welded bridges of St. 37:

Ia, Ib	— unjointed members in tensile and compressive zones.
IIa	— jointed members in tensile zone, close to butt welds and in the butt welds themselves, when the roots of the seams have been re-welded and the seams worked over.
IIb	— same as IIa, in compressive zone.
IIIa, IIIb	— same as IIa and IIb when it is not possible to re-weld at the root.
IVa, IVb	— permissible principal stresses according to the formula $$\sigma = \frac{\sigma_I}{2} + \frac{1}{2}\sqrt{\sigma_I^2 + 4\tau_I^2}.$$
Va, Vb	— members close to end fillet welds and at the beginnings of side fillet welds, the transitions to the former and the ends of the latter not being worked over.
VIa, VIb	— same as Va and Vb with optimum treatment of the transitions to end fillets and ends of side fillets.

Bridges" (*Vorläufigen Vorschriften für geschweißte vollwandige Eisenbahnbrücken*). The values of γ, which depend on the ratio $\frac{\min S_I}{\max S_I}$ or $\frac{\min M_I}{\max M_I}$ may be

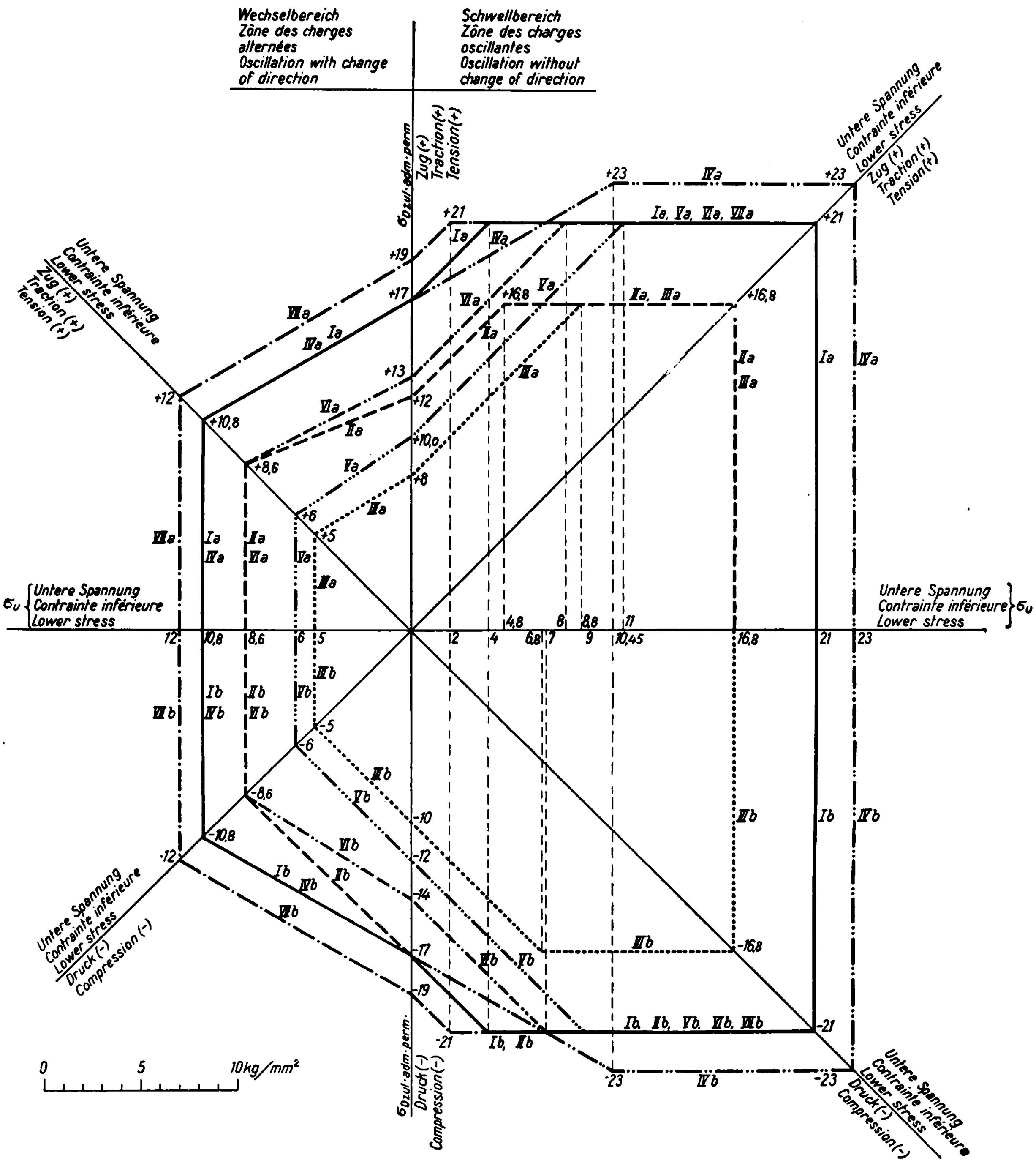

Fig. 2b.

Diagram of permissible stresses $\sigma_{D_{zul}}$ in welded bridges of St. 52.

Ia, Ib — unjointed members in tensile and compressive zones under heavy traffic (more than 25 trains a day on each track).

IIa — jointed members in tensile zone, close to butt welds and in the butt welds themselves, when the roots of the seams have been re-welded and the seams worked over.

IIb — same as IIa, in compressive zone.

IIIa, IIIb — same as IIa and IIb when it is not possible to re-weld at the root.
IVa, IVb — permissible principal stresses according to the formula

$$\sigma = \frac{\sigma_I}{2} + \frac{1}{2}\sqrt{\sigma_I^2 + 4\tau_I^2}.$$

Va, Vb — members close to end fillet welds and at the beginning of side fillet welds, the transitions to the former and the ends of the latter not being worked over.
VIa, VIb — same as Va and Vb with optimum treatment of the transitions to end fillets and ends of side fillets.
VIIa, VIIb — same as Ia, Ib under light traffic (up to 25 trains a day on each track).

inferred from the stress diagram for $\alpha = 1$. The values of α in relation to $\frac{\min S_I}{\max S_I}$ or $\frac{\min M_I}{\max M_I}$ may appear as slightly curved lines, which in the regulations are represented approximately by straight lines.

This form of calculation does not lead to simplification of the design. The reduction in the permissible stresses, clear and un ambiguous in the stress diagrams, is somewhat obscured by the introduction of the coefficients. The natural procedure is to check that the maximum stress σ_{max} as obtained from the statical calculations always falls within the portion of the diagram indicated as permissible. Through the reduction imposed on the value of the permissible stresses, the designing engineer is guided to adopt economical and correct design from the point of view of welding: for instance to adopt butt welds instead of cover straps, to place the joints of girders in the neighbourhood of the points of inflexion of the moments, and to separate the top and bottom welds of plate web girders from the flange plates.

If the amplitude is assumed to have the same values throughout, the general formula for the amplitude falling off in accordance with the yield point or crushing stress, takes the following form:

$$\sigma_{D\,zul} = \frac{\sigma_{U\,zul}}{1 - \frac{\sigma_{U\,zul} - \sigma_{W\,zul}}{\sigma_{W\,zul}} \cdot \frac{\min S}{\max S}} = \frac{\max S}{F_{erf}}$$

With $\sigma_S = \sigma_U = 2\,\sigma_W$ we obtain the simple design formula

$$F_{erf} = \frac{\max S - \min S}{\sigma_{U\,zul}}$$

as given originally by *Wöhler*. In this way the process of calculation is greatly simplified and there is no necessity to make use of tables. It is only necessary to know the basic stresses, that is to say the permissible amplitudes $\sigma_{S\,zul}$ and the reduction coefficients for a butt joint (= 0.8) and for fish plated connections with end and side fillet seams (0.65 — 0.75) respectively. If these values are borne in mind design may be carried out without reference to tables.

Under these simplified assumptions (according to present knowledge) the lines Ia and Ib are applicable to an amplitude of $\sigma_{S\,zul} = \sigma_{U\,zul} = 2\,\sigma_{W\,zul} = 14$ or 16 kg/mm² respectively for tension or compression in continuous unjointed members made of St. 37 or St. 52. Similarly the line IIa applies to butt jointed members (wherein the root of the seam has been welded over and wherein transitions of the seam have been after-treated) for the tensile zone with an amplitude of $0.8 \times 14 = 11.20$ kg/mm² for St. 37 or $0.8 \times 16 = 12.80$ kg/mm² for St. 52. Butt welds in the compression zone may be stressed to the same extent as continuous members without joints (line IIb). In the case of members in the proximity of end fillet seams and at the beginning of the end of side fillet seams (with the transitions carefully worked) the lines IIIa and IIIb are applicable ($0.75 \times 14 = 10.5$ kg/mm² for St. 37, $0.75 \times 16 = 12.0$ kg/mm² for St. 52). If, however, the seams are not worked, the lines to use are IVa and IVb ($0.65 \times 14 = 9.1$ kg/mm² for St. 37; $0.65 \times 16 = 10.4$ kg/mm² for St. 52). (Figs. 3a and 3b.)

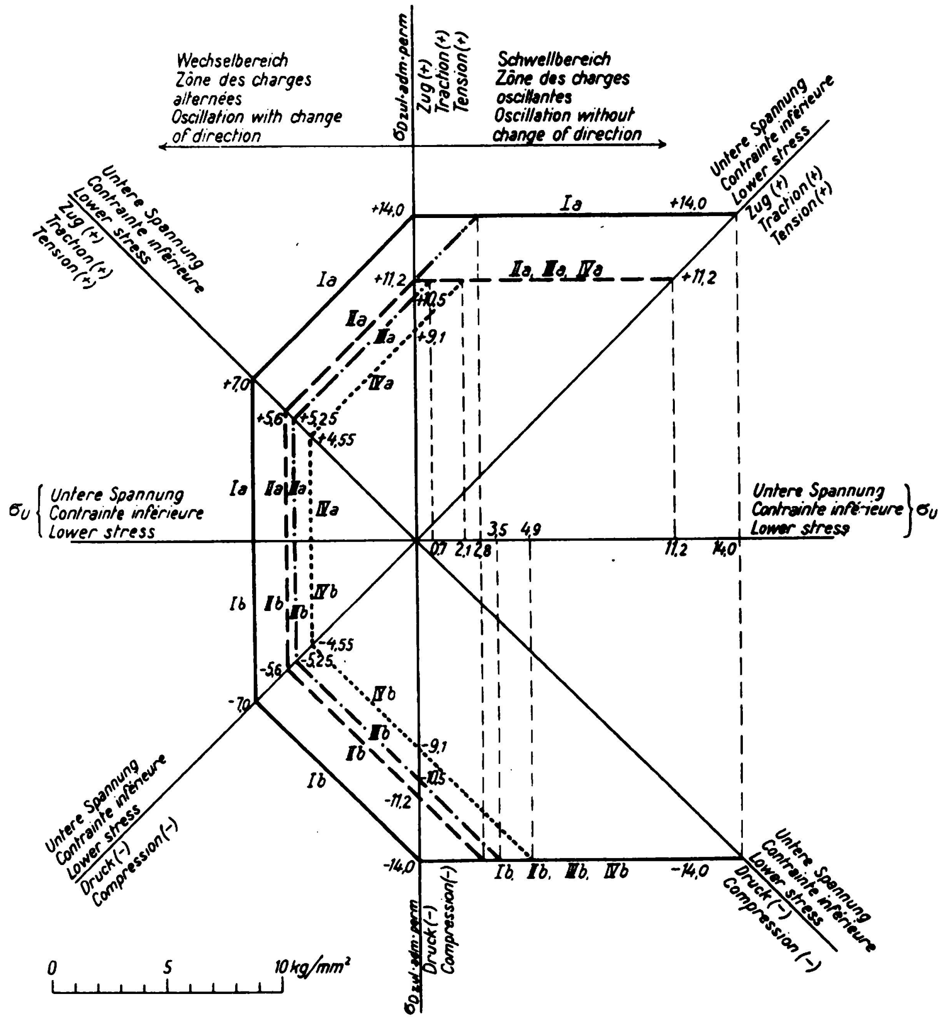

Fig. 3a.

Diagram of permissible stresses $\sigma_{D_{zul}}$ in welded bridges of St. 37.

Ia, Ib — unjointed members in tensile and compressive zones $\sigma_u = 14$ kg/mm².

IIa, IIb — jointed members in tensile and conpressive zones, close to butt welds and in the butt welds themselves, when the roots of the seams have been re-welded and the seams worked over. $\sigma_u = 0{,}8 \times 14 = 11{,}2$ kg/cm².

IIIa, IIIb — members in tensile and compressive zones close to end fillet welds and at the beginnings of side fillet welds, the transitions to the former and the ends of the latter being carefully worked over. $\sigma_u = 0{,}75 \times 14 = 10{,}5$ kg/mm².

IVa, IVb — same as above, in tensile and compressive zones, transitions to end fillets and ends of side fillets not being worked over. $\sigma_u = 0{,}65 \times 14 = 9{,}1$ kg/mm².

II. Turning now to the application of this research work to practical design, the fatigue tests have made it possible to make better use of the properties of the material by careful design and in this way to save weight, while at the same time

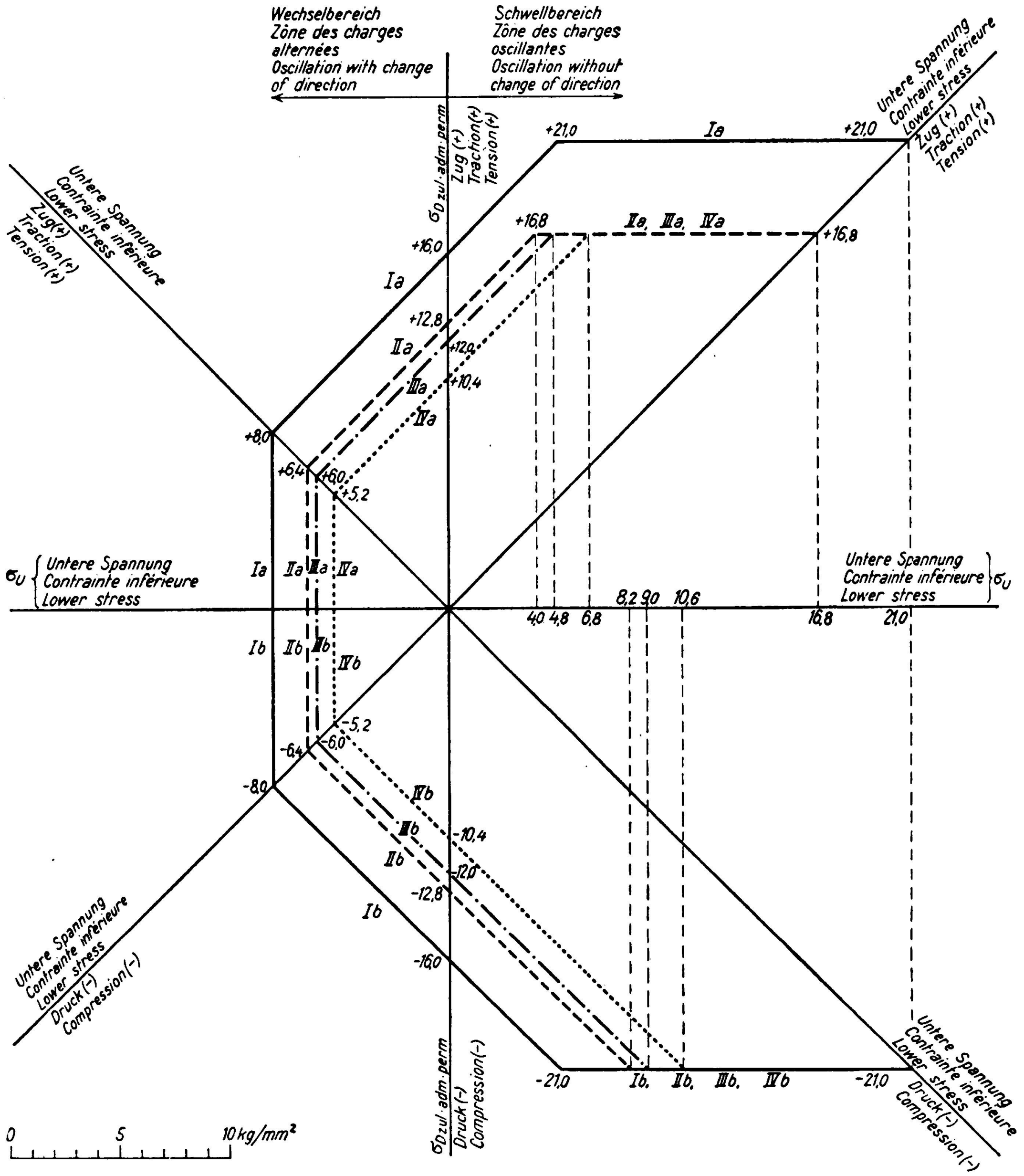

Fig. 3b.

Diagram of permissible stresses $\sigma_{D\,zul}$ in welded bridges of St. 52.

Ia, Ib — unjointed members in tensile and compressive zones. $\sigma_u = 16$ kg/mm².

IIa, IIb — jointed members in tensile and compressive zones, close to butt welds and in the butt welds themselves, when the roots of the seams have been re-welded and the seams worked over. $\sigma_u = 0{,}8 \times 16 = 12{,}0$ kg/mm².

IIIa, IIIb — members in tensile and compressive zones close to end fillet welds and at the beginnings of side fillet welds, the transitions to the former and the ends of the latter being carefully worked over. $\sigma_u = 0{,}75 \times 16 = 12{,}0$ kg/mm².

IVa, IVb — same as above, in tensile and compressive zones, transitions to end fillets and ends of side fillets not being worked over. $\sigma_u = 0{,}65 \times 16 = 10{,}4$ kg/mm².

increasing the safety and reliability of the welded structure. The design of welded connections on the basis of statical tests has often led to incorrect conclusions and faulty construction, and in this respect it may be remembered that proposals were put forward to strengthen rivetted connections by side fillets and butt welds to cover by welded — on fish plates; but the real improvement in correct welding is especially due to fatigue tests. The first rule to be followed in correct welding design is to avoid everything which may prejudice fatigue resistance.

The knowledge obtained from fatigue experiments has led to a change in the features of welded structures; the designer of welded work has freed himself to an increasing extent from rivetted forms. He has learnt to produce bridges from the smallest possible number of easily weldable elements; to avoid undesirable multiplication of weld seams, deviations of the flow of stress through cover straps and sudden changes of direction, concentrations of stress due to sudden changes in cross section-or at any rate to reduce these features to a minimum.

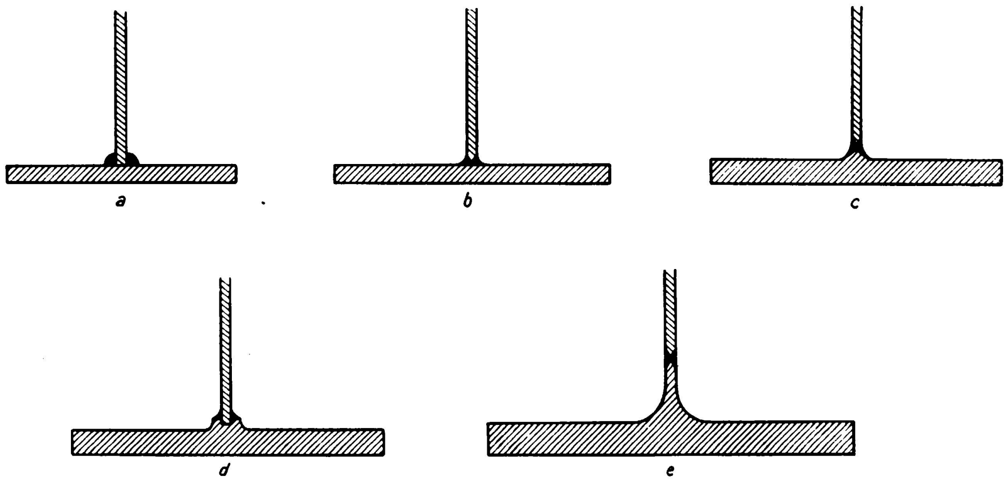

Fig. 4.

And finally, he has learned to make the weld seams as short and small in cross sections as possible with a view to minimising the anuvoidable thermal and shrinkage stresses.

The most important structural form in bridge work at the present time is the solid-webbed girder made by welding together plates and special rolled sections of differing widths and thicknesses. The suitability and economy of the plate web girder has been proved numerous examples, and through the work of *Dr. Schaper* girders of this type have been built up to 54 m in span (for instance in the Rügendamm bridge).

Welded girders of I-section, built after the manner of a rolled joist, are used with advantage in small and medium span of bridges. This type gives a freer choice of dimensions, not being limited by considerations of rolling, as is the joist type of bridge. Besides the I-section, use is made of box and other hollow cross sections.

In the earliest welded plate web girders (Fig. 4) the flange plates were connected to the web plate by full fillet seams (Fig. 4a), but experience and research have led to the bevelling the edges of the web plate and welding with concave fillet seams on either side (Fig. 4b). With thick web plates this, however, led to difficulties in ensuring penetration at the root of the weld, and the further step was taken of adopting special shapes of rolled section for the flange plates, among which may be mentioned the "nosed section" *(Nasenprofil)* of the Dortmunder Union for use with fillet seams (Fig. 4d), the spine-plate *(Wulstprofil)* of *Dr. Dörnen* (Fig. 4c), and also Krupp's ribbed plates which are butt-welded to the web plate. The displacement of the weld connecting the web plate to the flange plate into a region of lower stress, and the improved stress conditions brought about by the more uniform transition, are among the advantages obtained in the last mentioned, while at the same time notching effects in the flanges is avoided (Fig. 4e). There is the further advantage that bringing the seam some distance away from the flange renders it accessible for examination, and it can be easily tested by X-rays.

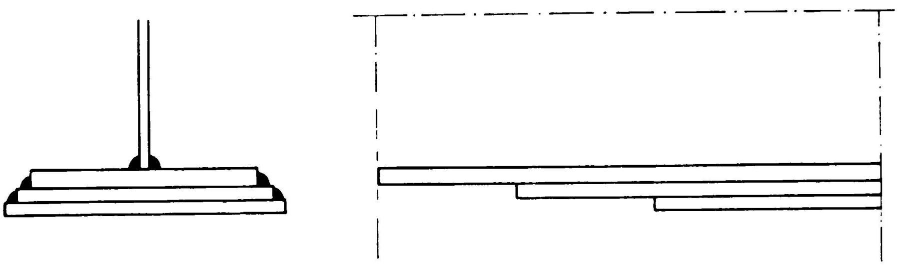

Fig. 5.

For economic reasons it is not possible, in girders of long span, to avoid the necessity for curtailing the cross section of the flanges in accordance with the bending moment diagram. In such cases the additional flange plates have been stacked one above the other, and welded by means of fillet seams (Fig. 5). From a constructional point of view this arrangement is open to some objection, because only the innermost plate of the stack is connected to the web plate and

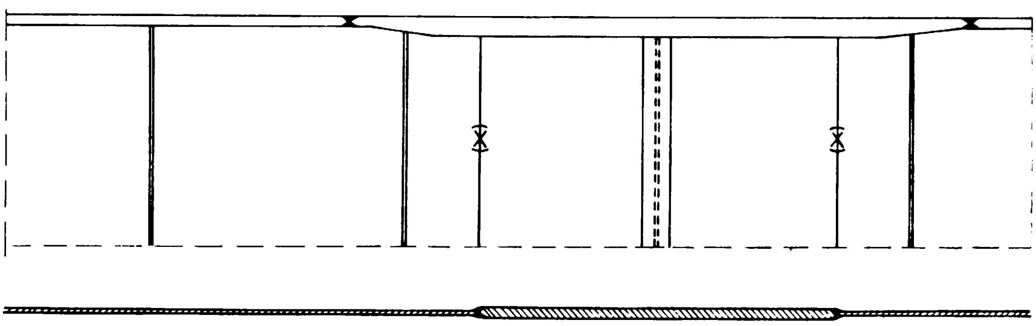

Fig. 6.

fillet seams are inadequate as a means of connecting wide plates; when the fillet seams are welded the wide plates tend to arch, and in this way the risk of buckling in the compression zone is increased. The disadvantage is reduced if the flange plates are made of differing thicknesses and are connected by butt welds so as to give a gradual transition (Fig. 6). Usually the thickening of the flanges is placed on the inside so as to obtain flat surfaces on the outside.

A good arrangement is obtainable by the use of nosed and ribbed sections with reinforcing plates welded onto the inside (Figs. 7a to c). If this design is adopted the rib of the web plate should be made high and strong enough to make the weld easily accessible and to allow of the additional plate for strengthening the flanges being connected to the web plate by butt welds; in

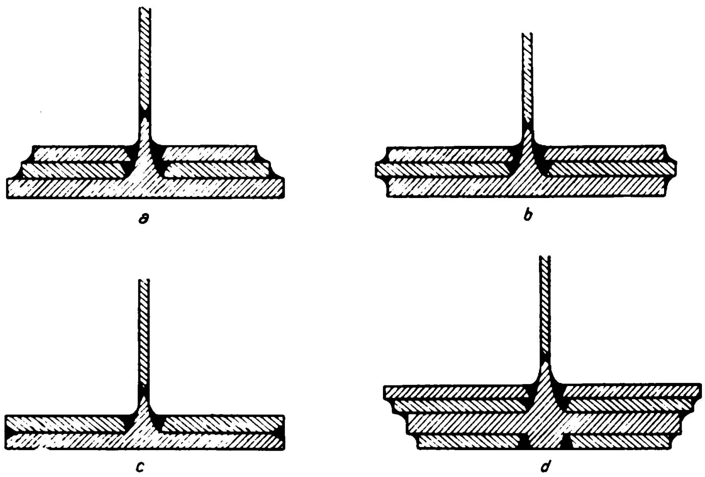

Fig. 7.

this way a perfectly uniform flow of stress from the flanges to the web may be ensured. A further improvement towards making the cross section of the flanges conform to the curve of maximum moment is obtained by the use of cross rib plates (Fig. 7d) with additional flange reinforcing plates connected to the ribs

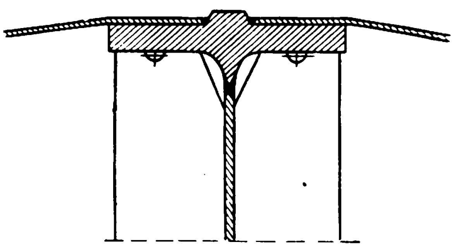

Fig. 8.

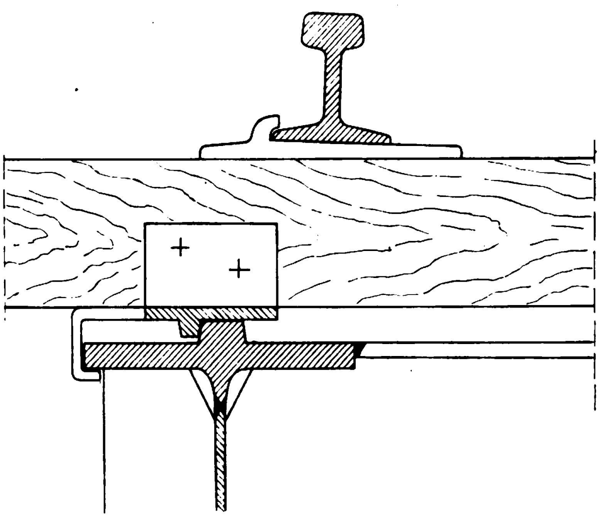

Fig. 9.

by butt welds on either side. In the case of road bridges it is advantageous to make use of trough plates or buckler plates welded into position by butt seams (Fig. 8), and in railway bridges the adoption of cross ribbed plates enables the sleepers to be arranged centrally (Fig. 9).[1]

[1] *Schaechterle:* Der geschweißte Vollwandträger. Beitrag zur Gestaltung von geschweißten Brücken. Bauingenieur, Vol. 17, Nos. 15/16, pages 131 follg.

Another important consideration in the design of welded plate web girders is the stiffening of the sides of the web. Where the latter is of considerable depth the stiffening may be provided by flat plates, T or I sections. If the web plate is more than 16 mm thick it is not necessary that the seams should be staggered, but to avoid crossing the welds which connect the flange to the web the inner corners of the stiffeners are cut away. In the tension zone packing plates are driven in, and are connected to the stiffeners by fillet welds in order to prevent any weakening of the tensile flange by notches burnt out. In deck bridges it is advisable to connect the upper flange of the cross girders or cross frames to the main girders by butt welds, and also to weld the web plates of the cross girders to the main girder completely at the corner of the frame; in this way the forces in the flange of the cross framing will be transferred to the main girder and there will be no lack of rigidity (Fig. 10).

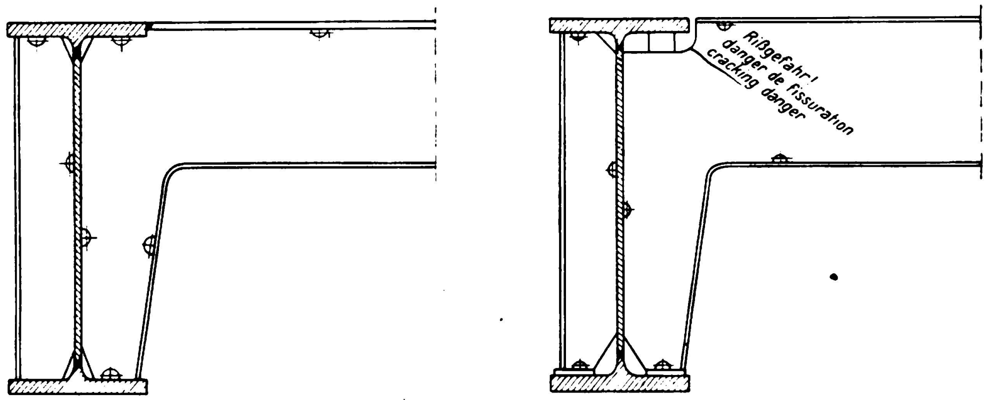

Fig. 10.

III. A problem which still awaits solution is that of the shrinkage stresses which arise in welding. In the regulations no account is taken of these, although it is a matter of experience that they may reach high values, and where the design or sequence of welding are faulty may even lead to cracking. The study of shrinkage stresses has been somewhat neglected even in research, and the next task to be undertaken must be that of ascertaining the magnitude of these shrinkage stresses in bridges, and their effect on the fatigue resistance. The existence of internal stresses in welded structures is a danger, if by the welding operation fine cracks are generated.

Recorded failures are attributable to the following causes:

1) High carbon or high silicon welds with defects, such as very fine notches in the form of cracks, pores, laps, doublings, etc.

2) Faulty workmanship, such as the cold bending of stiff sections.

3) Unsuitable heat treatment, with the result that cooling and shrinking stresses are superimposed on one another.

4) Concentrations of stress due to weld seams being crowded together, sudden changes of cross section, etc.

It is necessary to learn first of all to master shrinkage stresses in actual practize, and from this point of view the welding process itself should be properly planned on the basis of experience and research.

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Shrinkage stresses arise when the free movement of structural parts which are being welded together is hindered. Up to a certain point shrinkage stresses can be eliminated by cold stretching, but the conditions are not the same as those in joists subjected to rolling and cooling stresses, and under unfavourable conditions the shrinkage stresses in welds may reach considerably higher values. Hence every precaution must be taken in the welding of bridge work to reduce the shrinkage stresses to a minimum, and these precautions should begin even in the selection of plates or pieces of material for the welded connections, which should be governed by the possibility of forming perfect weld seams. Since plates less than 8 mm thick are only exceptionally used in bridge construction the continuous one layer welding is not practised. For forming V, X or U seams the abutting edges of the plate should be carefully prepared so as to ensure that the cross section of the seam will extend uniformly over the whole length and that thickenings of weld metal will be avoided. The welding gap should be at least 2 mm, in order to ensure perfect penetration of the weld at the root and to avoid too deep scraping out of the root when finishing the seam afterwards. While the seams are being formed, the different parts of the work must be so arranged that movement in the principal direction of shrinkage is freely possible, while at the same time deformations are limited to a minimum. In selecting the welding rods the thickness of the plates to be connected must be considered; the thicker the plates the thicker the electrodes to be used. The risk of cracking is greatest while depositing the first layer of the seam, and great care should, therefore, be exercised when making the first run using an electrode too thin. The butt joint with a V seam is adopted for thin plates, and also for thick plates where it is not possible to turn the work over.

On thick plates the U seam has the advantage over the V seam of requiring a smaller amount of weld metal. Either kind of seam allows of the root being cleaned out and rewelded overhead. In both cases the matter of angular distortion requires attention, and this may be met by the precaution of placing the two plates to be joined at a small angle to one another so that after the angular distortion has taken place they will lie in the same plane. The X form of seam is the one which requires the smallest amount of weld metal and is used where the welding can be performed in a revolving jig; also in vertical joints and in cases where overhead welding is permitted, angular distortion being balanced out by welding the runs on either side simultaneously or alternately. To ensure that the two halves of the seam shall be as nearly as possible equal in size when completed, the portion of the cross section which is to be welded first must be made somewhat larger than the other in order to allow for scraping out and rewelding the root. If the lower portion of the seam is to be welded overhead, it is desirable to arrange the root of the weld in the lower third.

Butt welds should be arranged as nearly as possible at right angles to the axis of the girder in order to require a minimum of weld metal. In some cases, where joints in wide flanged girders have been welded with inclined butt welds, a great deal of contraction has occurred and part of the web plate has been torn away from the seam (Fig. 11), but if square butt welds are used these defects do not occur. Double sided stiffeners and cross girders are connected to the web by means of thin fillet welds. Double fillet welds, in the same way as X welds, allow of the angular distortion being reduced or compensated by attention to the sequence of welding. Apart from this angular effect, transverse and longitudinal shrinkage occurs. Long seams are usually begun in the middle and run towards both ends simultaneously. The intermittent method is less used in bridge work as the number of beginnings that have to be made is liable to cause defects. It is possible to reduce transverse stresses and shrinkage by heat treatment of the completed section of weld metal. If the artificial heating of the seam ceases on the completion of the welding operation, transverse shrinkage may occur over the whole length of the weld and this can only result in a shortening of the member; this presupposes that all parts of the member are free to move easily and without restraint.

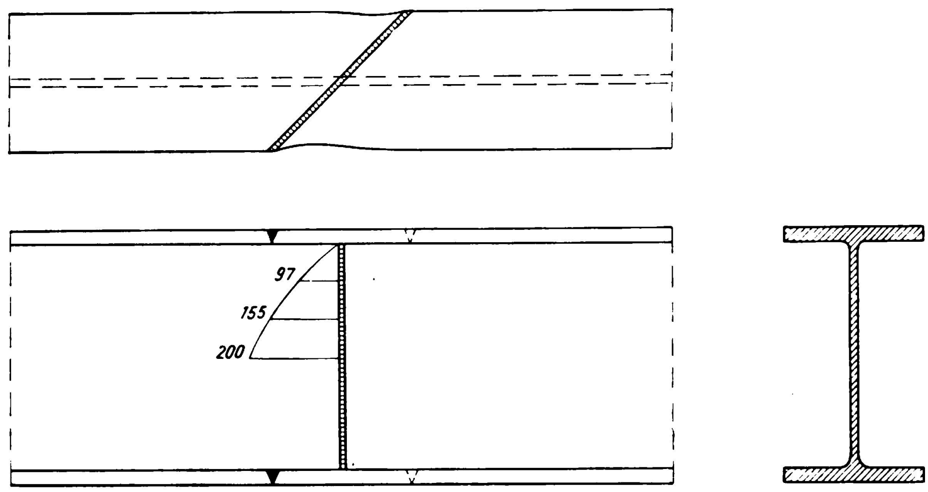

Fig. 11.

Apart from the shrinkages of the weld seam itself, expansion takes place in the adjacent zones on either side of the seam during the welding operation and is later followed by contraction. These effects cannot be mitigated by additional heating before and after the operation but all that can be done in this way is to equalise the stress. Unrestrained expansion during the process of pre-heating, and contraction afterwards, may be ensured by the use of moveable bearings.

The requirement of free movement for the work while being welded is easily secured in the workshop, and this being the case the web plate and flanges are welded together in long pieces, which are connected finally to form the girder. The device of welding the stiffeners onto the web plate before making the welds between the web and the flanges frees the latter from stress, whereas if this sequence is reversed the greater distortion of the web causes additional stresses in the seams. On the other hand, the accurate fitting of stiffeners serves to prevent cross shrinkage at the places where they are affixed, with the result that the flange is apt to assume an undulating shape, a defect which cannot be prevented by leaving the middle portion of the web plate to be inserted later, with pre-heating. If, however, the stiffeners are arranged with a certain amount of play, it is desirable to have available a number of packers of different thicknesses so as to be able to take up the slight changes in length wich result from the shrinkage. If there is a gap between the flange plate and a stiffener it is wrong to form a connection by welding this gap, as such a procedure would have the effect of drawing down the flange plate. When the girder is assembled the longitudinal contractions of the seam become apparent as a shortening of the work as a whole, and since the magnitude of the longitudinal shrinkage depends on the length of the seams it is customary to begin the weld at the middle and work towards either end. It is an advantage to connect both the flanges to the web plate simultaneously, or else to weld the top and bottom seams alternately, so as to prevent the web plate becoming curved towards one side.

In some types of bridge sections — for instance, in bridges where the decking is formed from welded on buckler plates, trough plates, or flat plates — weld seams are crowded together in the upper flange of the road girders, and since the result of the longitudinal contraction is to shorten the upper boom, bending of the girder takes place this effect must be allowed for in advance by giving an additional camber.

Continuous girders of large size and length require to be assembled on the site, which has hitherto been effected on rollers. This arrangement, however, has the disadvantage that any movement perpendicular to the rollers encounters great frictional resistance which hinders shrinkage in that direction. A method has, therefore, recently been adopted whereby the pieces of the girder are supported, while being welded, in such a way that they can move longitudinally and that each piece added to what is already in position can freely follow the shrinking movement. This arrangement necessitates the occasional use of overhead welds, but under the present conditions of the art of welding these offer no difficulty. The sequence in which the various seams of the girder joint are to be welded is determined by the rule that whatever seam offers the greatest resistance to the free movement erection of the work piece should be welded last.

Flange joints are usually arranged on either side of the web joints. The top and bottom welds, connecting the flanges to the web plate, are left open over a considerable length, so that shrinkages in the flanges and web plate may occur independently. The welding of both the flange seams is begun at the same time, and the gap at the root of the web seam is increased in such a way that after shrinkage in the first layers has taken place, it is still possible to ensure perfect penetration of the web seam. By alternately welding the flange and web seams it is possible to combat the accumulation of stresses, and to obtain a balance between compressive and tensile stresses. Thus the shrinkage of each successive layer of the flange seam causes compression in the layers already completed of the web seam, and in this way the tensile stresses set up by the shrinkage of the web seam are partially compensated. This process is repeated for each layer, so that finally the only stresses remaining are those which are produced by the final layers of the seams, in particular by a weld layer with a larger amount of weld metal. It is a common practice to weld first one-third of the flange seams, then two-thirds of the web plate seams, finally completing the flange seams and the web seams simultaneously ultimately forming the top and bottom neck seams between web and flanges.

IV. In the last few years a number of all-welded bridges have been built, characterised by the straightness and continuity of the girders and extreme simplicity of design. From the aesthetic point of view welded structures are superior to rivetted.

IIIa 2

Dynamic Stresses on Welded Steel Structures.

Dynamische Beanspruchungen bei geschweißten Stahlkonstruktionen.

Actions dynamiques sur les constructions soudées.

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The object of this contribution is to draw attention to a difficulty which, in the writer's opinion, arises in regard to the study of dynamic action on welded structures.

Generally speaking the effect of moving live loads on bridges and building frameworks is to bring into play forces of inertia. Such forces call for a "live" resistance, or resilience, in place of the usual kind of statical resistance which is exerted by materials. It may be observed that in the case of butt welded connections the resilience is always considerable, amounting to at least 8 kg/cm². In practice still higher vlues are obtained, being of the same order as the resilience of the parent metal, approximately 12 kg/cm². From this point of view it would appear, then, that the average resistance of a welded structure, taking full account of the presence of joints, is high enough to ensure that such a structure will behave well under live loads of the kind which may give rise to impact.

The writer has in fact found this to be true as regards the swing bridge at Brest, which, after having undergone strengthening operations, was subjected to tests under the direction of Mons. *Cavenel,* Ingénieur en Chef des Ponts et Chausseés, and Mons. *Lecomte,* Ingénieur des Ponts et Chausseés. These tests yielded very satisfactory results, it being found that after the strengthening work the vibration was considerably reduced.

For some time past a good deal of importance has also been attached to fatigue tests. It is known that if a solid — particularly steel — is subjected to forces which are repeated a great number of times fracture may occur even though the limiting resistance or even the elastic limit has not been reached. This fact obviously implies a serious danger due to dynamic action and the question is indisputably one which deserves the closest examination. The writer is of opinion, nevertheless, that this danger should not be exaggerated, for it is a fact that the majority of framed structures are not subjected to repeated loading of the kind that occurs in mechanical engineering.

A very complete investigation of this problem which has been carried out in France by Mons. *Dutilleul,* Ingénieur du Génie Maritime, indicates that lack of fatigue resistance in welds is always attributable to the presence of air bubbles, or in other words, to the porosity of the metal. In the present state of knowledge on the subject it appears, to the writer, dangerous to adopt as a criterion of quality of welding something which depends partly on chance, and he is of opinion that the resilience is the most important characteristic to consider.

In any case, whether it is resilience or fatigue resistance that is the most important element, there remains the over-riding necessity of ensuring that the weld bead shall be given a shape which will not tend notably to increase the

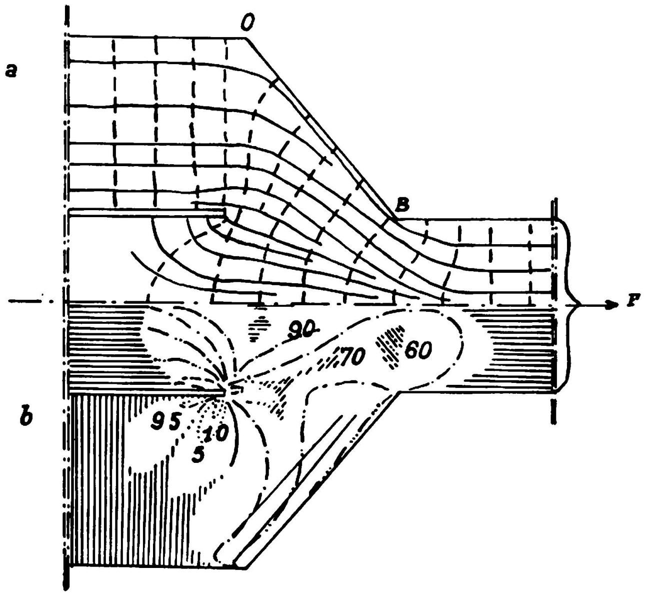

Fig. 1.

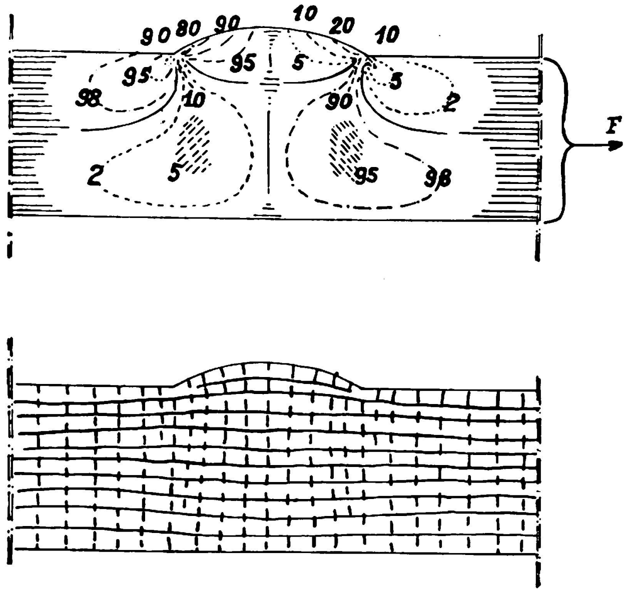

Fig. 2.

risk of rupture. On this subject, as Messrs. *Kommerell* and *Graf* have shown, valuable data can be inferred from the study of lines of principal stress under static loads.

The two illustrations here given go to confirm this argument, representing as they do the lines of force in a transverse weld and in a butt weld (Figs. 1 and 2). These were obtained by the use of polarised light applied to a celluloid model shaped and stressed in the same way as the welded piece. The photographs show very clearly that it is important to minimise any disturbance of the flow of the lines of force. For instance, in the case of the butt weld it may be observed that an excess of thickness may be harmful, in that it perceptibly increases the amount of strain suffered at one of the faces of the specimen.

IIIa 3

Questions for Discussion on Fundamental Relationships and Principles Governing the Fatigue Strengths of Welded Connections.

Diskussionsfragen über Grundbeziehungen und Begriffsfestsetzungen für die Dauerfestigkeit geschweißter Stabverbindungen.

Thèmes de discussion concernant les relations fondamentales et la détermination des notions se rapportant à la résistance à la fatigue des assemblages soudés.

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During the last five years numerous experiments on fatigue have been carried out in the German laboratories for testing materials, with a view to the formulation of official regulations for welded connections of structural members. The interpretation of these experiments has given rise to a series of questions, some of which it has been possible to clear up, while others constitute problems regarding which an international exchange of ideas is desirable and which demand further research.

1) *Representation of the fundamental dimensions force, space and time (Fig. 1).*

a) The ordinary force-space plane X—Y (or stress-strain plane) serves to represent the results obtained in the ordinary statical breaking test, which lies at the basis of the theory of strength and elasticity. The effect of the time factor on the breaking test is usually ignored, but may be recognised in the fact that if the experiment is carried out more quickly the usual line 1 in Fig. 1 merges into line 2, or in the case of impact tests may even merge into line 3.

b) If the third axis of coordinates Z is adopted as the time axis, then the Y—Z plane shows the transition to the region of vibration, or to time-strength relationships, and the result of fatigue tests may be represented in it by a fatigue-time line (known as the *Wöhler* line).[1] Here the abscissae z represent the duration of the experiment, though not according to the usual time scale, for it is expressed by the number of alternations of load (for instance 2 million alter-

[1] *Wöhler:* Zeitschrift für Bauwesen, Berlin 1860, 1863, 1866 and 1870.

nations of 4 seconds each, or 8 million seconds in all). If time is measured in this way, by the number of similar vibrations instead of by the clock, the *Wöhler* line shows how many vibrations can be withstood by the specimen under a given stress. If the experiment continues long enough this line finally becomes parallel to the Z axis at a distance σ_D where it represents the fatigue stress *(Dauerfestigkeit)* that is to say, the amount of stress that can be supported indefinitely.[2]

c) The range of vibration covers a system of coordinates X′ Y′ Z′ which has been displaced parallel to the XYZ system, the stresses being again indicated by the ordinates y′ measured along the direction of the load axis. In the Y′ Z′ plane (the load-time plane) it is possible to visualise the following three fundamental concepts: I) alternating strength for equal positive and negative stresses (*Wechselfestigkeit* σ_W), II) fatigue strength for stresses pulsating between zero and a maximum (*Ursprungsfestigkeit* σ_U), and III) fatigue range (*Schwellfestigkeit* $\sigma_{D\,max}$), which is the fatigue strength subject to a pre-existing stress σ_m (In reference to I) the limiting values of the stress wave are $\sigma_o = +\sigma_w$ and $\sigma_u = -\sigma_w$ hence $\sigma_m = 0$; in reference to II) they are $\sigma_o = \sigma_U$ and $\sigma_u = 0$, hence $\sigma_m = \frac{1}{2} \cdot \sigma_U$).

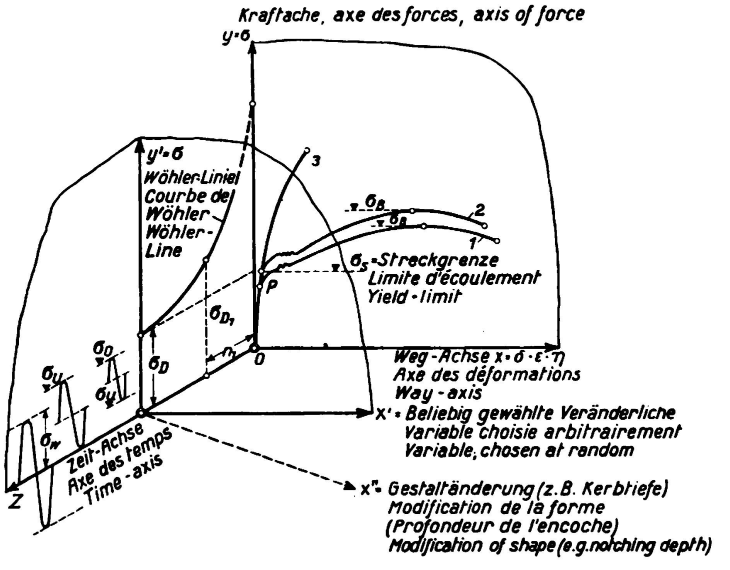

Fig. 1.

d) A plane X″ of X′ rotated about the Y-axis may serve for the representation of any other function of fatigue strength, the stresses σ_D always being measured by the ordinates y′. In this way, for instance, it is possible to indicate the relations of the stresses to the depths of notches in notched bars, or to describe the γ — method of the Reichsbahn, wherein $x' = \sigma_{min} : \sigma_{max}$ is introduced, or to indicate the relationship to statical pre-stressing (*Haigh*'s construction).

This form of representation (Fig. 1) allows all parts of the theory of strength of materials to be indicated with their symbols.

2) *Permissible stresses and design coefficients for steel bridges.*

A) *Rivetted railway bridges.*[3]

Whereas, under stationary or statical loading, safety is determined by the proportions between the stress σ_s which corresponds to the yield point, and the

[2] The first two of the questions for discussion enumerated in Section 8 arise here.

[3] The tern "railway bridges" is used to denote bridges which carry railway tracks in contradistinction to roadways.

stress σ_{zul} which arises under live load (so that $v = \sigma_s : \sigma_{zul}$), under conditions of alternating load safety is determined by the *number of alternations*, and this is a statistical problem.

Case 1. Side members of a trussed main girder in a rivetted railway bridge (Fig. 2).

On an old trussed bridge of 39 m span[4] a tensile stress $\sigma_{max} = + 215$ kg/cm² and a compressive stress $\sigma_{min} = - 70$ kg/cm² were recorded graphically during the passage of two test vehicles of $4 \times 8 = 32$ tons weight, and the curve of stress was determined graphically as in Fig. 2, corresponding to the total influence line of the test crane. If, during the passage of a locomotive the designed

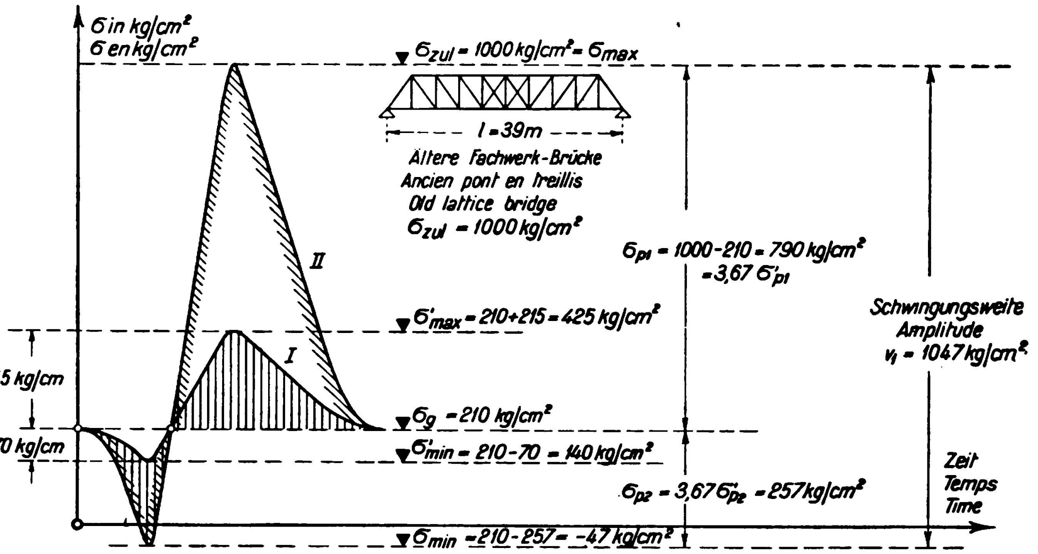

Fig. 2.

Amplitude and static safety of connections of railway bridges.

permissible stress of $\sigma_{zul} = \sigma_{max} = 1000$ kg/cm² is to be utilised, then the curve of stress corresponding to a uniform permanent stress of $\sigma_g = 210$ kg/cm² must be magnified in the ratio of $\frac{1000 - 210}{215} = 3.67$, giving at the trough of the wave $70 \times 3.67 = 257$ kg/cm², and $\sigma_{min} = 210 - 257 = - 47$ kg/cm² (compression). The total amplitude then works out at

$$v_1 = 1000 + 47 = 1047 \text{ kg/cm}^2 \doteq 10.5 \text{ kg/cm}^2. \tag{1}$$

If the problem now arises of determining the fatigue strength σ_{D1} of this member and its connections experimentally, the amplitude must in the same way be fixed at $w_1 = v_1 = 10.5$ kg/cm² in the experiment. If only a small portion of the stress curve for the tensile member in question falls within the compression zone, the fatigue test may properly be based upon the pulsating strength *(Ur-*

[4] *W. Gehler:* Nebenspannungen eiserner Fachwerkbrücken, p. 67 (Wilh. Ernst & Sohn, Berlin 1910).

sprungsfestigkeit) ($\sigma_{min} = 0$; $\sigma_{max} = 10.5$ kg/cm²). For instance, if the fatigue test shows breakage at $n_{D1} = 2000000$ changes of load,[5] and if the bridge is traversed daily by $n_1 = 25$ trains, then this number becomes n_{D1}, and failure need not be apprehended until at the earliest 80000 days or 220 years have elapsed (but under tramway traffic with $n_T = 250$ vehicles per day failure might be expected in only 22 years). Thus the criterion of safety may be expressed in the form of the life of the bridge in days and we have the relationship[6]

$$\nu_T = n_D : n_T \tag{2}$$

A fatigue experiment of this kind, however, does not yield a true picture of the situation, because it is carried through without interruption, whereas in the actual structure long pauses intervene, especially at night, and in these pauses it is conceivable that a recovery of the material may take place. Even if no influence due to rest pauses has hitherto been disclosed by the experiments as regards the fatigue resistance of the materials themselves, it may well be possible that the conditions in regard to rivetted or welded *connections* are more favourable in this respect.[7]

Case 2. Boom of a continuous solid-webbed or openwebbed main girder.

This example shows the necessity of separating, in the longitudinal section of the girder, the zone in which the calculated limiting stresses σ_{max} and σ_{min} have the same sign from the alternating zone in which they have different signs. Thus according to the ratio

$$\xi = \frac{\min S}{\max S} \quad \text{or} \quad \frac{\min M}{\max M} \tag{3}$$

of the statical load S and moment M in the bars, a vibration coefficient

$$\gamma = \frac{\sigma_{zul}}{\sigma_{D\,zul}} > 1 \tag{4}$$

$\left(\text{corresponding to the buckling coefficient } \omega = \frac{\sigma_{zul}}{\sigma_{D\,zul}}\right)$ is to be introduced, because in the fatigue experiments the alternating strength, the pulsating strength and the fatigue range have different values for St. 37 and St. 52. In this way the different γ — ξ lines of the Reichsbahn are obtained (B.E., Berechnungsgrundlagen für deutsche Eisenbahnbrücken, Section 36, Table 17). In a similar way to the assumption of $\frac{\omega \cdot S}{F} \leqq \sigma_{zul}$ for buckling, the stress is here to be taken as

$$\sigma_I = \frac{\gamma \cdot \max S}{F} \leqq \sigma_{zul} \tag{5}$$

and the calculation may then be performed in exactly the same way as for members under purely statical loading.[8]

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[5] Compare first question for discussion.

[6] Compare fourth question for discussion.

[7] Here the third question for discussion arises.

[8] *Kommerell:* Erläuterungen zu den Vorschriften für geschweißte Stahlbauten, Part II, page 39 (Wilhelm Ernst & Sohn, Berlin 1936).

Case 3. Connections between longitudinal and cross girders.

It is a matter of experience that rivets at these connections easily work loose in service, and in the new regulations of the Reichsbahn (B.E., Section 46) it has been sought to promote safety not only by making the design assumptions more severe (increased thrust max $A' = 1.2\ (A_g + \varphi\, A_p)$ and increased bending moment in St. 52 compared with St. 37), but also by the requirement of special constructional precautions (such as provision, in every case, of a plate running through on top). The reduced span of the longitudinal girder is also safeguarded by adopting a higher value of the impact coefficient φ, such as for instance $\varphi = 1.6$ under permanent way with sleepers and $l = 5.0$ m instead of $\varphi = 1.4$ for the main girders of medium span. The only way to estimate the true magnitude and effect of the stress variations in this complicated special case, where the distribution of loading is influenced by the superstructure, and to compare it with the results of fatigue experiments on similar types of connection, would be to carry out exact measurements on actual bridges — a problem which still awaits research.

B) *Welded railway bridges.*

Account is taken of live loading effects in the following ways:

a) By placing stationary train loads in unfavourable positions and plotting influence lines.

b) Impact coefficients of $\varphi \geqq 1$ are adopted (wherein $S = S_g + \varphi \cdot S_p$ or $M = M_g + \varphi\, M_p$) in order to allow for the effect of impact and vibration through the movement of the loads, by comparison with stationary loads (for instance as a result of driving wheel action, rail joints, etc.). Such effects tend to increase the statical deflection. (This is destined to be a principal problem of bridge investigation in the future).

c) The *vibration coefficient* $\gamma \geqq 1$ is expressed as a function of the calculated statical limits min S and max S in order to allow for the difference in fatigue effects on the structural member under alternating and pulsating loads, and apart from this, different values are used for St. 37 and St. 52; also different values according as the traffic is heavy or light ($n_T = 25$ or $n_1 \geqq 25$ trains per day).

d) The *design reduction coefficient* $\alpha \geqq 1$. Whereas the coefficients γ may be fundamentally the same for rivetted and welded bridges the permissible stresses for welded railway bridges have been still further reduced in accordance with the German fatigue experiments,[9, 10] becoming (see equation 5):

$$\sigma'_I = \frac{\sigma_I}{\alpha} = \frac{\gamma \cdot \max S}{\alpha \cdot F} \leqq \sigma_{zul} \quad \text{or} \quad \frac{\gamma}{\alpha} \cdot \frac{\max M}{W} \leqq \sigma_{zul}, \tag{6}$$

wherein the design factor α is given a different value according to the form of seam (whether a butt weld or a fillet weld) and according to the quality of

[9]) Dauerfestigkeitsversuche mit Schweißverbindungen, 1935, VDI-Verlag, Berlin. Joint report of Staatl. Materialprüfungsamt Berlin-Dahlem and Versuchs- und Materialprüfungsamt Dresden. By *K. Memmler, G. Bierett* and *W. Gehler*.

[10] See footnote 8, *Kommerell*, page 44.

workmanship (e. g., whether the root of the seam has been re-welded or not, and whether the finished seams have been improved by further working). Reduction coefficients of this kind are already in use for welded building frames (DIN 4100, Section 5) wherein, for instance, butt welds may be stressed in tension to $\rho_{zul} = 0.75\ \sigma_{zul}$, hence $\alpha = 0.75$.

C) *Rivetted and welded road bridges.*

Road bridges are much less frequently exposed to sustained alternating loading than is the case with railway bridges, and since, moreover, the German loading assumptions (DIN 1073) already ensure ample safety as regards weight and

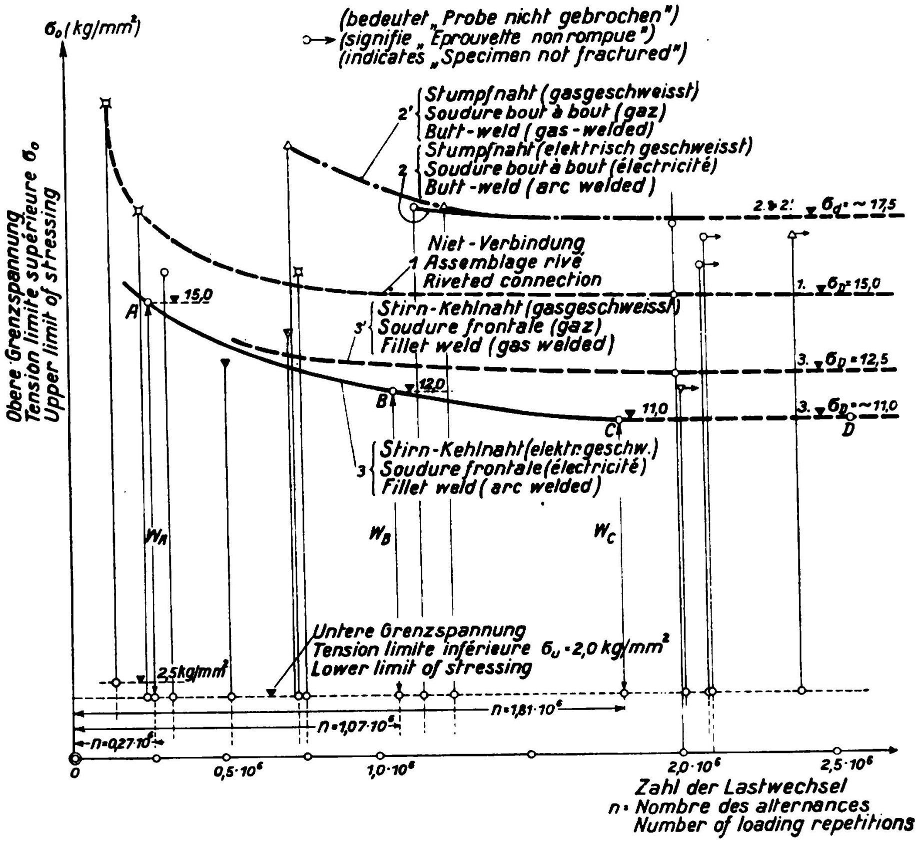

Fig. 3.

Wöhler lines according to Report of Board of Administrators with welded and riveted connections of members stressed in Tension.

traffic density of the loads, these bridges are nearly always regarded as being statically loaded. Consequently the vibration coefficient may be omitted from among the four effects enumerated under B) above and γ may be put equal to unity, while retaining the impact coefficient φ, and also certain design reduction coefficients α in the case of welded road bridges.

3) *The limiting stress-time curve (Wöhler's curve)* (Fig. 3).

Since the fatigue strength σ_D depends on a number of variables (such as n, σ_o, σ_u or σ_m) it is desirable to represent these in different planes with the axes Z, X' and X'' respectively as shown in Fig. 1. The first requirement is the recording of experimental results in the form known as the *Wöhler* curve. Suppose, for instance, that the stress σ_D is to be determined in the pulsator by means of tensile tests for electrically welded side fillet welds[11] (Fig. 3, Line 3). An upper limiting stress is first fixed arbitrarily at, for instance, $\sigma_o = 15$ kg/mm² with a lower limiting stress of $\sigma_u = 2.0$ kg/mm² and it is found that breakage takes place at n = 270000 changes of load (Point A). On a second attempt

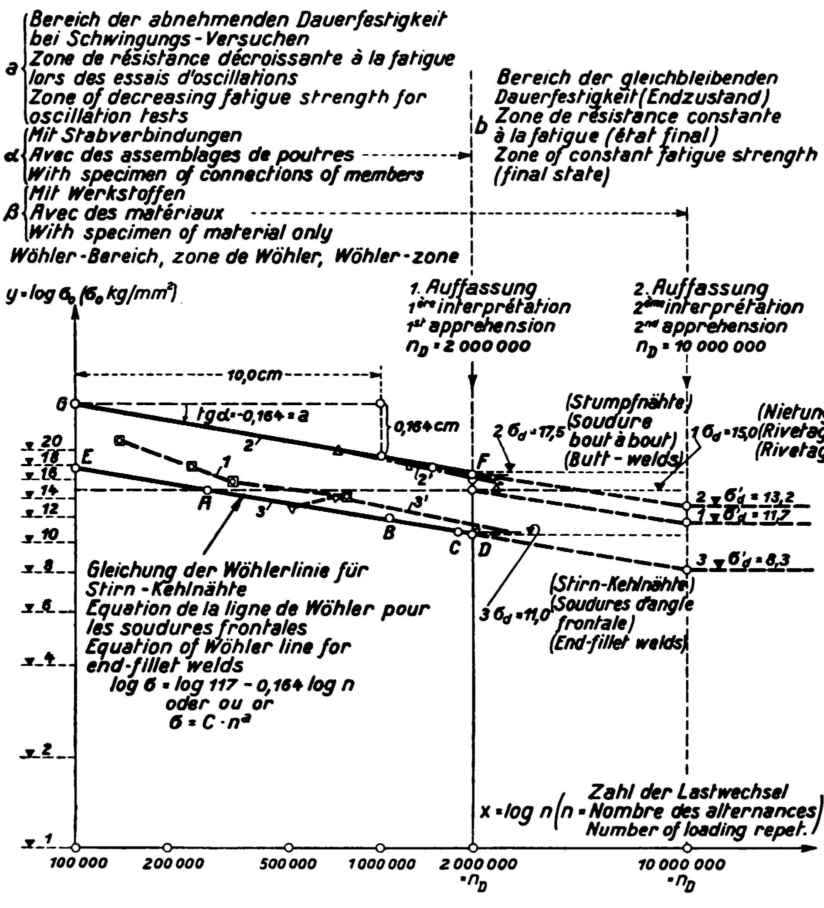

Fig. 4.

Zone of decreasing fatigue strength (Wöhler zone) of the stress-time curve (plotted logarithmically) for riveted and welded connections.

being made with $\sigma_o = 12$ kg/mm² and the same value of $\sigma_u = 2.0$ kg/mm² there is obtained n = 1.07 million (Point B) and finally in a third experiment with $\sigma_o = 11.0$ kg/mm² there is obtained n = 1.81 million (Point C). Since the portion CD of the line ABC is already approximate horizontal, the final value of the fatigue strength may be assumed at $\sigma_D = \lim \sigma_o$ = approximately 11 kg/mm².[12]

This experiment may now be plotted as in Fig. 4 with $y = \log \sigma_o$ as ordinates and $x = \log n$ as abscisse, the logarithmic scale being adopted along both coordinate axes *(and not only along the X axis, as is commonly done)*. It is then

[11] See footnote 9.

[12] First question for discussion.

found that the line ABC approximates closely enough to a straight line ED, which with the parallel line GF indicates the trend of direction of the remaining experimental lines. The equation for the straight line ABC is as follows:

$$\log \sigma = \log 117 - 0.164 \log n \tag{7a}$$

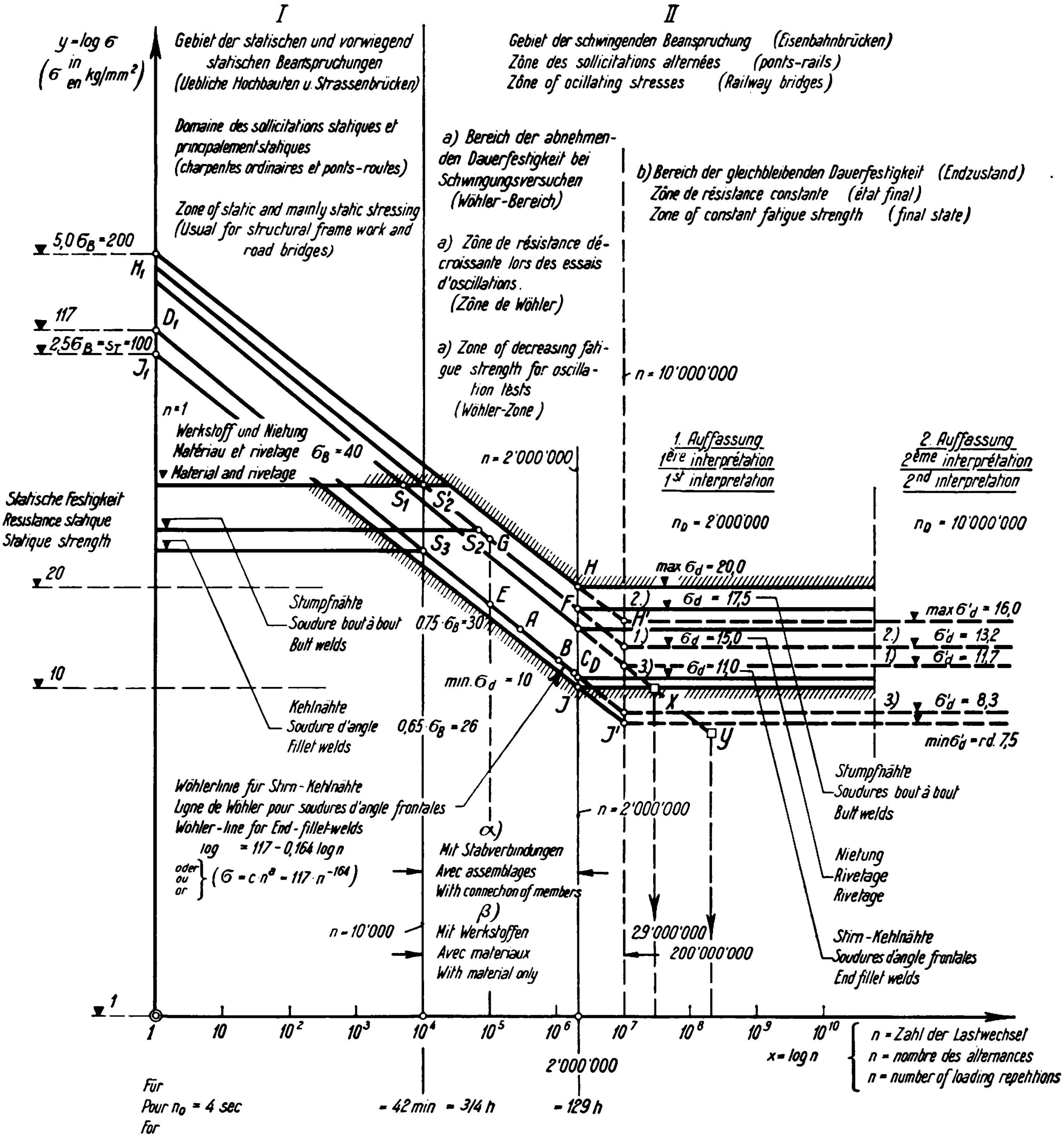

Fig. 5.

The zones of the Stress-time lines and the Wöhler-line for connections of members (in logarithmic scale).

and this corresponds to the exponential curve[13] as represented in Fig. 3:

$$\sigma = C \cdot n^a \tag{7b}$$

[13] See also Moore, Am. Soc. Test. Mat. 1922, p. 266 and Basquin, ditto, 1910, p. 625.

wherein $a = \tan \alpha$ 0.164 gives the slope of the lines and $C = 117$ kg/mm² gives the value of σ for $n = 1$.

In Fig. 5 the whole of the limiting stress-time curve is shown in the same logarithmic form. For *the region of falling fatigue strength,* which will here be designated the *Wöhler* region, we have thus set limits at the respective points $n = 10000$ and $n = 2000000$. On the left and right these connect with other horizontal portions, so that the limiting stress-time curve when represented in this logarithmic form consists of straight lines with two bends. The line DE cuts the coordinate axis ($n = 1$) with the ordinate $C = 117$ kg/mm². If, now, a line is drawn through the point J with $\sigma_D = 10$ kg/mm² and $n = 2000000$, and a line through the point J_1 to the axis of coordinates with $\sigma_D = 100$ kg/mm², and if a further line HH_1 is drawn parallel to J_1 with the point H corresponding to the value $\sigma_D = 20$ kg/mm², the result is to enclose a figure which contains practically the same values as are found in fatigue experiments on connections of bars.

The stress-time curve may then be divided up as follows. Firstly there is the region of statical or mainly statical stresses, such as occur in the usual forms of building frames and in road bridges. Secondly there is the region of alternating stresses, such as occur in railway bridges — further divisible into a portion where the fatigue strength is falling away as in vibration experiments (the *Wöhler* region) and a portion where the fatigue strength remains constant and may be looked upon as a final condition (IIa and IIb).

The sub-division into these two portions IIa and IIb is in itself arbitrary, and is the subject of Question 1 for discussion.[14] The knowledge hitherto available from fatigue experiments carried out on *connections* between structural members has led to the adoption of $n_D = 2000000$ (first assumption). If, however, as is usual in testing *materials,* $n_D = 10\,000000$ is substituted (second assumption), then the rectilinear projection of the lines in the *Wöhler* region (for instance, as far as the point H′ and J′ in Fig. 5) to correspond with rivetted practice, would give $\sigma_D = 11.7$ kg/mm² instead of 15 kg/mm² (see points V and W). The fact that rivetted railway bridges have given good performance under railway traffic when designed with $\sigma_{zul} = 10$ kg/mm² would then be difficult to reconcile with experimental results.

The relation to the fatigue strength of $\sigma_D = 15$ kg/mm² as found for rivetted connections in the fatigue experiments, to the permissible stress $\sigma_{zul} = 14$ kg/mm² in rivetted railway bridges, is

$$\nu_w = \frac{\sigma_D}{\sigma_{zul}} = \frac{15}{14} = 1.07$$

This affords a further margin of safety which, although small, may be relied upon to compensate for any possible lack of uniformity and quality of the material or other inaccuracies in execution.

Since in the testing of materials for $n > 10000000$ the stress strain line was assumed to be horizontal (Phase IIb) it is surprising that in the experiments carried out at Dresden the fatigue failure of a controlled specimen should have

[14] See Section 5 (First question for discussion).

taken place after 29000000 changes of load (see point X in Fig. 5) while in the testing machine at Berlin-Dahlem the fatigue failure at a rivet hole of a truss member should not have taken place until after 200000000 changes of load (see point Y). The second point for discussion is whether values of this order ($n > 10000000$) have been observed elsewhere, either in experiments or in railway service.[15]

In Fig. 6 the *Wöhler* line is again indicated without distortion of scale at the points V, W, X and Y. Referring to the fatigue test on structural connections, if the line had been terminated at $n = 10000000$ changes of load (point W) instead of at $n = 2000000$ (point V), then a fatigue strength approximately 20% lower would have been obtained. In the exceptionally long-delayed fatigue

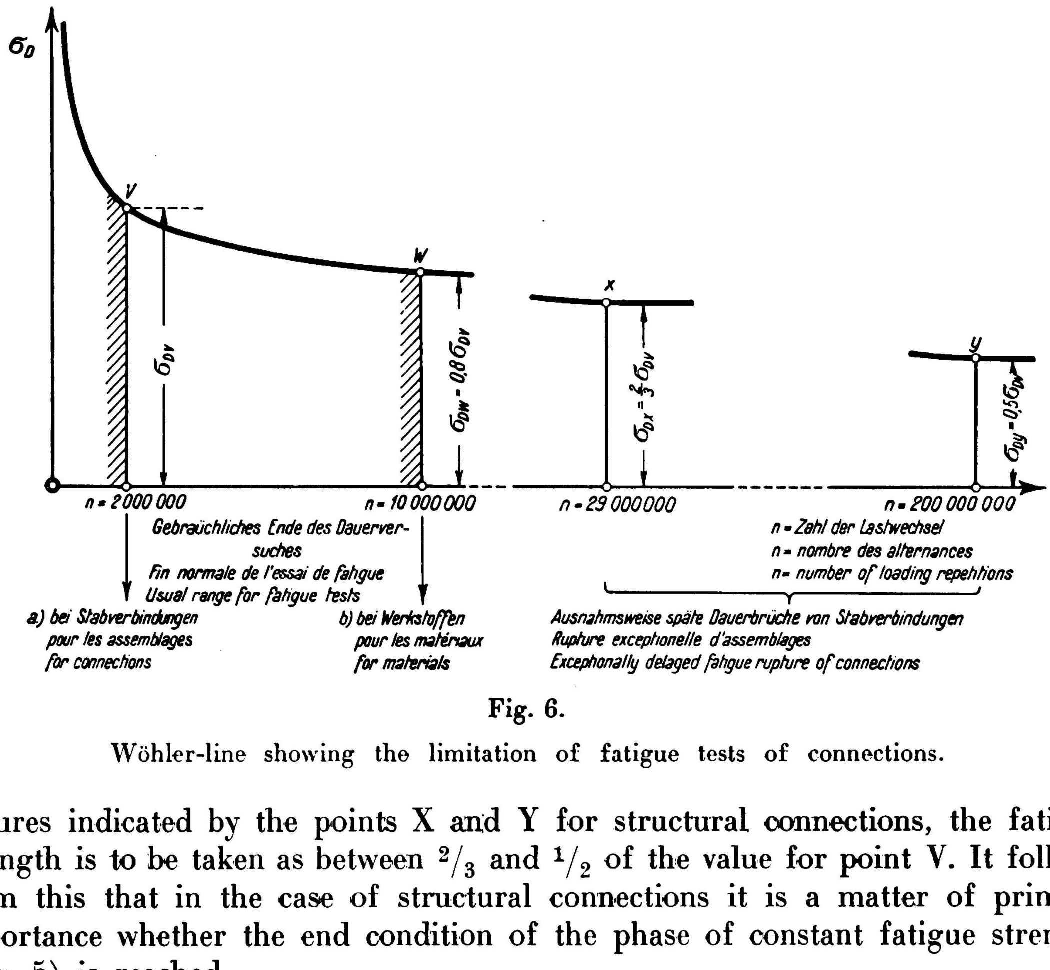

Fig. 6.

Wöhler-line showing the limitation of fatigue tests of connections.

failures indicated by the points X and Y for structural connections, the fatigue strength is to be taken as between $^2/_3$ and $^1/_2$ of the value for point V. It follows from this that in the case of structural connections it is a matter of primary importance whether the end condition of the phase of constant fatigue strength (Fig. 5) is reached.

By adopting this logarithmic presentation of the stress-time line, the whole field of statical and fatigue tests results may be incorporated in the stress-time plane. It should also be noticed that the ordinate of the point J_1, belonging to the lower limiting line JJ_1, corresponds approximately to the critical strength

$$s_T = 2{,}5 \cdot \sigma_B = 2{,}5 \times 40 = 100 \text{ kg/mm}^2 \qquad (8)$$

and that the intersection points S_1 and S_2 also indicate a certain uniformity in their relationship to the horizontal lines of the statical region and the inclined

15 See Section 5 (Second question for discussion).

lines of the *Wöhler* region. The test results obtained in the statical and fatigue tests would agree much better if stresses for butt welds were to be taken in future as 1.0 σ_{zul} instead of as 0.75 σ_{zul}; that is to say, if the points S_2 and S'_2 were to be shifted.

Physically, C in Equation (7b) may be regarded as a *coefficient of cohesion* (compare Equation 8). A meaning for the other coefficient *a* may then be inferred from the equation (7b) as follows:

$$y' = \frac{d\sigma}{dn} = c \cdot a \cdot n^{a-1} = 117 \cdot 0.164 \cdot n^{-0.836} = \sim \frac{19.2}{n}, \tag{9}$$

which for a first approximation corresponds to a rectangular hyperbola. In the *Wöhler* region the y′ line at first falls steeply downwards and finally it runs parallel to the axis of the abscissae (Fig. 3). If the ordinates $\sigma \left(\frac{kg \cdot mm}{mm^3} = \frac{kg}{mm^2}\right)$ be regardet as representing specific energy, or loading per unit volume of 1 mm^3, then the ordinates give a measure of the efficiency (work/time), and the y′ line represents the drop in this efficiency, or the fatigue experienced during the experiment. According to Fig. 5 the trend of this line is similar for the structural connections investigated and for the *Wöhler* region. The value of *a* may thus be designated as a fatigue coefficient ($a = 0$ in region I and IIb of Fig. 5).

4) *Comparison between the stresses actually arising in the structure according to statical calculations, and the stresses occurring in the test bars.*

Since the experiment has to be arranged as simply as possible it is confined to sinussoidal waves rising and falling between the upper and lower limits of σ_o and σ_u on either side of the average stress σ_m. Actually however — as indicated in Fig. 2 — the range of live load stress above and below the dead load stress σ_g is usually very different; in side members, for instance, being σ_{p1} above $> \sigma_{p2}$ below. In the boom members of girders σ_{p2} may even be equal to zero. Hence the experiment differs from reality not only as regards the shape of the waves, but also as regards their lack of symmetry in the different amplitudes (σ_{p1}) above and (σ_{p2}) below, Unfortunately, again, the effect of this discrepancy has not yet been investigated, and this suggests a further field for research.

5) *Questions for discussion.*

Question 1) Is it expedient to refer fatigue tests on connections of bars to a number of changes of load $n_D = 2000000$ instead of to $n_D = 10000000$ as is customary in the testing of materials? (Compare footnotes 2, 5, 12 and 14; also Figs. 5 and 8, points S and W).

Question 2) Have the exceptionally long delayed fatigue breakages, after 29 and 200 million changes of load, as observed in the German experiments, been confirmed elsewhere either in fatigue experiments on structural connections or in actual railway service? (Compare footnote 2 and 15; also Figs. 5 and 6, points X and Y).

Question 3) Is there any experimental evidence that a favourable effect on the fatigue strengths of structural connections may be exercised by rest pauses? (Compare footnote 7).

Question 4) Seeing that rivetted connections, which form the basis for investigating the behaviour of welded connections, give an average fatigue strength of $\sigma_D = 15$ kg/mm² with a permissible stress (including impact allowance) of $\sigma_{zul} = 14$ kg/mm², may the criterion for the safety of railway bridges properly be taken as the proportion between the number of changes of stress applied in the experiments and the number of trains per day? Or in other words, is the question answered by stating the life of the bridge in years $\nu_T = n_D : n_T$ making use of a *statistical* conception of safety? Can any other suitable suggestions be put forward for designating safety?

IIIa 4

Characteristic Features of Welding.

Charakteristische Merkmale der Schweißung.

Caractéristiques propres à la soudure.

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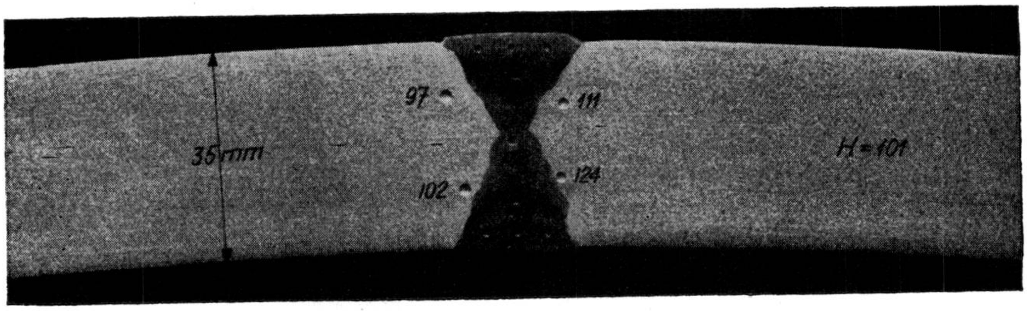
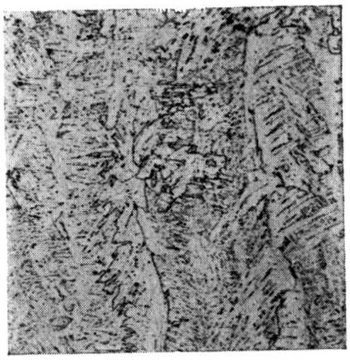
As regards the strength and deformation properties, the quality of the structure of the weld material, including that of the transition zone, is fundamental. Weld metal is in fact a form of steel casting; its structure is heterogeneous and anisotropic — Fig. 1 —, and the theory of constant strain-energy of deformation holds good only if account is taken of the anisotropic condition. In practice, no guarantee can be given that *weld material* is *free from pores* and *slags*. *Shrinkage-cracks* in the outer surfaces and in the interior of the weld material are, as a matter of fact, rare occurrences, but occasionally they may be present — Fig. 2 —. Pores, slag inclusions and cracks must be considered equivalent to a *mechanical imperfection. Thermal influences* give rise, on the surface, to the structure known by the name of Widmannstaetten with transcrystallization — Fig. 3 — and in the transition zone to the formation of sorbite, troostite — Fig. 1 — and — in steels containing less than 0.15 % carbon — also martensite — Fig. 4 — which on account of its brittleness promotes the formation of cracks. So far as the material itself is concerned, welding *cannot* therefore be considered equivalent to riveting — Fig. 5 —.

By metallurgical means such as proper choice of the electrodes, correct procedure with pre-heating of the steel when necessary — Fig. 6 —, heat treatment without stress (up to the lower transition-temperature at the most) or annealing (beyond the upper transition-temperature) combined with suitable precautions in design — i. e. reduction of thermal stresses — Fig. 7 —, proper choice of type of connection — Fig. 8 — and shape — Fig. 9 — the mechanical-caracteristics of the welded connection may be made to approximate to those of the riveted connection.

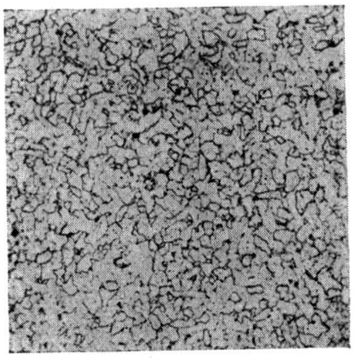
The series of tests and precautions is completed by examination of the welders based on the results of mechanical and deformation tests, either on plates specially welded both in the normal and overhead position or on specimens of suitable shape (round or oval) removed from work carried out in actual practice, combined with X-ray examinations.



$\beta_z \cong 38$ kg/mm² $\sigma_u \cong 16$ kg/mm²

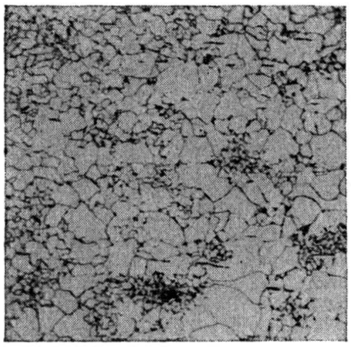


Coarse Widmannstaetten structure in weld metal.



Fine, normalised structure in weld metal.

Thermally altered structure, ferrite and sorbite, transition zone.



Thermally altered structure ferrite and degenerated perlite, transition zone.

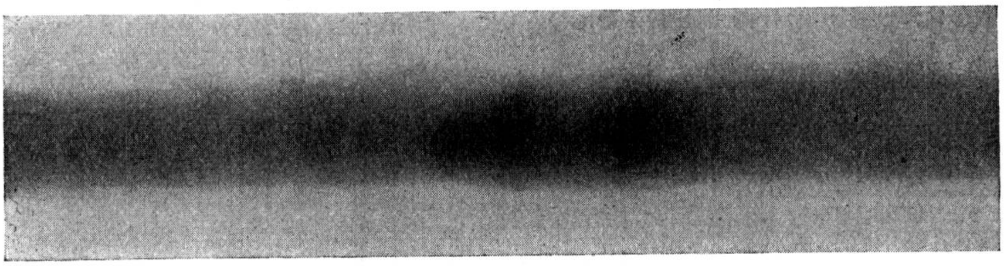
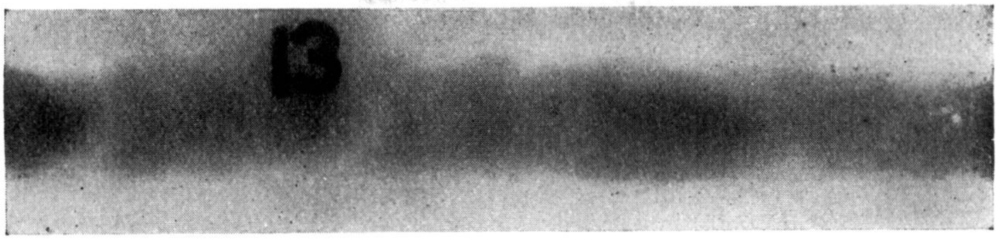
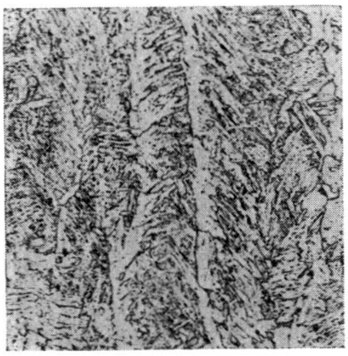


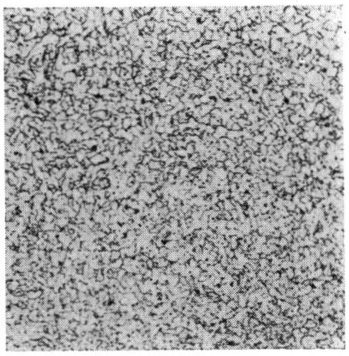
Fig. 1.

Arc weld free from defects, in normal structural steel. Heterogeneity of the weld metal structure.

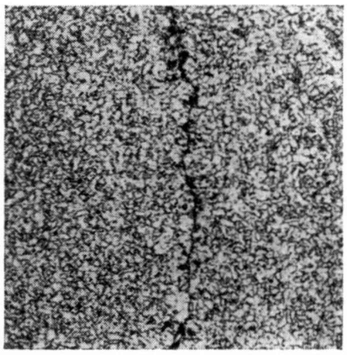




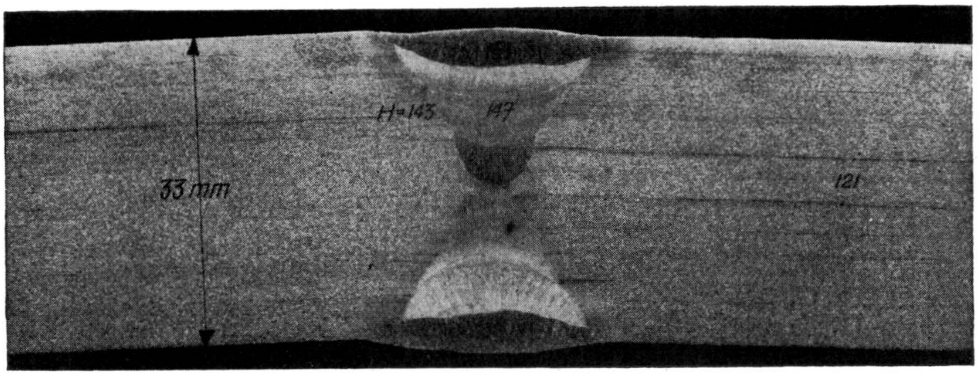
Coarse Widmannstaetten structure, weld metal, last layer.



Fine, normalised change in structure, welt metal.



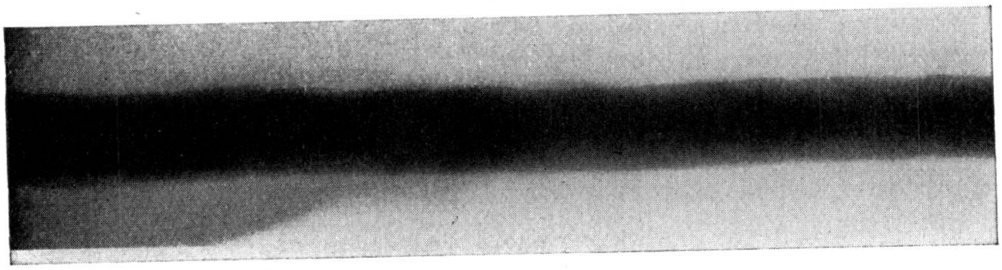
Fine, normalised change in structure with microscopic crack, weld metal.

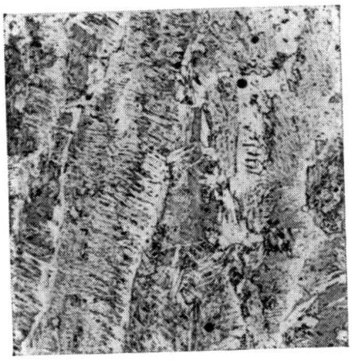


$\beta_z \cong \mathbf{44}$ kg/mm² $\qquad \sigma_u \cong 17$ kg/mm² $\qquad \frac{1}{2}\sigma_w \cong 27$ kg/mm²

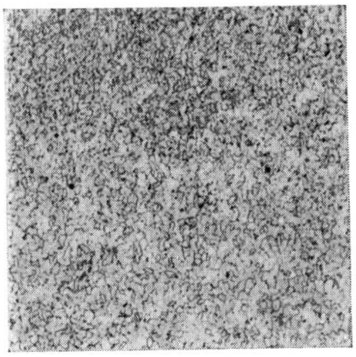
Fig. 2.

Arc weld of superior quality in steel containing 0.25 % C. Microscopic cracks in weld metal.

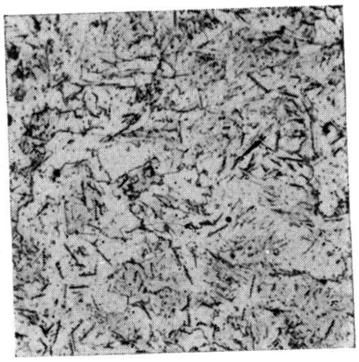




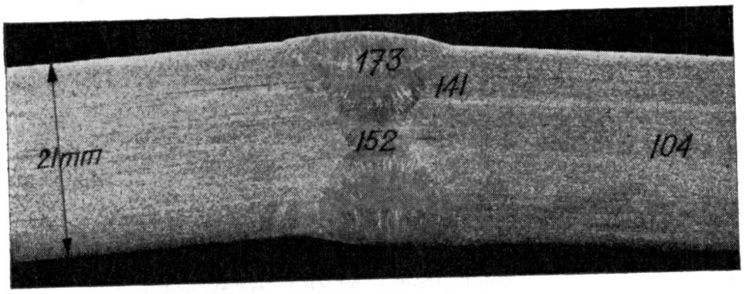
Widmannstaetten structure, weld metal.



Fine alteration in structure, weld metal.



Local enrichment of nitride inclusions, weld metal.

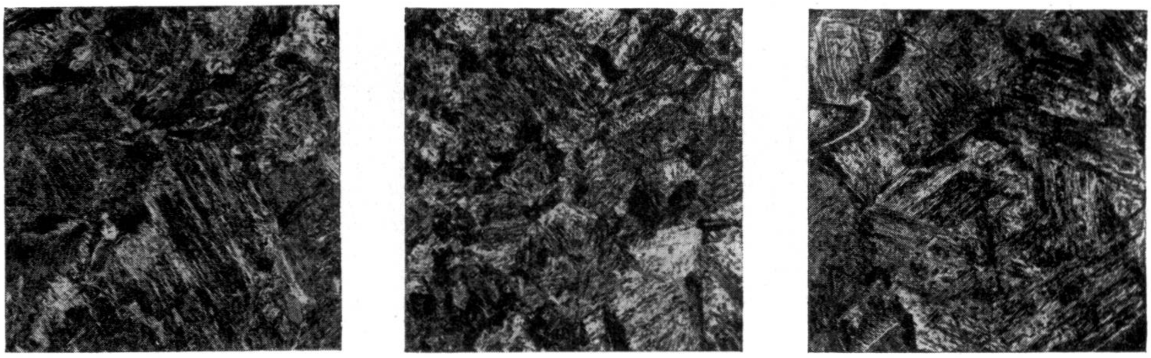


$\beta_z \cong 38\ \text{kg/mm}^2 \quad \sigma_u \cong 18\ \text{kg/mm}^2 \quad \frac{1}{2}\,\sigma_w \cong 26\ \text{kg/mm}^2$

Fig. 3.

Arc weld free from defects, in normal structural steel. Widmanstaetten structure with transcrystallisation.

Siemens-Martin steel with 0.20—0.25 % C.



Temperature: — 10° C. Martensite with traces of troostite due to quenching effect.

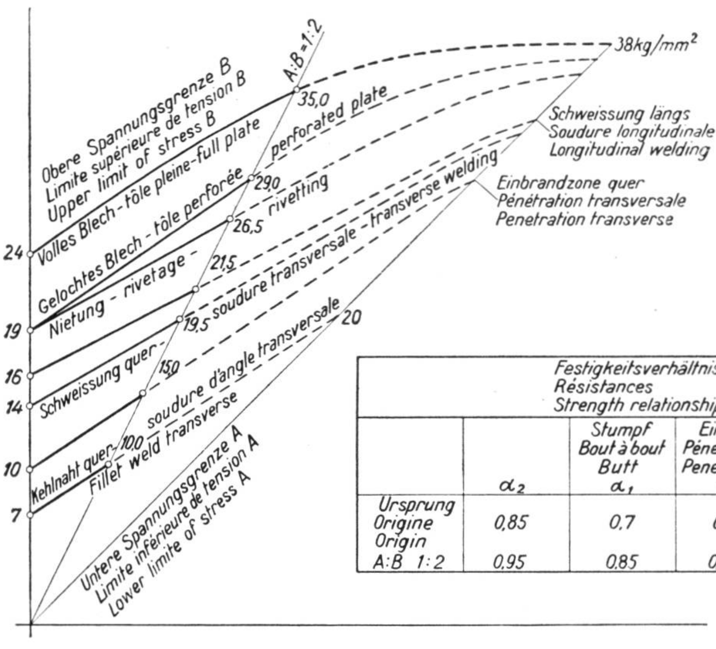
Temperature: 25° C. Martensite with troostite due to quenching effect.

Temperature: 50° C. Martensite with little troostite, traces of ferrite.

Fig. 4.

Formation of Martensite in the transition zone immediately at the junction of the weld metal with Siemens-Martin steel plate.

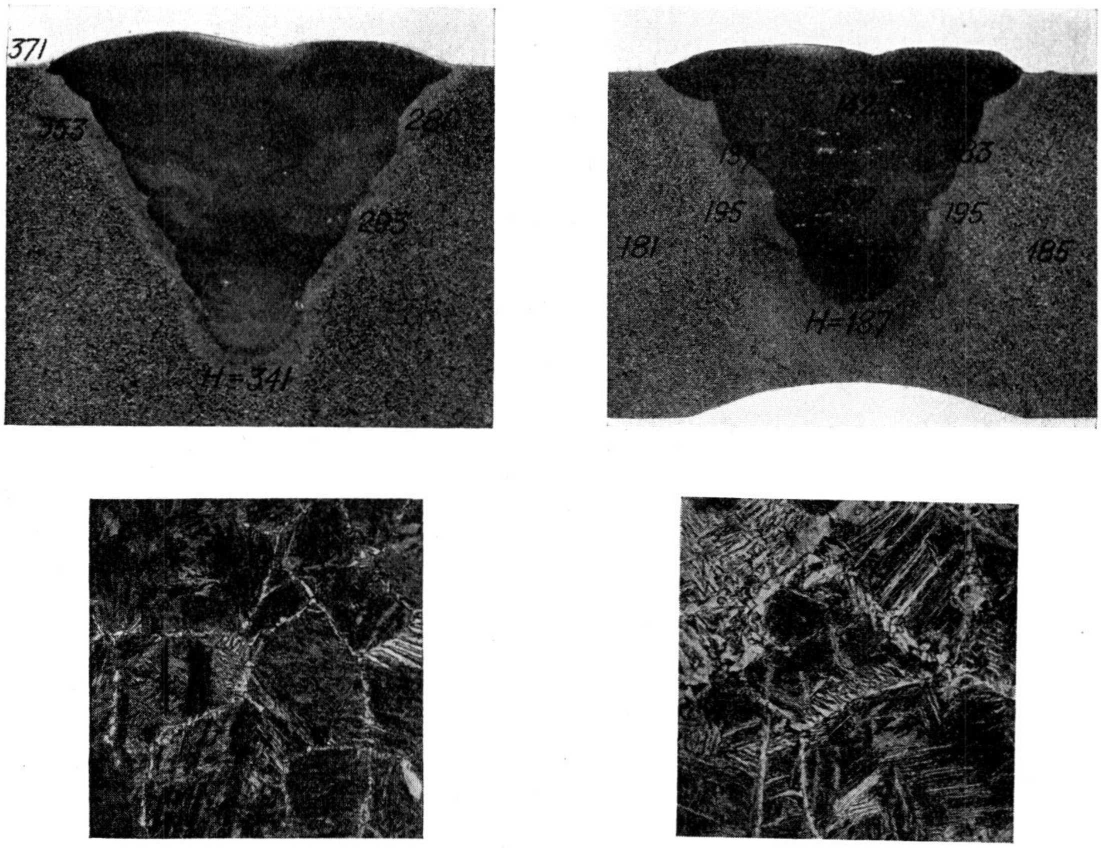
Temperature of the Siemens-Martin steel, when welded, — 10°, + 25° and 50° C.



Festigkeitsverhältnis Résistances Strength relationships					
	α_2	Stumpf Bout à bout Butt α_1	Einbrand Pénétration Penetration α_1	Kehl D'angle Fillet α_1
Ursprung Origine Origin	0,85	0,7	0,6	0,35
A:B 1:2	0,95	0,85	0,65	0,45

Fig. 5.

Tensile fatigue strength in relation to the lower limit of stress.



Not pre-heated, formation of Martensite.

Pre-heated, no formation of Martensite.

Fig. 6.

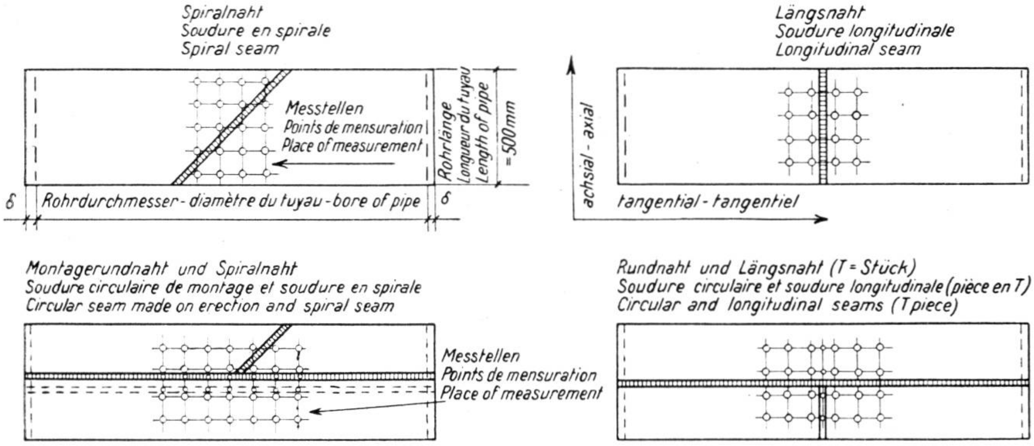
Electrically welded cast steel, carbon content 0.28 %.

Welding without pre-heating:

High hardness number in the transition zone, formation of Martensite.

Welding with pre-heating:

Normal hardness number, no formation of Martensite.



Nature of weld seams, where measured.

Fig. 7 a.

Internal stresses of weld seams, annealed and not annealed.

Maximum values of internal stresses as measured.

annealed or not	Nature of weld seams where measured	Reduced internal tensile stresses in kg/cm²	
		axial	tangential
annealed	Longitudinal X-seam	+ 1010	+ 1060
not annealed	Longitudinal X-seam	+ 1620	+ 2460
annealed	Spiral X-seam	+ 280 + 447	+ 727 + 336
not annealed	Circular and longitudinal U-seams	+ 2070	+ 2070

Fig. 7 b.

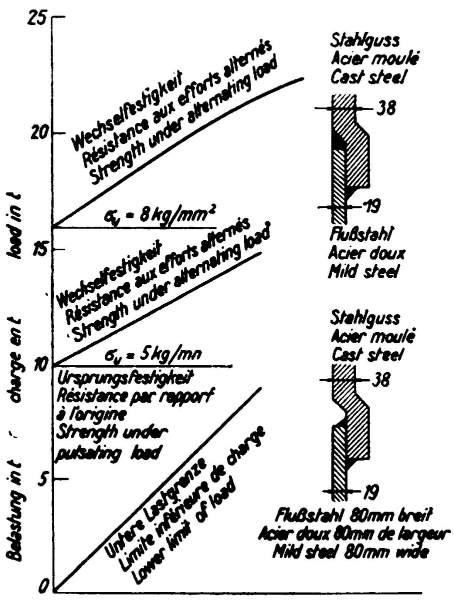
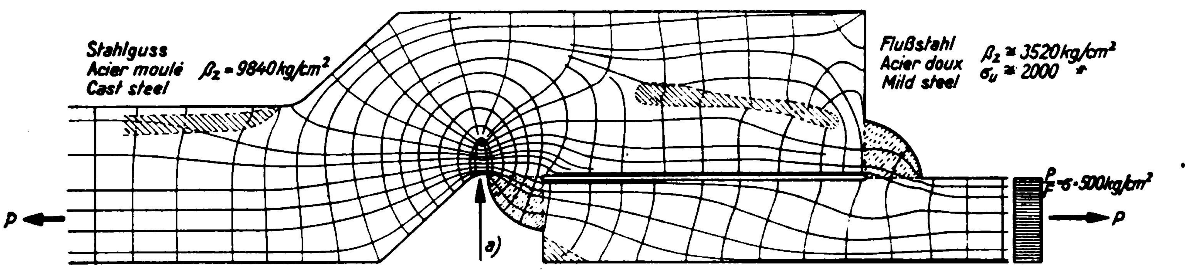


Fig. 8 a.

Increased resistance to alternating stress through suitable weld connections between cast steel and mild steel.



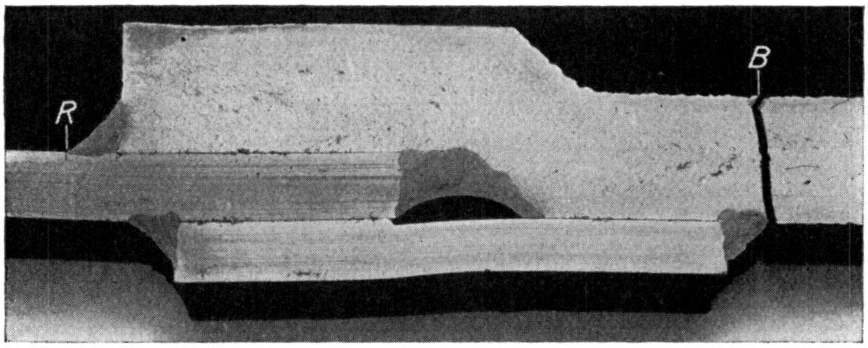
a) Navier-Hooke stresses, by calculation $\frac{P}{F} + \frac{M}{W}$: $\sigma_{max} = 2{,}8\,\sigma = 1400$ kg/cm².

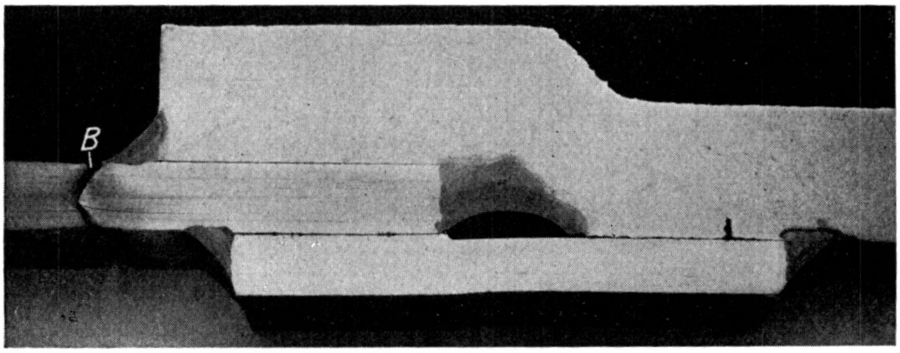
Resistance of the connection to pulsating stress: $\sigma_u \cong 500$ kg/cm².

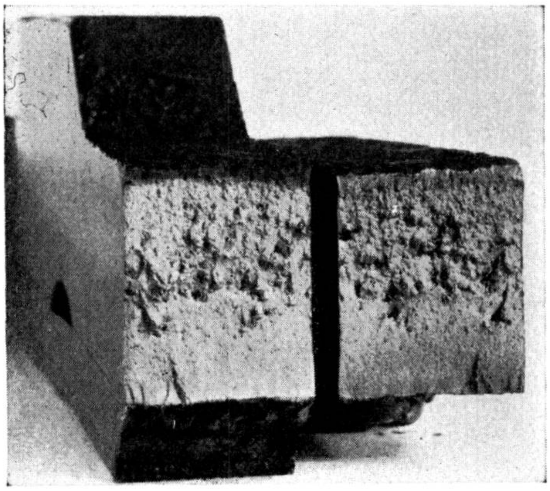
Concentration of stress at base of notch as determined optically: $\sigma_{max} = 5\,\sigma = 2500$ kg/cm².

Fig. 8 b.

Stress conditions at base of notch.







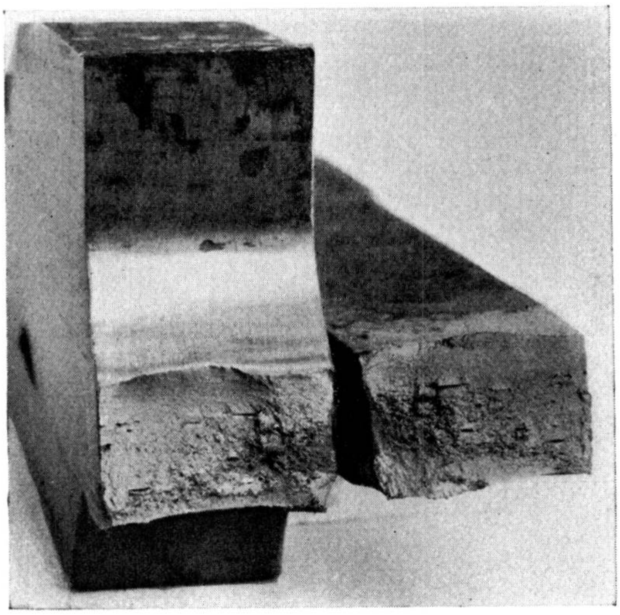


Fig. 9.

Welded connections between cast steel and mild steel.
Increased resistance to pulsating stress by grinding the weld seams.

IIIa 5

The Fatigue Strengths of Welded Connections in St. 37 and St. 52.

Dauerfestigkeit von geschweißten Verbindungen von St. 37 und St. 52.

La résistance à la fatigue des assemblages soudés en acier St. 37 et St. 52.

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i. Willem Smit & Co's Transformatorenfabriek N. V. Nijmegen, Holland.

It has been shown in previous experiments that the fatigue strengths obtained in laboratory experiments with round and flat bars of St. 37 which have been worked smooth, are the same as those of the unwelded material as rolled, but in the case of St. 52 they are lower than the latter. Hence in St. 37 the fractures occur outside and in St. 52 within the weld, but in spite of this the fatigue strength of the welded connection of St. 52 is at least 30 % higher than that in St. 37. Since, moreover, the permissible stresses in St. 52 are 30 % higher than those in St. 37, the same should be true of the welded connections, provided that heavy coated, high quality electrodes have been used.

The values determined are shown in Table 1.

Table 1.

Bending fatigue strengths for welded and unwelded St. 37 and St. 52.

Material	Fatigue strength kg/mm²	Breakage
(a) Bending fatigue tests on round specimens.		
St. 37 — unwelded	$\sigma_{wb} = 20.1$	—
— welded	$= 20.1$	Outside
St. 52 — unwelded	$\sigma_{wb} = 30.8$	—
— welded	$= 26.4$	Inside
Deposited weld metal	$\sigma_{wb} = 24.3$	—
(b) Bending fatigue tests on flat specimens.		
St. 37 — unwelded	$\sigma_{wb} = 17.8$	—
— welded	$= 17.8$	Partly in weld
St. 52 — unwelded	$\sigma_{wb} = 30.5$	—
— welded	$= 22.5$	In weld
(c) Torsion fatigue test on round specimens.		
St. 37 — unwelded	$\sigma_{w} = 11.5$	—
— welded	$= 11.5$	Outside
St. 52 — unwelded	$\sigma_{w} = 17.2$	—
— welded	$= 15.5$	Insidc
Deposited weld metal	$\sigma_{w} = 15.3$	—

These results, while interesting for purposes of comparison, are of little practical importance, for the conditions in practice are quite different, the majority of connections in welded bridges and building structures not being worked over, with the result that non-uniform distribution of stress occurs and these effects are further increased by notch action at the edges of the weld or at the base of fillet seams. There are, therefore, two factors which play a decisive part, namely 1) the execution and workmanship of the weld. 2) The design of the connection.

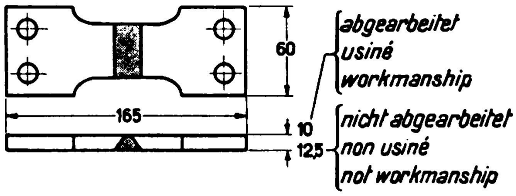

Fig. 1.
Shape and dimensions of flat bending specimen for fatigue tests.

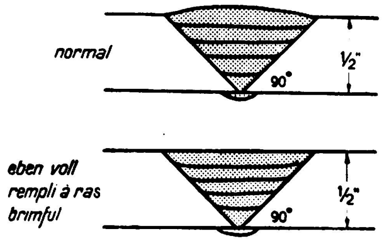

Fig. 2.
Execution of welds.

The effect attributable, to the execution and workmanship of the weld was examined by the authors with the aid of flat bending test bars as indicated in Fig. 1, the weld itself being carried out either in the usual way or smoothed (see Fig. 2), while in a third experiment the upper surface of the bead was filed off. The results of these experiments are shown in Table 2 together with values for unwelded material of three different types of St. 52 — a chrome-copper steel (I), a manganese-silicon steel (II), and a manganese steel (III).

Table 2.

Fatigue strengths of welded connections in St. 52.

Steel	Not welded. Upper surface filed	Welded		
		Weld filed	Not worked	
			Welded in ordinary way	Filled smooth
I	31.0 kg/mm²	23.0 kg/mm²	11.0 kg/mm²	15.0 kg/mm²
II	29.0 „	21.5 „	9.5 „	16.5 „
III	31.5 „	22.5 „	8.0 „	14.0 „

In the case of all the bars which have not been smoothed fracture occurs at the junction of the weld metal and the plate material, either on the upper side of the V seam (Fig. 3) or on the root side at the edge of the backing bead (Fig. 4); a circumstance which may be explained by changes in the micro-structure caused by the more or less remarkable hardening effect at these places (Fig. 5). The efforts of the steel maker will, therefore, be directed towards limiting this increase in hardness as much as possible, but as the phenomenon is connected with the increased strength of these steels it cannot be entirely avoided. The most favourable results were found in the chrome-copper, chrome-molybdenum and manganese-silicon steels when the amount of alloy element present

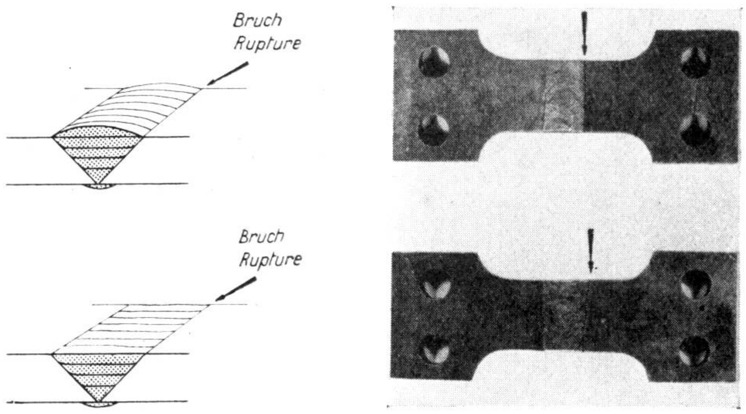

Fig. 3.

Fatigue failures at transition from weld to plate.

was as low as possible, and especially when the carbon content did not exceed 0.15 to 0.20 %.[1]

The effect of shape was examined in a T-connection carried out in several different ways and tested under dynamic loading simultaneously with static pre-

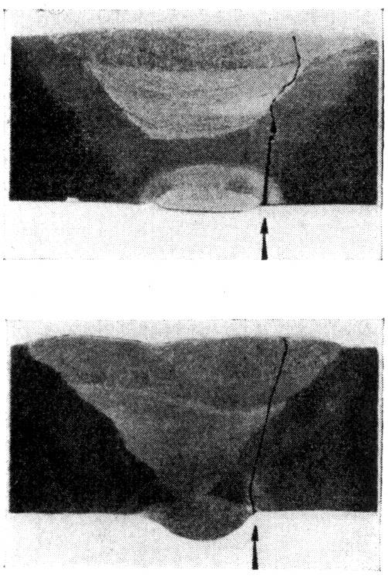

Fig. 4.

Beginning of fatigue failure at edge of reverse bead.

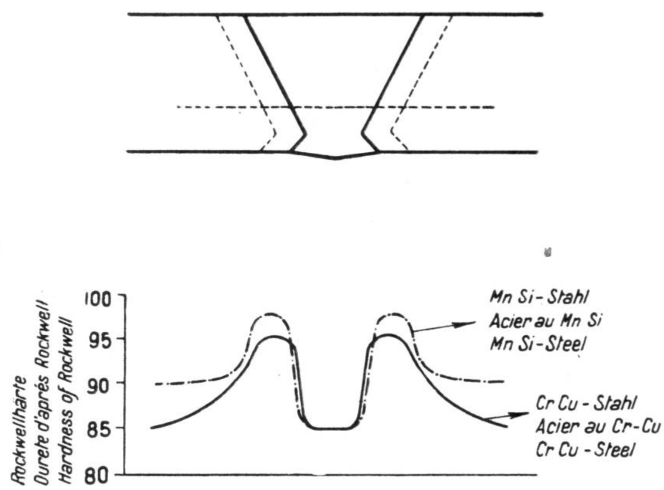

Fig. 5.

Distribution of hardness in a welded connection in St. 52.

stressing. In accordance with the method of calculation adopted by the bridge construction bureau of the Netherlands Railways the dynamic stress was taken as

[1] See Smit-Laschtydschrift, Vol. 1, N° 2 (1937).

30 % of the static pre-stress, and it was sought to obtain in each connection the maximum value of this pre-stress which did not result in fracture after two million changes of load (see Fig. 6).

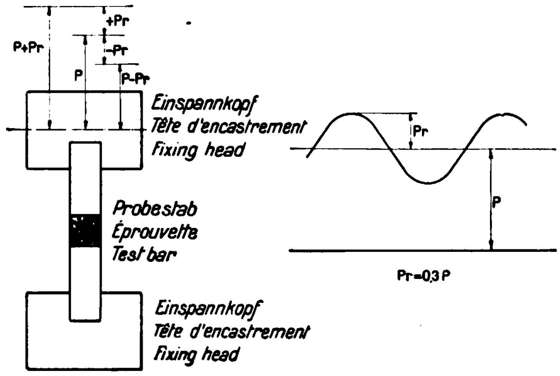

Fig. 6.

Variations of stress in experiments on stress alternating between tension and compression.

The experiments were carried out in a Losenhausen pulsating machine. The T-connection was made in the form of a double-sided fillet weld and as an X-weld (Fig. 7 and 8). The results of the experiments are given in Table 3.

Table 3.

Results of fatigue tests on welded connections in St. 37 and St. 52.

Material	Connection	Maximum statical pre-stress kg/cm²	Dynamic stress	Stress changes without fractures
St. 37	X-weld (Fig. 7 a)	1900	± 30 %	$2 \cdot 10^6$
	Fillet weld (Fig. 7 b)	1250	± 30 %	$2 \cdot 10^6$
St. 52	X-weld (Fig. 7 a)	2000	± 30 %	$2 \cdot 10^6$
	Fillet weld (Fig. 7 b)	1000	± 30 %	$2 \cdot 10^6$

These results indicate that the fillet seams of the connections in St. 37 possess a higher dynamic strength than those in St. 52, but in the case of the X-seams the values are approximately equal. This clearly shows that the fatigue strength of welded connections in St. 52 is not greater than in St. 37, a fact which may be explained by the greater notch sensitiveness of St. 52. Moreover in both cases the strength of X welds is much greater than that of the fillet seams, and the general rule holds good, therefore, to *adopt butt welded connections wherever possible.*

When fillet welds cannot be avoided they are best made as shown in Fig. 9, the weld having the maximum possible depth, and being made smooth at the edges with a gradual transition into the parent metal without any notches.

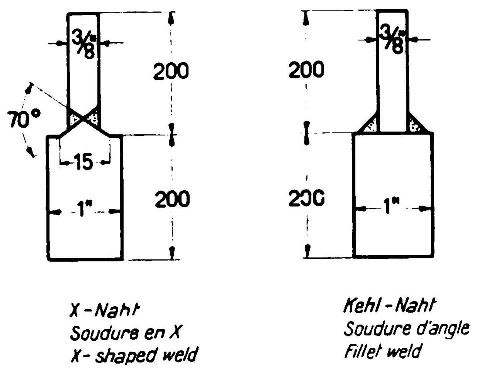

Fig. 7.

Shape and dimensions of specimens for tensile-compressive alternating stress experiments.

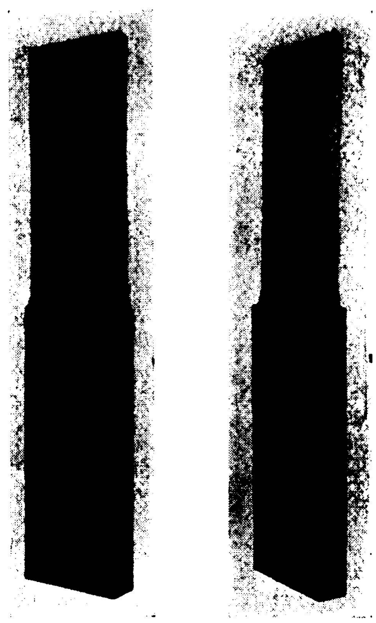

Fig. 8.

Welded specimens for experiments as in Fig. 7.

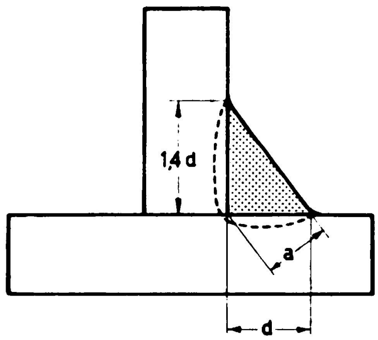

Fig. 9.

Ideal form of fillet seam offering maximum resistance to static and dynamic loading.

The author's experiments were further extended to fatigue tensile tests with shear stresses, but as these have not yet been completed the results will be reported later.

IIIa 6

Notes on the Method of Expression of Allowable Unit Stress as determined by the Pulsation or Reversal of the Stresses.

Angaben über die Methode des Ausdrucks einer zulässigen Spannung, die durch Schwingung oder Wechsel im Vorzeichen der Spannungen bestimmt ist.

Notes sur la méthode d'expression de la contrainte admissible déterminée à partir de la pulsation ou de l'alternance des efforts appliqués.

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In standard American specifications for riveted bridges, a reduction of allowable unit stress has been prescribed in the case of reversed stresses, but not in the case of pulsating stresses (fluctuating but without change of sign).

However, the committee assembled by the American Welding Society to prepare a specification for welded bridges (issued 1936), decided from the available data, largely the published reports of Professors *Graf* and *Schaechterle*, to make a reduction in the allowable unit stress on certain types of welded joints for stresses pulsating through considerable range, as well as for those actually reversing.

These notes do not discuss the actual values selected for allowable unit stress under various conditions, but only the manner of their expression. As most of the important members of any bridge, and their connections, are subject to pulsating stress, it is important to keep to a minimum the arithmetical labor involved in carrying out any prescribed rules.

The previous American specifications applicable, as noted above, in the case of reversed stresses only, require the calculation from the minimum and maximum total stresses of a third or hypothetical stress, greater than the maximum, to which the normal unit stress is applied, thus giving an increase of required area. An identical method is, for geometrical reasons, not possible of employment when reduction of unit stress is to be made for pulsation as well as for reversal.

The official German method ("gamma method") is similar to the foregoing, and requires the calculation from the maximum and minimum total stresses of a multiplier "gamma" to be applied to the maximum stress.

Each of these methods introduces an auxiliary step, the calculation of a modified or hypothetical maximum stress, before proceeding to the determination of the required area. The method adopted by the American Welding Society eliminates the preliminary step and yields the required area by the direct application of a simple formula, the derivation of which will now be given.

Let the line A B C be a graph of permissible unit stresses, plotted so that any minimum is an abscissa and the corresponding maximum is the corresponding ordinate. Thus at A, min = max, and the ordinate S_{-1} is the unit stress selected on the basis of the test data to be permitted in the case of full reversal. So at B, min = 0, and the ordinate S_0 is the unit stress selected to be permitted in the case of pulsation from zero.

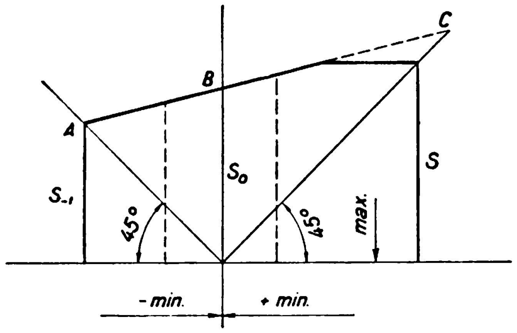

Fig. 1.

For all practical purposes, A B C may be taken as a straight line. It is undesirable to complicate design requirements by introducing any other form of variation, considering how small the percentages of error, even if they could be definitely known, must be.

The full range of the line B C is not utilizable, because the values of maximum unit stress increase beyond the value S which has been established for static conditions (max = min). The sloping line must therefore be neglected above its intersection with a horizontal through the value S.

Consider now a bridge part which undergoes some other degree of reversal than complete reversal, or some other degree of pulsation than return to zero, as indicated by dotted ordinates in the sketch. For this member the unknown "max" and "min" permissible unit stresses are of course proportional to the known "Max" and "Min" or calculated total stresses.

Then:

$$\text{max.} = S_0 + \frac{S_0 - S_{-1}}{S_{-1}} \text{ min.} \tag{1}$$

$$= S_0 + \varnothing \cdot \text{min.},$$

"$\varnothing$" being the slope of the line A B $= \dfrac{S_0 - S_{-1}}{S_{-1}}$.

$$\text{Required area A} = \frac{\text{Max.}}{\text{max.}} = \frac{\text{Max.}}{S_0 + \varnothing \cdot \text{min.}} = \frac{\text{Max.}}{S_0 + \varnothing \cdot \dfrac{\text{Min.}}{\text{A}}}$$

$$\text{A} \cdot S_0 + \varnothing \cdot \text{Min.} = \text{Max.}$$

$$\text{A} = \frac{\text{Max.} - \varnothing \cdot \text{Min.}}{S_0} \tag{2}$$

This, then, is the form in which the specification is cast. For each particular type of stress or type of joint, the committee has selected a permissible value of S_0 and a value of S_{-1}. From these, by Eq. (1), $\varnothing$ is derived and then Equation (2) is written into the specification. The designer, having from the prescribed loding calculated "Max" and "Min" derives his area "A" in the simplest possible fashion.

In the future, as further test results become available, and as for other reasons the necessary factors of safety are re-considered, future committees may modify "S_0" or "S_{-1}", or both. The form of the several formulas need not be disturbed, and a simple modification of "$\varnothing$" or "S_0", or both, will embody the desired change or changes.

As an example the American Welding Society specification for the area of fillet welds is:

$$\text{Area} = \frac{\text{Max.} - \frac{1}{2}\,\text{Min.}}{7200} \text{ but not less than } \frac{\text{Max.}}{9600}.$$

(The second expression embodies the portion of the foregoing diagram in which the sloping line is replaced by the horizontal line through the ordinate "S".)

Example 1. Max. $= + 80000$ Min. $= - 80000$

$$A = \frac{80\,000 + 40\,000}{7200} = 16.7 \text{ sq. in.}$$

Example 2. Max. $= + 80000$ Min. $= - 40000$

$$A = \frac{80\,000 + 20\,000}{7200} = 13.9 \text{ sq. in.}$$

Example 3. Max. $= + 80000$ Min. $= 0$

$$A = \frac{80\,000}{7200} = 11.1 \text{ sq. in.}$$

Example 4. Max. $= + 80000$ Min. $= + 16000$

$$A = \frac{80\,000 - 8000}{7200} = 10.0 \text{ sq. in.}$$

$$\text{but not less than } \frac{80\,000}{9600} = 8.33 \text{ sq. in.}$$

Example 5. Max. $= + 80000$ Min. $= + 64000$

$$A = \frac{80\,000 - 32\,000}{7200} = 6.67 \text{ sq. in.}$$

$$\text{but not less than } \frac{80\,000}{9600} = 8.33 \text{ sq. in.}$$