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The Behaviour of Reinforced Concrete Framed Structures at Incipient Failure.

Das Verhalten von Eisenbeton-Rahmenkonstruktionen bei beginnender Zerstörung.

Comportement des portiques en béton armé à l'amorce de la rupture.

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At working stresses it is probable that the distribution of moments throughout a reinforced concrete framework is reasonably well given by calculations according to the elastic theory. It has been shown by prolonged loading tests at the Building Research Station[1*] and in America[2] that the creep of concrete at working stresses has no important effect on the moment distribution in a frame.

When incipient failure is reached at any part of the structure, however, the inelastic movements of either the steel at its yield point or of the concrete near its ultimate strength are so large that the elastic theory no longer holds. Deformation of the affected part is limited by the movements of the rest of the structure so that collapse of this part may not occur until considerable elastic deformation has occurred elsewhere. That is, further load can be carried by the structure without collapse, the maximum stress at the affected part tending to remain practically constant whilst the moments and stresses at other parts increase. For convenience the change in the distribution of bending moments from that in a purely elastic framework will be called "redistribution of bending moments" in this paper.

Tests on two-span continuous beams by *Kazinczy*[3] have shown that when steel is the deciding factor for failure, variation of the amount of steel in the span or over the central support from that required by the elastic theory leads to redistribution of moments such that the full strengths of both the span and support sections are reached. Similar results were obtained for built-in beams by the German Reinforced Concrete Committee[4] for the condition of failure due to steel yield. Such redistribution is to be expected because of the large inelastic deformation of steel at its yield, but the extent to which it can be relied upon without causing concrete failure is unknown. No previous tests are known in which the effect of inelastic deformation of the concrete at incipient failure on the ultimate strength of a framework has been studied.

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* These figures relate to the list of references at the end of the paper.

The tests described in this paper were part of an investigation undertaken at the Building Research Station in conjunction with the Reinforced Concrete Association to obtain definite information on the importance of inelastic deformations at highly stressed parts of a reinforced concrete framework. The ivestigation included tests to destruction on, A, two span continuous beams and B, portal frames.

A. Tests to Destruction on Two-span Continuous Beams.

Tests were carried out on two-span continuous beams designed as follows:

1) With weakness over the central support due to the use of a low percentage of tension steel.
2) With weakness over the central support due to the use of a low strength concrete without compression reinforcement.
3) As (2) except that compression reinforcement was provided.
4) As (2) but with an increased span length in order to reduce shear stresses.
5) As (2) except that a low strength concrete was used at an age of about 6 months instead of 7 days.

All tests were made in duplicate, and river aggregates were used throughout.

1) *Primary Failure in Tension Steel.*

Details of the beams and the positions of the loading used in the tests to determine the effect of using insufficient steel when calculated according to the ordinary elastic theory are given in Figure 1.

The notation used in the table of Figure 1 and subsequent tables of stresses is as follows:

t denotes the stress in longitudinal tension reinforcement.

t' denotes the stress in longitudinal compression reinforcement.

M denotes the bending moment.

n denotes the depth of neutral axis.

a denotes the arm of the resistance moment.

S denotes the total shear.

s denotes the shear stress.

t_w denotes the stress in web reinforcement.

s_b denotes the bond stress.

W denotes the Load.

ξ_B denotes the Distance of point of inflexion from B.

ξ_F denotes the Distance of point of inflexion from column face.

s_E denotes the Bond at E (lower bars).

R_A, R_B, R_C denotes the reactions at A, B and C.

It will be seen that over the central support where the moment is normally greatest, there are only two $^3/_8$ in. diameter bars whereas in the span four $^5/_8$ in. diameter bars are provided. At quite a low load therefore, the yield point stress of the $^3/_8$ in. diameter bars would be expected; it would be anticipated that yield of these bars would lead to a redistribution of moments whereby the section over the central support would be continuously relieved, enabling the load carried by the system to be further increased until failure in the span.

The actual moments during the tests were determined by measuring the strain in the supporting steel joist at a fixed distance from the end supports and hence

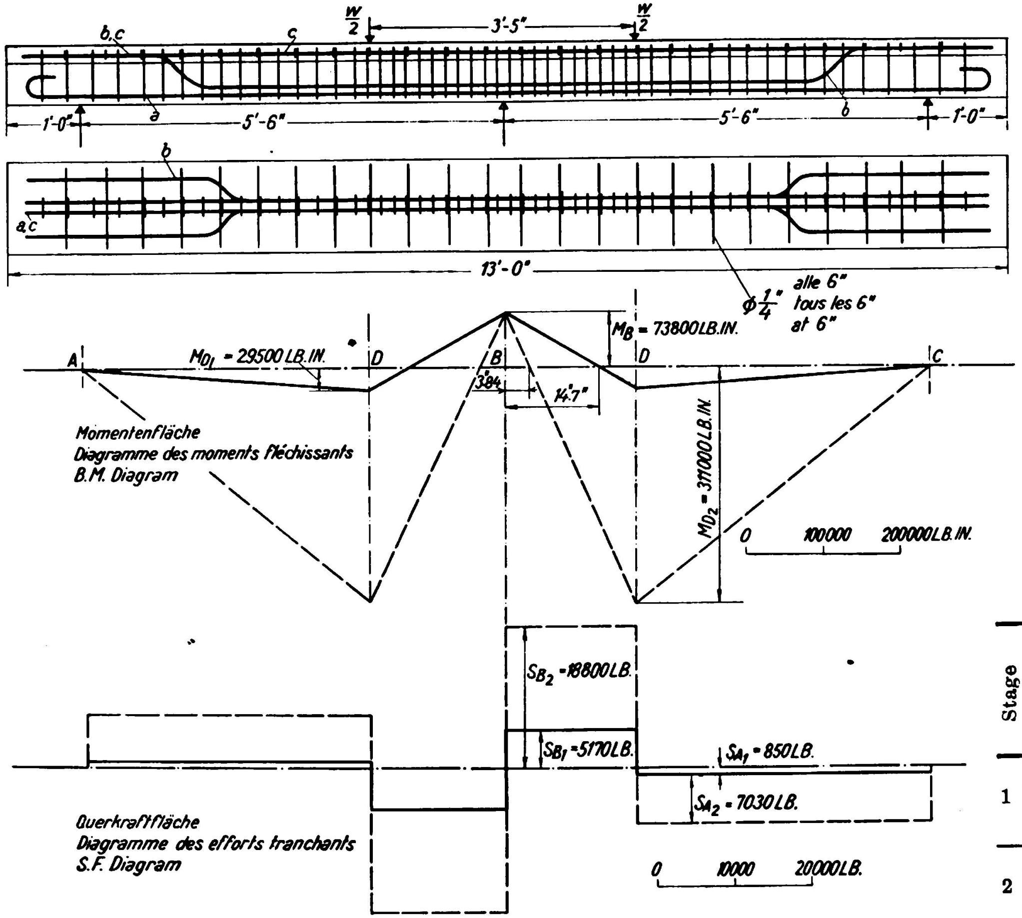
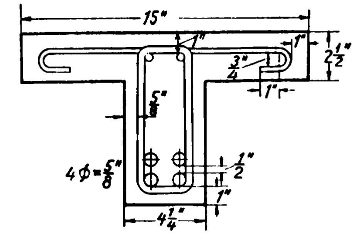


Fig. 1.

Redistribution of moments in continuous beams.
Steel failure.

R.M.2.



Eisenliste – Liste des Fers – Bar schedule			
Eisen Armature Bar	φ	No.	Form – Forme
a	5/8"	2	12'-6"
b	5/8"	2	1'-9" 9" 7'-6" 9" 1'-9"
c	3/8"	2	12'-6"

Calculated Stresses (lb. inch units)

Stage	at B									at D						at A				W	ξ_B	R_B	$R_A = R_C$
	t	c	M	n	a	S	s	t_w	s_b	t	c	M	n	a	s_b	S	s	t_w	s_b				
1	44700	2545	73800	1.85	7.47	5170	162	6255	293	3740	285	29500	2.09	6.40	103	852	28	2170	30	10980	14.7	10340	852
2			73800			18800	592	22700	1073	39100	3000	311000			374	7030	231	17800	251	50750	3.94	37600	7030

Dead Load Effects

$M_B = 3800$ lb in $\qquad R_A = 204$ lb

$M_D = 0$ lb in $\qquad R_B = 476$ lb

$u = 8000$ lb/in² $\qquad m = 5.5$

calculating the end reactions from a previous calibration of the joist. The results for one of the two beams tested are shown in Figure 2.

Incipient failure over the central support is clearly indicated by a sudden decrease in moment at that point, after which the moment increased somewhat.

On the assumption that the central support moment remains constant after yield begins, the subsequent moments in the span have been calculated and the theoretical curves are shown on the diagram. It is evident that the assumption leads to a very fair estimate of the actual span moments for the present test.

The concrete used for this test was made in the proportions 1:1:2 (by weight) using rapid hardening Portland cement, and the beam was tested at an age of 44 days. For the second beam a high alumina cement 1:2:4 concrete (by weight) was used, and the beam was tested at an age of 6 days. In the second beam, as a result of the higher tensile strength of the high alumina cement concrete, the help afforded by the concrete in tension was such that the stress in the continuity steel increased from a very low value to its yield point value at the occurrence of the first crack over the support. Apart from this effect, there was apparently no important difference in behaviour, resulting from the use of the two types of cement.

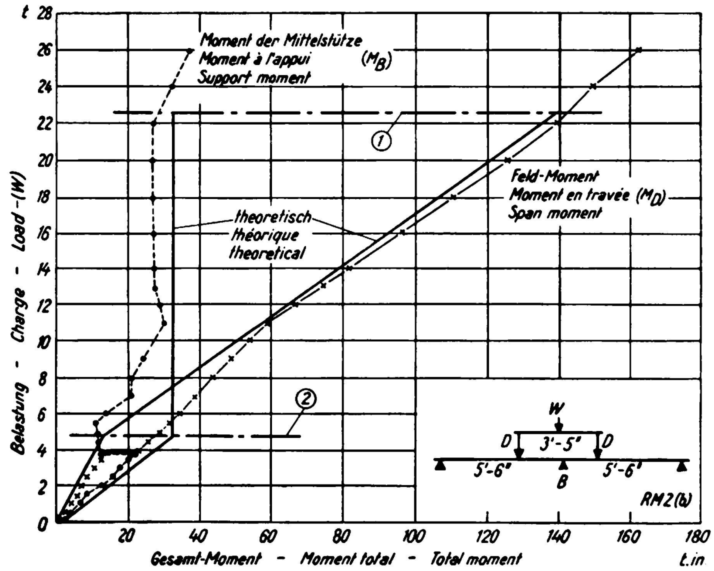


Fig. 2.

Tests on continuous beams. Steel failure (b).
Rapid-hardening portland cement 1 : 1 : 2 concrete (by wt.)
Water/cement ratio = 0.44 (by wt.). Age at test — 44 days.
Cube strength of concrete = 6660 lb. per sq. in.
(1) Theoretical load for general failure.
(2) Theoretical load for support failure.

The deflections at midspan relative to the central support were measured throughout by means of dial gauges. There was no appreciable difference between the deflections of the two beams, and at three-quarters of the failing load the maximum deflection was only about 0,1 in. The supporting steel joist deflected during the test and the sinking of the end supports relative to the central support was therefore also measured. This sinking affects the moments during the elastic stage of the test and has therefore been taken into account in calculating the theoretical curves and stresses given in Figures 1 and 2.

The maximum crack widths, measured with a portable microscope, are given in Table 1. The cracking over the central support increased considerably during the second part of the test, i. e. after the steel had commenced to yield, and shortly before final failure the cracks were from 0.06 to 0.08 in. wide. These cracks are approximately ten times the width usually observed just before the commencement of steel yield.

The loads calculated for failure, (1) on the elastic theory, and (2) on the basis that both the support and span sections develop their full strengths after redistribution, are given in Table 2 together with the actual failing loads. It will be seen that the effect of the redistribution of moments on the load carrying capacity of a continuous beam may be considerable in cases of steel weakness over the central support. However, the cracking accompanying the increased load is very marked, so that in practice, advantage can be taken of moment redistribution due to steel yield only in cases where the increased cracking is not a matter of importance.

Table 1.

Maximum Crack Widths in Continous Beams.

Series	Load — tons	Maximum Crack Width — inch × 10−3											
		Over Central Support						In Span					
		5	10	15	20	25	Yield[1]	5	10	15	20	25	Yield[1]
1. Steel Failure	(a)	0	15	30	42	60	0	0	1.3	2.3	2.6	3.8	0
	(b)	6	15	34	55	79	5	0	1.5	2.6	4.6	6.6	0
2. Concrete Failure	(a)[2]	0	1.5	2.4	3.1	3.3	0.5	0	1.9	3.5	6.0	10.0	0.6
(No compression steel)	(b)	0	1.3	2.2	2.6	2.6	1.2	0	1.3	2.2	3.3	3.9	0.7
3. Concrete Failure	(a)	1.0	3.1	3.7	4.6	5.5	3.4	0	0.9	1.6	2.4	3.5	1.3
(With compression steel)	(b)	1.6	4.0	5.2	9.2	10.5	4.8	0	1.3	1.7	2.6	5.2	1.5
4. Concrete Failure	(a)	3.3	3.7	—	—	—	1.6	1.5	4.0	—	—	—	0.8
(Increased span length)	(b)	0.1	1.0	—	—	—	0	1.3	4.2	—	—	—	0
5. Concrete Failure	(a)	0	1.6	2.7	2.6	1.5	—	0	1.3	2.5	3.6	5.0	—
(Weak concrete at about 6 months)	(b)	0	0.7	1.0	1.1	1.2	—	0	1.4	2.4	3.5	7.2	—

[1] The yield load is the theoretical load for support failure according to the elastic theory (see Table 2).

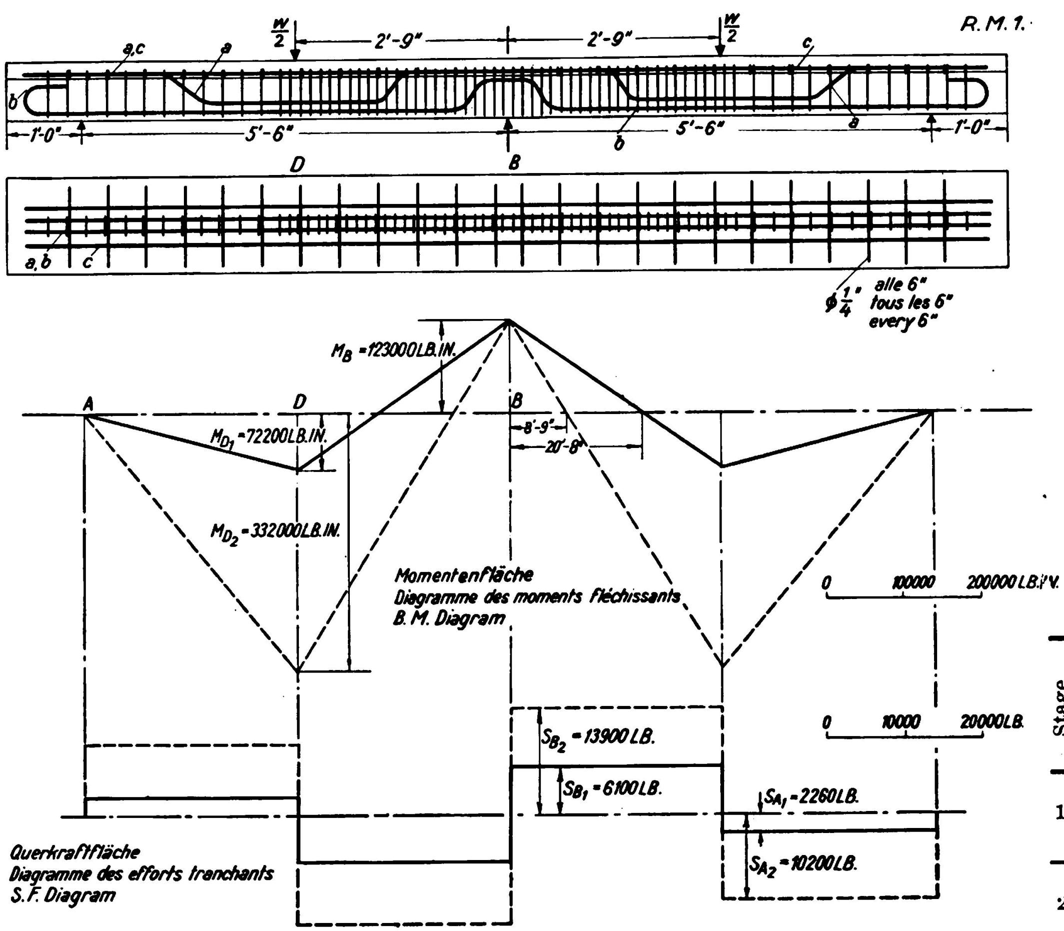
[2] The maximum crack widths in beam (a) of series (2) was measured at the depth of the most highly stressed edge of the tension steel; in all other beams the measurements were at the depth of the centre of the most highly stressed bar.

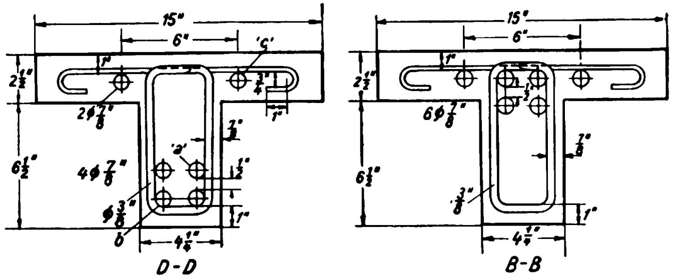
2) *Primary Concrete Failure. No Compression Steel Provided over Central Support.*

In the beams designed to fail by crushing of the concrete, all the tension reinforcement in the span was taken up over the support so that the compression at that point was taken wholly by the concrete in the rib. Details of the beams, spans and positions of loads are given in Figure 3. The concrete was made with ordinary Portland cement using a $1:2^1/_2:3^1/_2$ mix (by weight) and a water-cement ratio of 0,66 (by weight). Te tests were made at an age of 7 days, the strength aimed at being the lowest (2250 lb. per sq. in.) allowed by the Rein-

Table 2. Failing Loads of Continuous Beams.

Basis of Bending Moment Calculations	Basis of Resistance Moment Calculations		Failing Loads — tons									
			1. Steel Failure		2. Concrete Failure (No compression steel)		3. Concrete Failure (Compression steel)		4. Concrete Failure (Increased span length)		5. Concrete Failure (age $5^1/_2$ months)	
		Test No: —	RM 2 (a)	RM 2 (b)	RM 1 (a)	RM 1 (b)	RM 3 (a)	RM 3 (b)	RM 4 (a)	RM 4 (b)	RM 5 (a)	RM 5 (b)
Elastic theory: i. e. No redistribution of moments. Loads are for support failure	No Stress Redistribution	True "instantaneous" modular ratio used	4.9	4.9	7.0	7.2	13.0	14.2	2.7	2.3		
	Stress Redistribution	$m = \frac{40000}{\text{cube strength}} = \frac{40000}{u}$	5.0	4.9	7.6	7.8	19.5	19.8	3.0	2.5		
		Steel Failure: Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	7.8	6.5	8.0	8.2	25.4	26.2	3.2	2.7		
Theory of Redistribution of Moments: i. e. Simultaneous failure at central support and in span	No Stress Redistribution	True "instantaneous" modular ratio used	22.7	22.6	20.8	21.4	25.7	28.1	9.8	8.6		
	Stress Redistribution	$m = \frac{40000}{u}$	23.0	22.6	27.8	28.5	35.0	36.3	13.0	11.8		
		Steel Failure: Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	26.1	24.0	32.6	32.8	40.1	40.5	14.2	13.9		
Actual load at which signs of distress were first noticed in the concrete			—	—	20.8	24.0	23.0	24.0	9.0	9.5	18.8	16.5
Actual ultimate load carried by beam			29.1	28.7	27.5	28.6	27.6	28.9	13.4	13.0	33.0	27.5





Eisenliste - Liste des fers - Bar schedule			
Eisen Armature Bar	ϕ	No.	Form - Forme
'a'	$\frac{7}{8}$"	2	1'-9" — 2'-3½" — 2'-8" — 2'-3" — 1'-9"
'b'	$\frac{7}{8}$"	2	5'-8" — 9" — 5'-8"
'c'	$\frac{7}{8}$"	2	12'-6"

Fig. 3.

Redistribution of moments in continuous beams.
Concrete failure.

Calculated Stresses (lb. inch units)

Stage	at B									at D						at A				W	ξ_B	R_B	$R_A = R_C$
	t	c	M	n	a	S	s	t_w	s_b	t	c	M	n	a	s_b	S	s	t_w	s_b				
1	6400	2050	123000	5.35	5.32	6100	270	7800	70	5180	445	72200	3.09	5.80	35	2260	80	4660	63	15800	20.8	12200	[illegible]
2			123000			13900	615	17700	159	23800	2050	332000	3.09	5.80	159	10200	366	21000	283	47300	8.9	27800	[illegible]

Dead Load Effects

$M_B = 4120$ lb in	$R_A = 200$ lb
$M_D = 910$ lb in	$R_B = 480$ lb
$u = 2050$ lb/in²	$m = 9{,}5$

forced Concrete Code of Practice. Actually the strength was about 10 per cent. less than this value (see Appendix 1).

In order to reduce the shear stresses with this weak concrete, the loads were applied at midspan, instead of nearer to the central support as in the case of the previous beams.

The results are given in Figure 4. It will be noticed that there is not such a well defined point at which failure over the support commences as in the case of the previous beams in which the steel yielded, but rather a gradual change from the elastic to the inelastic stages of the test.

The concrete at the support continued to carry load in an apparently undistressed condition long after the load calculated to produce a fibre stress equal to the cube strength had been reached. In fact there was no evidence of crushing over the central support until the load was more than twice this value.

The measured span moments were again in fair agreement with those calculated on the assumption of a constant support moment after passing the elastic stage.

Throughout the test the crack widths were small (see Table 1) so that redistribution of moments in the case of concrete weakness may be considered without reference to cracking. The beam deflections were of the same order as those measured in the previous series.

3) *Primary Concrete Failure. Compression Steel Provided over the Central Support.*

In tests designed to show weakness in compression in the presence of a limited amount of compression steel the reinforcement was the same as in the previous beams except that the lower bars were continuous throughout the beam, thus providing help in compression over the central support. The concrete mix used was again $1 : 1^1/_2 : 3^1/_2$ (by weight) using ordinary Portland cement, and the tests were made at an age of 7 days; the strength (see Appendix 1) was a little higher than that obtained in the previous tests.

The moments throughout the system were measured, and it was again found that there was a gradual change between the two stages of the test, and it is interesting that the final loads attained (see Table 2) were almost exactly the same as for the beams in which no compression reinforcement was provided.

There was no evidence of compression failure over the central support until just before final collapse of the system. The main tensile crack at that section gradually closed towards the end of the test until it extended only about 2 in. from the top surface of the beam, indicating that the whole of the rib and even some of the flange was bearing compression forces.

The maximum crack widths are given in Table 1.

4) *Primary Concrete Failure. Beams with Increased Span Length.*

The beams in series (2) were provided with closely spaced stirrups over the central support in order to avoid shear failure with the weak concrete used. It was suggested that this reinforcement gave lateral support to the concrete, thus increasing its ability to carry longitudinal compression. In order to show whether this was the case, two further beams were prepared similar to those of series (2) except that the span length was increased to 12 ft. so that the failing moments would be reached at a lower load, hence reducing the amount of shear reinforcement required.

The results showed conclusively that the central support section was not weakened by the wider spacing of the stirrups. The percentage increase in load due to redistribution was approximately the same as before (see Table 2), and the support moment carried at failure was actually greater than had been obtained in the previous tests of series (2).

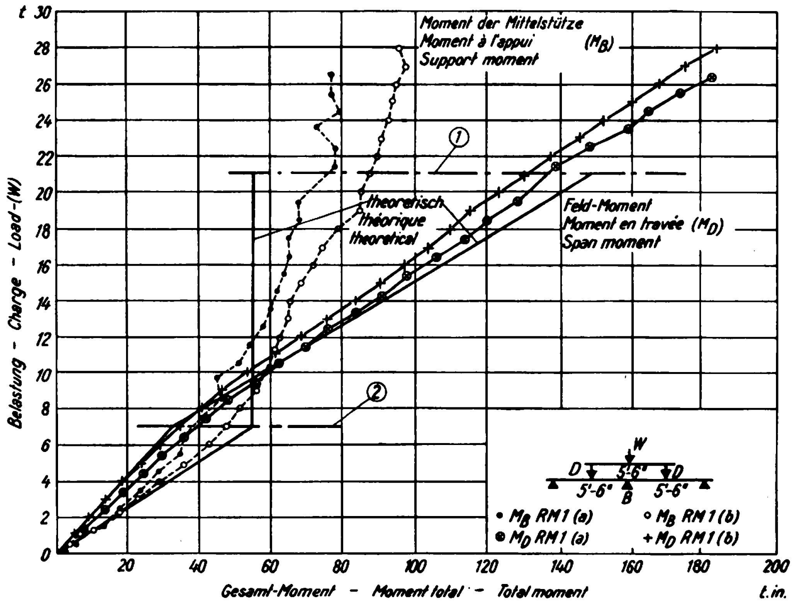


Fig. 4.

Tests on continuous beams. Concrete failure (No compression steel). Normal Portland cement $1:2^1/_2:3^1/_2$ concrete (by wt.) Water/cement ratio = 0.66 (by wt.) Age at test — 7 days. Cube strength of concrete = 2050 lb. per sq. in.

① Theoretical load for general failure.

② Theoretical load for support failure.

5) *Primary Concrete Failure at an Age of $5^1/_2$ Months.*

The tests previously carried out with weak concretes were made at an age of 7 days in all cases, and although it seemed probable that the amount of redistribution that occurred as a result of inelastic deformation of the concrete would depend on the strength of the concrete rather than its age, it was thought advisable to test two beams similar to those of series (2) (no compression reinforcement) at a greater age. In order to obtain a low strength at about 6 months an ordinary Portland cement was used in a mix of proportions 1 : 4 : 7 (by weight) for the first beam; this was changed to 1 : 5 : 6 for the second to give a better mix with the same water-cement ratio of 1,05.

The failing loads, given in Table 2, were as great and in one case greater than those obtained previously. The concrete strength was, however, not known very accurately as the cubes cast with the beams could not be relied upon to give a fair estimate of the concrete quality in the beam itself for such a poor quality concrete. Samples were cut from the ends of the beams and tested, and the results indicated that if anything the concrete was somewhat weaker than that used for the earlier tests. There is no doubt, therefore, that the redistribution obtained with the richer concrete was not attributable to the fact that it had hardened for only a comparatively short period.

B. Tests on Portal Frames.

Tests were made to determine to what extent the load-bearing capacity of a simple reinforced concrete portal frame may be increased as a result of redistribution of stress and moment when high stresses are reached at the column head.

The conditions tested were:

1) Primary failure of the tension steel in the column.

2) Primary failure of the concrete in compression in the column.

For each condition two frames were tested.

1) *Primary Failure of the Tension Steel in the Column.*

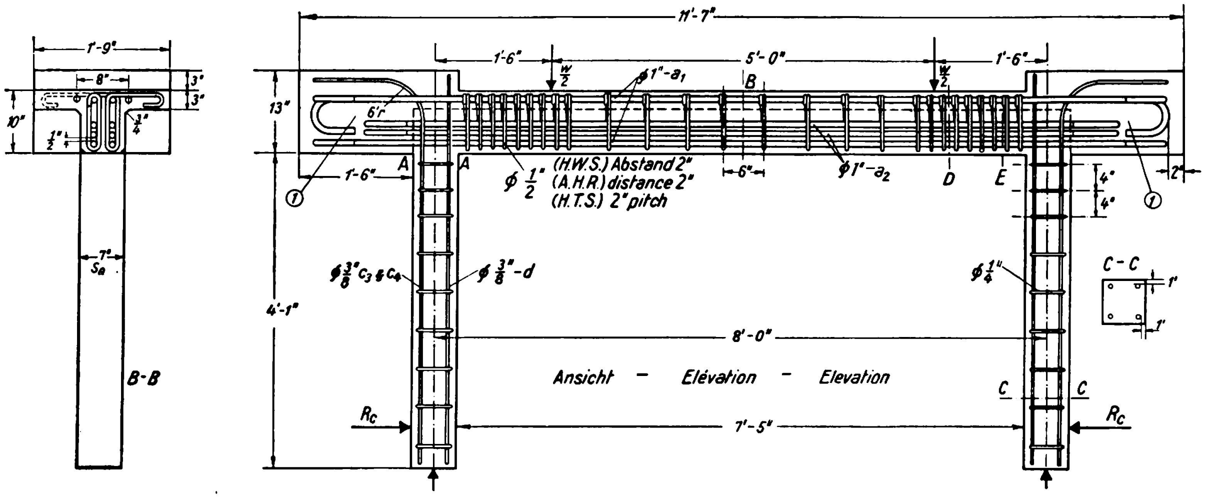
Details of the frames and the positions of loading are given in Figure 5. The design of the reinforcement and the method of loading was such that the beam was considerably stronger than the column. At incipient failure of the weak column there was a considerable reserve of strength in the beam.

In order to ensure that the frame should fail by bending, and not in shear or by slip of the bars, it was necessary to give special attention to the design of the shear reinforcement and the anchorage of the bars. It is clear that redistribution of moment can increase the ultimate load of a structure only when the conditions of bond and shear that result from such redistribution are amply provided for. The large blocks at the column-beam junctions were provided solely for the purpose of giving ample anchorage to the reinforcement of the beams and columns in order that the yield point of the steel could be reached.

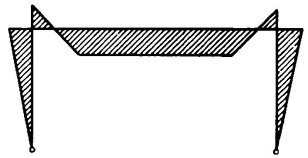
A high strength high alumina cement concrete was used for these tests; details of this are given in Appendix 2.

The horizontal load was applied by two helical tension springs stretched between the column feet, the load being transmitted to the column faces through knife edges. The load on the beams was applied through cylindrical bearings and rollers to allow free rotation and translation of the beam. In the first test the column feet were supported on similar bearings but it was found that the frictional force due to the rollers was sufficient to affect appreciably the horizontal spring load required to prevent outward movement of the feet, and a special knife edge link system was used for subsequent tests.

During the test, gauges were set up at the column feet to measure the movement outwards, and the horizontal load due to the springs was continually adjusted so that the feet were brought back to their original position. That is,



R.M.F.2. & R.M.F.3.



Momentenfläche (nicht masstäblich)
Diagramme des moments (non à l'échelle)
Moment diagram (not to scale)

Eisenliste Liste des fers Bar schedule			
Eisen Armature Bar	ϕ	Masse - Dimensions	Anzahl Nombre No. Off.
a_1	1"	7"; 11'-3"; 6"	6
a_2	1"	7"; 9'-11"; 6"	4
c_3	3/8"	4"; 2 1/4"; 18"; 5'-1"	2
c_4	3/8"	4"; 2 1/4"; 18"; 5'-1"	2
d	3/8"	5'-1"	4

All main bars in beam 1" dia.
Vertical cover 1".
(1) All hooks of internal dia. 4".
Length of straight 4".

Calculated Stresses (lb. inch units) R.M.F.3

Stage		Column						Beam at B			Beam at D				ξ_F	s_E	W	R_C
		c	t	M_A	S	s	s_b	c	t	M_B	S	s	s_b	t_w				
1.	I_1	4200	47300	102000	2380	65	190	920	7000	206000	17300	415	155	7600	2.6	190	34500	2380
	I_2	4800	47300	114000	2650	70	210	1200	9000	266000	21900	525	200	8700	2.3	242	43700	2650
2.		11000	47300	283000	6590	175	520	11000	40600	1240000	86000	2120	775	37600	0 2	—	172000	6590

u = 11000 lb. in² m = 5

I_1 Moments of inertia, for moment distribution calculations, based on whole area of Concrete Ignoring Steel.
I_2 Ditto, based on whole area of Concrete Including Steel.

Fig. 5.

Redistribution of moments in frames. Steel failure.

the conditions of restraint were those of a portal, position fixed and pin-jointed at the feet of the columns.

A view of one of the frames whilst the test was in progress is given in Figure 6. A special framework was arranged to prevent any rotation or lateral movement of the supporting beam relative to the upper loading beam, so that no torsional or lateral bending stresses should be set up in the columns.

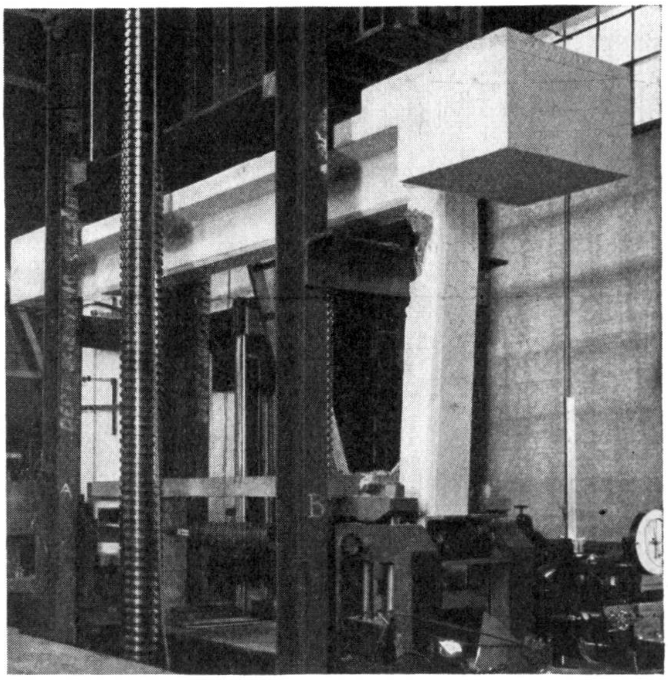


Fig. 6.
Test on reinforced concrete portal frame (Concrete failure).

The main results for the second test are shown in Figure 7. In this figure the applied loads are plotted against the horizontal reactions which are proportional to the moments at the column head, and on the same figure some theoretical curves are also given. One of these curves shows the load-reaction relationship expected for the frame from calculations based on the elastic theory; a series of curves are given for the relationship between the loads and reactions which produce steel yield on the following assumptions:

1) The "instantaneous" modular ratio determines the stress distribution,

2) the modular ratio is taken to be $m = \frac{40\,000}{u}$, and

3) the maximum concrete stress is assumed to reach the cube strength (u).

The point where the first mentioned curve intersects each of the steel failure curves determines the load at which the frame should have failed according to the elastic theory, with or without allowance for stress redistribution according to which assumption the curve represents. These loads are given in Table 3.

On the simplest theory of moment redistribution (i. e. assuming that the column tension steel remains continuously at its yield point) the horizontal reactions and therefore the moments will, after yield of the column steel, conform to the relationship shown by one of the steel failure lines in Figure 7, according to the amount of stress redistribution that occurs. The experimental results gave horizontal reactions which were initially somewhat lower than expected, redistribution beginning at quite a low load, soon after the appearance of cracks at the column head. The curve showing the experimental results gradually approaches the steel failure lines as the load is increased and crosses the line based on $m = \frac{40\,000}{u}$. Incipient concrete failure caused a sudden drop in the rate of increase in moment, and finally failure was reached as a result of concrete crushing.

The beam failure lines indicate the values for the applied load at which beam failure would occur for the degrees of fixity afforded by the various horizontal reactions and it will be seen that if the concrete in the column had not failed a slight increase in load could have been obtained before beam failure.

Throughout the test, measurements were made of the strains at the column heads. The strains were measured on the faces of the column; no direct readings were taken on the steel itself, the steel strain being deduced on the usual assumption that plane sections remain plane. This assumption will probably not

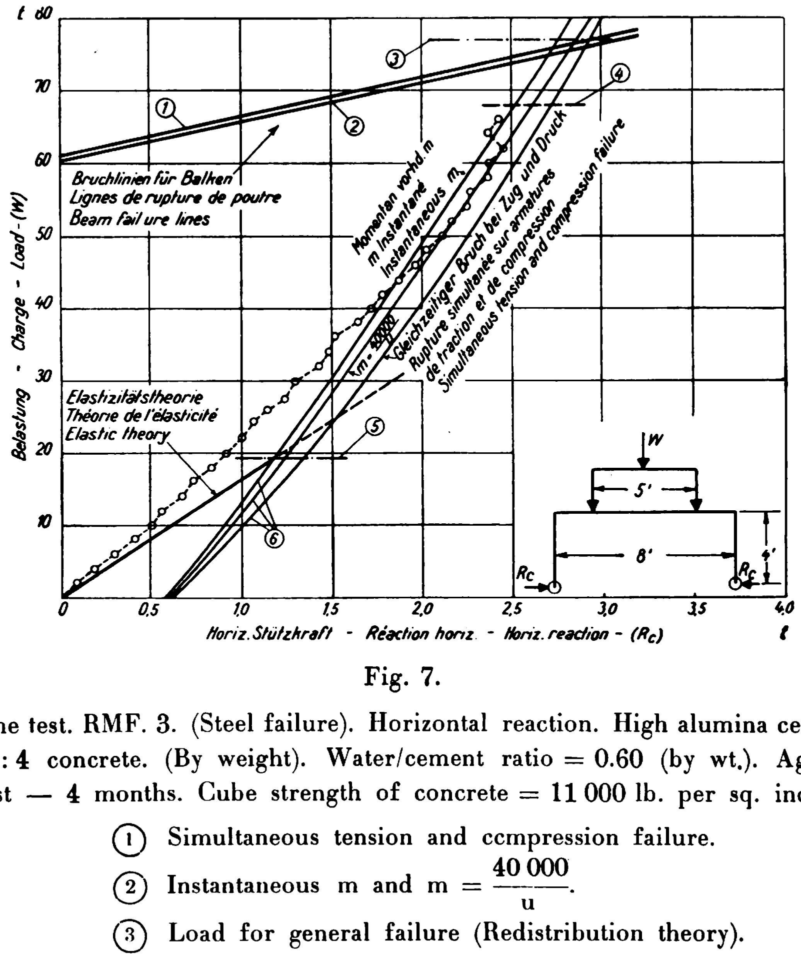


Fig. 7.

Frame test. RMF. 3. (Steel failure). Horizontal reaction. High alumina cement 1 : 2 : 4 concrete. (By weight). Water/cement ratio = 0.60 (by wt.). Age at test — 4 months. Cube strength of concrete = 11 000 lb. per sq. inch.

(1) Simultaneous tension and ccmpression failure.

(2) Instantaneous m and $m = \frac{40\,000}{u}$.

(3) Load for general failure (Redistribution theory).

(4) Actual failing load.

(5) Load for column failure (Elastic Theory).

(6) Column tension failure lines.

lead to very great error except in the final stages of the test. The strain for a steel stress of 47.300 lb. per sq. in. (the yield stress, see Appendix 2) was reached at a load of just over 20 tons, and the strain increased to over four times this value before collapse became imminent. The concrete strain at first signs of crushing was about 32×10^{-4}.

The beam deflection was measured relative to the loading points by means of dial gauges. This deflection was only one-thousandth of the span at about three quarters of the failing load. The overall longitudinal extension of the beam soffit was also measured; as failure of the frame was approached this

Table 3.

Failing Loads of Portal Frames.

Basis of bending Moment Calculations	Basis of Resistance Moment[1] Calculations		Failing Loads — tons			
			Steel Failure		Concrete Failure	
		Test No.: —	RMF 2	RMF 3	RMF 4	RMF 5
Elastic theory: i. e. No redistribution of moments, Loads are for column head failure.	No Stress Redistribution	True "instantaneous" modular ratio used	19.5	19.5	21.2	15.0
	Stress Redistribution	$m = \frac{40000}{\text{cube strength}} = \frac{40000}{u}$	21.3	21.3	24.0	18.3
		Steel Failure. Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	25.0	25.0	27.5	21.4
Theory of Redistribution of Moments: i. e. Simultaneous failure at column head and in beam span.	No Stress Redistribution	True "instantaneous" modular ratio used	75.0	75.0	46.0	41.7
	Stress Redistribution	$m = \frac{40000}{u}$	75.5	75.5	46.8	42.6
		Steel Failure. Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	77.0	77.0	47.8	43.6
Actual load at which signs of distress were first noticed in the concrete			65.0	64.0	40.0	38.0
Actual ultimate load carried by frame			65.0	67.8	47.1	43.2

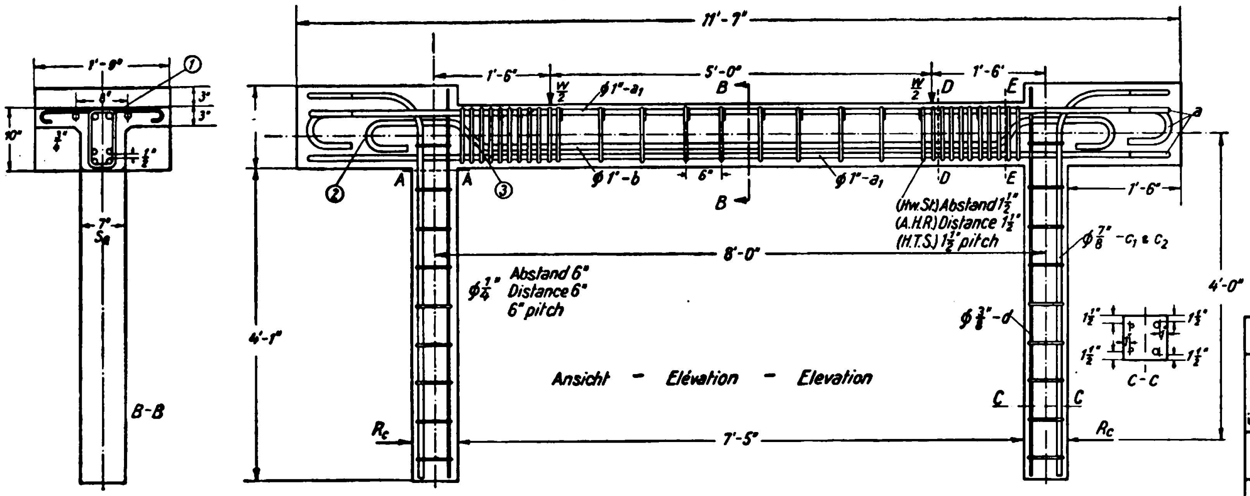
[1] By resistance moment in these tables is meant the ultimate moment the section can carry.

movement was about one-twelfth of an inch at each column head. This movement is insufficient, as an added eccentricity, to have an appreciable effect on the stress at the column head.

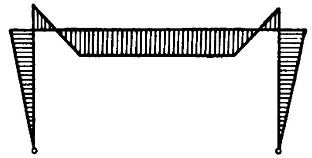
Cracks at the column head appeared at a load of about 5 tons, widened steadily throughout the test, and just before failure were about twice as wide as the cracks usually obtained when steel reinforcement reaches its yield point.

2) *Primary Failure of Concrete in the column.*

Details of the reinforcement used for the second type of frame are given in Figure 8. Again the design was arranged to give a reserve of strength in the beam. The tension steel in the column was increased to two $^7/_8$ in. diameter bars instead of $^3/_8$ in. bars and the concrete used was an ordinary Portland



R.M.F.4 & R.M.F.5



Momentenfläche (nicht masstäblich)
Diagramme des moments (non à l'échelle)
Moment diagram (not to scale)

Eisenliste - Liste des fers - Bar schedule			
Eisen Armature Bar	ϕ	Masse - Dimensions	Anzahl Nombre No. Off.
a_1	1"	11'-3"	6
b	1"	9'-9"	2
c_1	$\frac{7}{8}$"	5'-1"	2
c_2	$\frac{7}{8}$"	5'-1"	2
d	$\frac{3}{8}$"	5'-1"	4

Calculated Stresses (lb. inch. units) R.M.F.4

Stage		Column						Beam at B			Beam at D				ξ_F	s_E	W	R_C
		c	t	M_A	S	s	s_b	c	t	M_B	S	s	s_b	t_w				
1.	I_1	2850	15000	127000	2960	90	115	860	12300	245000	21500	465	260	12400	3.1	460	43000	2960
	I_2	2850	13000	124000	2890	89	114	1010	14500	289000	23800	515	285	13800	2.3	510	47500	2890
2.		2850	7500	131000	3050	94	120	2850	41100	817000	53500	1160	645	31000	—	1150	107000	3050

I_1 Moments of inertia for moment distribution calculations based on whole concrete area ignoring steel.

I_2 Ditto, based on whole concrete area and including steel.

u = 2850 lb/sq. in.
m = 9 (Stage 1)
$m = \frac{80000}{u}$ (Stage 2)

① Stirrups welded.

② Hook internal radius = 3", all other hooks of internal radius = 4 × bar dia.

③ Internal radius of bend for main bars, $3\frac{1}{2}$".

(All main bars in beam 1" Dia. Vertical Cover 1").

Fig. 8.

Redistribution of moments in Frames. Concrete failure.

cement of $1:2^1/_2:3^1/_2$ mix (by weight). Details of the strengths of the steel and concrete are given in Appendix 2.

The method of test was identical with that used in the second frame of the previous series, and the values for the horizontal reaction for the first frame are given in Figure 9. It will be seen that the initial relationship between vertical load and the horizontal reaction is in good agreement with that expected from the elastic theory. According to this theory the concrete should crush at a load of about 21 tons, i. e. at the load when the initial line in Figure 9

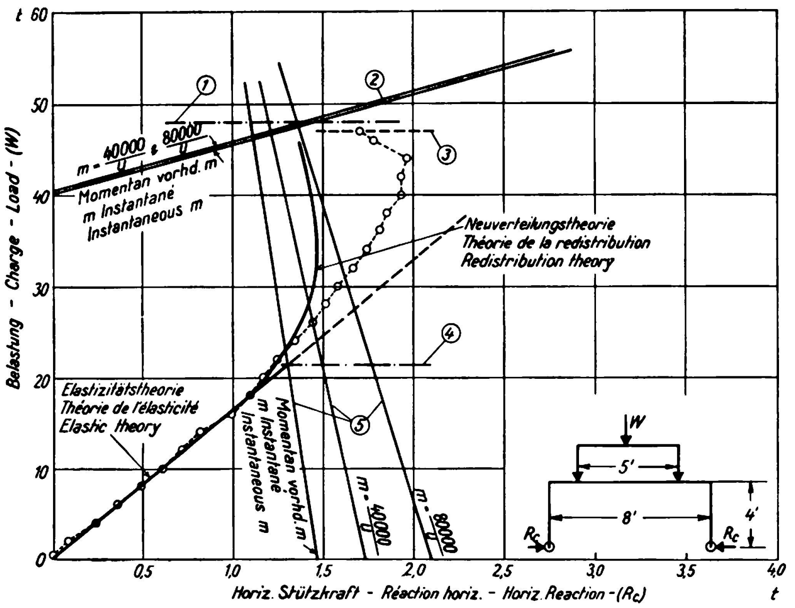


Fig. 9.

Frame test. RMF. 4. (Concrete failure). Horizontal Reaction. Normal Portland cement $1:2^1/_2:3^1/_2$ concrete (by weight). Water/cement ratio = 0.66 (by wt.). Age at Test = 9 days. Cube strength of concrete = 2850 lb. per sq. inch.

(1) Load for general failure (Redistribution theory).
(2) Beam failure lines.
(3) Actual failing load.
(4) Load for column failure (Elastic theory).
(5) Column compression failure lines.

reaches the compression failure line for a modular ratio of m = 9, which is the true value for the concrete used when inelastic deformations are disregarded. Curves representing compression failure are also given based on modular ratios of $\frac{40.000}{u}$ and $\frac{80.000}{u}$. The redistribution of stress in the column head section was even more favourable than is assumed by this last line, probably due to the increased stresses taken by the concrete above those assumed by a linear distribution of stress from the neutral axis to the compressed face. However, assuming this last compression failure line as a safe guide, it is seen that unless moment redistribution occurs there will be signs of distress in the concrete at a load of about 28 tons. If moment redistribution does take place, then the load will increase with a reduction of horizontal reaction until beam failure is reached at a load of about 48 tons. Actually redistribution will start before signs of distress can be seen and the approximate theoretical change in load and moment is indicated in Figure 9. The actual curve shows that the theory is on the safe side, the moments increasing more than expected from the simple theory of redistribution, with a sudden drop in moment after signs of crushing first appeared. The failing load, 47.1 tons, agrees well with the expected value (see Table 3) and was the result of simultaneous crushing of the concrete in the column and yield of the steel in the beam.

The strains at the column head were measured as before; the interpolated steel strains showed that the tension stresses were low throughout, but that the compression bars were working at their yield load towards the end of the test. The deflection of the beam and the extension of the soffit were again small; the column cracking was also of little importance whilst the beam cracks increased to a width of about 6 or 7 thousandths of an inch, a width usually associated with a steel stress of about 40.000 lb. per sq. in.

In the case of the second frame of this series, the concrete strength was somewhat less than that used for the first frame (see Appendix 2) but apart from the reduced values of load and moment due to this cause the results were very similar to those already discussed. Again the use of a modular ratio of $\frac{80.000}{u}$, together with the assumption that the column will continue to deform so as to redistribute the moments until beam failure occurs, leads to an accurate estimate of the failing conditions (see Table 3).

Discussion of Results.

A. Continuous Beam Tests.

The actual failing loads of the continuous beams, together with values calculated on various assumptions are summarised in Table 2. It is apparent that with all the beams, the ultimate load carried before failure of the system was greater than the theoretical load for support failure calculated on the elastic theory. The increased load can be considered as due to two factors, both resulting from inelastic deformations of either concrete or steel:

1) Redistribution of moments throughout the system tending to give simultaneous failure both at the central support and in the span.
2) Redistribution of stress at the highly stressed sections, increasing the moments these sections are capable of taking above the values as calculated by the ordinary theory.

In Table 2 the calculated loads are based on three sets of resistance moments. The first is obtained by the use of the true or "instantaneous" modular ratio, that is, the ratio which neglects all inelastic deformation of the concrete. The second is obtained by assuming that inelastic deformation of the concrete will lead to an increase of the modular ratio to a value $m = \frac{40.000}{\text{cube strength}}$, the value suggested for design purposes in the Code of Practice for the Use of

Reinforced Concrete in Buildings.[5] The third set of resistance moments were calculated on the following assumptions:

a) In the case of primary tension steel failure the steel will yield until the maximum concrete stress reaches the cube strength of the concrete.

b) In the case of primary concrete failure, the modular ratio will effectively increase to a value given by $m = \frac{80.000}{\text{cube strength}}$. If, however, tension steel yield occurs when this higher value is used, the resistance moment is calculated as for (a). If the calculated stress in the compression steel exceeds its yield value when the higher modular ratio is used, the calculations are modified so that the compression bars do not exceed thier yield value.

From Table 2 it will be seen that if the elastic theory is used for calculating the moments at failure, the theoretical failing loads are less than the actual ultimate loads, even when allowance is made for redistribution of stress.

On the other hand, if redistribution of moments is allowed for, the theoretical loads for simultaneous failure at the central support and in the span, when no redistribution of stress is taken into account, are also less than the actual loads carried, though the margin of safety is not so great.

If allowance is made for both moment and stress redistribution the use of a modular ratio of $\frac{40.000}{u}$ leads to theoretical loads which are not greatly different from the actual ultimate loads except in the case of the beams in which compression reinforcement was used over the central support with a weak concrete [series (3)]. The use of the third method of allowance for stress redistribution, when moment redistribution is also allowed for, is clearly unsafe except in the case of primary steel failure, for which it must be remembered that the redistribution of moments is accompanied by widening of the tension cracks, see Table 1.

The results of the tests on the beams in which compression reinforcement was provided are important. The use of a very high modular ratio for estimating the resistance moment of a section leads to increased computed stresses in the compression bars and it does not appear advisable to rely upon this. In order to investigate this aspect more fully, some simple beam tests were carried out to measure the resistance moments of the sections similar to those used over the central support in the main tests. From these tests, it was found that the use of the highest modular ratio $\frac{80.000}{u}$ is reasonable in all cases of concrete failure except those in which compression reinforcement was provided. In these cases the simple beam tests indicated that redistribution of stress may occur to the extent indicated by the use of the lower modular ratio of $\frac{40.000}{u}$, whereas the support moments measured in the continuous beam tests are not appreciably greater than those calculated on the basis of the "instantaneous" modular ratio. It is possible, however, that the higher shear stresses in the continuous beams with compression reinforcement may have been the reason for the low moment carried over the central support. It appears therefore that when compression reinforcement is provided at the support its effect should be ignored in making calculations taking moment redistribution into account. If this is done for the present beams of series (3), the calculated loads $\left(\text{using a modular ratio of } \frac{40.000}{u}\right)$ are 28.9 and 31,6 tons, 5 and 9 per cent. greater respectively than actually obtained. If the effect of the compression reinforcement in the span is also ignored, the calculated loads are 23.4 and 25.2 tons respectively and these are on the safe side.

B. Portal Frame Tests.

It is clear from the tests that there may be considerable divergence between the actual ultimate load-carrying capacity of a frame and the load which, according to calculations based on the elastic theory, produced a stress in the concrete or steel, at the column head, equal to the ultimate strength of the concrete or the yield strength of the steel. It is important to note that in the tests special precautions were taken to prevent shear failure, closely spaced high tensile steel stirrups being provided in the beams, and special anchorage blocks at the beam-column junctions. Redistribution of moments cannot occur unless the secondary reinforcement and the anchorage of the steel are sufficient for the conditions resulting from the redistribution.

In the case of primary steel failure, the increase in load due to redistribution of moment and stress was over 200 per cent. However, in this case complete moment redistribution did not occur, beam failure not being reached, owing to the earlier crushing of the concrete in the column, even though the cube strength was 11 000 lb. per sq. in. In such cases it is not at present possible to calculate accurately the load at which the concrete will fail as it depends on the deformation of the column after yield of the tension steel. Since the extent to which redistribution can take place as a result of steel yield is not clearly defined and redistribution leads to increased cracking it would be wise to ignore it until further experimental evidence has been obtained.

In the case of primary concrete failure, there are again considerable increases in the ultimate loads carried by the frames as a result of redistribution of stress and moment. If we consider that the useful limit of load increase is when signs of crushing first appear on the column faces it will be seen from Table 3 that the load increase above the value calculated on the elastic theory was 90 per cent. for the first frame and 150 per cent. for the second frame.

In both cases the increase in beam load-carrying capacity as a result of the column moment was less than 20 per cent. whereas the columns would, if loaded axially, have been able to withstand about twice the load that they took in the frame test. The need for taking bending in columns into account is evident.

It would appear that an estimate of the effects of redistribution can be made in simple cases where concrete failure is the deciding factor on the following assumptions:

1) The modular ratio can be taken as $\frac{80.000}{u}$.
2) Both column head and span develop their full strengths before failure of the system occurs.

In any cases where the use of the higher modular ratio leads to calculated stresses in the tension steel greater than the yield point of the steel, the particular section should be calculated on the assumption that both steel yield and the full concrete strength are developed.

It is clear from Figure 9 that stress redistribution occurred in the column head section to a greater extent than that indicated by the use of a modular ratio of $\frac{80.000}{u}$ and from this figure and Table 3, it is seen that the effect of stress redistribution, if moment redistribution is ignored, is to increase the failing load by about 30 per cent. for the particular section used. The increase may not be so great in other cases. For example in the continuous beam tests described earlier in this report the increase in resistance moment due to stress redistribution was only about 13 per cent. for the central support section of the beams of series (2) and (4). In the columns of the portal frames designed for concrete failure, the compression steel used was much less than the tension steel whereas normally the section would be symmetrically reinforced. In view of the smaller amount of stress redistribution that occurred in beam sections reinforced in compression, it would therefore be unwise to use the higher modular ratio, and a value of $m = \frac{40.000}{u}$ is likely to lead to more satisfactory results.

General.

It has been shown that as a result of inelastic deformation of either the steel or the concrete at incipient failure, moment redistribution will usually occur in reinforced concrete structures before final collapse.

The amount of moment redistribution that can occur depends on many factors but to a large extent on the amount of deformation possible at weaker sections. Where weaker sections are capable of developing sufficient deformation, redistribution will be complete and failure simultaneous at principal sections. Further investigation is necessary to fix the safe limits of deformation. Until this is done it would appear wise not to deviate greatly in design from the requirements of the elastic theory.

Design of reinforced concrete structures on the basis of redistribution of moments must take into account the higher bond and shear stresses that accompany redistribution.

List of References.

[1] *W. H. Glanville* and *F. G. Thomas:* „The Redistribution of Moments in Reinforced Concrete Beams and Frames." Journal of the Institution of Civil Engineers, 1936, No. 7, pp. 291—329.

[2] *F. E. Richart, R. L. Brown* and *T. G. Taylor:* „The effect of Plastic Flow in Rigid Frames of Reinforced Concrete." Journal Am. Conc. Inst., Vol. 5, pt. 3, 1934, pp. 181—95.

[3] *G. von Kazinczy:* „Das plastische Verhalten von Eisenbeton." Beton und Eisen, Vol. 32, pt. 5, 1933, pp. 74—80.

[4] *C. Bach* and *O. Graf:* „Versuche mit eingespannten Eisenbetonbalken." Deutscher Ausschuß für Eisenbeton, Heft 45, 1920.

[5] "Report of the Reinforced Concrete Structures Committee of the Building Research Board, with Recommendations for a Code of Practice for the Use of Reinforced Concrete in Buildings." H. M. Stationery Office, 1933.

Appendix 1.

Quality of concrete and steel used in connection with continuous beam tests.

a) *Concrete.*

Series	Beam	Concrete Mix (by wt.)	W/z Ratio.	Age at Test	Cube Strength — lb/sq. in.	True "Instantaneous" Modular Ratio.
1. Steel Failure	RM2 (a)	H.A. 1 : 2 : 4	0.60	6 days	10.140	5.0
	RM2 (b)	R.H.P. 1 : 1 : 2	0.44	44 days	6.660	6.0
2. Concrete Failure (No compression steel)	RM1 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.020	10.0
	RM1 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.070	10.0
3. Concrete Failure (With compression steel)	RM3 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.250	9.5
	RM3 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.470	9.1
4. Concrete Failure (Increased span length)	RM4 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.130	9.7
	RM4 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	1.830	10.4

P. = Ordinary Portland Cement. H.A. = High Alumina Cement.
R.H.P. = Rapid-Hardening Portland Cement.

b) *Steel.*

Series	Bar diameter — inch.	Yield Stress — lb/sq. in.[1]	Failing Stress — lb/sq. in.[1]
1. Steel Failure	$\frac{5}{8}$	39.400	—
	$\frac{3}{8}$	44.700	62.200
2. Concrete Failure (No compression steel)	$\frac{7}{8}$	40.200	56.500
	$\frac{3}{8}$	46.100	61.500
3. Concrete Failure (With compression steel)	$\frac{7}{8}$	39.800	53.800
	$\frac{3}{8}$	46.700	62.700
4. Concrete Failure (Increased span length)	$\frac{7}{8}$	37.900	53.300
	$\frac{3}{8}$	46.700	61.800
5. Concrete Failure (Weak concrete at about 6 months)	$\frac{7}{8}$	36.600	51.500
	$\frac{3}{8}$	45.800	61.400

[1] The stresses are in all cases based on the nominal original area of the bar.

Appendix 2.

Quality of concrete and steel used in connection with portal frame tests.

a) *Concrete.*

Series	Beam	Concrete Mix (by wt.)	W/z Ratio	Age at test	Cube Strength lb/sq.in.
Steel Failure	RMF 2 RMF 3	H.A. 1 : 2 : 4 H.A. 1 : 2 : 4	0.60 0.60	48 days 4 months	10.500 11.000
Concrete Failure	RMF 4 RMF 5	P. 1 : 2 1/2 : 3 1/2 P. 1 : 2 1/2 : 3 1/2	0.66 0.66	9 days 7 days	2.850 1.850

P. = Ordinary Portland cement. H.A. = High Alumina Cement.

b) *Steel.*

Series	Beam	Bar diameter inch.	Yield Stress — lb/sq.in.[1]	Failing Stress — lb/sq.in.[1]
Steel Failure	RMF 2	$\frac{3}{8}$ 1 1/2[2]	49.200 41.500 66.900	60.800 63.700 106.000
	RMF 3	$\frac{3}{8}$ 1 1/2[2]	47.300 40.600 63.800	59.700 65.700 107.000
Concrete Failure	RMF 4 et RMF 5	$\frac{7}{8}$ 1 1/2[2] $\frac{3}{8}$	38.600 41.100 64.700 48.300	53.800 63.000 107.000 60.300

[1] The stresses are in all cases based on the nominal original area of the bar.
[2] High tensile steel used for web reinforcement of beam.