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The Calculation of Reinforced Concrete Sections Subject to Bending.

Berechnungsverfahren von auf Biegung beanspruchten Eisenbetonquerschnitten.

Les méthodes de calcul des sections de béton armé sollicitées à la flexion.

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A. The German and Austrian Regulations.

I. Decisions of the German Committee for Reinforced Concrete.

1) *Carrying capacity.*

According to the German Regulations of 1932, Paragraph 17, reinforced concrete sections subject to bending are to be calculated on the assumption that strain is proportional to distance from the neutral axis and that co-operation by the concrete on the tension side is entirely neglected. (Condition II b in

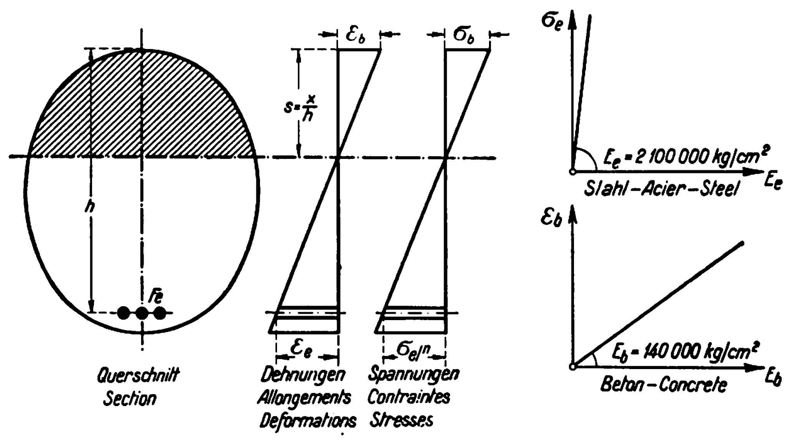

Fig. 1.
German Regulations.

Fig. 1.) The ratio of the moduli of elasticity of steel and concrete is taken as n = 15, and both in the steel and in the concrete a straight line relationship is assumed to hold good between stress and strain (*Hooke*'s Law).

In what follows below, the carrying capacity will be denoted by

$$T = \frac{M \cdot h}{J_i}$$

where J_i is the "ideal" moment of inertia. The expression $\frac{J_i}{h}$ is independent of the shape of the cross section of the beam, and in case of an homogeneous cross section it corresponds to the section modulus W and represents concrete stress plus $\frac{1}{n}$ times steel stress. A picture of the conditions governing the strength of a reinforced concrete section is obtained when T is expressed as a function of the distance from the neutral axis $s = \frac{x}{h}$. When plotting this function the abscissae are so divided that successive values $\frac{1}{s}$ occupy equal distances.

Following the method of calculation hitherto used (in accordance with Type IIb) the carrying capacity in the concrete portion of the beam becomes

$$T = \frac{W_b}{s}$$

(where W_b is the cube strength) and in the steel portion

$$T = \frac{\sigma_s}{n} \; \frac{1}{1-s}$$

(where σ_s is the yield point of the steel). If this system of co-ordinates is adopted the curve of carrying capacity becomes a straight line for the concrete and a hyperbola as regards the steel.[1]

2) *Comparisons with experimental results.*

Fig. 2 represents the results of experiments carried out on rectangular cross sections reinforced with St 37, wherein the cube strength of the concrete $W_b = 110$ kg per sq. cm as nearly as possible, and wherein the yield point of the steel was $\sigma_s = 2800$ kg per sq. cm. The cross sections were so chosen as to develop a carrying capacity corresponding to a considerable range of s. In Fig. 2 the calculated carrying capacities according to the German regulations (broken line) and the capacities determined by experiment are juxtaposed, and the following comparisons emerge:

a) In the region where failure depends on the yield point of the steel: —
   - α) The experimental results always work out approximately 10 % higher than those found by calculation.
   - β) The curves of carrying capacity obtained by calculation and by experiment are entirely similar. It is not possible, however, to justify an increase in the permissible stress in the steel region, and the risk of cracking would in itself be an objection to such a course, nor is there good cause for altering the method of calculation in the region a.

[1] *E. Friedrich:* „Über die Tragfähigkeit von Eisenbetonquerschnitten." Beton und Eisen, 1936, No. 9.

b) In the region where breakage is governed by the strength of the concrete: —

A) The first point that arises is that the curve of carrying capacity extends to values of much higher percentage of reinforcement than the steel carrying capacity line [or as in Fig. 2 up to much higher s-values].

B) In the whole of the second region the carrying capacity is much higher according to experimental values, than according to the calculated values.

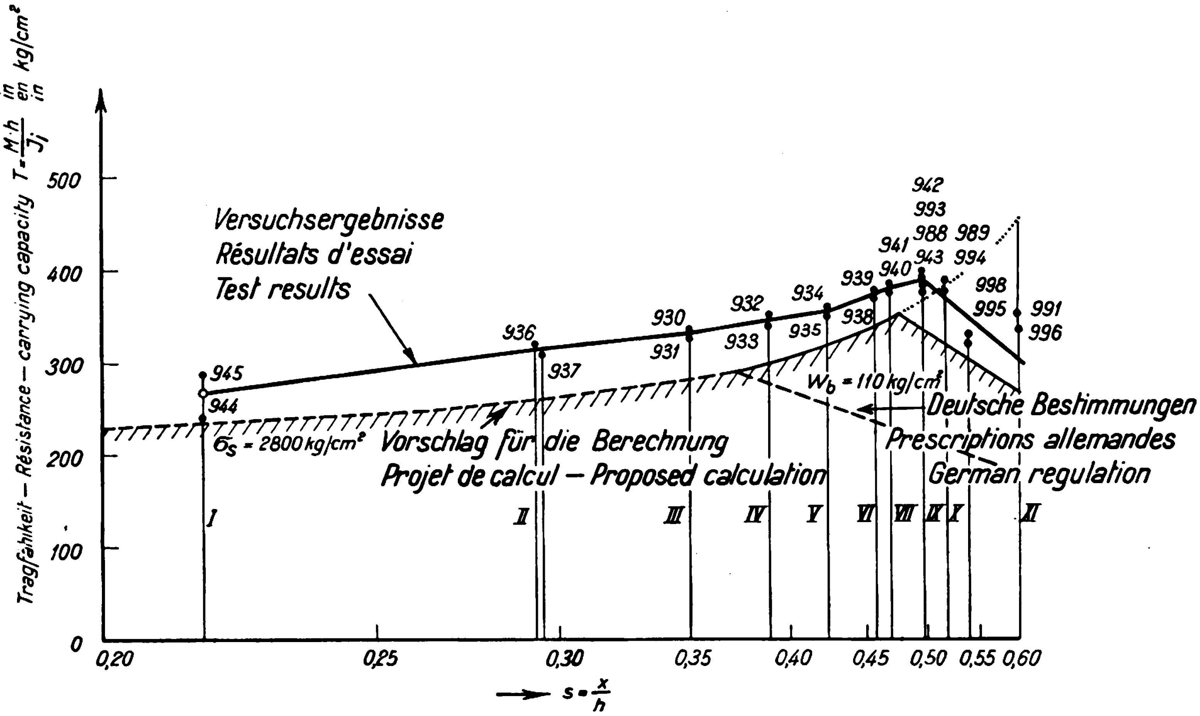

Fig. 2.

Curve of carrying capacity as determined by experiment (full line), by calculation (broken line) and as proposed for St. 37 (hatched border).

II. The Austrian regulations.

In the Austrian regulations an attempt has been made to overcome the defects of the method of calculation hitherto in use. According to the suggestion made by *von Emperger* and *Haberkalt* the limits for the steel and the concrete region is to be raised to an extent corresponding with an increase in the permissible concrete stress to 15 to 25% above that allowed hitherto. Since, however, the existing values of permissible stress have in fact been retained, it follows that the curve of carrying capacity shows a break at the point which marks the limit of reinforcement, and two disadvantages arise in consequence of this:

a) Cases may occur in which the calculated carrying capacity is reduced on an addition being made to the reinforcing steel.

b) Since the limit of reinforcement is made dependent on the percentage of reinforcement provided, the suggestion can be applied only to rectangular cross sections.

Fig. 3 shows the curve of carrying capacity in accordance with the Austrian regulations.

B. New suggestions for the calculation of reinforced concrete sections subject to bending.

The tendency in reinforced concrete design, both in buildings and bridge work, is to avoid both sloping undersides to the beams and compression reinforcement. A suggestion is now made whereby this tendency can be satisfied while retaining the same degree of safety as at present, and while conforming with the lessons of experiments.

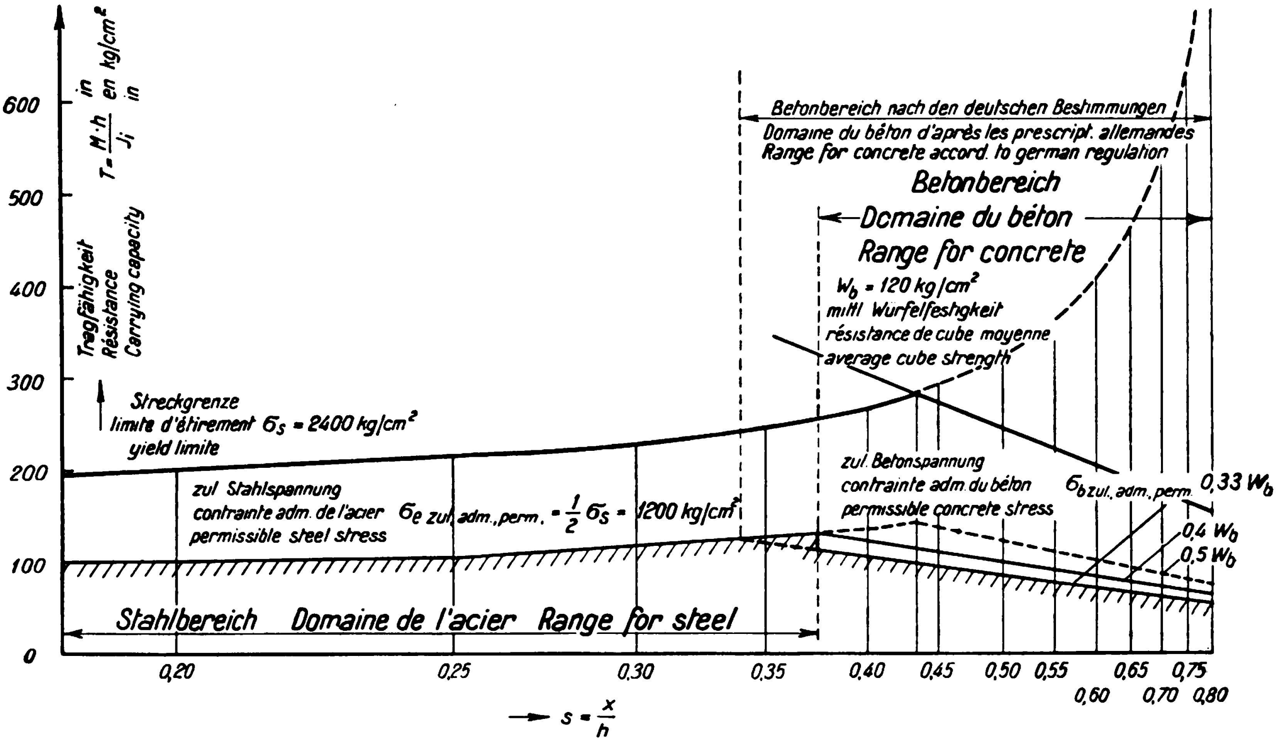

Fig. 3.

Carrying capacity line according to Austrian regulations.

I. Region where failure is determined by the yield point of the steel.

In this region the present method of calculation, as described, will be retained, and provided the required cube strength of the concrete is satisfied the stress in the latter need not be calculated.

II. Region where failure is determined by the strength of concrete.

1) Basis of calculation.

a) Determination of the neutral axis.

Where the bending moment is moderate, the condition indicated by IIb will obtain, as assumed by the method of calculation hitherto in use. That is to say the concrete will tend to crack in the tension zone once the stress in its outermost fibre becomes equal to the breaking stress (which is here equated to the cube strength) but, instead of the beam at once breaking as implied by the calculations hitherto in use, the condition IIb will give way to a new condition IIc, characterized by the fact that the concrete on the compression side becomes plastic. The neutral axis remains in its original position and distance from the neutral axis may, therefore, best be calculated on the same assumptions as hitherto.

$$s^2 + 2s\varphi - 2\psi = 0 \tag{1}$$

$$\left(\text{where } \varphi = \frac{f}{b \cdot h}, \quad \psi = \frac{\gamma}{b \cdot h^2} \quad \text{and} \quad f = n\,F_e + n\,F'_e \right.$$

$$\left. \gamma = n\,F_e\,h + n\,F'_e\,h' \right).$$

b) Stress-strain curve for the steel.

To be calculated on the basis of *Hooke*'s Law (Fig. 4).

$$\sigma_s = E_e \cdot \varepsilon_e$$

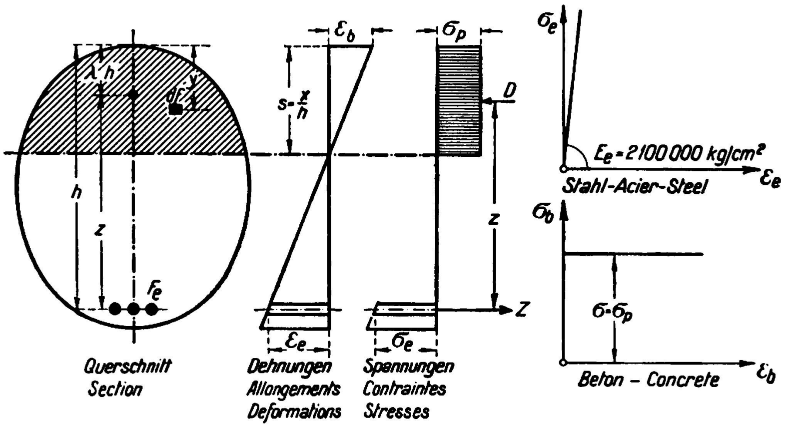

Fig. 4.
Basis of calculation for proposed calculation based on the strength of concrete.

c) Stress-strain curve for the concrete.

To be calculated on the basis of the law of plasticitiy:

$$\sigma_p = \text{const. (independent of E)}$$

d) Navier's assumption.

The calculation assumes that cross sections remain plane.

e) Equilibrium.

At every cross section the external and internal forces must be in equilibrium.

2) Details of calculation.

The stresses and carrying capacity of reinforced concrete sections can be calculated from these assumptions, the total compression D being obtained from the equation.

$$D = \int^{F_W} \sigma_p \cdot df = \sigma_p \cdot \int^{F_W} df = \sigma_p \cdot F_W$$

in which F_w is the effective area of concrete.

The total tension is

$$Z = F_e \cdot \sigma_e = F_w \cdot \sigma_p$$

and for equilibrium we have $Z = D$, or

$$F_e \cdot \sigma_e = F_W \cdot \sigma_p$$

whence

$$\sigma_e = \sigma_p \cdot \frac{F_W}{F_e}. \tag{2}$$

The statical moment of the effective concrete area about the extreme fibre is

$$S_W = \int^{\text{effec. concr. area}} y \cdot df,$$

and hence the distance $\lambda \cdot h$ to the centre of gravity of the effective concrete area is given by

$$\lambda \cdot h \cdot F_W = S_W.$$

The lever arm for the internal forces is

$$z = h - \lambda \cdot h = h \frac{h \cdot F_W - S_W}{h \cdot F_W}.$$

and since the internal and external moments must be equal we have

$$D \cdot z = M$$

$$\sigma_p \cdot F_W \cdot h \cdot \frac{h \cdot F_W - S_W}{h \cdot F_W} = M$$

$$\frac{S_W}{h} - F_W + \frac{M}{h \cdot \sigma_p} = 0. \tag{3}$$

Equation I serves to fix the neutral axis and Equation III enables the carrying moment M to be calculated.

3) Comparison with experimental results.

The formulae explained under (2) above will now be compared with the experimental results for rectangular beams given in Fig. 2, reinforced with St. 37.

For a rectangular cross section —

$$F_W = s \cdot b \cdot h$$

$$S_W = s^2 \cdot h^2 \cdot \frac{b}{2}.$$

In order to allow a comparison between the method of calculation now put forward and that hitherto in use the value of

$$T = \frac{M \cdot h}{J_i}$$

will now be calculated.

In the case of a simply reinforced rectangular section we have

$$\frac{J_i}{h} = b h^2 \frac{\left(1 - \frac{s}{3}\right) \cdot s^2}{2}$$

and it follows from Equation III that

$$T = \frac{M \cdot h}{J_i} = 2 \frac{\sigma_p}{s} \cdot \frac{1 - s/2}{1 - s/3}.$$

Fig. 2 also contains a line marked by hatching, which shows the results obtained from the proposed method of calculation.

In Fig. 5 a comparison is made between the experimental results as indicated in Fig. 2 and the newly suggested method of calculation.

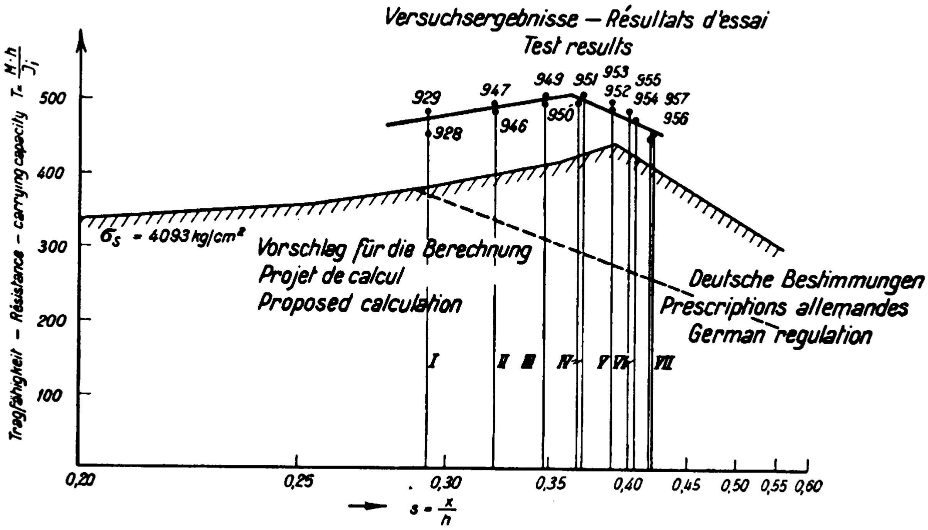

Fig. 5.

Carrying capacity line according to proposal for high grade reinforcing steel.

Fig. 5 shows a further series of experimental results obtained with constructional steel of high yield point (Isteg steel for which $\sigma_s = 4100$ kg per sq. cm). In these experiments the prism strength was ascertained to be $\sigma_p = 94$ kg per sq. cm. Comparison with the method of calculations hitherto in use indicates that the new suggestion gives much better agreement with the experimental results. Fig. 6 shows the fracture of the beam 957, which had taken place in the concrete region; Fig. 7 shows the fracture of beam 947 which has taken place in the steel region, and these two illustrations will enable the two separate regions to be clearly distinguished.

C. Suggestions in regard to the "Regulations".

It has now been shewn how the actual carrying capacity can be reconciled with the carrying capacity as calculated, and suggestions will be made for amending the regulations accordingly.

1) Stress in steel.

The permissible stress in the steel will be as hitherto $\sigma_{e\,perm} = \frac{\sigma_s}{2}$ unless the risk of cracking makes it desirable to prescribe a lower value.

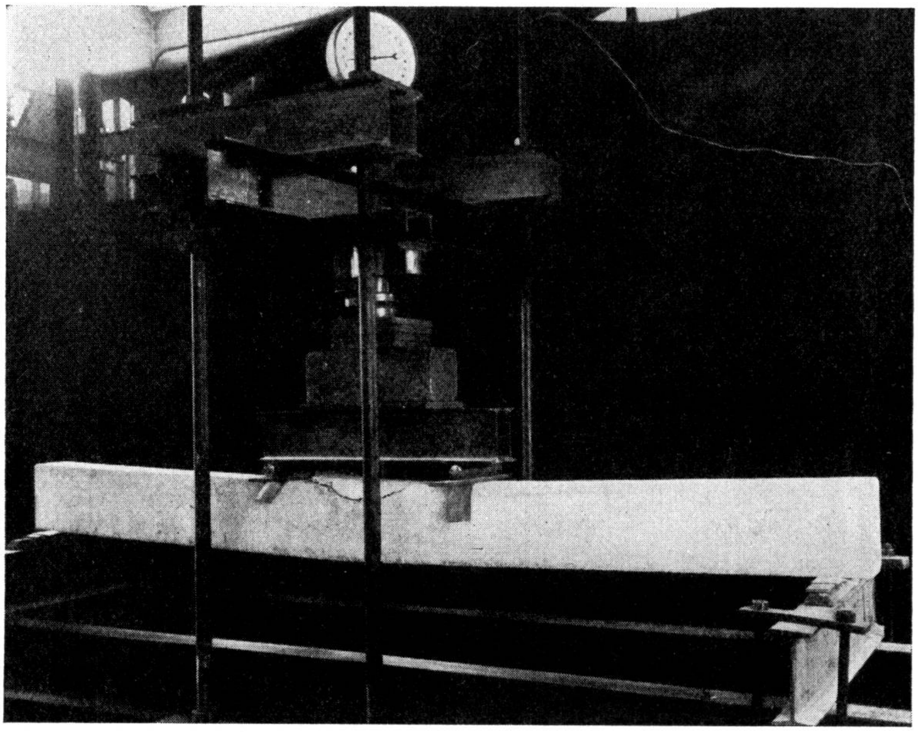

Fig. 6.
Fractured beam No. 957 (fracture due to reaching ultimate strength of concrete).

2) Stress in concrete.

At present a factor of safety of three in relation to the cube strength W_b is adopted. Since, however, the prism strength is to enter into the calculations,

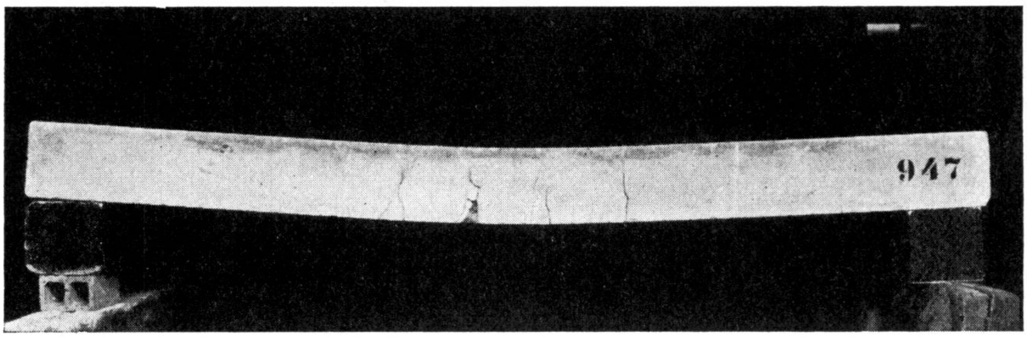

Fig. 7.
Fractured beam No. 947 (fractures due to reaching yield limit of steel).

the permissible concrete stress as hitherto allowed must be reduced. As a rule the prism strength can be taken as 0.75 of the cube strength, (conversion factor for cube strength assuming stationary loading) and on equating the permissible stress to one quarter of the cube strength we obtain ($\sigma_{b\,perm} = \frac{1}{4} W_b$)

permissible stress = 30 kg per sq. cm when cube strength = 120 kg per sq. cm
permissible stress = 40 kg per sq. cm when cube strength = 160 kg per sq. cm
permissible stress = 56 kg per sq. cm when cube strength = 225 kg per sq. cm

3) Limit of reinforcement in rectangular cross sections.

This limit can be determined by equating the carrying capacity in the steel region to that in the concrete region. The carrying capacity of the steel region is

$$\frac{M \cdot h}{J_i} = \frac{\sigma_s}{n} \cdot \frac{1}{1-s};$$

and that in the concrete region is

$$\frac{M \cdot h}{J_i} = 2 \cdot \frac{\sigma_p}{s} \cdot \frac{1-s/2}{1-s/3}.$$

Writing

$$k = \frac{\sigma_s}{n\,\sigma_p},$$

it becomes possible to obtain s_G from the equation

$$s_G = \frac{3}{2} - \frac{1}{2} \cdot \sqrt{\frac{3\,(1+3k)}{3+k}}. \qquad (4)$$

Conclusion.

A great many suggestions for reconciling the results of calculation and of experiment have already been made but if these suggestions are to be embodied in actual regulations they must be perfectly open to experimental confirmation. The suggestion here put forward for determining the limit of reinforcement in rectangular cross sections reinforced with St. 37 is one which appears to be fully supported by experiment, and similar tests on high tensile reinforcing steel are in hand.

The series of experiments has further been extended to cover the case of beams with compression reinforcement, so as to investigate the change in carrying capacity that results from the use of the latter.

The suggested procedure favours a much more uniform utilisation of the material than is obtained by the existing methods, and since, to a considerable extent, it renders inclined soffits and compression reinforcement unnecessary, it offers improved possibilities of adapting reinforced concrete construction to modern requirements in design: in buildings, for instance, a flat under-surface can in this way be given to reinforced concrete floors covering several spans, and in bridge work the girders can be made of equal thickness throughout. At the same time the proposal is attended by economic advantages, in that shuttering and reinforcing steel are saved.