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Rubrik: II. Stressing and degree of safety in reinforced concrete structures, from the designer's point of view

II

Stressing and degree of safety in reinforced concrete structures, from the designer's point of view.

Beanspruchungen und Sicherheitsgrad im Eisenbetonbau vom Standpunkt des Konstrukteurs.

Sollicitations et coefficients de sécurité dans les constructions en béton armé, au point de vue du constructeur.

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II

General Report.

Generalreferat.

Rapport Général.

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Part I: The influence of stationary, permanent and repeated loading.

1) The carrying capacity of reinforced concrete beams in relation to the reinforcement provided has been the subject of investigation during the last few years by the Austrian Reinforced Concrete Committee, notably through the work of *Emperger, Haberkalt* and *Gebauer,* to whom great credit is due.

If, as in Fig. 1, the carrying capacity $\frac{M}{b \cdot h^2}$ is plotted as an ordinate and the amount of reinforcement $\mu = \frac{F_e}{b \cdot h}$ as an abscissa, two well known regions may be clearly distinguished in the resulting diagram, namely: 1) The region representing weakly reinforced beams — the more usual condition — wherein failure is determined by the yield point of the steel, and 2) the region representing the rarer case in which failure is determined by the compressive strength of the concrete.

It is indicated in Fig. 1 at the point marked II that according to the present method of calculation the first of these regions is not fully exploited. The suggestion put forward by *Emperger* and *Haberkalt* was intended to overcome this disadvantage by providing that the stress in the concrete might be increased by 20 % for the purpose of calculating the limiting amount of reinforcement which divides the two regions from one another. This solution, however, is not altogether satisfactory, because it is applicable only to rectangular cross sections, and because cases may arise in which the carrying capacity so calculated decreases when more reinforcing steel is added. This occurs, for instance, when on adding reinforcement the appropriate reading in the diagram is made to fall further to the right of point III, and is on a lower step in the line which denotes carrying capacity.

The experiments lately carried out at Dresden have served to bring out this point very clearly (Fig. 2). A series of reinforced concrete beams were examined in which merely the amount of reinforcement μ was altered. In the case of the usual commercial steel St. 37 the line AC in the diagram, Fig. 2, became practically straight, fixing the point C for the limiting amount of reinforcement, which divides the first mentioned region AC from the second region CD. The parabola which represents the carrying capacity as calculated by the ordinary

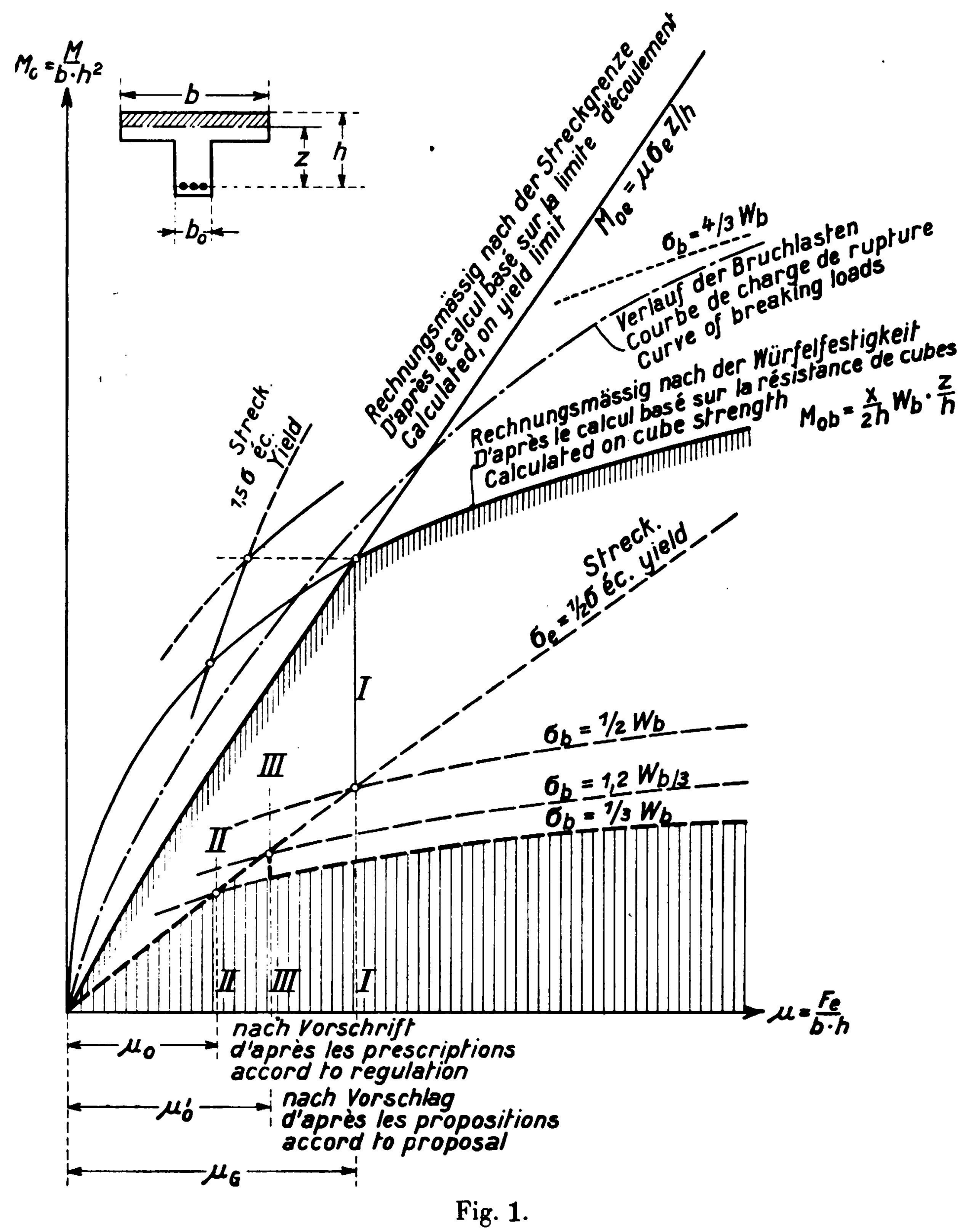

Fig. 1.

The carrying capacity of T-beams in dependence on the percentage of reinforcement (according to Emperger and Haberkalt).

method indicates that the first region is far from being fully utilised, and this is true, in principle, also as regards the lines AEF and AGH which refer to high tensile steel such as Isteg. The conclusions which emerge from these experiments are the following:

So far as concerns the first region in the diagram, representing the usual case of weakly reinforced beams in which failure is conditioned by the yield point of the steel, there appears no justification to depart from the methods of calculation hitherto in use. Once the critical amount of reinforcement which marks the boundary between the two regions has been established by the experiments now in hand it will be permissible to extend the practical utilisation of the first

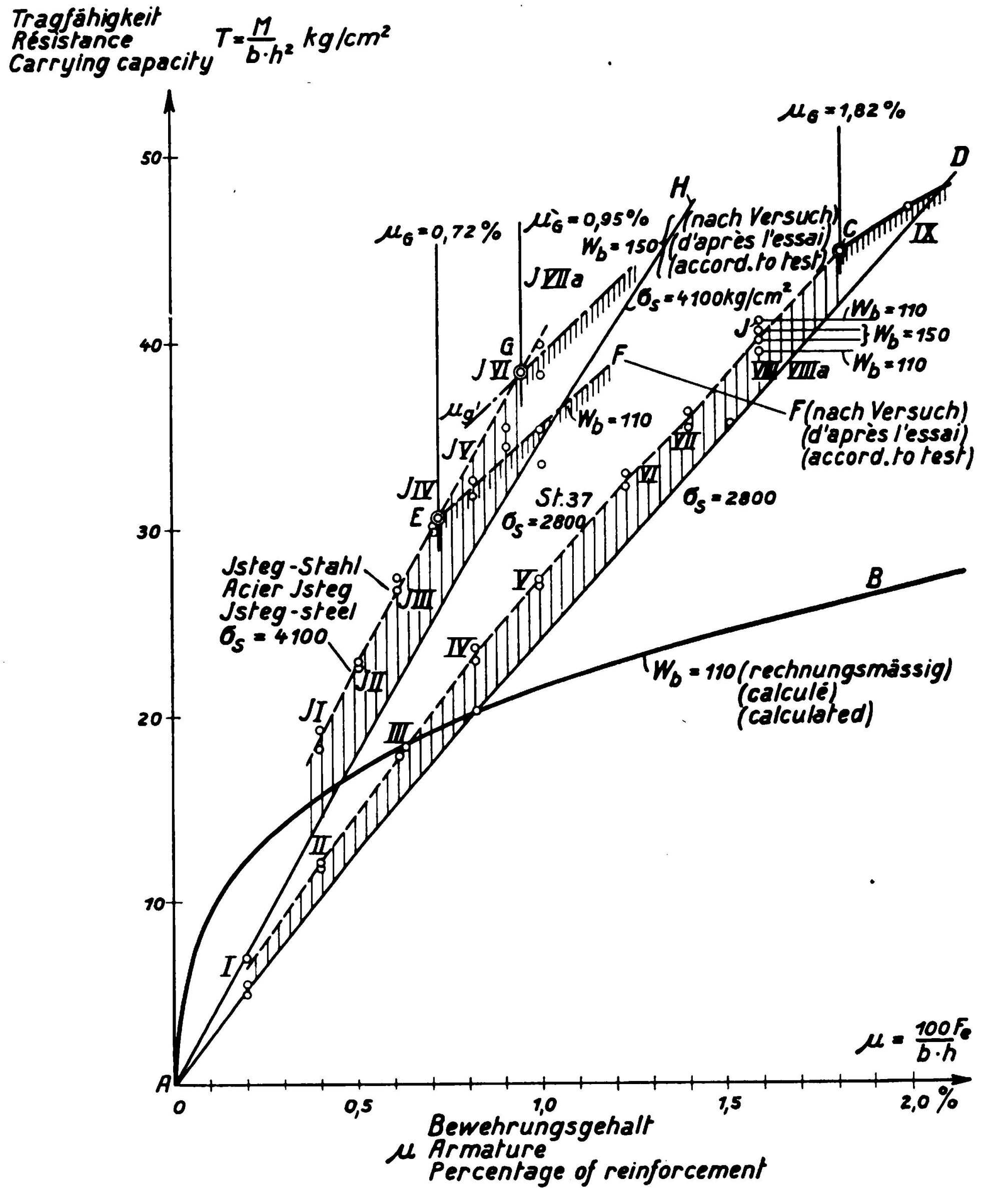

Fig. 2.

The carrying capacity of rectangular beams in dependence on the percentage of reinforcement, according to Dresden tests.

region up to the limit so ascertained, and the usual simple method of calculation will be applicable accordingly.

As regards the second region, representing the rarer case in which the governing condition is the compressive strength of the concrete, a new method may be introduced which will ensure more complete utilisation of the material and will serve the purpose of fixing the critical amount of reinforcement which separates the two regions. In this way the necessity for compression steel and for inclined haunches to the beams can be reduced to a minimum, and the design thereby improved.

Referring again to the first region — the case of lightly reinforced beams — a number of writers have joined in attacking the current methods of calculation, and especially the assumption of a fixed value for n whether this be 10, 15 or 20. *Emperger*, for instance, has shown in his latest paper that on the basis of the Austrian experiments the value of n as found graphically may be anything from 1 to 100, so that no justification exists for preferring a figure such as 10 or 15. Such a calculation, as indicated in the paper by *Saliger*, can be relevant only to the particular case where, at the moment of failure, the yield point of the steel in tension happens to be reached simultaneously with the prism strength of the concrete in compression. Even so the problem of the second region in the diagram, wherein failure is governed by the compression in the concrete, remains unsolved, for hitherto there has been no means of evaluating the stress that exists in the steel at the moment of failure.

Even if, as suggested by *Emperger*, the calculation for a rectangular cross section is to be simplified by the arbitrary assumption that the neutral axis is situated at the middle of the depth of the beam, the problem remains unanswered as regards tee beams and as regards the case where bending is combined with a longitudinal force.

The German Committee for Reinforced Concrete has not, up to the present, seen any justification for abandoning the old method of calculation with the assumption n = 15, particularly since the results so obtained are in very satisfactory agreement with those indicated by the Dresden experiments on beams of rectangular cross section.

In a paper by *Brandtzaeg* of Norway a notable suggestion is put forward for determining the safety of reinforced concrete sections subject to eccentric loading, based on experiments carried out at Stuttgart, in America, and by the author himself. The values of the stresses in members so loaded, calculated by this method, have led to the Norwegian regulations limiting the permissible compressive stress in such cases more strictly than the German rules. In this method, again, an arbitrary value is adopted for the elastic ratio, determined by the tangent to Talbot's parabola at the zero point; the problem does not admit of solution from considerations of equilibrium alone.

On the basis of the Dresden experiments mentioned above, *Friedrich* of Dresden has proposed a new method of calculation for beams comprised in the second of the two regions; a method which further offers the advantage of providing a simple determination of the limiting reinforcement which differentiates the two regions.

2) In the paper by Brice of Paris special emphasis is laid on the fact that from the point of view of the safety of a structure the dead load and the live load possess an entirely different significance, and that this significance is determined by the nature of the material used. As long ago as 1910 Caquot of Paris put forward the consideration that a structural member is to be regarded as sound when it not only takes up the elastic and reversible strains below its fatigue limit, but is also adequately resistant to those permanent deformations which

progressively increase over a notable period of time before, finally, reaching a limit at which their amount remains steady. The effect of this plastic type of deformation is to bring about an equalisation of the stresses, and it is, therefore, to be looked upon as a process of the structural member adapting itself to the loading imposed.

Reinforced concrete, however, is able to adapt itself, in this sense, only to the permanent or dead loading and not to the variable or live loading. Consequently,

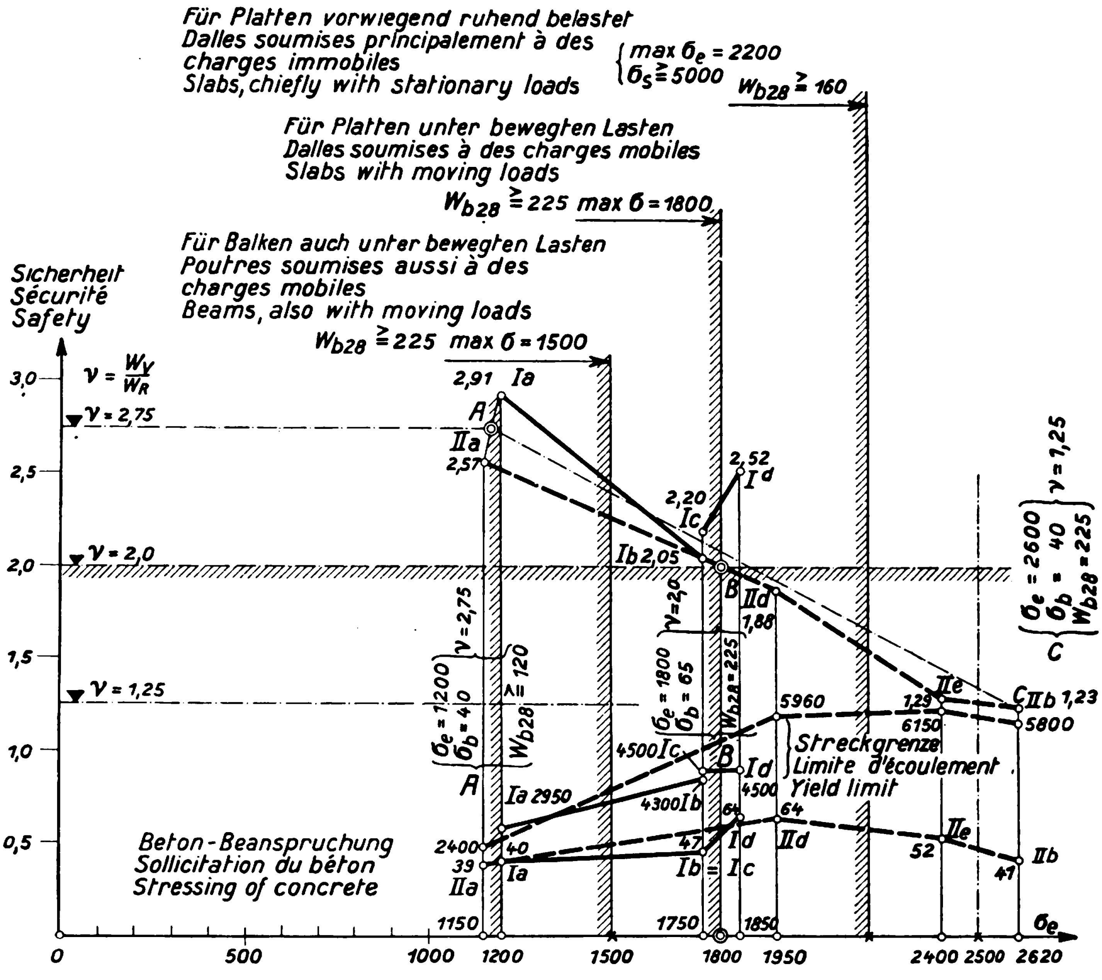

Fig. 3.

Results of Stuttgart fatigue tests on slabs with Isteg — and steel fabric reinforcement (I and II respectively).

in the code adopted by the French Association of Reinforced Concrete Constructors the allowance for live load in the design is increased in relation to the dead load, as a means of taking account of the unfavourable circumstance described.

The experiments indicate that under frequently repeated loading the elastic deformations may be permitted to occur a large number of times but the plastic deformations only a relatively small number. This leads to the corollary that once a member has been fully loaded any further variable live loading to which it may be subjected by traffic should be such as to provoke only elastic deformations. Massive structures such as the heavier types of floor, mushroom floors, and bridge works come into a different category since in these the proportion of dead load is relatively high, compared with other types of construction. The smaller the fluctuations in stress attributable to live load, in proportion to the dead load, the more durable will be the construction.

3) The paper by *Graf* of Stuttgart deals with the effects of permanent and frequently repeated loading. According to the Stuttgart experiments the resistance to a permanent and stationary load may be taken as equal to at least 80 % of the strength as determined in ordinary breaking tests.

It is found, in confirmation of earlier experiments by *Probst* and *Mehmel* of Karlsruhe and *Roš* of Zurich, that resistance of concrete to frequently repeated loading in tension, compression or bending amounts to at least one half of the strength as determined by the ordinary compression test. As in the case of structural steelwork, however, if the frequently repeated load is superimposed upon a pre-existing static load the effect is to reduce the range of stress which can be withstood an indefinite number of times to below the value which it would have if there were no permanent load. Where a beam is exposed to this kind of dynamic stress care should be taken to make the radius to which the reinforcing bars are bent as large as possible and to anchor their hooks as carefully as possible in the concrete at the ends.

In the paper by *Gehler*, the Stuttgart fatigue tests on variously reinforced slabs are evaluated, and the concept of the "factor of safety under traffic", $v = w_v : w_r$, is introduced (Fig. 3). Here w_v denotes the maximum amplitude that can be resisted an indefinite number of times, as determined by the fatigue tests, and w_r denotes the greatest amplitude possible on which the statical calculations could be based. The value $v = 2$ was adopted as a suitable value for the slabs tested. It was found that this double factor of safety was realised in slabs reinforced with high tensile steel subject to a permissible stress of 1800 kg/cm² (25600 lbs. per sq. in) and made with concrete showing a minimum cube strength of 225 kg/cm² (3200 lbs. per sq. in).

Part II: Means for increasing the tensile strength of concrete and for reducing the liability of cracking.

1) Results of experiments on the usual method of preparation of concrete are given in the paper by *Bornemann* of Berlin. The tensile strength of concrete is governed, in the first place, by the tensile strength of the cement, and according to the new method of testing the latter should be determined by reference to a tensile-bending test carried out on a sample of mortar made by using the cement together with fine aggregate of varying size of grain. The tensile strength of the concrete depends also on the quality of the concrete (as indicated by the cube strength), on the granulation of aggregates (as indicated by the fineness modulus, or proportion of dust in the sand and sand in the aggregate as a whole), on the cement content, and finally on the water-cement ratio. Tensile tests on samples of concrete afford no satisfactory criterion, but good results have been obtained from tensile-bending tests carried out on concrete beams 70 cm long spanning a distance of 60 cm between supports, 15 cm wide by

10 cm deep, subjected to two symmetrical point loads at 20 cm distance. *Gehler* suggests that using the cube strength as a basis the tensile-bending strength can be approximate evaluated from the formula:

$$K_b = \sqrt[3]{W^2}$$

The results obtained by the formula have to be increased by up to 10 % in case of damp plastic concrete of best granulation and decreased by up to 20 % in case of very soft concrete.

Assuming a given cement content and a given granulation, the attainment of a particular degree of workability will depend upon the amount of water used in relationship to the shape and surface properties of the aggregate. The more compacted the aggregate the smaller will be the amount of water required and

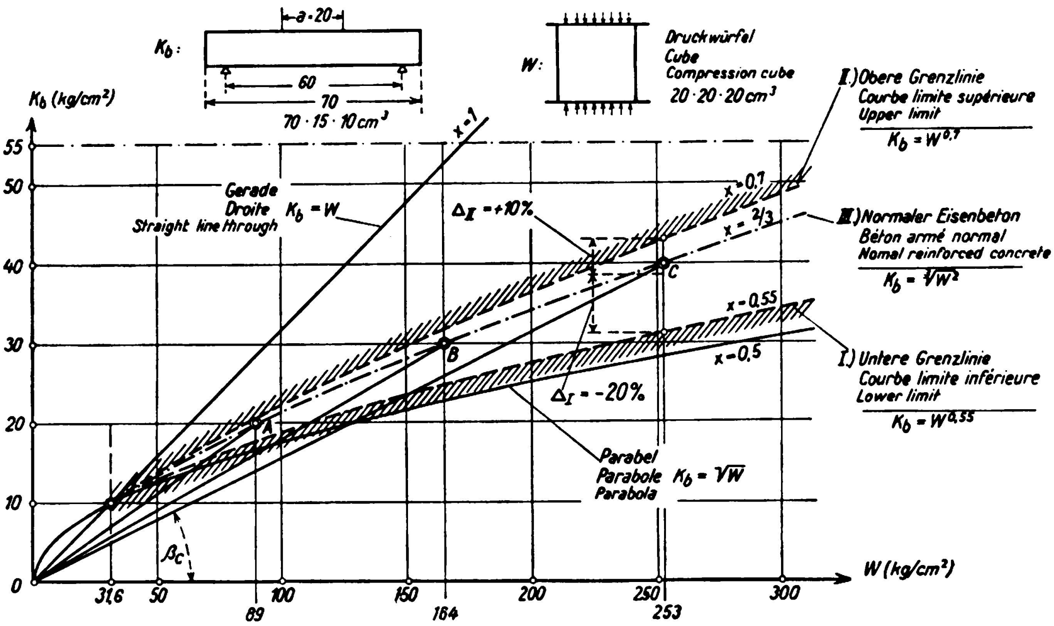

Fig. 4.

Relationship between the bending tensile strength K_b and the compressive strength W of concrete: $K_b = W^x$.

the higher will be the cube strength obtained. In the instructions laid down for the formation of roadway slabs on the Reichsautobahnen the proportion of length: width: thickness of the aggregate are prescribed as varying from 1: 1: 1 where the aggregate is of rounded shape, down to 1.0: 0.6: 0.2 where the shape is closely packed. Where concrete is worked in a wet condition the presence of chippings in the aggregate may have the effect of considerably reducing the tensile-bending strength. In order to increase the tensile strength a recent requirement is that the proportion of coarse sizes, above 7 mm in diameter, shall be graded by the employment of suitable sieves, even though from the point of view of compressive strength this measure is not so important.

The three usual qualities of concrete which give minimum cube strengths of 120, 160 and 225 kg/cm² (1707, 2276 and 3200 lbs. per sq. in) respectively showed average bending-tensile strengths of 20, 30 and 40 kg/cm² (285, 427 and 569 lbs. per sq. in), or, in the most favourable instance, 55 kg/cm² (782 lbs. per sq. in). Where, however, reinforced concrete pieces are being produced under factory conditions, it is possible by the use of special methods such as vibration to attain a considerably greater density, attended by bending tensile strengths up to 80 or 120 kg/cm² (1138 or 1707 lbs. per sq. in).

2) In the paper by *Colonetti* of Turin it is shown by calculations of strength that the grip is made much more dependable by the use of reinforcing bars of small diameter than by the use of a smaller number of larger bars.

3) The paper by *F. G. Thomas,* of England, deals with the development of cracks in reinforced concrete. It was shown by means of careful measurements of shrinkage stresses in concrete specimens that the risk of such cracks deve-

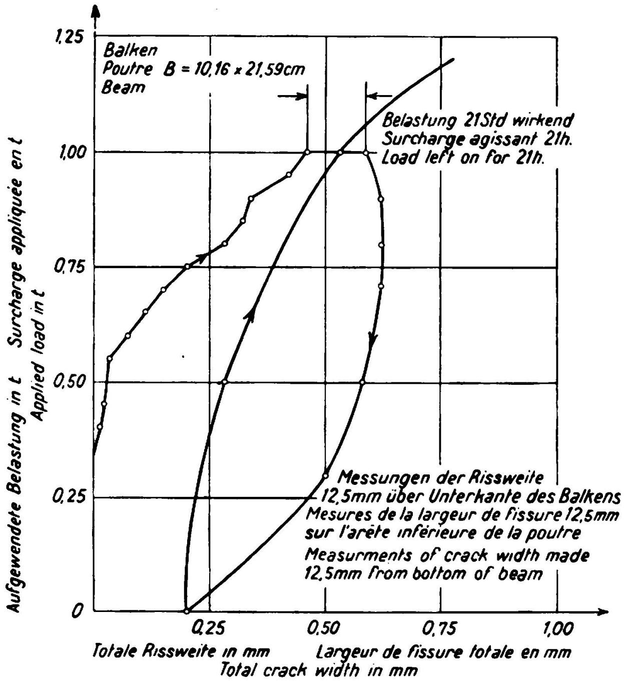

Fig. 5.

Measurement of width of cracks by F. G, Thomas.

loping increases according to the rate of hardening of the cement. In the diagram, Fig. 5, the load is shown plotted vertically and the width of the cracks plotted horizontally. It will be seen that if the load is reduced to zero the width of the cracks becomes less, but not, as frequently assumed, in direct proportion, the relationship being that indicated by a strongly bent curve; if, for instance, the load was reduced by one half no change at all could be detected in the width of cracks. The complicated problem of relating the width of cracks to the tension in the steel under increasing and permanent loading is exhaustively studied in the paper. One notable result obtained is a confirmation of the fact observed by Professor Duff Abrams that very fine cracks existing in concrete may, in the course of time, completely heal up; this occurs not only where the specimens are stored under water but even where they are stored in air.

4) The paper by *Freyssinet,* in its present form, constitutes the third section of the book published by that author in the same year as the Congress, which has already been widely discussed: it may, therefore, be expedient to give here a brief summary of the main principles treated in the first two parts of the book.

Improvement in the materials employed must be a matter of decisive importance for the further development of reinforced concrete construction, and this is true not only as regards what may be called the raw materials — the cement, aggregate and steel — but also as regards the methods of preparing and using concrete. A parallel to this is to be seen in steel construction where, at the present time, much attention is being paid to the further improvement of high tensile structural steel with special reference to welding and to fatigue effects. In his new book, "Une Révolution dans les Techniques du Béton" [Léon Eyrolles, Paris 1936] *Freyssinet* has performed the service of putting forward a thermodynamical basis for explaining the preparation of concrete, in the shape of a hypothesis from which he has deduced a large number of conclusions that have a bearing on the improvement of concrete.

At the time of the Bridge Congress in Vienna, *Freyssinet* showed from his experience in the construction of the bridge at Plougastel in 1928 that the law governing elastic deformation and the law governing non-reversible plastic deformation were altered when reinforced concrete members were stored under load in the open air so as to be exposed to the action of heat and humidity. He divided the total amount of shrinkage into two portions, one depending on warmth and dampness and a portion called the water-elasticity part. It is a question, therefore, of properties of material which are functions of the time T, of the temperature t, of the relative humidity of the air ε, and, especially, of any compressive stress that may have been preimposed. By the application of known principles of thermo-dynamics it becomes possible to establish certain fundamental equations:

A) I) The term "pseudo-solid" is applied to a material such as cement or concrete which appears outwardly to be a solid but which actually consists of a network of very fine pores containing water and air, the presence of which confers upon it mechanical properties different from those of a true solid or dense body. It is proposed to derive a mathematical expression for the condition under which the water will evaporate out of these capillary pores.

II) *The capillary phenomenon of the meniscus in the saturated pores of such a body.* According to Laplace the surface tension in the meniscus is given by:

$$\pi = A \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where R_1 and R_2 represent the principal radii of curvature of the meniscus. In the case of a very small interstice bounded by parallel walls of width D, putting $R_2 = \infty$ and $R_1 = \frac{1}{2} D$ we obtain:

$$\pi = \frac{2A}{D}. \tag{1}$$

III) *Influence of the humidity of the air contained in the pores.* According to *Carnot* and *Lord Kelvin*, a second relationship may be derived from the conditions of equilibrium existing between the capillary stress in the meniscus and the relative humidity of the surrounding atmosphere, the ratio between the vapour pressure at any given level and at zero level being denoted by ε. For water at 15° C the equation is

$$\pi = \frac{2\,A}{D} = 1300 \log e \frac{1}{\varepsilon} \tag{2}$$

or

$$\frac{1}{\varepsilon} = e^{\left(\frac{2\,A}{1300\,D}\right)} \tag{2 a}$$

Here A denotes what is known as the capillary constant, the value of which can be determined by experiment and may amount to, for instance, $A = 8$ mg/mm. It should also be noted that the width of pores as calculated by this formula to correspond with a condition of equilibrium when evaporation is taking place under a relative humidity of $\varepsilon = 20$ to 95 % is extremely small, being of the order of one to 25 millionths of a millimetre or between three and 100 times the diameter of a molecule of water, which measures 0.26 millionths of a millimeter.

IV) *Hygrometric equilibrium in a pseudo-solid.* In accordance with Equation (2) the hygrometric condition of a body may be stated in terms of the ultimate surface tension π_ε of the meniscus, or of the limiting width $D = D_\varepsilon$ at which evaporation is in equilibrium, or, finally, of the relative degree humidity ε. In the limiting case for an atmosphere saturated with water vapour we have $\varepsilon = 1$; hence $D = D_\varepsilon = \infty$ and $\pi = 0$.

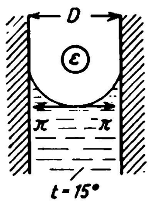

Fig. 6: Notation used by M. Freyssinet:

t = temperature in ° C.
T = time.
π = tension at the surface of the meniscus of a pore filled with water (in kg/mm²).
A = capillary constant.
R_1 et R_2 = principal radii of curvature of the meniscus.
D = width of a lamelar interstice with parallel walls (in millionths of a mm).
D_ε = limiting thickness of pores (Definition: when $D > D_\varepsilon$ the water vanishes from the pore).
ε = relative humidity in %, or ratio of vapour pressures between a given level and zero level.
H_1 = vapour pressure in concrete saturated with water.
H_{max} = saturation pressure in the pores of the concrete at the temperature of the experiment t_1.

Pseudo-solid body (cement, concrete) = 1) Externally solid, 2) Internally a network of infinitely small pores filled with air *or* water.

Principles: molecular theory. Velocity of gaseous molecules.

It has been shown by the experiments of *Berthelot* and *Laplace* that a liquid in an air tight container is capable of retaining a considerable surface tension provided there are no air bubbles in the liquid; such tension may in fact amount to several tonnes per sq. cm. Thus the tensions in the meniscus are in equilibrium and the evaporation also is in equilibrium. If, then, a pseudo-solid such as a piece of concrete is placed in an atmosphere saturated with water vapour the small channels will become completely filled, with water only if the surrounding air is at a higher temperature. In the case of concrete, subject to the effect of setting heat, the contrary is always true and the temperature of the concrete t_1 is higher than the temperature t_2 of the surrounding air. It may then be supposed that the concrete will assume a moisture condition ε equal to the ratio of the vapour pressures H_1 and H_{max}, and we have:

$$\varepsilon = H_1 : H_{max} \leqq 1 \tag{3}$$

where H_1 is the vapour pressure of the atmosphere in the pores of the sample and H_{max} is the saturation pressure, at temperature t_1,for the liquid contained in the pores of the concrete. The liquid will, therefore, disappear from all those pores having a diameter D greater [see Equation (2)] than the limiting diameter

$$D_\varepsilon = \frac{2\,A}{1300 \cdot \log_e \frac{1}{\varepsilon}} = \frac{2\,A}{1300\, \log_e \frac{H_{max}}{H_1}}. \tag{4}$$

In Fig. 7, Equations (1) and (2) have been represented for $A = 8$ mg/mm using a system of three coordinates π_ε, D_ε and ε so as to give a space-curve. ABC for the relationship $f(\pi_\varepsilon, D_\varepsilon, \varepsilon) = 0$. This representation serves to illustrate the three gradations which are of capital importance in considering the effect of climate on reinforced concrete:

Stage I: Continental climate, dry and keen with $\varepsilon = 20\,\%$ (very small), $\pi_\varepsilon = 2100$ kg/cm² (very large).

Stage II: Semi-Continental climate with $\varepsilon = 60\,\%$ and $\pi_\varepsilon = 665$ kg/cm² (average value).

Stage III: Maritime climate, very damp and mild, with $\varepsilon = 95\,\%$ and $\pi_\varepsilon = 65$ kg/cm².

If, for instance, in a concrete of high water content we have $\varepsilon = H_1 : H_{max} = 0.9$ the pores larger than $D_\varepsilon = 11.4$ millionths of a millimeter will be completely dried out for this value of the coefficient of cohesion A; on the other hand for $\varepsilon = 0.5$ we obtain $D_\varepsilon = 5.5$ millionths of a millimeter so that a much larger number of pores will completely dry-out. The smaller the limiting value of D_ε found in accordance with Equation (4) and the greater the number of pores drying out having a diameter $D > D_\varepsilon$ in any given distribution, the faster, in proportion, the whole test piece will dry-out. There is a risk, therefore, of insufficient water for setting. Thus an aluminous cement may easily dry out under water, its great setting heat operating to accelerate the process of drying.

V) *Calculation of shrinkage.* Consider a cross section of 1 cm² of a pseudo-solid body and assume that this contains a portion ω_p of solid matter in addition to a portion ω_s of voids and a portion ω_m of space filled with water, or partially so filled. The surface tension in the meniscus will then be represented by a force

$$P = \pi \cdot \omega_m$$

and, considering two such sections, the forces P and P′ will cause a contraction equal to

$$\delta = \frac{P}{1\,E_1} = \frac{\pi \cdot \omega_m}{E_1} = \left(\frac{\omega_m}{E_1}\right)\left(1300 \log_e \frac{1}{\varepsilon}\right) = a \cdot b$$

where P is known as the „shrinkage pressure“ and E, as the contraction modulus which amounts to 10^9 kg/cm².

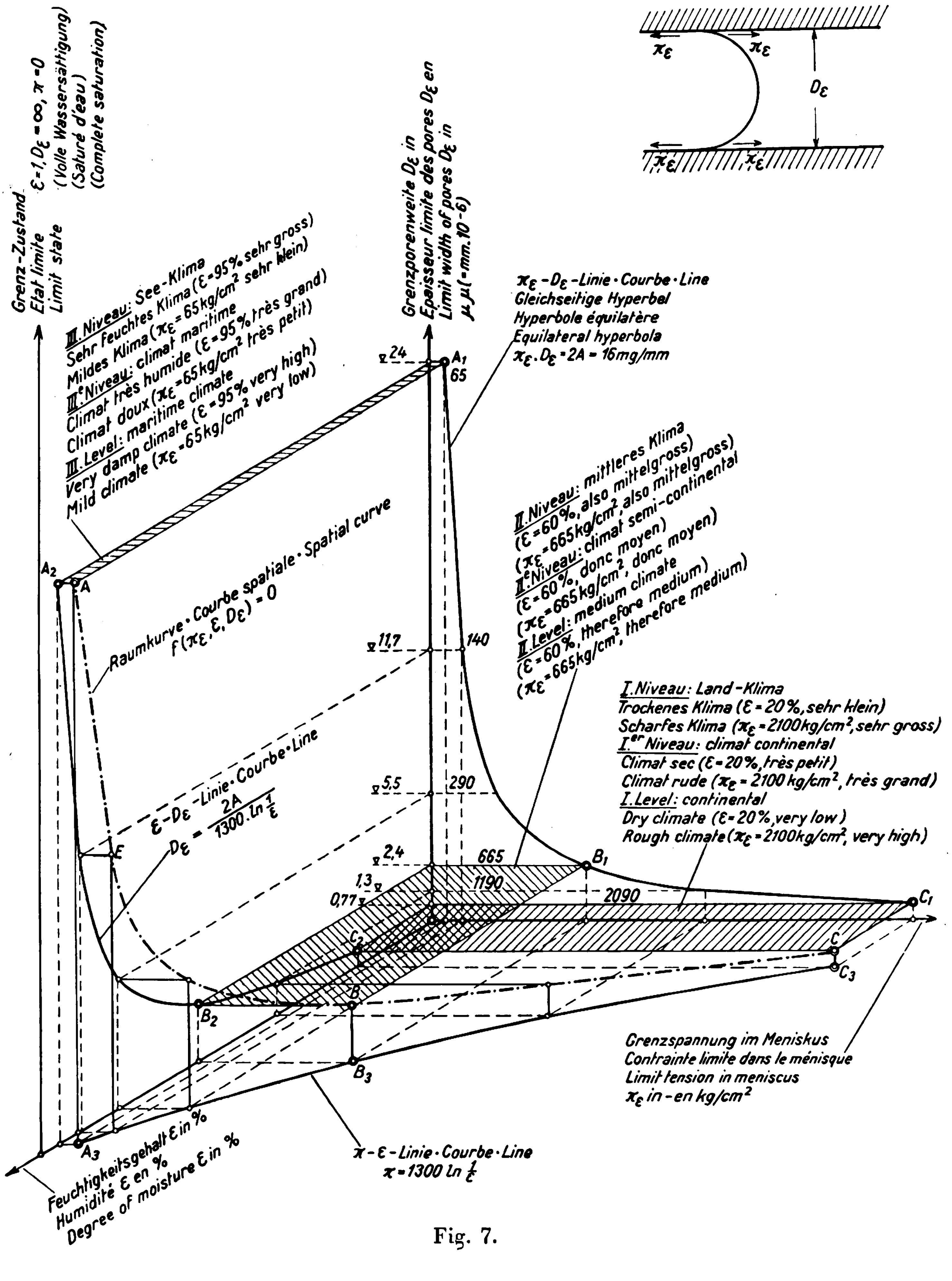

Fig. 7.

The total shrinkage is made up of two components: the hydro-elastic factor $a = \frac{\omega_m}{E_1}$ and the thermo-hygrometric factor $b = 1300 \log_e \frac{1}{\varepsilon}$. (See also *Freyssinet*, Vienna Congress, 1930; *Gehler*, Zurich Congress, I.A.T.M., 1932, page 1118; also *Gehler* and *Amos*, 1934, German Comittee on Reinforced Concrete, Publication N° 78.)

The only difference between shrinkage and creep is that the former occurs in consequence of a permanent external load whereas the latter takes place under dead load alone.

VI) The suggestion is made that the considerations arising from *Freyssinet*'s hypothesis, and their practical implications for improving the quality of concrete, should be checked by experiment.

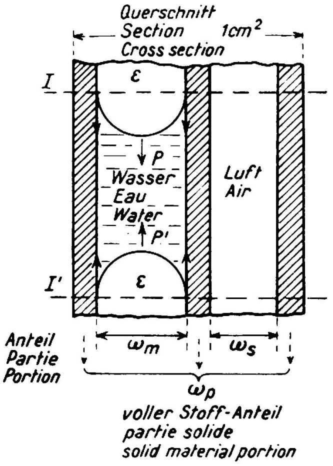

Fig. 8.

B) The present paper by *Freyssinet* contains a few indications of ways in which the foregoing might be applied in practice, derived from the third part of his book. *Freyssinet* envisages three methods of improving the quality of concrete work: the use of a very dense concrete, the adoption of prestressing as a means of ensuring that in any given cross section of a beam in bending there shall occur (as far as possible) only compressive stresses in the concrete, and, finally, the application of heating, perhaps by means of steam. He carried out experiments on concrete poles under repeated alternating stress and obtained extraordinarily high strengths. Railway sleepers subjected to a pressure at the time of casting of 100 to 300 kg/cm² (1422 to 4267 lbs. per sq. in) gave cube strengths as high as 1000 kg/cm² (14220 lbs. per sq. in), with a perfectly hard smooth surface to the concrete. The paper contains a detailed description of the very difficult construction work involved in strengthening the port station at Le Havre wherein the method of pre-stressing followed by rapid hardening of the concrete was used with great success. *Freyssinet* expresses the opinion that this method will prove particularly useful for application to girders of large spans, and also to mushroom floors and to roadway slabs.

In this connection attention may be drawn to the paper by *Dischinger* of Berlin, presented at the fifth meeting of the Congress, and dealing with the question of long span bridges. This paper puts forward what appears, in principle, a sound solution to the problem of constructing long span bridges. Following the analogy of an arch bridge with a pre-stressed tie it is proposed to make use of a suspension chain under the beam composed of round steel bars of 60 to 100 mm diameter which will be structurally quite separate from the compression boom. The concrete compression member will receive only concentric stresses due to the dead weight of the bridge itself, and the steel will be pre-stressed to such an extent that when the shuttering is struck the whole system will conform as closely as possible to the proper intended geometrical shape. The reinforced concrete box section will be subjected to bending only on account of the live load. The effects of shrinkage and creep are to be taken up by subsequent further tightening of the suspended rod-system. It should be possible in this way to build girder bridges up to 100 or 150 m in single span.

This remarkable and indeed revolutionary proposal must give rise to numerous questions in the minds of both designers and constructors. One obvious question will be in regard to the nature of the steel to be used. *Freyssinet* has hitherto made use of round bars of up to 16 mm diameter, having the elastic limit raised by cold stretching from 24 to 80 kg/mm^2 (from 34,140 to 113,790 lbs. per sq. in). For the purpose of these long span concrete girder bridges *Dischinger* envisages round steel rods up to 10 cm diameter and 100 m in length, the joints in which would be formed by electric resistance welding. This suggests further problems from both the welding and the metallurgical points of view.

Part III. Application of high tensile steel.

1) The paper by *Gehler* of Dresden makes reference to the following conclusions emerging from the exhaustive experiments of the German Committee for Reinforced Concrete:

The introduction of high tensile steel into reinforced concrete practice has entirely fulfilled the expectations. The advantages, as indicated in Table I, include an increase in the permissible stress of the steel from 1200 kg/cm^2 (17,000 lbs. per sq. in) for commercial St. 37 to as much as 1800 kg/cm^2 (25,600 lbs. per sq. in) according to the elasticity and quality of the concrete, and in exceptional cases 2200 kg/cm^2 (31,290 lbs. per sq. in). In the case of tee beams carrying mainly stationary load and reinforced with St. 52, stressed up to 1800 kg/cm^2, the margin of safety against cracking is the same as by using St. 37 stressed to 1200 kg/cm^2, assuming a minimum cube strength of 225 kg/cm^2 (3200 lbs. per sq. in) in the concrete in both instances. In slabs of rectangular cross section reinforced with St. 52 and showing a cube strength of 225 kg/cm^2 the permissible strength in the steel may be raised to 1800 kg/cm^2 even under moving loads, but in tee beams only to 1500 kg/cm^2 (21,340 lbs. per sq. in).

These increases in the permissible stresses which may be realised where high grade concrete and high tensile steel are used result in a number of advantages: for instance, in a reduction of the cross section of steel necessary in the tensile boom of a girder, and therefore of its total width, and of its weight. Further, the disadvantage of having a large number of bent bars crowded together in the cross section over the supports is reduced to a minimum.

In the experiments at Dresden the cracking was recorded photographically using a magnification of 23, thus enabling the widths to be accurately measured, and the depths were also ascertained. In this way the question of safety against cracking has been placed on a sound scientific basis.

The criterion of safety against cracking, namely the ratio between the load at the time when the first crack appears and the load actually arising in service, amounts to 1.8 in the case of slabs supported along all four edges and reinforced

Paper I.

Table of permissible stresses in reinforcements of high yield points for reinforced concrete slabs and beams.

1	2	3	4	5	6	7	8
Manu- facturer No.	Type of Steel	Minimum yield point[1]	Minimum elongation at fracture	Minimum cube strength of concrete	in slabs σ_e perm	in T-beams	Range of validity
	—	kg/cm²	%	kg/cm²	kg/cm²	kg/cm²	—
1	St. 52	3600	20	120 225	1500 *1500*	1200 *1500*	Also movable loads[3]
2	St. 52	3600	20	120 160 225	1500 1800 1800	1200 1200 1500[4] 1800[5]	Mainly stationary loading and in building frames not exposed to the weather. (Nos. 2–4)
3	Special steel[2]	3600	14[6]	120 160 225	1200 1800 1800	1200 1200 1500[4] 1800[5]	
4	Special steel[2]	5000	14[7]	120 160 225	1200 2200 2200	1200 1200 1500[4] 1800[5]	

[1] *Yield points.* In accordance with the reinforced concrete regulations, Section 7, account must be taken of the properties of the steel. In the case of reinforcements which have no definite yield points this may be taken as 0.4 % of the limit of total elongation instead of at 0.2 % as laid down in DIN 1602, pending the final decision of the question on the basis of experiments now in hand.

[2] Special steel arranged in a particular way in accordance with permission granted by the building authorities.

[3] Corresponds with the regulations hitherto in force.

[4] When the cross section of each reinforcing bar exceeds 3.14 cm² (in the case of twisted steel the total cross section of the twisted bar is to be taken).

[5] When the cross section of each reinforcing bar is not greater than 3.14 cm² (otherwise as for 23).

[6] In the case of slabs a steel having a minimum elongation at fracture of 10 % is also permissible.

[7] In the case of slabs a steel having a minimum elongation at fracture at 8 % is also permissible.

crosswise; to 1.4 in that of slabs supported at the four corners (preliminary experiments on mushroom floors); to 0.75 in that of slabs reinforced only in one direction and to 0.5 in the case of tee beams. It is, therefore, for use in slabs that high tensile steel is most emphatically to be recommended. Safety against cracking increases also with improvement in quality of the concrete, but unfortunately only to a small extent on account of the greater brittleness of high grade cement. The purely statistical conclusions reached from these experiments suggest the physical interpretation that when a crack occurs the cross section thereby broken (depth t, width of rib b_o) ceases to act, and the tension in the concrete which previously existed therein disappears. Its magnitude may amount to 4, 8 or 12 % of the tension in the steel at the moment of formation of the crack, according as the quality of the concrete is poor, medium or good. In the case of tee beams subject mainly to stationary loading and reinforced with St. 52, allowing a stress of 1800 kg/cm² in the steel and assuming the use of high quality concrete, the degree of safety against cracking will be the same as if St. 37 were used with a stress of 1200 kg/cm², embedded in ordinary concrete.

As regards the form of cross section of reinforced concrete beams it may be inferred, from the Dresden experiments, that reinforced concrete members produced under factory conditions for use in long span bridges may, with advantage, be given an I-shaped or box-shaped cross section, from the point of view both of freedom from cracking and of carrying capacity.

2) The paper by *Saliger* of Vienna refers in the first place to columns with high tensile steel reinforcement, and leads to the surprising conclusion that as regards columns having both longitudinal and lateral bindings the law of superposition (which has been so much under discussion) does not hold good where high tensile steel is used. This is attributed to the fact that the compression undergone by the concrete on fracture is not as great as the reinforcing bars themselves would permit, with the consequence that the concrete suffers disturbance earlier than it should be from the buckling of the longitudinal bars. It is only in the case of columns provided with spiral binding that the requisite amount of contraction before breakage is attainable, so as generally to justify the use of high tensile steel. For columns of this type, the author gives a formula based on his experiments, which corresponds to the law of superposition ordinarily used.

The second part of this paper by *Saliger* deals with beams having high tensile reinforcement, and confirms the indications given in the paper by *Gehler* as regards the parabolic nature of the relationship existing between the stress in the steel when the first crack appears and the percentage of reinforcement. The method of calculations has already been partly dealt with in Part I.

3) *Dr. Olsen* of Munich, who proposes to refer to this matter in the course of the discussion, has already published an account of numerous experiments in his book which appeared in 1923 under the title „Über den Sicherheitsgrad von hoch beanspruchten Eisenbetonkonstruktionen". The results which he then obtained on concrete beams reinforced with steel bars agree very well with those of the Dresden experiments so far as concerns the relationship between percentage of reinforcement, quality of concrete and safety against cracking. These experi-

ments by *Olsen* also indicate, that where high tensile steel is used, the amount that the stress in the steel exceeds the limit of elasticity at the moment of breakage increases with the compressive strength of the concrete. One particularly notable point which he has established is that when a strength of 100 kg/cm^2 in the concrete and 2000 kg/cm^2 in the steel was assumed and concrete was used having a cube strength of 250 kg/cm^2 with steel having a yield point of 4000 kg/cm^2, it was possible to maintain at least a factor of safety of 2 against rupture.

4) The paper by *Brebera* of Czechoslovakia discusses experiments carried out on two of the steels most used in that country, namely "Roxor" and "Isteg", and gives an account of some notable applications.

Part IV. Effect of construction and expansion joints.

In the paper by *Baravalle* of Vienna it is recommended that the principle followed in Lamellae construction for arches should be applied also in other structural work such as floors, water tanks, etc.: that is to say joints should be provided which will remain open only during the constructional period or for some weeks at least, before being filled up with concrete, these being additional to the permanent expansion or contraction joints which serve to separate one part of the structure from another and allow the necessary freedom of breathing.

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IIa

Influence of stationary and of repeated loading.

Einfluß dauernder und wiederholter Belastung.

Endurance — Résistance aux efforts répétés statiques ou dynamiques.

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IIa 1

Permissible Concrete Stresses in Rectangular Reinforced Concrete Sections under Eccentric Loading.

Zulässige Betondruckspannungen in rechteckigen Eisenbetonquerschnitten bei außermittigem Druck.

Contraintes de compression admissibles dans les sections de béton armé rectangulaires sollicitées excentriquement.

Dr. techn. A. Brandtzaeg,
Professor an der Technischen Hochschule Trondheim.

Several investigators have raised objections to the usual method of designing reinforced concrete sections in bending or bending combined with compression, by the method based on the assumption of a straight line relation between stresses and strains in the concrete under compression. Nevertheless, the method is still in general use, and the Building Regulations of nearly all countries are based thereon.

In previous publications[1] and[2] the author has presented a method wherewith the *ultimate moments* or the *ultimate loads* of reinforced concrete members with rectangular cross-section may be computed in fair agreement with the results of actuals tests. On the basis of the ultimate carrying capacity of any rectangular section, determined in this way, the usual method of design may be tried out. Investigation will show how far the method meets the fundamental requirement that the same desired *factor of safety* should be maintained with different grades of concrete, different percentages of reinforcement and different eccentricities of load, and the most suitable working stresses may be determined. The case of simple bending has already been treated[3]; here the case of bending combined with compression will be investigated. Only short members with negligible deflections are considered.

1) Computation of Ultimate Loads.

The usual distinction must be made between *over-reinforced* and *normally reinforced* sections. Failure of the former starts on the compression side of the

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[1] *A. Brandtzaeg:* „Der Bruchspannungszustand und der Sicherheitsgrad von rechteckigen Eisenbetonquerschnitten unter Biegung oder außermittigem Druck." Norges Tekniske Höiskole, Avhandlinger til 25-ars jubileet 1935, F. Bruns Bokhandel, Trondheim, page 677 to 764.

[2] *A. Brandtzaeg:* Det kgl. norske Videnskabers Selskabs Skrifter 1935, Nr. 31, F. Bruns Bokhandel, Trondheim.

[3] *A. Brandtzaeg:* „Die Bruchspannungen und die zulässigen Randspannungen in rechteckigen Eisenbetonbalken." Beton und Eisen, Vol. 35, No. 13, July 5, 1936, pages 219 to 222.

section; no yielding of the tensile reinforcement occurs during failure. With normally reinforced sections the failure starts with yielding of the tensile steel; through opening of the crack of failure the compression area is subsequently reduced and finally crushed. In intermediate cases the two types of failure overlap. While in the case of simple bending the type of failure depends only on the properties of the materials and the percentage of reinforcement, it is, in the case of bending combined with compression, dependent also upon the eccentricity of the load.

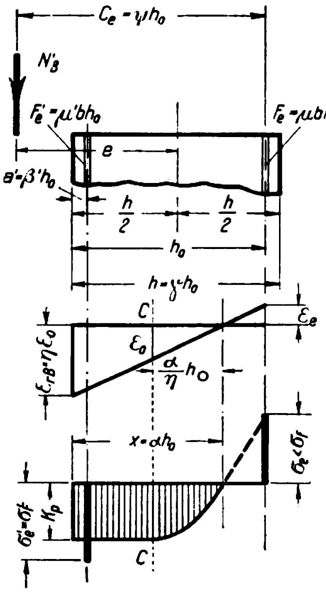

Fig. 1.

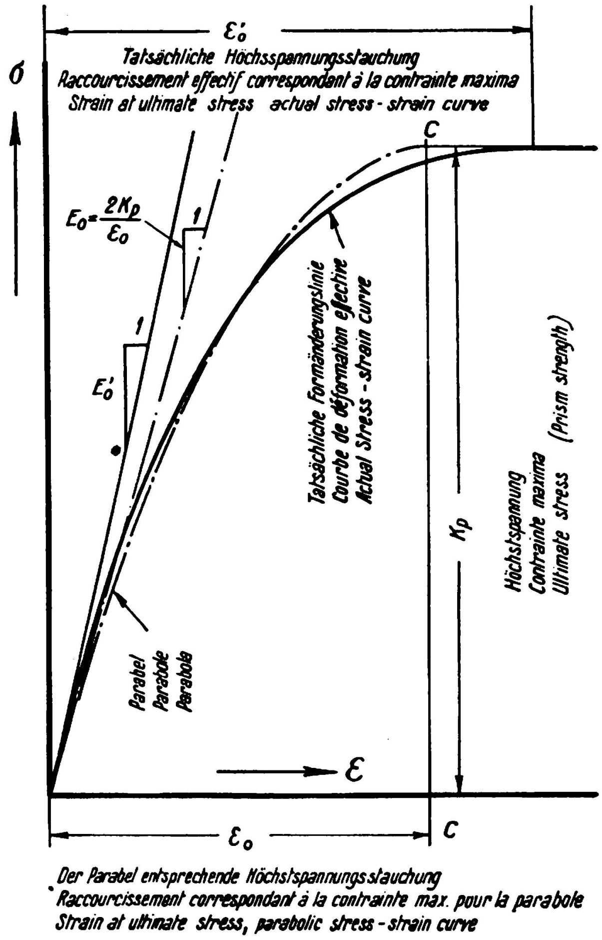

Fig. 2.

a) *Over-reinforced Sections.*

At the failure of a reinforced concrete member in bending or bending and direct compression the ultimate strain on the compression side of the member, ε_{rB}, is very much larger than the strain, ε_0, at which the same concrete under axial compression would reach its ultimate stress, the prism strength, K_P. The size of this *ultimate strain on the compression side* determines to some extent the ultimate carrying capacity of the member. It may be conveniently expressed by means of the *ultimate strain ratio*, $\eta = \dfrac{\varepsilon_{rB}}{\varepsilon_0}$.

In Figs. 1 and 3 is shown the distribution of stress which is assumed for a section at the stage of failure in bending or bending with compression. Where the compressive strain is smaller than ε_0 (to the right of the lines C-C in Figs. 1

and 3) the stresses vary according to the stress-strain curve of the concrete in simple compression (Fig. 2). Where the strain is larger, the stress remains constant equal to the prism strength of the concrete, K_P. The steel stresses also correspond to the strains. Compression steel of mild or intermediate grade generally will have passed its yield point before the stage of failure is reached. No account is taken of tension in the concrete.

The above assumptions are in agreement with the author's own tests (See [1], pages 728 to 735 and [2], pages 54 to 61). *Saliger* has made similar assumptions on the basis of his tests.[4]

For the purpose of the analytical treatment the following equation, proposed by *Talbot,* is substituted for the actual stress strain curve of the concrete:

$$\sigma = E_0 \varepsilon \left(1 - \frac{1}{2}\frac{\varepsilon}{\varepsilon_0}\right) \tag{1}$$

Here σ is the compressive stress and ε the corresponding strain, ε_0 is the abscissa of the vertex of the parabola (Fig. 2) and E_0 defines the slope of the tangent to the parabola at zero stress. By suitable choice of the values of E_0 and ε_0 the parabola is fitted as well as possible to the actual stress-strain curve. Generally E_0 should then be chosen somewhat smaller than the actual modulus of elasticity of the concrete, E'_0, and ε_0 somewhat smaller than the strain, ε'_0, at which the concrete actually reaches its ultimate stress, K_p (Fig. 2). (See [1], pages 738—739 and [2], pages 64—65.)

Other curves, as for instance the one proposed by *von Emperger,*[5] agree somewhat more closely with the actual stress-strain curve. With the curve proposed by Talbot, however, the analysis is simpler, and the curve is sufficiently accurate for the present purpose. In 9 tests made by the author, the error arising from the use of Talbot's curve instead of the actual stress strain diagram of the concrete amounted for the ultimate loads to $\div$ 4.6 to + 1.0 per cent, average $\div$ 0.48 per cent, and for the ultimate moments to $\div$ 0.7 to + 0.7 per cent, average + 0.13 per cent (See [1], page 732 and [2], page 58, Table 8, Columns 13 and 14).

The computation should be made separately for the two cases, Fig. 1 und Fig. 3, with the neutral axis inside and outside the cross-section, respectively.

In the first case the distance to the neutral axis, defined by the ratio $\alpha = \frac{x}{h_0}$, is given by the equation:

$$\left[\frac{1}{2} - \frac{1}{3\eta} + \frac{1}{12\eta^2}\right]\alpha^3 - (1-\psi)\frac{3\eta - 1}{3\eta}\alpha^2 + [2\,n\eta\psi\mu - (1-\psi-\beta')\,m'\mu']\,\alpha - 2\,n\eta\psi\mu = 0 \tag{2}$$

[4] *R. Saliger:* „Versuche über zielsichere Betonbildung und an druckbewehrten Balken." Beton und Eisen, Vol. 34, No. 1 and 2, Jan. 5 and 20, 1935, pages 12 to 18 and 26 to 29.

[5] *F. v. Emperger:* „Die Formänderung des Betons unter Druck." International Association for Testing Materials, Congress in Zürich 1931, pages 1149 to 1159. — See also Beton und Eisen, Vol. 35, No. 10, May 20, 1936, page 179.

Here $n = \frac{E_o}{E_e}$ (E_e is the modulus of elasticity of the tensile steel) and $m' = \frac{\sigma'_F}{K_p}$ (σ'_F is the yield point of the compression steel). The other notation is shown in Figs. 1 and 3.

The ultimate load then is:

$$N'_B = \frac{1}{\psi}\left[\alpha\left(1-\frac{\alpha}{2}\right) - \frac{\alpha}{3\eta}\left(1-\alpha+\frac{\alpha}{4\eta}\right) + m'\mu'(1-\beta')\right] bh_0 K_P \quad (3)$$

The unit stress in the tension reinforcement is found to be:

$$\sigma_e = 2\,n\eta\,\frac{1-\alpha}{\alpha}\,K_P \quad (4)$$

In the second case, $\alpha > 1$, Fig. 3, we obtain two equations for the ultimate load. Equilibrium of the axial forces requires:

$$N'_B = \left[\frac{3\eta-1}{3\eta}\,\alpha - \frac{\eta}{\alpha}(\alpha-\gamma)^2\left(1-\frac{\eta}{3\alpha}(\alpha-\gamma)\right) + m'\mu' + 2\,n\eta\mu\,\frac{\alpha-1}{\alpha}\right] bh_0 K_P \quad (5a)$$

and the equilibrium of moments about the center of gravity of the tension reinforcement gives:

$$N'_B = \frac{1}{\psi}\left\{\alpha\left(1-\frac{\alpha}{2}\right) - \frac{\alpha}{3\eta}\left(1-\alpha+\frac{\alpha}{4\eta}\right) + m'\mu'(1-\beta') + \frac{\eta}{\alpha}(\alpha-\gamma)^2\left[\frac{\alpha+2\gamma}{3} - 1 + \frac{\eta}{3}\left(1-\frac{\alpha+2\gamma}{4}\right) - \gamma\frac{\eta}{\alpha}\left(\frac{1}{3}-\frac{\gamma}{4}\right)\right]\right\} bh_0 K_P \quad (5b)$$

From the equations (5a) and (5b) N'_B may be determined graphically.

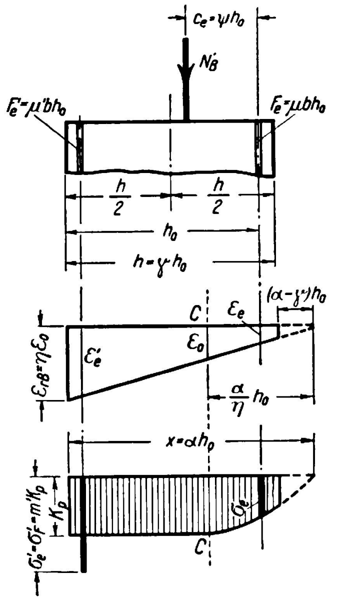

Fig. 3.

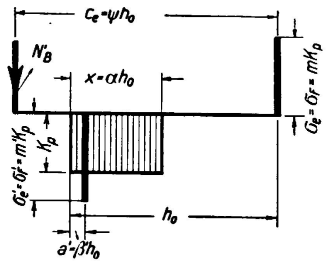

Fig. 4.

b) *Normally Reinforced Sections.*

In the vicinity of the crack that opens up at failure, the compressive stress in the concrete may be taken as constant over the entire compression area of the section (Fig. 4). The resulting error is very small, as is shown in [1], page 698 and [2], page 24. The stress in the tensile steel is assumed to be equal to the yield point, σ_F, as discussed in [3], Sections 4 and 6. After the steel has started to yield, there can be no influence of shrinkage or other tensile stresses in the concrete on the steel stress at the crack of failure.

With these assumptions we have:

$$\alpha = -(\psi - 1) + \sqrt{(\psi - 1)^2 + 2\,m\mu\psi - 2\,m'\mu'(\psi - 1 + \beta')} \tag{6}$$

and

$$N'_B = \frac{1}{\psi}\left[\alpha\left(1 - \frac{\alpha}{2}\right) + m'\mu'(1 - \beta')\right] bh_o K_P \tag{7}$$

where $m = \frac{\sigma_F}{K_P}$.

With the load acting inside the cross-section, the ultimate load, N'_B, according to Equation (7), is quite large, and it increases very rapidly with decrease of the eccentricity and increase of the percentage of reinforcement. In actual fact, therefore, with $\psi < 1$ nearly all cross-sections conforming to ordinary practice are to be classed as *fully* reinforced, as discussed below, Articles 5 and 6.

2) Values of the Constants K_P, n and η.

By means of the above equations we may compute the ultimate load on any rectangular reinforced concrete member under bending combined with direct compression, provided the constants K_P, n (E_o) and η are known for the particular concrete in question. To make the equations applicable in all cases, the constants should be known as direct functions of some known numerical criterion of the quality of the concrete, as for instance the cube strength, K_W. No such functions, correct under all conditions, are, however, available. The relation of the prism strength, the modulus of elasticity and the ultimate strain ratio to the cube strength varies with a series of conditions, as for instance with the moisture content and the porosity of the concrete, the properties of the cement and the aggregates, etc. Nevertheless it seems possible to state general relations which will be sufficiently accurate for the purpose of a general investigation of the variation of the ultimate load with the quality of the concrete, the percentage of reinforcement and the eccentricity of the load. Better agreement with actual tests in any particular case may, of course, be obtained by determining at least K_P and E_o experimentally. The following relations are based mainly on the tests described in papers [1] and [2]:

$$K_P = 0{,}77\,K_W \tag{8}$$

$$E_o = 95\,500 + 390\,K_W \text{ kg/cm}^2 \tag{9}$$

$$\eta = 1{,}25 + \frac{400}{K_W} - \frac{K_W}{400} \tag{10}$$

These relations have been used in the computations to follow, for concretes with $K_W = 100$ kg/cm² to 300 kg/cm².

Equation (10) represents fairly well the *lowest values of the ultimate strain ratio* found in tests by the author and by *Saliger.*[4] More extensive experiments are, of course, needed to determine what wider field of application the relation may be given. The fact that the ultimate strain ratio decreases with increase of the concrete strength is particularly important (see [3], page 221). One might, perhaps, expect η to decrease with the eccentricity of the load. The tests, however, have shown no regular variation of η with variation of the eccentricity (see [1], page 739 and [2], page 65, Table 9, Column 9).

3) Comparison of Computed with Actual Ultimate Loads.

The tests described in the papers [1] and [2] included the testing of 9 over-reinforced and 4 normally reinforced specimens with eccentrically applied axial loads, with $\psi = 0.661$ to 1.855. The specimens had 0.70 to 4.64 per cent of tensile reinforcement. The concrete used in the tests gave rather unusual values of the ratio K_P/K_W. When the actual values of K_P, as found in the tests, and also the test values of E_o and η (which, however, are in fair agreement with equations (9) and (10)) are entered in the computations according to equations (2) to (5), ultimate loads are found, which for two of the three groups of over-reinforced specimens agree well with the test results. The greatest deviation is 12 per cent and the average deviation for the 6 specimens of these groups is 5 per cent. On account of differences in the compacting of the concrete in different kinds of specimens, the tests with the third group of over-reinforced specimens gave no basis for such comparison. Also for these specimens, however, the influence of variations in the eccentricity of load and the percentage of reinforcement seems to be well represented by the equations of Section 1 (see [1], page 744 and [2], page 70, Table 10, Column 8).

The actual ultimate loads of the four normally reinforced specimens were on the average 8,8 per cent greater than computed on the basis of the actual values of K_P. When the cube instead of the prism strength is entered in Equations (6) and (7), the actual ultimate loads are on the average 1.7 per cent smaller than the computed ones. It does, in fact, seem probable that during a local failure like that taking place in normally reinforced specimens, the compressive stress in the concrete may well reach a value equal to the cube strength. For the sake of safety, however, the prism strength is used in the computations.

The most complete series of tests of reinforced concrete in bending combined with compression, known to the author, is the one carried out by *Bach* and *Graf*.[6] In Table 1, Column 14, are given the average ultimate loads of the 15 groups of test specimens. The average dimensions, percentages of reinforcement and eccentricities of load are listed in columns 2 to 12, according to the report in paper.[6] The average cube strength of the concrete was $K_W = 225\ \mathrm{kg/cm^2}$, consequently $K_P = 0.77\ K_W = 173\ \mathrm{kg/cm^2}$, which agrees with the test results for plain specimens in centric compression (see [6], Table 24). According to the equations (9) and (10) we then have $n =$ about 11.5 and $\eta =$ about 2.5. The ratios m and m′ have been determined from the values of the yield point of the steel shown in Table 3 of paper [6]. With the constants thus determined, the ultimate loads of the 15 groups of specimens have been

[6] *C. Bach* and *O. Graf*: „Versuche mit bewehrten und unbewehrten Betonkörpern, die durch zentrischen und exzentrischen Druck belastet wurden.“ Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, No. 166 to 169, 1914.

computed, see Table 1, Column 13. As seen from Column 15 the agreement between the computed and the actual ultimate loads is good. For one of the groups of plain concrete specimens the computed load fell 15.3 per cent below the actual value, otherwise the deviations vary between — 3.98 per cent and + 5.15 per cent. The average deviation of the computed from the actual loads for all 15 groups amounts to — 1.13 per cent.

Slater and *Lyse* have tested two plain concrete prisms under eccentric loading.[7] The dimensions of the prisms were $20.3 \times 20.3 \times 30.5$ cm, the prism strength of the concrete was K_P 285 kg/cm² and consequently the probable cube strength about $K_W = 370$ kg/cm². When η is computed from Equation (10), α from Equation (2) and N'_B from Equation (3) with $\mu = \mu' = 0$, we obtain $N'_B = 74.4$ t. The actual average ultimate load was 70.5 t, that is 5.3 per cent smaller than computed.

In the above cases it is seen that the ultimate loads computed from the equations of Article 1 agree fairly well with the results of tests. It seems, therefore, that the equations may at least be used as the basis of a general investigation of the variation of the ultimate load of eccentrically loaded members with the eccentricity of the load and the percentage of reinforcement.

4) The Factor of Safety.

The permissible or *safe loads* may be computed by dividing the ultimate loads by the *factor of safety*. The proper choice of the factor of safety has been discussed at some length in previous publications (see [1], pages 688 to 693; [2], pages 14 to 19 and [3], pages 221 to 222). If an *actual* factor of safety of 2 is desired, the *nominal* factor of safety for simple compression should be raised to 3.3 or 3.4, on account of the influence of long-time or repeated loads and on account of the difference in strength due to difference in size between ordinary structural members and usual test specimens. *The proposed new Norwegian Building Regulations for Reinforced Concrete*, designated as NS 427, the first part of which was published for discussion in the autumn of 1935,[8] are based on factors of safety in simple compression of 4.13, 3.85, 3.65 and 3.60 respectively for the four Standard Concretes A to D with cube strengths of 290 kg/cm², 230 kg/cm², 180 kg/cm² and 140 kg/cm² respectively.

Certain differences in the manner of failure of concrete in simple compression and in bending or bending with compression, make it seem desirable to have a factor of safety 10 % higher for bending and bending with compression than for simple compression. (See [1], pages 751 to 754; [2], pages 77 to 80 and [3], page 222.) The factors of safety for bending and bending with compression to correspond with the above values then should be 4.54, 4.24, 4.02 and 3.96 respectively for the Standard Concretes A to D. These values are used in the computations referred to below.

For the reinforcement there is no such difference between the actual and the

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[7] *W. A. Slater* and *Inge Lyse*: „Compressive Strength of Concrete in Flexure as Determined from Tests of Reinforced Beams." Proceedings, American Concrete Institute, Vol. 26. 1930. in particular pages 852 to 859.

[8] „Forslag til Norsk Standard: Regler for utførelse av arbeider i armert betong — NS 427, utarbeidet av Den Norske Ingeniørforening." Supplement to Teknisk Ukeblad No. 38, 1935.

Table I.

Actual and calculated rupture loads for eccentric loading according to tests by Bach and Graf 1914.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Test piece No.	Rein-forcement	Eccentricity of load measured from the axis of the test piece	Mean dimensions					Mean percentage of reinforcement		Position of load (fig. 1, 3, 4)		Rupture load			Notes
			b	h	a	h_o	a'	μ	μ'	c_e	$\psi = \frac{c_e}{h_o}$	Calculated N'_B	Mean test result N_B	$\frac{N'_B - N_B}{N_B}$	
		cm	cm	cm	cm	cm	cm	%	%	cm		tons	tons	%	
75, 88, 142	0	10	40.1	40.2	0	40.2	0	0	0	30.1	0.749	138.0	136.0	+ 1.47	without reinforcement
76, 89, 143	0	15	40.1	40.1	0	40.1	0	0	0	35.05	0.874	69.3	81.8	— 15.30	„ „
82, 90, 97	4 ø 16	0	40.1	40.1	3.4	36.7	0	0.559	0	16.65	0.454	277.0	280.3	— 1.18	heavy reinforcement
85, 91, 94	„	20	39.9	40.1	3.6	36.5	0	0.564	0	36.45	0.999	93.6	93.0	+ 0.65	normal „
86, 92, 95	„	30	40.0	40.1	3.6	36.5	0	0.567	0	46.45	1.272	57.9	60.3	— 3.98	„ „
87, 93, 96	„	50	40.0	40.1	3.9	36.2	0	0.570	0	66.15	1.830	28.9	30.0	— 3.67	„ „
107, 108	8 ø 16	10	40.0	40.1	3.7	36.4	3.1	0.558	0.560	26.35	0.724	198.3	202.5	— 2.07	heavy reinforcement
99, 102, 118	„	20	40.1	40.1	3.6	36.5	3.3	0.558	0.556	36.45	0.999	119.3	124.0	— 3.79	normal „
119, 120, 121	„	20	40.1	40.2	3.6	36.6	3.3	0.558	0.555	36.50	0.998	119.0	123.3	— 3.49	„ „
100, 103	„	30	40.1	40.3	3.5	36.8	3.3	0.554	0.552	46.65	1.269	69.3	69.6	— 0.43	„ „
101, 104	„	50	40.2	40.2	3.6	36.6	3.3	0.558	0.552	66.50	1.818	33.3	32.4	+ 2.78	„ „
140, 141	8 ø 22	10	40.0	40.3	3.7	36.6	3.8	1.045	1.043	26.45	0.723	236.6	225.0	+ 5.15	heavy reinforcement
63, 122, 137	„	20	40.1	40.1	3.8	36.3	3.7	1.047	1.050	36.25	0.999	164.8	157.5	+ 4.63	„ „
123, 138	„	30	40.1	40.1	3.7	36.4	3.8	1.044	1.045	46.35	1.272	105.5	105.0	+ 0.48	normal „
65, 124, 139	„	50	40.1	40.1	3.8	36.3	3.7	1.050	1.048	66.25	1.825	55.1	53.5	+ 3.00	„ „

Constants of material: $\eta = 2{,}5$; $n = 11{,}5$. For rounds of 16 mm ø: $\sigma_F = 3773$ kg/cm², $\sigma'_F = 3680$ kg/cm², $K_P = 173$ kg/cm².
For rounds of 22 mm ø: $\sigma_F = 3672$ kg/cm², $\sigma'_F = 3754$ kg/cm².

nominal factor of safety, since for mild and intermediate steel the tensile strength which can be relied upon under such repetition of loading as occurs in most reinforced concrete structures, will come very close to the yield point of the steel, which is the stress used in computing the ultimate loads of normally reinforced members according to Equations (6) and (7). Consequently the nominal factor of safety may be chosen equal to the actual factor of safety desired. In the computations referred to below a factor of safety of 1.8 was used for normally reinforced sections. This should be fully sufficient for a uniform material like steel.

5) Safe Loads and Limiting Points.

In Fig. 5 are shown the safe loads, N_{zul}, computed as described above, for a section with tensile reinforcement only ond for a symmetrically reinforced section. The computation was made for the following case: Position of load 1.5 h_o from the tensile reinforcement (moment arm ratio, $\psi = 1.5$), $\gamma = 1.08$, $\beta = 0.08$ (See Figs. 1 and 3), cube strength of concrete, $K_W = 180$ kg/cm², $\eta = 3.03$, $n = 12.7$ (Standard Concrete C according to NS 427), yield point of steel $\sigma_F = \sigma'_F = 2000$ kg/cm²,[9] $m = m' = 14.4$. With the percentage of reinforcement as abscissa the safe unit loads, $\frac{N_{zul}}{bh_o}$, have been plotted as well for over-reinforced sections [Equations (2) to (5)] as for normally reinforced sections [Equations (6) and (7)]. At any particular value of μ, the lower one of the two corresponding values of N_{zul} does, of course, represent the actual value of the safe load. (Heavily drawn lines in Fig. 5.)

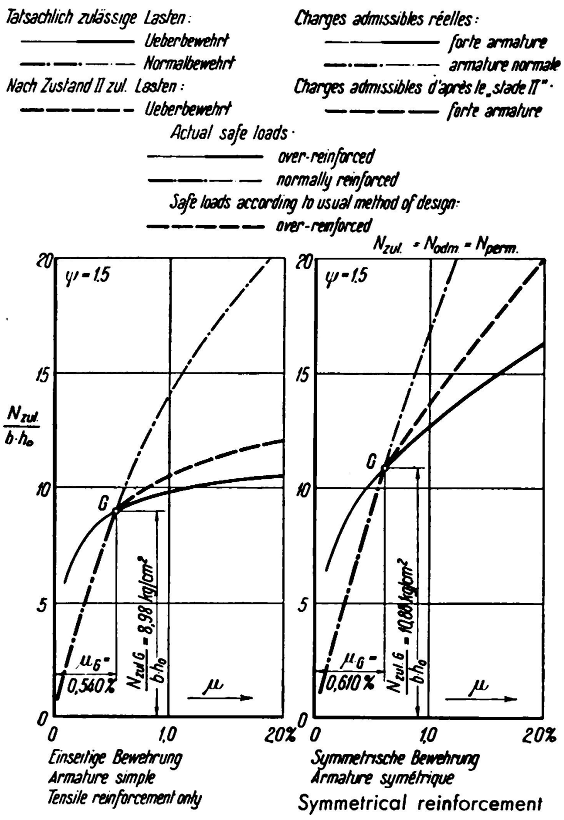

Fig. 5.

Safe loads for concrete C with $\psi = 1.5$ as actually obtained and according to usual method of design.

The point G, where the two lines for N_{zul} intersect, is the *limiting point* separating the two ranges of reinforcement, one range of *partly reinforced* sections, where the reinforcement determines the safe load, and one of fully *reinforced sections,* where the safe load is dependent mainly upon the strength of the concrete.

Lines like those in Fig. 5 might well be used as a means of designing eccentrically loaded rectangular reinforced concrete sections. However, the ordinary method of calculation may as well be used, *provided only that the working*

[9] This is considered as the lower limit for ordinary mild reinforcing steel as used in Norway.

stresses are so chosen that the ordinary calculation will in every case lead to the correct value of the safe load.

For partly reinforced sections, where the reinforcement determines the safe load, this can generally be attained by the use of one single value of the working steel stress for all percentages of reinforcement. For fully reinforced sections, however, the case is different. With one single value of the allowable concrete fibre stress the ordinary method of calculation gives nominal safe loads which increase far more rapidly with increase in the percentage of reinforcement than

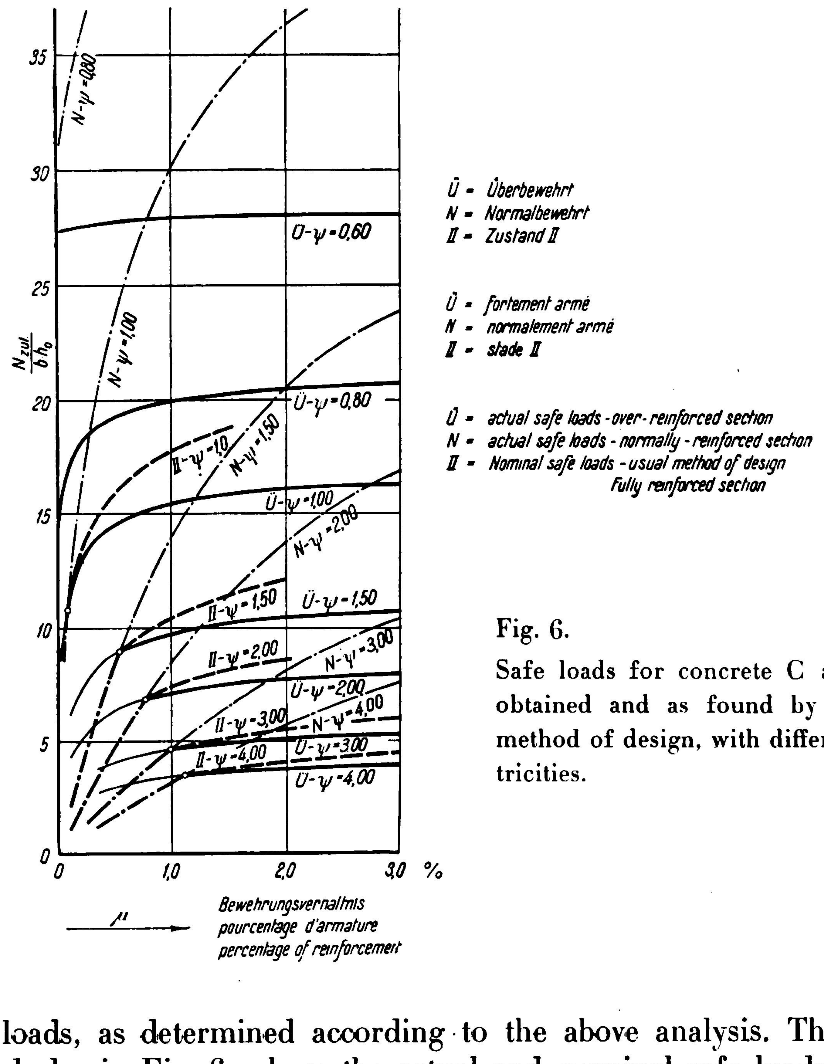

Fig. 6.

Safe loads for concrete C as actually, obtained and as found by the usual method of design, with different eccentricities.

actual safe loads, as determined according to the above analysis. This is shown in Fig. 5 and also in Fig. 6, where the actual and nominal safe loads are plotted for several values of ψ, the assumptions being as for Fig. 5. Consequently, one definite factor of safety can only be maintained throughout the range of fully reinforced sections if the working stress for concrete is varied with the percentage of reinforcement.

It has been shown previously that in the case of pure flexure the correct allowable fibre stress in concrete is the stress corresponding to the limiting point, G. (See [1], page 688, [2], page 14 and [3], page 222). The same applies to the case of bending with compression, provided that the eccentricity of load is large. With smaller eccentricities allowable concrete stresses other than those corresponding to the limiting points may be of practical interest. In the first place, when the load is acting inside the cross-section, there hardly is any limiting point to be found, since practically all sections are fully reinforced (See Article 1, b and Fig. 6). In the second place, even with the load acting well outside the cross-section, the percentages of reinforcement corresponding to the limiting points are so small, that in practice very often more reinforcement must be used (Fig. 6).

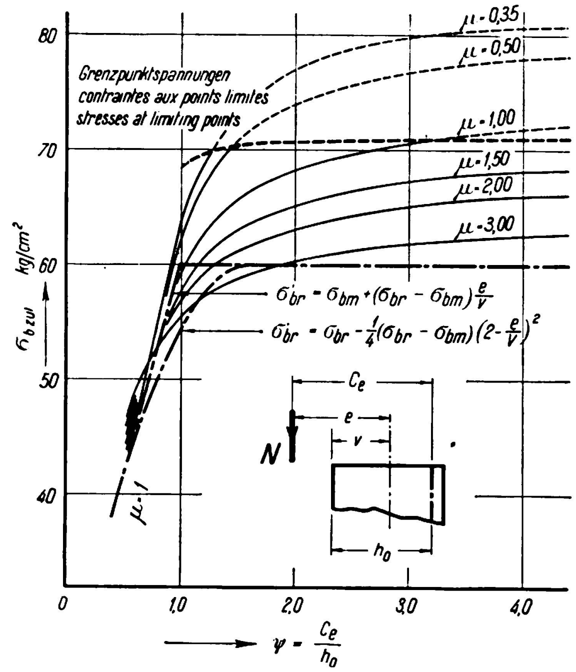

Fig. 7.

Correct permissible stresses for concrete C with different eccentricities, using reinforcement on one side only.

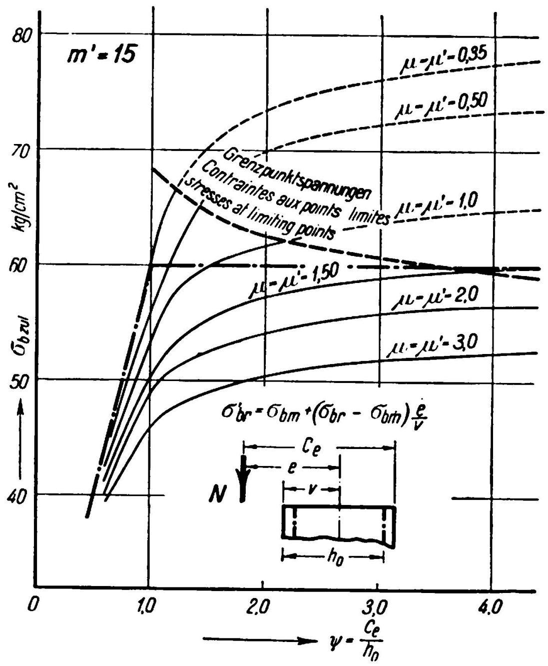

Fig. 8.

Correct permissible stresses for concrete C with different eccentricities, using symmetrical reinforcement (assuming m' = 15).

6) Correct Working Stresses for the Concrete.

From the actual safe loads, determined as described above, the corresponding correct working stresses for concrete, to be used with the ordinary method of calculation, can be computed for different eccentricities of load and different percentages of reinforcement. Working stresses, thus determined for Standard Concrete C with assumptions as in Article 5, have been plotted in Figs. 7 and 8 with $\psi = \frac{C_e}{h_o}$, as a measure of the eccentricity, as abscissa. In addition, the concrete stresses corresponding to the limiting points, discussed in Article 5, have been plotted in the diagrams. Concrete working stresses exceeding the stresses at limiting points, are of no significance, since they correspond to sections for which the steel, not the concrete stress determines the safe load (partly reinforced sections).

As one might expect, the diagrams show that the *correct working stresses for concrete decrease very rapidly with decrease in the eccentricity of the load. As the load approaches the centre of gravity of the cross-section, the correct working stresses approach those valid for simple compression.*

Thus, under the assumptions stated above, the allowable fibre stresses for standard concrete C with tensile reinforcement only should be as follows:

In pure bending, at limiting point $\sigma_{b\,zul\,1} = 71.0$ kg/cm².

In bending with compression, with 1 per cent of reinforcement:

With the load at the edge of the cross-section

($\psi = 1.0$) . . . $\sigma_{b\,zul} = 59.6$ kg/cm² $= 0.84\ \sigma_{b\,zul\,1}$.

With position of load so that the stress at the far edge of the cross-section is zero

($\psi = 0.63$) . . . $\sigma_{b\,zul} = 49.0$ kg/cm² $= 0.69\ \sigma_{b\,zul\,1}$.

With the load acting at a distance of 0,135 h_o from the centre of gravity of the section

($\psi = 0.54$) . . . $\sigma_{b\,zul} = 44.8$ kg/cm² $= 0.63\ \sigma_{b\,zul\,1}$.

It is seen that if the *same* working stress is used in actual design in all these cases, the factor of safety will actually be very much less with small eccentricities of load than in the case of pure flexure.

7) Effectiveness of Compression Steel.

As the Figures 7 and 8 show, the correct working stresses for concrete vary much with the quantity of reinforcement, and in particular with the quantity of compressive reinforcement. With symmetrical reinforcement the correct working stresses are appreciably lower than with tension reinforcement only. The same applies to the case of pure flexure, as the dotted line in Fig. 9 shows. The correct concrete working stress for Standard Concrete C at limiting points is about 21 per cent lower with symmetrical reinforcement than with tension reinforcement only.

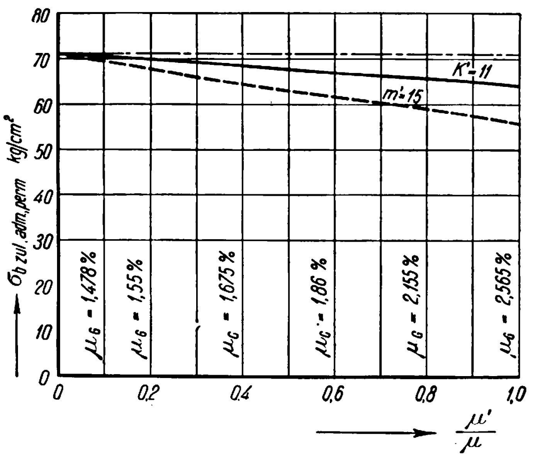

Fig. 9.

Correct permissible stresses for concrete C under pure bending with different amounts of compression reinforcement (calculated partly with $m' = 15$ and partly with $k' = 11$).

The correct concrete working stresses represented in Figures 8 and 9 have been computed from the safe loads by the ordinary method of computation, whereby the stresses in concrete and steel have been assumed to be distributed as indicated in Fig. 10. The stress in the compressive reinforcement has been computed from the equation:

$$\sigma'_e = m' \sigma_{br} \frac{\alpha - \beta'}{\alpha} \tag{11}$$

where σ_{br} is the allowable fibre stress in concrete in the case considered, and m' is equal to $\frac{\sigma'_F}{K_P}$, as defined in Article 1, a. For the concrete assumed here, with

$K_w = 180$ kg/cm^2 and $K_P \cong 138$ kg/cm^2 (cylinder strength at 28 days (f'_c) about 2000 lb. per sq. in.) and for steel with a minimum value of the yield point, $\sigma'_F = 2000$ kg/cm^2 (about 28400 lb. per sq. in.), we have m' approximatively equal to 15, which is the value used in computing the curves of Figures 8 and 9.

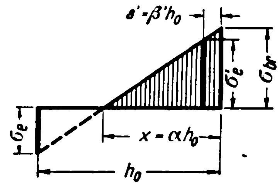

Fig. 10.

Now, while the actual stress in the concrete at failure is equal to K_P (Figures 1 and 3), the nominal stress corresponding to the load at failure according to the stress-distribution of Fig. 10, will be much larger than K_P. Correspondingly, at that load the nominal stress in the compressive reinforcement according to Equation (11) will be much larger than σ'_F. That equation thus leads to an exaggeration of the effect of the compression steel upon the ultimate load. To correct this, a factor k', smaller than m', should be used in Equation (11). The factor should be chosen so as to make the computed stress in the compression steel at failure equal to $m' \cdot K_P = \sigma'_F$. The result should be about the same if k' is taken as given by the equation

$$k' = m' \frac{\sigma_{bm}}{\sigma_{br}} \frac{\alpha}{\alpha - \beta'} \tag{12}$$

where σ_{bm} is the allowable concrete stress in simple compression. At the working load, σ'_e will then be equal to $m'\sigma_{bm}$.

Most building regulations specify the use of the factor n instead of m' in Equation (11). Usually, however, $n = 15$ is used, at least for the grade of concrete considered here, and since that was the value of m' used in computing the curves of Figures 8 and 9, computation according to most building regulations would give the same results as are shown there, with the same exaggeration of the effect of the compression steel.

In that portion of the proposed new Norwegian Regulations, NS 427, which has not yet been published, values of k' approximately in agreement with Equation (12) are specified for use in the cases of bending and bending with direct stress. In simple compression, the ratio between stresses in steel and concrete is taken to be $m' = \frac{\sigma'_F}{K_P}$. For the grade of concrete considered here, $k' = 11$ and $m' = 15$ are the specified values. For the stress in the tensile reinforcement, the ratio $n = 15$ is used in all cases.

The full line in Fig. 9 and the curves in Fig. 11 show the correct concrete working stresses obtained by using $k' = 11$ instead of $m' = 15$ in Equation (11). It is seen that there is still a difference between sections with and without compression steel. This is due mainly to the fact that according to NS 427, σ_{br} for the concrete considered is only 60 kg/cm^2, while according to our computations $\sigma_{br} = 71$ kg/cm^2 would be the correct value. However, much of the difference is eliminated with the use of k' instead of m'.

8) The Working Stresses for Concrete as Specified in Building Regulations.

In the building regulations of most countries very little account is taken of the great influence of the eccentricity of load on the correct concrete working stresses for structural members in bending with compression, which is demonstrated in Figures 7, 8 and 11. According to the regulations of several countries, the full working stress for pure bending may be applied also in the case of bending with compression, provided only that the working stress for simple compression is not exceeded when the load is considered as acting centrally. If, for instance, the allowable fibre stress in flexure is 60 kg/cm² and in simple compression 38 kg/cm², as specified for Standard Concrete C in NS 427,[8] the full bending stress could in the case treated in Article 6, assuming 1 per cent of tensile reinforcement only, be applied with the load acting only 0.105 h_0 from the center of gravity of the section, that is, with $\psi = 0.508$. The correct working stress would in that case be about 43.5 kg/cm², as against 71.0 kg/cm² in pure bending. That is, the factor of safety would be about 39 per cent less than in the case of pure bending.

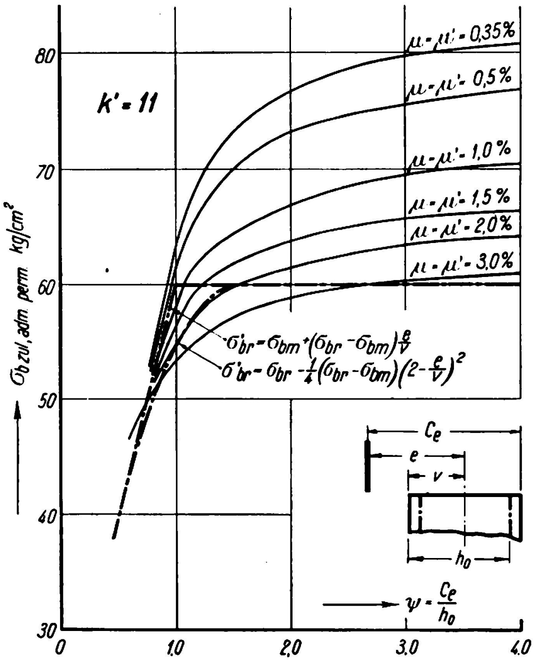

Fig. 11.

Correct permissible stresses for concrete C with different eccentricities, using symmetrical reinforcement (calculated with $k' = 11$ instead of m') (compare Fig. 8).

The latest American regulations[10] provide for an increase in the working stress for eccentrically loaded as compared to centrically loaded columns, trough multiplying the working stress for simple compression with a factor, which for instance with $\psi = 1.0$ and 1 per cent of reinforcement on either side of the cross-section, amounts to about 1.163. In a column without spirals the working stress would then be $0.154\, f'_c \cdot 1.163 \cong 0.18\, f'_c$. ($f'_c$ is the minimum ultimate compressive strength of test cylinders at 28 days, for the concrete considered about 2000 lb per sq. in.) Now, the allowable unit stress in pure flexure is specified to be $0.40\, f'_c$. From Fig. 8 we find the correct concrete working stress with $\psi = 1.0$ and $\mu = \mu' = 1.0$ per cent to be 53.2 kg/cm², or about 75 per cent of the correct working stress in pure flexure with no compression steel (71.0 kg/cm²) which has been determined with the same factor of safety. That means that the factor of safety in the case considered would be the same as in pure flexure, if the working stress were fixed at $0.75 \cdot 0.40\, f'_c = 0.30\, f'_c$. Actually only $0.18\, f'_c$ is allowed, and hence the American Concrete Institute's Regulations provide in this case for a factor of safety which is about 67 per cent greater than the factor of safety actually used in pure flexure.

As seen, the case of bending combined with compression is treated very differently in the building regulations of different countries. According to some

[10] *Building Regulations for Reinforced Concrete* (A.C.I. 501—36 T) tentatively adopted, Feb. 25, 1936, Journal American Concrete Institute, March-April 1936, Vol. 7, pages 407—444.

regulations, the factor of safety is much smaller in the case of bending with compression than in the case of pure flexure, according to others, it is larger.

In the proposed new Norwegian Regulations, NS 427[8], an attempt has been made to adapt the working stresses for concrete in bending with compression somewhat better to the correct values. The allowable unit fibre stress for concrete in bending with compression is specified as follows:

a) With the load acting inside the cross-section ($\psi < 1,0$):

$$\sigma'_{br} = \sigma_{bm} + (\sigma_{br} - \sigma_{bm}) \frac{e}{v}; \qquad \frac{e}{v} < 1 \tag{13}$$

where: σ_{br} = allowable unit fibre stress in pure flexure,
σ_{bm} = allowable unit fibre stress in simple compression,
e = eccentricity of load, measured from the gravity axis of the equivalent concrete section,
v = distance from gravity axis to extreme fibre in compression.

b) With the load acting outside the cross-section ($\psi \geqq 1$):

$$\sigma'_{br} = \sigma_{br}; \qquad \frac{e}{v} \geqq 1 \tag{14}$$

The allowable unit stresses according to Equations (13) and (14) have been plotted in Figures 7 and 11 for comparison with the correct values. It is seen that although the working stresses specified in the proposed NS 427 do not lead to the same factor of safety in all cases, nevertheless much of the variation implicit in other specifications has been eliminated.

The agreement between correct and specified working stresses would be improved, if the full allowable fibre stress for pure flexure were to be applied only with $\frac{e}{v} > 2$ or $\psi >$ about 1,6, and if a parabolic instead of a linear variation of the working stress for smaller eccentricities were specified, for instance as given by Equation (15):

$$\sigma'_{br} = \sigma_{br} - \frac{1}{4} (\sigma_{br} - \sigma_{bm}) \left(2 - \frac{e}{v}\right)^2; \qquad \frac{e}{v} < 2 \tag{15}$$

The corresponding curves are shown in Figures 7 and 11, they agree quite well with the smaller values of the correct working stresses as here determined.

IIa2

The Calculation of Reinforced Concrete Sections Subject to Bending.

Berechnungsverfahren von auf Biegung beanspruchten Eisenbetonquerschnitten.

Les méthodes de calcul des sections de béton armé sollicitées à la flexion.

Dr. techn. Ing. E. Friedrich,
Dresden.

A. The German and Austrian Regulations.

I. Decisions of the German Committee for Reinforced Concrete.

1) *Carrying capacity.*

According to the German Regulations of 1932, Paragraph 17, reinforced concrete sections subject to bending are to be calculated on the assumption that strain is proportional to distance from the neutral axis and that co-operation by the concrete on the tension side is entirely neglected. (Condition II b in

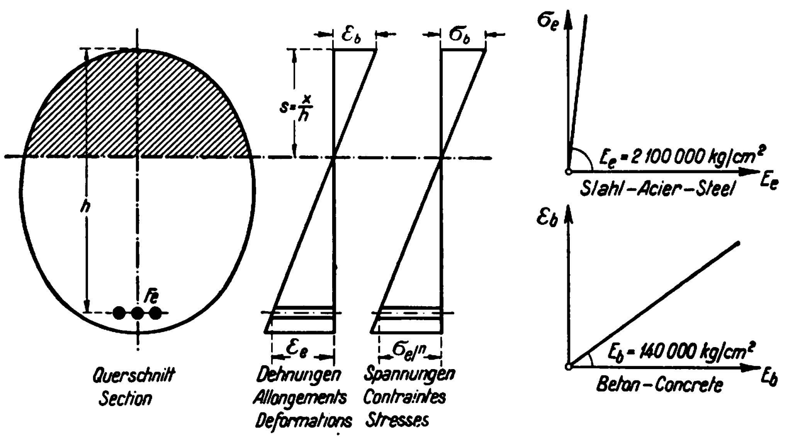

Fig. 1.
German Regulations.

Fig. 1.) The ratio of the moduli of elasticity of steel and concrete is taken as n = 15, and both in the steel and in the concrete a straight line relationship is assumed to hold good between stress and strain (*Hooke*'s Law).

In what follows below, the carrying capacity will be denoted by

$$T = \frac{M \cdot h}{J_i}$$

where J_i is the "ideal" moment of inertia. The expression $\frac{J_i}{h}$ is independent of the shape of the cross section of the beam, and in case of an homogeneous cross section it corresponds to the section modulus W and represents concrete stress plus $\frac{1}{n}$ times steel stress. A picture of the conditions governing the strength of a reinforced concrete section is obtained when T is expressed as a function of the distance from the neutral axis $s = \frac{x}{h}$. When plotting this function the abscissae are so divided that successive values $\frac{1}{s}$ occupy equal distances.

Following the method of calculation hitherto used (in accordance with Type IIb) the carrying capacity in the concrete portion of the beam becomes

$$T = \frac{W_b}{s}$$

(where W_b is the cube strength) and in the steel portion

$$T = \frac{\sigma_s}{n} \, \frac{1}{1-s}$$

(where σ_s is the yield point of the steel). If this system of co-ordinates is adopted the curve of carrying capacity becomes a straight line for the concrete and a hyperbola as regards the steel.[1]

2) *Comparisons with experimental results.*

Fig. 2 represents the results of experiments carried out on rectangular cross sections reinforced with St 37, wherein the cube strength of the concrete $W_b = 110$ kg per sq. cm as nearly as possible, and wherein the yield point of the steel was $\sigma_s = 2800$ kg per sq. cm. The cross sections were so chosen as to develop a carrying capacity corresponding to a considerable range of s. In Fig. 2 the calculated carrying capacities according to the German regulations (broken line) and the capacities determined by experiment are juxtaposed, and the following comparisons emerge:

a) In the region where failure depends on the yield point of the steel: —

  α) The experimental results always work out approximately 10 % higher than those found by calculation.

  β) The curves of carrying capacity obtained by calculation and by experiment are entirely similar. It is not possible, however, to justify an increase in the permissible stress in the steel region, and the risk of cracking would in itself be an objection to such a course, nor is there good cause for altering the method of calculation in the region α.

[1] *E. Friedrich:* „Über die Tragfähigkeit von Eisenbetonquerschnitten." Beton und Eisen, 1936, No. 9.

b) In the region where breakage is governed by the strength of the concrete: —

A) The first point that arises is that the curve of carrying capacity extends to values of much higher percentage of reinforcement than the steel carrying capacity line [or as in Fig. 2 up to much higher s-values].

B) In the whole of the second region the carrying capacity is much higher according to experimental values, than according to the calculated values.

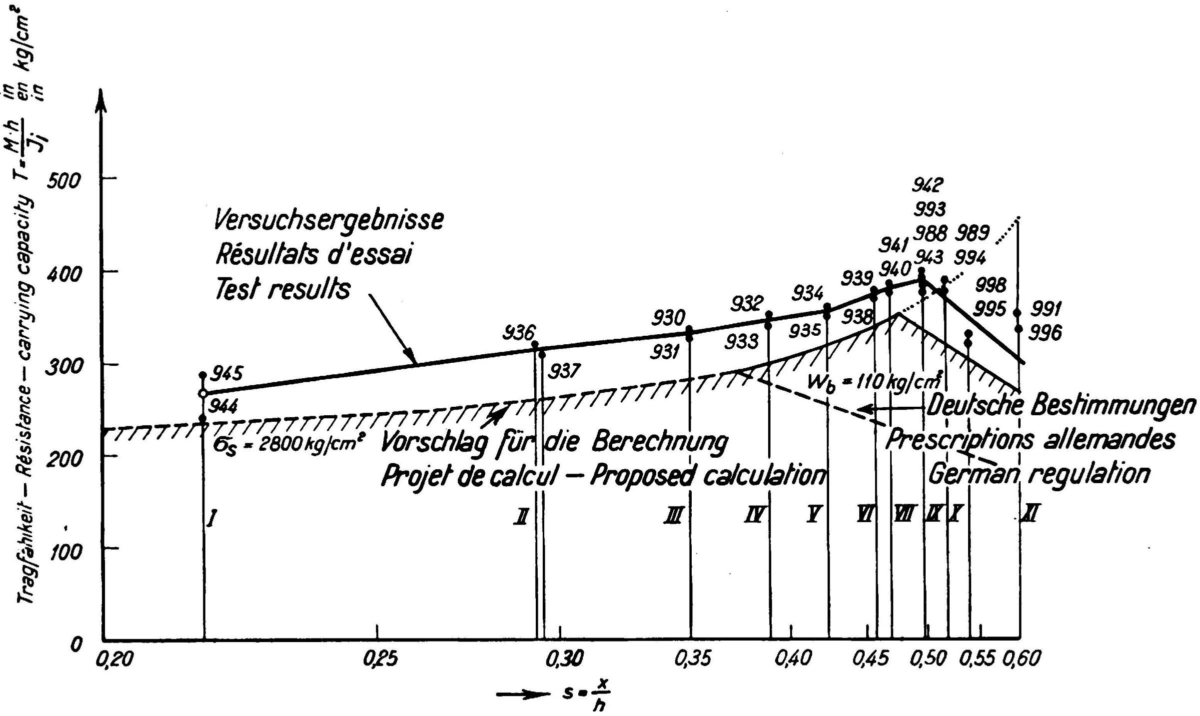

Fig. 2.

Curve of carrying capacity as determined by experiment (full line), by calculation (broken line) and as proposed for St. 37 (hatched border).

II. The Austrian regulations.

In the Austrian règulations an attempt has been made to overcome the defects of the method of calculation hitherto in use. According to the suggestion made by *von Emperger* and *Haberkalt* the limits for the steel and the concrete region is to be raised to an extent corresponding with an increase in the permissible concrete stress to 15 to 25% above that allowed hitherto. Since, however, the existing values of permissible stress have in fact been retained, it follows that the curve of carrying capacity shows a break at the point which marks the limit of reinforcement, and two disadvantages arise in consequence of this:

a) Cases may occur in which the calculated carrying capacity is reduced on an addition being made to the reinforcing steel.

b) Since the limit of reinforcement is made dependent on the percentage of reinforcement provided, the suggestion can be applied only to rectangular cross sections.

Fig. 3 shows the curve of carrying capacity in accordance with the Austrian regulations.

B. New suggestions for the calculation of reinforced concrete sections subject to bending.

The tendency in reinforced concrete design, both in buildings and bridge work, is to avoid both sloping undersides to the beams and compression reinforcement. A suggestion is now made whereby this tendency can be satisfied while retaining the same degree of safety as at present, and while conforming with the lessons of experiments.

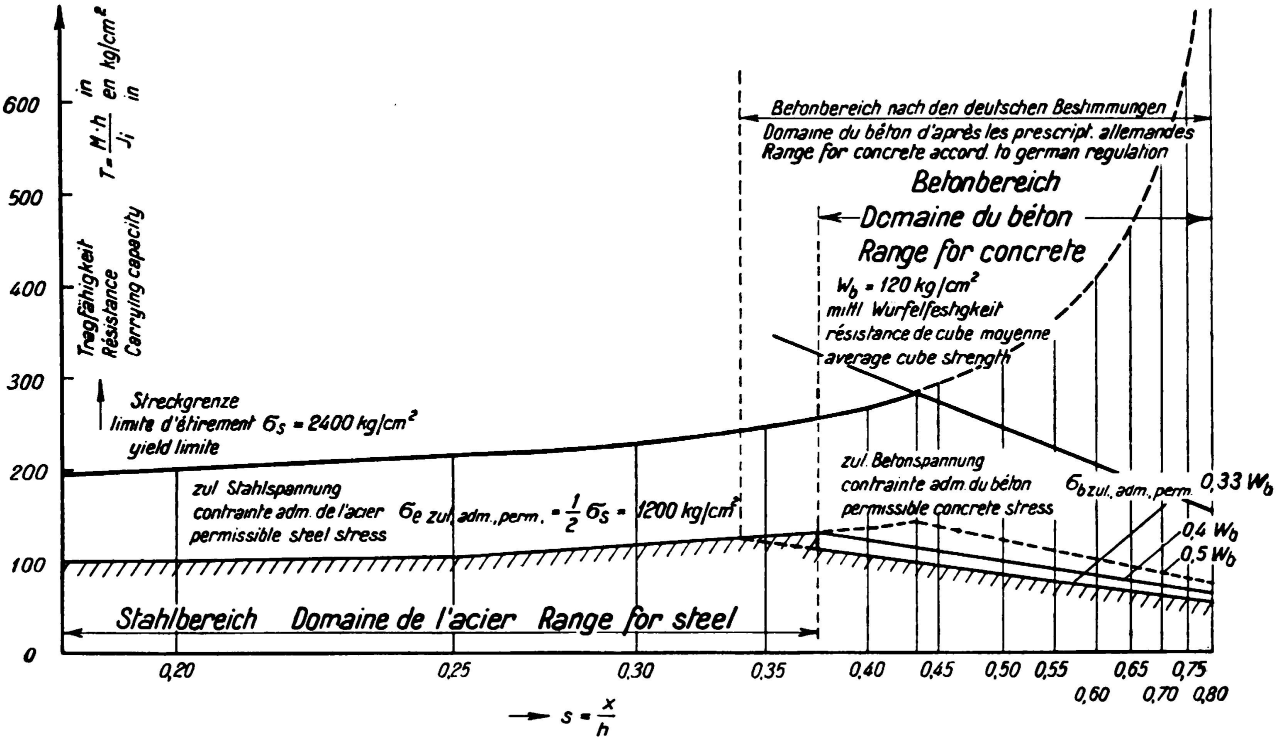

Fig. 3.

Carrying capacity line according to Austrian regulations.

I. Region where failure is determined by the yield point of the steel.

In this region the present method of calculation, as described, will be retained, and provided the required cube strength of the concrete is satisfied the stress in the latter need not be calculated.

II. Region where failure is determined by the strength of concrete.

1) Basis of calculation.

a) Determination of the neutral axis.

Where the bending moment is moderate, the condition indicated by IIb will obtain, as assumed by the method of calculation hitherto in use. That is to say the concrete will tend to crack in the tension zone once the stress in its outermost fibre becomes equal to the breaking stress (which is here equated to the cube strength) but, instead of the beam at once breaking as implied by the calculations hitherto in use, the condition IIb will give way to a new condition IIc, characterized by the fact that the concrete on the compression side becomes plastic. The neutral axis remains in its original position and distance from the neutral axis may, therefore, best be calculated on the same assumptions as hitherto.

$$s^2 + 2s\varphi - 2\psi = 0 \tag{1}$$

$$\left(\text{where } \varphi = \frac{f}{b \cdot h}, \quad \psi = \frac{\gamma}{b \cdot h^2} \quad \text{and} \quad f = n\,F_e + n\,F'_e \right.$$

$$\left. \gamma = n\,F_e\,h + n\,F'_e\,h'\right).$$

b) Stress-strain curve for the steel.

To be calculated on the basis of *Hooke*'s Law (Fig. 4).

$$\sigma_s = E_e \cdot \varepsilon_e$$

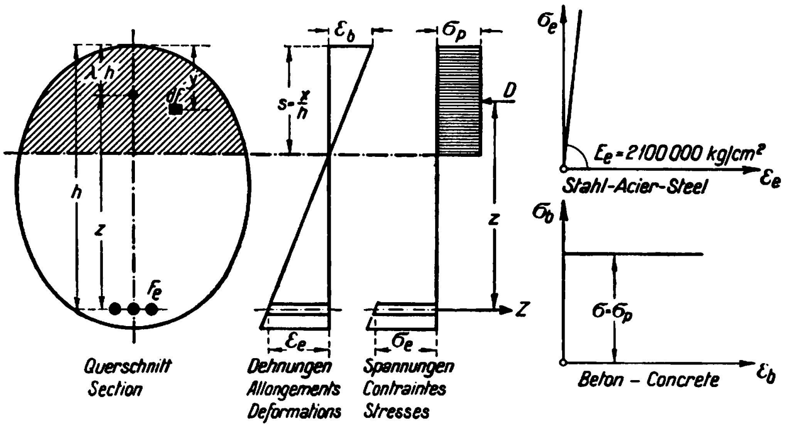

Fig. 4.
Basis of calculation for proposed calculation based on the strength of concrete.

c) Stress-strain curve for the concrete.

To be calculated on the basis of the law of plasticitiy:

$$\sigma_p = \text{const. (independent of E)}$$

d) Navier's assumption.

The calculation assumes that cross sections remain plane.

e) Equilibrium.

At every cross section the external and internal forces must be in equilibrium.

2) Details of calculation.

The stresses and carrying capacity of reinforced concrete sections can be calculated from these assumptions, the total compression D being obtained from the equation.

$$D = \int^{F_W} \sigma_p \cdot df = \sigma_p \cdot \int^{F_W} df = \sigma_p \cdot F_W$$

in which F_w is the effective area of concrete.

The total tension is

$$Z = F_e \cdot \sigma_e = F_w \cdot \sigma_p$$

and for equilibrium we have $Z = D$, or

$$F_e \cdot \sigma_e = F_W \cdot \sigma_p$$

whence

$$\sigma_e = \sigma_p \cdot \frac{F_W}{F_e}. \tag{2}$$

The statical moment of the effective concrete area about the extreme fibre is

$$S_W = \int^{\text{effec. concr. area}} y \cdot df,$$

and hence the distance $\lambda \cdot h$ to the centre of gravity of the effective concrete area is given by

$$\lambda \cdot h \cdot F_W = S_W.$$

The lever arm for the internal forces is

$$z = h - \lambda \cdot h = h \frac{h \cdot F_W - S_W}{h \cdot F_W}.$$

and since the internal and external moments must be equal we have

$$D \cdot z = M$$

$$\sigma_p \cdot F_W \cdot h \cdot \frac{h \cdot F_W - S_W}{h \cdot F_W} = M$$

$$\frac{S_W}{h} - F_W + \frac{M}{h \cdot \sigma_p} = 0. \tag{3}$$

Equation I serves to fix the neutral axis and Equation III enables the carrying moment M to be calculated.

3) Comparison with experimental results.

The formulae explained under (2) above will now be compared with the experimental results for rectangular beams given in Fig. 2, reinforced with St. 37.

For a rectangular cross section —

$$F_W = s \cdot b \cdot h$$

$$S_W = s^2 \cdot h^2 \cdot \frac{b}{2}.$$

In order to allow a comparison between the method of calculation now put forward and that hitherto in use the value of

$$T = \frac{M \cdot h}{J_i}$$

will now be calculated.

In the case of a simply reinforced rectangular section we have

$$\frac{J_i}{h} = b h^2 \frac{\left(1 - \frac{s}{3}\right) \cdot s^2}{2}$$

and it follows from Equation III that

$$T = \frac{M \cdot h}{J_i} = 2 \frac{\sigma_p}{s} \cdot \frac{1 - s/2}{1 - s/3}.$$

Fig. 2 also contains a line marked by hatching, which shows the results obtained from the proposed method of calculation.

In Fig. 5 a comparison is made between the experimental results as indicated in Fig. 2 and the newly suggested method of calculation.

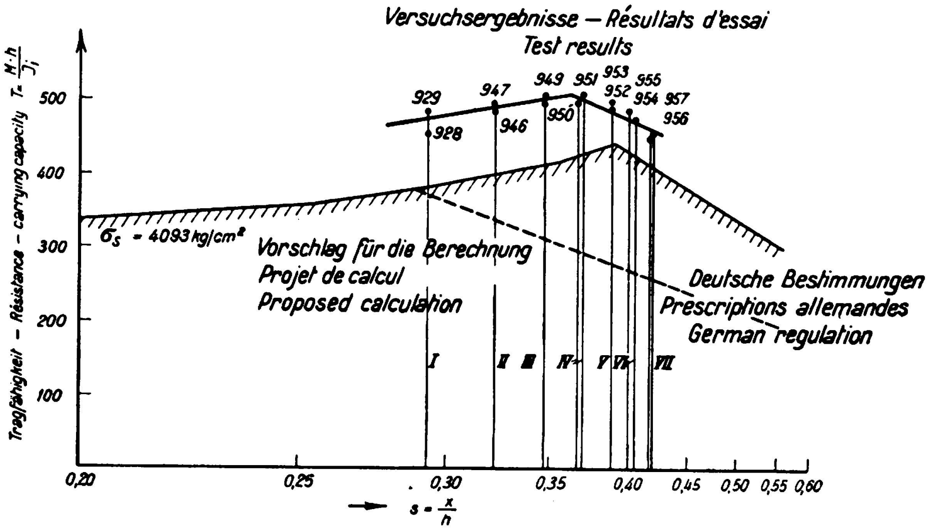

Fig. 5.

Carrying capacity line according to proposal for high grade reinforcing steel.

Fig. 5 shows a further series of experimental results obtained with constructional steel of high yield point (Isteg steel for which $\sigma_s = 4100$ kg per sq. cm). In these experiments the prism strength was ascertained to be $\sigma_p = 94$ kg per sq. cm. Comparison with the method of calculations hitherto in use indicates that the new suggestion gives much better agreement with the experimental results. Fig. 6 shows the fracture of the beam 957, which had taken place in the concrete region; Fig. 7 shows the fracture of beam 947 which has taken place in the steel region, and these two illustrations will enable the two separate regions to be clearly distinguished.

C. Suggestions in regard to the "Regulations".

It has now been shewn how the actual carrying capacity can be reconciled with the carrying capacity as calculated, and suggestions will be made for amending the regulations accordingly.

1) Stress in steel.

The permissible stress in the steel will be as hitherto $\sigma_{e\,perm} = \frac{\sigma_s}{2}$ unless the risk of cracking makes it desirable to prescribe a lower value.

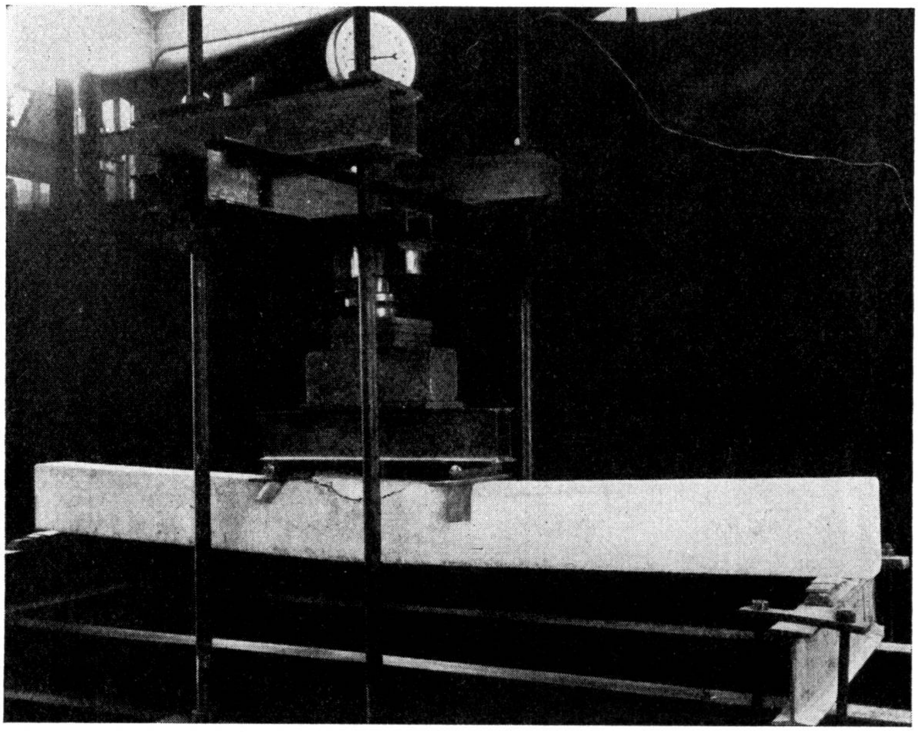

Fig. 6.
Fractured beam No. 957 (fracture due to reaching ultimate strength of concrete).

2) Stress in concrete.

At present a factor of safety of three in relation to the cube strength W_b is adopted. Since, however, the prism strength is to enter into the calculations,

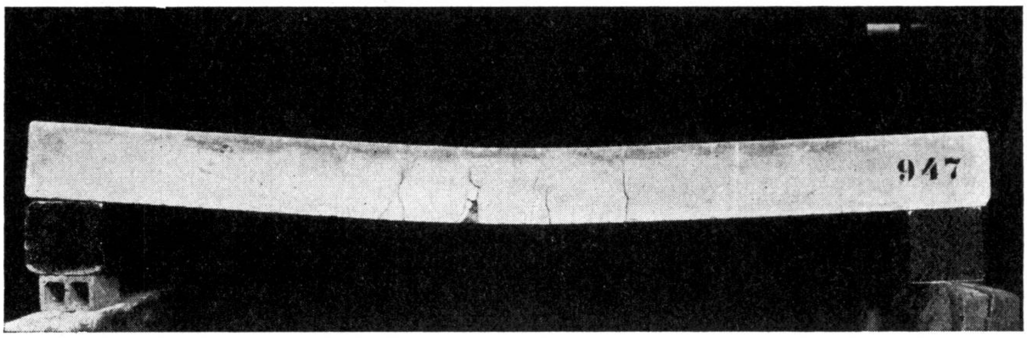

Fig. 7.
Fractured beam No. 947 (fractures due to reaching yield limit of steel).

the permissible concrete stress as hitherto allowed must be reduced. As a rule the prism strength can be taken as 0.75 of the cube strength, (conversion factor for cube strength assuming stationary loading) and on equating the permissible stress to one quarter of the cube strength we obtain ($\sigma_{b\,perm} = \frac{1}{4} W_b$)

permissible stress $= 30$ kg per sq. cm when cube strength $= 120$ kg per sq. cm
permissible stress $= 40$ kg per sq. cm when cube strength $= 160$ kg per sq. cm
permissible stress $= 56$ kg per sq. cm when cube strength $= 225$ kg per sq. cm

3) Limit of reinforcement in rectangular cross sections.

This limit can be determined by equating the carrying capacity in the steel region to that in the concrete region. The carrying capacity of the steel region is

$$\frac{M \cdot h}{J_i} = \frac{\sigma_s}{n} \cdot \frac{1}{1-s};$$

and that in the concrete region is

$$\frac{M \cdot h}{J_i} = 2 \cdot \frac{\sigma_p}{s} \cdot \frac{1-s/2}{1-s/3}.$$

Writing

$$k = \frac{\sigma_s}{n\,\sigma_p},$$

it becomes possible to obtain s_G from the equation

$$s_G = \frac{3}{2} - \frac{1}{2} \cdot \sqrt{\frac{3\,(1+3k)}{3+k}}. \tag{4}$$

Conclusion.

A great many suggestions for reconciling the results of calculation and of experiment have already been made but if these suggestions are to be embodied in actual regulations they must be perfectly open to experimental confirmation. The suggestion here put forward for determining the limit of reinforcement in rectangular cross sections reinforced with St. 37 is one which appears to be fully supported by experiment, and similar tests on high tensile reinforcing steel are in hand.

The series of experiments has further been extended to cover the case of beams with compression reinforcement, so as to investigate the change in carrying capacity that results from the use of the latter.

The suggested procedure favours a much more uniform utilisation of the material than is obtained by the existing methods, and since, to a considerable extent, it renders inclined soffits and compression reinforcement unnecessary, it offers improved possibilities of adapting reinforced concrete construction to modern requirements in design: in buildings, for instance, a flat under-surface can in this way be given to reinforced concrete floors covering several spans, and in bridge work the girders can be made of equal thickness throughout. At the same time the proposal is attended by economic advantages, in that shuttering and reinforcing steel are saved.

IIa 3

New Experiments on Reinforced Concrete Beams.

Neue Eisenbetonbalkenversuche.

Nouveaux essais effectués sur des poutres de béton armé.

Ministerialrat Dozent Dr. Ing. F. Gebauer,
Wien.

Comparative Experiments with Different Amounts of Cover to the Steel and Different Arrangements of Stirrups, and Experiments on Very Heavily Reinforced Beams.

The degree of safety possessed by a reinforced concrete structure cannot be correctly assessed by designing it on the ordinary "n" method.[1] The experimental results show great differences between the actual degree of safety and that assumed from calculation or that which it is desired to ensure.[2] If the stresses in the material are calculated from the breaking moment by the aid of the "n" method, values are obtained which differ considerably above or below the values which are to be regarded as governing the properties of the material, namely the cube strength of the concrete and the elastic limit of the steel.[3] In particular, consideration of the curves for extension of the steel and compression of the concrete indicates that no justification can be put forward for using the "n" method of calculation.[4]

The author, continuing his investigation of the correctness of his views, has carried out a further series of experiments on beams. In one set of these experiments beams with different amount of cover to the steel (from 2 to 5 cm) were made the subject of comparative tests, and beams with ordinary stirrups were compared with those having the stirrups inclined at 45°.[5]

The dimensions of the beams were b : h = 20 : 20 cm. The reinforcement consisted of three round bars of St. 37 of 10 mm dia. and the proportion of steel

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[1] *Stüssi:* The Safety of Simply Reinforced Rectangular Beams. Publications I.A.B.S.E., Vol. I, Zürich 1932.

[2] *Abeles:* Über die Verwendung hochwertiger Baustoffe im Eisenbetonbau. Beton und Eisen, 1935, Nos. 8 and 9.

[3] *Gebauer:* Berechnung der Eisenbetonbalken unter Berücksichtigung der Schwindspannungen im Eisen. Beton und Eisen, 1934, No. 9.

[4] *Gebauer:* Das alte n-Verfahren und die neuen n-freien Berechnungsweisen des Eisenbetonbalkens. Beton und Eisen, 1936, No. 2.

[5] *Gebauer:* Vergleichsversuche über den Einfluß der Dicke der Eisenüberdeckung und den Einfluß der Bügellage auf das Tragvermögen von Eisenbetonbalken. Beton und Eisen, 1937, No. 8.

was 0.59 %. The cube strength of the concrete was between 416 and 425 kg per sq. cm; the elastic limit of the round steel bars was between 2859 and 2959 kg per sq. cm, and the span of the beam 2.00 m. In the case of the beams of 22 cm total depth carrying two isolated loads at 80 cm centres, the average breaking load was 5.725 tonnes, and in that of the beam of 25 cm total depth it was 6.06 tonnes; taking account also of the shrinkage stresses in the reinforcement the calculated breaking loads work out at 5.70 and 5.93 tonnes respectively. Taking no account of the shrinking stresses in the steel and of tensile stresses in the concrete, but having regard only to the actual dimensions of the beams, the calculated breaking loads were between 4.50 and 4.57 tonnes. Whereas the actual breaking loads differ from those calculated by the first method by only 0.4 or — 2.1 %, using the latter method of calculation they differed by — 21 and — 25 % respectively. Calculating with the aid of the "n" metod, the elastic limit of the steel bars should be attained under a load of 4.05 tonnes regardless of the depth of cover, and the difference by comparison with the actual breaking load amounts in this case to between — 29 and — 33 %.

Using the "n" method the depth of the compression zone amounts to $x = 6.82$ cm, whereas in the trial beam the cracks extended to within about 1 cm of the compression face. According to the method of calculation which does not involve "n" the calculated compression depth works out $x = 0.82$ cm.

The Steuermann method of calculation,[6] which takes no account of the depth of cover to the steel and assumes a triangular compression diagram for the concrete, likewise implies considerably greater depths of the compression zone than appear from the bending test of the beam: for instance with $\sigma_{bz} = 25$ kg per sq. cm, $x = 2.66$ cm and the breaking load is 6.27 tonnes. Since in this instance the tensile strength of the concrete was not ascertained no more accurate comparison could be made.

In these cases[6] the shapes of the elongation curve for the steel and of the compression curve for the concrete show particularly well that the "n" method cannot be regarded as a proper method of calculating either the breaking condition or, still less, the stresses that arise under working loads.

The author carried out a further series of tests on experimental beams with a view to examining the effect of exceptionally heavy reinforcement.[7] Three pairs of beams were tested containing respectively 3.14, 4.91 and 6.53 % of steel. The dimensions were b : h = 20 : 20 cm, total depth 25 cm, span 2.00 m. The reinforcement was of St 37, namely four round bars of 20 mm dia, four of 25 mm dia, and, in the last example, three round bars of 30 mm with one of 25 mm dia. To prevent the beams failing prematurely through shear stresses their end portions were furnished with heavy inclined stirrups in addition to the bent-up main bars. The elastic limit of the reinforcing steel was 2.580 kg per sq. cm without any notable deviation. One beam from each pair was tested after four weeks and the other after six weeks. The concrete strengths at four weeks

[6] *Steuermann:* Das Widerstandmoment eines Eisenbetonquerschnittes. Beton und Eisen, 1933, Nos. 4 and 5.

[7] See also *Gebauer:* „Neue Balkenversuche zur Klärung der Schwindspannungsfrage und des Verhaltens von Balken bei außergewöhnlich starken Bewehrungen." Monatsnachrichten des österr. Betonvereins 1937, Heft 5.

amounted respectively to 466, 458 and 410 kg per sq cm and after six weeks to 473, 512 and 514 kg per sq cm. The breaking loads on the beams (stated in the same sequence as above) were 22.0 and 22.0 tonnes; 28.9 and 29.9 tonnes; and 32.9 and 36.0 tonnes. The decisive part played by the concrete strength is easily recognisable in the breaking loads.

Using the method of calculation which take no account of "n" but which is based on the elastic limit of the steel, on the cube strength of the concrete and on the assumption of a uniform distribution of compressive stress with (or without) taking account of the shrinkage stresses, the breaking loads in the several beams after hardening for four weeks work out at 21.5 (20.0) tonnes, 30.8 (28.7) tonnes and 33.1 (30.7) tonnes. The corresponding loads after six weeks hardening are 22.9 (20.4); 32.8 (29.7); and 40.4 (37.1) tonnes.

Comparison between the calculated and the experimental results shows that in the case of the beams reinforced with 3.14 % of steel, a better agreement is obtained when the shrinkage stresses are taken into account than when they are ignored, but generally speaking the discrepancies in the case of beams containing more than 4 % of reinforcement are not large, whether the shrinkage stresses have been considered or not. For beams containing 4.91 % and 6.53 % of reinforcement the experimental results approximate to those found by calculation regardless of the shrinkage stresses, though if the latter are taken into account the difference amounts to 12.2 % in the case of only one of the beams (N° 64). Hence the tolerance of 10 % which is usually regarded as acceptable is only slightly exceeded. This deviation of 12 % is easier to explain in view of the uncertainty which attends the calculation of shrinkage stresses in any case, and of the difficulty of constructing heavily reinforced beams in which the spaces between the reinforcing bars are very narrow. Moreover a yielding of the concrete at the end hook was observed to occur immediately before the actual breakage, so that the full resisting moment of the beam could not be developed.

From the experiments hitherto carried out it may also be inferred that where the reinforcement is particularly heavy the shrinkage stresses exert a smaller influence because of the smaller proportion between the circumference and the area of the cross section of the bars; on the other hand thinner reinforcing bars have a proportionately larger area of contact and with these the effect of shrinkage is consequently greater.

Supported by the experimental results explained above, the author has advocated the abandonment of the "n" method before the Second International Congress on Bridge and Structural Engineering in Berlin. It is to be noticed that Prof. *Saliger*, also, has taken up this point of view in the Preliminary Report of the Congress, though he has left the question of shrinkage stresses out of account and instead of using the cube strength has worked on the prism strength of the concrete which is about one quarter lower, with the result that calculation gives breaking loads somewhat lower than are determined in these experiments.

IIa 4

The Behaviour of Concrete and Reinforced Concrete under Sustained Loading.

Das Verhalten von Beton und Eisenbeton unter dauernder Belastung.

Comportement du béton et du béton armé sous l'action des charges permanentes.

R. Dutron,

Directeur du Groupement professionnel des Fabricants de Ciment Portland Artificiel de Belgique, Bruxelles.

This short contribution will deal only with the effect of sustained loading on reinforced concrete constructions, on the basis of numerous data obtained experimentally in the laboratory regarding the behaviour of concrete and reinforced concrete under compressive, tensile and bending loads maintained in action over a period of two to three years. The slow changes of shape which result from shrinkage and expansion will be considered together with those attributable to sustained loading.

Special emphasis will be laid on the importance of the conditions under which the concrete is stored, for if the deformations and other properties of the concrete are to be recorded numerically account must be taken of whether the structure is immersed or buried, or is exposed to the weather and seasonal changes, or whether it is under cover or heated during a great part of the year.

The strength R_b, the modulus of elasticity E_b, the plastic strain under sustained loading, and the amount of shrinkage, all vary considerably according to these conditions of exposure. The following are relative average values for concrete stored under permanent conditions for three years.

	under water	in air: relative humidity 70 %	in air: relative humidity 45 to 50 %
R_b	1.00	0.75	0.60
E_b	1.00	0.80 to 0.85	0.65 to 0.70
Plastic strain	1.00	2.00 to 2.25	3.00 to 4.00
Shrinkage	+ 1.00	—3.50 to —4.50	—5.00 to —6.00

It must be understood that numerical values for the properties of concrete also vary a great deal in accordance with such well-known factors as the proportions of the mix, the granulation and the age of the concrete.

Once plastic deformation has taken place under the action of the dead load and permanent live load, everything proceeds as if the value of the modulus of elasticity were reduced, and (as is well known) one of the results of this condition is a corresponding change in the distribution of stress between the concrete and the reinforcement. This change takes place slowly, and, as is true of the strain, it tends in the course of time towards a limiting value.

In reinforced concrete *compression members,* stored in dry air and loaded to between 22 and 24 % of the cube strength of the concrete, the compressive stress in the reinforcing bars may reach 15 to 20 kg per sq. mm, or 19 to 27 kg per sq. mm if the compression due to shrinkage is added. When the stress in the concrete amounts to between 30 and 32 % of the cube strength, stresses of 20 to 30 kg per sq. mm may arise in the reinforcing bars under certain conditions of testing in dry air, and when augmented by the compression due to shrinkage they may considerably exceed the elastic limit of mild steel.

In *members subject to bending* the compression zone may behave in a similar way to the above, and in dry air the compressive stresses in the reinforcement, including those due to shrinkage, may in exceptional cases approach the elastic limit of mild steel. In the tensile zone, however, the increase in stress of the reinforcing bars is relatively small, and consequently the lever arm of the resisting couple is not greatly reduced despite the plastic strain undergone by the concrete.

It is of interest to note that the compression in tensile reinforcement due to initial shrinkage was found to have disappeared during the long period that the bending load was maintained in being, and a similar observation has been made on bars embedded in specimens of reinforced concrete exposed permanently to simple tensile and compressive loads.

In all the beams subjected to bending (concrete at 60 kg per sq. cm, steel at 12 kg per sq. mm, $m = 15$) while permanently exposed to dry air, the cracks in the concrete under tension appeared as a result of the shrinkage stresses of the concrete while the load was being applied, and the cracking continued to increase while the load was maintained, though the cracks did not open at all conspicuously.

After long periods under load, neither the compressive nor the tensile strength of plain concrete, nor the compressive nor the bending strength of reinforced concrete, was found to be less than the strength of the corresponding members stored under the same conditions without having been subjected to the loads. Once the permanent strains had taken place the elastic character of the reinforced members continued to be manifested after repeated loading and unloading which followed upon two or three years of maintenance under permanent load.

The conclusion may be drawn that the strength of reinforced concrete is not reduced by its being kept under heavy permanent loads during a very long period. It does not appear that a lower breaking stress in the concrete need be assumed to meet such conditions, nor does the usual coefficient of 28/100 need to be diminished. Less importance attaches to the elastic limit of the steel being exceeded in the case of compressive reinforcement than in that of tensile reinforcement, but it would, nevertheless, appear desirable to make use of high elastic limit steels in the compression zone of the concrete under special conditions, where the magnitude of the permanent load and the conditions to which the structure is exposed are liable to cause large plastic strains in the concrete with the passage of time, and where, consequently, there is a risk of excessive stresses in the reinforcing bars. In such a case special attention should be paid to the effectivences and the spacing of the cross stirrups, and the danger of cracking should be the object of special care.

IIa 5

Effect of Plasticity of Concrete and Steel on the Stability and Endurance of Reinforced Concrete.

Der Einfluß der Plastizität des Betons und des Stahles auf Stabilität und Dauerhaftigkeit des Eisenbetons.

Rôle de la plasticité du béton et de l'acier sur la stabilité et la durée du béton armé.

R. L'Hermite,

Directeur Adjoint des Laboratoires du Bâtiment et des Travaux Publics, Paris.

Before investigating how the endurance of structures is affected by the plasticity of their materials — particularly as regards reinforced concrete — it may be well to consider exactly what is meant by "plasticity" and what influence this property has on variations in stress. In an earlier paper the author has attempted, by means of a simple mathematical theory, to show that the magnitude of elastic and plastic strains depends not only on the momentary load but also on antecedent conditions other than loading. The theory cannot pretend to finality, but consists merely of a series of syllogisms which derive from certain simple experimental premises and which lead to conclusions that are difficult to establish experimentally.

If a sample of steel be subjected to a load in excess of its elastic limit the steel will suffer a permanent deformation and if the load remains constant the deformation will continue to increase, to a greater or less extent, with the passage of time, according to a law of creep which depends on the quality of the steel and on the temperature. This creep, while extremely small in the case of loads close to the creep limit, is nevertheless not zero. If the load fluctuates between two definite limits the permanent deformation grows very appreciably in course of time, and if the upper limit of the varying load exceeds the critical fatigue load (or natural limit of elasticity as defined by *Bauschinger*) this phenomenon takes place even below the limit of elasticity (yield point]: it is the result of an exchange of energy which arises from the elastic and plastic hysteresis between the elastic and plastic strains. A rapidly applied load may, therefore, produce contrary effects according to its mode of application: thus a single shock may cause diminution of the plasticity, whereas repeated loading, sustained vibration or slowly applied loads may bring about an increase in this quality.

The practical importance of this phenomenon appears when alternating or pulsating loads are applied to reinforced concrete members which have been pre-stressed in accordance with the system of *Freyssinet.* When this occurs the steel bars may be observed gradually to extend, and at the same time the pre-imposed compression in the concrete is reduced; in certain cases, where the amount of the pre-stressing is small by comparison with the load applied, cracks may appear in the concrete under tension. The destructive effects produced by the repeated loadings on the steel bars becomes greater in proportion as the bars are irregular, embrittled or rusted; and the fatigue limit is much lower for the hook at the end of a bar than for the straight portion.

There appears to be no method, other than experiment, of determining the stress-strain curve for a sample of concrete beforehand, for this material has no fixed elastic limit; the latter varies according to age and depends on the rate at which loads are applied. Everything which has been said above regarding the plasticity of steel is applicable even more strongly to concrete. The constants of hysteresis which define the plastic and elastic viscosity are small, and the amount of such hysteresis large. Hence, in calculations relating to reinforced concrete, the concept of a "coefficient of elasticity" has no meaning unless it is associated with constants which define the conditions of plasticity, creep and hysteresis. That is why agreement has never been reached as to the proper value for the modular ratio m.

The effect of accelerated creep under repeated loading is found also in concrete, to a very high degree. The phenomenon of plastic creep is attended by phenomena of irreversible friction; these are additive, and tend to hasten adaptation because of the effect of bond on the elastic strain, to which the author has referred in his earlier paper.[1] Moreover this process of adaptation is associated with all those factors which usually accompany ageing: increased stiffness, increased strength, and reduced shrinkage, etc. The concrete is liable to exhibit all those phenomena of fatigue after repeated loading which occur in the metal. For instance, a concrete which has a breaking strength of 350 kg per sq. cm and is subjected to loads varying 500 times a minute between 50 and 300 kg will break at the end of one hour; during this time its modulus of elasticity will have changed and the length of the specimen will have decreased. For any such specimen there is a fatigue limit which determines the pulsating load that will cause breakage after a limited number of alternations; but below this limit, on the other hand, the effect of repeated loading is to bring about an increase in the statical strength.

A number of experiments on the behaviour of bent beams subjected to repeated loads have been carried out in the laboratoires[2] in Paris to which the author is attached, and it has been observed that even in these cases there exists a characteristic fatigue limit, so that by making successive experiments on a series of similar beams it becomes possible to construct a *Wöhler* curve in which the first limb is much more steeply inclined than would be the case for concrete or steel by itself. Finally, it has been noticed that the principal effect produced by successive loadings was to accelerate the occurence of plastic strain, and it has been possible to devise a method of accelerated experiment for the study of the adaptation that takes place in a reinforced concrete member under load, the effects

[1] See Theme I.

[2] Laboratoires du Bâtiment et des Travaux Publics.

of the repeated loadings being practically equivalent to an artifical ageing of the work in question. This has led to the observation that adaptation does not take place equally in the parts under tension and those under compression. Again, it appears that the fatigue limit in relation to the static breaking load is much lower for concrete in tension than for concrete in compression. Account must also be taken of those mutual forces between steel and concrete which constitute the bond: the experiments go to show that bond is very sensitive to repeated loadings, and a large number of beams failed through slipping of the bars, probably because the latter had been unable to adapt themselves to the deformations caused by plasticity. In yet other cases the stabilisation of the bar, after its first slip, brought about a considerable degree of cracking in the concrete without actually leading to failure of the member.

The upshot of these considerations is that any attempt to calculate the strain in a piece of concrete by referenec to elementary data must be a very complex matter, and can, in the present state of our knowledge, be attempted only as a rough approximation. When all is said and done, the scope for the occurrence of adaptation appears very great, and however rough this approximation may be its effect is to suggest that when the earliest designers of reinforced concrete introduced the idea of partial continuity they came nearer to the truth than do all the hyperstatical calculations which have since been developed.

IIa 6

The Behaviour of Reinforced Concrete Framed Structures at Incipient Failure.

Das Verhalten von Eisenbeton-Rahmenkonstruktionen bei beginnender Zerstörung.

Comportement des portiques en béton armé à l'amorce de la rupture.

W. H. Glanville, D.Sc., Ph.D., M.Inst.C.E., M.I.Struct.E.

and

F. G. Thomas, B.Sc., Assoc.M.Inst.C.E., Garston.

At working stresses it is probable that the distribution of moments throughout a reinforced concrete framework is reasonably well given by calculations according to the elastic theory. It has been shown by prolonged loading tests at the Building Research Station[1]* and in America[2] that the creep of concrete at working stresses has no important effect on the moment distribution in a frame.

When incipient failure is reached at any part of the structure, however, the inelastic movements of either the steel at its yield point or of the concrete near its ultimate strength are so large that the elastic theory no longer holds. Deformation of the affected part is limited by the movements of the rest of the structure so that collapse of this part may not occur until considerable elastic deformation has occurred elsewhere. That is, further load can be carried by the structure without collapse, the maximum stress at the affected part tending to remain practically constant whilst the moments and stresses at other parts increase. For convenience the change in the distribution of bending moments from that in a purely elastic framework will be called "redistribution of bending moments" in this paper.

Tests on two-span continuous beams by *Kazinczy*[3] have shown that when steel is the deciding factor for failure, variation of the amount of steel in the span or over the central support from that required by the elastic theory leads to redistribution of moments such that the full strengths of both the span and support sections are reached. Similar results were obtained for built-in beams by the German Reinforced Concrete Committee[4] for the condition of failure due to steel yield. Such redistribution is to be expected because of the large inelastic deformation of steel at its yield, but the extent to which it can be relied upon without causing concrete failure is unknown. No previous tests are known in which the effect of inelastic deformation of the concrete at incipient failure on the ultimate strength of a framework has been studied.

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* These figures relate to the list of references at the end of the paper.

The tests described in this paper were part of an investigation undertaken at the Building Research Station in conjunction with the Reinforced Concrete Association to obtain definite information on the importance of inelastic deformations at highly stressed parts of a reinforced concrete framework. The ivestigation included tests to destruction on, A, two span continuous beams and B, portal frames.

A. Tests to Destruction on Two-span Continuous Beams.

Tests were carried out on two-span continuous beams designed as follows:

1) With weakness over the central support due to the use of a low percentage of tension steel.
2) With weakness over the central support due to the use of a low strength concrete without compression reinforcement.
3) As (2) except that compression reinforcement was provided.
4) As (2) but with an increased span length in order to reduce shear stresses.
5) As (2) except that a low strength concrete was used at an age of about 6 months instead of 7 days.

All tests were made in duplicate, and river aggregates were used throughout.

1) *Primary Failure in Tension Steel.*

Details of the beams and the positions of the loading used in the tests to determine the effect of using insufficient steel when calculated according to the ordinary elastic theory are given in Figure 1.

The notation used in the table of Figure 1 and subsequent tables of stresses is as follows:

t denotes the stress in longitudinal tension reinforcement.

t' denotes the stress in longitudinal compression reinforcement.

M denotes the bending moment.

n denotes the depth of neutral axis.

a denotes the arm of the resistance moment.

S denotes the total shear.

s denotes the shear stress.

t_w denotes the stress in web reinforcement.

s_b denotes the bond stress.

W denotes the Load.

ξ_B denotes the Distance of point of inflexion from B.

ξ_F denotes the Distance of point of inflexion from column face.

s_E denotes the Bond at E (lower bars).

R_A, R_B, R_C denotes the reactions at A, B and C.

It will be seen that over the central support where the moment is normally greatest, there are only two $^3/_8$ in. diameter bars whereas in the span four $^5/_8$ in. diameter bars are provided. At quite a low load therefore, the yield point stress of the $^3/_8$ in. diameter bars would be expected; it would be anticipated that yield of these bars would lead to a redistribution of moments whereby the section over the central support would be continuously relieved, enabling the load carried by the system to be further increased until failure in the span.

The actual moments during the tests were determined by measuring the strain in the supporting steel joist at a fixed distance from the end supports and hence

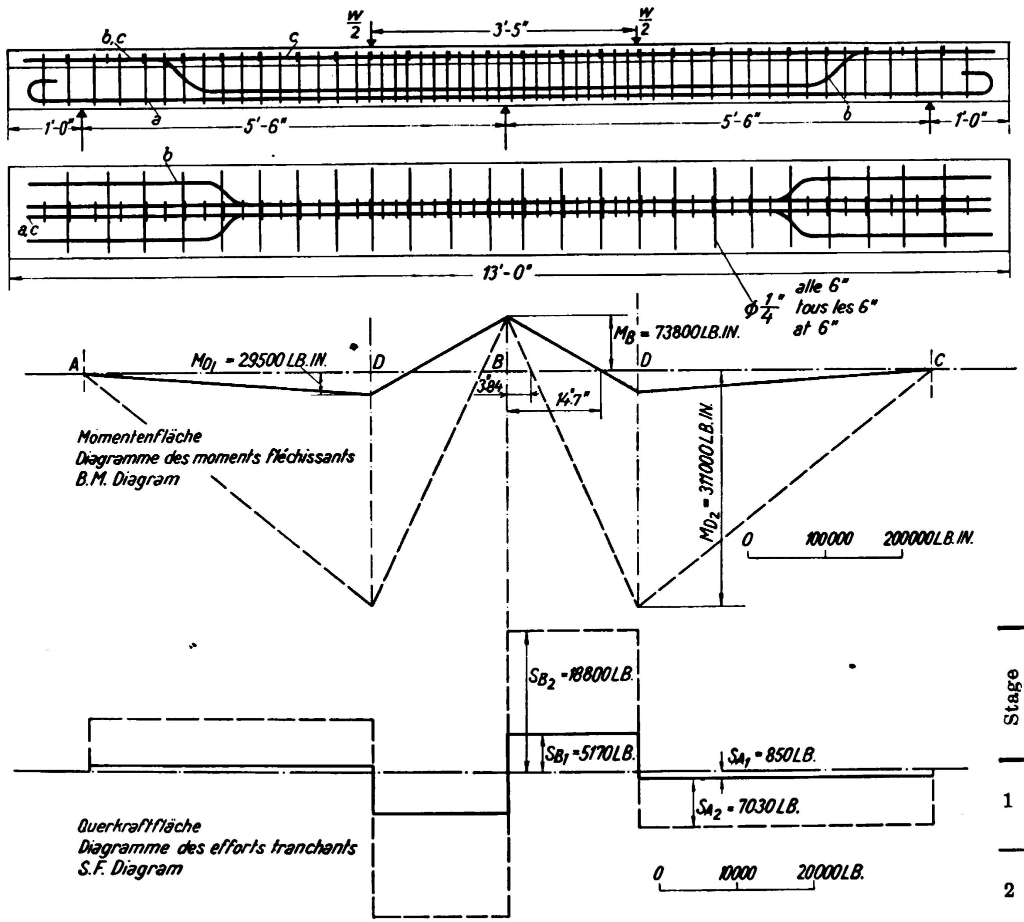
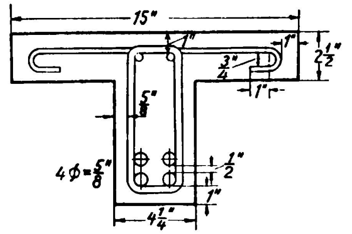


Fig. 1.

Redistribution of moments in continuous beams.
Steel failure.

R.M.2.



Eisen / Armature / Bar	ϕ	No.	Form - Forme
a	5/8"	2	12'-6"
b	5/8"	2	1'-9" 9" 7'-6" 9" 1'-9"
c	3/8"	2	12'-6"

Eisenliste - Liste des Fers - Bar schedule

Calculated Stresses (lb. inch units)

Stage	at B t	at B c	at B M	at B n	at B a	at B S	at B s	at B t_w	at B s_b	at D t	at D c	at D M	at D n	at D a	at D s_b	at A S	at A s	at A t_w	at A s_b	W	ξ_B	R_B	$R_A = R_C$
1	44700	2545	73800	1.85	7.47	5170	162	6255	293	3740	285	29500	2.09	6.40	103	852	28	2170	30	10980	14.7	10340	852
2			73800			18800	592	22700	1073	39100	3000	311000			374	7030	231	17800	251	50750	3.94	37600	7030

Dead Load Effects

$M_B = 3800$ lb in $R_A = 204$ lb

$M_D = 0$ lb in $R_B = 476$ lb

$u = 8000$ lb/in² $m = 5.5$

calculating the end reactions from a previous calibration of the joist. The results for one of the two beams tested are shown in Figure 2.

Incipient failure over the central support is clearly indicated by a sudden decrease in moment at that point, after which the moment increased somewhat.

On the assumption that the central support moment remains constant after yield begins, the subsequent moments in the span have been calculated and the theoretical curves are shown on the diagram. It is evident that the assumption leads to a very fair estimate of the actual span moments for the present test.

The concrete used for this test was made in the proportions 1:1:2 (by weight) using rapid hardening Portland cement, and the beam was tested at an age of 44 days. For the second beam a high alumina cement 1:2:4 concrete (by weight) was used, and the beam was tested at an age of 6 days. In the second beam, as a result of the higher tensile strength of the high alumina cement concrete, the help afforded by the concrete in tension was such that the stress in the continuity steel increased from a very low value to its yield point value at the occurrence of the first crack over the support. Apart from this effect, there was apparently no important difference in behaviour, resulting from the use of the two types of cement.

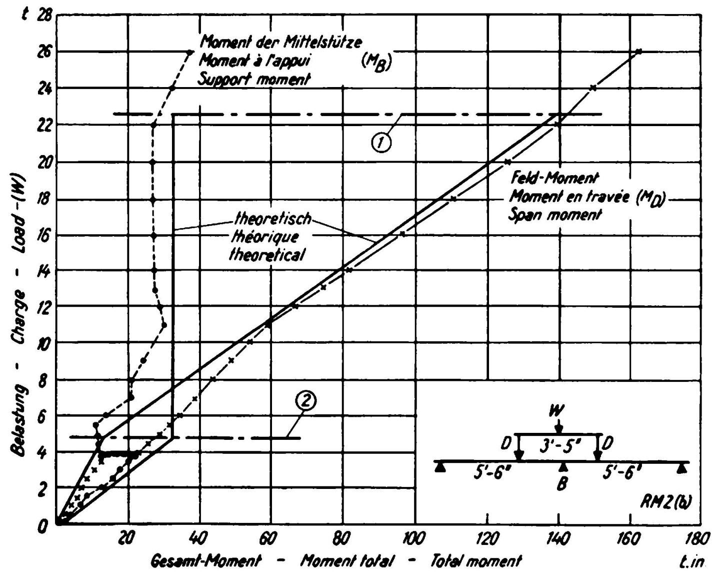


Fig. 2.

Tests on continuous beams. Steel failure (b).
Rapid-hardening portland cement 1 : 1 : 2 concrete (by wt.)
Water/cement ratio = 0.44 (by wt.). Age at test — 44 days.
Cube strength of concrete = 6660 lb. per sq. in.
(1) Theoretical load for general failure.
(2) Theoretical load for support failure.

The deflections at midspan relative to the central support were measured throughout by means of dial gauges. There was no appreciable difference between the deflections of the two beams, and at three-quarters of the failing load the maximum deflection was only about 0,1 in. The supporting steel joist deflected during the test and the sinking of the end supports relative to the central support was therefore also measured. This sinking affects the moments during the elastic stage of the test and has therefore been taken into account in calculating the theoretical curves and stresses given in Figures 1 and 2.

The maximum crack widths, measured with a portable microscope, are given in Table 1. The cracking over the central support increased considerably during the second part of the test, i. e. after the steel had commenced to yield, and shortly before final failure the cracks were from 0.06 to 0.08 in. wide. These cracks are approximately ten times the width usually observed just before the commencement of steel yield.

The loads calculated for failure, (1) on the elastic theory, and (2) on the basis that both the support and span sections develop their full strengths after redistribution, are given in Table 2 together with the actual failing loads. It will be seen that the effect of the redistribution of moments on the load carrying capacity of a continuous beam may be considerable in cases of steel weakness over the central support. However, the cracking accompanying the increased load is very marked, so that in practice, advantage can be taken of moment redistribution due to steel yield only in cases where the increased cracking is not a matter of importance.

Table 1.

Maximum Crack Widths in Continous Beams.

Series	Load — tons	Maximum Crack Width — inch × 10⁻³: Over Central Support 5	10	15	20	25	Yield[1]	In Span 5	10	15	20	25	Yield[1]
1. Steel Failure	(a)	0	15	30	42	60	0	0	1.3	2.3	2.6	3.8	0
	(b)	6	15	34	55	79	5	0	1.5	2.6	4.6	6.6	0
2. Concrete Failure (No compression steel)	(a)[2]	0	1.5	2.4	3.1	3.3	0.5	0	1.9	3.5	6.0	10.0	0.6
	(b)	0	1.3	2.2	2.6	2.6	1.2	0	1.3	2.2	3.3	3.9	0.7
3. Concrete Failure (With compression steel)	(a)	1.0	3.1	3.7	4.6	5.5	3.4	0	0.9	1.6	2.4	3.5	1.3
	(b)	1.6	4.0	5.2	9.2	10.5	4.8	0	1.3	1.7	2.6	5.2	1.5
4. Concrete Failure (Increased span length)	(a)	3.3	3.7	—	—	—	1.6	1.5	4.0	—	—	—	0.8
	(b)	0.1	1.0	—	—	—	0	1.3	4.2	—	—	—	0
5. Concrete Failure (Weak concrete at about 6 months)	(a)	0	1.6	2.7	2.6	1.5	—	0	1.3	2.5	3.6	5.0	—
	(b)	0	0.7	1.0	1.1	1.2	—	0	1.4	2.4	3.5	7.2	—

[1] The yield load is the theoretical load for support failure according to the elastic theory (see Table 2).

[2] The maximum crack widths in beam (a) of series (2) was measured at the depth of the most highly stressed edge of the tension steel; in all other beams the measurements were at the depth of the centre of the most highly stressed bar.

2) *Primary Concrete Failure. No Compression Steel Provided over Central Support.*

In the beams designed to fail by crushing of the concrete, all the tension reinforcement in the span was taken up over the support so that the compression at that point was taken wholly by the concrete in the rib. Details of the beams, spans and positions of loads are given in Figure 3. The concrete was made with ordinary Portland cement using a $1:2^1/_2:3^1/_2$ mix (by weight) and a water-cement ratio of 0,66 (by weight). Te tests were made at an age of 7 days, the strength aimed at being the lowest (2250 lb. per sq. in.) allowed by the Rein-

Table 2. Failing Loads of Continuous Beams.

Basis of Bending Moment Calculations	Basis of Resistance Moment Calculations		Failing Loads — tons: 1. Steel Failure		2. Concrete Failure (No compression steel)		3. Concrete Failure (Compression steel)		4. Concrete Failure (Increased span length)		5. Concrete Failure (age $5^1/_2$ months)	
		Test No: —	RM 2 (a)	RM 2 (b)	RM 1 (a)	RM 1 (b)	RM 3 (a)	RM 3 (b)	RM 4 (a)	RM 4 (b)	RM 5 (a)	RM 5 (b)
Elastic theory: i. e. No redistribution of moments. Loads are for support failure	No Stress Redistribution	True "instantaneous" modular ratio used	4.9	4.9	7.0	7.2	13.0	14.2	2.7	2.3		
	Stress Redistribution	$m = \frac{40000}{\text{cube strength}} = \frac{40000}{u}$	5.0	4.9	7.6	7.8	19.5	19.8	3.0	2.5		
		Steel Failure: Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	7.8	6.5	8.0	8.2	25.4	26.2	3.2	2.7		
Theory of Redistribution of Moments: i. e. Simultaneous failure at central support and in span	No Stress Redistribution	True "instantaneous" modular ratio used	22.7	22.6	20.8	21.4	25.7	28.1	9.8	8.6		
	Stress Redistribution	$m = \frac{40000}{u}$	23.0	22.6	27.8	28.5	35.0	36.3	13.0	11.8		
		Steel Failure: Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	26.1	24.0	32.6	32.8	40.1	40.5	14.2	13.9		
Actual load at which signs of distress were first noticed in the concrete			—	—	20.8	24.0	23.0	24.0	9.0	9.5	18.8	16.5
Actual ultimate load carried by beam			29.1	28.7	27.5	28.6	27.6	28.9	13.4	13.0	33.0	27.5

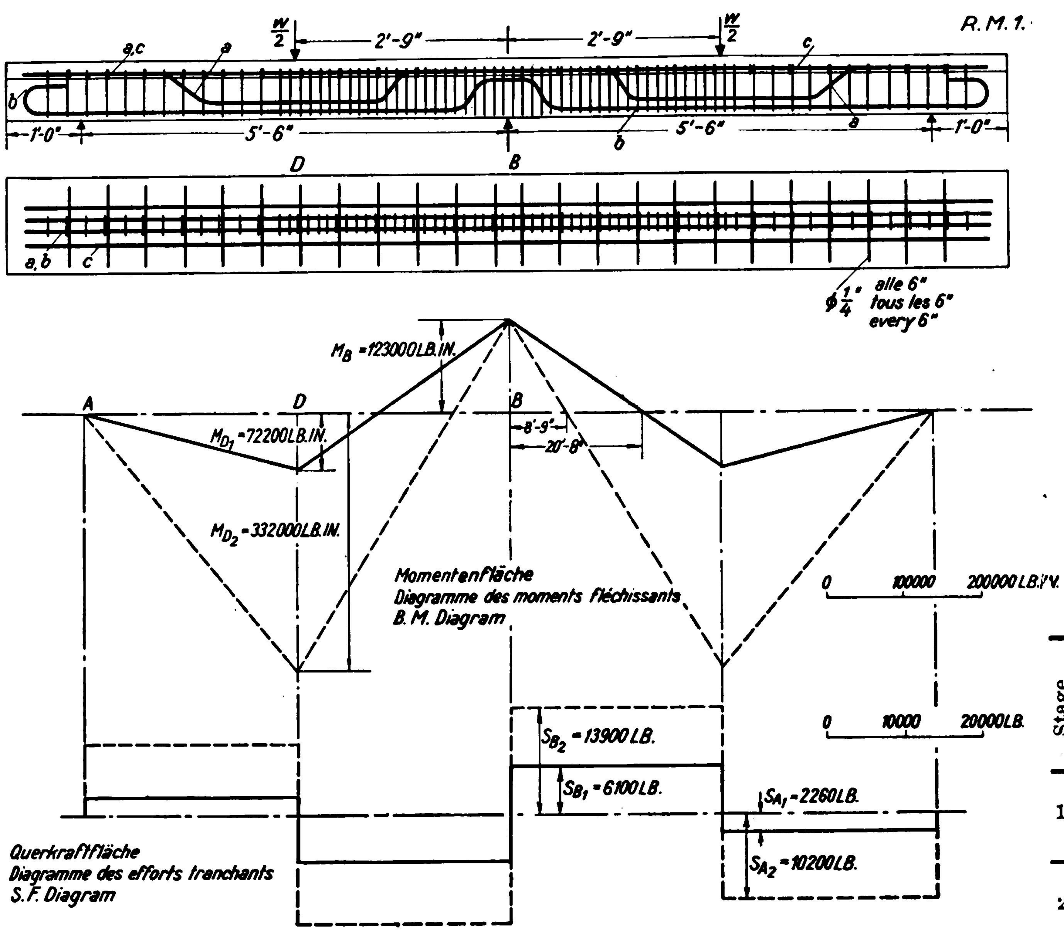
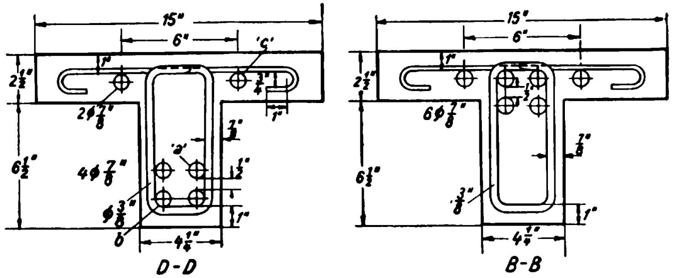


Fig. 3.

Redistribution of moments in continuous beams.
Concrete failure.



Eisenliste - Liste des fers - Bar schedule			
Eisen Armature Bar	ϕ	No.	Form - Forme
'a'	7/8"	2	1'-9" – 2'-3 1/2" – 9 1/2" – 2'-8" – 9 1/2" – 2'-3" – 1'-9"; 7"; 3 1/2"
'b'	7/8"	2	5'-8" – 9" – 9" – 5'-8"; 1 6 1/2"
'c'	7/8"	2	12'-6"

Calculated Stresses (lb. inch units)

Stage	at B									at D						at A				W	ξ_B	R_B	$R_A = R_C$
	t	c	M	n	a	S	s	t_w	s_b	t	c	M	n	a	s_b	S	s	t_w	s_b				
1	6400	2050	123000	5.35	5.32	6100	270	7800	70	5180	445	72200	3.09	5.80	35	2260	80	4660	63	15800	20.8	12200	2260
2			123000			13900	615	17700	159	23800	2050	332000	3.09	5.80	159	10200	366	21000	283	47300	8.9	27800	10200

Dead Load Effects

M_B = 4120 lb in	R_A = 200 lb
M_D = 910 lb in	R_B = 480 lb
u = 2050 lb/in²	m = 9,5

forced Concrete Code of Practice. Actually the strength was about 10 per cent. less than this value (see Appendix 1).

In order to reduce the shear stresses with this weak concrete, the loads were applied at midspan, instead of nearer to the central support as in the case of the previous beams.

The results are given in Figure 4. It will be noticed that there is not such a well defined point at which failure over the support commences as in the case of the previous beams in which the steel yielded, but rather a gradual change from the elastic to the inelastic stages of the test.

The concrete at the support continued to carry load in an apparently undistressed condition long after the load calculated to produce a fibre stress equal to the cube strength had been reached. In fact there was no evidence of crushing over the central support until the load was more than twice this value.

The measured span moments were again in fair agreement with those calculated on the assumption of a constant support moment after passing the elastic stage.

Throughout the test the crack widths were small (see Table 1) so that redistribution of moments in the case of concrete weakness may be considered without reference to cracking. The beam deflections were of the same order as those measured in the previous series.

3) *Primary Concrete Failure. Compression Steel Provided over the Central Support.*

In tests designed to show weakness in compression in the presence of a limited amount of compression steel the reinforcement was the same as in the previous beams except that the lower bars were continuous throughout the beam, thus providing help in compression over the central support. The concrete mix used was again $1 : 1^1/_2 : 3^1/_2$ (by weight) using ordinary Portland cement, and the tests were made at an age of 7 days; the strength (see Appendix 1) was a little higher than that obtained in the previous tests.

The moments throughout the system were measured, and it was again found that there was a gradual change between the two stages of the test, and it is interesting that the final loads attained (see Table 2) were almost exactly the same as for the beams in which no compression reinforcement was provided.

There was no evidence of compression failure over the central support until just before final collapse of the system. The main tensile crack at that section gradually closed towards the end of the test until it extended only about 2 in. from the top surface of the beam, indicating that the whole of the rib and even some of the flange was bearing compression forces.

The maximum crack widths are given in Table 1.

4) *Primary Concrete Failure. Beams with Increased Span Length.*

The beams in series (2) were provided with closely spaced stirrups over the central support in order to avoid shear failure with the weak concrete used. It was suggested that this reinforcement gave lateral support to the concrete, thus increasing its ability to carry longitudinal compression. In order to show whether this was the case, two further beams were prepared similar to those of series (2) except that the span length was increased to 12 ft. so that the failing moments would be reached at a lower load, hence reducing the amount of shear reinforcement required.

The results showed conclusively that the central support section was not weakened by the wider spacing of the stirrups. The percentage increase in load due to redistribution was approximately the same as before (see Table 2), and the support moment carried at failure was actually greater than had been obtained in the previous tests of series (2).

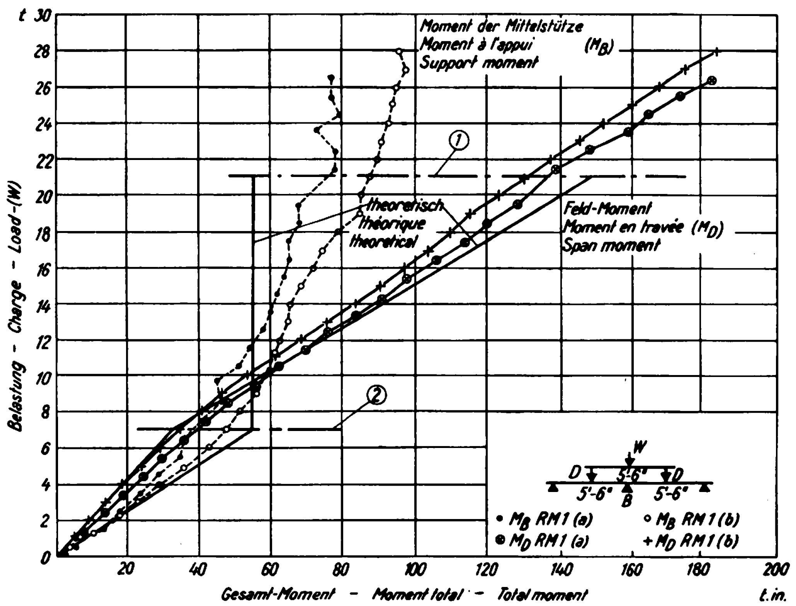


Fig. 4.

Tests on continuous beams. Concrete failure (No compression steel). Normal Portland cement $1:2^1/_2:3^1/_2$ concrete (by wt.) Water/cement ratio = 0.66 (by wt.) Age at test — 7 days. Cube strength of concrete = 2050 lb. per sq. in.

① Theoretical load for general failure.

② Theoretical load for support failure.

5) *Primary Concrete Failure at an Age of $5^1/_2$ Months.*

The tests previously carried out with weak concretes were made at an age of 7 days in all cases, and although it seemed probable that the amount of redistribution that occurred as a result of inelastic deformation of the concrete would depend on the strength of the concrete rather than its age, it was thought advisable to test two beams similar to those of series (2) (no compression reinforcement) at a greater age. In order to obtain a low strength at about 6 months an ordinary Portland cement was used in a mix of proportions 1:4:7 (by weight) for the first beam; this was changed to 1:5:6 for the second to give a better mix with the same water-cement ratio of 1,05.

The failing loads, given in Table 2, were as great and in one case greater than those obtained previously. The concrete strength was, however, not known very accurately as the cubes cast with the beams could not be relied upon to give a fair estimate of the concrete quality in the beam itself for such a poor quality concrete. Samples were cut from the ends of the beams and tested, and the results indicated that if anything the concrete was somewhat weaker than that used for the earlier tests. There is no doubt, therefore, that the redistribution obtained with the richer concrete was not attributable to the fact that it had hardened for only a comparatively short period.

B. Tests on Portal Frames.

Tests were made to determine to what extent the load-bearing capacity of a simple reinforced concrete portal frame may be increased as a result of redistribution of stress and moment when high stresses are reached at the column head.

The conditions tested were:

1) Primary failure of the tension steel in the column.

2) Primary failure of the concrete in compression in the column.

For each condition two frames were tested.

1) *Primary Failure of the Tension Steel in the Column.*

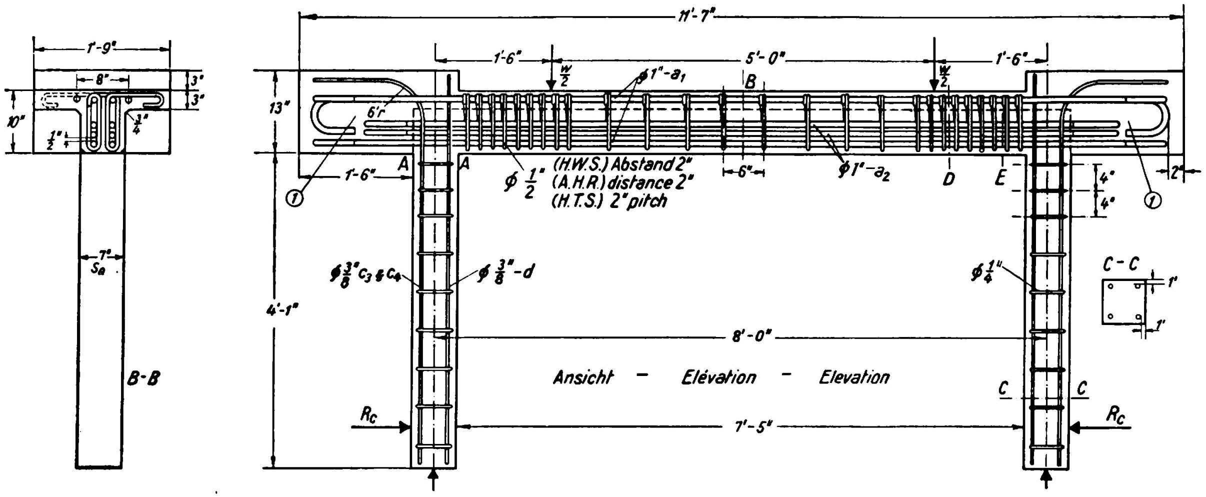
Details of the frames and the positions of loading are given in Figure 5. The design of the reinforcement and the method of loading was such that the beam was considerably stronger than the column. At incipient failure of the weak column there was a considerable reserve of strength in the beam.

In order to ensure that the frame should fail by bending, and not in shear or by slip of the bars, it was necessary to give special attention to the design of the shear reinforcement and the anchorage of the bars. It is clear that redistribution of moment can increase the ultimate load of a structure only when the conditions of bond and shear that result from such redistribution are amply provided for. The large blocks at the column-beam junctions were provided solely for the purpose of giving ample anchorage to the reinforcement of the beams and columns in order that the yield point of the steel could be reached.

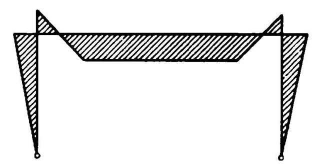
A high strength high alumina cement concrete was used for these tests; details of this are given in Appendix 2.

The horizontal load was applied by two helical tension springs stretched between the column feet, the load being transmitted to the column faces through knife edges. The load on the beams was applied through cylindrical bearings and rollers to allow free rotation and translation of the beam. In the first test the column feet were supported on similar bearings but it was found that the frictional force due to the rollers was sufficient to affect appreciably the horizontal spring load required to prevent outward movement of the feet, and a special knife edge link system was used for subsequent tests.

During the test, gauges were set up at the column feet to measure the movement outwards, and the horizontal load due to the springs was continually adjusted so that the feet were brought back to their original position. That is,



R.M.F.2. & R.M.F.3.



Momentenfläche (nicht masstäblich)
Diagramme des moments (non à l'échelle)
Moment diagram (not to scale)

Eisenliste Liste des fers Bar schedule			
Eisen Armature Bar	ϕ	Masse - Dimensions	Anzahl Nombre No. Off.
a_1	1"	7"; 11'-3"; 6"	6
a_2	1"	7"; 9'-11"; 6"	4
c_3	$\frac{3}{8}$"	4"; $2\frac{1}{4}$"; 18"; 5'-1"	2
c_4	$\frac{3}{8}$"	4"; $2\frac{1}{4}$"; 18"; 5'-1"	2
d	$\frac{3}{8}$"	5'-1"	4

Calculated Stresses (lb. inch units) R.M.F.3

Stage		Column						Beam at B			Beam at D				ξ_F	s_E	W	R_C
		c	t	M_A	S	s	s_b	c	t	M_B	S	s	s_b	t_w				
1.	I_1	4200	47300	102000	2380	65	190	920	7000	206000	17300	415	155	7600	2.6	190	34500	2380
	I_2	4800	47300	114000	2650	70	210	1200	9000	266000	21900	525	200	8700	2.3	242	43700	2650
2.		11000	47300	283000	6590	175	520	11000	40600	1240000	86000	2120	775	37600	0 2	—	172000	6590

$u = 11000$ lb. in² $m = 5$

I_1 Moments of inertia, for moment distribution calculations, based on whole area of Concrete Ignoring Steel.
I_2 Ditto, based on whole area of Concrete Including Steel.

All main bars in beam 1" dia.
Vertical cover 1".
(1) All hooks of internal dia. 4".
Length of straight 4".

Fig. 5.

Redistribution of moments in frames. Steel failure.

the conditions of restraint were those of a portal, position fixed and pin-jointed at the feet of the columns.

A view of one of the frames whilst the test was in progress is given in Figure 6. A special framework was arranged to prevent any rotation or lateral movement of the supporting beam relative to the upper loading beam, so that no torsional or lateral bending stresses should be set up in the columns.

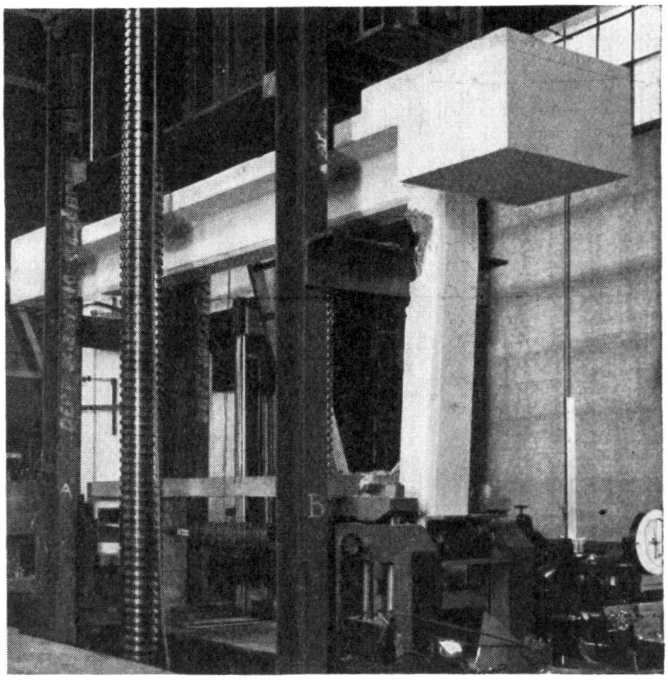


Fig. 6.
Test on reinforced concrete portal frame (Concrete failure).

The main results for the second test are shown in Figure 7. In this figure the applied loads are plotted against the horizontal reactions which are proportional to the moments at the column head, and on the same figure some theoretical curves are also given. One of these curves shows the load-reaction relationship expected for the frame from calculations based on the elastic theory; a series of curves are given for the relationship between the loads and reactions which produce steel yield on the following assumptions:

1) The "instantaneous" modular ratio determines the stress distribution,

2) the modular ratio is taken to be $m = \frac{40\,000}{u}$, and

3) the maximum concrete stress is assumed to reach the cube strength (u).

The point where the first mentioned curve intersects each of the steel failure curves determines the load at which the frame should have failed according to the elastic theory, with or without allowance for stress redistribution according to which assumption the curve represents. These loads are given in Table 3.

On the simplest theory of moment redistribution (i. e. assuming that the column tension steel remains continuously at its yield point) the horizontal reactions and therefore the moments will, after yield of the column steel, conform to the relationship shown by one of the steel failure lines in Figure 7, according to the amount of stress redistribution that occurs. The experimental results gave horizontal reactions which were initially somewhat lower than expected, redistribution beginning at quite a low load, soon after the appearance of cracks at the column head. The curve showing the experimental results gradually approaches the steel failure lines as the load is increased and crosses the line based on $m = \frac{40\,000}{u}$. Incipient concrete failure caused a sudden drop in the rate of increase in moment, and finally failure was reached as a result of concrete crushing.

The beam failure lines indicate the values for the applied load at which beam failure would occur for the degrees of fixity afforded by the various horizontal reactions and it will be seen that if the concrete in the column had not failed a slight increase in load could have been obtained before beam failure.

Throughout the test, measurements were made of the strains at the column heads. The strains were measured on the faces of the column; no direct readings were taken on the steel itself, the steel strain being deduced on the usual assumption that plane sections remain plane. This assumption will probably not

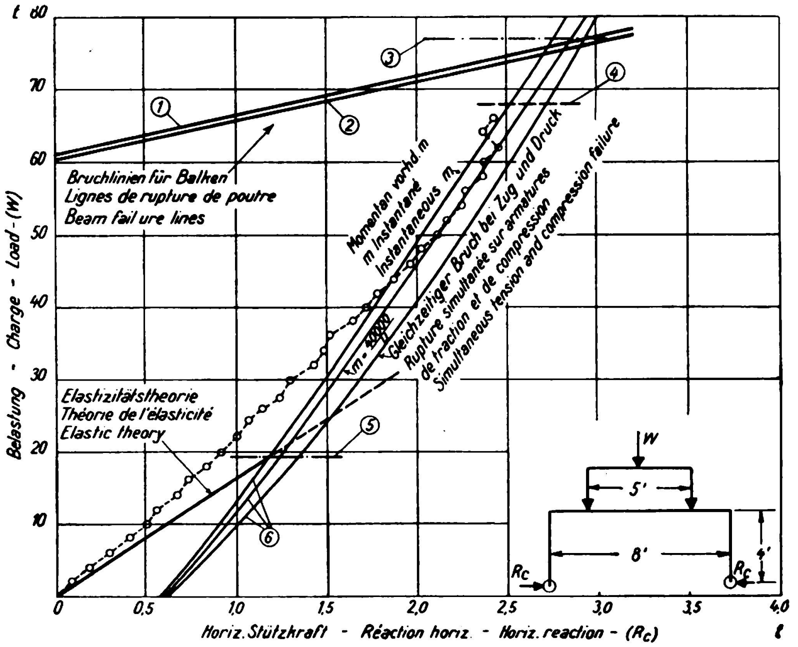


Fig. 7.

Frame test. RMF. 3. (Steel failure). Horizontal reaction. High alumina cement 1 : 2 : 4 concrete. (By weight). Water/cement ratio = 0.60 (by wt.). Age at test — 4 months. Cube strength of concrete = 11 000 lb. per sq. inch.

(1) Simultaneous tension and ccmpression failure.

(2) Instantaneous m and $m = \frac{40\,000}{u}$.

(3) Load for general failure (Redistribution theory).

(4) Actual failing load.

(5) Load for column failure (Elastic Theory).

(6) Column tension failure lines.

lead to very great error except in the final stages of the test. The strain for a steel stress of 47.300 lb. per sq. in. (the yield stress, see Appendix 2) was reached at a load of just over 20 tons, and the strain increased to over four times this value before collapse became imminent. The concrete strain at first signs of crushing was about 32×10^{-4}.

The beam deflection was measured relative to the loading points by means of dial gauges. This deflection was only one-thousandth of the span at about three quarters of the failing load. The overall longitudinal extension of the beam soffit was also measured; as failure of the frame was approached this

Table 3.

Failing Loads of Portal Frames.

Basis of bending Moment Calculations	Basis of Resistance Moment[1] Calculations	Test No.: —	Failing Loads — tons: Steel Failure RMF 2	Steel Failure RMF 3	Concrete Failure RMF 4	Concrete Failure RMF 5
Elastic theory: i. e. No redistribution of moments, Loads are for column head failure.	No Stress Redistribution	True "instantaneous" modular ratio used	19.5	19.5	21.2	15.0
	Stress Redistribution	$m = \frac{40000}{\text{cube strength}} = \frac{40000}{u}$	21.3	21.3	24.0	18.3
		Steel Failure. Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	25.0	25.0	27.5	21.4
Theory of Redistribution of Moments: i. e. Simultaneous failure at column head and in beam span.	No Stress Redistribution	True "instantaneous" modular ratio used	75.0	75.0	46.0	41.7
	Stress Redistribution	$m = \frac{40000}{u}$	75.5	75.5	46.8	42.6
		Steel Failure. Maximum concrete stress reaches cube strength. Concrete Failure: $m = \frac{80000}{u}$	77.0	77.0	47.8	43.6
Actual load at which signs of distress were first noticed in the concrete			65.0	64.0	40.0	38.0
Actual ultimate load carried by frame			65.0	67.8	47.1	43.2

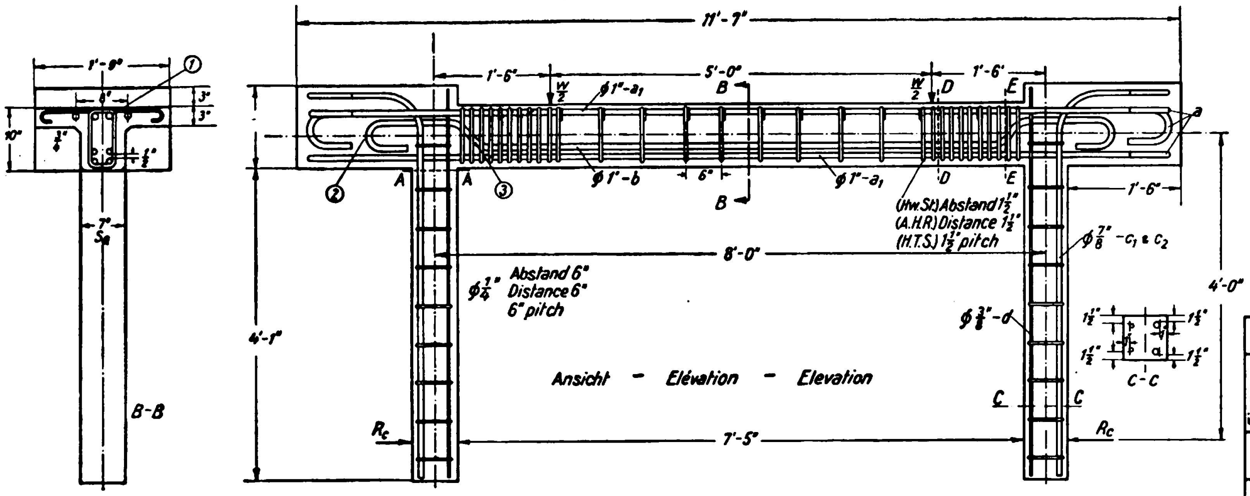
[1] By resistance moment in these tables is meant the ultimate moment the section can carry.

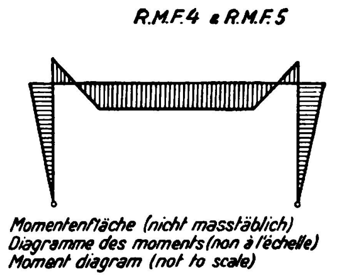
movement was about one-twelfth of an inch at each column head. This movement is insufficient, as an added eccentricity, to have an appreciable effect on the stress at the column head.

Cracks at the column head appeared at a load of about 5 tons, widened steadily throughout the test, and just before failure were about twice as wide as the cracks usually obtained when steel reinforcement reaches its yield point.

2) *Primary Failure of Concrete in the column.*

Details of the reinforcement used for the second type of frame are given in Figure 8. Again the design was arranged to give a reserve of strength in the beam. The tension steel in the column was increased to two $^7/_8$ in. diameter bars instead of $^3/_8$ in. bars and the concrete used was an ordinary Portland





Eisenliste - Liste des fers - Bar schedule

Eisen Armature Bar	ϕ	Masse - Dimensions	Anzahl Nombre No. Off.
a_1	1''	11'-3''	6
b	1''	9'-9''	2
c_1	$\frac{7}{8}''$	5'-7''	2
c_2	$\frac{7}{8}''$	5'-7''	2
d	$\frac{3}{8}''$	5'-1''	4

① Stirrups welded.

② Hook internal radius = 3'', all other hooks of internal radius = 4 × bar dia.

③ Internal radius of bend for main bars, $3\frac{1}{2}''$.

(All main bars in beam 1'' Dia. Vertical Cover 1'').

Calculated Stresses (lb. inch. units) R.M.F.4

Stage		Column						Beam at B			Beam at D				ξ_F	s_E	W	R_C
		c	t	M_A	S	s	s_b	c	t	M_B	S	s	s_b	t_w				
1.	I_1	2850	15000	127000	2960	90	115	860	12300	245000	21500	465	260	12400	3.1	460	43000	2960
	I_2	2850	13000	124000	2890	89	114	1010	14500	289000	23800	515	285	13800	2.3	510	47500	2890
2.		2850	7500	131000	3050	94	120	2850	41100	817000	53500	1160	645	31000	—	1150	107000	3050

I_1 Moments of inertia for moment distribution calculations based on whole concrete area ignoring steel.

I_2 Ditto, based on whole concrete area and including steel.

u = 2850 lb/sq. in.
m = 9 (Stage 1)
$m = \frac{80000}{u}$ (Stage 2)

Fig. 8.

Redistribution of moments in Frames. Concrete failure.

cement of $1: 2^1/_2 : 3^1/_2$ mix (by weight). Details of the strengths of the steel and concrete are given in Appendix 2.

The method of test was identical with that used in the second frame of the previous series, and the values for the horizontal reaction for the first frame are given in Figure 9. It will be seen that the initial relationship between vertical load and the horizontal reaction is in good agreement with that expected from the elastic theory. According to this theory the concrete should crush at a load of about 21 tons, i. e. at the load when the initial line in Figure 9

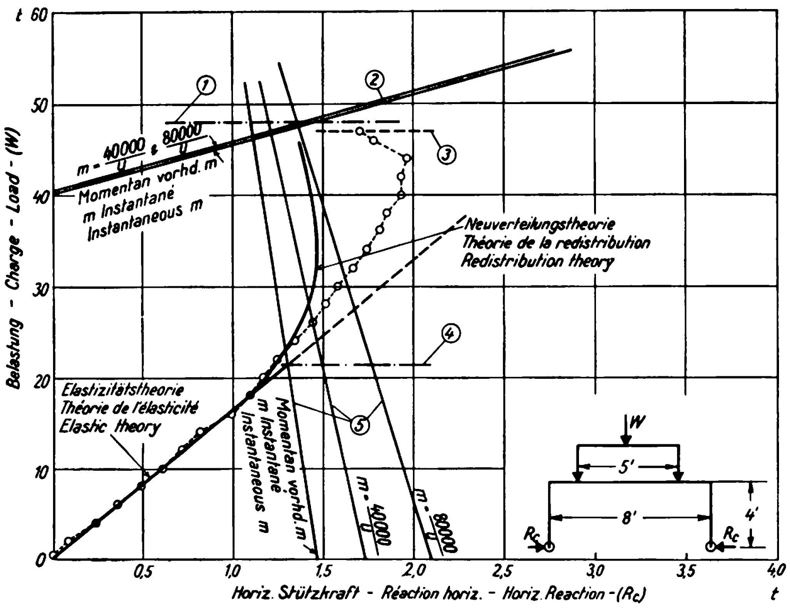


Fig. 9.

Frame test. RMF. 4. (Concrete failure). Horizontal Reaction. Normal Portland cement $1 : 2^1/_2 : 3^1/_2$ concrete (by weight). Water/cement ratio = 0.66 (by wt.). Age at Test = 9 days. Cube strength of concrete = 2850 lb. per sq. inch.

(1) Load for general failure (Redistribution theory).

(2) Beam failure lines.

(3) Actual failing load.

(4) Load for column failure (Elastic theory).

(5) Column compression failure lines.

reaches the compression failure line for a modular ratio of m = 9, which is the true value for the concrete used when inelastic deformations are disregarded. Curves representing compression failure are also given based on modular ratios of $\frac{40.000}{u}$ and $\frac{80.000}{u}$. The redistribution of stress in the column head section was even more favourable than is assumed by this last line, probably due to the increased stresses taken by the concrete above those assumed by a linear distribution of stress from the neutral axis to the compressed face. However, assuming this last compression failure line as a safe guide, it is seen that unless moment redistribution occurs there will be signs of distress in the concrete at a load of about 28 tons. If moment redistribution does take place, then the load will increase with a reduction of horizontal reaction until beam failure is reached at a load of about 48 tons. Actually redistribution will start before signs of distress can be seen and the approximate theoretical change in load and moment is indicated in Figure 9. The actual curve shows that the theory is on the safe side, the moments increasing more than expected from the simple theory of redistribution, with a sudden drop in moment after signs of crushing first appeared. The failing load, 47.1 tons, agrees well with the expected value (see Table 3) and was the result of simultaneous crushing of the concrete in the column and yield of the steel in the beam.

The strains at the column head were measured as before; the interpolated steel strains showed that the tension stresses were low throughout, but that the compression bars were working at their yield load towards the end of the test. The deflection of the beam and the extension of the soffit were again small; the column cracking was also of little importance whilst the beam cracks increased to a width of about 6 or 7 thousandths of an inch, a width usually associated with a steel stress of about 40.000 lb. per sq. in.

In the case of the second frame of this series, the concrete strength was somewhat less than that used for the first frame (see Appendix 2) but apart from the reduced values of load and moment due to this cause the results were very similar to those already discussed. Again the use of a modular ratio of $\frac{80.000}{u}$, together with the assumption that the column will continue to deform so as to redistribute the moments until beam failure occurs, leads to an accurate estimate of the failing conditions (see Table 3).

Discussion of Results.

A. Continuous Beam Tests.

The actual failing loads of the continuous beams, together with values calculated on various assumptions are summarised in Table 2. It is apparent that with all the beams, the ultimate load carried before failure of the system was greater than the theoretical load for support failure calculated on the elastic theory. The increased load can be considered as due to two factors, both resulting from inelastic deformations of either concrete or steel:

1) Redistribution of moments throughout the system tending to give simultaneous failure both at the central support and in the span.
2) Redistribution of stress at the highly stressed sections, increasing the moments these sections are capable of taking above the values as calculated by the ordinary theory.

In Table 2 the calculated loads are based on three sets of resistance moments. The first is obtained by the use of the true or "instantaneous" modular ratio, that is, the ratio which neglects all inelastic deformation of the concrete. The second is obtained by assuming that inelastic deformation of the concrete will lead to an increase of the modular ratio to a value $m = \frac{40.000}{\text{cube strength}}$, the value suggested for design purposes in the Code of Practice for the Use of

Reinforced Concrete in Buildings.[5] The third set of resistance moments were calculated on the following assumptions:

a) In the case of primary tension steel failure the steel will yield until the maximum concrete stress reaches the cube strength of the concrete.

b) In the case of primary concrete failure, the modular ratio will effectively increase to a value given by $m = \frac{80.000}{\text{cube strength}}$. If, however, tension steel yield occurs when this higher value is used, the resistance moment is calculated as for (a). If the calculated stress in the compression steel exceeds its yield value when the higher modular ratio is used, the calculations are modified so that the compression bars do not exceed thier yield value.

From Table 2 it will be seen that if the elastic theory is used for calculating the moments at failure, the theoretical failing loads are less than the actual ultimate loads, even when allowance is made for redistribution of stress.

On the other hand, if redistribution of moments is allowed for, the theoretical loads for simultaneous failure at the central support and in the span, when no redistribution of stress is taken into account, are also less than the actual loads carried, though the margin of safety is not so great.

If allowance is made for both moment and stress redistribution the use of a modular ratio of $\frac{40.000}{u}$ leads to theoretical loads which are not greatly different from the actual ultimate loads except in the case of the beams in which compression reinforcement was used over the central support with a weak concrete [series (3)]. The use of the third method of allowance for stress redistribution, when moment redistribution is also allowed for, is clearly unsafe except in the case of primary steel failure, for which it must be remembered that the redistribution of moments is accompanied by widening of the tension cracks, see Table 1.

The results of the tests on the beams in which compression reinforcement was provided are important. The use of a very high modular ratio for estimating the resistance moment of a section leads to increased computed stresses in the compression bars and it does not appear advisable to rely upon this. In order to investigate this aspect more fully, some simple beam tests were carried out to measure the resistance moments of the sections similar to those used over the central support in the main tests. From these tests, it was found that the use of the highest modular ratio $\frac{80.000}{u}$ is reasonable in all cases of concrete failure except those in which compression reinforcement was provided. In these cases the simple beam tests indicated that redistribution of stress may occur to the extent indicated by the use of the lower modular ratio of $\frac{40.000}{u}$, whereas the support moments measured in the continuous beam tests are not appreciably greater than those calculated on the basis of the "instantaneous" modular ratio. It is possible, however, that the higher shear stresses in the continuous beams with compression reinforcement may have been the reason for the low moment carried over the central support. It appears therefore that when compression reinforcement is provided at the support its effect should be ignored in making calculations taking moment redistribution into account. If this is done for the present beams of series (3), the calculated loads (using a modular ratio of $\frac{40.000}{u}$) are 28.9 and 31,6 tons, 5 and 9 per cent. greater respectively than actually obtained. If the effect of the compression reinforcement in the span is also ignored, the calculated loads are 23.4 and 25.2 tons respectively and these are on the safe side.

B. Portal Frame Tests.

It is clear from the tests that there may be considerable divergence between the actual ultimate load-carrying capacity of a frame and the load which, according to calculations based on the elastic theory, produced a stress in the concrete or steel, at the column head, equal to the ultimate strength of the concrete or the yield strength of the steel. It is important to note that in the tests special precautions were taken to prevent shear failure, closely spaced high tensile steel stirrups being provided in the beams, and special anchorage blocks at the beam-column junctions. Redistribution of moments cannot occur unless the secondary reinforcement and the anchorage of the steel are sufficient for the conditions resulting from the redistribution.

In the case of primary steel failure, the increase in load due to redistribution of moment and stress was over 200 per cent. However, in this case complete moment redistribution did not occur, beam failure not being reached, owing to the earlier crushing of the concrete in the column, even though the cube strength was 11 000 lb. per sq. in. In such cases it is not at present possible to calculate accurately the load at which the concrete will fail as it depends on the deformation of the column after yield of the tension steel. Since the extent to which redistribution can take place as a result of steel yield is not clearly defined and redistribution leads to increased cracking it would be wise to ignore it until further experimental evidence has been obtained.

In the case of primary concrete failure, there are again considerable increases in the ultimate loads carried by the frames as a result of redistribution of stress and moment. If we consider that the useful limit of load increase is when signs of crushing first appear on the column faces it will be seen from Table 3 that the load increase above the value calculated on the elastic theory was 90 per cent. for the first frame and 150 per cent. for the second frame.

In both cases the increase in beam load-carrying capacity as a result of the column moment was less than 20 per cent. whereas the columns would, if loaded axially, have been able to withstand about twice the load that they took in the frame test. The need for taking bending in columns into account is evident.

It would appear that an estimate of the effects of redistribution can be made in simple cases where concrete failure is the deciding factor on the following assumptions:

1) The modular ratio can be taken as $\frac{80.000}{u}$.
2) Both column head and span develop their full strengths before failure of the system occurs.

In any cases where the use of the higher modular ratio leads to calculated stresses in the tension steel greater than the yield point of the steel, the particular section should be calculated on the assumption that both steel yield and the full concrete strength are developed.

It is clear from Figure 9 that stress redistribution occurred in the column head section to a greater extent than that indicated by the use of a modular ratio of $\frac{80.000}{u}$ and from this figure and Table 3, it is seen that the effect of stress redistribution, if moment redistribution is ignored, is to increase the failing load by about 30 per cent. for the particular section used. The increase may not be so great in other cases. For example in the continuous beam tests described earlier in this report the increase in resistance moment due to stress redistribution was only about 13 per cent. for the central support section of the beams of series (2) and (4). In the columns of the portal frames designed for concrete failure, the compression steel used was much less than the tension steel whereas normally the section would be symmetrically reinforced. In view of the smaller amount of stress redistribution that occurred in beam sections reinforced in compression, it would therefore be unwise to use the higher modular ratio, and a value of $m = \frac{40.000}{u}$ is likely to lead to more satisfactory results.

General.

It has been shown that as a result of inelastic deformation of either the steel or the concrete at incipient failure, moment redistribution will usually occur in reinforced concrete structures before final collapse.

The amount of moment redistribution that can occur depends on many factors but to a large extent on the amount of deformation possible at weaker sections. Where weaker sections are capable of developing sufficient deformation, redistribution will be complete and failure simultaneous at principal sections. Further investigation is necessary to fix the safe limits of deformation. Until this is done it would appear wise not to deviate greatly in design from the requirements of the elastic theory.

Design of reinforced concrete structures on the basis of redistribution of moments must take into account the higher bond and shear stresses that accompany redistribution.

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[2] *F. E. Richart, R. L. Brown* and *T. G. Taylor:* „The effect of Plastic Flow in Rigid Frames of Reinforced Concrete." Journal Am. Conc. Inst., Vol. 5, pt. 3, 1934, pp. 181—95.

[3] *G. von Kazinczy:* „Das plastische Verhalten von Eisenbeton." Beton und Eisen, Vol. 32, pt. 5, 1933, pp. 74—80.

[4] *C. Bach* and *O. Graf:* „Versuche mit eingespannten Eisenbetonbalken." Deutscher Ausschuß für Eisenbeton, Heft 45, 1920.

[5] "Report of the Reinforced Concrete Structures Committee of the Building Research Board, with Recommendations for a Code of Practice for the Use of Reinforced Concrete in Buildings." H. M. Stationery Office, 1933.

Appendix 1.

Quality of concrete and steel used in connection with continuous beam tests.

a) *Concrete.*

Series	Beam	Concrete Mix (by wt.)	W/z Ratio.	Age at Test	Cube Strength — lb/sq. in.	True "Instantaneous" Modular Ratio.
1. Steel Failure	RM2 (a)	H.A. 1 : 2 : 4	0.60	6 days	10.140	5.0
	RM2 (b)	R.H.P. 1 : 1 : 2	0.44	44 days	6.660	6.0
2. Concrete Failure (No compression steel)	RM1 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.020	10.0
	RM1 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.070	10.0
3. Concrete Failure (With compression steel)	RM3 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.250	9.5
	RM3 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.470	9.1
4. Concrete Failure (Increased span length)	RM4 (a)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	2.130	9.7
	RM4 (b)	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	1.830	10.4

P. = Ordinary Portland Ccment. H.A. = High Alumina Cement.
R.H.P. = Rapid-Hardening Portland Cement.

b) *Steel.*

Series	Bar diameter — inch.	Yield Stress — lb/sq. in.[1]	Failing Stress — lb/sq. in.[1]
1. Steel Failure	$\frac{5}{8}$	39.400	—
	$\frac{3}{8}$	44.700	62.200
2. Concrete Failure (No compression steel)	$\frac{7}{8}$	40.200	56.500
	$\frac{3}{8}$	46.100	61.500
3. Concrete Failure (With compression steel)	$\frac{7}{8}$	39.800	53.800
	$\frac{3}{8}$	46.700	62.700
4. Concrete Failure (Increased span length)	$\frac{7}{8}$	37.900	53.300
	$\frac{3}{8}$	46.700	61.800
5. Concrete Failure (Weak concrete at about 6 months)	$\frac{7}{8}$	36.600	51.500
	$\frac{3}{8}$	45.800	61.400

[1] The stresses are in all cases based on the nominal original area of the bar.

Appendix 2.

Quality of concrete and steel used in connection with portal frame tests.

a) *Concrete.*

Series	Beam	Concrete Mix (by wt.)	W/z Ratio	Age at test	Cube Strength lb/sq. in.
Steel Failure	RMF 2	H.A. 1 : 2 : 4	0.60	48 days	10.500
	RMF 3	H.A. 1 : 2 : 4	0.60	4 months	11.000
Concrete Failure	RMF 4	P. 1 : 2 1/2 : 3 1/2	0.66	9 days	2.850
	RMF 5	P. 1 : 2 1/2 : 3 1/2	0.66	7 days	1.850

P. = Ordinary Portland cement. H.A. = High Alumina Cement.

b) *Steel.*

Series	Beam	Bar diameter inch.	Yield Stress — lb/sq. in.[1]	Failing Stress — lb/sq. in.[1]
Steel Failure	RMF 2	$\frac{3}{8}$	49.200	60.800
		1	41.500	63.700
		1/2[2]	66.900	106.000
	RMF 3	$\frac{3}{8}$	47.300	59.700
		1	40.600	65.700
		1/2[2]	63.800	107.000
Concrete Failure	RMF 4 et RMF 5	$\frac{7}{8}$	38.600	53.800
		1	41.100	63.000
		1/2[2]	64.700	107.000
		$\frac{3}{8}$	48.300	60.300

[1] The stresses are in all cases based on the nominal original area of the bar.
[2] High tensile steel used for web reinforcement of beam.

IIa 7

Stressing and Factor of Safety of Reinforced Concrete Trussed Girders.

Beanspruchung und Sicherheitsgrad der Eisenbeton-Fachwerke.

Sollicitations et degré de sécurité des poutres réticulées en béton armé.

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It was found in the author's own experiments on reinforced concrete trusses that structures of this type offer exceptional resistance to impact and dynamic stresses.

The tests were carried out in the laboratory for testing materials at the Swiss Federal Institute of Technology at Zürich,[1] the specimens being two reinforced concrete trusses such as are used in bridge work (Fig. 1). The span of these girders was 6 m, their height 1.50 m and they were subject to an isolated load of 50 tons at the centre.

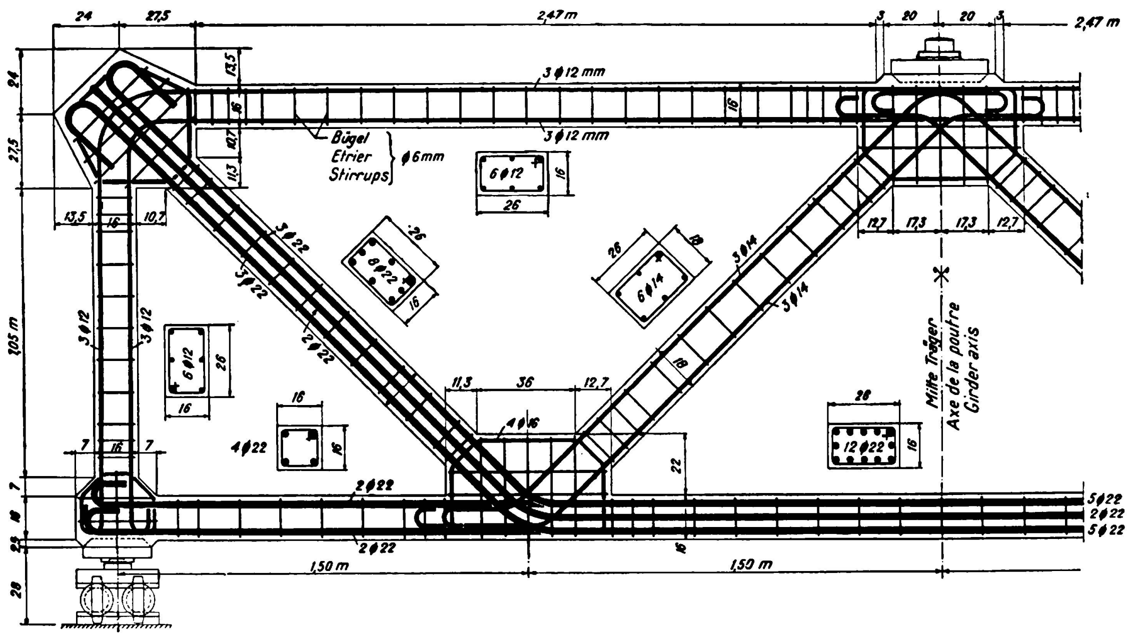

Fig. 1.

Details of the Test-Girder.

[1] *Mortada:* Beitrag zur Untersuchung der Fachwerke aus geschweißtem Stahl und Eisenbeton unter statischen und Dauerbeanspruchungen. Dissertation, Zürich, 1936.

The age of the concrete at the time of the experiment was 90 days, its prism strength ${}_p\beta_d$ was 360 kg/cm^2 and its fatigue strength $\sigma_u = 220$ kg/cm^2 amounting to approximately 0.6 of the prism strength. The reinforcement consisted

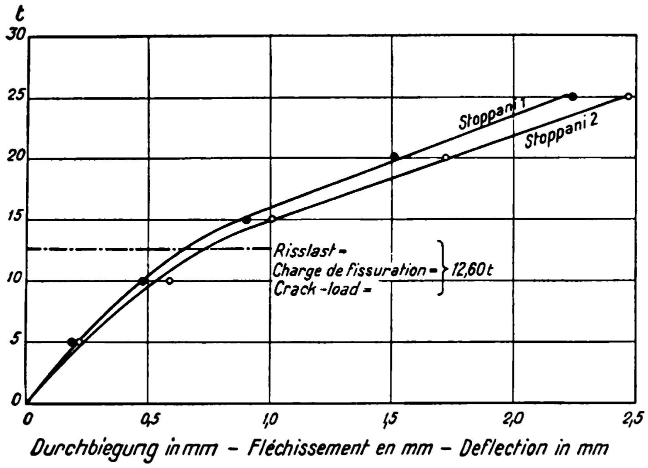

Fig. 2.

Determination of crack-load.

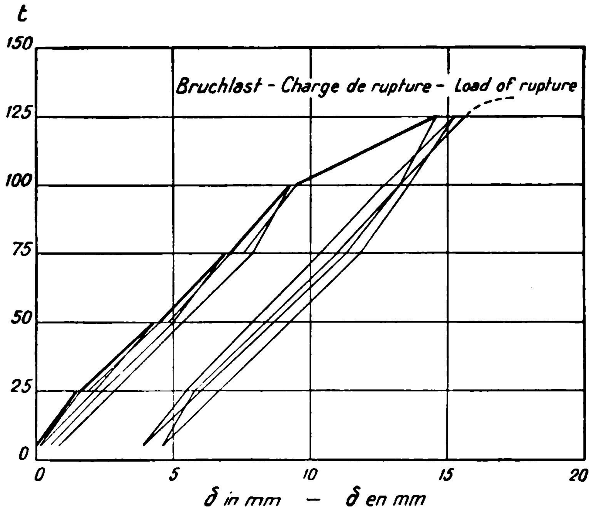

Fig. 3.

Break-test: Loading-Unloading-Deflection Diagram.

of round bars of ordinary steel having a yield point of 2700 kg/cm^2, a tensile strength of 4200 kg/cm^2 and a fatigue strength of 2500 kg/cm^2.

One of the two girders was subjected only to statical tests, in order to study its behaviour under static loading and finally its static breaking load. The second girder, however, was subjected to fatigue tests before being tested statically in exactly the same way as the first. In this way it was possible to ascertain how far the effect of fatigue influenced the statical behaviour and carrying capacity of a structure of this kind.

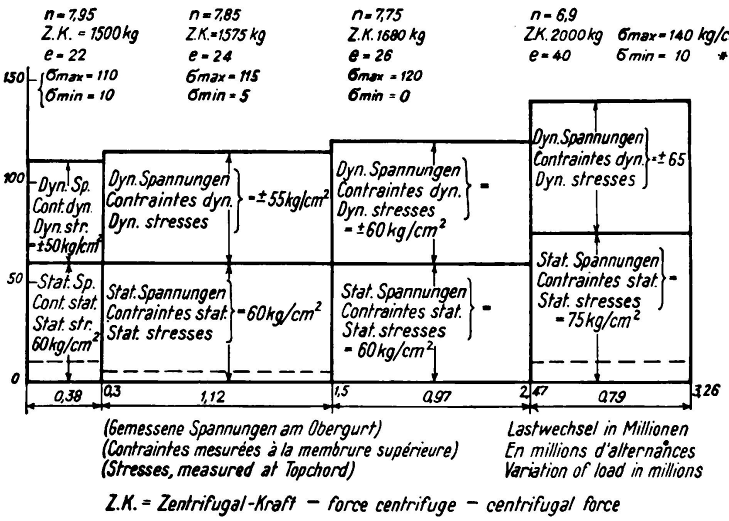

Fig. 4.

Fatigue-test: Measured stresses at different test-phases.

Preliminary experiments were made to ascertain the cracking load and the amount of permanent deformation which occurred after the concrete had cracked. The effect of cracking at various points in the concrete is to introduce irregularities in the stress-strain curve and this enables the cracking load to be determined (Fig. 2), amounting in this case to approximately $^1/_4$ of the calculated live load. The average breaking stress of the concrete in tension corresponding to the cracking load amounted to 17 kg/cm² though the tensile strength of the concrete itself was 40 kg/cm². The large difference between these two values is to be explained on the following grounds:

a) Pre-stressing of the concrete in tension as the result of shrinkage.
b) Incompleteness of the cracking of the concrete when related to the whole of the cross section.

The cracking of the concrete naturally resulted in large permanent deformations, amounting to about 25 % of the elastic deformation under live load. In structures of this kind the secondary stresses (especially those in the compression members) are exceptionally high, amounting to as much as 110 % (on the average they may be put at 70 %) while at the same time the bending stresses in tensile members are very low.

It was found that at the moment when the stress in the reinforcing steel reached the yield point the maximum compressive stress in the concrete was of the order of 220 kg/cm², which is equal to the fatigue strength of the concrete. The load corresponding to this was twice the live load. Since the fatigue strength of the concrete and the yield point of the steel are the criteria for resistance to repeated loading, it follows that in reinforced concrete trusses there exists a factor of safety of 2 against fatigue effects. Under stresses of this order the permanent deformations amount to 5.5 % of the total (Fig. 3) and may, therefore, be taken as accurate for practical purposes.

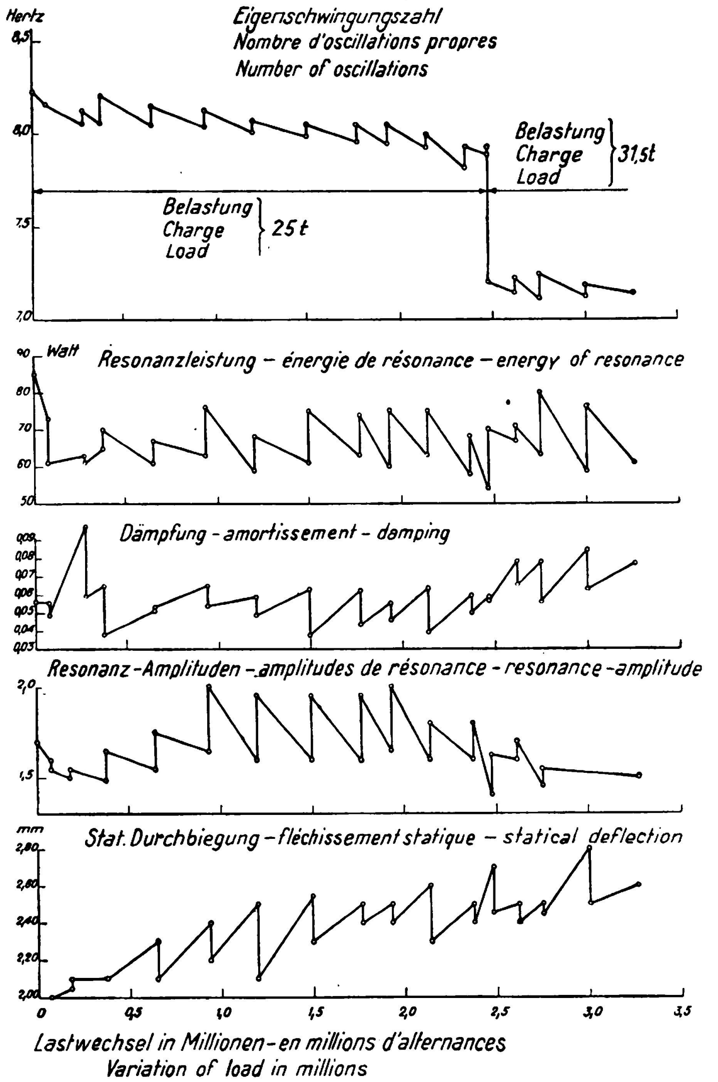

Fig. 5.

Fatigue-test: Change of dynamic-values with the Fatigue.

The factor of safety against statical breakage amounted to 2.6. The ratio between the respective factors of safety against repeated and statical loading is, therefore, that of $2/2.6 = 77\,\%$.

The ranges of stress and corresponding numbers of changes of load in the fatigue tests are indicated in Fig. 4. After a very large number of repetitions of load ($3^1/_4$ millions) within the permissible limits of stress (or even slightly in excess of those limits) no appreciable change in the statical or dynamical properties of the test girders could be observed and neither their carrying capacity nor their factor of safety were in any way affected.

A number of notable observations were made in the process of the fatigue tests (Fig. 5). Thus it was found that in the course of the process the damping and the static deflection increased, while at the same time, the restoring force and natural frequency of vibration decreased. Resonance tests carried out under the same experimental conditions showed that the power consumed by the vibration testing machine (Fig. 6), as well as the amplitude and the magnification, decreased with the fatigue, to be followed by subsequent recovery of stiffness. During certain periods of the tests a state of inertia was found to be established after a certain number of repetitions of load.

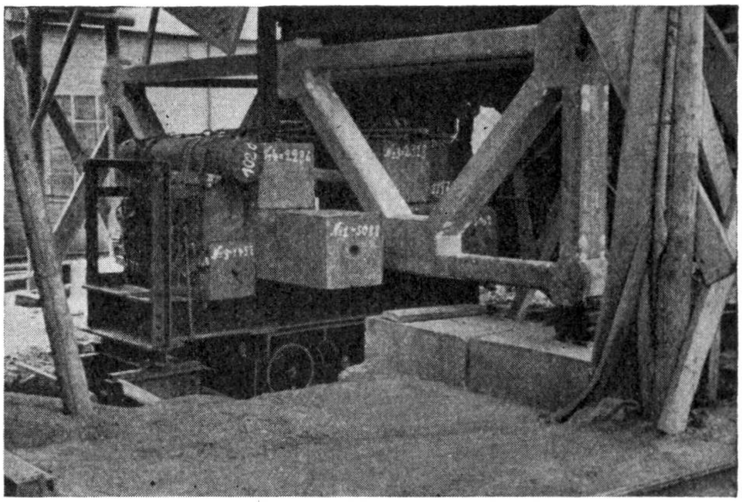

Fig. 6.
Arrangement of Fatigue-test.

From a practical point of view the most important conclusions to be drawn from these experiments on reinforced concrete girders are the following:

Frequently repeated stress within the observed limits (fatigue strength) does not adversely affect either the elasticity or the carrying capacity or the dynamical properties of reinforced concrete trusses, and in structures of this kind safety as regards static loading may also be taken to imply adequate safety against the effects of repeated loading.

IIa8

The Factor of Safety of Reinforced Concrete Structures.

Über die Sicherheiten der Eisenbetonbauten.

La sécurité des ouvrages de béton armé.

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1) *Present definition of factor of safety.*

According to present usage the factor of safety of static structures is defined by reference to the permissible stresses, being, as a rule, the proportion between the breaking stress or yield point of the material and the permissible stress.

This definition, however, is not an adequate one, and in the course of time it has gradually been found necessary to supplement it by statements of special requirements. For instance in the case of retaining walls it is necessary to insure not only against excessive pressure on the foundations but also against the risk of overturning; much the same thing applies to cantilever slabs; and brick chimneys are made subject to the special requirement that the theoretical tensile stresses must not occur beyond the centre of gravity of the cross section. In all these instances the special requirements are those which have reference to stability.

An even more notable fact is that the concept of permissible strength has no meaning in application to columns. It is true that the permissible stress is now prescribed as a function of the buckling length, but this practice amounts to no more than a restatement (or, as it were, a tabular solution) of the column formulae, and the crux of the matter is that columns are in fact dimensioned to carry a load which has already been multiplied by a factor of safety, there being no regular relationship between load and stress. Thus in the design of columns the whole idea of a breaking load is abandoned, and this is contrary to the practice followed in the design of tie bars wherein the cause of failure may always be taken as some defect in the material independent of any increase in load.

To sum up, it is impossible to give any concise definition of what is meant by the factor of safety in static structures as the term is used at present. It may be noted, also, that at the present time safety against dynamic stresses is partly covered by the introduction of an impact coefficient, and this again implies a different idea than the original one of depending on permissible stresses.

2) *Disadvantages of the present factor of safety as applied to structures.*

A general disadvantage of using the factor of safety in its present form is that it does not admit of concise definition. A further disadvantage is that the principal criterion of safety, namely the permissible stress, should be one which in many cases (such as problems relating to stability, column design and dynamic stresses) is of little or no importance. It is again a disadvantage that this principal criterion should need to be supplemented by a variety of extra requirements which have no relation to one another, the factor of safety being made to refer now to the loading, now to the conditions of fracture of the material, and now to the yield point of the latter. As materials increase in strength, problems of stability will tend to become ever more important, and this may entail an even greater variety of special conditions than at present. It must be counted a defect that the main criterion of safety should not be one which in itself guarantees stability from every aspect, and that the form of guarantee should not allow of different weight being attached to different kinds of stress and loading: thus certain stresses due to the dead weight of the structure and to the process of erection might reasonably be treated in a different way from the stresses that will arise when the completed structure is in service. It is a disadvantage, further, that the "own weight" of the structure should have to be multiplied by the same maximum value of the factor of safety whether its action is favourable or unfavourable to stability.

The most frequent occasion for special conditions, tending in this way to take the place of the permissible stress as the criterion of safety, is found in the absence of proportionality between load and stress. In columns this lack of proportionality is due to buckling, but in most other cases it is due to the fact that the dead load and the live load produce stresses which have no common measure: that is to say the dead load stresses and the live load stresses cannot directly be added together.

3) *Special disadvantages attending the application of the present form of the factor of safety to reinforced concrete structures.*

The disadvantages noted above are of general application to most forms of construction and to all kinds of material, but reinforced concrete possesses certain characteristics which render the present criteria of safety especially unsuitable.

In the first place, reinforced concrete is a heterogeneous material: as a rule the steel reinforcement is arranged, with the greatest possible nicety, to carry the tensile stresses, and the result is that when such stresses arise at unintended places the material is particularly ill suited to resist them. That is to say, in the case of reinforced concrete the lack of proportionality between load and stress is particularly marked, and reinforced concrete is much more sensitive than homogeneous materials to changes in the proportion between stationary and moving loads. Changes in the proportion between dead and live loads are particularly dangerous in the case of reinforced concrete arches, and from this point of view, indeed, the arched form of structure is at some disadvantage compared with the beam, whatever the material used.

As an example, the following are the stresses that arise in a two-hinged roof arch of 24 m span, 4 m rise and 15 cm thickness, reinforced on each side with five rods of 10 mm diameter per metre width, subject to a dead load of 400 kg/m^2 and a live load of a 100 kg/m^2:

Steel stress $\sigma_j \sim 943$ kg/cm^2.
Concrete stress $\sigma_b \sim 44.8$ kg/cm^2.

If the live load be increased by 50 % to 150 kg/cm^2 the stresses become

$$\sigma_j \sim 1770 \text{ kg/cm}^2$$
$$\sigma_b \sim 65.9 \text{ kg/cm}^2$$

In other words, σ_j is increased by 87.5 % and σ_b by 47.2 %.

On the other hand, in a simply supported reinforced concrete slab designed for the same dead and live loads, the increase both of σ_j and σ_b when the live load is increased by 50 % is only 10 %.

These figures speak for themselves. Structures which have been designed with special reference to the characteristics of a stationary load are particularly sensitive to changes in the relationship between stationary and moving loads, and generally speaking reinforced concrete structures are less favourably conditioned in this respect than either steel or timber structures — partly because reinforced concrete is in itself heavier, and partly on account of its heterogeneity.

A further reason for abandoning the present criteria of safety as applied to reinforced concrete constructions is to be found in the greater importance attaching to the conditions of breakdown of this material. In concrete and reinforced concrete *Hooke*'s Law is not valid, and for economic reasons the principles followed in dimensioning the cross section are those derived from breaking tests. Also in calculating shear forces (such as those due to moments, transverse loads, etc.) the tendency is always in the direction of laying emphasis on the conditions of fracture. This makes if all the more important that breakdown should be logically defined, which cannot be done by reference to the usual permissible stresses.

Yet a third reason for abandoning the permissible stress as a criterion of safety lies in the great dead weight of concrete structures. It may be observed that structures in which the dead weight is large may more safely be overloaded than structures in which it is relatively small. That is to say, a stationary load which is incapable of increasing above its assumed amount, which cannot vary, and which can exercise no dynamic effect, may be regarded more favourably than a moving load and from the point of view assessing the degree of safety possessed by the structure the former should be regarded in a different category from the latter. Indeed, so far as dynamic effects are concerned, this difference is already recognised by the introduction of impact coefficients, but otherwise the customary method of design by reference to permissible stresses is too severe in its treatment of stationary loads. This statement applies generally to all constructional materials, but the disadvantage is greatest in the case of mass structures and from this point of view reinforced concrete is prejudiced by comparison with steel and timber.

The customary method of calculation is illogical in yet another sense. In most countries, if any noticeable defects appear during the course of construction the work in question is not immediately pulled down, but a test is made under load, and if the defects seem to be serious the test loads are increased so as to produce an overload of perhaps 50 % at the most dangerous places; if the structure succesfully withstands these test loads it is regarded as acceptable for use. Dependence is, therefore, placed on a construction because it has been found amply safe to resist live load: but no regard is taken of its untested degree of safety to resist dead load. It should, however, here be observed that a structure which has been designed to carry a particular ratio of live to dead load may be dangerous when subjected to a form of loading in which this proportion is noticeably increased.

The heavy dead weight possessed by a reinforced concrete structure is a valuable characteristic, and one which ought not to be needlessly penalised.

4) *What should the factor of safety cover?*

The following points will be briefly mentioned:

a) Errors and inaccuracies in the assumed basis of design.
b) Defects of material.
c) Inaccuracies of execution.
d) Inaccuracies of the imposed loading.

In other words, the following items should all be covered: secondary stresses, internal stresses, certain fluctuating stresses, imposed stresses, erection stresses, inaccuracies of calculation, faulty material, inaccuracies in sections (such as steel bars) as delivered from the workshops, inaccuracies in erection and workmanship, inaccuracies in the "own weight", divergences of the live load from that assumed in the design, exceptional overloads such as test loads, and other contingencies.

It is not possible, however, to fix a factor of safety of ordinary magnitude which will cover all these contingent errors and inaccuracies individually: the most that can be done is to take account of their *probable* combinations.

It is true that the latter may equally well be expected to consist of a few high values as of a larger number of small or medium values, but it may be shown that several of the categories of defects named above can only be covered — or can most economically be covered — by the assumption of an increase in the live load. Generally speaking, it may be said that a stationary load can always be assumed to be replaced by a moving load, but a moving load cannot be taken as replaced by a stationary load.

Certain defects in material form an exception to this statement, in that the best way to allow for them is to assume a reduced value for the breaking stress or yield point. Here it is necessary to be clear what purpose is actually served by the use of a factor of safety. In the author's opinion what matters most is safety against breakage, whereas safety against cracking — important as it may be — is secondary.

5) *Proposed new form of the factor of safety for practical use.*

The factor of safety in its present form is expressible as follows:

$$(1)\quad \sigma_p + \sigma_g + \sigma_w + \sigma_t \leqq \sigma_{zul} = \frac{1}{n}\sigma_B$$

In the case of colums:

$$(2)\quad P_{zul} \leqq \frac{1}{n} P_{\text{breakage}}.$$

In application to problems of stability:

(3) $M_{\text{favourable}} \geqq n' \cdot M_{\text{unfavourable}}$

where zul. *(zulässig)* denotes "permissible"

p refers to live load

g „ „ dead load

w „ „ wind load

t „ „ temperature stresses, etc.

σ_B is the nominal breaking stress or yield point.

n and n' are factors of safety.

The first and most general rule can be re-written

(4) $n \cdot \sigma_p + n \cdot \sigma_g + n \cdot \sigma_w + n \cdot \sigma_t = \sigma_B$

or (5) $\sigma_{(n \cdot p)} + \sigma_{(n \cdot g)} + \sigma_{(n \cdot w)} + \sigma_{(n \cdot t)} = \sigma_B$

referring to the stresses caused by the loads multiplied by n. Equation (5) gives the nominal breaking condition which agrees with Equation (2) but is contradictory to Equation (3), seeing that n' is usually smaller than n. In other words, the definition of breakdown is not consistent; moreover it is impossible really to imagine the "own weight" as being multiplied by n, which, in the case of columns, is an abstraction that has to be made.

In the proposal now put forward, the three conditions numbered (1), (2) and (3) above are combined as follows:

$$\sigma_{(n_g \cdot g)} + \sigma_{(n_p \cdot p)} \leqq n_B \cdot \sigma_B = \sigma'_B \qquad \text{(I)}$$

where n_g is the factor of safety for dead load, and

n_p is the factor of safety for live load, while

n_B, which is less than unity, is the factor of safety of the material as such.

If, now, the coefficients n_p and n_g are so chosen that the ratio n_p/n_g is sufficiently great — for instance, 1.5 — then safety against overturning (the problem of stability) is automatically assured and no additional requirements need be stipulated. σ_B is the breaking stress or yield point as determined by experiment, as, for instance, the compressive strength of concrete at 28 days. The lower value $\sigma'_B = n_B \cdot \sigma_B$ is *defined* as the nominal breaking stress; it may, therefore be used as a definite basis for subsequent calculations.

Similar proposals have already been put forward by *Gerber* and others, but have never been fully worked out.

The nominal breaking load is definitely given by $n_p \cdot p + n_g \cdot g$ etc. and the nominal conditions for the breakdown of a structure are determined from the nominal breaking stresses and nominal breaking loads. If *Hooke*'s Law is to be abandoned as a basis for design — as has already been done in many respects for reinforced concrete — it must be replaced by other working principles, and since it is known that the properties of materials as determined experimentally cannot be directly applied to materials as used in actual structures it is better to distinguish certain safe "nominal" properties which can be attributed to the materials, so as to serve as a consistent and logical basis for design, rather than to rely on an arbitrary factor of safety.

When a number of external loads are present, such as, for instance, a vertical live load, a wind load, and additional loads due to shrinkage, temperature, settlement of the supports, etc., probable combinations can be allowed for in the following way:

$$\sigma_{(n'_g \cdot g)} + \sigma_{(n'_p \cdot p)} + \sigma_{(n_w \cdot w)} + \sigma_{(n_x \cdot x)} = n_B \, \sigma_B \tag{II}$$

wherein n'_g and n'_p are given lower values than n_g and n_p in Equation (I).

This principle can, of course, be carried further, but for practical purposes it is sufficient to lay down conditions (I) and (II). Additional stresses resulting from statical indeterminacy are less dangerous, from the point of view of breakage, than stresses due to loads, and generally speaking these are smaller than as calculated by *Hooke*'s Law because in the constructional materials adopted the line of stress is bent towards the axis of deformation, and moreover the additional stresses become smaller when the deformation is permanent. Thus n_x may be given a lower value than n_p and n'_p.

Where one particular moving load predominates over the others, as for instance, where the horizontal live load is much greater than the wind and braking loads, it is sufficient to satisfy one condition of form (II), and this is in fact the general case.

The present practice of requiring two separate conditions to be satisfied — one with and one without the additional loads — is inconsistent. In the case of statically indeterminate structures the usual requirement that $\sigma_g + \sigma_p \leqq \sigma_{zul}$ is apt to be applied in conjunction with certain assumed additional loads, instead of $\sigma_g + \sigma_p + \sigma_{addnl} \leqq \sigma'_z$ (wherein σ'_z denotes an increased permissible stress) being taken as the criterion which governs the dimensions, and as a result the degree of safety possessed by a statically indeterminate structure is often made to appear smaller than that of a structure which is determinate.

It is preferable, as here proposed, to adopt a lower factor of safety in respect of the additional loads than in respect of principal loads, seeing that the former cannot by themselves cause breakage, and the probability of maximum additional loads occurring simultaneously with maximum live load is smaller than that of the occurrence of maximum live loads by themselves.

Two different groups of factors of safety may be used according as the calculations are required to be more or less accurate: for instance $n_{g,1}$ — $n_{p,1}$ — $n_{x,1}$ and $n_{B,1}$ will give a higher degree of accuracy than $n_{g,2}$ — $n_{p,2}$ and $n_{B,2}$.

Considerations of this kind can be practically applied in structural designing. There is good justification for equating certain stresses, such as the erection stresses in the completed structure, to the "own weight" stresses, and in many cases if this is done the calculations are still further simplified — as for instance in the case of *Melan* structures where it is required to take account of the preimposed stresses in the rigid reinforcement. The general effect of the conditions of safety here proposed is to make it possible to take special account of special stresses without complicating the calculations. This fact is very important, for with the old method of calculation there was no way of making allowance for differences in liability to increase as between the different kinds of stress.

A further peculiarity of the *Melan* system of construction will now be mentioned. If, for instance, the pre-existing stress in the rigid steel reinforcement amounts to two-thirds of the permissible stress, then, according to the usual method of calculation, the total cross section may only be stressed up to $\frac{\sigma_{j,\,zul}}{3 \cdot 15} \cdot (F_b + 15\,F_j)$ — but this limitation is unjustified, for it would imply that if the pre-stress has been equal to $\sigma_{j,\,zul}$ the total cross section (concrete + rigid reinforcement + round reinforcement) is unable to carry any further load at all.

Using the old method of calculation, very arbitrary distortions have to be introduced in order to avoid an increase in the pre-imposed stress, and still more exception made from the ordinary rules of design. By the proposed method, however, the calculations are simplified as follows:

$$n_g \cdot \sigma_{j,\,\text{pre-stress}} + n_g \cdot \sigma_{j,\,g,\,\text{completed}} + n_p \cdot \sigma_{j,\,p,\,\text{completed}} \leqq n_B \cdot \sigma_B$$

(and similarly as regards the concrete stress): that is to say the calculations may be based upon the separate loadings which will actually arise, and finally all the stresses may be added together. Care need only be taken not to fix the ratio n_p/n_g too small.

It may happen that the dead load is imposed in the form of a live load, either as regards the proportion it bears to the assumed values, or because it is actually movable. There might, therefore, be a temptation to assume part of the fixed load as being movable, but such a procedure is unpractical because it introduces an unnecessary complication into the calculations by implying that there are two movable loads, differently constituted, instead of one, and because there is a limit to the movability of the "fixed" load. Moreover it is difficult to look upon large cross girders as movable. On the other hand it is easy to visualise a slab of varying thickness, so that the dead load will not be uniformly distributed over its area as assumed.

It is better to allow this freedom of movement of the "fixed load" to be covered by the factor of safety applied to the moving load, but where the fixed loads are very large in proportion to the moving loads such an assumption becomes insufficient. To meet this exceptional case it is both logical and practical to require that the total movable load must be taken as not less than a certain fraction of the total stationary load (for instance 10 %) in each structural member. This question, however, will only arise as regards the principal members of large structures subject to small live loads.

6) *The principal advantages of using the new proposals.*

a) The scope of the new proposals is more general than that of the usual methods of calculation.

b) The two main groups of defects which should be covered by the factor of safety — namely defects in material and defects in load — are each covered by their separate coefficients.

c) Safety as regards stability is automatically assured without the need for stating special requirements.

d) The existence of a large dead weight, which in general is to be looked upon as an advantage (being, for instance a protection against explosion risks, dynamic effects, noise, etc.) will not needlessly be penalised.

e) Where a structure is subsequently found, by accurate investigation, to have been particularly well built, it may without risk be more heavily loaded.

f) Test loadings, involving the imposition of increased live loads at the most dangerous places, may be carried out without undue risk.

g) It ought to be possible to identify the true factor of safety of any given structure with the ratio between the absolute maximum live load that can be brought to bear thereon at the moment of breakage and the live load that has been assumed in the calculations. This definition is not of course an entirely satisfactory one, but the nominal factor of safety against breakdown ought not to be too different from the true value in this sense. Such consistency can be obtained by the method of calculation now proposed, but not by that ordinarily followed.

h) The wide significance here attributed to the factor of safety can, in this way, at least be given a logical basis, and need not merely be regarded as a vague and unpractical symbol, as it must be when the ordinary method, based on permissible stresses, is used.

i) The nominal breaking stresses, the nominal breaking load and therefore the nominal breaking conditions can all be worked out.

k) The deviations from *Hooke's* Law, etc., which are admissible in approximate calculations, can be made subject to definite and consistent rules.

l) Safety against cracking, against repeated loading etc., may be attained by the same means, and more convincingly than by the usual methods.

m) The basis of calculation is rendered more consistent, and the statical calculations themselves are made simpler and more reliable, especially as regards structures in which questions of stability, pre-stressing, etc. are involved. The values finally adopted for the safety coefficients must be consistent with the rules governing both design and erection.

IIa 9

Tests on the Slow Buckling of Concrete Sticks.

Versuche über das langsame Knicken an Betonkörpern.

Essais de flambement lent de baguettes en béton.

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When a prism-shaped body is loaded at the end its condition is one of unstable equilibrium if the load equals or exceeds the limit given by *Euler's* expression $\frac{\pi^2 E I}{l^2}$.

The mechanism of buckling can in fact be visualised as follows: any slight eccentricity of the load gives rise to a bending moment which causes an initial deformation, this deformation has the effect of increasing that moment so as to cause a second deformation, and so on. If the deformations thus obtained constitute a divergent series the piece buckles. This is the meaning of *Euler's* equation, which further implies that the limit of stability is independent of the amount of initial eccentricity.

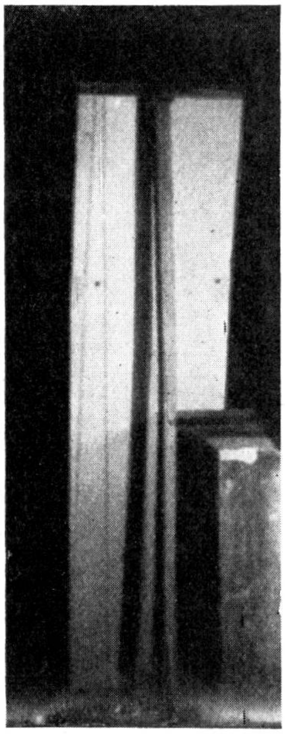

Fig. 1.
Photograph showing the deflection assumed by a test bar 135 · 3 · 3 cm the day before its collapse.

According to the customary rules governing the strength of materials these deformations occur immediately the load is applied, and as a corollary to this the series of phenomena described below take place almost instantaneously, and fracture occurs suddenly without any warning.

Concrete, however, behaves in a different way: the initial deformation is almost instantaneous, but its subsequent growth is slow. There exists, therefore, an *a priori* possibility that under certain values of load the piece may assume a condition of stable equilibrium under the action of the first deformation, and that it is the subsequent of slow deformations which constitutes the divergent series characteristic of buckling. In other words, the reasoning on which *Euler's* formula is based is independent of time, and what takes place is exactly as if the modulus of elasticity E were to decrease as the stress and its duration increased. In order, then, to arrive at the true criterion of buckling, it becomes necessary to introduce the final value of E into *Euler's* formula.

It was thought that the matter might be illustrated in a particularly striking way if it could be reproduced in the laboratory, and attempts were accordingly made to bring about slow buckling in concrete pieces. These pieces took the form of sticks measuring 135 cm by 3 cm by 3 cm, made from fine gravel concrete using 350 kg per cu. m of "superciment" or of aluminous cement (Photograph N° 1). The sticks were loaded by means of a lever-operated press, and the results are given in the following table:

No. of Specimen	Kind of Cement	Age of Concrete when tested (days)	Strength of concrete as measured on 20 cm cubes (kg per sq. cm)	Load Applied kg	Load Applied kg per sq. cm	Results of Tests	Deflection	Modulus of elasticity as calculated from buckling
1	Artificial	130	260	780	86	Instantaneous buckling		210.000
2	do.	130		580	64	A deformation begins to occur, but after 6 days shows no perceptible further increase; the load is then increased:		
				650	72	Buckling at 14 days	3 mm	175.000
3	Superciment	19		1120	124	Instantaneous buckling		300.000
4	do.	19		720	80	Buckling after 15 minutes		195.000
5	Aluminous	3	430 at 3 days	1520	170	Instantaneous buckling		410.000
6	do.	8		1070	118	Buckling after 5 days	4 mm	290.000
7	Aluminous	4	360 at 4 days	1140	126	Instantaneous buckling		310.000
8	do.	4		960	106	do.		260.000
9	do.	4		900	100	do.		240.000
10	do.	4		780	86	Buckling after 5 minutes		210.000
11	do.	5		650	72	Buckling after 7 days	3 mm	175.000

It was found that the piece sometimes broke at once and sometimes resisted the load indefinitely, but between these two extremes it was found possible, after a few attemps, to bring about the desired phenomenon in sticks Nos. 2 (second test) 4, 6, 10 and 11.

These experiments amount to no more than a first approach to the study of a problem which deserves closer attention, and however incomplete the results given above it seemed worth while to publish them as evidence of the existence of this phenomenon of slow buckling and as forming a broad outline of its nature.

It is impossible to lay too much emphasis on the danger that may arise, in practice, from this cause, and on the necessity for adopting a very low value of the modulus of elasticity in applying *Euler's* formula. At the same time, it should be observed that the deformation of a member placed in this condition of unstable equilibrium becomes apparent a short time after the load is applied and then increases progressively to a high value. Since, therefore, failure through slow buckling is preceded by these visible phenomena, it is less dangerous than failure by instantaneous buckling, though the actual occurrence of fracture is in fact equally sudden in both cases.

II b

Means for increasing the tensile strength of concrete and reducing cracking.

Mittel zur Erhöhung der Zugfestigkeit und zur Verminderung der Rissebildung des Betons.

Moyens d'augmenter la résistance à la traction et de diminuer la formation des fissures dans le béton.

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IIb 1

The Elimination of Tension in Concrete, and the Use of High Tensile Steel by the Freyssinet Method.

Der Ausschluß von Betonzugspannungen und die Verwendung hochwertigen Stahles durch das Freyssinet-Verfahren.

L'élimination de la traction dans le béton et l'application de l'acier à haute résistance suivant la méthode Freyssinet.

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Knowledge as to the causes and importance of crack formation in reinforced concrete has made great progress. Precautions against cracking extend both to the choice of suitable material and to proper constructional procedure, but so far it does not appear that advances made in producing cements of higher tensile strength will play an important part in these precautions; nor does attention to the choice of aggregates and of the water-cement ratio, and to the after-treatment of the concrete, offer more than partial guarantees of succes; and these controls are not practicable in all circumstances. The adoption of specially shaped reinforcing bars, which has become customary in conjunction with the higher stresses in certain kinds of steel (Isteg and other types of deformed bars) indeed offers some improvement from the point of view of bond, and therefore some reduction in the risk of the steel slipping and giving rise to large cracks; but the greater part of the advantage so gained is discounted by the heavier permissible stresses in such steel, which are attended by heavier than normal stresses in the concrete. The fact that the unavoidable shrinkage is in itself sufficient to cause tensile stresses without any load being on the structure when (as is most often the case) the reinforcing bars are placed eccentrically, and that the calculated values of such stresses may be up to the limit of the available tensile strength, is only slightly mitigated by plastic tensile strain; for such strain can be significant only in cases where the concrete is relatively weak, and this in turn implies that the tensile strength is low.

The possibility of improving matters by suitable choice of materials would appear, therefore, to have reached its limit. As a rule the criterion for distinguishing between dangerous and harmless cracks is taken to be that the width of the crack shall not exceed an amount variously given as 0.2 to 2/3 rds mm. In principle it may be laid down that such cracks as cannot be prevented from arising when the permissible stresses in the concrete and steel are attained (the maximum elongation at fracture of the concrete, under bending, being approximately 0.3 mm per metre) are not to be regarded as dangerous. The distinction between harmless and dangerous cracks is however, very vague, being influenced by the situation of the job and by the effects of weathering, and above all by the effects of repeated stressing and impact.

In view of this fact the regulations which are operative in various countries continue to fix limits for the tensile stresses, although it is known that these limitation are of little value for the reason that the initial stresses cannot be estimated. In the German regulations for reinforced concrete bridges, for instance, the tensile stress is limited to $^{1}/_{5}{}^{\text{th}}$ of the cube strength, and the French regulations of 1934 are similar in their implication, though applicable only to oblique principal stresses.

It is therefore an advance, the importance of which it would be impossible to exaggerate, that a reliable means altogether different from those suggested above has now been found for eliminating all tensile stresses (including those which are not dangerous), and of doing this without exceeding the limits of what is economically feasible. The methods proposed by *M. Freyssinet* in the Preliminary Publication combine the following advantages:

1) Certainty of eliminating all tensile stresses in a member exposed to bending or to eccentric pressure, with consequent elimination of all risk of cracking.

2) Apart from the matter of freedom from cracking, which is germane to the present report, there is the circumstance that the whole of the cross section of the concrete can be utilised for compression so that a member is made to resist bending to the full extent of its cross section and resisting moment, and can be calculated as if it were a homogeneous body.

3) This makes it possible for the same compressive stresses in the concrete to take up much heavier bending moments, or alternatively, if higher permissible stresses can be accepted in the concrete, it is possible considerably to reduce the size of cross section and therefore the amount of material required.

4) Variations and repeated alternations in load no longer play any appreciable part in determining the risk of cracking, and variations in stress in the reinforcing bars are minimised by contrast with what is true of structures designed in accordance with Stage IIb.

As is well known, *M. Freyssinet* has obtained these revolutionary advantages in reinforced concrete construction mainly through the adoption of pre-stressing both of the longitudinal steel and of the stirrups. This principle has long been understood, but earlier applications, as for instance those attempted by *Koenen* and *Lund*, have been defeated by two difficulties:

1) The pre-stressing was so light that the whole of its effect was lost by shrinkage, plastic deformation and drop in temperature.

2) It was not possible for the engineers concerned in these attempts, and for those who later took up the problem, to devise practical means of pre-stressing which would be certain in their effect and at the same time economical.

Freyssinet himself succeeded not by developing the idea of pre-stressing by itself, but by combining it intimately with the following measures:

1) By making the amount of pre-stressing very heavy, somewhere between 4000 and 7000 kg/cm^2, and by the use of steels with yield points of 8000 to 12 000 kg/cm^2.

2) By adopting a constructional arrangement so designed as to ensure uniform pre-stressing of the steel reinforcements, the necessary anchorages being connected either to the shuttering or to its sub-structure in such a way that the pre-stressing operation would be perfectly carried out and the apparatus afterwards disconnected by simple means.

3) Neither of these two conditions could be fulfilled without radical improvement in the production of the concrete itself. It would be both difficult and uneconomic to maintain these heavy pre-imposed stresses in the steel long enough for the concrete to harden sufficiently to pick up the stresses. *Freyssinet,* therefore, devised a means of making concrete of medium and high compressive strength in less time than ever before, using a process which he has described as "virtually instantaneous hardening" *(endurcissement quasi instantané)*; a process which offers the further advantage that concrete beams, posts, piles and pipes can be formed in short lamellar layers, whereby the expense of complicated shuttering is reduced to a fraction. The principle of this "virtually instantaneous hardening" has been described in the Preliminary Report, and depends on subjecting the concrete to vibration, compression and heat; the practical significance of these treatments has been discussed in detail in the Publications of the I.A.B.S.E., Vol. IV, and here it need merely be stated that vibration has the effect of so arranging the small particles of aggregate as to reduce the voids; the effect of pressure is further to reduce these voids; and after both these processes have been carried out it is permissible to apply heat, since the capillarity of the voids effectively reduces the loss of moisture by evaporation.

It is to be noticed that none of these three processes by itself gives the desired result. It is known, for instance, from the vibration tests carried out by *Graf* and *Walz* at Stuttgart, that the vibration of plastic watery mixtures such as are used in reinforced concrete work, containing large quantities of sand, does not by itself cause any reduction in the time required for hardening nor any improvement in quality. It is only applying pressure to increase the density, that the limited void content resulting from the packing of the small particles makes it possible to apply heat. This additional heat considerably increases the total present due to the setting of the cement.

This process has been carried out not only in the laboratory, but also on actual work for making masts, solid and hollow piles, pressure pipes and beams of large dimensions. The results vary according to the shape of cross section; with closed and compact sections strengths corresponding to those obtained after 28 days in ordinary reinforced concrete work, or even considerably higher, are obtained in a very short time. With other forms of cross section, as for instance I-beams, the shuttering is removed after a few hours and an initial strength of 150 to 200 kg/cm^2 is secured, which is enough to allow either of another piece being begun or of the pre-stress being transferred into the concrete.

Owing to the fact that longitudinal tensile stresses are entirely eliminated it becomes possible by suitably pre-stressing the stirrups to render the principal stresses entirely compressive, and the question of permissible compressive stresses for concrete made on the site assures a fundamentally new aspect.

In the customary form of reinforced concrete design, assuming the presence of a tensile zone, the permissible compressive stress in the concrete is quite properly based upon a higher factor of safety than that which governs the steel stress referred to the elastic limit. As a rule this implies a considerably higher factor of safety as regards the permissible compressive stress in the concrete; a differentiation which is justified only by the greater uncertainty which attends the production of concrete by comparison with that of steel in the steel works, but also, more pertinently, by the question of freedom from cracking. If, then, the compressive stress in the concrete is notably increased, the calculated tensile stress in the concrete will likewise be increased, and the limitation of cracks will become more difficult. Apart from this, in T-beams an increase in the compressive stress in the concrete is associated with a crowding together of the steel bars, which may itself tend to cause cracking. Both these components of safety as affecting concrete compressive stresses are eliminated by the *Freyssinet* system, and the only remaining reason for the concrete being made subject to a somewhat higher factor of safety than is possessed by the steel referred to its elastic limit lies in the different conditions of their manufacture; not on any effect of the increased concrete compressive stress on the design. There is, therefore, no point in fixing the permissible compressive stress in the concrete at about $^1/_3$rd of the 28-day cube strength, because when the *Freyssinet* system is applied compressive stresses in the concrete of 150 kg/cm^2 are obtained even in unfavourable cases, and these may straight away be looked upon as permissible values.

As regards alterations occurring in the steel stresses under live load (particularly under moving loads) it is to be remembered that since the neutral axis lies approximately at the lower edge of the beam the whole cross section co-operates, so that variations in the steel stress will amount only to n times the variations in the concrete compressive stress. This is relatively quite a small fraction of the initial stress, contrasting in this respect with reinforced concrete construction with a tension zone.

The use of high elastic limit steels, without pre-stressing, as hitherto practised, is known to have resulted in increased safety against breakage but not in any increase in safety against cracking, though deformed bars have been proved efficacious with regard to bond and to sub-division of the cracks. The increased steel stresses adopted with these kinds of steel, with correspondingly greater elongation, increase the danger of crack formation, thus contrasting with the effect described above. With the *Freyssinet* system the high tensile steel which is pre-stressed no longer performs the same function as is performed by reinforcement in ordinary construction; it merely serves to take up the heavy eccentric pre-imposed stress, and contrary to what happens with high tensile steel used without pre-stress it results in an avoidance of cracks. Hence all the arguments against the adoption of high tensile steel become invalid, and this applies particularly to the objections which have been raised to the use of cold-stretched steels. Any apprehension is easily overcome by the fact that these qualities of steel may be produced by suitable alloying without cold stretching. By reason of the high stress in the steel it is not necessary to make use of exceptionally large diameters; hence anchorage by the usual form of hooks involves no risk.

The application of the method extends to a great many types of construction. It is already well known how hollow piles of almost unlimited length have been made in this way, an operation of which *M. Freyssinet* has given an account in the literature (see also the Preliminary Report of the Congress), in the reconstruction of the marine station at Le Havre. In addition, the production of thin walled high tension pylons for power transmission has been practised for some years. Quite recently two applications of the method have been developed both in the laboratory and on actual work which deserves special attention here, namely (1) the production of long span beams, and (2) the production of high pressure pipes of large diameter for heavy pressures both in service and under test.

There is neither space nor occasion here to give details of the constructional arrangements adopted for producing pre-stress, or of the qualities of the "virtually instantaneous hardened" concrete; these matters must be left for later description. The underlying principle of the system may, however, be briefly mentioned: the pre-stressing is carried out either with the aid of the shuttering itself or with that of its supporting structure, and usually by means of hydraulic jacks. Contrary to what was found in the earlier experiments by other workers, it is possible with safety to make use of pre-stressing loads as high as 1000 tonnes. Another method, which is particularly interesting, is to make use of the concrete itself for pre-stressing the steel reinforcement. This method is quite new and is being widely adopted. The idea of using the concrete itself to pre-stress the steel bars may at first sight appear difficult to understand, but it may be readily understood by recalling the heavy shear and compressive resistance offered by concrete which has been placed while in a fluid condition, and has been systematically dewatered and subjected to heavy pressure on all sides. Here, as a rule, such pressure is imposed by means of tight inflatable rubber sheaths connected to the shuttering. The theoretical principle underlying this process (which has met with great succes in practice) have been explained by *Freyssinet* by reference to considerations recently put forward by Caquot in his *Equilibre des Massifs à Frottement Interne* (Paris, 1934).

A brief outline of the stress conditions obtaining in long span beams will now be given, reference being made to the treatment of the subject of "Long Span Girder Bridges" at the Paris Congress in 1932 in which the economical constructional limits of span were found to be determined by the tensile stress of the concrete and the space taken up by the reinforcing bars. In this consideration of long span beam bridges the unfavourably small ratio between live load and dead load moments plays a vital part, and the ratio in question is one which becomes smaller still in proportion as the span increases. At the same time it is proper to observe that this circumstance, unfavourable as it is from an economic point of view, has frequently been put forward as an advantage of reinforced concrete construction, by reason of the fact that variations in load and in the stress of the reinforcing bars are in this way reduced. In the case of beams constructed by the pre-stressing method much more favourable (that is to say much greater) ratios between the live load and dead load moments are obtained, while at the same time the advantage of limited variation in stress under repeated fluctuations of load is ensured, because — as explained above — the whole cross section collaborates.

Fig. 1 shows an experimental beam constructed in Germany on the *Freyssinet* system, in which all the dimensions of the steel reinforcing bars and of the

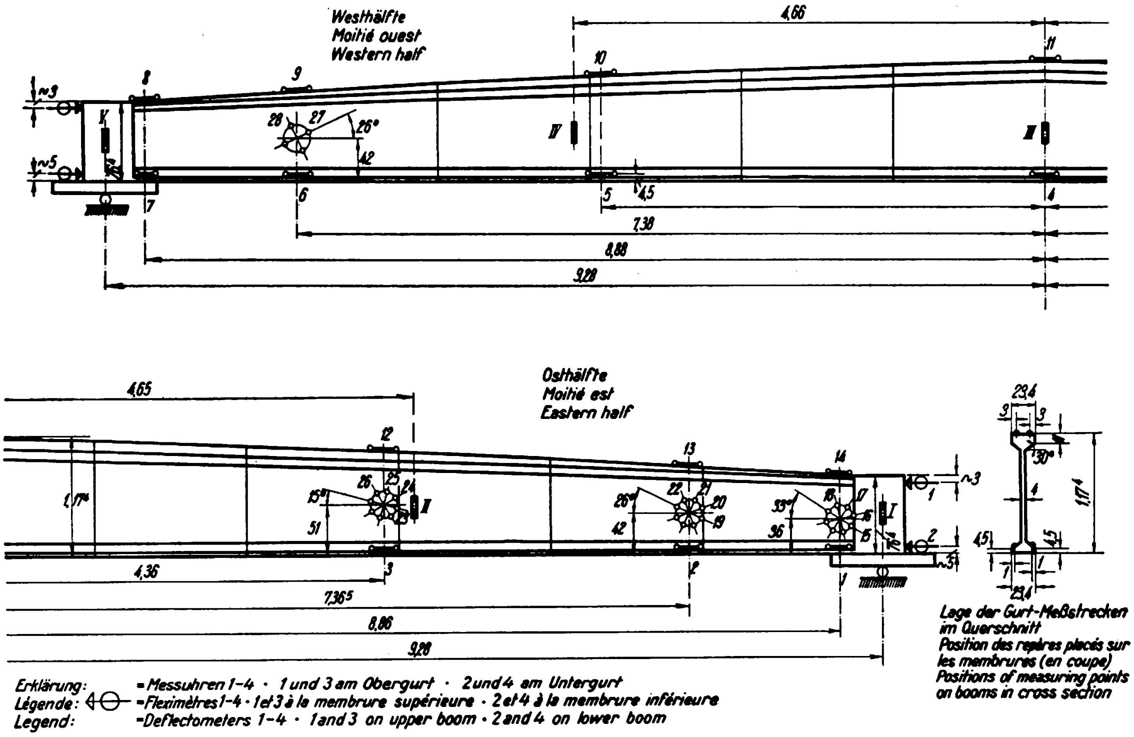


Fig. 1.

Arrangement of measuring points.

spaces between are made exactly to a scale of 1:3, representing a roof girder over a hall of 60 m span. The model reinforced concrete beam has, therefore, a span of approximately 20 m; the pre-imposed stress in the reinforcing bars is 5500 kg/cm², which gives the following moments at the central section:

From own weight, and loading frame .	9.67	m-tonnes
From live load	59.5	m-tonnes
Total, approximately	69,2	m-tonnes.

The maximum concrete stress due to own weight plus maximum live load plus loading frame amounts to:

In the upper flange	143.7 kg/cm²	compression
In the lower flange	18.9 kg/cm²	compression.

The neutral axis thus falls below the lower edge of the beam. With a pre-stress of only 3500 kg/cm² the stress in the upper flange amounts to 160 kg/cm² and that in the lower flange to 40 kg tension. With a pre-stress of about 5000 kg/cm² the lower flange stress at the soffit of the girder is exactly zero; the total compression in the lower flange amounts to 205 kg/cm² due to the pre-stress alone without counting the "own weight".

The girder has a cross section of 1008 cm² at the centre, and its lower boom is reinforced with 64 round bars of 5.4 mm dia., corresponding in scale to bars of 16 mm dia. in the full sized construction. As already mentioned above, the girder was constructed in lamellar stages in such a way that altogether twelve pieces are arranged in sequence along the length. The construction and hardening of such a member required only a few hours. Before the pre-imposed stress was transferred onto the concrete shrinkage cracks appeared at the horizontal boundaries between the lamellae, the width of which was estimated at 1/20th mm. After the transfer of the pre-imposed stress these shrinkage cracks closed up so that they could no longer be detected with the naked eye, and the lime wash which had been painted over for purposes of measurement was observed to flake off. Measurements of elongation in the flanges, made in the direction of the principal stresses and in that of the bisectors between the principal stresses and the axes, were carried out by the Materials Testing Laboratory of the Technische Hochschule at Stuttgart, and these showed that *all the principal stresses were compressive.*

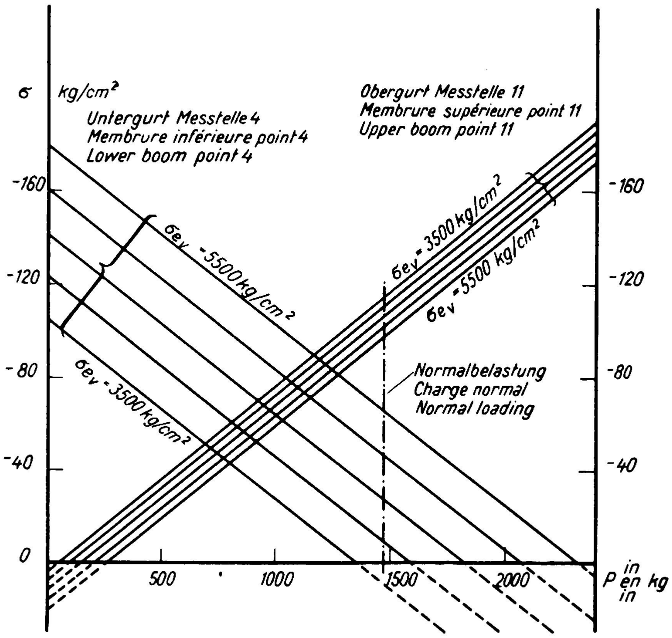


Fig. 2.

Section on centre line.

The following observations may be recorded (Fig. 3). With a pre-imposed stress of 5500 kg/cm² the girder acted under full load like a homogeneous girder, the whole cross section co-operating to provide resisting moment against bending. No tensile stresses arose either in the lower boom or in the directions of the principal stresses, nor in any other direction. The deflection of the girder was relatively very small as a result of the large resisting moment; it amounted to only 1:750, corresponding to that of a steel girder of similar cross section stressed to only 500 kg/cm².

The explanation of this exceptionally favourable distribution of stress is to be found in the pre-stressing of the longitudinal bars and stirrups. As is well known, the principal stresses corresponding to the stresses σ_x and σ_y are given by

$$\sigma_{I,\,II} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}\,; \qquad \operatorname{tg} 2\varphi = \frac{2\tau}{\sigma_y - \sigma_x}$$

With ordinary reinforced concrete construction at the neutral axis $\sigma_x = \sigma_y = 0$; hence the inclined principal stress $\sigma_{II} = -\tau$ is at an angle of $\varphi = 45^0$. It is clear that the effect of the normal stress σ_x (pre-imposed compression) is to diminish the principal tensile stress. If Mohr's diagram be drawn (Fig. 4) with $\sigma_x = \tau$ there is obtained a principal tensile stress of

$$\sigma_{II} = \frac{\tau}{2}(1 - \sqrt{5}) = -0.618\,\tau$$

If $\sigma_x = 2\,\tau$ the maximum principal tensile stress becomes $\sigma_{II} = -0.414$. If, finally, besides $\sigma_x = \tau$ a vertical pre-stress is imposed by means of stirrups, the principal tensile stress becomes $\sigma''_{II} = 0$ and the principal compressive stress $\sigma''_{I} = 2\,\tau$. This condition can easily be obtained by stressing the stirrups, as was done in the present case. As regards shear effect, tensile stresses are entirely eliminated and the shear stress may therefore be brought up to one half of the permissible compressive stress.

With concrete compressive stresses of approximately 145 kg/cm² as already stated, the girder suffices to carry a live load amounting to seven times its own weight.

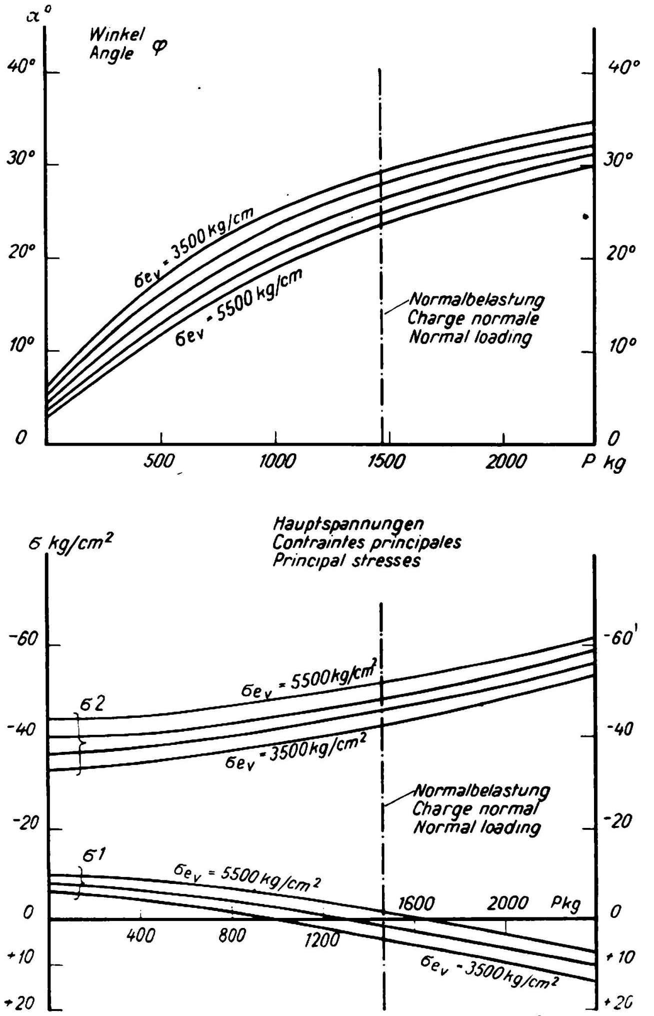


Fig. 3.

Section 1—14.

The girder to be actually constructed would have a cross section of 0.84 m² at the centre, and the total moment at the centre of the span would then amount to about 2000 m-tonnes, the ratio between the live load moment and the dead load moment being approximately 1.2. If these spans, not hitherto realised with solid webbed girders, are compared with those of existing bridges, the very favourable relationship of live load to dead load moment will become evident.

A girder of 100 m span subject to the same stresses would have an average "own weight" of 5 tonnes per m run, and would be able to carry a live load of 3 tonnes per metre run, it being assumed that the permissible compressive stresses are kept suitably low.

High pressure pipes have been constructed on the same principle, the steel reinforcing bars being pre-stressed by compression of the concrete itself while supported all round and subjected to an increase of internal pressure. The following results were obtained:

Small test pipes of 440 mm internal dia., and 37 mm thickness of wall, were exposed to a test pressure of 90 atmospheres without any detectable loss of water. By contrast with this a pipe of the same dimensions and containing the same amount of reinforcement, but not pre-stressed, was unable to withstand a pressure in excess of 6 atmospheres without loss of water-tightness. It is particularly notable that the empty pipe had to withstand a compression of 550 kg/cm² due to the pre-stressing at the instant of its removal from the forms, and this happened within a few hours of its manufacture.

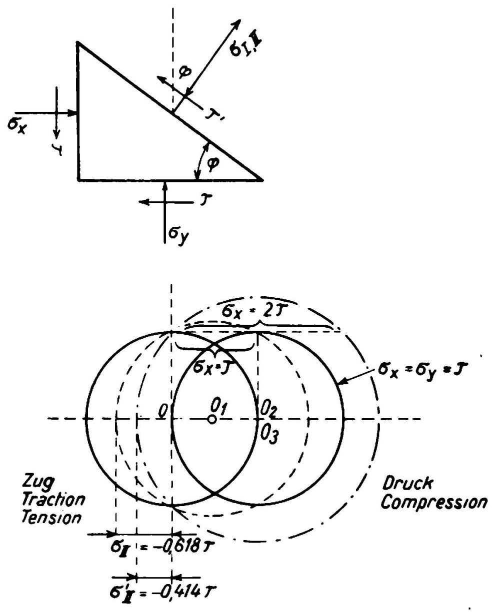


Fig. 4.

The same principle is now being applied, in actual practice, to the continuous production of high pressure pipes of large diameters. Thus pipes of 800 mm dia., with walls 5 cm thick, for test pressures up to 16 atmospheres, have been produced without any defects, and are removed from their shuttering a few hours after the beginning of the process of manufacture. Under these conditions the concrete when removed from the moulds is subject to a compressive stress of 140 kg/cm² about three hours after it has been deposited. It is particularly notable that if such pipes are purposely exposed to pressure in excess of the usual test pressure, so as to cause them to sweat, they again become perfectly water-tight after the pressure has been relieved, and remain so when the pressure is again increased up to the permissible amount. This shows that the effect of pre-stressing is completely to close up any cracks that may have been formed by statical effects; a result which is of great importance as regards resistance to impact effect (from water hammer).

IIb 2

Reducing the Risk of Cracks in Reinforced Concrete Structures.

Die Erhöhung der Rißsicherheit bei Eisenbetonbauten.

L'amélioration de la sécurité à la fissuration dans les ouvrages en béton armé.

Regierungs- und Baurat a. D. Dr. Ing. W. Nakonz,
Vorstandsmitglied der Beton- und Monierbau A.-G., Berlin.

The difficulty of ensuring that reinforced concrete structures, however well reinforced, shall be entirely free from defects and cracks is known to every specialist. Fine hair cracks are in fact present in the majority of reinforced concrete beams, and are due to the tensile strength of the concrete having been exceeded by the bending stresses which result from the dead weight of the beam and the imposed loads in addition to temperature and shrinkage stresses, or, in most cases, to a combination of such stresses.

These fine hair cracks have no effect on the carrying capacity of the structure, as the tensile strength of the concrete has been left out of account in the statical calculations and all stresses on the tension side are carried by the steel embedded therein. The hair cracks may, however, afford a channel whereby in course of time the surrounding air may penetrate to the steel and cause rusting if it is damp or acidic. Twenty-five years ago this danger was the subject of lively discussion among engineers, but experience has since shown that in carefully executed reinforced concrete work it does not exist and that no fear need be entertained of the reinforcing steel being gradually destroyed through rust in this way.

In the last few years the question of the freedom of concrete from cracking has, however, again come to the fore in connection with the use of high tensile steels and with the execution of structures of ever increasing span. In accordance with the German regulations for reinforced concrete the type of commercial steel which has hitherto chiefly been used may be stressed up to 1200 kg/cm^2 in ordinary cases, and more recently a permissible stress of 1500 to 1800 kg/cm^2 has been authorised for St. 52. Generally the adoption of these higher stresses in the steel is associated with higher tensile stresses in the concrete, with the result that the margin against cracking becomes smaller.

During the past ten years the spans of girder bridges have continually been increasing. Thus the bridge across the Danube et Grossmehring, completed in 1930, has a span of 61.50 m over the central opening, and the bridge of the SA in Bernburg crosses the Saale by a span of 61.78 m. Both of these are girder bridges with a supended span in the central opening over the water. Large halls have been built as two-hinged frames of approximately 53.0 m span, and at the end of this paper reference will be made to a statically determinate roof structure carried on two supports at a clear distance of 50.0 m with a span of 50.80 m between centres of bearings.

It is to be anticipated that this process of development will continue, and that in the future even greater spans will be bridged by reinforced concrete structures subject to bending. In these large spans it becomes of cardinal importance to reduce the dead weight to a minimum, and the sections of the reinforced concrete members must, therefore, be made as light as possible. As a result, the portion of the cross section of concrete which is stressed in tension will be reduced in area; the tensile stress therein will be correspondigly greater and the safety against cracking will be smaller.

The direct tensile strength of concrete such as is used in reinforced concrete structures lies between 12 and 25 kg/cm^2 according to the quality of the work. The bending tensile strength, which as a rule affords a better standard of comparison, may be taken as 25 to 30 kg/cm^2, but it is to be noticed that the upper limit is reached only with the best possible workmanship, using aggregates of the highest possible quality and a correspondingly small water content.

The extensibility of concrete in tension lies between 0.1 and 0.2 mm per m; that is to say when this amount of elongation is exceeded the concrete begins to crack. In selected types of concrete it may be possible to increase the amount to 0,3 mm per m. In making this statement no account has been taken of plastic strain, the magnitude of which, in concrete under tension, has hitherto been little investigated, and the possible effect of which may be to increase the total elongation two or three times.

The shrinkage of concrete suitable for use in reinforced concrete structures is usually given as about 0.4 mm per m, a large proportion of the total shrinkage being attained by the end of a few months. The concrete continues, however, to shrink slowly for a further period, and does not reach its final dimensions till the end of about five years. The rate at which shrinkage proceeds is greatly influenced by the degree of dampness or dryness of the surrounding air. It is known that concrete will shrink very rapidly in warm dry rooms whereas under water it will not shrink at all, but on the contrary, will swell.

The shrinkage value of 0.4 mm per m as stated above can only be taken as a laboratory figure. In massive structures, and indeed in all-work out of doors, the shrinkage is less, being reduced by the natural dampness of the surrounding air. If the amount of shrinkage be reckoned at 0.15 to 0.20 mm per m a considerable part of the extensibility available in the concrete will already have been utilised even if it be assumed that plastic deformation has operated to relieve the load. It has been shown that reinforced concrete structures inside closed buildings and exposed to rapid drying may show fine hair cracks from shrinkage alone.

In the case of reinforced concrete girders of long span the tensile stress imposed on the concrete by shrinkage is usually smaller than the stresses caused by the external loads, particularly those due to dead weight and live load, and possibly the temperature stresses. Elongations of 0.2 to 0.4 mm per m must be reckoned with if the construction is to be economically feasible, and in most cases this in itself implies that the extensibility of the concrete is exceeded, with the result that the fine cracks on the tension side will become more pronounced and be apparent even to the unpracticed eye.

It is easy to understand the desire that these cracks should, as far as possible be eliminated, even though in most cases they are merely defects of appearance. The most effective solution to this problem would be for the cement industry to produce a cement capable of conferring a higher tensile strength on the concrete, or alternatively one which would reduce the modulus of elasticity E for concrete in tension, thereby increasing the elongation.

It is obvious that comparison with natural stones affords no great hope of this, for the latter all possess much higher compressive than tensile strengths. Another warning against exaggerated expectations may be found in the fact that during the past few decades scarcely any advance has been made in the matter of increasing the tensile strength of concrete. If, however, it became possible merely to increase the *bending* tensile strength of a good concrete, which may to-day perhaps be put at 40 kg/cm², by some 50% to 60 kg/cm², a great step forward would have been made, and many forms of structure would become possible which at present are excluded by the risk of cracking. Having regard to the low value of the tensile strength of concrete as such, a 50% increase within a reasonable time may not perhaps, be outside the bounds of possibility.

A further measure that might be generally adopted would be so to regulate the final distribution of stress, by the use of pre-stressing, that the tension in the concrete would be limited to an acceptable amount. Such pre-stressing might be applied either to the concrete or to the steel, but in the former alternative it would be necessary to use a type of cement which expands instead of contracting as does the cement now in use. Whether the cement industry is in a position to produce a cement of this kind is an open question: according to an article by *Henry Lossier*, «Les Fissures du Beton Armé»,[1] the French industry appears likely to do so in the near future.

Attempts to pre-impose a stress in the concrete through the medium of the steel are almost as old as reinforced concrete construction itself, having been suggested by *Koenen* in a paper entitled „Verfahrung zur Erzeugung einer Anfangsdruckspannung in Zuggurtbeton von Eisenbetonbalken" which appeared in the Zentralblatt der Bauverwaltung in 1907.

To ensure that no cracks shall appear in the concrete, which implies that the tensile strength of the latter must never be exceeded, it is necessary to calculate and dimension the structure in such a way that the bending tensile stress σ_{bz} is kept within reasonable limits. According to the present practice of reinforced concrete design it is quite rightly customary to take no account of the bending tensile stress, for this affords no proper basis for calculation, and is affected by shrinkage, which in turn depends on the arrangement and cross section of the steel. The regulations for reinforced concrete are so framed that if they are carefully and correctly followed there is no chance whatever of hair cracks arising. According to DIN 1075, „Berechnungsgrundlagen für massive Brücken" an

[1] Le Génie Civil, 1936, pages 182 following.

estimation of the bending tensile stress is necessary in reinforced concrete girder bridges of more than 20 m span, and the matter is governed by the requirement that σ_{bz} must not be greater than one fifth of the calculated compressive strength of the concrete, failing which special precautions are to be taken against dangerous cracking. This regulation is important, and might well be applied in a similar way to roof trusses of large span or to other such structures in reinforced concrete building work.

In an article entitled „Die Donaubrücke Großmehring"[2] the author has calculated the maximum tensile bending stresses in a series of long span reinforced concrete girder bridges and obtained values between 37 and 47 kg/cm². In the bridge over the Saale near Bernburg, already mentioned, the corresponding

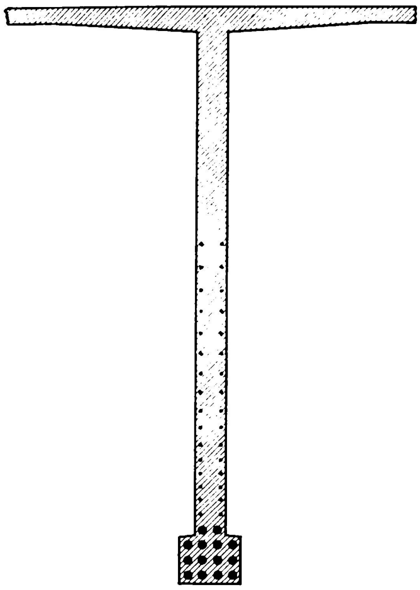


Fig. 1.

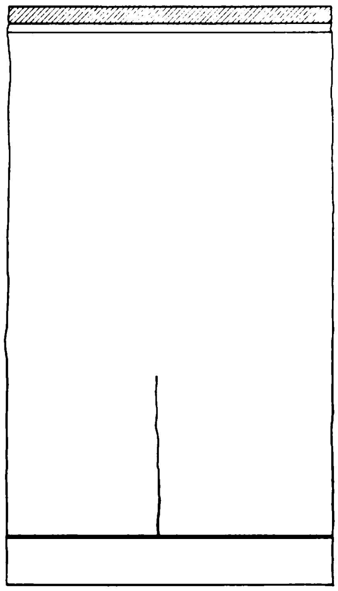


Fig. 2.

maximum value of σ_{bz} is 55 kg/cm². In the present state of concrete technique it would appear inadvisable to exceed this upper limit without quite special precautions.

In reinforced concrete structures of long spans the girders are even now being made of considerable height, for instance in the bridge over the Danube near Grossmehring which has already been mentioned several times the depth is 2.75 m at the middle of the span and 5.40 m over the bearings. When the cross section is of this magnitude the calculated tensile reinforcement placed close to the extreme fibre may properly be supplemented by adequate longitudinal reinforcement below the surface covering the whole of the tension zone, with a view to preventing the formation of cracks between the actual tensile reinforcement and the neutral axis, or at any rate to lessen their concentration. In the case of a section like that shown in Fig. 1, wherein the lower boom has been widened to accomodate the necessary tensile steel, it has been found that whereas the concrete shows no tendency to crack in the enlarged section at the bottom,

[2] Zentralblatt der Bauverwaltung, 1931, pages 123 following.

it is apt to do so in the relatively thin web portion above, and the purpose of the longitudinal bars indicated in Fig. 2 is to combat this tendencey. The presence of a large number of steel bars in the lower boom portion has the effect of increasing the extensibility of the concrete there, and this is supplemented by some measure of plastic deformation, which explains the absence of cracking.

Two mistakes that are frequently made may be mentioned at this juncture: the first is that of crowding many hooks into the same cross section, and the second is the occurrence of changes in the cross section due to openings or offsets. Wherever possible hooks should be avoided in the tension zone of the concrete, though this may not always be possible in cross sections subject to great variation in stress, as for instance at the corners of frames; in any case however, it is undesirable that a number of bars should terminate in hooks at the same place. The presence of the bent hooks is equivalent to a considerable

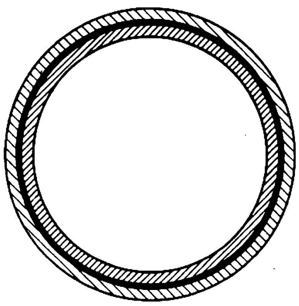


Fig. 3.

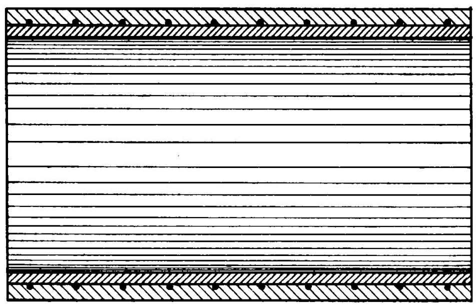


Fig. 4.

reduction in the cross section of concrete which may originate a crack at the point affected.

For similar reasons, openings or sudden changes in section are undesirable and even small openings like those required for the passage of cables and the like should not, if possible, be placed in the tension zone. If such an arrangement is unavoidable additional steel should always be placed around them in order to prevent the formation of cracks.

The expedient of subjecting the steel to a preliminary stress as mentioned above has frequently been followed with success in the case of reinforced concrete members produced under factory conditions. An interesting example of this is provided by the Ruml pipe, in which the annular reinforcements are pre-stressed so as to avoid the occurrence of tension in the concrete under the tangential forces which result from heavy hydraulic pressure inside, and on account of the pre-stressing only compressive stress arises.[3] Fig. 3 shows a cross section and Fig. 4 a longitudinal section through such a pipe. The inner portion, as far as the steel, is concreted first in suitable shuttering, and when this has hardened the steel reinforcement heated in an oil furnace is wound tightly over the concrete core, the third stage of the process being the formation of the outer portion of the concrete. It is claimed that these pipes are completely water-

[3] See „Eisenbetonrohre R. T. System Ruml", by Dr. *F. Emperger*, Beton und Eisen, 1931.

tight even under pressures of ten atmospheres, and they have been widely used in Czechoslovakia and several other countries.

Where reinforced concrete structures are built on the site, pre-stressing has been used with success for purely tension members such as for instance the anchorage of a roof truss or an arch bridge. At the suggestion of *Dischinger*, tie-bars for taking up the horizontal thrust of reinforced concrete arch bridges have been pre-stressed by the use of hydraulic jacks. *Pujade-Renaud* in a paper entitled «Les hangars triples à hydravions de la base maritime de Karouba (Tunisie)»[4] has described French aeroplane hangars with roofs of arched construction wherein the horizontal thrust, if too great to be resisted by the ground itself, is withstood by circular steel bars extending from one bearing to another and embedded in the ground. In this instance the pre-existing stress has been obtained by forcing the bars apart at their centre.

Pre-stressed tie-bars of this kind have also been successfully applied in Germany for large covered hall structures built in the form of arches. Usually hydraulic jacks have been employed, by means of which the desired amount of load can be accurately imposed.

Figs. 5 and 6 show a wide span roof over a hall without intermediate supports consisting of relieved arches of 100 m span between the abutments. The arch ribs are placed at 5 m centres and bear at each end on continuous abutments, to which the anchorage of 40 m diameter bars is attached in the usual way by means of hooks embedded in the concrete. The tie-bars were interrupted at the centre of the hall in order to insert the hydraulic jacks used for pre-stressing, an arrangement which offers the advantage that the full half length of the tie-bars, measuring 53 m, could be obtained ready made from the rolling mills and did not therefore need to be welded on the site. Details of the joints in the tie-bars and the method of stressing are shown in Fig. 7: the bars connected to the left hand abutment are secured at their right hand ends in an anchoring beam to the right of the centre of the hall, and those which connect with the right hand abutment are similarly embedded at their left hand ends in a second anchoring beam placed to the left of the centre line. The respective tie-bars from either side thus overlap by about 3 m at the middle; those coming from the left pass through the left hand anchoring beam, and those from the right through the right hand anchoring beam, by the way of gas pipes. Hydraulic jacks of 50 tons lifting capacity were placed in the space between the two anchoring beams, enabling the latter to be forced apart and the required stress imposed in the tie-bars. This pressure was applied simultaneously with the removal of the scaffolding which was carried on screw jacks, and during the operations the relative positions of the abutments were read on Zeiss dials giving an accuracy of $\frac{1}{100}$ mm. A certain amount of tension was first applied to the tie-bars and the lowering of the scaffold was then begun: as soon as the Zeiss instrument on the abutments began to indicate a movement the pre-stress was increased and the scaffold was lowered by a further amount, and so on alternately until the calculated amount of pre-stress had been obtained in the

[4] "La technique des travaux", 1934, pages 85 following.

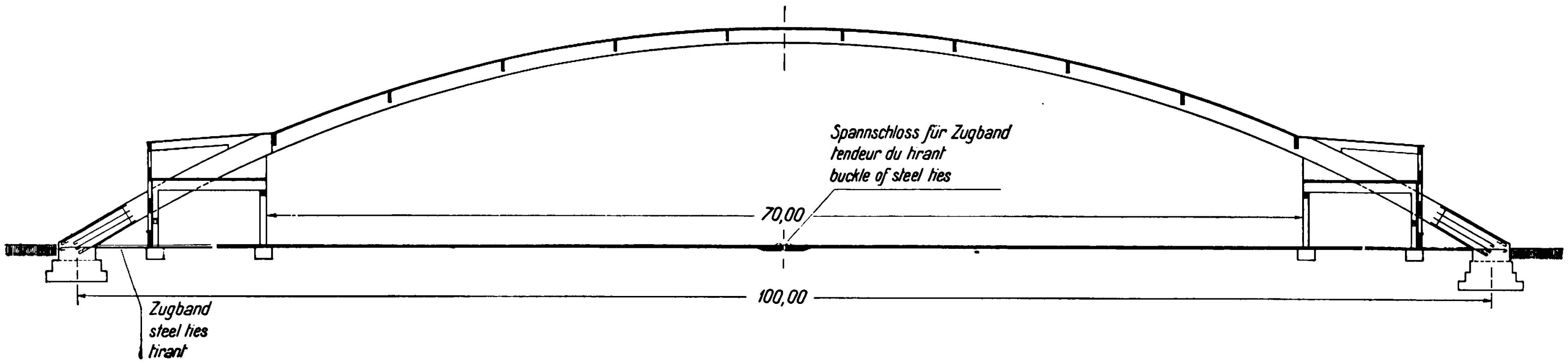


Fig. 5.

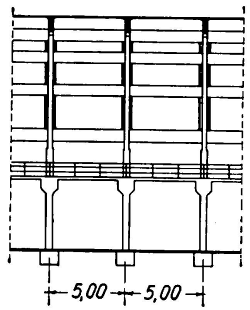


Fig. 6.

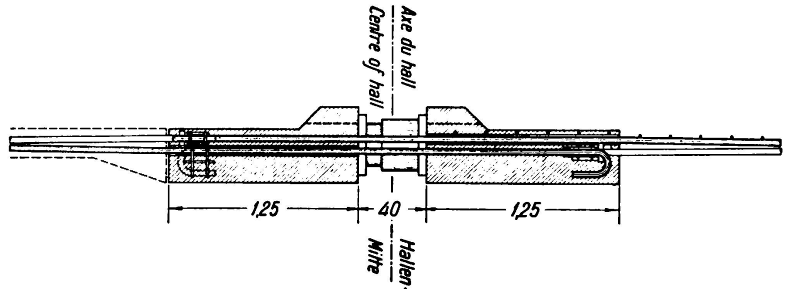


Fig. 7.

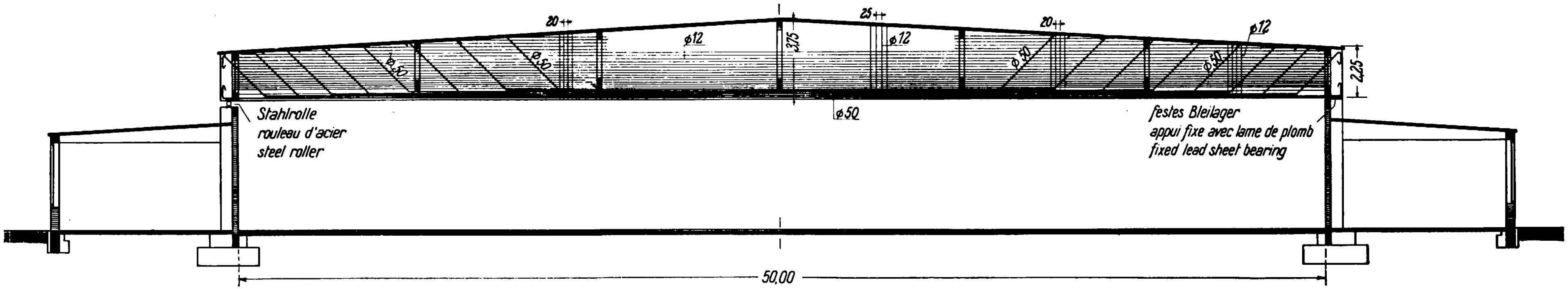


Fig. 8.

tie-bars and the scaffold had been completely struck. The displacement of the abutments during the operation amounted to less than 1 mm (an amount which was, of course, quite insignificant in a span of 100 m) and the extension of the tie-bars to 58 mm.

The pre-stressing of the tensile reinforcement in a member subject to bending which is to be concreted on the site constitutes a more difficult problem, and in recent years many proposals in this direction have been made. *Freyssinet* in his recent book «Une révolution dans les techniques du béton» describes a reinforced concrete girder which was pre-stressed so as to place all the steel in tension by the amount necessary to take up the thrust.[5] The process is well conceived but takes up a good deal of time, and its economy does not appear to be established. Hitherto it has always been found that such proposals are difficult to carry out by the means available on the site, and it remains to be seen whether the future will bring a change in this respect.

Apart from the constructional devices described above, all necessary precautions must be observed to increase the tensile strength of the concrete and to make full provision against the formation of undesirable cracks. It is important to take all possible steps to secure a concrete of high tensile resistance, such as the choice of suitable cement and aggregates, careful concreting and after-treatment of the concrete.

No such wealth of experimental results is available for the tensile or bending tensile strength of concrete as for its compressive strength. For the present, therefore, we must be content with the broad generalisation that an increase in the tensile strength follows similar laws to that of compressive strength. In other words the stronger the concrete in tension the stronger it will be in compression, though the two strengths will not increase in the same proportion. It is not to be recommended that reinforced concrete structures of long span, which are exposed to bending stresses, should be made from concrete of the usual quality having an average compressive strength of perhaps 150 to 180 kg/cm^2, but if certainty against cracking is to be ensured the concrete must show a compressive strength of 250 to 300 kg/cm^2 or preferably more.

To obtain strengths of this order it is necessary to use cements of high quality (indeed of exceptionally high quality), giving the maximum possible tensile strength. They must also be so chosen as to cause a minimum amount of shrinkage, at any rate initially. Every technician knows that the liability of cement to shrinkage is very variable; there are some cements which give good results in this respect and others which have been found to shrink excessively. Unfortunately the use of this knowledge must remain a matter of hit and miss until more perfect methods of testing for shrinkage are developed and until cement manufacturers can be called upon to supply their customers with information not only of the fineness and as to the compressive strength that may be anticipated but also as to the shrinkage properties of their product.[6]

[5] See also Volume 4 of the "Publications" of the I.A.B.St.E. and the "Preliminary Publication" of the Berlin Congress.

[6] In this connection see also the present author's paper „Entwicklungsrichtungen im Eisenbetonbau", Bautechnik, 1936, p. 141.

It should further be remembered that in accordance with the explanations given above the tensile bending stress in the concrete may be combined with the tensile stress arising from shrinkage, even though the latter may be reduced by the plastic deformation of the concrete. The amount of cement should, therefore, be liberally proportioned even at the risk of increasing the shrinkage, for the advantage of the greater strength of high quality cement outweighs the disadvantage of the increased shrinkage which may result from the larger cement content and from the greater fineness of the high quality cement.

The modulus of elasticity of concrete is subject to wide variations, depending mainly on the nature and proportions of the aggregates and binding material, the water content, the methods of working and after-treatment followed, as well as on the position and magnitude of the stress. Some detailed numerical data on this matter have been published by *Hummel*,[7] who obtained the following figures for the elongation due to tensile bending of concretes prepared from different aggregates with an admixture of 350 kg of high quality cement per m^3 and a slightly plastic consistency:

Aggregate	σ_{bz} kg/cm²	Breaking values		Compressive strength
		Specific elongation $1 \cdot 10^{-4}$	E kg/cm²	kg/cm²
Red quartz porphyry . .	48	2.94	163 000	479
Quarzite	49	2.89	169 000	483
Grey stone chippings . .	50	2.66	188 000	485
Broken stone	44	1.98	222 000	488
Basalt chippings	48	1.93	249 000	555

With the same bending tensile strength of $\sigma_{bz} = 48$ kg/cm² the specific elongation at fracture of concrete made with red quartz porphyry is 2.94×10^{-4} and that of concrete made with basalt chippings only 1.93×10^{-4}. It is understandable that concrete with a larger value of ε and a smaller value of E may be less susceptible to cracking than a concrete with a smaller ε and a larger E, and it is important that the implications of these experiments should be applied in the selection of aggregates.

The arrangement and sections of the reinforcing bars also are important from the point of view of crack formation. The bars should be placed as far as possible apart, though this rule is difficult to follow in the case of wide span structures wherein the section of concrete has to be reduced to a minimum and where, therefore, a smaller number of larger bars must be used. In Germany it is customary to use round bars almost exclusively. A few years ago the Isteg steel was placed on the market, and it is possible that bars of this kind, or bars with protrusions, may contribute to a finer distribution of the cracks even if they do not increase the carrying capacity.

In roof trusses of long span it will frequently not be possible to dispense with construction joints, having regard to the working conditions in concreting and

[7] „Beeinflussung der Betonelastizität" by Dr. Ing. *A. Hummel*, Zement, 1935, pages 665 following.

to the availability of suitable plant, the erection of scaffolding, etc. So far as possible such construction joints should be placed only in the compression zone of the concrete and should run at right angles to the direction of pressure. If it is impracticable to avoid their presence in the tension zone they will entail a reduction in the available concrete section and it is essential that this should be made up by "knitting" the concrete together across the gap by a large number of additional bars.

The concrete increases in strength during the first few weeks and months, so that the later it receives its full stress the better its quality and the greater the guarantee against the formation of cracks. It is important, in this connection, that the concrete should be protected from premature drying immediately after it has set, by covering it over and keeping it damp. A concrete which is permanently kept damp shrinks only a small amount or may even swell, and by this means the additional stresses due to shrinkage may be avoided during the period in question and for some time afterwards, the final shrinkage stresses being

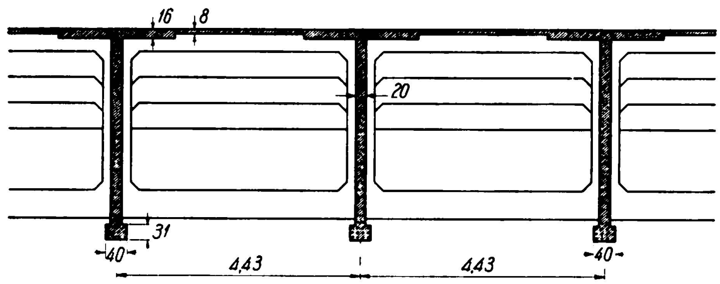


Fig. 9.

thus considerably reduced. It is also important to delay the removal of shuttering as long as possible, or if this cannot be done to provide temporary supports capable of carrying the whole weight of the structure.

In conclusion a description may be given of one more roof girder of large span which was built with due regard to the considerations mentioned above and has proved entirely successful.

The roof system shown in Figs. 8 and 9 covers a clear space of 50.0 m and consists of statically determinate girders on two supports with spans of 50.80 m. So far as the author knows, this is the first occasion that a structure of the type in question has been employed over so great a span. The spacing of the girder is 4.43 m and the roof covering is a reinforced concrete slab 8 cm thick. The latter is thickened to 16 cm over the girders in order to provide an adequate compression boom at these places. The cross bracing of the girders is provided by framed purlins at 8.40 m centres. The depth of the girder at the centre is 3.75 m and tapers to give a suitable fall to the roof being 2.25 m at the end. The thickness of the web is only 20 cm and in order to accomodate the tensile reinforcement the lower edge is thickened to 40 cm with a height of 31 cm. The bearing at the fixed end is on lead and that at the movable end on steel rollers.

The reinforcement of a girder is represented in Fig. 8 the tension bars being of 50 mm diameter and stressed up to a maximum of 1200 kg/cm^2. In addition to these and the vertical stirrups, which are 12 mm in diameter, longitudinal reinforcing bars also of 12 mm diameter were provided in both the faces of the web with a view to preventing the formation of cracks. In the compression zone at the middle of the span it was possible to omit these, but at the two ends they were provided over the whole depth in order to co-operate with the bent-up longitudinal bars and the vertical stirrups in carrying the shear forces. In girders of this depth, so limited in thickness, special care is needed to ensure neatness and accuracy in arranging the steel according to plan.

The cement used was a high grade type supplied by the Thysen works under the name of Novo, and the aggregate was made up of sand and quartz porphyry chippings. Test cubes made during the concreting operations showed compressive strengths of over 400 kg/cm² at 28 days. Special precautions were necessary in pouring the concrete in view of the large number of reinforcing bars and also because the Novo cement began to harden after a few hours in the warm summer temperature. These difficulties, however, were entirely over-

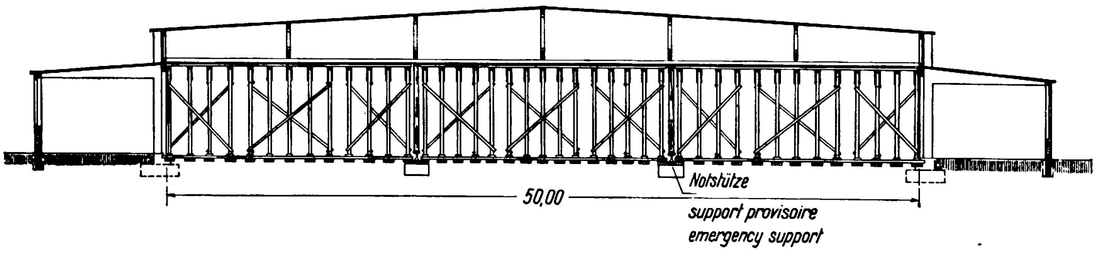


Fig. 10.

come with the result that all of the girders were concreted in such a way that on removal of the shuttering not a single cavity was found, nor did any bad place need to be patched.

To allow of the concrete being kept properly damp during the first few weeks provision was made for sprinkling it from water pipes close to either side of each girder, the pipes having small holes at 20 cm intervals through which the water was squirted against the sides of the girders. It was possible in this way to keep the concrete uniformly and continuously damp for about six weeks, after which the sprinkling had to be discontinued on account of the progress of other operations in the construction of the hall.

The shuttering for one of the girders is shown in Fig. 10, consisting of a simple framing of round scaffold poles arranged to transmit the load to the ground in the most direct way, but in view of the necessity for completing the floor as early as possible it was not feasible to allow these supports to remain in position as long as might have been wished. In order, therefore, to permit the shuttering to be struck from the girders as late as possible, temporary supports were provided in advance under the third points, each consisting of two heavy round timbers resting on screw jacks at the foot and capable of carrying the whole dead load of the girder. These temporary supports did not interfere with further building work of the hall — particularly of the floor — and they did not, therefore, have to be removed until about six weeks after concreting, whereas the other supports had had to be taken away after about three weeks.

The total deflection of the girders on the removal of the temporary supports amounted to about 5 cm at the middle, representing approximately $\frac{1}{1000}$ of the span, and in the course of years this deflection will increase to some extent on account of plasticity. To provide for this increase, and particularly with a view to good appearance, the lower boom of the girder has been given a camber of 24 cm on the centre line beforehand. All the work was carried out to perfection and none of the girders showed cracks of any kind.

Finally, it is proper to lay every emphasis on the fact that cracks which result merely from the tensile strength of the concrete having been exceeded, and which do not imply any structural defect such as insufficient steel reinforcement, are not dangerous to the work except in so far as they may allow rusting of the tensile reinforcing bars under load. Apart from this contingency such cracks have no significance, and in order to prevent the rusting of the bars it is sufficient that the cracks should be pressed out or covered over with a spray of cement grout, an elastic paint or an elastic compound after three or four years have elapsed and the shrinkage is approaching its final amount.

IIb3

Effect of Petrographical Properties of Aggregates on the Strength of Concrete.

Einfluß der petrographischen Eigenschaften der Zuschlagstoffe auf die Betonfestigkeit.

Influence des propriétés pétrographiques des matériaux additionnels sur la résistance des bétons.

Dr. Ing. A. Král,
Professor der techn. Fakultät an der Universität Ljubljana.

Arising out of the papers published in Section IIb of the Preliminary Publication of the Congress, it would appear useful to refer to some straightforward but characteristic series of experiments on concrete which have been carried out in the Materials Testing Laboratory of the Technical Faculty of the University of Ljubljana in Yugoslavia, with a view to closer investigation of the stone found in large quantities within the jurisdiction of the Banat of Drave, from the point of view of its suitability for making high quality concrete.

The district in question lies at the north west corner of the country and includes the eastern chains of the southern limestone mountains in addition to the northern or Carinthian region of the Dinaric Alps. From these orographical characteristics it will be at once apparent that the whole region contains mainly limestone with some dolomite. In the central Drave valley and in the transition to the eastern central mountains there exists, however, a fairly pronounced massif of foothills known as the Pohorje which consists mainly of primary rocks, and which in addition to some softer sedimentary rocks contains an excellent plutonic rock known as *Tonalite*, which is a special form of diorite and is typical of the boundary region between the central and southern alps. This rock differs from granite in having a smaller quartz content, varying between 16 and 31 %; its main constituent is plagioclase. The stone is uniformly of medium to fine grain and is well compacted.[1]

In the alpine chains there occur porphyritic intrusions, notably *Keratophyr*, which are intermediate in quartz content between the granitic and syenitic groups, and are classed as magmatic rocks; these show a fine porphyritic structure. In the spurs of the alps which form the boundary of the Panon plain many veins and blocks of andesite occur; here again the main constituent is plagioclase with occasional grains of magnesite and volcanic glass. The structure varies from fine-grained to amorphous, and as a result of the low degree of crystallisation and of

[1] This and all the following mineralogical and petrographical details have been supplied through the great kindness of the Mineralogical-Petrographical Institute of the University of Ljubljana under Professor *V. Nikitin.*

the presence of the volcanic glass the stone is fairly brittle, but apart from this it must be regarded as a good material which is suitable for the purpose named.

In the past, use has been made not only of these local magmatic rocks but also of a basalt, found in the Lavan Valley of Carinthia in immediate proximity to the Yugoslav frontier, although in Austrian territory. The existence of favourable railway connections has made possible a fairly extensive use of this material in Yugoslav territory; the stone shows the usual characteristics of a material of normal quality, is very uniform, and has a fine grained structure.

These four kinds of magmatic rocks were the subject of the experiments mentioned above, and two further limestones and two dolomites were examined at the same time for purposes of comparison. The first of these limestones comes from the northern edge of the Carinthian region at Verd, south of Ljubljana; it is a palaeozoic material with a fairly high content of silicate admixtures. The second limestone comes from Trbovlje and belongs stratigraphically to the trias formation; it is mainly pure and contains only very small admixtures. The two dolomites also come from triassic strata in the foothills of the eastern alps, and differ only as regards their origins at Trbovlje and Senovo respectively.

The sand and small material obtained from these rocks was put together as far as possible in accordance with *Fuller's* sieve curve. In the tonalite, and in one series of tests with basalt, pure quartz sand of less than 1 mm gauge was added as a filler. High grade cement to the following specification was employed: —

At two days 27 kg/cm^2 tension, 377 kg/cm^2 compression.

At seven days 36 kg/cm^2 tension, 636 kg/cm^2 compression.

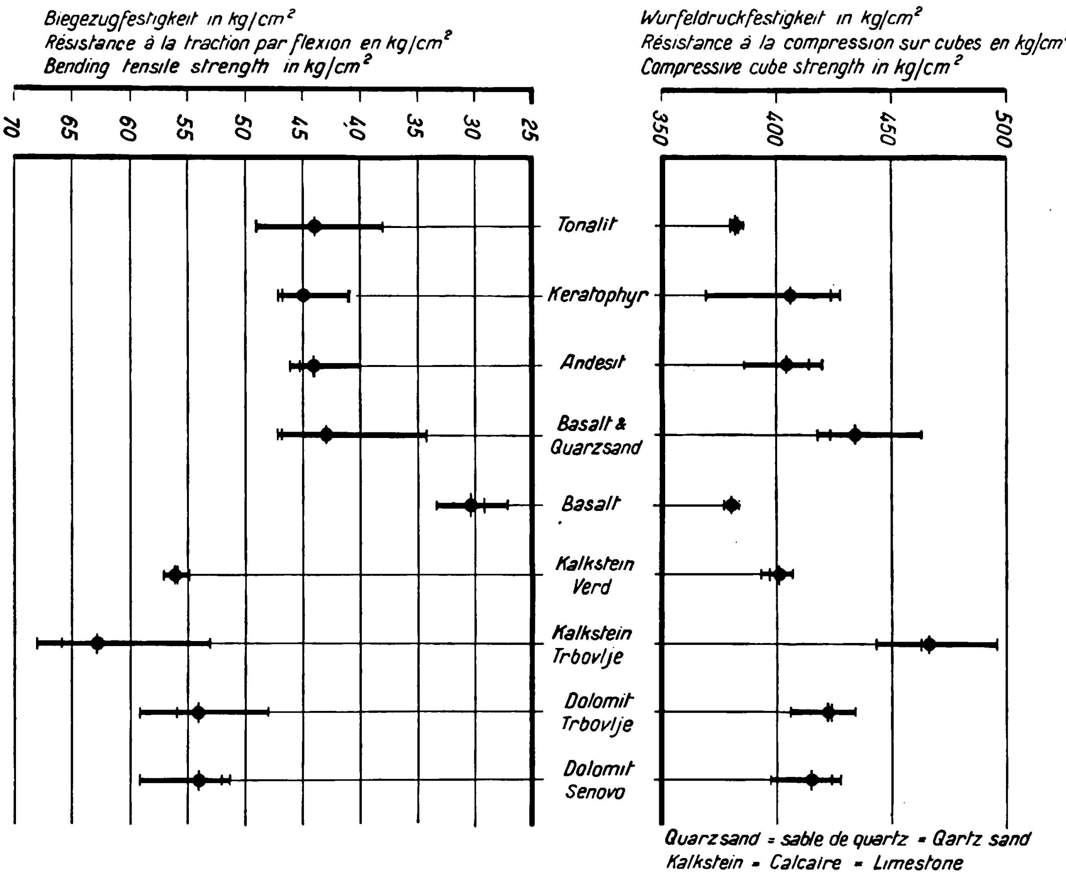
The test specimens were prepared in accordance with the Yugoslav standards using the *Tetmajer-Klebe* tamping apparatus.

The cement admixture of 400 kg/m^3 of concrete was made with a water-cement ratio of 0.5; and the consistency was further controlled by the American slump method, in order to ensure the greatest possible uniformity of the concrete made with all kinds of stone.

From the wide range of experiments carried out with these materials, only the cube and bending strengths at 28 days are given below, in a graphical form for the sake of convenience in perusal. Despite the relatively small scope of the statistical data, the following conclusions may be drawn:

From the point of view of compressive strength no appreciable difference is disclosed between the concrete made with magmatic rock and that made with calcareous rock, most of the average values being 400 to 450 kg/cm^2 and the deviations from the average being for the most part less than 10 %, or in many of the series of experiments inappreciable. The diagram of tensile strength is more instructive: the tensile strengths of concretes made with magmatic rock all lie close to the average value of 45 kg/cm^2, whereas it is clearly apparent in the case of the limestone groups that the tensile strength averages close to 55 kg/cm^2. Even the relatively large scattering of values for the bending strength here obtained cannot impair this interesting indication, and the minimum strengths in the calcareous groups are also noticeably higher than the maximum strengths in the magmatic groups.

A further interesting comparison may be made between the strength of the concrete made from the Verd limestone and that made from the Trbovlje limestone. The whole of this region, the geography of which was explained at the beginning of this note, lies in the region of contact between the Alps and the Dinarides. Owing to the orogenetic processes which are known to have taken place there the earth's crust has been violently compressed, and this is clearly apparent in the microscopical structure of the rocks in question. They all show consequences of such pressure having been exerted in a variety of orientations, and throughout the region the cohesion of the stone depends to a large degree on whether this orogenetic pressure has been exerted on it from all sides at a great



depth or whether, on the other hand, it has been the result of later infiltrations of calcite into the cracks and cleavages by a secondary process. The Verd limestone being older, and having clearly been produced in lower strata by pressure in addition to having undergone this secondary infiltration of calcite, is better bonded and much more uniform than the Trbovlje limestone, but on the other hand the latter is a good deal purer. This is the probable explanation of the higher strength obtained in the latter material and also of the greater scattering of values both for compressive and bending strengths. The diaclase apparent in its micro-structure shows every sign of having been a further disturbing cause in the coherence of the concrete mass.

In spite of this lack of uniformity, it seems justifiable to conclude that calcareous rocks offer a higher degree of adherence for cement mortar than is afforded by the otherwise stronger magmatic rocks, and the result of this is a higher tensile (though not compressive) strength in the concrete for which they form the mineral skeleton.

IIb 4

Means of Increasing the Tensile Strength and Reducing Crack Formation in Concrete.

Mittel zur Erhöhung der Zugfestigkeit und zur Verminderung der Rissebildung im Beton.

Moyens d'augmenter la résistance à la traction et de diminuer la formation des fissures dans le béton.

M. Coyne,
Ingénieur en Chef des Ponts et Chaussées, Paris.

The writer has had occasion during the past few years to construct a large number of retaining walls of the following type: the face is entirely of masonry or of reinforced concrete of limited thickness, regardless of the height of the

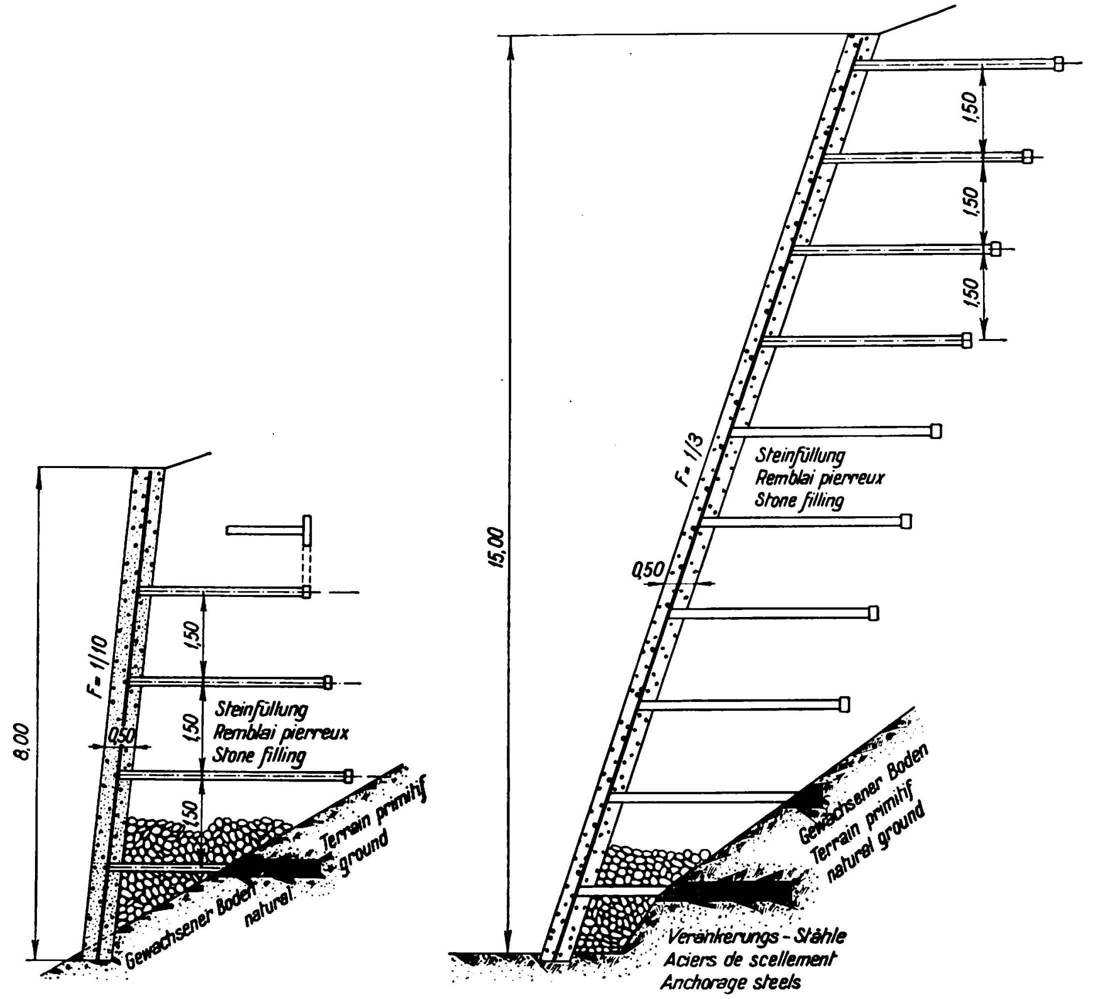


Fig. 1. Retaining walls on Coyne's ladder system — Cross sections.

wall, stability being obtained through the agency of relatively short tie-bars which are contained almost entirely within the prism of pressure.

An explanation of the mechanism whereby the stability of these structures is ensured will be found in an article in Le Génie Civil dated 29th October, 1927.

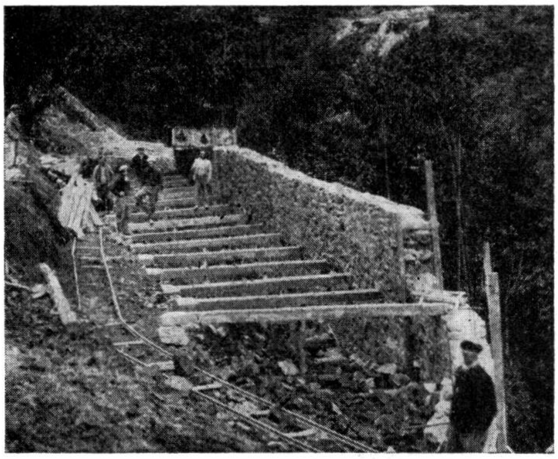


Fig. 2.
Retaining wall, ladder system (8 m high).

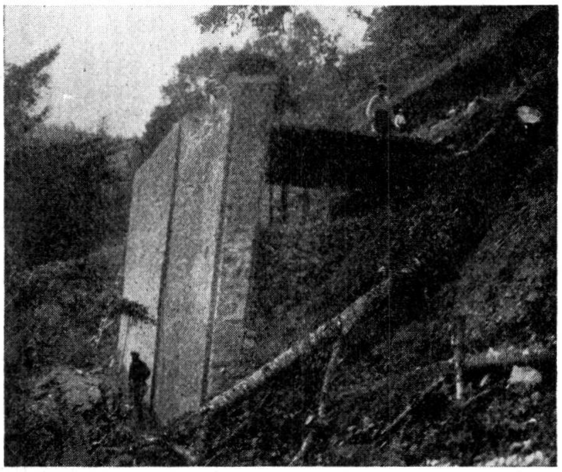


Fig. 3.
Retaining wall, ladder system (8 m high).

The name "ladder retaining walls" has been applied to them. A few examples are shown in Figs. 1, 2 and 3.

The construction of the tie-bars, which are of reinforced concrete, involves a special problem, in that the settlement of the ground causes the tie-bars to deflect, whereas as indicated in Fig. 4 the wall itself undergoes no settlement.

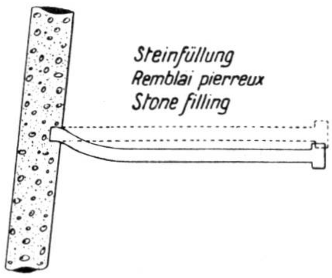


Fig. 4.
Diagram showing the bending of a tie bar due to settlement of the back-fill.

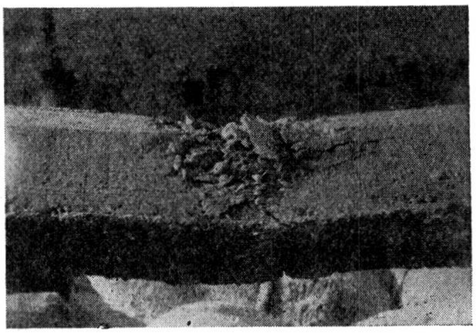


Fig. 5.
The concrete around the tie bar subjected to tension and bending develops cracks which expose the reinforcement to rusting, even though ordinary hooping steel is provided.

The concrete being thus subjected to tension in addition to bending is apt to crack and the steel is thereby exposed to corrosion (Fig. 5). The problem is to reduce this tendency to crack; hence the justification for mentioning the matter under the present heading. The solution is as follows:

The steel is situated at the centre of the tie-bar, and the concrete sleeve which encloses it is in turn surrounded by a steel hoop, the object of which is to prevent or restrain the formation of cracks. If however, this hooping is done in the ordinary way it is useless, since a crack may be formed between successive turns

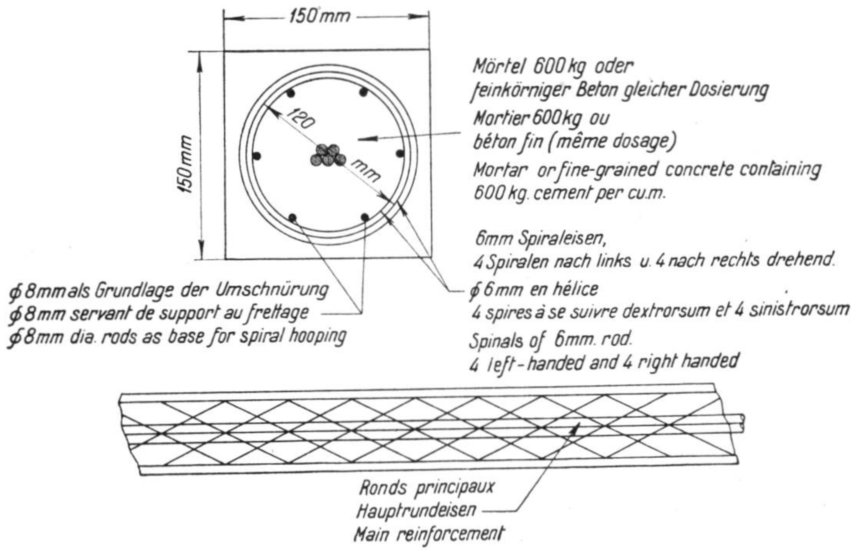


Fig. 6.

Tie bar with special hooping (of coarse pitch).

(Fig. 5). The turns must therefore be arranged in a spiral (Fig. 6) so that, firstly, the cracks are rendered discontinuous, and secondly, the longitudinal tension of the tie-bar is transformed by the agency of the spiral into a lateral

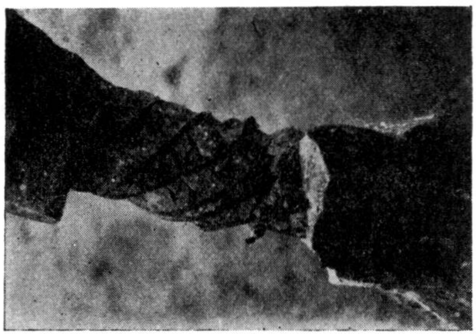


Fig. 7.

Tie bar with special hooping.

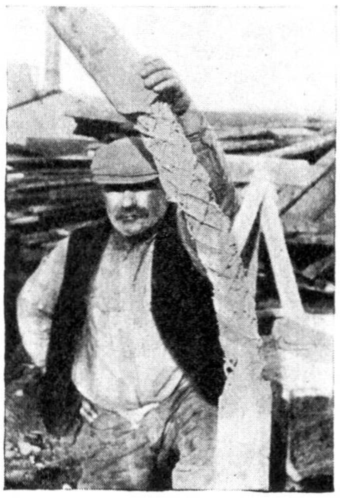


Fig. 8.

Tie bar with special hooping.

restraint. In this way tie-bars are formed which are capable of carrying very heavy bending moments without the concrete core suffering damage (Fig. 7 and 8).

This new method of forming *tensile* joints in reinforced concrete would doubtless be capable of many other applications also.

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II c

Use of high-tensile steel.

Anwendung von hochwertigem Stahl.

Utilisation des aciers à haute résistance.

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IIc 1

Examples of the Application of High Tensile Steel in Reinforced Concrete Slabs.

Beispiele für die Anwendung von hochwertigem Baustahl bei Plattenträgern aus Eisenbeton.

Exemples d'application de l'acier à haute résistance dans les systèmes en dalles de béton armé.

Dr. Ing. H. Olsen,
München.

Hitherto the development of reinforced concrete construction in bridges has been concerned almost exclusively with the design of arch and beam bridges, but in view of the great improvement attained in the mechanical properties of concrete, the production of a structural steel of high yield point, and the endeavours now being made to utilise these two high-grade materials consistently, further constructional development of the slab girder is to be anticipated. This is a type of structure with the merit of clear and simple statical conditions, because as a rule the bending moments are in one direction only, while moreover the shuttering work, the arrangement of the reinforcement, and the placing of the concrete are all notably simplified. Again, slab shaped members, in consequence of the great width of the concrete in the tension zone, show much greater freedom from cracking than is the case with the shallow ribs of T-beams.

The structural possibilities of slab girders when account is taken of the increased permissible stresses will be illustrated here in a few practical examples. The bridges in question were designed by the author and were completed as part of the work on the eastern section of the German Alpine Road in the spring and summer of 1936.

Fig. 1 shows a reinforced concrete slab designed as a *Gerber* girder over three openings of 12.4 m span each, with a roadway width of 8.5 m. The piers and abutments make a wide angle with the axis of the road. The thickness of the slab, only 0.60 m at the side and 0.68 m at the axis of the road, shows the extent that the constructional depth can be reduced through the use of high-grade materials. In the present case the proportion of the mix was 300 kg of ordinary Portland cement per cubic metre, and at 28 days the cube strength of the concrete was 405 and 513 kg/cm^2, allowing a permissible compressive stress in the concrete up to 70 kg/cm^2, while the permissible stress in the steel in the round bars of St. 62 was taken as 1500 kg/cm^2. Loading was assumed in accordance with the German regulations for Class I bridges, including a 24 tonne steam

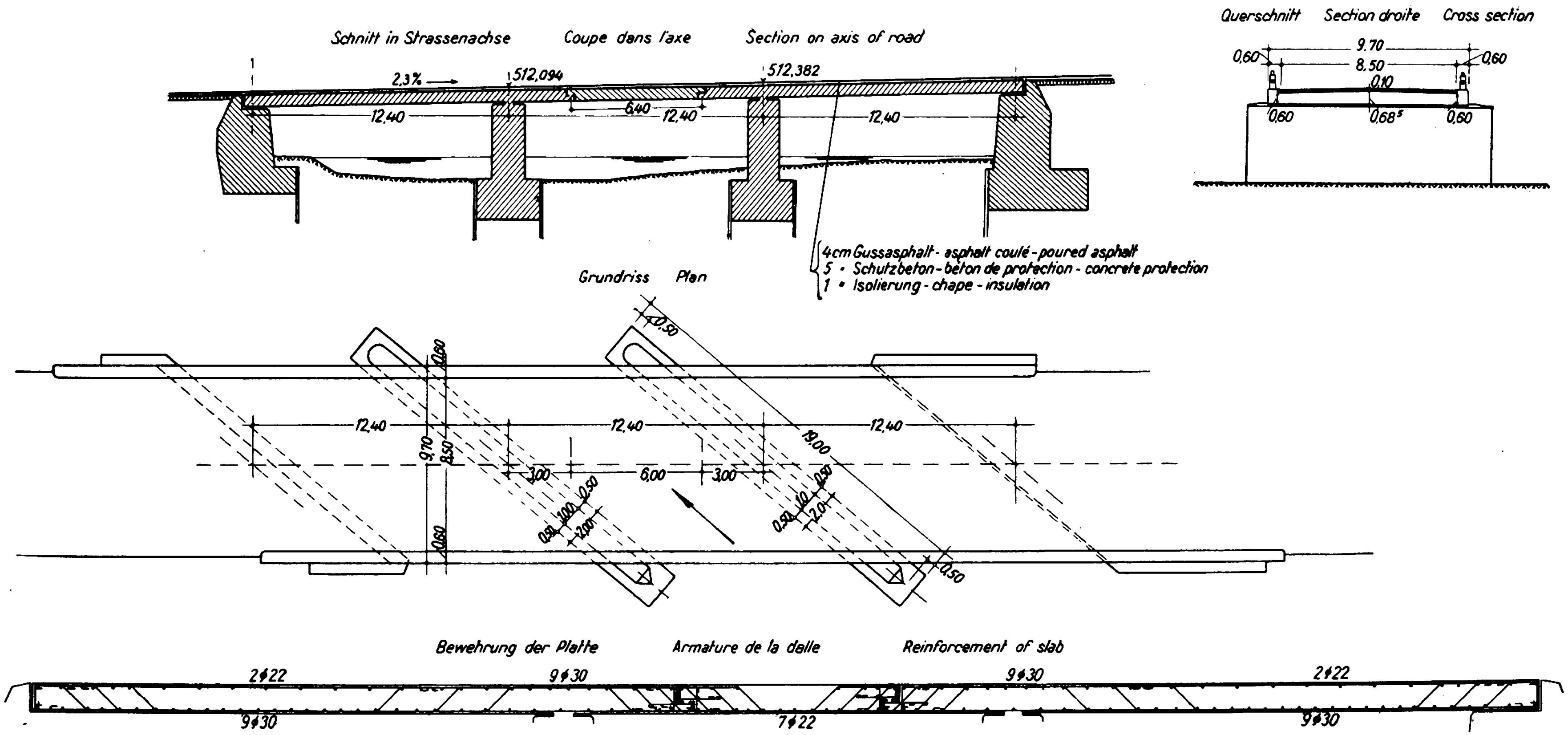


Fig. 1.

Weissbach Bridge II.

roller and a 12 tonne motor lorry uniformly distributed over two lanes of traffic totalling 5.0 m in width, with an impact coefficient of 1.4.

The reinforcement in the direction of the length of the bridge is shown in Fig. 1. With a maximum moment of 51.7 mt/m at the centre of the outside spans and of 60.3 mt/m over the supports, and taking $\sigma = 70/1500$ kg/cm², the design provides nine round bars of 30 mm diameter in each unit of width over the supports. The suspended slab in the central field, which is 6.4 m long and receives a maximum moment of 20.3 mt/m with $\sigma = 42/1500$ kg/cm², is reinforced with seven round bars of 22 mm diameter.

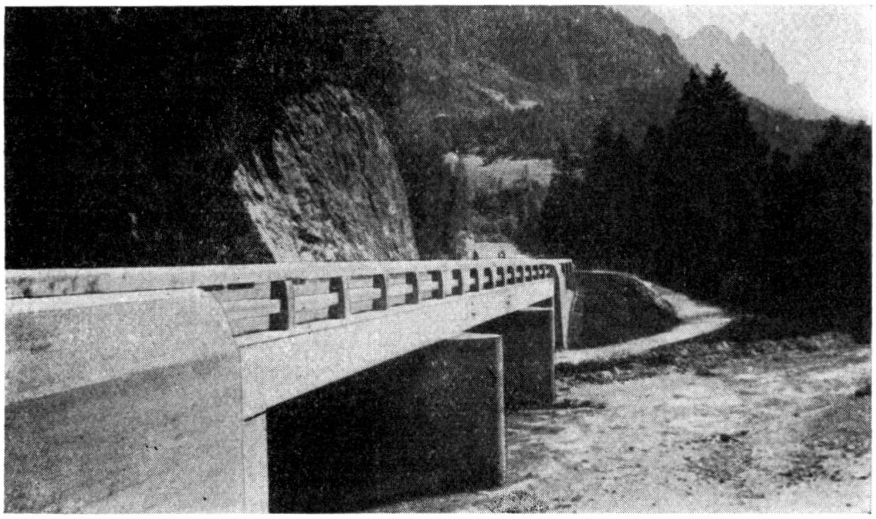


Fig. 2.

Fig. 2 shows the flowing lines in which the bridge crosses the river. The adoption of a timber railing on reinforced concrete posts notably improves the architectural unity of the structure, and this railing runs into massive parapets carried on the wing walls.

Fig. 3 shows another reinforced concrete slab built as a *Gerber* girder over three openings, each of 11.5 m span, with a road width of 8.5 metres. In this case again the two piers and the abutments are askew with the axis of the road. The roadway slab is cambered at 1.5 % and is uniformly 60 cm thick.

Here again the slab was reinforced with round bars of St. 62 subject to a stress of 1800 kg/cm². This stress was justified, among other factors, by the conclusion drawn from the Dresden experiments that such slabs possess notably greater safety against cracking than T-beams, and also by adequate safety against breakage. In the Stuttgart fatigue tests on high tensile steel the further conclusion was drawn that a permissible stress in the steel of 1800 kg/cm² is suitable in slabs, even under moving loads, in cases where the concrete shows a cube strength of not less than 225 kg/cm².

The reinforcement required to resist the standard loading for Class I bridges is shown in Fig. 3. With a maximum moment of 47 mt at the centre of the outside spans and of 45.5 mt over the supports, seven round bars of 13 mm diameter were adopted, the stresses in the cross sections being then respectively

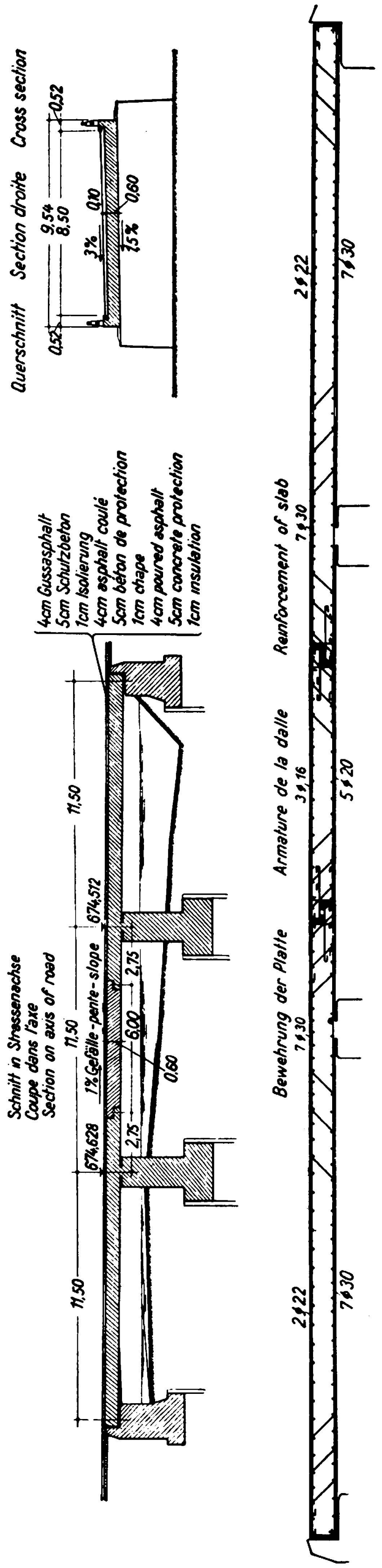


Fig. 3.
Traun Bridge Hinterpoint.

$\sigma = 74/1800$ and $71/1680$ kg/cm². The suspended slab in the central field is of 6.0 m span, and under a maximum moment of 18.8 mt requires five round reinforcing bars of 20 mm diameter for $\sigma = 77/1800$ kg/cm². The cube strength of the concrete made with 300 kg of ordinary Portland cement per cu. metre was 661 kg/cm² at 28 days. This exceptionally high cube strength indicates — as do the cube strengths mentioned above — the particular care with which the concreting of bridges on the German Alpine Road has been carried out. In view of this circumstance the adoption of reinforcing steel of 30 mm diameter was permitted, and provision was made for its firm anchorage in the concrete by means of suitable end hooks.

Fig. 4 shows how well this bridge merges into the surrounding landscape.

By the adoption of framed designs of slab girders it is possible considerably to reduce the constructional depth. Fig. 5 shows a two-hinged slab frame of 10.6 m span with an average depth of 3.25 m. The thickness of the slab of the lintel portion varies from 0.33 m at the side to 0.46 m on the centre line, and the thickness of the side members is 0.60 m. The frame was reinforced with round bars of St. 52 and once again a stress of 1800 kg/cm² was adopted.

Fig. 6 shows the reinforcement of the frame with a maximum moment at the centre of the lintel amounting to 17.9 mt/m, combined with the normal force of 5.5 tonnes, the necessary reinforcement, assuming permissible stresses of $\sigma = 75/1800$ kg/cm², being ten round bars of 20 mm diameter. The fixing of the lintel into the vertical members is calculated for a maximum moment of — 21 mt/m, and taking $\sigma = 50/1800$ kg/cm², eight round bars of 20 mm in diameter are necessary. In the upper portion of the verticals, with $\sigma = 1800$ kg/cm², the necessary reinforcement is seven round bars of 20 mm diameter, and in the middle of the verticals four round bars of that diameter.

Fig. 7 shows the finished bridge, the external lines of which are derived directly from the statical conditions.

By making proper use of the mechanical properties peculiar to high-grade concrete, it becomes possible to construct slab bridges even over large spans, and at the same time the amount of steel required can be much reduced by adopting

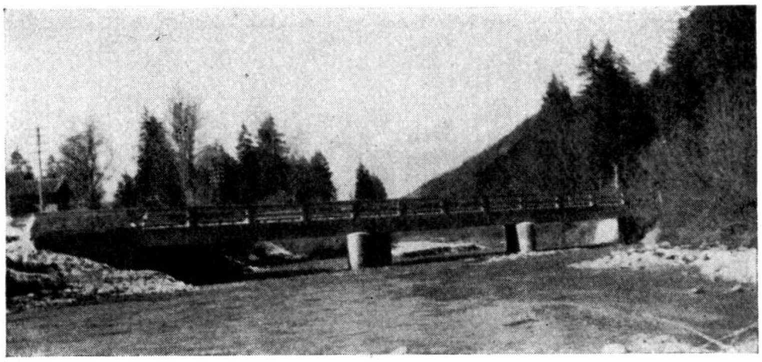


Fig. 4.

high tensile steel and taking advantage of the increased permissible stress therein.

The bridges just described are the first in Germany in which permissible stress in the steel of 1800 kg/cm² has been adopted; this figure exceeds what is

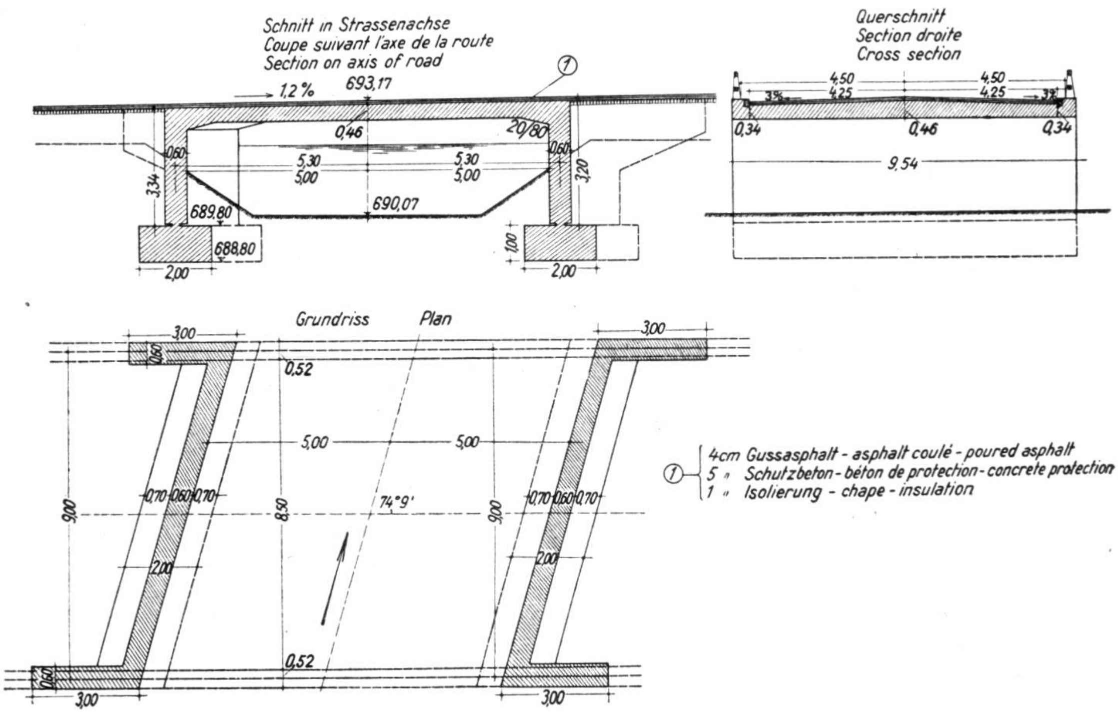


Fig. 5.

Bridge over Grosswaldbach.

allowed by the current regulations, but in view of the knowledge now made available by the testing laboratories its adoption was held to be justified. Moreover the peculiar mechanical properties of high tensile structural steel are confirmed by practical experience in actual work, particularly by the excellent performance noted after six months service under heavy traffic.

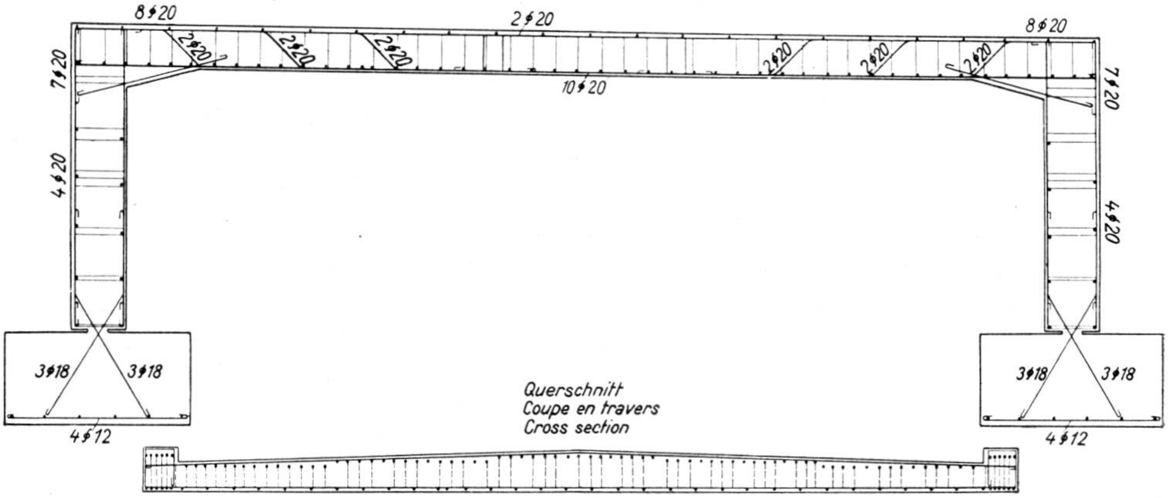


Fig. 6.

Reinforcement of frame.

It may be deduced from these descriptions of structures that slab girders are in fact a method of construction which offers scope for development. Seeing that the scantlings, and therefore the "own weights" of the structure, depend on

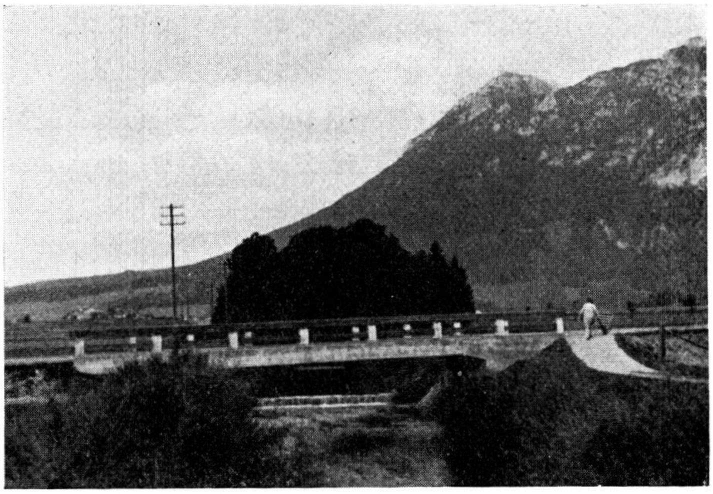


Fig. 7.

the magnitude of the permissible stresses the question arises what is the maximum span to which bridges of this type can be built with constructional and economic advantage; the answer to this depends, above all, on improving the qualities of high-grade concrete and high tensile steel.

IIc 2

The Welding of „Roxor" High Tensile Steel.

Das Schweißen von hochwertiger Stahlbewehrung „Roxor".

Le soudage de l'acier à haute résistance „Roxor".

A. Brebera,

Ingenieur, Obersektionsrat im Ministerium für öffentliche Arbeiten, Prag.

The introduction of the "Roxor"[1] high tensile steel represents a great advance in reinforced concrete construction, and its use as reinforcement for high strength concrete has made it possible to erect reinforced concrete structures of so large a span that the usual length of the bars has frequently to be exceeded. It follows, therefore, that in any large structure of this kind a large number of joints have to be provided in the reinforcing bars.

Hitherto it has been customary to make such joints by the simple process of allowing the bars to overlap over a certain length, but if a joint of that kind is to be made completely effective it is essential that the bars should be completely embedded in concrete, which entails an increased distance between them with the result that the width of the beam has to be increased accordingly. According to the official specifications it is not permissible to allow bars to overlap of more than 32 mm diam., and this places a limit on the span of the structures which can be built.

Hence the necessity to find some other type of joint which would allow of the ends of the two bars being completely bonded together over the whole of their cross sections without any weakening, and the only satisfactory solution to this important problem is that of welding the bars. Most of the existing regulations for welded work apply only to ordinary qualities of steel, and as no regulations were available for the welding of high tensile steels such as "Roxor" the problem had to be studied from first principles.

"Roxor" high tensile steel is made by slightly increasing the carbon content above that of the ordinary steel C 37 (maximum 0.22 % C), which is done by introducing certain elements such as silicon (maximum 0.90 % Si), manganese (maximum 0.50 % Mn) and copper (maximum 0.50 % Cu). The amounts of sulphur and phosphorus remain the same. The following are the mechanical properties of "Roxor" steel produced in this way.

Minimum yield point	38 kg/mm^2
Minimum tensile strength	50 kg/mm^2
Minimum elongation	20 %.

[1] Preliminary Publication, page 240.

In the welding of "Roxor" steel, the choice of electrodes plays a very important part, as it is necessary that the weld metal should be of the same quality as the parent metal. To ensure this, "Arcos-Superend" electrodes were adopted, the yield point, tensile strength, elongation and tenacity of these having been determined in accordance with the special regulations for the welding of metal bridges issued by the Ministry of Public Works. The results of these tests are given in Table I:

Table I.

Tests of weld metal deposited by "Arcos-Superend" electrodes.

	Minimum	Average	Maximum
Yield point	40.7	49.7	56.4 kg/mm^2
Tensile strength	50.7	57.0	63.3 kg/mm^2
Elongation (gauge length = 5 diameters	18.0	19.5	20.4 %
Reduction in area	29.9	39.2	50.1 %
Tenacity (Mesnager)	5.0	6.6	7.9 mkg/cm^2

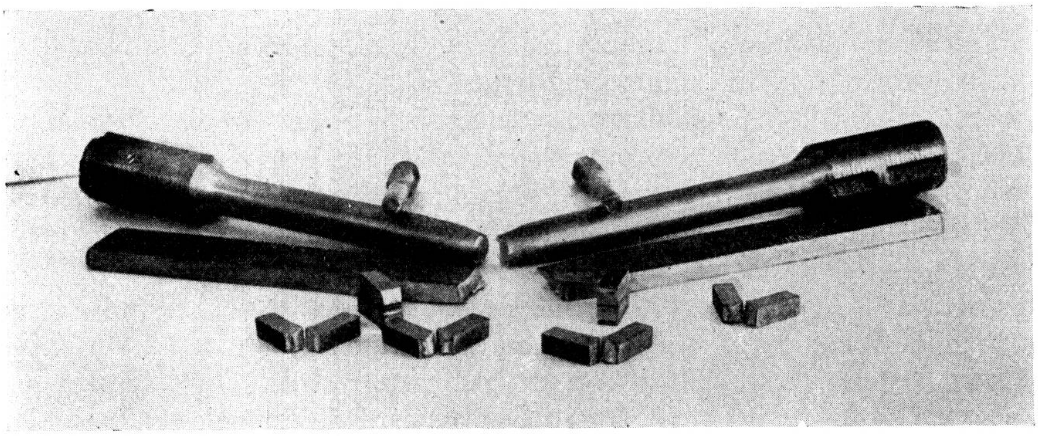


Fig. 1.
Tests of parent metal.

These figures indicate a quality of weld metal which, when deposited between two sheets of "Roxor" steel having the characteristics given in Table II below, will be the same as that of the parent metal (Fig. 1).

Table II.

Tests on "Roxor" steel.

	Average	Specified
Yield point	41.7	38 kg/mm^2
Tensile strength	58.5	50 kg/mm^2
Elongation (gauge length = 10 diameters)	22.6	20 %
Reduction in area	53.7	— %
Tenacity (Mesnager)	11.0	— mkg/cm^2

In addition to the tests on the electrodes, the tensile strength of the weld produced by them was tested on specimens consisting of two "Roxor" steel plates 12 mm thick joined by a weld at right angles to the direction of rolling (Fig. 2); the angle of folding and elongation of the extreme fibres of the weld

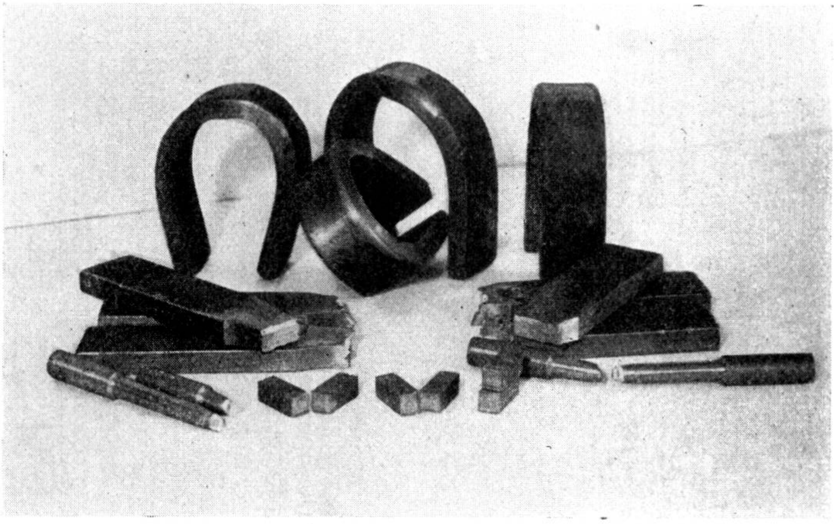


Fig. 2.
Tests of welds.

were also tested, giving results which were used as a check on the welders. The results are given in Table III:

Table III.

Tests on welded "Roxor" plates.

	Minimum	Average	Maximum
Tensile strength	50.5	59.8	65.7 kg/mm²
Folding angle	180°	180°	180°
Elongation of extreme fibres	16.0	18.8	22.0 %

These tests were followed by the welding of "Roxor" reinforcing bars, which are of cruciform section with ribs to improve the adhesion. The welding tests were carried out on "Roxor" bars of 60 mm diameter (circumscribed circle), which is a size frequently used in long span bridges. The ends of the bars to be joined were first bevelled, then held in a special form of clamp (Fig. 3) while being welded with a V — seam using the "Arcos-Superend" electrodes (Fig. 4).

The results of all the tests carried out on welded "Roxor" bars are shown graphically in Fig. 5. It should be noticed that the welds were not machined. The average values obtained in the tests, together with the maxima and minima, are given in Table IV. In every case breakage of the bar took place outside the weld.

Table IV.

Tests on welded "Roxor" bars.

	Minimum	Average	Maximum
Yield point	39.4	40.1	40.6 kg/mm^2
Tensile strength	55.7	57.4	58.5 kg/mm^2
Elongation (gauge length $= 11.3 \sqrt{A}$ [A = area])	22.8	26.1	28.5 %
Reduction in area	33.8	47.9	51.4 %
Folding angle	180°	180°	180°
Elongation of the extreme fibres	6.2	10.3	12.1 %

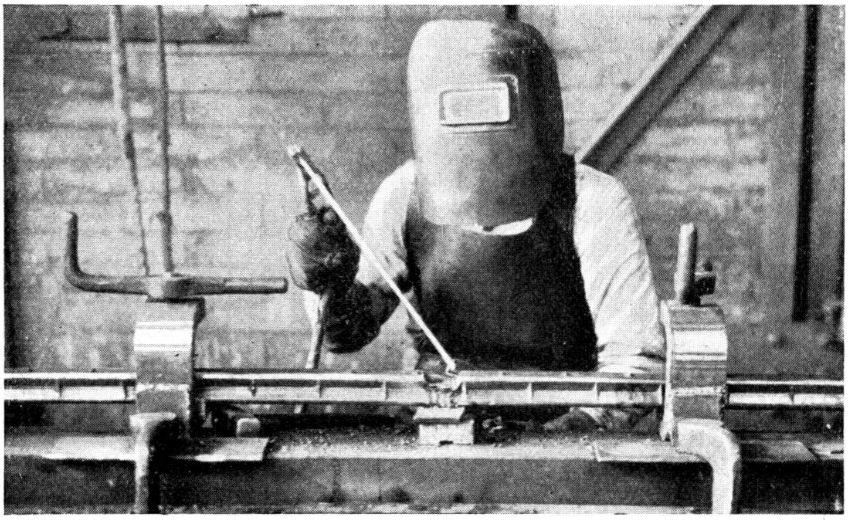


Fig. 3. "Roxor" bar ready for welding.

Test bars measuring $2 \times 1.25 = 2.50$ m in length were adopted in order that when the "Roxor" bars were applied in an actual job the heat should not spread

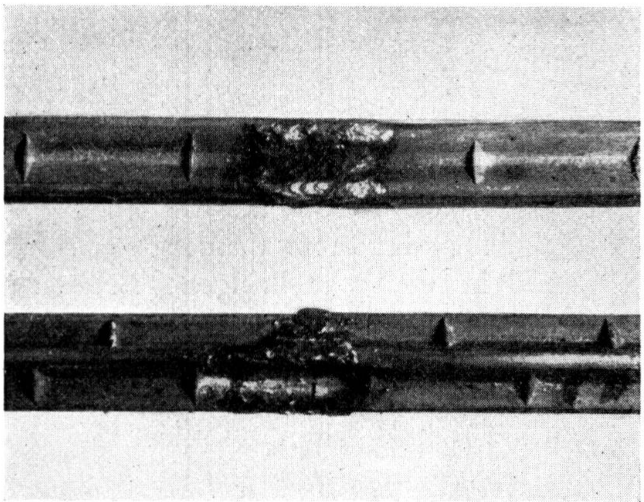


Fig. 4. Welded "Roxor" bars.

more rapidly than in the tests, as would have been the case if the usual small sized specimens had been used. It was ascertained, in the course of welding, that the heating of the bars did not extend beyond 0.50 m from the weld, a result which may be attributed to the large cross section of the specimens (17.34 cm²).

In addition to the tensile tests the "Roxor" welded bars were subjected in cold bending tests (Fig. 6), and the results of these were found to be in accordance with the specifications. To determine the elongation of the extreme fibres, and the effectiveness of the bond with the parent metal, a few of the bars were machined in the neighbourhood of the weld before testing, and the results so obtained are included in Table IV. The internal diameter of the bend is equal to five times (or occasionally six times) the diameter of the "Roxor" bar.

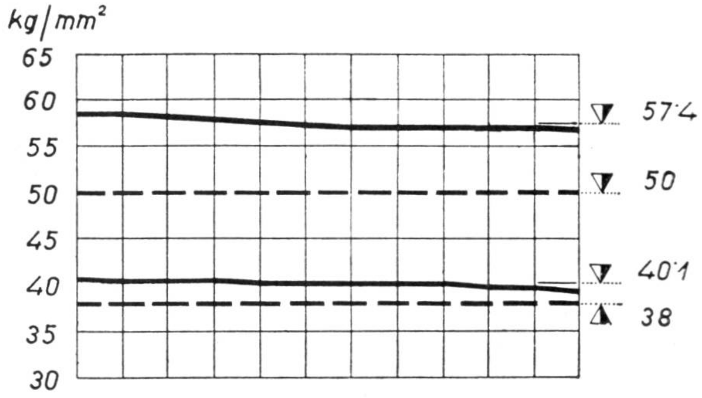


Fig. 5.

Results of tensile tests on welded "Roxor" bars, showing tensile strength and apparent elastic limit in kg/mm.

Figs. 7 and 8 are macroscopic and microscopic views of the longitudinal sections of a 60 mm "Roxor" welded bar. Fig. 7 shows part of the band of

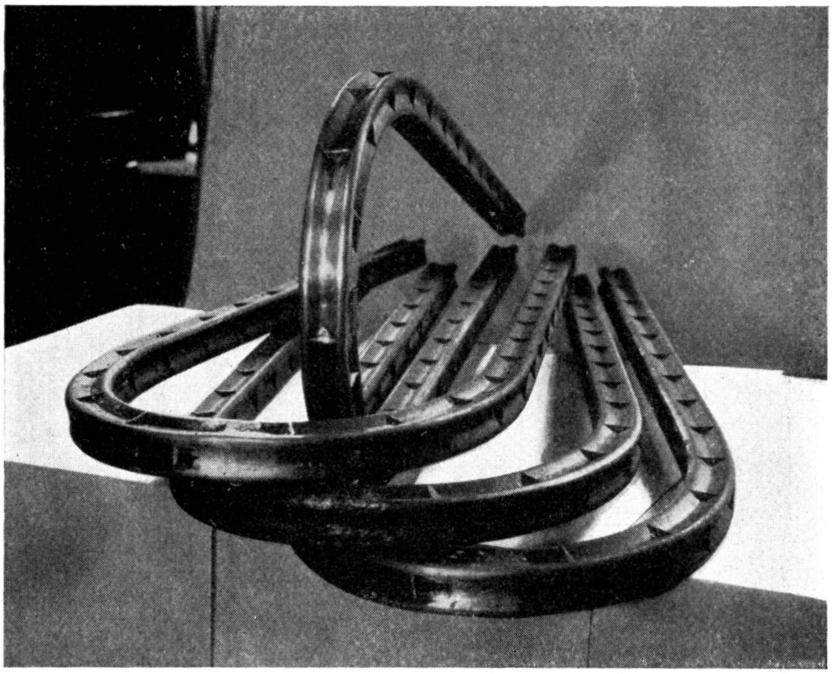


Fig. 6.

Bending tests of "Roxor" bars.

recrystallised metal between the weld metal and the parent metal. The crystals of sulphur and phosphorus are uniformly distributed in the parent metal without forming a coagulation, and the weld itself is completely free from such crystals.

The white crystals appearing in Fig. 8 practically all consist of pure iron (ferrite) and the dark spots around them are perlite (ferrite plus cementite, F_{e3} C). The

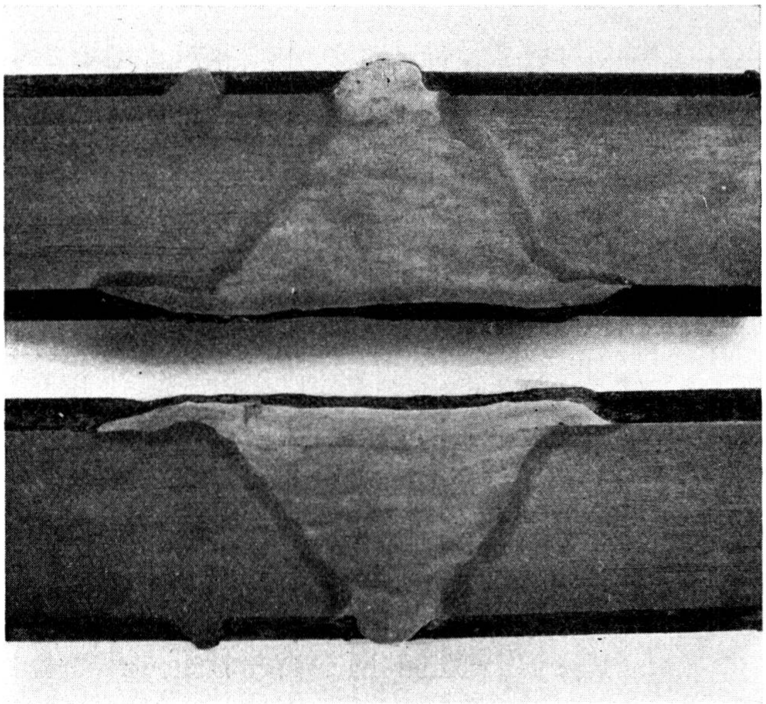
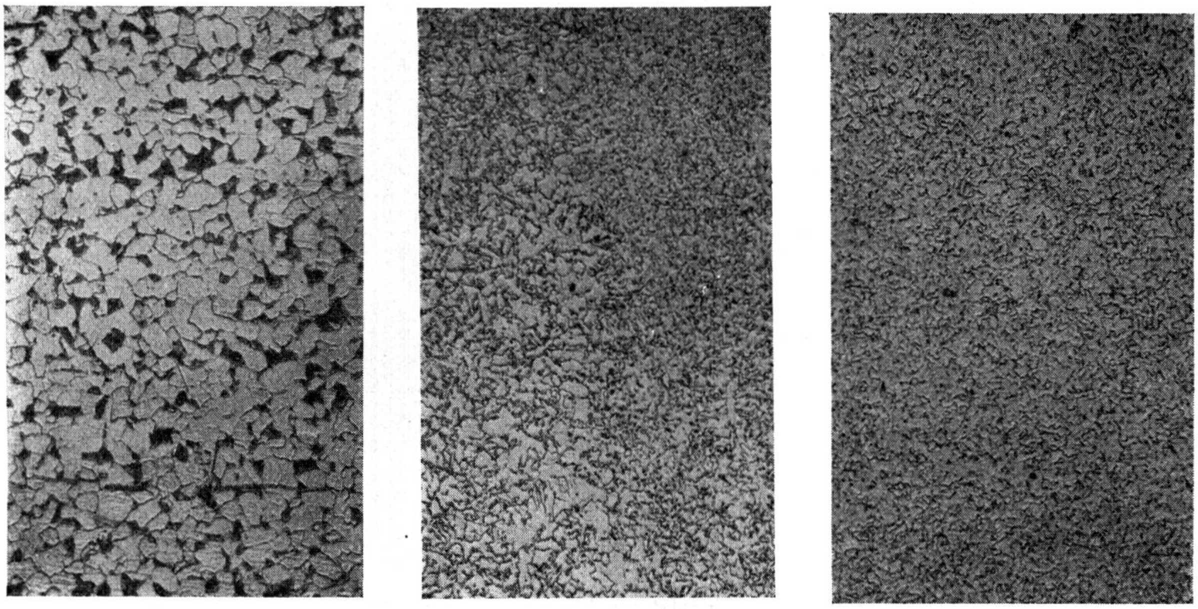


Fig. 7.
Etched and polished longitudinal section through a welded "Roxor" bar.

structure of "Roxor" steel is characterised by the presence of small uniform grains of ferrite, and the good mechanical properties are a consequence of this.



Parent metal   Transition zone   Weld metal

Fig. 8. Structure of welded "Roxor" bar, × 100.

In the transition zone, the structure varies uniformly and changes into a completely regular fine-grained ferrite structure, the transition from the parent metal to the weld metal being imperceptible. The modification undergone by the parent metal as a result of welding was determined by the Brinell test, using a ball of 10 mm diameter under a pressure of 3000 kg (Fig. 9), which gave the following results.

$$\text{Diameter of indentation} < 4.80 \text{ mm} : P = 0.345 \text{ H}$$
$$\text{Diameter of indentation} > 4.80 \text{ mm} : P = 0.342 \text{ H}$$

where P is the tensile strength in kg/mm^2 and H is the Brinell hardness in kg/mm^2. It may be seen from Table V, which gives the results of these ex-

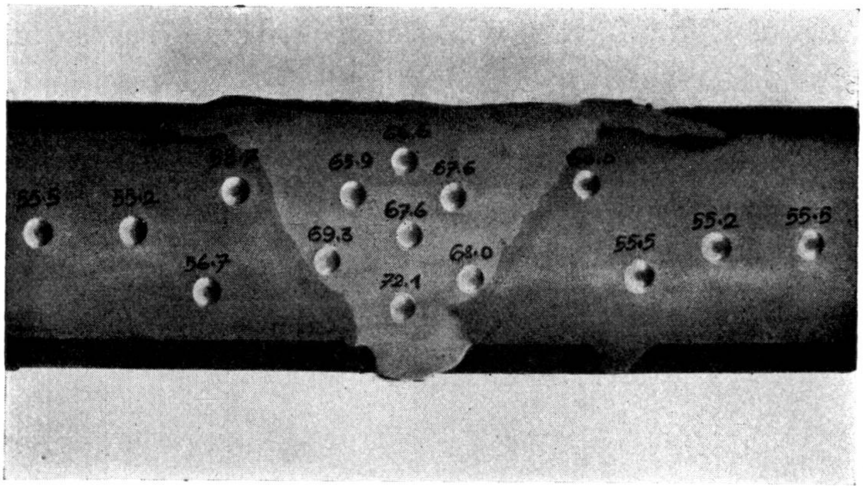


Fig. 9.
Brinell test on a welded "Roxor" bar.

periments, that the hardness of the unmodified parent metal corresponds to its tensile strength as determined in the breaking tests, whereas in the transition zones an increase in the hardness of the metal in relation to the change of structure is observed, this being brought about by the heat developed by the electric arc. The hardness of the metal increases as the structure of the weld becomes finer. This explains the small elongation of the weld (average 10.3 %) by comparison with that of the bars (average 26.1 %).

Table V.

Effect of welding on "Roxor" steel.

Tensile strength —

Of the parent metal (kg/mm^2):
55.5 55.2 56.7 55.5 55.2 55.5 Average 55.7

Of the metal in the transition zone (kg/mm^2):
58.7 69.3 68.0 60.0 Average 65.9

Of the weld metal (kg/mm^2):
65.9 66.6 72.1 67.6 67.6 Average 67.0

These favourable test results justify acceptance of the principle that welded joints may be formed in the "Roxor" bars without detriment to the factor of safety prescribed for unjointed bars, and for the purpose of calculation the cross section of a welded "Roxor" bar may betaken as unimpaired even in the neighbourhood of the joint.

The welding of "Roxor" bars must be carried out only by officially authorised welders and only by the use of "Arcos-Superend" electrodes as adopted in the tests; new tests would be necessary before other electrodes could be used. In order to avoid distortion of the welded bars as a result of the great shrinkage which takes place at the surface of the V-seam, it is recommended that a slight clearance should be allowed when fixing the bars in the clamp.

An incidental outcome of these tests was the acquisition of knowledge regarding the behaviour of other high tensile steels when welded, as, for instance, steel C 52.

IIc 3

High Tensile Steel in Reinforced Concrete Structures.

Verwendung des hochwertigen Stahls in Eisenbeton-Konstruktionen.

Les aciers à haute résistance dans les constructions de béton armé.

Dr. Ing. A. Chmielowiec.
Lwów, Pologne.

According to the regulations in force in various countries the permissible stress of mild steel such as is ordinarily used in the reinforcement of concrete amounts to 1200 kg/cm², and that of high tensile steel to 1800 kg/cm². The cross section of the tensile reinforcement may, therefore, be reduced by one third if the "1800" or high tensile steel is adopted instead of the "1200" or ordinary mild steel, without altering the depth of the beam. This involves a slight increase in the compression of the concrete, but this is always permissible as *Saliger* has shown in his paper N° II c 3 before the present Congress. If, then, it is desired to replace n round bars of diameter d of mild steel by n_1 round bars of diameter d_1 of steel "1800" we obtain —

$$n\, d^2\, \pi \cdot 1200 = n_1\, d_1^2\, \pi \cdot 1800$$

It is desired that the adhesion stress should remain the same in both cases, therefore

$$n\, d\, \pi = n_1\, d_1\, \pi$$

The above equations give the condition

$$n : n_1 = d_1 : d = 1200 : 1800 = 2 : 3$$

It follows that we can, for instance, replace two bars of 9 mm diameter of steel "1200" by three bars of 6 mm diameter of steel "1800". This entails very thin bars, which are expensive and which are not stiff enough to retain their straightness.

These disadvantages may be avoided by giving the reinforcing bars a regular triangular section. Of all the possible regular polygons of equal area, the triangle is the one which has the largest perimeter and the circle is the one which has the smallest. If d is the diameter of the circle and a = 1.1 d is the side of an equilateral triangle, then the perimeter of the triangle is 3 a = 3.3 d and that of the circle is π d = 3.14 d, and the difference between them is $3a - 3d = 0 \cdot 16d$. Thus the perimeter of the triangle is 5 % greater than that of the circle.

The area of the circle is $A_0 = \frac{d^2\pi}{4}$ and that of the triangle is $A_\Delta = a^2 \frac{\sqrt{3}}{4}$

Hence $$\frac{A_0}{A_\Delta} = \frac{d^2\pi}{a^2\sqrt{3}} = \frac{\pi}{1.21\sqrt{3}} = 1.5 = \frac{1800}{1200}$$

Thus a round bar of diameter d of steel "1200" may be replaced by a triangular bar of steel "1800" if the side of the triangle is 1.1 d. This gives a saving of 33 % of steel without reducing the bond.

It would, therefore, be advantageous to adopt bars of steel "1800" of triangular section, and if the rolling of such sections were decided upon the following advantages would also accrue:

1) The danger of confusion between round bars of steel "1800" and those of steel "1200" would be eliminated.

2) Of all regular figures of equal area, the triangle has the largest moment of inertia and the circle has the smallest. (Figures bounded by perimeters containing re-entrant angles or concave curves, such as for instance a star, are here not considered. A bar of which the section is shaped like a star can be drawn out of the concrete within a cylindrical space which has no concavity, this cylinder being the smallest that can be circumscribed around the star in question.) The area of the circle being $A_0 = \frac{d^2\pi}{4}$ the corresponding moment of inertia will be $I_0 = A_0 \frac{d^2}{16}$. The area of the triangle being $A_\Delta = a^2 \frac{\sqrt{3}}{4}$ its corresponding moment of inertia will be $I_\Delta = A_\Delta \frac{a^2}{24}$.

From the equation $A_0 = A_\Delta$ we have $\frac{a^2}{d^2} = \frac{\pi}{\sqrt{3}}$

whence $$\frac{I_\Delta}{I_0} = \frac{2a^2}{3d^2} = \frac{2\pi}{3\sqrt{3}} = 1.21.$$

Thus the moment of inertia of the triangle is 21 % greater than that of the circle having the same area. Triangular bars are therefore more rigid than round bars and are not as easy to curve and bend, but retain their straightness better in course of handling, both in the store and on the site. This is a matter of some importance, for curved bars must straighten themselves before they can begin to act in tension, and meanwhile those bars which are already straight are overworked. In the case of compression reinforcement the stiffness of the bars is still more important, and round bars not being very rigid tend to buckle easily. There is, therefore, no object in using round bars of steel "1800" in compression.

3) Triangular bars take up less room in the store than round bars because they fill the whole of any given space without wastage: six triangles form

a regular hexagon, and such hexagons may be stacked closely against one another without losing any space.

4) A triangular bar can easily be twisted so as to obtain a special shape as in Ransome's system. In this way the grip between the bar and the concrete may be still further improved, seeing that the circumference of the circle enclosing the regular triangle is 21 % greater than the perimeter of the triangle itself. A twisted bar cannot be pulled out of the concrete without first having to strip the latter from the cylindrical surface circumscribing the bar, or from a surface which is still larger. This has been shown by experiments on Isteg steel. In such experiments at Warsaw, carried out by *Bryla* and *Huber*, two round bars of 7 mm diameter twisted into a spiral around one another gave an adhesion 20 % greater than a single equivalent round bar of 12 mm diameter. The circle circumscribed around the two twisted bars each 7 mm in diameter is itself of 14 mm diameter, and its circumference is, therefore, 16.67 % greater than that of the round bars. The difference of 20—16.67 % is attributable to the fact that the imaginary tube enclosing the twisted Isteg is a little larger than twice 7 mm, and does not form a precisely regular cylinder.

Instead of rolling triangular bars of steel 1800 they might be rolled from steel 1200, and their quality subsequently improved by stretching and twisting as is done for Isteg steel.

IIc 4

On the principles of calculation for reinforced concrete.

Zu „Berechnungsgrundlagen des Eisenbetons".

Les principes de calcul du béton armé.

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Professor an der Eidg. Techn. Hochschule und Direktionspräsident der Eidg. Materialprüfungs- und Versuchsanstalt für Industrie, Bauwesen und Gewerbe, Zürich.

On the question of "n" — which has been so much discussed and mainly on quite erroneous grounds — it is to be remarked that within certain limits this factor has no influence if the permissible stresses are chosen in relation to the value of n selected (n = 10, 15 or 20). The very latest tendency to do away with the n figure altogether is to be regarded as a mistake, leading not to simplification and clarity but rather to trouble and confusion. It is possible to hold different opinions on the value of n, but as a basis for the calculation of reinforced concrete it is impossible to dispense with this figure, and for practical purposes it lies at the basis of reinforced concrete theory.

The classsical theory of reinforced concrete, which relies on the *Navier-Hooke* law as regards compression, tension and bending and on the generalised *Euler* formula as regards buckling, has been extended in the last few years by knowledge won in the testing of materials — particularly as regards plastic strain — and these extensions are of great value for the more accurate estimation of the degree of safety possessed by reinforced concrete structures.

They cover the following aspects of the problem of deciding what stresses are to be regarded as permissible:

The stress-strain law of the concrete and reinforcing steel.[1]

The modular ratio $n = \frac{E_e}{E_b}$ within the elastic region.[2]

The relationship between the prism compressive strength ${}_p\beta_d$ and elastic modulus of the concrete ${}_bE_e$.[1]

The danger of breakage in concrete stressed along more than one axis. (Experiments at the Swiss Federal Institute for Testing Materials in the light of *Mohr*'s theory of fracture.)[3]

The fatigue resistance to a pulsating load in the concrete and in the reinforcing steel,[4] and

The laws governing stability against buckling in columns loaded centrally and eccentrically. (Experiments and theory developed at the Swiss Federal Institute for Testing Materials.)[5], and by maintaining the closest possible contact between the drawing office, the laboratory and the job itself *it is justifiable to proceed by calculating reinforced concrete structures in accordance with the classical theory of elasticity,* and at the same time to plan the organisation in such a way, and take such constructional measures as

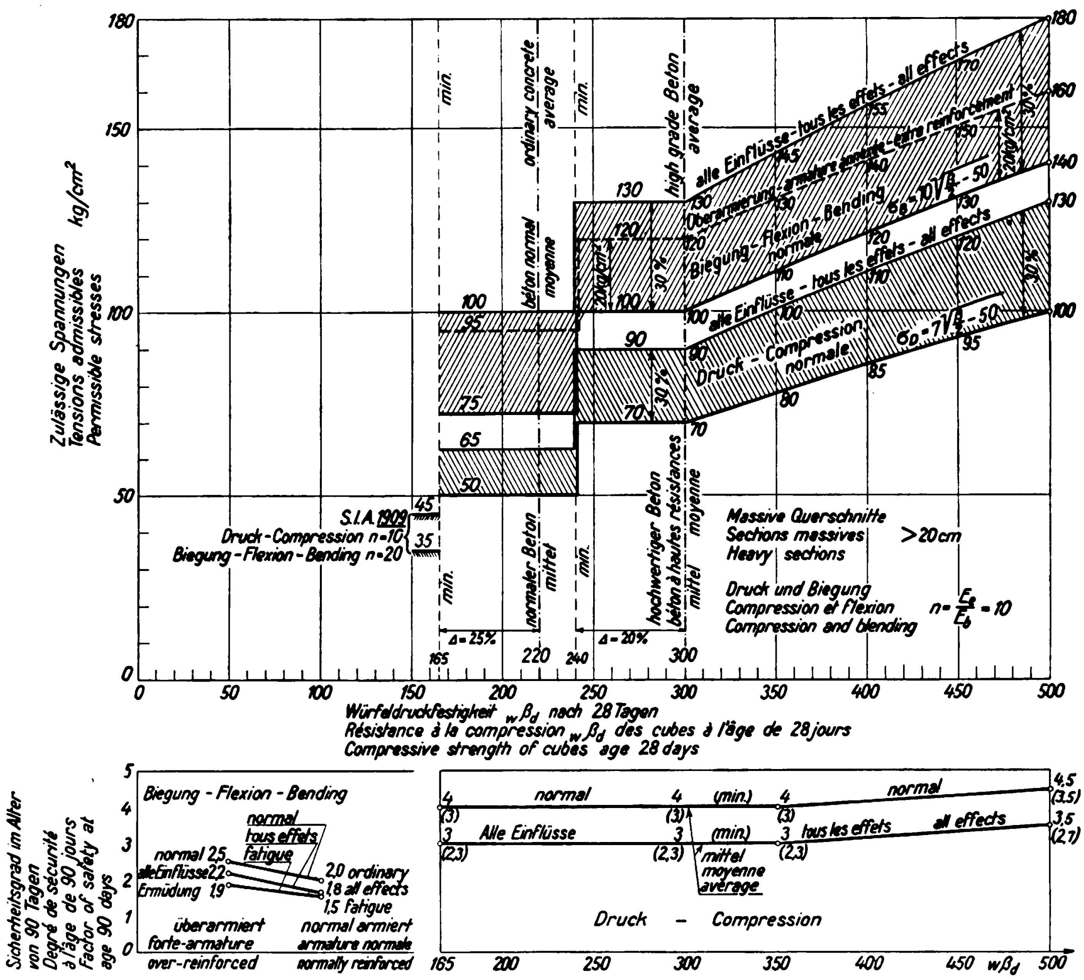


Fig. 1.

Swiss regulations for reinforced concrete of May 14 th, 1935: permissible stresses in the concrete and reinforcing steel in relation to the compressive stress of the concrete and yield point of the steel.

Oblique principal stresses.

Ordinary concrete $\tau_{zul} = 4$ kg/cm² High grade concrete 5 kg/cm²

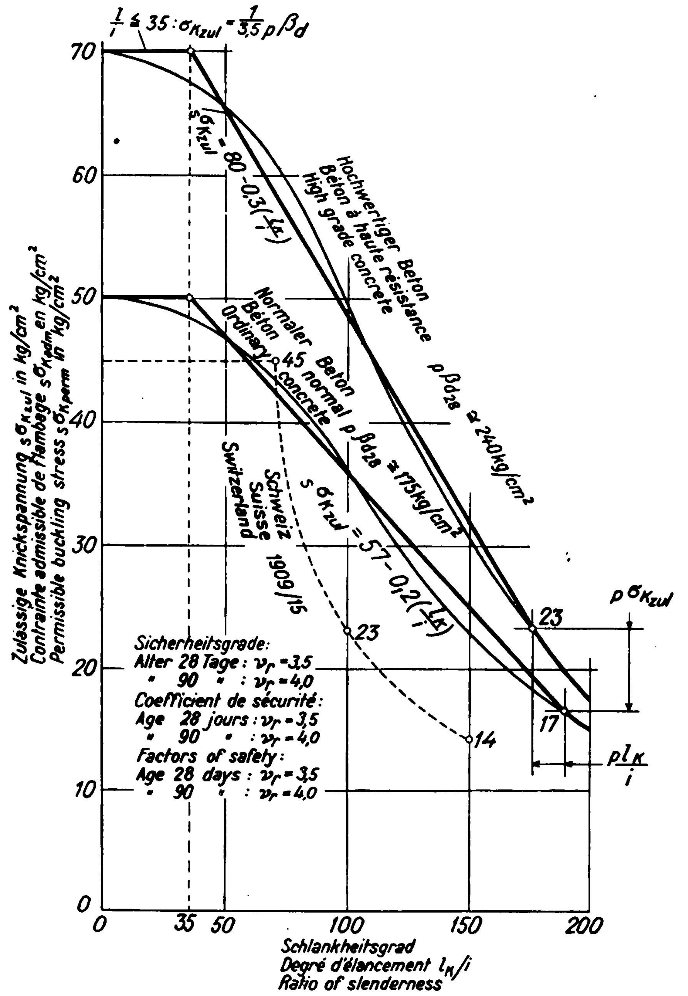
Reinforcing steel: Permissible stresses $_{perm}\sigma_e$.

All effects:

	ordinary	high grade
Excluding temperature and shrinkage stresses	1400 kg/cm²,	1700 kg/cm²
Including temperature and shrinkage stresses	1500 „	1900 „

regards both general arrangement and details, that certainty may be won as to the conditions and effects of stress, which, in its turn, will yield exact indications as to the true factor of safety.

Exhaustive experiments on completed reinforced concrete structures go to show that they behave in accordance with the elastic representation.[6] Despite all that is being argued to the contrary, the theory of elasticity will continue in the future to serve as the basis for dimensioning and estimating the safety of reinforced concrete structures — due account being taken of the effect of plasticity of the concrete on the carrying capacity,[7] but without encroaching on



$_p\beta_d$ Prism compressive strength of concrete

σ_q Yield point of reinforcing steel

F_b Cross section of concrete

F_e Cross section of steel

μ Percentage of reinforcement $F_e : F_b$

Buckling load:

$$P_K \cong {}_s\sigma_K F_b \left(1 + \frac{\sigma_q}{{}_p\beta_d} \mu\right)$$

$$_{perm}P_K = {}_s\sigma_{K_{perm}} \cdot F_b (1 + n\mu)$$

Limit of proportionality:

$$\sigma_P \cong 0.33\ {}_p\beta_d$$

$$_p\sigma_{K_{perm}} \cong \frac{1}{3.5} \cdot 0.33\ {}_p\beta_d$$

$$\frac{{}_p l_K}{i} = \pi \sqrt{\frac{E'\min}{0.33\ {}_p\beta_d}}$$

$$E'_{\min} \cong 0.75 \cdot 550000 \frac{{}_p\beta_d}{{}_p\beta_d + 150}$$

Fig. 2.

Concrete columns without hoop reinforcement, with longitudinal reinforcement $\mu \cong 1\,\%$.

Permissible concentric buckling stresses $_s\sigma_{K_{perm}}$ for $m = 0$.

Ordinary and high grade concrete.

the final reserves possessed in this way by the material.[8] It may be said that the axial and shear forces and bending moment which result from the imposition of external loads are now determined in all countries by fundamentally the same rules, so that if definite principles for estimating of the conditions of breaking, fatigue and buckling can be agreed upon on an international basis, the international regulation of calculated factors of safety will be a matter only of careful and wellfounded understandig.

The *unification of principles* would have to be based on characteristics of materials which, while not yet expressed quantitatively in all countries, are nevertheless understood in the same sense. Such characteristics, as regards reinforced concrete, include the following:

The modular ratio $n = \frac{E_e}{E_b}$.

The yield point of the reinforcing steel σ_f

The yield point σ_s under tension.

The breaking point σ_q under compression.

The fatigue resistance to a pulsating non-alternating stress in the reinforcing steel $\sigma_u \cong 0.85\ \sigma_f$.

The prism compressive strength of the concrete ${}_p\beta_d \cong 0.8\ {}_v\beta_d$, ${}_w\beta_d$ = cube compressive strength.

The limit of proportionality of the concrete $0.33\ {}_p\beta_d \cong {}_b\sigma_{zul} \cong \sigma_p$ = *Euler*'s buckling stress.

The fatigue strength of the concrete $\sigma_u \cong 0.6\ {}_p\beta_d$ = resistance to pulsating stresses without change of sign.

The buckling modulus T_K.

The percentage of reinforcement $\mu = \frac{F_e}{F_b}$.

It would appear desirable to observe a calculated factor of safety of beetween ~ 1.8 and ~ 2.5 as regards static failure, a factor of safety of ~ 1.5 to ~ 2.0 against fatigue, and one between ~ 3 and ~ 4 against buckling, having reference in each case to the total load. The following permissible stresses might then be adopted as a basis:

	All Effects	
	Excluding shrinkage and heat	Including shrinkage and heat
	σ_{zul}-values	σ_{zul}-values
Concrete: normally reinforced .	$\sim 0.4\ {}_p\beta_d$	$\sim 0.5\ {}_p\beta_d$
Concrete: over-reinforced ${}_{eff}\sigma_e < {}_{zul}\sigma_e$	$\sim 0.4\ {}_p\beta_d + 0.05\ ({}_{zul}\sigma_e - {}_{eff}\sigma_e)$	$\sim 0.5\ {}_p\beta_d + 0.065\ ({}_{zul}\sigma_e - {}_{eff}\sigma_e)$
Reinforcing steel: normal quality $\sigma_s \cong 2400$ kg/cm²	~ 0.5 to $0.6 \cdot \sigma_s$	$\sim 0.65\ \sigma_s$
Reinforcing steel: high-tensile $\sigma_s \cong 3500$ kg/cm²	~ 0.45 to $0.5 \cdot \sigma_s$	$\sim 0.55\ \sigma_s$

The higher values as here proposed for the permissible stresses in the concrete in over-reinforced sections (wherein the permissible stresses of the steel reinforcement is not fully utilised) are based on the results of a great many breaking tests, and are also supported by theoretical considerations such as the greater depth of the neutral axis in such cases, and the plasticity of the concrete. The same "n" figure is retained as in normally reinforced sections, as it is more correct to adopt this method than to increase "n".

In view of the smaller risk of breakage the permissible extreme fibre stress of the concrete under bending may be fixed 40 % higher than the stress under direct compression.

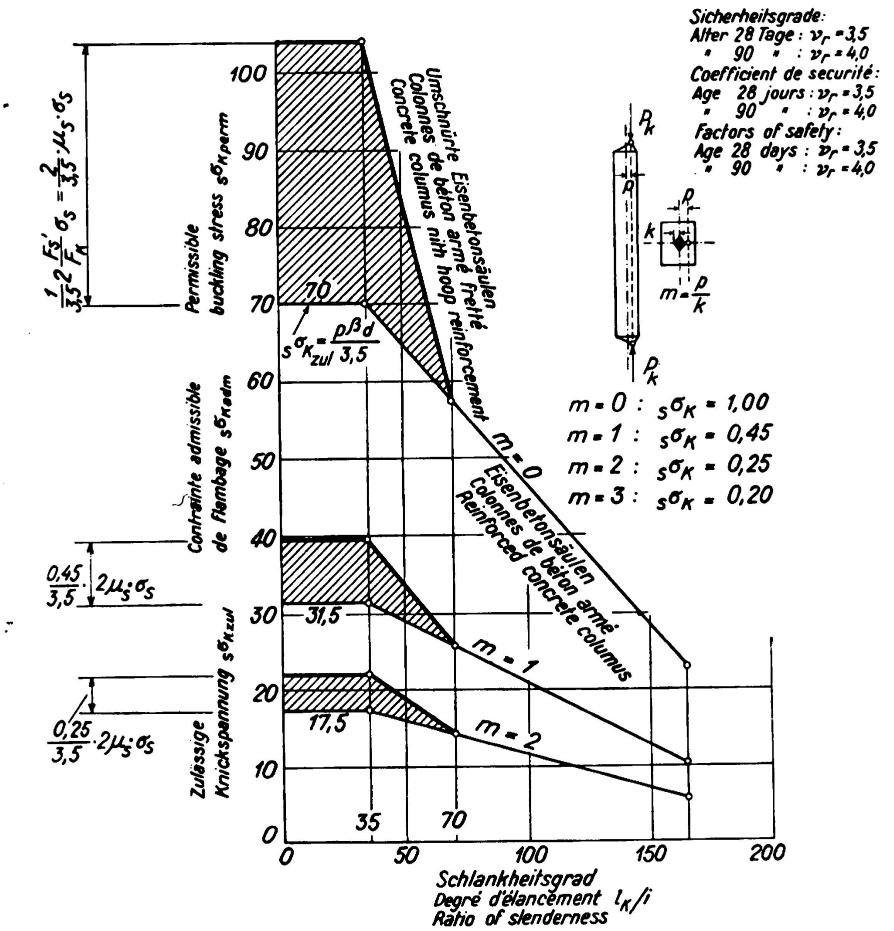


Fig. 3.

Concrete columns with hoop reinforcement and longitudinal reinforcement.

Permissible concentric buckling stresses ${}_s\sigma_{K_{perm}}$.

For m = 0, m = 1 and m = 2. High grade concrete.

Concrete columns with longitudinal reinforcement:

$$\text{Buckling load:}\quad P_K \cong {}_s\sigma_K\, F_b \left(1 + \frac{\sigma_q}{{}_p\beta_d} \cdot \mu\right); \quad \left(\frac{l_K}{i}\right) \geqq 70$$

Concrete columns with longitudinal and hoop reinforcement:

$$\text{Breaking load:}\quad P_{failure} = {}_bF_K\,({}_p\beta_d + 2\,\mu_s \cdot \sigma_s)\left(1 + \frac{\sigma_q}{{}_p\beta_d}\,\mu\right); \quad \left(\frac{l}{i}\right) \leqq 35$$

$$\text{Buckling load:}\quad P_K \cong {}_s\sigma_K \left({}_bF_K + \frac{\sigma_q}{{}_p\beta_d}\,F_e + 2\,\frac{\sigma_s}{{}_s\sigma_K}\,F'_s\,\frac{70 - \frac{l_K}{i}}{35}\right);$$

$$35 \leqq \left(\frac{l_K}{i}\right) \leqq 70.$$

The effect of the principles explained in the preceding paragraph on the new Swiss regulations for reinforced concrete, dated 14th of May 1935, is represented in Fig. 1, showing graphically:

The permissible stresses in the concrete ${}_{zul}\sigma_b$ under compression and bending,[9] in relation to the quality of the concrete (cube strength), and

the factor of safety referred to an age of 90 days.

The permissible stresses in resistance to buckling, for columns with and without hoop reinforcement in normal and high grade concrete, may be taken from Figs. 2 and 3. The contents of Figs. 1, 2 and 3 may serve as an indication of the great advances made in reinforced concrete construction in the last few years, and of the new possibilities of design.

The knowledge now available, based on theoretical and technical conceptions drawn from the field of testing materials — such as strength and deformation — and also on experience gained in practice,[10] offers a starting point for international cooperation to aim at *unification of the interpretation of the laws governing the strength of materials* and at *the establishment of definite factors of safety in reinforced concrete construction.*

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9 The permissible oblique concrete stresses should not exceed one-twelfth to one-fourteenth of the permissible compressive stresses and the whole of any excess is to be taken up by inclined reinforcing bars.

10 *M. Roš:* „Erfahrungen mit ausgeführten Eisenbeton-Bauwerken in der Schweiz und deren Lehren für die Portlandzementindustrie." Separate reprint from the 24th Annual Report, 1934, of the Swiss Association of Manufacturers of Cement, Lime and Gypsum.

11 *M. Roš:* „Vereinheitlichung der material-technischen Erkenntnisse und des Sicherheitsgrades im Stahlbetonbau." Monatsnachrichten des Österreichischen Betonvereins. Vol. IV, Vienna 1937.

IIc 5

Tests with Concrete Beams Reinforced with Isteg Steel.

Versuche mit Eisenbetonbalken mit Isteg-Stahl-Bewehrung.

Essais de poutres en béton armé d'acier Isteg.

Dr. Ing. St. Bryła and Dr. Ing. M. T. Huber,
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An account will be given below of the results of experiments on special reinforcing bars carried out in Poland. It is known that the yield point of such bars can be considerably increased by mechanical treatment (pre-stretching) and that in this way the tensile breaking stress may be raised; the most advantageous amount of pre-stretching appears, from experience, to be approximately 6 %. In reinforced concrete members subject to bending the yield point of steel — or, the stress corresponding to an elongation of $\varepsilon = 0.4$ % — is a matter of primary importance, the failure of such members being almost always the result of the carrying capacity of the reinforcement being destroyed when ε reaches 0,4 %. Two kinds of reinforcement which have received preliminary treatment in this way are in practical use; namely Isteg steel and expanded metal.

a) Isteg steel.

Isteg steel is manufactured by twisting together two round bars of equal diameters. In 1934 experiments on the reinforced concrete members indicated in Table I were carried out in the testing laboratory of the Technical University of Warsaw, and these disclosed some valuable properties of Isteg steel for the reinforcement of beams and slabs.

The elements marked A were reinforced with Isteg and those marked B with ordinary round bars, the reinforcement being so designed that the cross section of the Isteg steel was 33 % smaller than that of the round bars in the corresponding members. The tests carried out on these materials gave the following average values:

Table 2.

Material	Yield point	Breaking stress	Modulus of elasticity
A. Isteg steel 5.5 mm	3738 kg/cm^2	4261 kg/cm^2	1 630 000
Isteg steel 7 mm	3723 „	4339 „	1 600 000
B. Round bars	2640 „	3630 „	2 101 000

It will be seen from these figures that the Isteg steel has a yield point averaging 41.3 % higher and that its strength is 18.5 % higher. The results obtained in the experiments with reinforced concrete elements led to the following conclusions.

1) *Bending strength.*

The breaking loads for elements reinforced with Isteg steel of 33 % smaller cross section were almost the same as those for members reinforced with larger section of round bars. If the amount of reinforcement was small, the first crack appeared earlier in the beams reinforced with Isteg than in those reinforced with round bars, but this difference disappeared when the amount of reinforcement was increased. With the round bars the first hair cracks spread almost immediately after their appearance into wide open fissures.

With the Isteg steel the cracks were at first almost imperceptible. They opened very slowly, even when the load was considerably increased, and did not lose the character of hair cracks. The reason for this is probably to be found in the better bond of the concrete on to the spirally wound steel rods.

The further conclusion may be drawn that the compressive stress conditions in the concrete at the instant of breakage are more favourable where Isteg reinforcement is adopted, as the deformation of the concrete takes place more uniformly, whereas with round bars this change of shape is strongly concentrated in a few short sections.

2) *Deflections.*

The deflections of the concrete elements containing Isteg steel were much greater than those of the corresponding elements containing round bars. This is easily understood on the following grounds:

a) The stresses in the Isteg steel are some 50 % higher under the same loading than those in the round bars of the control elements, and assuming a modulus of elasticity of the same value in both cases this would imply 50 % greater elongation of the Isteg bars.
b) Apart from this, the modulus of elasticity of the Isteg bars if lower, being $E = 1\,615\,000$, and this leads to a further increase in the elongations of approximately 30 %.

For these two reasons combined, the elongation of the Isteg steel is multiplied by $1.5 \times 1.3 = 1.95$, that is, it is increased by 95 %, and the greater deflections obtained are the result. Generally speaking, however, no disadvantage is to be apprehended from this as all reinforced concrete structures are in fact very stiff.

3) *Actual stresses.*

In the experiments carried out with elements IV and IVa the deformations ε in the steel and concrete respectively were measured by means of *Huggenberger* tensometers and the stresses were then calculated from the equation $\sigma = E \cdot \varepsilon$ by inserting the mean values of E already determined. These stresses may be regarded as directly measured, and therefore as actual stresses.

Table 1.

Summary of

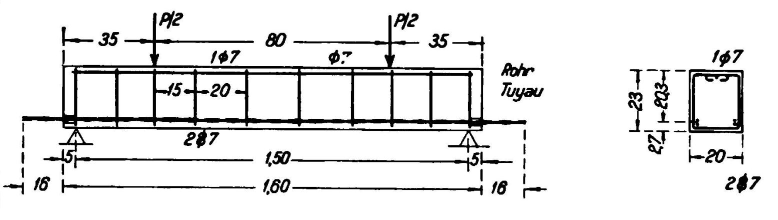
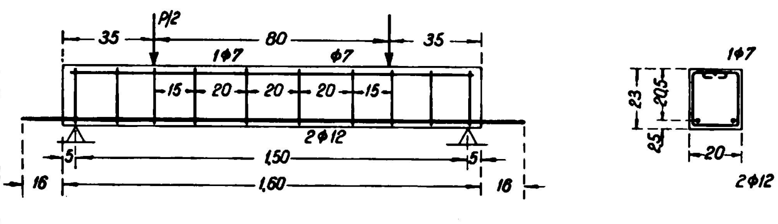
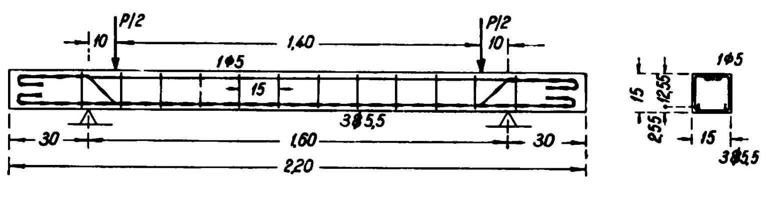
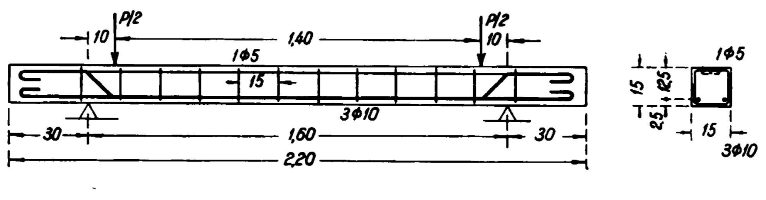
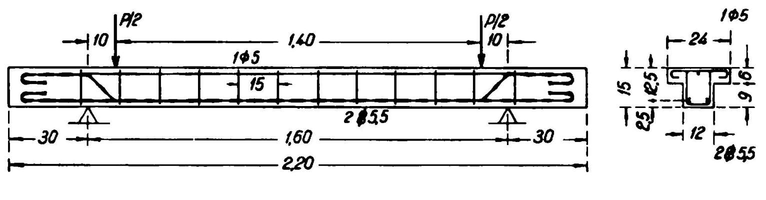
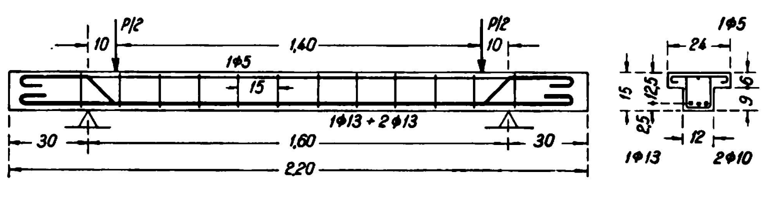
Nr.	Dimensionen — Dimensions	Beton Nr. / Béton No. / Concrete Nr.	Ausgeführt / Executé / Executed	Geprüft / Essayé / Tested	Zweck der Probe / But de l'essai / Purpose of testing
II-A		2	27/IX.	22/XI.	Haftung — Grip
II-B		2	27/IX.	22/XI.	
III-A		2	27/IX.	24/XI.	Druck — Compression
III-B		2	27/IX.	24/XI.	
IIIa-A		2	27/IX.	24/XI.	Druck — Compression
IIIa-B		2	27/IX.	24/XI.	

Table 1.

specimens tested.

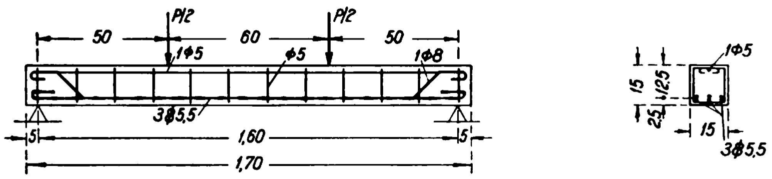
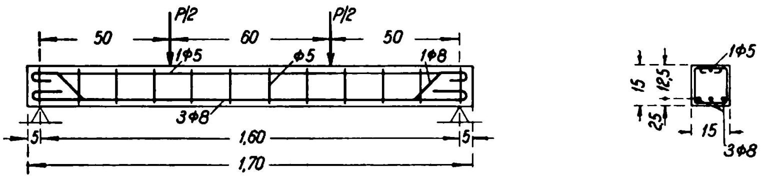
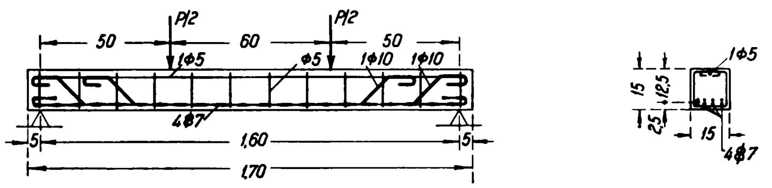
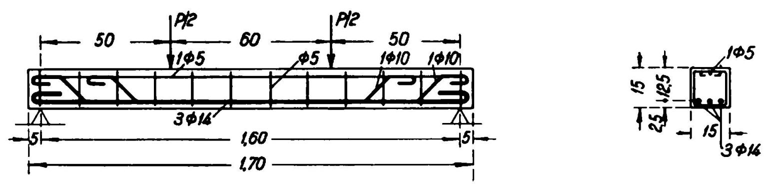
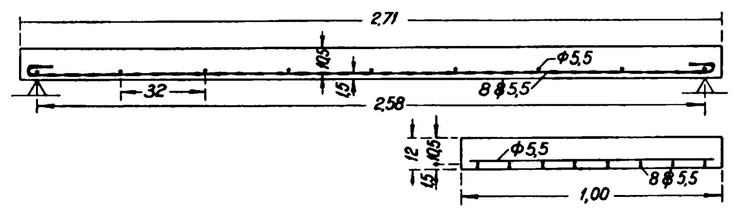
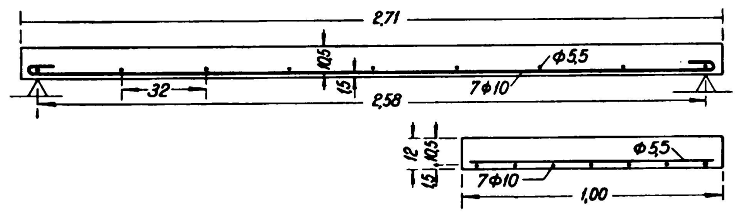
Nr.	Dimensionen — Dimensions	Beton Nr. / Béton No. / Concrete Nr.	Ausgeführt / Exécuté / Executed	Geprüft / Essayé / Tested	Zweck der Probe / But de l'essai / Purpose of testing
IV-A		2	27./IX.	21./XI.	Druck — Compression
IV-B		2	27./IX.	21./XI.	Druck — Compression
IVa-A		2	27./IX.	21./XI.	Durchbiegung — Flèche — Deflection
IVa-B		2	27./IX.	21./XI	Durchbiegung — Flèche — Deflection
I-A		1	18./IX.	18./X.	Durchbiegung — Flèche — Deflection
I-B		1	18./IX.	18./X.	Durchbiegung — Flèche — Deflection

Table 3. Comparison between calculated and measured stresses.

Beam	Reinforcement	Concrete (measured)		Steel (measured)		Concrete: calculated for			Steel: calculated for		
		total ε	elastic ε	total ε	elastic ε	Phase I	Phase II with		Phase I	Phase II	
							n = 15	true n		n = 15	true n
IV B	Round bars	30.1	26.8	903	420	21.4	31.9	37.9	105	785	772
IV A	Isteg	49.2	35.2	536	363	24.6	34.9	45.3	120	772	748
IVa B	Round bars	24.3	21.8	307	202	19.3	22.1	24.9	82	258	249
IVa A	Isteg	29.7	23.6	377	194	19.7	24.5	30.8	90	380	360

In Table 3 two sets of values of the "measured" stresses are compared, namely those calculated from the total elongations and those calculated from the elastic elongations within the range of load of 500 kg. Corresponding stresses are calculated for Phase I with $n = 8$ and for Phase II with $n = 15$. Also with

$$\text{true } n = \frac{\text{true value of E for steel}}{\text{true value of E for concrete}}.$$

It must be stated that although the measurements were actually made in Phase I the measured stresses correspond more closely with those calculated for Phase II. In the concrete the agreement between the measured and the calculated stresses is fairly good with $n = 15$, especially if account is taken only of the elastic elongations. Reasonably good agreement between the measured and the calculated stresses for the total deformation is also obtained, especially if the true value of n is used.

As regards the reinforcement, however, only those stresses which are calculated from the total elongation approximate to the calculated stresses in Phase II, the measured stresses for the round reinforcing bars being a little higher and those for the Isteg bars a little lower. Stresses calculated on the basis of elastic deformation alone worked out lower, without any exception, by about 50 % in comparison with the calculated stresses for Phase II, but from two to four times higher than the calculated stresses according to Phase I. The true stresses therefore lie between those found from Phase I and Phase II. This can only be explained on the assumption that n is considerably greater for the tensile zone in Phase I than the usually assumed value $n = 8$.

It may be assumed as probable that the measured stresses in the reinforcement agree with the unknown actual stresses, but as regards the stresses in the concrete matters are different for the following reasons:

1) Within the scope of the measurements the reinforced concrete sections are working according to Phase I, so that the statical behaviour is not like that which would correspond to Phase II.

2) The actual distribution of stresses is very different from the *Navier* distribution, and — especially in the case of the round bars — the stresses around the bars are smaller and the stresses close to the neutral axis greater than is implied by the linear diagram. It may be concluded from this that the actual stresses are lower than is implied by calculation from the measurements as above. The mean value of E in bending must be smaller than in pure com-

pression. Several foreign experimenters have given the following values for concrete:

$$E_{\text{for bending}} = \frac{2}{3} \text{ to } \frac{1}{2} E_{\text{for axial compression}}.$$

The close agreement of the measured stresses with the calculated stresses (according to the usual formula for Phase II) is, therefore, relevant to the present case.

4) *The coefficient n.*

The results of the present experiments do not entail the adoption of another value of n than that which is now usual in calculations where Isteg steel is adopted, although direct measurement of the elastic characteristics of Isteg steel by comparison with concrete gives an average value of $n = 9$. In practice, however, the calculated stresses are almost independent of n. Moreover, according to a number of experiments which have been carried out, the true value of n varies a great deal and depends upon the stresses, even assuming the same kind of concrete.

5) *Gripping stresses.*

The Isteg steel with 33 % smaller cross section gave more than 20 % higher grip resistance than the ordinary round bars, and if the load was further increased the Isteg steel was found to slip more slowly than was the case with round bars.

6) *Shear.*

There can be no doubt that in the experiments carried out on beams III and IIIa the governing factor was not the compressive strength of the concrete but the shear forces. Under bending loads the weakest part of each beam was the cross section immediately below the concentrated load, because at this point most of the reinforcing bars provided for the purpose of resisting the bending moment were bent up at a place where the magnitude of this moment was still at a maximum.

The compressive stresses in the concrete were calculated for Phase I and Phase II. The stresses in the reinforcing bars were calculated both for the bent up bars and in reference to all the bars.

Table 4 shows the stress values in kg/cm² when the first crack appeared, and it will be seen that this happened at practically the same time with either methods of reinforcement.

Table 4. Shear stresses.

Beam	Reinforcement	Stress in Concrete		Stresses in reinforcement	
		Phase I	Phase II	Bent-up bars	all bars
III B	Round bars	21.0	30.8	4780	1970
IIIa B	„	18.7	37.6	2930	1604
III A	Isteg steel	21.2	29.7	7260	3010
IIIa A	„	18.1	34.9	4675	2450

It is a matter of great difficulty to estimate the stresses in the bent-up bars. The usual method of calculation, which assumes that in the absence of stirrups the whole of the shear force is carried by the bent-up bars, is:

$$\text{Stress in bent-up bars} = \frac{\text{Total shear}}{\text{Area of bent-up bars} \times \sqrt{2}}$$

but this led to quite impossible values in the present case, greatly exceeding the breaking stress of the material. This explains the effective cooperation of the straight bars on account of the good anchorage provided outside the supports.

When, however, the stresses in the reinforcement are calculated in reference to the straight bars by the formula:

$$\text{Stress in bars} = \frac{\text{Total shear}}{\text{Area of straight bars} + \text{Area of bent-up bars} \times \sqrt{2}}$$

values are obtained which correspond almost exactly with the bending stresses. It follows from the comparison of the breaking load stresses calculated in relation to the whole of the bars that in this case, also, the carrying capacity of the Isteg steel is 1.5 times as great as that obtained with ordinary reinforcement.

b) Expanded metal.

Expanded metal, as is well known, consists of a network of rhomboidal spaces which is produced by special machines from annealed steel sheets. The smaller angle of each rhomboid is about 41°, this optimum value having been determined by experiments. The side strips of such a rhomboid undergo an elongation amounting to

$$\frac{1}{\cos 20.5^{0}} - 1 = 0.067 = 7\,\%.$$

This value practically agrees with the elongation of the material used for Isteg steel, which is about 6%.

Expanded metal is manufactured from sheet thicknesses of 0.5 to 4.5 mm. The width of the strips varies from 2.5 to 10 mm and the sizes of the rhomboid are 10/42, 20/62, 40/115, 75/200 and 150/400 mm respectively. Expanded metal has already been in use for some forty years and has frequently been examined in testing stations.

Table 5. Tests on Expanded Metal.

Plate			Expanded metal			% change through working		
σ_p kg/cm²	σ_s kg/cm²	ε %	σ_p kg/cm²	σ_s kg/cm²	ε %	σ_p kg/cm²	σ_s kg/cm²	ε %
2848	3375	22.1	3736	3993	11	+ 30.1	+ 18.1	— 50.3
3042	4205	26.2	4544	4715	10.9	+ 49.2	+ 12.2	— 58.4
3129	4204	23.9	4728	5001	12.1	+ 51.1	+ 18.8	— 49.4
3234	3787	23	4607	4667	7.7	+ 42.4	+ 23.3	— 66.5

In the autumn of 1934 experiments were carried out in the testing laboratory of the Technical University of Warsaw to determine the increase in the yield point which results from the permanent elongation of the sheet metal strips in the production of expanded metal, and the results of these experiments are given in Table 5 above.

It was found, in agreement with foreign experiments, that the yield point of the expanded metal may be in excess of 3600 kg/cm^2 and that the best results are obtained with soft sheets having a maximum amount of extensibility ε. Reinforced concrete elements containing expanded metal have been in practical use for years. The cooperation between the expanded metal and the concrete is very similar to that of Isteg steel. The deflections obtained are greater than with round bar reinforcement A 35, but the cracks are smaller, more numerous and more uniform, with the result that the stress on the concrete in compression is more uniform. The greater resistance to slip possessed by Isteg steel is easily explained by its special shape, each of the many intersections acting as a separate hook. Expanded metal by itself would be subject to a great deal of deformation. Embedment in the concrete has the effect of considerably stiffening the intersections of the network, and thus hinders deformation of the spaces enclosed. In order to render this stiffening effective the sizes of the openings should not be too small. The conclusions obtained in regard to Isteg are fundamentally valid also for expanded metal.

IIc 6

Experiments on Tubular Beams of Centrifugally Cast Concrete.

Versuche mit Schleuderbeton=Rohrbalkenträgern.

Essais effectués sur des poutres tubulaires en béton centrifugé.

Dr. Ing. A. Král,
Professor der techn. Fakultät an der Universität Ljubljana.

In the summer of 1936 a large job was completed in the Yugoslavian textile mills at Duga Resa near Karlovac (Banat of Save), in which roof girders took the form of pipe beams made of centrifugal concrete. The arrangement of the roof construction may be seen in Fig. 1. The opportunity was seized to make several series of exhaustive experiments at the materials testing station of the University of Ljubljana on differently designed and differently reinforced pipe beams.

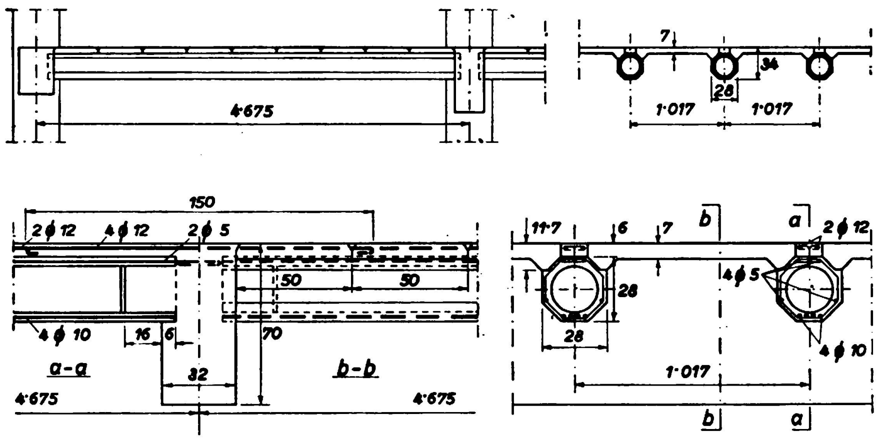


Fig. 1.

The pipes were made in three different shapes, namely:

1) an octagonal form as in Fig. 2a with a constructional depth of 28 cm;
2) the same form with a constructional depth of 22 cm; and
3) a polygonal form widened in the tension zone as shown in Fig. 2b.

Individual pipes, intended for heavy isolated loads, were provided with transverse stiffeners at the load points and at the supports, in order to prevent premature damage through the pipe collapsing.

As shown in Fig. 2 the reinforcement consists of four bars of 5 mm diameter in the upper and middle corners and tensile reinforcement in the lower side, in addition to spiral hooping of steel wire 3 mm in diameter fixed by the contractor which (with the exception of pipe 17 and pipe 18) was welded to the compression and tensile reinforcements at particular points.

The following materials were used for the reinforcements:

1) Structural steel C 37 obtained from the Kranjska Industrijska Družba at Jesenice (corresponding to the German steel St. 37).
2) Isteg steel, supplied by the same firm.

The pitch of the spiral hooping was made variable, and in places two spirals were used.

High grade Portland cement of the "Stockbrand" mark, supplied by the Portland cement factory at Split, was used for all the pipes. The aggregate consisted partly of limestone of 13 mm gauge, obtained from the quarries of the

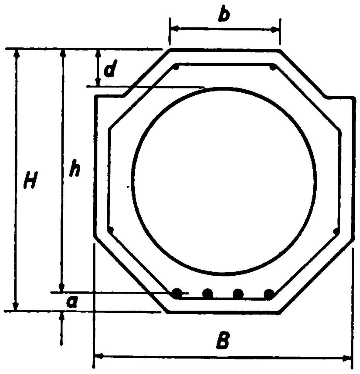


Fig. 2a.
Beams Nos. 1 to 12, 17 to 22, I to III.

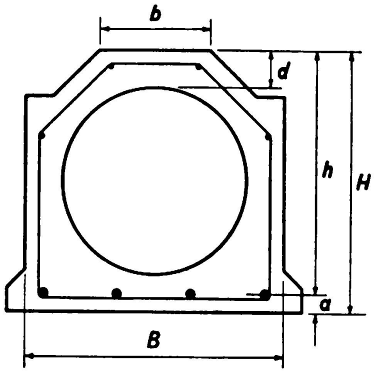


Fig. 2b.
Beams Nos. 13 to 16.

textile works at Duga Resa, and partly gravel and sand taken from the Save-river, up to 13 mm in gauge.

The following three mixes were used:

1) Broken limestone with 410 kg of cement per cubic metre of finished concrete, water cement ratio 0.45—0.515.
2) Save gravel and sand with 410 kg of cement per cubic metre of finished concrete, water cement ratio 0.45—0.50.
3) Broken limestone with 300 kg of cement to one cubic metre of finished concrete, water cement ratio 0.69—0.72.

The reinforcing steel C 37 showed mechanical properties considerably better than the minima laid down in the standard. Its yield point varied on an average between 29.52 and 33.07 kg/mm^2; the tensile strength between 40.41 and 42.43 kg/mm^2; the specific elongation at fracture, in a gauge length of ten diameters, between 27.3 and 40.7 %.

The Isteg steel had an ultimate strength of 44.7 to 47.4 kg/mm^2, an elastic limit of 37.9 to 40.3 kg/mm^2 for 4 % elongation, and an elongation of breakage of 5.5 to 8.5 %.

The average strength of the different kinds of concrete at an age of four weeks may be seen from the following table:

Nature of aggregate	Cement kg per m³ of concrete	Cube strength per kg/cm²	Tensile bending strength per kg/cm²
Broken limestone . . .	410	630	62.3
Save gravel and sand . .	410	585	56.9
Broken limestone . . .	300	639	54.4

The test specimens were prepared not in the laboratory but on the job, and the dates of production are those given by the supervisors of the work. In the laboratory exact dimensions and weights were recorded, and in the case of the pipe beams which were tested to destruction the reinforcement was afterwards exposed and remeasured.

The investigation extended to 21 series of variously designed types of beam, each represented by two samples, so that altogether it covered 42 pipe beams, from which only a few characteristics results will be given below.

Fig. 3 shows the relationship between the breaking moment and the amount of tensile reinforcement, using the normal round steel C 37 and the Isteg steel, for two different types of beam of 28 and 22 cm depths respectively.

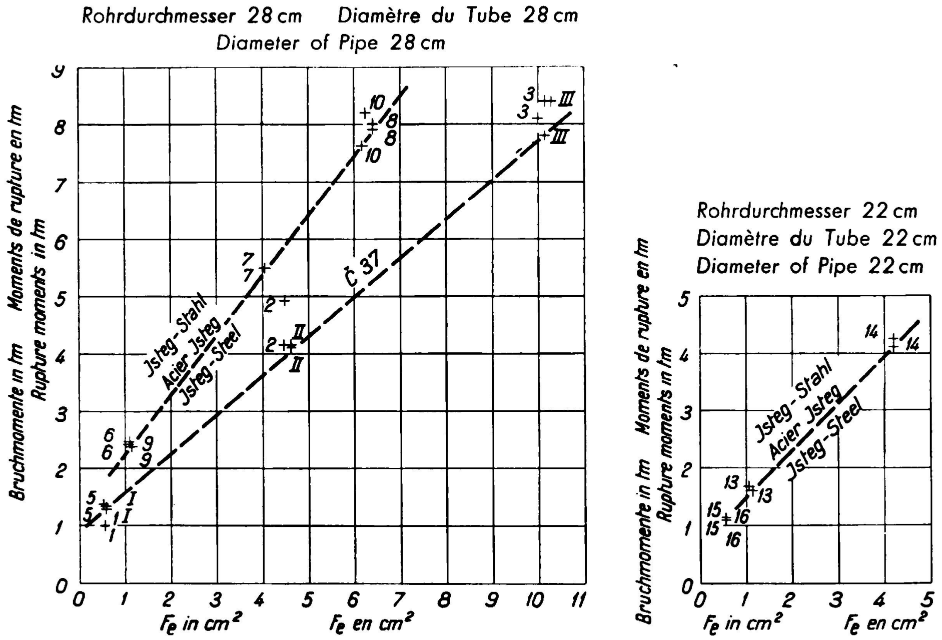


Fig. 3.

In this summary the great uniformity (or small amount of scattering) of the experimental results is apparent, and this is true not only as regards individual pairs of beams but also as regards the uniform increase in breaking moment according to the reinforcement provided.

In order to show details of the experiments, and the kind of results obtained, the following is a table of the results for six characteristic beams with weak, medium and strong tensile reinforcement respectively, both with round steel C 37 and with Isteg steel.

No.	Reinforcement			Bending moment		Calculated steel stress at fracture
	Type of steel	∅	Area	at cracking	at fracture	
		mm	cm^2	tonne-metres	tonne-metres	kg/cm^2
5	C 37	2 ∅ 6	0.58	0.78	1.29	9460
2	,,	4 ∅ 12	4.48	1.79	4.16	4205
3	,,	4 ∅ 18	9.99	3.32	8.10	3883
6	Isteg	2 ⊖⊖ 6	1.08	0.82	2.43	9490
7	,,	4 ⊖⊖ 8	4.07	1.79	5.49	5885
8	,,	5 ⊖⊖ 10	6.41	2.66	7.91	5575

The summary shows that in the case of the lightly reinforced beams no cracking appeared under a load equal to half the working load, and the same result was obtained in the other series of experiments. In beams containing heavier reinforcement, or in those with Isteg reinforcement, fine cracks made their appearance earlier; but under a load equal to half the breaking load the distribution of these fine cracks was in one series within the region of the maximum bending stresses, and when the load was removed they closed up again so as to be scarcely perceptible with the naked eye. Open cracks did not appear until fracture.

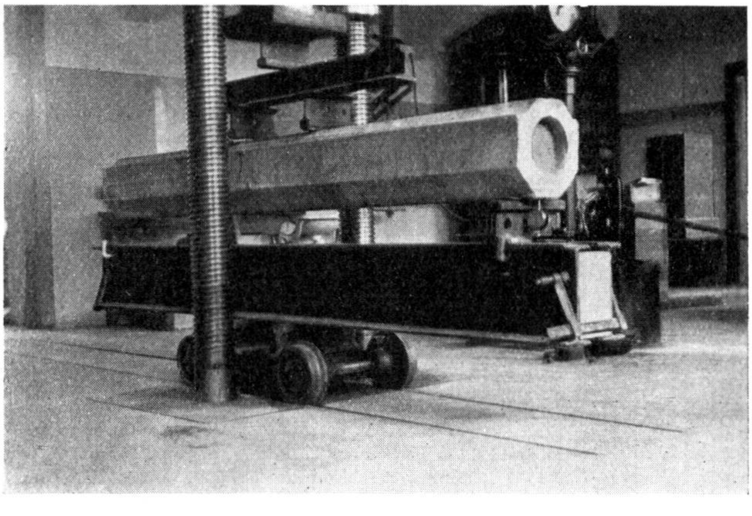


Fig. 4.
Arrangement of experiments.

In the case of the lightly reinforced pipes only tensile cracks were visible, even in the breaking condition. With the heavier tensile reinforcement and single spiral hooping shear cracks arose which in some cases spread into pipe-bursting-cracks or combined with the latter. With the heavier hooping a bulging of the concrete occurred in the compression zone, usually in the neighbourhood of the load point. With lighter hooping, failure occurred by bursting of the pipes, especially in the case of the pair of pipes marked "1" (Fig. 3).

The steel stresses given in the table, which were calculated on the assumption of a modular ratio of 10 as for Condition II, indicate that — especially in the case of lightly reinforced beams — the theoretical steel stresses assume illusory values. This is attributable to the fact that in these beams (even when in the breaking condition) the concrete in the tension zone continues to co-operate very considerably in spite of its continuity being impaired by cracks. The heavier the reinforcement the more closely did the calculated stress in the steel just before fracture approximate to the yield point of the reinforcement.

Measurements of bending under repeated loading show excellent elastic performance in addition to the normal phenomena of plasticity; hence the classical

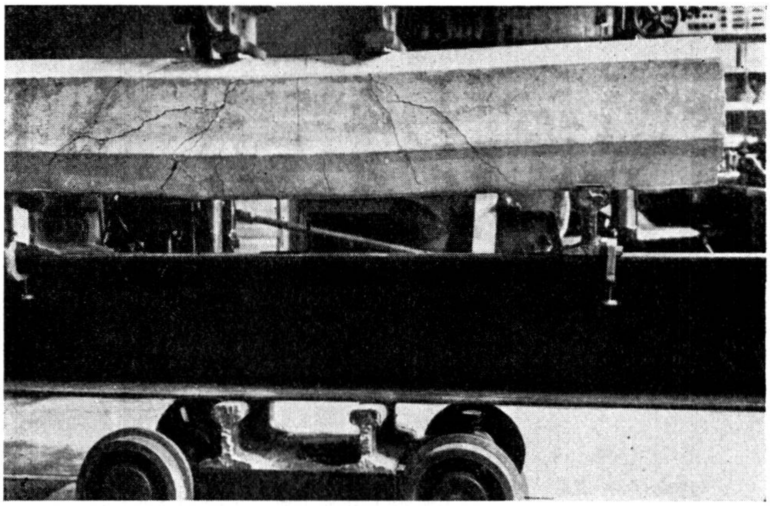


Fig. 5.

Cracks resulting from combined shear and crushing of tube.

theory of elasticity may be used for determining the stresses and strains in beams of this kind. As in all reinforced concrete work it is mainly a question of the correct assumption of elastic constants. As regards bending stresses some amplification or correction of the classical *Bernoulli-Navier* theory of bending would appear, if possible, to be desirable, but in any case there are no insuperable difficulties in determining the stresses at the most dangerous points by reference to the elasticity theory with sufficient approximation, and where pipe beams are to be built up as here into a three-dimensional system this is especially important. The fact that the results of calculations made by assuming a linear condition of stress should differ greatly from the true conditions determined by a three-dimensional elasticity-tensor field is no more than logical, and is illustrated in the results given above.

The great uniformity of the test results is doubtless attributable to the concrete being exceptionally dense and regular, as could be observed at the fractures. These properties, which have been found in a considerable number of beams produced under factory conditions, go to show that the centrifugal process — already long in use for making transmission poles and pressure pipes — may rationally be applied to the production of load bearing beams also, provided that careful workmanship can be counted upon.

IIc 7

The Safety of Reinforced Concrete Structures.

Zur Frage der Sicherheit im Eisenbetonbau.

La sécurité des constructions en béton armé.

Ing. A. Umlauf,
Wien.

If a review is made of the increasing use of high tensile structural steels in reinforced concrete structures, numerous reports on experiments with such steels will be found in the literature dating from soon after the War.

It is proper that attention should be directed first to the problem which is of most frequent occurrence and admits of easiest experiment: namely that of bending. In this connection, the most important and interesting of the experiments made to date are those to establish a comparison with St. 37, the type of steel hitherto customary for reinforcement. One of the best of the few summaries of such experiments that have appeared is that by *Dr. Emperger,* which led to the discovery that when various specimens containing samples of steels with abnormally high yield points were tested, much greater breaking strengths were obtained than was to be expected from the usual calculations for bending. It was found that the coefficient of $n = 15$ usually adopted in the bending calculations was too high for the ordinary steel St. 37. (In Switzerland and Yugoslavia $n = 10$ is employed.)

A certain variability in the coefficient n had already been recognised in Great Britain by making its value depend on the cube strength. In Austria a correction of the concrete stress, where high tensile steel is used, has been permitted since 1928 for a special cold stretched steel with a yield point of 3600 kg/cm^2 and similarly in Bulgaria an increase in the concrete stress of 15% has been sanctioned. Very recently a New York regulation has authorised an increase in the concrete stress of 15% in reinforced concrete beams containing high tensile steel.

In Germany, also, the knowledge now available has not been neglected, and since 1932 the Ministry of Public Welfare has allowed the concrete stress to be increased by 15% where the special steel mentioned above is employed. The Ministry of Finance in Berlin, in view of various problems which are still outstanding in this connection, has accepted the proposal put forward by the German Committee for Reinforced Concrete that special regulations should be considered, and these are expected to be ready in the spring of 1937 after the conclusion of the relative experiments now in hand by that Committee. When the results of these experiments are available, the authorities concerned will, no doubt, amend the relevant regulations in accordance with them as has been done in the latest

Austrian standard or "Oenorm". In the latter, the increase in breaking loads established by *Dr. Emperger*'s survey in comparison with the loads calculated on the basis of n = 15 has been accepted as justifying the use of higher steel stresses in the calculations (and also higher concrete stresses) than where ordinary steels are used, and these increases vary from 15 % for ordinary steels to 25 % or more for high tensile steels.

From this point of view the proposal made by *Dr. Friedrich* of Dresden before the present Congress is no doubt a welcome advance, involving, as it does, the substitution of a rectangular distribution of the compressive stress due to bending for the impracticable triangular distribution assumed at present. The proposal carries all the more weight because other workers — such as, for

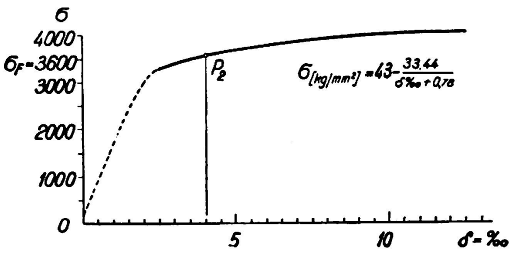


Fig. 1.
Isteg-Steel

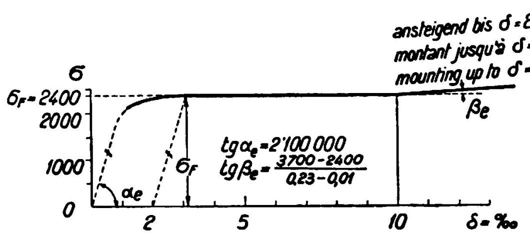


Fig. 2.
Steel St. 37.

instance, Hofrat *Saliger* of Vienna, Professor *Brandtzaeg* of Trontheim and *Dr. Bittner* of Vienna — have also arrived at this form of distribution of stress. By such a method the calculation is rendered very easy, *but in view of the many experimental results obtained in Germany, Austria, Switzerland, Czecho-Slovakia. U.S.A. and other countries, it would still appear expedient to make use of n = 10 for ordinary steels and n = 15 for high tensile steels in calculating the neutral axis assuming that the yield point of the steels is not less than 3600 kg/cm²*.

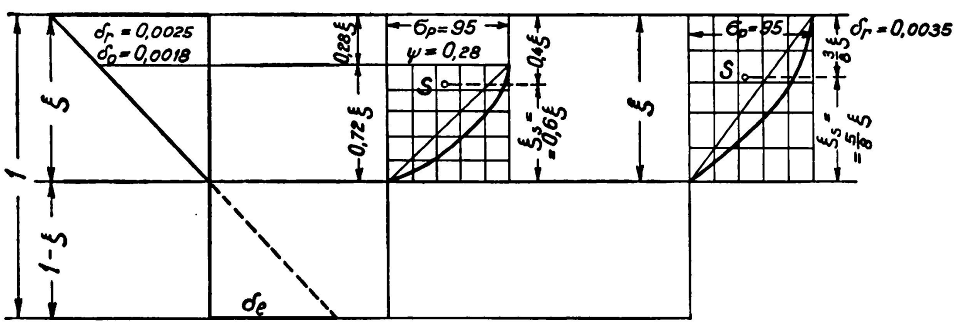


Fig. 3a. Fig. 3b. Fig. 3c.

In order to give the proposal a more definite form, advantage might be taken of the regularity of the stress-strain curve for structural steels under tensile stress by adopting the suggestion made by *Klockner* of Prague that the relationship in question is best represented as a hyperbola connecting to the straight line of *Hooke*'s Law (Fig. 1). The resulting curve for ordinary steel is shown in Fig. 2, and the elongations when the cross section is assumed to remain constant in Fig. 3. Fig. 3b shows the stress-strain curves plotted as parabolae and indicating an increase in compressive strain towards the edge of the beam but without any corresponding increase in stress in the final portion. Fig. 3c shows a similar parabolic shape of the stress-strain curve for concrete.

Adopting the hyperbolic form of equation for the shape of the stress-strain curve of the steel, and the parabolic form for that of the concrete (or a parabola combined with a rectangle), it becomes possible, on purely theoretical grounds, to calculate the curve of $\frac{M}{bh^2}$ in relation to the percentage of reinforcement.

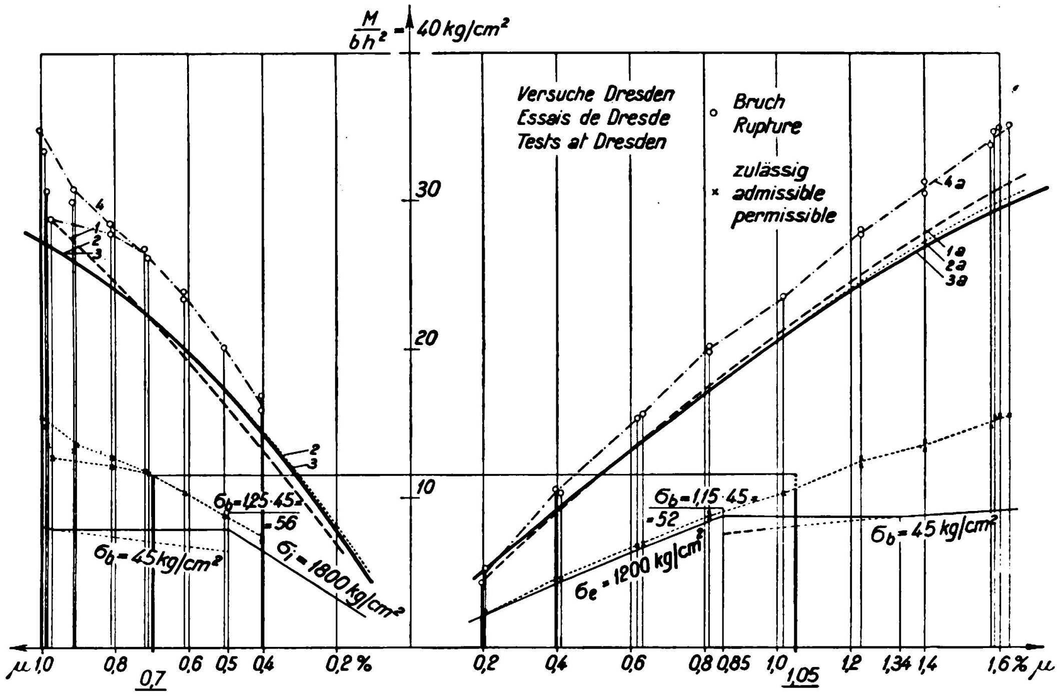


Fig. 4.

Isteg. St. 37.

Fig. 4 shows curves of resisting moments calculated by Professor *Roš* in accordance with the Swiss regulations mentioned above, using either of the above assumptions — on the left for Isteg steel which is chosen as an example of the high tensile steels, and on the right for St. 37, these being represented by the lines marked 1, 2 and 3 and 1a, 2a und 3a respectively, which it will be seen practically coincide. Lines 4 and 4a contain points representing a series of comparative experiments carried out at Dresden in which concrete of the lowest possible cube strength of 110 kg/cm² was purposely used, and in which the minimum cube strength required by the regulations, 160 kg/cm² was reduced by two-thirds, corresponding to twice the *degree of safety* in the steel and to three times that in the concrete. It is concluded that by halving the ordinates of the curve of resisting moment, values are obtained which represent the minima to be allowed.

The curves in bold lines show the effect of a correction of 25 % in the concrete stress where high tensile steel is used in accordance with the Austrian standards.

Fig. 5 indicates that assuming, for instance, a 15 % increase on account of high tensile steel, the degree of safety thus obtained is in no way less favourable than with round bars under the ordinary regulations. It is clear, however, that the degree of safety calculated in accordance with the regulations varies a great deal according to the percentage of reinforcement provided. This goes to show

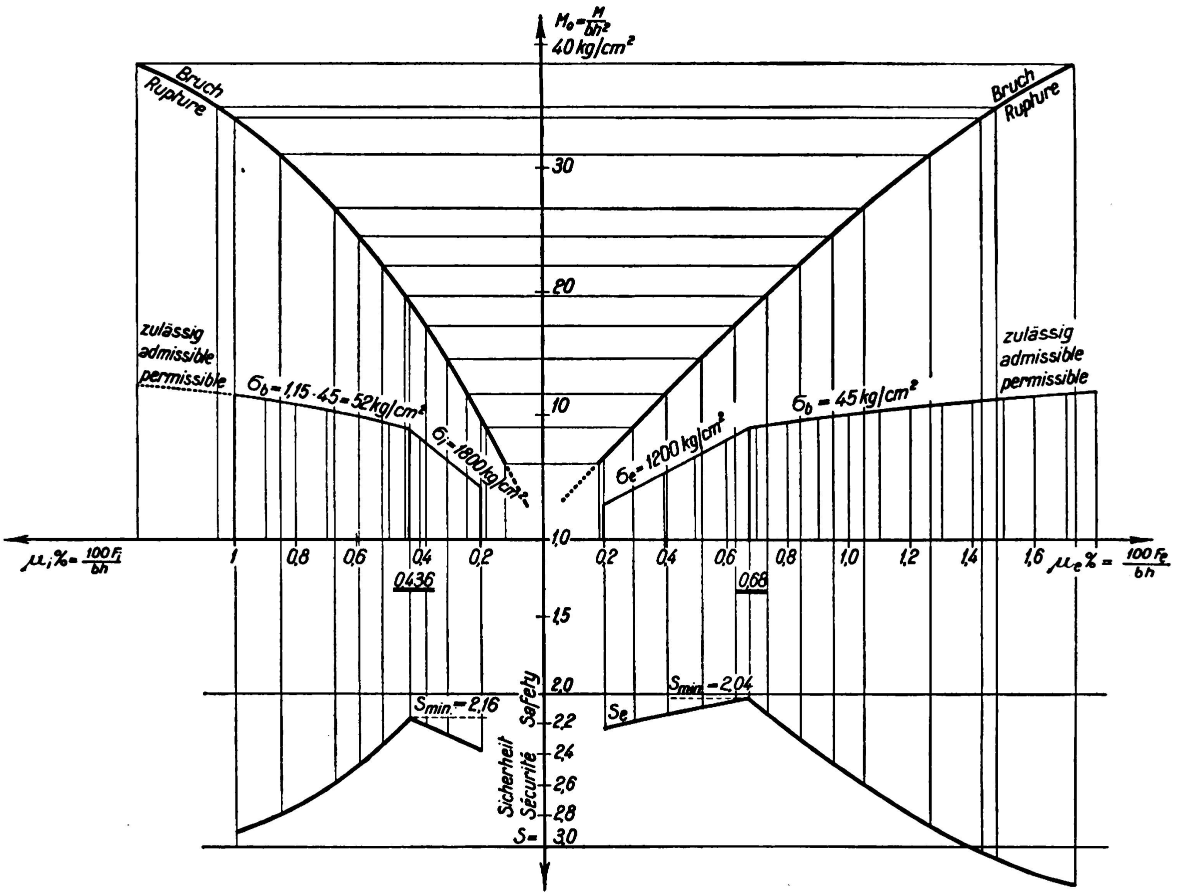


Fig. 5.

Isteg. St. 37.

how very important it is that methods of calculation should be adapted in such a way as to ensure the possibilities of high tensile steel being utilised to their full economic advantage, when designing members to resist bending, and at the same time to obtain closer agreement with the latest experimental knowledge than is possible with the present methods.

II d

Influence of concreting and dilatation joints.

Einfluß von Betonierungs- und Bewegungsfugen.

Influence des reprises de bétonnage et des joints de dilatation.

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IId 1

Reduction in Shrinkage and Expansion Stresses by the Systematic Use of Concrete Joints. — Application to the Philippe de Girard Bridge, Paris.

Verminderung der Wärme- und Schwindspannungen durch systematische Anwendung von Betonierungsfugen. — Anwendung für den Bau der Philippe de Girard-Brücke in Paris.

Diminution des efforts dus au retrait et à la dilatation par l'emploi systématique de reprises de bétonnage. — Application au cas du pont Philippe de Girard, à Paris.

J. Ridet,
Ingénieur en Chef Adjoint, Chemins de fer de l'Est, Paris.

The Philippe *de Girard* bridge, crossing the railway tracks outside the Gare de l'Est at Paris, consists of a concrete arch of 41 m opening (Fig. 1). The metal centreing was not erected underneath the arch but inside it, and later embedded in the concrete — an arrangement that was necessary in order to allow the passage of traffic while work was in progress.

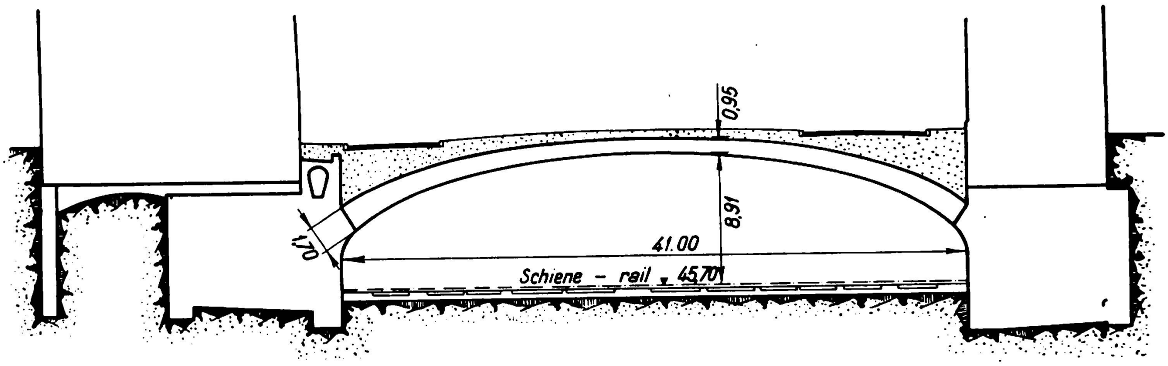

Fig. 1.
Philippe de Girard Bridge. Cross section.

The design of the arch was first attempted with due allowance for shrinkage on the assumption that the effect of this could be represented by a variation of temperature of 27° C. This implied considerable fixing moments at the springing, amounting to something of the order of 350 tonne-metres per m width of arch, and a considerable amount of reinforcement would have been necessary to resist them (Fig. 2). Such a design would have been complicated by the need to accomodate the reinforcing bars in the spaces left free by the ribs and bracings carrying the arch (Fig. 3), and the cost of the work would have been considerably greater in consequence.

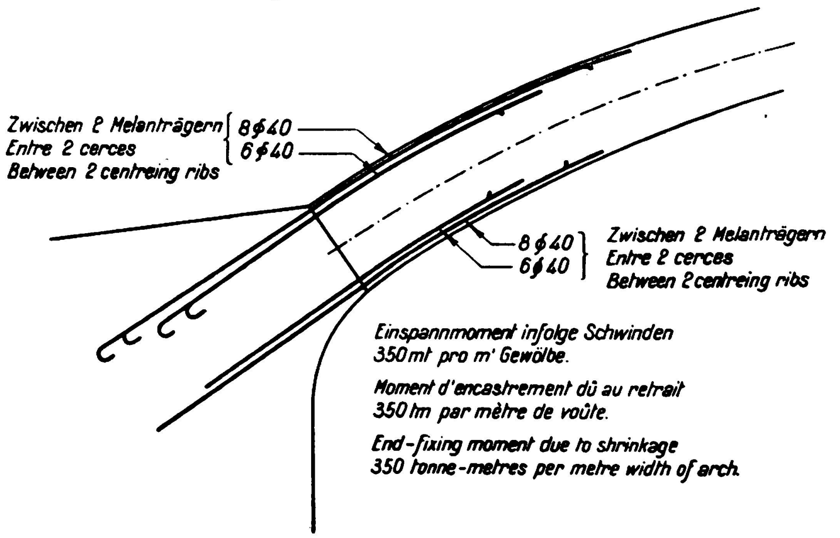

Fig. 2.

Section through haunch.

Ultimately, however, it was found possible to avoid the need for reinforcement by reducing shrinkage to a minimum, this being done by taking advantage of the fact that most of the shrinkage associated with the drying of the concrete occurs

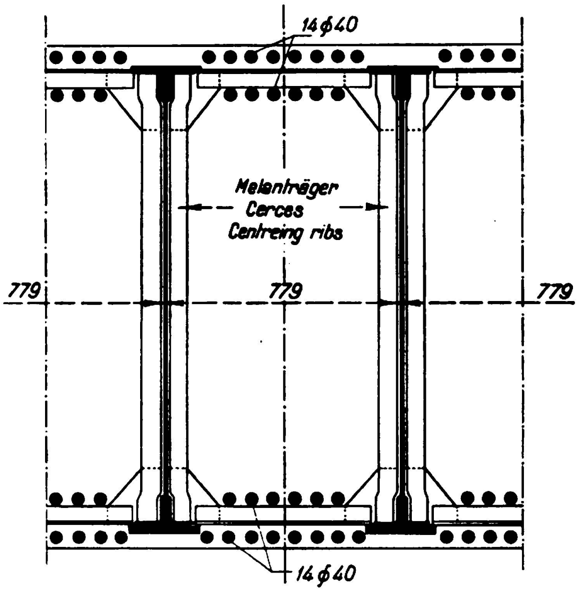

Fig. 3.

Section of arch.

when it first begins to harden: instead of pouring the whole of the arch in a continuous operation the latter was divided into voussoirs (Fig. 4), and these were concreted separately at a sufficient interval to allow of shrinkage taking place in each voussoir independently: in other words no concrete was placed in any given voussoir until a definite time had elapsed after completing the adjacent ones, during which the greater part of the shrinkage might be assumed to have occurred.

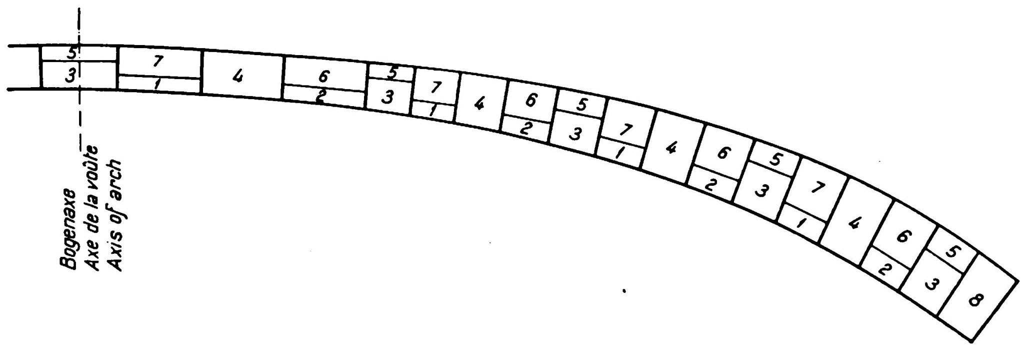

Fig. 4.
Sequence of concreting the voussoirs.

Moreover, the final closing of the arch at the haunches was delayed until the last voussoir was old enough to justify the assumption that its shrinkage had been practically finished.

By suitably planning the sequence of concreting on these lines it becomes possible considerably to reduce the effects of shrinkage, to lighten the construction, and to reduce its cost.

IId 2

The Effect of Concrete Joints.

Einfluß von Betonierungsfugen.

L'influence des reprises de bétonnage.

Ing. M. C. Fritzlin,
Rotterdam.

This effect has two principal aspects:

1) its effect on strength, and

2) its effect on weather resistance, water tightness, rust protection, etc.

The answer to the problem is essentially a practical matter and can never be reached entirely from considerations of theory; neither is any definite solution to be expected from laboratory experiments.

There is no uniformity of opinion among reinforced concrete engineers on this very important question, and the differences of opinion that exist find expression also in the regulations as to the treatment of construction joints that operate in various countries; regulations which may be looked upon as condensed experience. On this matter they all agree to the extent of requiring that the joints should be cleaned, roughened and wetted, and in reference to subsequent treatment they may be divided into two categories.

The first category — which includes Germany, Austria, Holland and Czechoslovakia — prescribes after-treatment on these lines: immediately before the new layer of concrete is placed a thin layer of mortar of the same composition as the mortar in the concrete is to be laid down; or in Italy the joint has to be wetted with cement grout.

In the second category — which includes France, Belgium and the United States — no such treatment is required.

As regards the French and American regulations, it would not be correct to speak of this matter having been overlooked but it should rather be assumed that the requirement has deliberately been omitted — and, in the present author's opinion, rightly so.

It may well be the case that when a thin layer is deposited the presence in it of a considerably higher proportion of cement than in the concrete itself may somewhat strengthen the bond, but on the other hand in placing this rich layer encouragement is given to the lifelong enemy of concrete, namely "shrinkage". Even if no overwhelming indictment can be sustained against a really thin layer, it will in most practical cases be impossible to deposit this uniformly over concrete which has already hardened, and usually it is necessary to be content with pouring the mortar or the cement grout from some distance away, through the reinforcement, onto the concrete already in place, making it impossible to avoid the collection of rich mortar at certain points which later show much more intense shrinkage than the normal concrete in the structure. Several instances are known to the author where, on the shuttering being struck, such layers spalled right off, so that with a pocket knife it was possible to scratch out long flat flakes which were glass hard in themselves. In this way the cure is worse than the disease. As a rule there is no great evil in a slight reduction in the bond, because in most cases concrete serves to transmit compression, and between two surfaces which are completely in contact with one another this is possible even without any bond at all.

In exceptional cases where the tensile strength of the concrete is taken into account it is always better to avoid the presence of concreting joints altogether — or, should this be quite impossible, to insert steel at the place where the joint occurs. As regards shear stresses, it is to be observed that the joints should be arranged at the point of minimum shear (a requirement specifically laid down in the American regulations), and further that it is now almost the universal practice for the whole of the transverse principal tensile stress at places where heavy shear loads occur to be taken by steel, so that even in this case only purely compressive stresses in the concrete come into question. In small jobs, provided ordinary care is taken (by roughening, cleaning, and wetting) the presence of a working joint in the concrete need cause no diminution in strength.

The problem of working joints in concrete is frequently affected by considerations other than those of strength which may be of great importance to the engineer: as, for instance, the questions of weather resistance and water tightness. It is well known that the denser the concrete the better its resistance to atmospheric influences, and the greater possibilities of weathering in a joint in the concrete can be attributed only to the smaller density at the joint, seeing that mechanical properties of the cement and aggregates are the same as elsewhere. Assuming that both masses of concrete are weather resistant, it is not a matter for apprehension that a small opening for attack may exist at the joint for all it means is that the neat surfaces in question are exposed like all neat surfaces to accelerated weathering. What may be dangerous, however, is a relative lack of density in the lower layers of the lift of concrete deposited last in order such as may result from improper workmanship. Apart from the usual unmixing process which occurs through the coarse aggregate falling to the bottom when the water content is too high (a matter to which insufficient attention is paid in most jobs at present) there may be a further weakening through loss of the fine materials for the first layers in the course of transport owing to the means of transport being in a cleaned condition. Such loss may be very considerable, but provided the necessary precautions are taken to avoid this defect, the presence of a working joint in the concrete is not attended by any risk whatever even as regards length of life.

There is not always complete assurance as to the water tightness of a joint in the concrete even if the work has been carefully done, particularly where heavy hydrostatic pressures occur as, for instance, in cellar work. Where the precautions against unmixing are adequate it is desirable to place as large a mass of concrete at one time as possible. But often it is not possible to erect the shuttering to above the water line, and in any case there nearly always has to be a joint in the concrete between the floors and the walls. The many large and deep cellars which have been constructed in recent years in Rotterdam have offered opportunities for studying this circumstance more closely, and have led to the conclusion that if the work is in other respects carefully carried out any lack of watertightness at the joint in the concrete is to be attributed to shrinkage.

In one of the earliest of the large cellars it was found that despite the great thickness of the walls, and the exceptional care taken to roughen and wet the joints and to cover them with a thin layer of mortar, the walls of the cellar became leaky at several places.

In a structure built shortly after this the cellar was formed of thinner but heavily reinforced walls, and special precautions were taken to ensure good watertightness by incorporating in the joints a continuous thin steel plate, in addition to extra shear reinforcement. In this way it was found possible to make the construction joints in the concrete quite watertight, but the method is attended by practical difficulties. The large amount of steel, including the plate, present in the middle of the surface to be cleaned, made it difficult to clean out the joint completely. The same objections arise to the method recommended in the American regulations in which the lower portion of the concrete is required to be dovetailed, and it is obvious that in heavily reinforced cellar work the execution of this dovetailing would be attended by great difficulties. Where such a method is suitable, however, it offers the advantage that the risk of shear is eliminated without necessitating extra reinforcement. At a later stage the method depending on the use of a steel plate was no longer applied, but in spite of this it was found possible to make large cellars, poured in lifts 1 to 2 metres high, completely watertight without the need for any subsequent treatment.

In Rotterdam all these cellars are founded on piles which have their heads in very yielding ground, with the result that the floor of the cellar carried on them has complete freedom to move. No cracking ever occurs in these floors.

If no special precautions are taken, the floor already deposited will have undergone a very large percentage of the total shrinkage to be anticipated in it during the time required for erecting the shuttering, placing the reinforcements and depositing the concrete on the next lift, and in the first few days after the new lift has been placed the old concrete will shrink considerably less than the new. It may be assumed that at the end of two weeks the shrinking will attain to $0.2\,^0/_{00}$, after four weeks to $0.3\,^0/_{00}$, and if the two layers are poured at an interval of two weeks there will, therefore, be a shear stress in the joint of the concrete, corresponding to a difference in length of $0.1\,^0/_{00}$. In very stiff construction of somewhat larger dimensions, the effect must necessarily be to displace one layer of concrete relatively to the other, and with greater dimensions still the tensile strength of the fresh concrete may be exceeded, with the result that vertical cracks will be formed at regular distances originating at the joint in the concrete. These cracks must necessarily be accompanied by horizontal cracks in the joint itself, due to the relative displacement of the layers, in exactly the same way as when the steel is loose in concrete subjected to tension. The presence of such cracks has, in fact, been confirmed, as is shown in the attached illustration which refers to two cellars approximately 25 m long wherein the cracks occurred in the walls at each end. At the joints they were as much as 1 or 2 mm wide, diminishing gradually upward, and it was established when the cellar was filled with water that horizontal cracks also occurred (although they could not be noticed with the naked eye) at the concreting joints on either side of the vertical cracks.

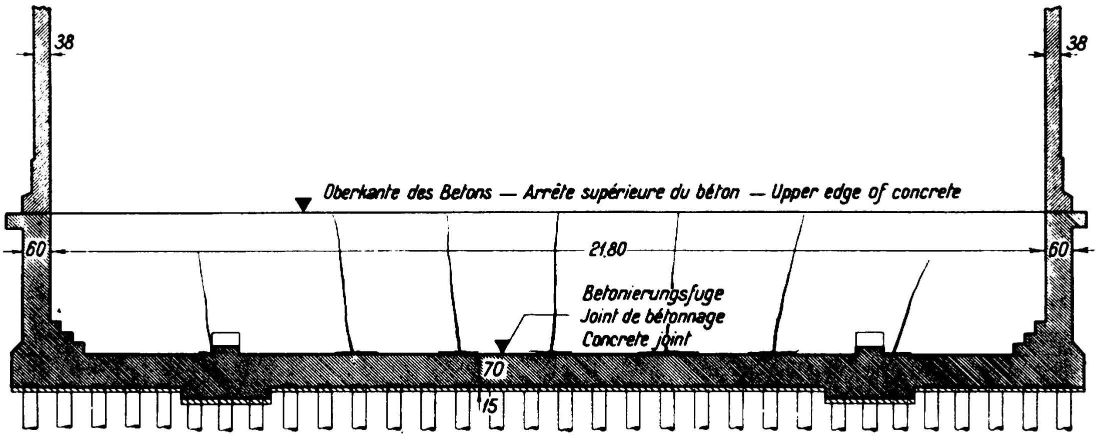

Fig. 1.

The proper precaution to take is, therefore, to let one stage of the work follow upon the other as quickly as possible, and to avoid shrinkage of the portion already deposited as much as possible.

In the construction of two cellars having exactly the same dimensions as those in which the cracks mentioned above occurred, using the same brand of cement and the same concrete mix, this principle was followed with notable success: the layer deposited first was not merely wetted just before the second layer was added but was kept sodden throughout the whole period from its production to the deposition of the next layer. An alternative (which is quite feasible in the case of the floor, for instance) is to keep it under water.

Conclusion.

In small jobs a joint in the concrete need have no effect on the safety of the structure, provided ordinary care is used. In large and stiff structures precautions must be taken to avoid irregular shrinkage, or where this is not possible, to combat the consequences of the latter.

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II

Free discussion.

Freie Diskussion.

Discussion libre.

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a)

Ministerialrat Dozent Dr. Ing. F. Gebauer,
Wien.

Qualifying his contribution to the discussion on Group IIa the author observes, on the one hand, that hitherto little has been done to investigate the subject of shrinkage stresses, and especially the changes undergone by these after a reinforced concrete structure has been in existence for a long time. On the other hand it is necessary that the method followed in the design of reinforced concrete should be as simple as possible and should rest upon an assured foundation, in order that the scope for such construction having a definite degree of safety may be promoted.

In the author's opinion the ideas which have found expression fail to correspond with the new status of scientific knowledge on the subject. Ultimately there is no difference between the behaviour of lightly and of heavily reinforced cross sections (Report ot the 2nd Congress: *Saliger,* page 316, last paragraph). Hence design according to the "n" method does not allow a correct estimate to be found of the factor of safety possessed by a structure. (Same reference: page 323, last paragraph.) Consideration of the test diagrams obtained with either lightly or heavily reinforced beams shows that calculation based upon permissible stresses has lost its significance. The calculation of beams must be referred, like that of columns, to the properties of the material used, and therefore to the conditions of equilibrium existing at the moment of failure, in which respect the yield point of the steel and the strength of the concrete are the determining values. These principles can be inferred from all the experiments that have hitherto been carried out, always provided that the beams have been reinforced accordingly. In particular this is indicated by the experiment mentioned by the author at the second meeting which relates to reinforcement between 0.5 % and 6.5 %.

In reference to his contribution under Group IIa the author would summarise the conclusions of the Congress in the following terms: —

An examination of the experimental results obtained with beams exposed to bending indicates that, with light and with heavy reinforcement alike, failure ultimately occurs through cracking of the concrete, and that the stresses in the steel exceed the yield point therein. Design can, therefore, be based upon the conditions of equilibrium at the moment of failure, account being taken of the desired degree of safety. Special attention is necessary in the constructional arrangement of beams wherein the steel bars are bent up at an angle. The "n" method of design, based upon the assumption of permissible stresses, does not afford a correct picture of the degree of safety secured in a reinforced concrete beam, and this method of calculation should be abandoned.

The experiments prove that the full carrying capacity, especially in the case of lightly reinforced beams, is not developed until the yield point of the steel is considerably exceeded, and a need arises for more exact investigations to interpret the reasons for this. Pending the attainment of further knowledge of this matter by research, the possibility of exceeding the yield point in the steel reinforcement should not be taken into account when making calculations for practical purposes.

b)

Dr. Ing. h. c. M. Roš,

Direktionspräsident der Eidg. Materialprüfungs- und Versuchsanstalt für Industrie, Bauwesen und Gewerbe, Zürich.

The author makes the following further observations in summing up the discussion on Group IIc: —

In the case of properly designed and carefully executed reinforced concrete structures experience supports the retention of the classical theory of elasticity as regards stresses within the permissible limits.[1]

Reinforced concrete structures which are suitably calculated, properly reinforced and carefully built show a practically elastic behaviour within the limits of the permissible stresses now usual,[2] namely $\sigma_{zul} = 0.4$ to $0.5\,{}_p\beta_d$.

The reconciliation of theory with actuality is a matter which must be referred to fundamental principles, account must be taken of the properties of the concrete or reinforced concrete as the case may be.

In regard to the technical properties of the concrete as such, attention must be paid to the total deformations divided as between elastic and plastic; to the influence of continued stresses on the deformation (time effect); to the effect of repeated loading (fatigue); to the changes undergone by the mechanical properties and power of deformation in the course of time, and to the consequences of shrinkage and temperature change. The influences of forces which exert their effect for a short period of time only (traffic load, wind, snow) should be separated from that of permanent loading (the dead weight of the structure itself) when considering the matter.

The effects, under these two headings, of the external load (the dead load and live load) and of temperature changes and shrinkage on the conditions of stress and strain, and therefore on the safety of the structure, should be worked out. Shrinkage and temperature are not external loads but exert an effect on the internal stresses alone.

It is only by making this separation in principle that clarity may be secured, allowing the theory of elasticity to be retained as a basis for the design dimensioning of reinforced concrete structures while making due allowance for the power of plastic deformation possessed by the concrete, so that as regards the

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[1] *M. Roš:* Versuche und Erfahrungen an ausgeführten Eisenbeton-Bauwerken in der Schweiz 1924—1937. Report N° 99, Swiss Federal Testing Institute, Zürich 1937.

[2] *M. Roš:* Vereinheitlichung der material-technischen Erkenntnisse und des Sicherheitsgrades im Stahlbeton. — Monatsnachrichten des Österreichischen Betonvereins, Festschrift, Vienna 1937.

factor of safety the significance of stresses due to shrinkage and temperature may not be confused with that effect of stresses due to the external loads (dead load and live load) — for the former may be diminished or may entirely disappear when the condition of failure is reached.

The statical and fatigue failure to which the factor of safety is referred is not dependent on the value of n, but upon *the compressive strength of the concrete* ${}_p\beta_d$, *the yield point of the reinforcing steel* σ, *the fatigue strength under pulsating load of the concrete* ${}_b\beta_u$, *the corresponding value for the steel* ${}_e\sigma_u$, *the percentage of reinforcement* μ *and the shape and dimensions of the cross section.*[2]

The endeavour to utilise the whole of the capacity for plastic strain possessed by the concrete in determining the degree of safety must be regarded as too far reaching. The material should not be deprived of its last reserves of strength; reserves which are not always present even under conditions of statical stress and which are only doubtfully present under conditions of repeated dynamic stress.[3]

c)

Dr. Ing. W. Gehler,

Professor an der Technischen Hochschule und Direktor beim Staatlichen Versuchs- und Materialprüfungsamt, Dresden.

In reference to this discussion the author desires to make the following additions:

1) In the calculation of reinforced concrete sections for bending a distinction should be drawn between the first zone of lightly reinforced beams (the usual case) wherein the exceeding of the yield point of the steel is attended by failure, and the second zone in which the criterion for breakage is the compressive strength of the concrete. As regards the first of these zones there is no justification for making any change in the method of calculation hitherto usual.
2) As soon as the limiting amount of reinforcement which separates these two zones shall have been determined by experiment, it will be possible for the first zone to be extended in practice right up to this limit. According to the Dresden experiments the limiting value in question has been determined for rectangular cross sections as $\mu_G = 1{,}8\,\%$, with a cube strength of $W_{b\,28} = 160$ kg/cm² and with steel St. 37, and as $\mu_G = 1{,}0\,\%$ with Isteg steel.
3) In the second zone, where failure is determined by the compressive strength of concrete, the usual method of calculation does not correctly indicate the factor of safety obtained. It is desirable, therefore, that a new method of calculation depending upon more complete utilisation of the material should be introduced.

[3] In the Swiss Regulations for Concrete and Reinforced Concrete of 14th May, 1935, plasticity has properly been taken into account in the case of hooped concrete columns and of beams subject to bending stresses.

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