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The Effect of Unequal Eccentricities on the Carrying Capacity of a Steel Column.

Der Einfluß einer Ungleichartigkeit der Fehlerhebel auf die Tragfähigkeit einer Stahlstütze.

L'influence des erreurs de centrage sur la résistance des colonnes métalliques.

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In actual steel work the incidence of a load in a column is in most cases quite uncertain and difficult to determine; the assumption of concentric loading, or of a loading which is eccentric by an equal amount at each end of the column, can be regarded only as a means of creating a standard of comparison of load by reference to which the effects exerted on the compressive force by the shape of cross section, length of column and magnitude of the eccentricities may be studied. The last mentioned of these quantities is determined, in the case of a column in a framework or in a steel skeleton, by its rigid connection with the adjacent members or by constructional details of such connections, and it can be precisely stated only if what are known as the secondary stresses in the frame are determined. The calculation of the latter is in any case very tiresome and requires lengthy calculations such as cannot be expected from a designing engineer; while moreover there is room for serious question whether its influence on the safety of the structure, having regard to the plastic behaviour of the material, can properly be compared with that of the primary stresses.

The approximate magnitude of the secondary stresses and the nature of their distribution within the column must now be regarded as established,[1] and the results of theoretical investigation have been confirmed by measurements of elongation on finished work from which it is known that the eccentricities of loading at the two ends are usually different — so much so, indeed, that the line of pressure in the compression flange usually intersects with the axis of the column. The solution to this more general problem, assuming no limit to the elastic behaviour of the material, offers no particular difficulty; the question is no more than an ordinary problem of stresses, and the safety of a column stressed in this way can be perfectly well guaranteed by ascertaining that the stresses

[1] *M. Roš:* Nebenspannungen infolge vernieteter Knotenpunktverbindungen eiserner Fachwerkbrücken. Report in Group V, Technical Commission of the Verband Schweiz. Brücken- und Eisenhochbaufabriken; June 1922.

lie within the permissible limits. It is only when the phenomena of plastic deformation are taken into account that the problem becomes complicated, giving rise to questions of critical loads; hence design by reference to a permissible stress is not a method which can be applied to all columns with equal assurance.

A predominant part in the treatment of any problem of plasticity by calculation is played by the "condition of yield", this being an analytical expression for the circumstances under which the steel changes from the elastically fixed to the plastically deformable condition. Under uniform conditions of stress, there is now agreement as to the nature of this phenomenon, but there is still room for question whether it can be assumed to operate in the same way where the condition of stress is not uniform. A newer hypothesis assumes that knowledge of the local conditions of stress is not sufficient for the prediction of yield phenomena, and that the question of risk of yielding can be decided only by considering the condition of stress over a larger region. On the basis of this new concept of "yield condition" it is now possible, by calculation, to obtain considerable knowledge as to the actual carrying capacity of a column which is subject to different accidental eccentricities of loading, taking account of the actual shape of its cross-section.

Assuming equal eccentricities at the two ends of the column the maximum moment occurs at the centre of the height and coincides, therefore, with the position of y_{max}. The carrying capacity of the column disappears if the resistance is so far weakened at this place by the sudden operation of the yield phenomenon that when an increase in the load occurs it can no longer contribute appreciably to the equilibrium between external loads and internal resistances. The lateral deflections thereupon increase very rapidly, and a new equilibrium can be reached only when the material has become set. When the eccentricities of load at the two ends are unequal y_{max} moves away from the middle of the column towards the end where the accidental eccentricity is the greater (Fig. 1). As long as it remains within the length of column l there is no important difference from the former condition, but when the ideal maximum of the elastic line falls outside the length of the column, so that the column receives its maximum bending moment at the end amounting to $P \cdot p_1$, then quite different phenomena appear (Fig. 2). The fact that the yield condition at the end of the column is satisfied then does not imply the disappearance of the carrying capacity, for the column cannot yet fail under load either laterally or in such a direction that unacceptable amounts of compression will occur along its axis. Yielding at the supported ends of the column cannot render its equilibrium unstable, for in such a case the column must for the most part retain its shape and length, no change in these magnitudes being possible without the expenditure of energy. The fact of the column being deformed over the whole of its length in an exclusively elastic way affects the deformation at the place where it has yielded, so that in this case the fulfilment of the "yield condition" means only that plastic deformation is about to take place.

As the load increases the support undergoes further deformation if the marginal conditions are different; and in view of what has been said above it is noteworthy that in such a case the carrying capacity does not reach its limit until the maximum moment at the end of the column coincides with the maximum of the elastic line, and the tangent to the former must then coincide there with the direction of the force.

It is of course difficult to give a correct and satisfactory explanation of the phenomena which now occur, and this can be done only approximately. If P_1 denotes the load which must exist at the end of the column in order to satisfy the "yield condition", then the stress at the point of yield is supplemented by

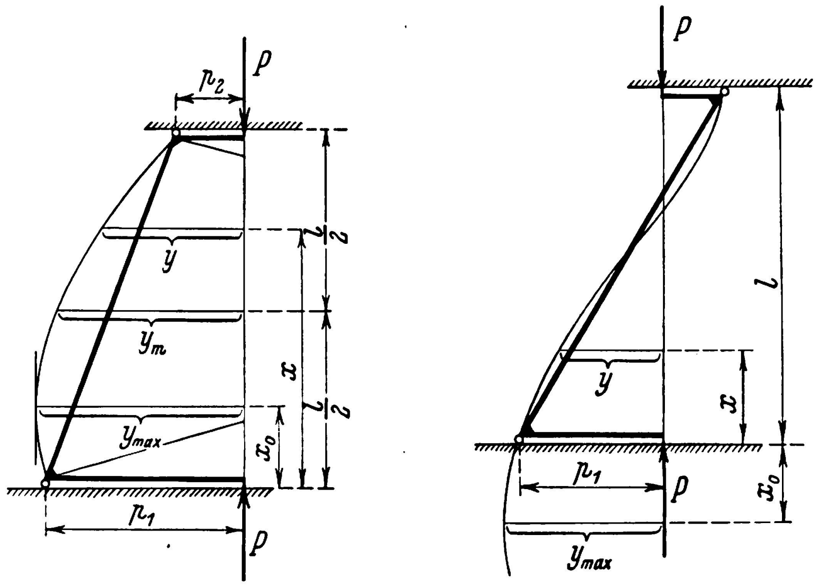


Fig. 1. Fig. 2.

the compressive force $P - P_1$, and on the basis of the experiments carried out by *Hohenemser*[2] and *Prager*[3] (who applied torsion as far as the yield point and caused this to be followed by tension) it is to be inferred that when bending occurs as far as the yield point and is followed by compression the bending moment supportable by the cross section is gradually diminished. In that case the load in the column could be increased only to the extent that its end has formed a "yield hinge". Thus the yielding has the effect of concentrating the increase of load along a particular direction, for owing to the failure that occurs at the point of yield the moment at the end of the column due to the increase in load $(P - P_1)\,p_1$, together with a gradually increasing share of the yield moment $P_1 p_1$ already sustained, must be resisted by the structure in some other way. The loss of bending resistance at the end of the column may be expressed by imagining that the pre-existing external load at the place where yielding occurs is supplemented by two moments which adapt themselves to the curvature of the elastic line corresponding to the new marginal conditions. The increase in the end moment of the column as P increases to P_1 opposes a moment of rotation in the opposite direction equal to $(P - P_1)\,p_1$, and the reduction in the yield moment through the imposition of the longitudinal force may be brought about

[2] *K. Hohenemser:* Neuere Versuchsergebnisse über das plastische Verhalten der Metalle. Zeitschrift für angew. Math. und Mech., 1931, p. 423.

[3] *K. Hohenemser* and *W. Prager:* Beitrag zur Mechanik des bildsamen Verhaltens von Flußstahl. Zeitschrift für angew. Math. und Mech., 1932, p. 1.

by a moment ΔM which may be calculated from the assumed condition of yield. Owing to lack of space it is not possible to quote here the very extensive calculations that arise, but these are given in the present author's contribution to

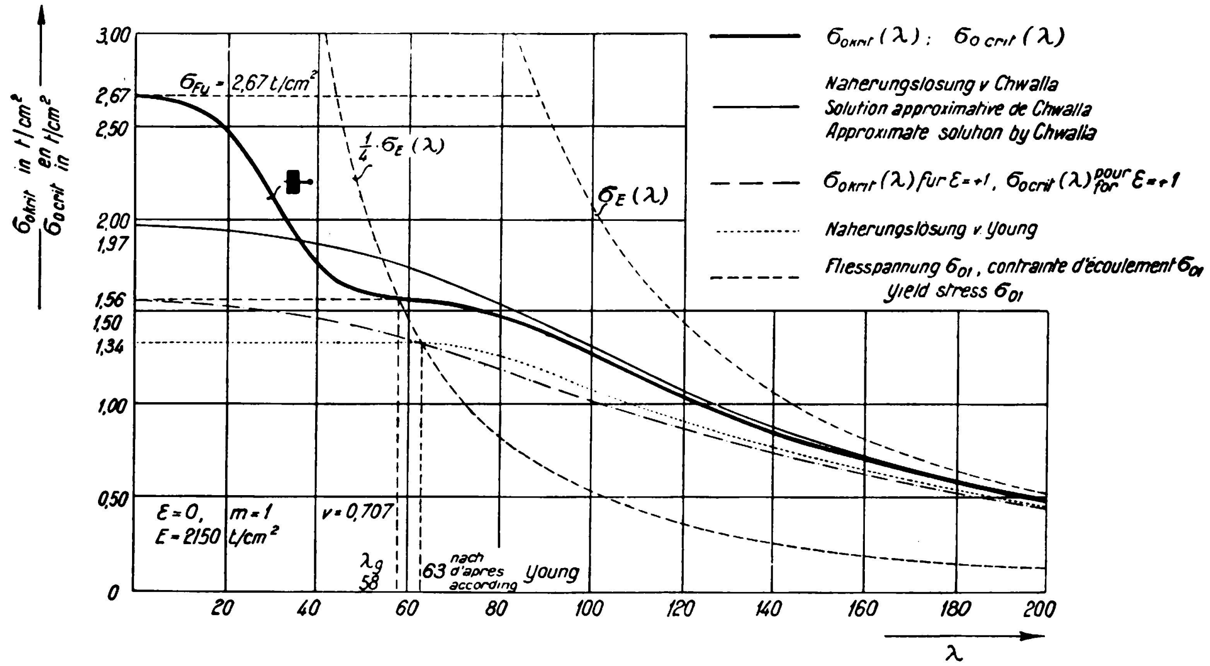


Fig. 3.

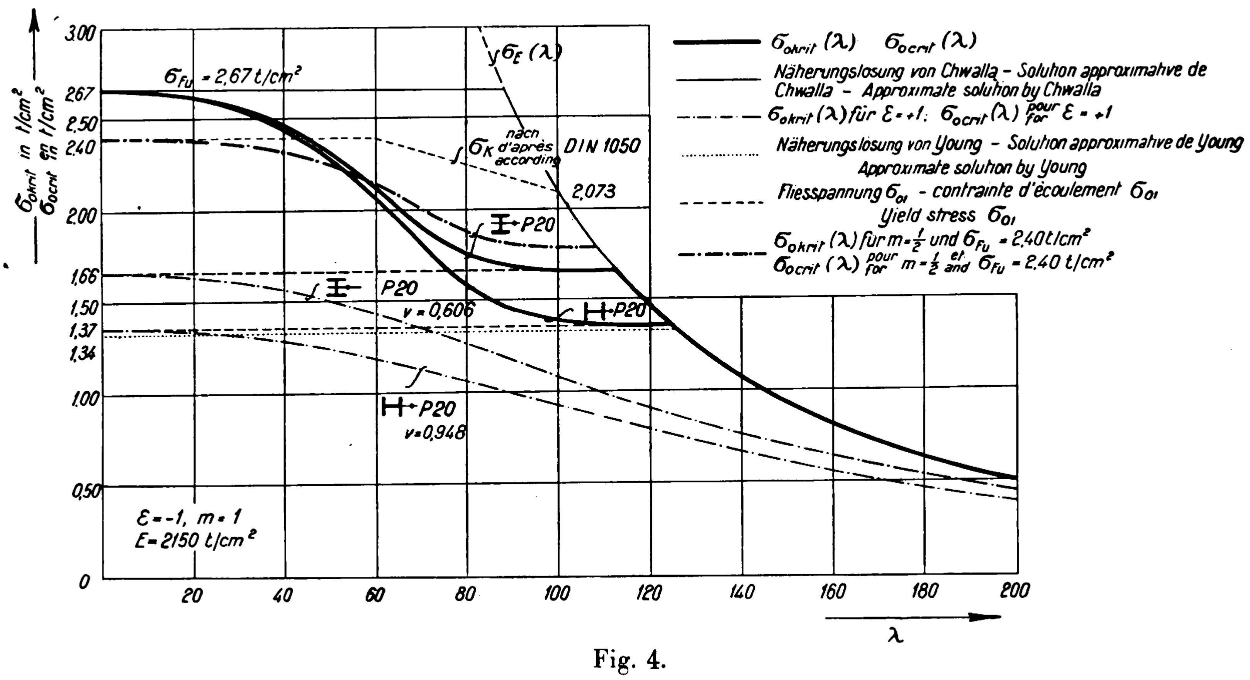


Fig. 4.

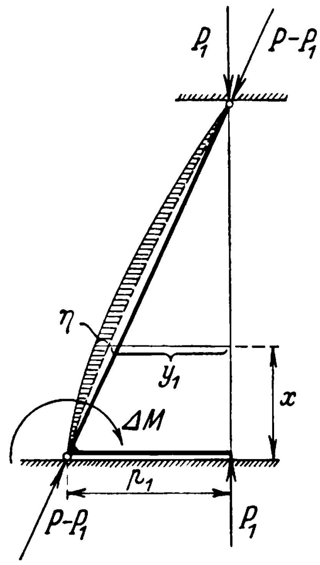
the journal, Der Stahlbau[4]. Figs. 3 and 4, taken from that publication, relate to the case where the eccentricities of loading at the two ends are equal but on opposite sides, and serve to show that the results so obtained differ quite

[4] *J. Fritsche:* Der Einfluß einer Ungleichartigkeit der Fehlerhebel auf die Tragfähigkeit außermittig gedrückter Stahlstützen. Der Stahlbau, 1936, Nos. 23 and 24.

appreciably from the cases hitherto examined in which the load is either concentric with the column or is equally eccentric at each end.

As already remarked, the conditions for a column which is built-in rigidly at the ends are such that $\varepsilon = \frac{p_1}{p_2}$ is approximately -1. The conditions where $\varepsilon > o$ or $\varepsilon = 1$ constitute rare exceptions, and moreover in such cases the eccentricities are usually small so that it is not correct to base the method of designing columns on exceptions of this kind. Where $\varepsilon = -1$ the lines $\sigma_{o\,krit}\,(\lambda)$ approximate very closely to the lines $\sigma_K\,(\lambda)$, the latter expressing the carrying capacity under concentric loading; hence it is correct, in the author's opinion, to make use of a "buckling stress line" — as is done, for instance, in the

Fig. 3a.



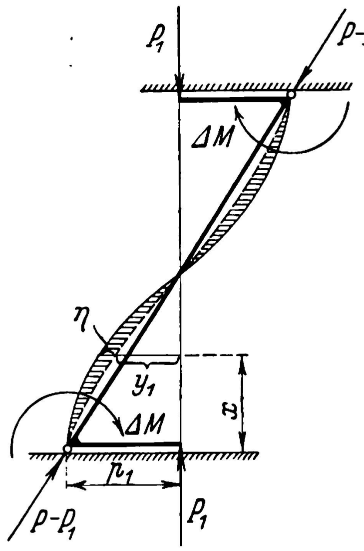


Fig. 4a.

German regulations DIN 1050. It would then be only a question of calculating maximum values for the usual eccentricities, or of estimating them from measurements, and of taking account of these when plotting the line $\sigma_{krit}\,(\lambda)$ for all columns. In such a case p would not normally be introduced, but the ratio, denoted by m, between p and the core radius of the cross section k. The value m = 1 would certainly be too high, and it would be preferable to use m = 0.5 in the present circumstances. In this way there would be obtained a "buckling stress line" which differs only slightly from that given in DIN 1050, but it would be advisable to deviate from the *Euler* line earlier than at 2073 kg/cm² — as, for instance, at about 1800 kg/cm².

So far as the author is aware no experiments for checking these calculated results are on record, and it is indeed not easy to carry out such experiments because in order to do so some method of supporting the experimental column must be devised which will enable the load to be centred within certain limits, as occurs in struts which are rigidly fixed at the ends. It is here a question of reproducing the varying marginal conditions of such a strut in the testing laboratory. How far this may be possible at all the author does not know; but it would be very desirable to attempt such experiments, as they would afford further insight into the actual behaviour of the compression members in a structure.