Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht

Band: 2 (1936)

Artikel: Rigid panel points of framed constructions

Autor: Campus, F.

DOI: https://doi.org/10.5169/seals-3205

V 4

Rigid Panel Points of Framed Constructions.

Steife Knotenpunkte bei Rahmenkonstruktionen.

Noeuds rigides de charpentes métalliques continues.

F. Campus,*
Professeur à l'Université de Liége,
Directeur du Laboratoire d'essais du Génie Civil.

I.

In 1929 the writer worked out a design for a continuous metal structure of staged frames, weighing 1817 tons, which was described in various publications[1]. The design, construction and tests of this structure showed the advantages of strictly applying the principle of continuity to the metal framework of multiple stage buildings and the progress thereby achieved in the design of such structures. The truth of these conceptions has been confirmed several times in Belgium by the reproduction of similar structures, accompanied by various improvements. These structures may be regarded as the most advanced attempts in this particular field as regards the degree of technical perfection. On the other hand, the big economic advantages of these structures have been shown by the marked success obtained in many competitive contracts among the best engineering firms of the country. Somewhat similar buildings have also been put up in other countries during the past few years and the particular problem has been investigated there in different directions. The object of this report is not to outline the principles of continuous metal structures, but it seemed useful to mention this point so as to show how the writer was led naturally to the study of rigid panel points. This particular subject may be regarded as the vital problem in the design and construction of this type of structures, besides being their *sine qua non*, their only difficulty, and, finally, their main feature and the basis of all their advantages. The calculation of these structures is not actually an obstacle or an essential element; what really matters is that the building be designed in terms of this calculation, and that depends simply on the constitution of the panel points. On the other hand, the elements and the operations of continuous construction, both at the workshop and on the erection site, are perfectly simple if the junction points are well designed, so that it is the latter which call for most attention and study. In them are based, directly and indirectly, all the causes of the technical and financial advantages of the system. The later part of this report will afford abundant proof in this respect.

---

* Vocabulary for illustration texts at end of article.

[1] Revue Universelle des Mines, 8th Series, Vol. IX., Nos. 5, 6 and 7 (1933). First International Congress of Bridge Building and Structural Engineering, Paris, 1932, Final Report. pp. 529—540.

This particular question was not new in 1929. An eminent compatriot of the writer, Eng. Prof. *A. Vierendeel*, had raised it in 1896 and solved it in practice in the design of truss which bears his name. However, up to 1929, this technique was, substantially, *M. Vierendeel's*. He himself had not carried out investigations on the panel points of his girders. Certain theoretical and experimental investigations had been carried out abroad, but they were incomplete and unsatisfactory in certain respects. On the other hand, *M. Vierendeel* considerably developed his system of panel points, which were aptly described by the term "arcade girders" which was applied to his method of bridge construction owing to the appearance of the large gussets. The conditions governing the rigid panel points in continuous multiple frame buildings are not identical to those governing the assemblage points in *Vierendeel* trusses. Again, if the system were adopted to too great an extent it would become impossible from the architectural standpoint. The dimensions and forms of panel points suitable for multiple frames of 16×5 m (these dimensions being greater than those of the largest rigid panels constructed) had to be investigated under heavy loads, while their size had to be limited as far as possible, or at all events below the dimensions used in the *Vierendeel* beams. Moreover, in this concrete case with which we are concerned, the panel points had to be dissymmetrical for architectural reasons.

This is the start of a set of investigations, some of which have already been described,[2] while others still remain unpublished. This was also the beginning of a number of applications of the method, not only to continuous metal frameworks, but also to *Vierendeel* girders. The investigations carried out on panel points had an immediate repercussion on the forms of numerous bridges of the *Vierendeel* system built since 1929 in Belgium and abroad. The dimensions of the panel points have been appreciably reduced in the majority of cases[3].

The object of this report is to outline the present state of the question, by mentioning the papers already published, giving the data as yet unpublished, and outlining the evolution of the problem in the light of actual examples.

II. Tests on Flat Models.

The first tests carried out under the writer's direction in 1929–30 utilized flat models of rigid panel points cut in 2.8 mm gauge steel plating, in accordance with the method adopted by the Dutch engineer *J. Schroeder van der Kolk*[4]. His researches were the only ones with which the writer was acquainted, except for *Wyss's* researches[5] mentioned later on. Certain references to previous work published in German will also be found in a paper by Prof. *A. Hawranek*[6]. *Wyss's* book also gives details of certain experiments on flat models which resemble the junction points of structures in some respects.

---

[2] Revue Universelle des Mines, 8th Series, Vol. IX., Nos. 1, 2 and 3, 1933.

[3] *A. Spoliansky:* Les ponts soudés en Belgique. Revue Universelle des Mines, 8th Series, Vol. XI., Nr. 8, 1935. Publications of the Int. Assoc. for Bridge and Struct. Engrg., Vol. 3, 1935.

[4] *N. C. Kist:* De vereischte dikte van knoopplaten van Vierendeellegers. De Ingenieur, 15th April 1916, The Hagua. — *A. Vierendeel:* Annales des Travaux Publics de Belgique, April 1924.

[5] *Th. Wyss:* Die Kraftfelder in festen, elastischen Körpern. Springer, 1926.

[6] *A. Hawranek:* Der Stahlskelettbau. Springer, 1931.

The reasons why new tests were carried out were as follows:

(1) All the tests known to the writer merely studied models of panel points or similar models subjected to transverse stresses, i.e., to bending. Both in the columns of frame buildings and in the booms (membrures) of *Vierendeel* trusses, the longitudinal stresses are just as important as the bending stresses. The normal axial stresses in vertical columns do not set up simple compression in the panel points with a horizontal transverse girder (three-branch assemblage point). As a matter of fact, the experiments have proved that, in the spread of the joint, the normal stress is excentric and itself sets up bending effects. The same thing must also apply to the *Vierendeel* trusses.

It was therefore deemed useful to carry out separate tensile and bending tests on the models. The stresses being applied within the elastic limit, the results were then combined by the principle of superposition. This combination was made by taking, as the ratio of the normal stresses and the transverse stresses (bending), the figure given by the preliminary calculation of 1929 for the main assemblage points of the particular building studied. As a matter of fact, the normal stresses to be considered in the columns were compressions. Tensions were applied to the flat model so as to obviate any increase in tension due to the plate buckling. The sign of the tensions or deformations was reversed. In this particular sense, these tests differ from and supplement those previously carried out by *J. Schroeder van der Kolk.* The results showed that the effects of the normal stresses must certainly be taken into account in the shape and dimensions of the panel points.

(2) The tests had a definite object in view, to afford a guide with regard to the shapes and dimensions to be adopted for the structure involved. The work of *Schroeder van der Kolk* and *Kist*[4] related to the panel point of a *Vierendeel* truss having curved symmetrical gussets (junction plates). In *Th. Wyss's* work[5] we find only one theoretical example of a stress trajectories in a three branch panel point, without junction plates (gussets).

The writer had no preconceived notion with regard to the shape to be given to the joints, and the elements mentioned could not serve as a guide owing to the fact that, where junction plates are employed, architectural requirements call for dissymmetrical assemblage points. It was therefore necessary to ascertain whether this dissymmetry was not unfavourable from the point of view of resistance. For these reasons, the researches were carried out on four models of panel points as Fig. 1, which summarises the results of the tests in the form of diagrams of the main stresses along the edges of the models under the combined effects of the normal stresses and bending.

As the work was urgent and the staff limited, and since the work was also increased by having four models to test, it was not possible, nor was it deemed necessary to undertake a complete examination of the joint in the way *Schroeder van der Kolk* had done. The results also showed that high stresses only were set up along the edges. It was therefore preferred to carry out a large number of concordant measurements for carefully ascertaining the stresses throughout the edges. However, the E S of bending were also ascertained inside the junction plates (noeuds) in a certain number of cross-sections. That these edge measurements are of vital importance is apparent from a paper by Prof. *G. C. J. Vreedenburgh*[7], of which the present writer

[7] De Ingenieur, 15th July, 1932, The Hague.

was unaware when making his own experiments. Describing the apparatus and equipment of his photo-elastic laboratory at the Bandoeng Technical College (Dutch East Indies), this writer states that, for the majority of investigations, it is sufficient to measure the edge stresses, because these are the highest and most characteristic. When made on steel plate models, these measurements present no difficulties if *Okhuizen* extensometers (*J. Schroeder van der Kolk*) or *Huggenberger* extensometers

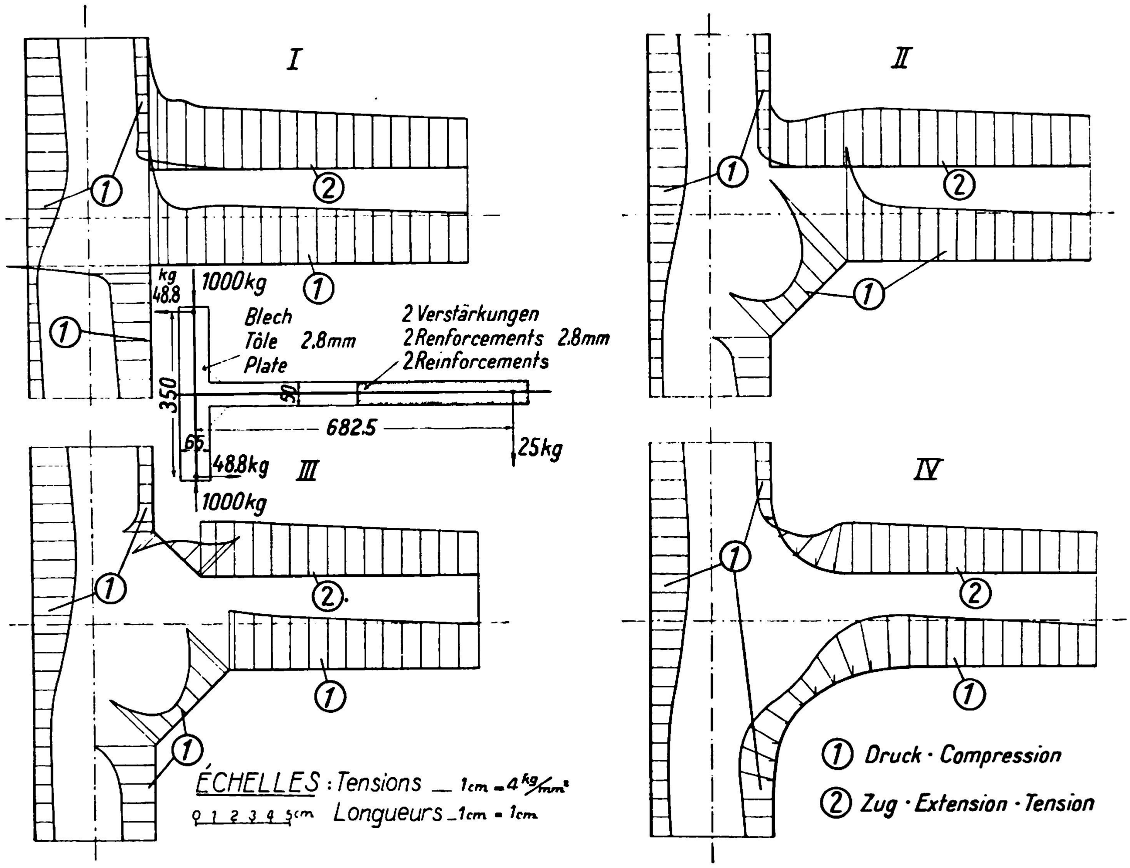

Fig. 1.

(the author's tests) are used. This justification may be regarded as valid, a fortiori, for the tests on threedimensional models referred to later on.

The results of the comparative tests on plane models were as follows:

1. The effects of normal stresses are equally as great as those of transverse stresses (bending).
2. The discontinuities give rise to supertensions, under the effect of both longitudinal and transverse stresses.
3. Panel points with junction plates are better than panel points without junction plates, and the double junction plates are superior to the single junction plates.
4. Panel points with double curved junction plates are superior to those with double triangular junction plates, as they lead to a general decrease in the stresses.
5. The dissymmetry of the junction plates (gussets) has not any adverse effect.
6. 45° Junction plates, and even circular curved junction plates are embedded too far in the columns or booms. The extent of this embedding must be pro-

portional to the bending moment of the corresponding member. Elliptical or 30^0 junction plates are probably best.

As regards 4, it should be noted (see Fig. 1) that the bending stresses taken in the vicinity of the panel point along the deflected member are lower for Model IV than they are for the others, and less than the figures calculated by the ordinary strength of materials formula. What is hard to conceive in terms of the strength of materials must be capable of explanation by the more precise conception of the theory of elasticity — a point to which the writer has already drawn attention[2]. This fact should be compared with the reinforcing effect of the assemblage points with curved junction plates which will be revealed several times at a later stage in this paper.

No attempt has been made to calculate the stresses in the junction plates (at the panel points) or to compare them with the results of the measurements, in view of the complication which the dissymmetry of the junction plates involved. Only Model I would have lent itself to this treatment, but this was a purely theoretical case, and was interesting mainly from the viewpoint of the over-stresses in the angles, which were not very accessible to calculation.

It should be noted that the models used bore certain approximate relations to the structure proposed, the girders and columns of which had double tee sections and the junction plates were borded with flanges. However, in view of the considerable difference in shape in the third dimension (transverse), it is quite certain that the results of these model tests must be regarded as purely qualitative. The results on three-dimensional models and on actual panel points, described further on, show that the agreement is satisfactory. Would these results have a more quantitative application for plate gussets without flanges? The author finds it difficult to express an opinion on this subject, because he has had no opportunity of testing similar junction plates, and he does not find it easy to conceive the transmission of the stresses in an assemblage of this kind.

III. Panel Points of the Rivetted Structure of the Institute for Metallurgical Chemistry at Val-Benoit, Liege.

As already mentioned above[1], this building has been described elsewhere.

Fig. 2 shows the standard panel point (of three branches) of this particular building. It was for this panel point that the tests on flat models were made as described above. The 520 mm radius of curvature of the top junction plate was the maximum allowed if no projecting parts had to be visible in the walls or floors. Due to mistakes of various kinds, certain of the junction plates (gussets) projected slightly. No trouble was found in concealing them by slabs and plinths. In the case of certain free columns, where the projection was more pronounced owing to bad design, the difficulty was overcome by putting in small soubassements, which gave a modern and pleasing architectural effect. Subsequent experience showed that the radius could have been still further reduced. Projections at the floors and walls can be avoided by suitably selecting the level of the concrete floors in relation to junction plates of suitable curvature. It should be noted that the total spread of the panel point on the columns is 2052 mm, the distance between the centre-lines of the joists being 5,000 mm. According to the recommendations of *M. Vierendeel*, however, the latter figure should have been 3333 mm. Finally, the distance between the centre-lines of the columns was 16 metres and the maximum spread 1335 mm with reference to the centre-line of each column. These arrangements seem bold enough for a first

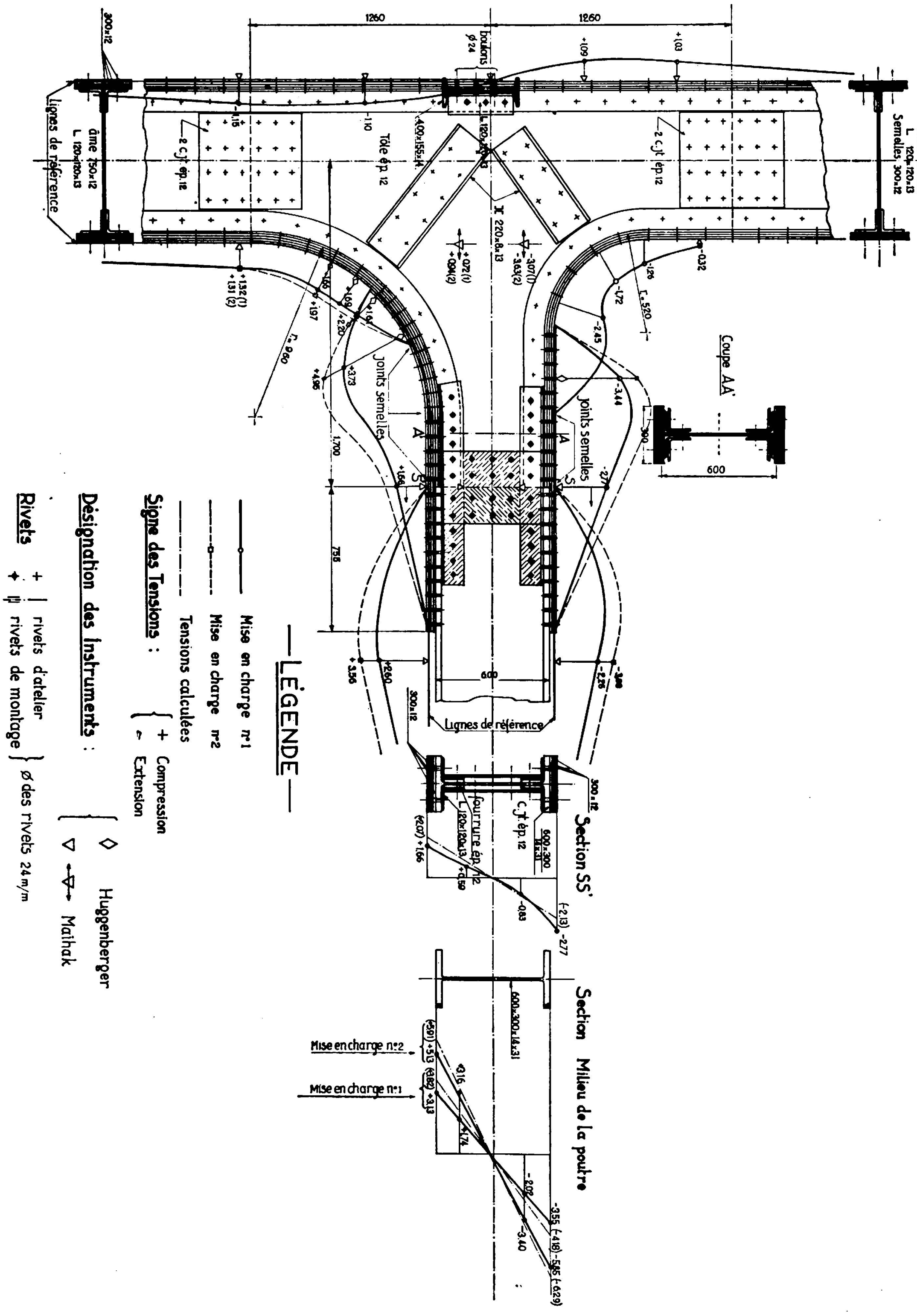

Fig. 2.

application, and certainly only slight changes could be made in view of the use of rivetting.

The results of the tests on plane models of course only formed one of the elements governing the final choice of the form of panel points adopted. The models had been cut entirely from a sheet, without discontinuity or assemblage. In the building as designed, rolled or built-up sections of different shapes had to be connected by rivets, and some method of continuity had to be ensured comparable in principle to the continuity of the models, i. e., the beams had to be rigidly secured in the columns. Moreover, certain of these assemblages had to form erection joints. With regard to the composition of the columns and joists, it appeared that the double curved junction plates (gussets) were more advisable still for reasons of correct and easy assembly and erection, than they were because of the favourable results of the model tests. Actually, this conclusion was not surprising. But the novelty of the arrangement called for every possible precaution, and some assurance had to be obtained as regards the possibility of reducing the spread of the joints as much as possible, independently of their shape.

Fig. 2 shows a type of assemblage point as actually carried out. It differs from the author's design only by two facilities allowed to the builder. The junction plates were made circular, whereas the design provided for the lower junction plate in the form of an equilateral hyperbola. The builder counted upon bending the angle irons by machine, whereas he bent them hot on templates. It would thus have been possible to conserve the original shape. Nevertheless, the circular form must be regarded as suitable. Finally, the joint between the junction plate and the beam was made complete in the angle irons, the webs and all the flats excepting one. This was asked for from reasons of transport and erection. The initial design provided for the classic overlapping of all the joints. No trouble was experienced from the latitude allowed, which made the work of erection accurate and easy.

As already stated, the curved angle irons were bent hot on templates. All the gus sets (junction plates) and all the framework of 1817 tons were identical. The gussets were cut with the acetylene torch and the edges trimmed off with the pneumatic chisel. All the rivet holes were drilled through the sheets in packets by multiple drilling machines. As a result, very little reaming had to be done on the site.

In the monograph relating to this building[1], the writer referred to the general ease of erection; the arrangement of the panel points contributed to this, and did not cause any trouble. It reduced to a minimum and also greatly facilitated rivetting up on the site. It will be noted in Fig. 2 that this system of panel point interrupts the continuity of the interior boom of the compressed column, which is perhaps a drawback of the system from the standpoint of the transverse rigidity. It was noticed whilst the columns were being transported and erected.[1] As a matter of fact, this arrangement became necessary because it was impossible, architecturally, to allow for assemblages comprising angle irons projecting on the outside of the flanges of the columns and girders. Calculation showed that the possible drawbacks were so slight that it was not found necessary to put interior angle irons in to ensure a certain continuity of this boom, in view of the stiffening by the U-irons rivetted on the junction plate (gusset) and the general concrete lining put in afterwards.

In the case of the 4-branch panel points, on the other hand, the two booms of the intermediate columns were made continuous throughout the joints, mainly for reasons of rigidity during transport and erection, but also for strength. Figs. 4 and 5 show the upper panel points of the end and intermediate columns. Fig. 6 shows a typical panel point. The illustration (see Fig. 1, p. 530 of the Final Report of the First Congress, Paris, 1932) shows the small amount of space taken up by the junction plates relative to the rest of the steel work.

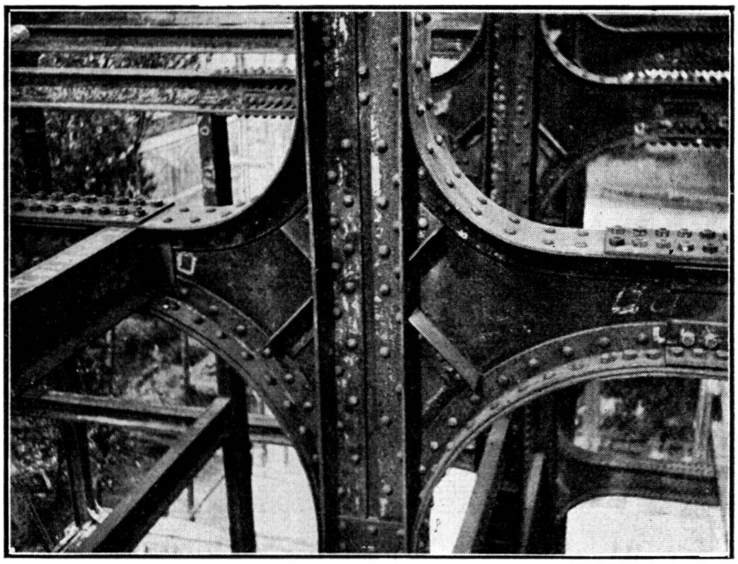

Fig. 3.

Loading tests of the type described in the monograph already mentioned[1] were carried out on this building in 1931. On this occasion a standard (typical) junction plate or panel point was thoroughly examined. Fig. 2 gives the results in a more

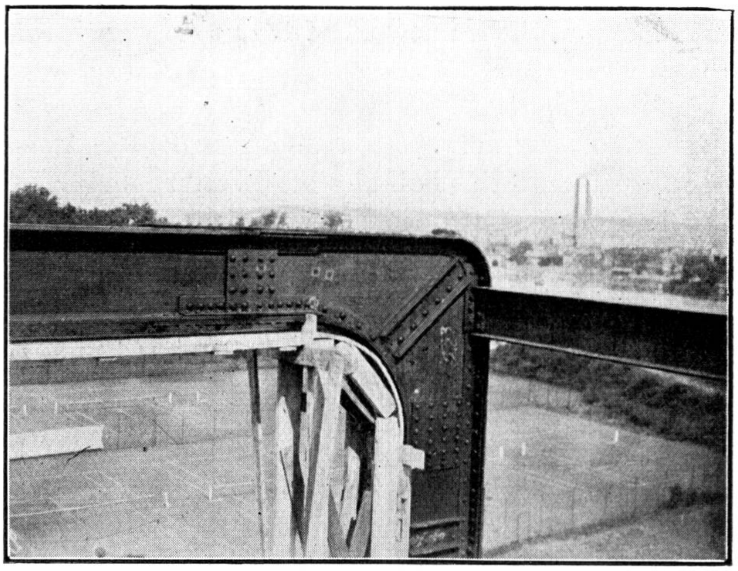

Fig. 4.

detailed and correct manner, having regard to the constitution of the assemblage, than the previous publications.

The stresses were applied by two symmetrical loads of 12.5 tons, suspended symmetrically from different parts of the middle of the girder (16.00 metres span), spaced 5.36 m apart for the first loading and 3.60 m for the second. It will be noted that the relative tensions are moderate and that, allowing for the discontinuity of the connecting plates in the joint, the shape of the stress curves is similar to that for the flat model. The tensometers were located along the longitudinal axes of the flanges (Fig. 6) and thus recorded maximum tensions.

Criticisms were afterwards raised regarding these panel points. *A. Vierendeel*[8] thinks the joints are very strong, but are fairly expensive to make. This engineer thinks it would have been better to use the *Vierendeel* joints ordinarily used hitherto, i. e., panel points comprising curved, projecting angle irons on the connecting plates of the columns and not joined up tangentially. As already stated, this was impossible for architectural reasons. This point will be discussed again later, but it may be

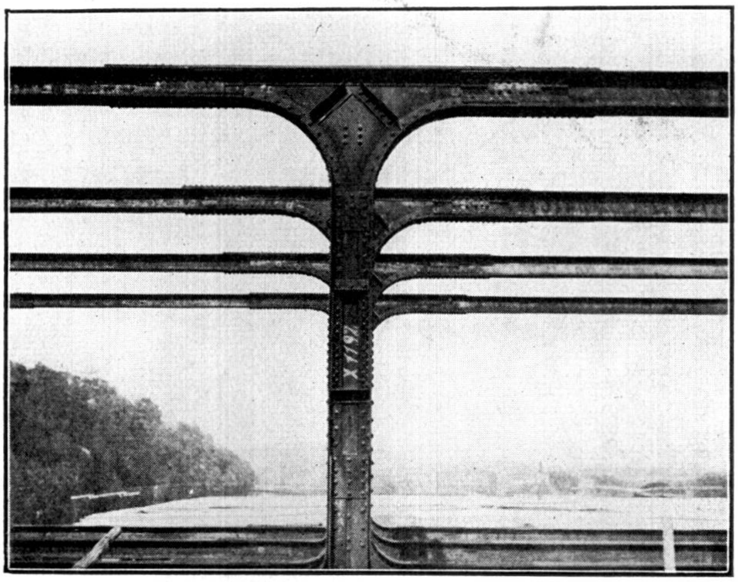

Fig. 5.

added that the present-day *Vierendeel* panel points include discontinuities whose effects can only be neglected owing to their considerable spread. If such a solution can cheapen the cost in cases where it is possible (say, for a bridge), it ought not to be rejected. But it is always doubtful whether these criticisms regarding economy are relevant, particularly as, in this case, the very numerous panel points, all identical and of small dimensions, only formed a very small fraction of the structure, and much less than in a *Vierendeel* girder.

Prof. *A. de Marneffe*[9] would have preferred triangular connecting members, and thinks they would have been cheaper. Compared to the criticism of *Vierendeel*, which is based on personal and practical knowledge, and who has no objection to the curvature of the gussets (junction plates), this criticism of the writer's colleague is purely ideological.

The shapes of the various models tested show that the panel points were studied

[8] Calcul d'une ossature gratte-ciel. Bulletin techn. de l'Union des Ingenieurs de Louvain, Nr. 4, 1932.

[9] *A. de Marneffe:* Les réactions de l'acier vis-à-vis de la concurrence du béton armé. Revue universelle des Mines, 8th Series, Vol. X, Nos. 5 and 6, 1934.

without any preconceived notions, and that junction plates (gussets) of triangular section were considered. The curved junction plates (gussets) were adopted after duly allowing for alle the elements of the problem involved in practice, and after a complete study of the precise problem to be solved. It was deliberately intended to achieve perfect continuity and all the advantages it brings. This result has been perfectly achieved, as shown by the results[1], which were fully recognised and appreciated by *Vierendeel*[8]. No accurate investigation of any kind has proved that this

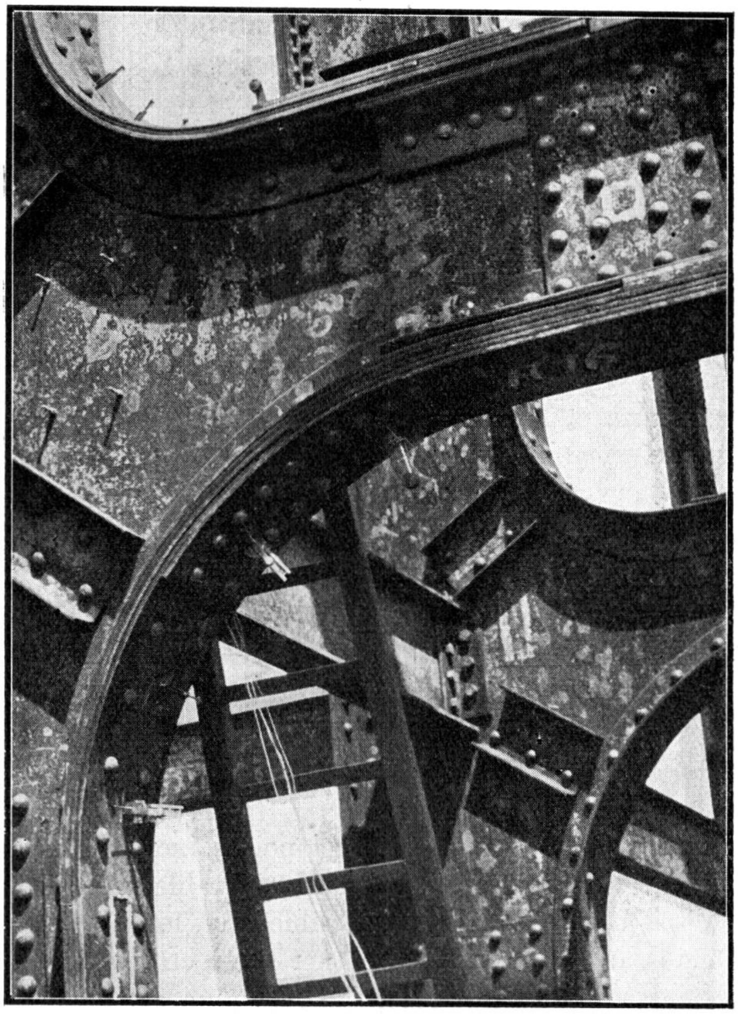

Fig. 6.

result could have been achieved more economically by adopting triangular gusset plates fulfilling the same conditions, whereas the writer's own investigations had led him to reject the triangular gusset plates for reasons which were practical rather than theoretical. *A. de Marneffe* also argues that, had they been free, the builders would have adopted simpler and cheaper connections. Apart from the fact that this point is not proved, any freedom of choice in the matter would only have had any bearing on the problem if the type of connections suggested by the contractors had satisfied conditions which were equal and adequate to the problem. The specification certainly did not call for the type of gusset plate proposed, but explicitly invited those tendering for the work to suggest other types. Eight well known firms of contractors in Belgium, Luxemburg and Germany, tendered for the work, and five of them submitted alternative designs. Some of these latter included triangular gusset plates with more complicated connections than the type suggested, involving a big increase in weight and cost. On counter-verification, all the less expensive schemes adopted purely and simply the type of panel point (joint) proposed. Actual experience therefore refutes the above argument. The abovementioned criticisms are not relevant, nor have they convinced the writer in any way that he ought to have done differently to what he has. Any real solution of the problem must necessarily be grained from a thorough and detailed investigation on paper. In this sense, the writer's own experience might have led him to make certain slight modifications which would not, however, have substantially modified the solution and the form. It must be remembered that rivetting imposes certain limits, which the joint described allows for.

It is interesting to note, that only one contractor took advantage of the opportunity allowed to submit an alternative scheme for a welded building, but the weight and the cost were too high. This seems very paradoxical in view of the typical successes achieved with this system of framework and panel points in the welded construction of steelwork.

IV. Welded Assemblage Points of the Vierendeel de Lanaye Road Bridge.

A first type of welded assemblage point was realised by *M. Spoliansky* (collaborator in the design of the building described above and in the tests on flat models) in connection with the alternative design for a welded road bridge (*Vierendeel* de Lanaye) put out to tender in 1931 by the Belgian Bridges and Highways Authorities. This structure has been described in several papers[2], [3], [10].

The joints connecting the rigid uprights to the booms have curved gusset plates and flanges meeting tangentially, like those of the previous structure. They were completely welded up in the shops. The flanges of the booms are not interrupted. The erection joints were rivetted. Fig. 1, p. 255, Final Report of the 1st int. Congress, Paris 1932, gives particulars of this joint. The details were designed before the tests described above, but were modified slightly in the dimensions on the actual job, due to reinforcement of the uprights. A preliminary test on a threedimension reduced scale model was made in January, 1932, at the contractors' works at the request of the authorities. An account of these tests has already been published[2], [10].

Fig. 7 shows the comparative results of the tests and of the calculation of the panel point. As in the case of Fig. 2, the representation of these results has been made more correct and more detailed by allowing for the rivetted assemblage, but this has not affected the results for the actual joint. On the scale of the model, the load applied represented 1.53 times the maximum stress of the upright. The measured stresses are lower than the calculated stresses practically everywhere, and at all events at the points where they are high. This is due mainly to the imperfect methods of calculation adopted, due to *Résal* and *Vierendeel*. The methods of calculation will be

[10] *Campus* and *Spoliansky:* Final Rep. of the 1st Int. Congress of Bridge and Struct. Engrg., Paris, 1932, pp. 254 et seq. *Santilman:* Le nouveau pont de Lanaye sur le Canal Albert. Annales des Travaux Publics de Belgique, December 1933.

discussed later. Another element must also have contributed to these divergences. The measurements were made with *Huggenberger* extensometers arranged in pairs, and symmetrically, at various points on the longitudinal plane of symmetry of the model. The apparatus were therefore not placed along the longitudinal axis of the flanges, but between this axis and the edges. Hence they did not record maximum tensions, but lower values.

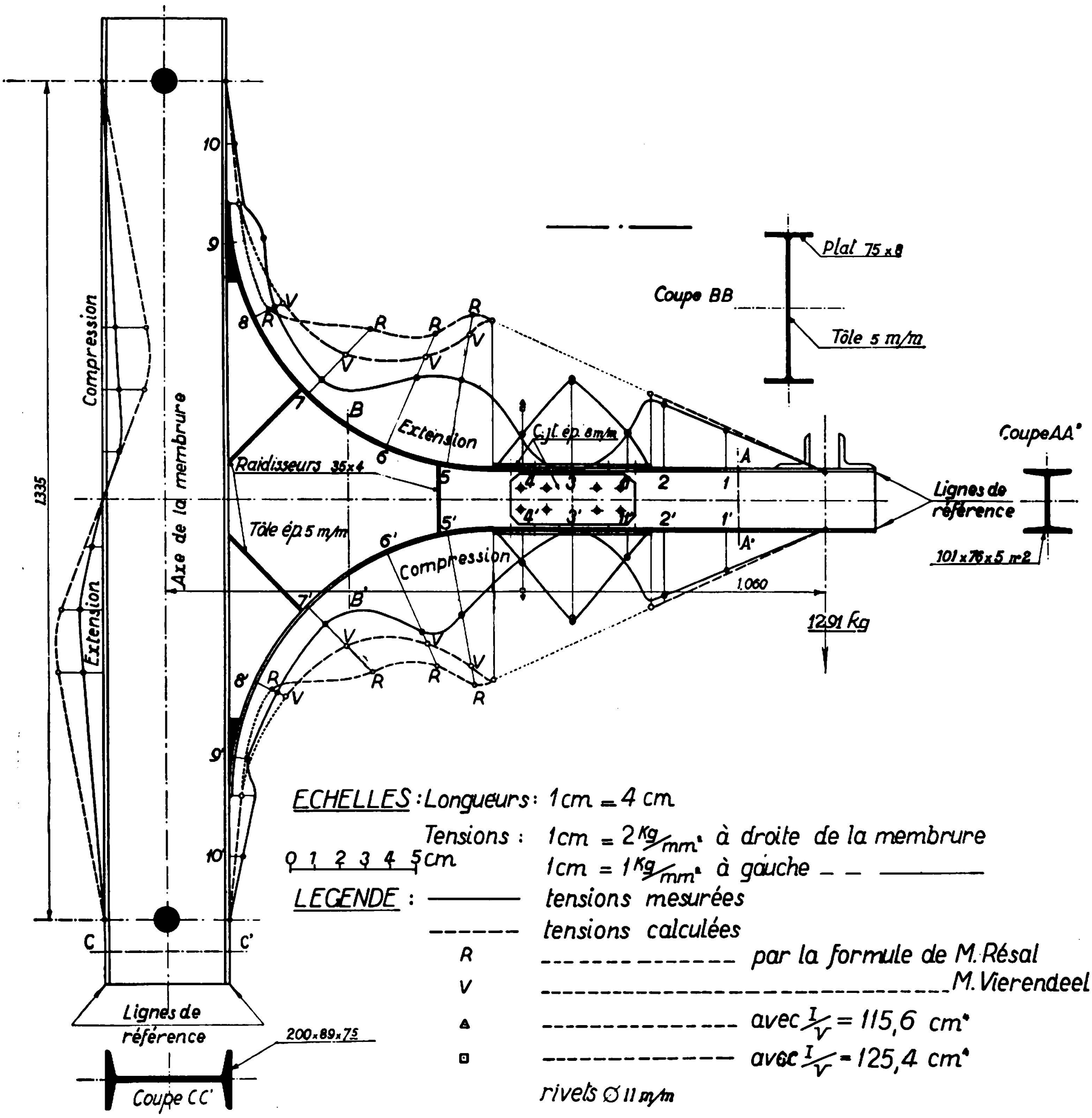

Fig. 7.

It will be seen farther on that the stresses in curved flanges are not constant, but decrease from the axis towards the edges. *Résal's* or *Vierendeel's* formulas must correspond to mean values of these tensions. The points of measurement may have been below the points at which average tensions were obtained, but owing to the small dimensions of the model, this difference could not be considerable in terms of the average. It certainly cannot explain the big difference between the measured and calculated figures, the main reason for which must be the first one mentioned.

At the point where the panel point engages on the boom, the calculated tensions become lower than the measured tensions, although both are very low. This is due to a discontinuity of the gusset plate in the tangential direction of the flanges. The curved flange is therefore free over a certain length, without being attached to the gusset plate. Because of this, a secondary bending must be set up, masked by an increase in the measured tensions, and which proves that normal to the curved flanges stresses must be set up where they are joined to the gusset plate. In the actual panel point, over this gap in the main gusset plate, small external gusset plates were welded to various parts of the flanges (see Fig. 1 of the Final Report of the First Congress, Paris 1932). While they must minimise the phenomenon mentioned, they have the advantage of retaining damp and dust and of complicating upkeep by setting up corrosion. This may be avoided by plugging up with cement or asphalt composition the small gap existing at the bearing end of each joint. It should be noted that, owing to the conditions of test and the dimensions of the model, the effect of the normal stress of the booms was not taken into account; only the edge effects intervene, i. e., bending.

Tension measurements were made on the two faces over the length of the gusset plate, and from these the main tensions were deduced. They were found to be very moderate, and lower than the tensions taken on the flanges, but agree very well with the calculated values.

Finally, the model was loaded dynamically several times, and then until breakage ensued. This took place at the rivets of the assemblage joint, and not at the panel point. The coefficient of safety in terms of this test was 6.36. Since the displacements of the model were higher than those of the actual joint, the true coefficient of safety must be still higher[2].

Loading tests on the bridge were carried out in May, 1933, under the direction of the Bridges and Highways Authorities, assisted by the writer and his usual collaborators. Tension measurements were made on the uprights and the panel points. Hitherto these have only been briefly described in the above-mentioned paper by *M. Santilman*, Chief Engineer of the Bridges and Highways Department[10]. On this test, the bridge was loaded by means of compression rollers and trolleys weighing 56 tons in all, located so that, according to the calculation, they would produce the maximum edge stress in the upright M. 4, on which the main measurements were made. According to the calculations given by *M. Spoliansky*, who designed the bridge, the tensions calculated for the parts of the upright M. 4, and certain adjacent boom sections, are shown on the diagrams of Fig. 8 along with the results of the tension measurements on the members and the panel points, the method of assembly being taken into account. It should be noted that, on the uprights, the tensions were measured by *Mahiak* extensometers placed along the longitudinal axes of the flanges. For all the other elements of the structure, *Huggenberger* tensometers were used, these also being located along the axes of the flanges, and thus recording maximum tensions throughout (Fig. 9).

The tensions, both measured and calculated, were set up by the combined effects of normal and edge stresses (bending). This was not the case for the model of the panel point, in which only the bending was considered. It will be noted that, for the upright M. 4, the stresses due to the measured and calculated longitudinal ten-

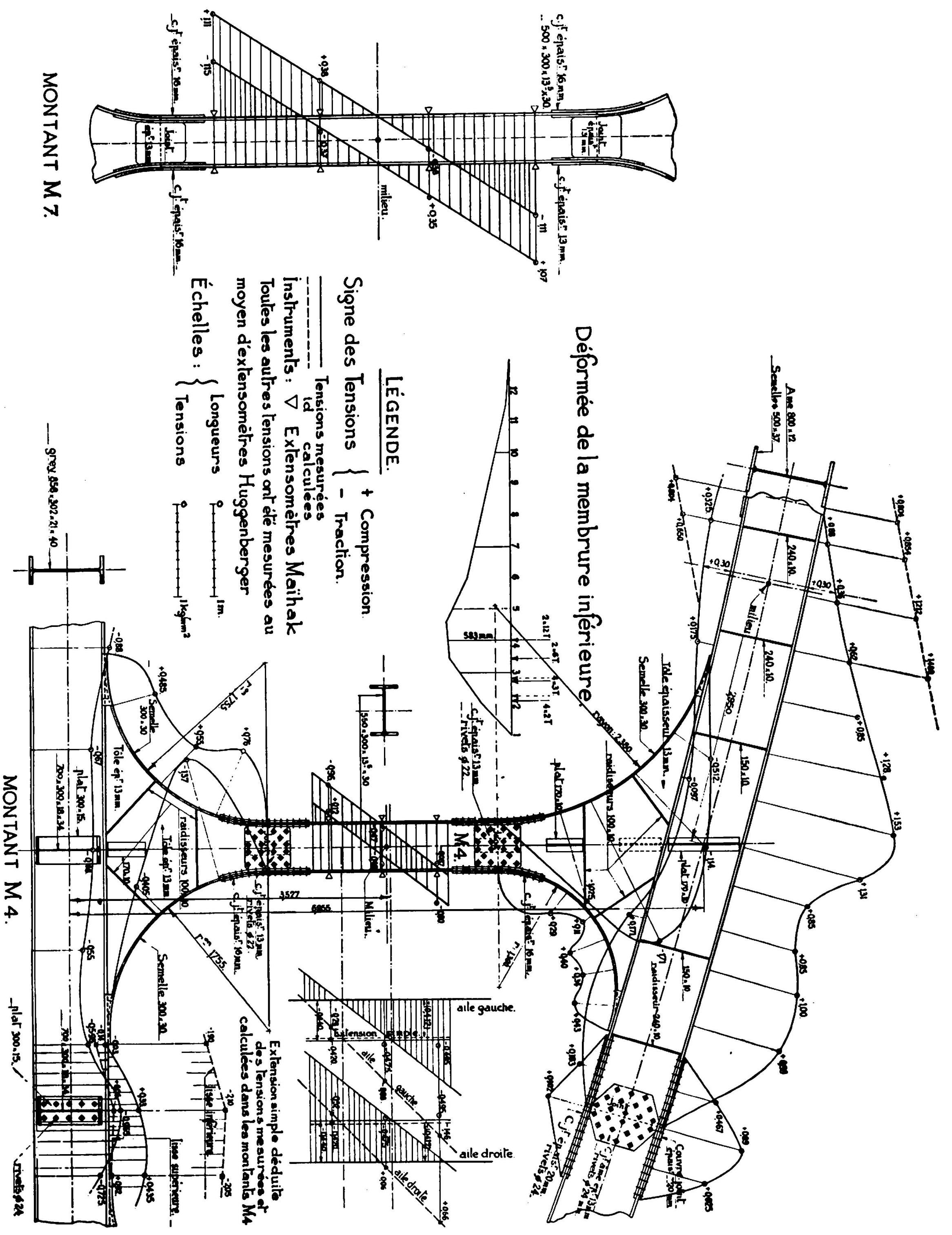

Fig. 8.

sions are nearly identical (measured value 0.440 kg/mm², calculated 0.412 kg/mm²). On the other hand, the bending stresses measured on the same upright are less than the calculated stresses. The panel points therefore reduce the bending moments. The points of inflection (peak points) of the uprights M. 4 and M. 7 are very close to the middles of the uprights, whereas they should have come elsewhere according to the calculation, i. e., nearer to the lower boom. This upward displacement of the point of inflection is logical in view of the stiffening action of the panel points and the roadway. However, the roadway did not include any longitudinal ties. The tensions measured on the flanges of the booms are considerably less than the calculated values. The maximum tensions recorded on the curved flanges of the panel points attain very moderate values having regard to the overloading of the bridge. The very regular variation of stresses over the length of the curved flanges will also be noted, showing that the welds were stressed very evenly in accordance with the calculation. The normal stress tensions in the two booms, deduced from the measurements, are very much lower than the calculated figures. For the upper boom 4—5, the calculated figure is $+ 0.804$ kg/mm^2 (compression) as against the measured value of

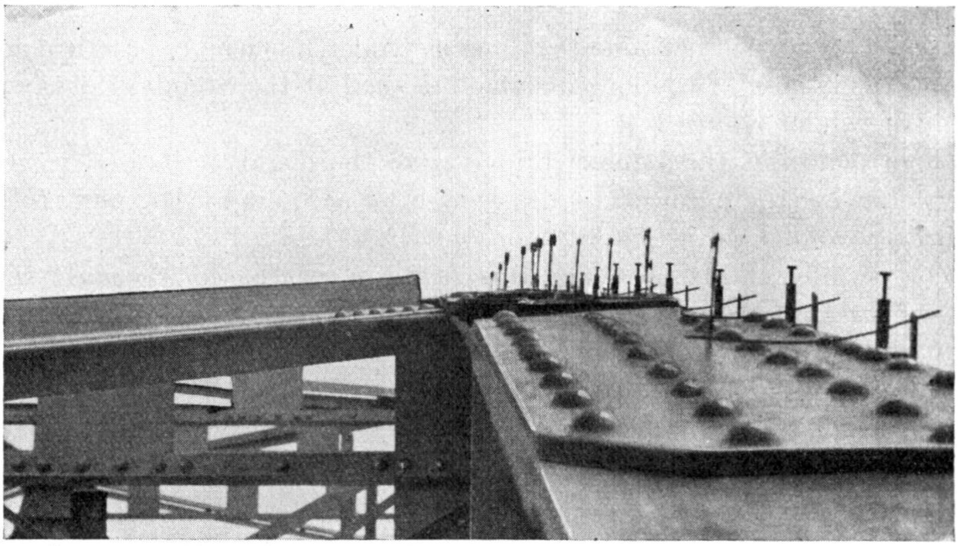

Fig. 9.

$+ 0.300$ kg/mm^2. For the lower boom 3—4, the calculated figure is $- 1.022$ kg/mm^2 (tension), and the measured value $- 0.312$ kg/mm^2. This is very difficult to explain. It must be admitted that these measurements are not sufficiently complete to permit of ascertaining the normal stresses in the flanges; nor was this what they were made for. Besides the influence of the panel points, which must be considerable, the distributing effect of the roadway must appreciably affect calculations based on the application of influence lines which imply the concentrated action of the loads. There are other causes as well for this variation in the tensions and compressions. On the whole, the bridge was found to be very stiff. The maximum deflection under the total load of 56 tons did not exceed 8 mm, and more probably 7 mm, for a span of 68 metres, or roughly $^1/_{10,000}$ th.

There si apparently a considerable analogy between the results of the test on the model panel point and the results of the tests on actual panel points, bearing in mind that the latter tests also allow for the effects of normal stresses in the booms and in the uprights.

As regards the panel points, the analogy is excellent with the results of Fig. 1, Model IV (flat model with curved gusset plates). It will be found that, as in Figs. 1, 2 and 7, 8, the maximum stress (tension) occurs, on the extreme fibres of the curved junction plates at a slight distance from where it joins the member that is subjected to the biggest bending moment. This phenomenon therefore assumes the character of a permanent rule.

The two tests, on the model and on the finished bridge, clearly show the exaggerated dimensions of the panel points of the Lanaye bridge, but they also indicate, by the regularity of the results, the excellent of the form of the panel points and the quality of their construction.

These panel points of the Lanaye bridge have a special significance, because they are the first rigid welded panel points and the essential elements of the largest welded bridge extant in 1933; or, briefly, the essential feature of the first welded bridge of 70 metres span for heavy traffic. The tests have definitely proved its high strength under mobile overloads. It has behaved perfectly over three years of normal service.

These panel points will be found to be much simpler, much more practical and more economical than those of the new marshalling shed at the Stendal railway station, built a little while before[11].

The panel points of the Lanaye bridge were thoroughly criticised by Dr. Ing. *Krabbe*[12] following the publication of a very brief account by Dr. phil. *Ihlenburg*[12] of the already condensed article by *Santilman*[10].

The lack of information on which this criticism was based is clearly shown by the fact that its author imagines that tensions were measured in the upper boom at various parts of the upright M. 4, whereas the corresponding stresses are compressions and were measured as such. *Dr. Krabbe's* judgment is based on a conventional difference in signs. The main points of this criticism will be dealt with later.

V. Welded Panel Points (Joints) of the Steelwork of the Civil Engineering Institute at Val-Benoit, Liége.

This building was put out to tender at the beginning of 1932 and finished at the beginning of 1933. The tests on the welded joints of the building were made in February-March, 1933, before the tests on the actual joints of the Lanaye bridge.

Fig. 10 shows the type of panel point (joint) designed for this building by the writer and upon which the tender was based, Its features are as follows:

(1) The radius of the upper circular gusset plate is reduced to 400 mm.
(2) The lower gusset plate is elliptical, with half-axes of 800 mm and 570 mm. (The total spread on the columns is thus reduced to 1488 mm, i. e., less than one-third the normal spacing between the girders, instead of two-thirds.)
(3) Alle the joints are welded; where the erection joint joins the columns with the beams, it is provided with their panel points. A small console-plate (taquet-console) welded on the column below the assemblage point acts as an adjusting support. All the erection welds are vertical.
(4) The curved flanges edging the gusset plates are in two parts, and welded by double fillets on to the gusset plates. In this way all discontinuity is avoided at the tangential connection between the gussets and the flanges, and the weld is perfect.

---

11 Der Bauingenieur, 6th Nov., 1931. Annales des Travaux Publics de Belgique, Feb. 1932.
12 Der Bauingenieur, 1934, pp. 307 and 460.

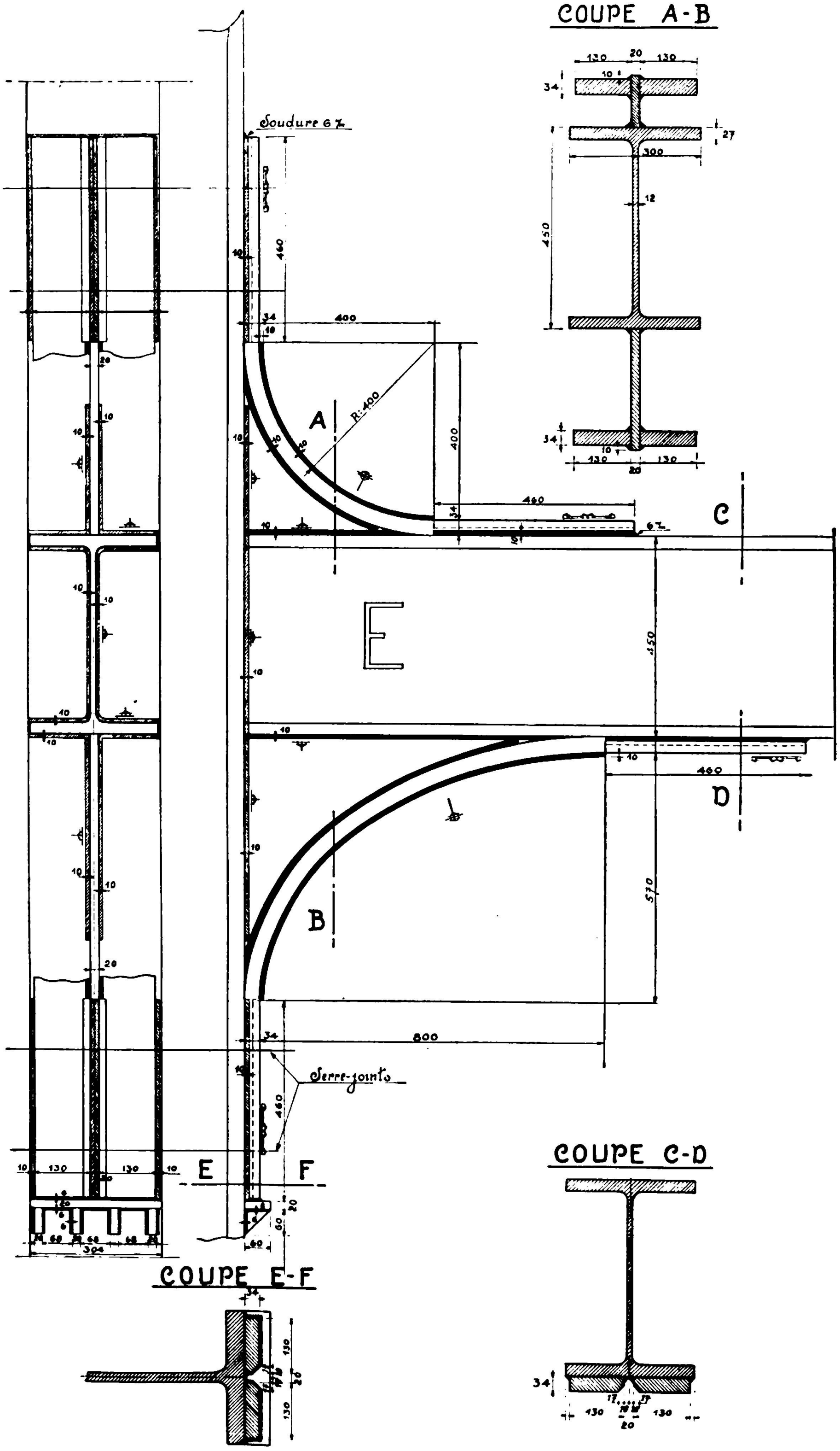

Fig. 10.

Because of their reduced dimensions and modified shapes, these joints are a big step forward compared to the joints of the Institute of Chemistry and Metallurgy building. This progress is based on the results of the 1931 tests mentioned in Section III., and on the results of the tests on two-dimensional models mentioned in Section II.

From the welding standpoint, their form is typical and very advanced, although they were designed independently of the joints of the Lanaye bridge. The accumulation of welds at the junction of the curved flanges and the gusset plate may perhaps be criticised. The welds are welds of 10 mm base. The symmetrical arrangement is rather favourable from the standpoint of the heat deformations. On the other hand, the flanges and gusset plates are extremely rigid. The type was combined to facilitate erection and welding on the site, and this result was fully achieved. This examination shows that, when designing a panel point of this kind, numerous technical factors must be considered, apart from the architectural and economic factors which affect the structure as a whole. It should be noted that the design simply called for the use of single laminated beams. Moreover, all the panel points or joints of the building were identical.

This building was designed to be constructed of Standard Belgian State Steel 42/50; the beams were of the Differdange section with wide webs. When the tender was being awarded, the Société Ame d'Ougrée-Marihaye suggested, as an alternative, a building of special steel 58/65, the girders and columns of which comprised joists of the standard section strengthened by welded webs. The engineering and economic advantages of this tender led to its acceptance. Nothing was modified in the essential overall dimensions of the trusses, nor in the dimensions, shapes and principles of the panel points. The only stipulation was that the curved flanges of the joints should be of 42/50 steel, as well as the actual gusset plates.

The welding operations were carefully supervised, both in the shops and on the site. An intelligent organisation of the work made the construction of the joints easy. The flanges were bent on templates, and the gusset plates prepared in the same way as for the rivetted steelwork described in Section III. Manufacture was organised on mass production principles, by using very ingenious methods of fixing and locking. The columns and the beams, complete with their junction plates, were forwarded to the site without any hitch. The panel points (joints) were erected and welded to the columns on the site in the depth of winter without any difficulty, and very accurately.

This work was carried out at practically the same time as the Lanaye bridge, because, although it was only begun afterwards, it was finished a few months before the latter job. The tests were carried out on it in February-March, 1933, whereas the tests on the Lanaye bridge were made in the month of May following.

From the engineering standpoint, this contract confirmed the success achieved by the Lanaye bridge, and also supplemented it by fresh engineering progress —welding throughout, without any hole, rivet or bolt—immediately followed the partial welding of the Lanaye bridge. On the other hand, the tests made on the structure of the Civil Engineering Institute, which have not yet been described, also confirm the teachings of all the experiments described above.

These tests were made on a completed structure under conditions of high precision and convenience, due to the load being applied by means of a hydraulic jack. The measurements were easy to repeat, and the stresses were applied very gradually and without shocks, thus preventing the apparatus getting out of adjustment; and, finally, the stresses as well as the points at which they were applied were known, and lent themselves to simple calculations. This enabled the testing of the various types of panel points.

The permissible calculated stresses amounted to 20 kg/mm². During the tests, measured stresses of roughly 15 kg/mm², equivalent to calculated stresses of 16 kg/mm² (approximately) were not exceeded in the girders. Under these conditions, the figure for the stresses measured on the joints did not usually exceed 10 kg/mm², or, exceptionally, 12 kg/mm².

Fig. 11 shows the results of a loading test on the three-branch assemblage points of an intermediate girder connected to two outside columns. The diagrams show the stresses recorded on various elements of the panel point, the outside edge of the curved gusset plates, the curved flanges, and the webs (sides) of the beams and columns. It will be noted that, locally, the stresses are certainly higher at the edges of the gusset plates than on the curved flanges. But the stresses recorded at the latter are more uniform, and not much below the average stresses taken at the edges. The gradual transmission of the stresses by the edges of the curved flanges is very doubtful. Allowing for the fact that the stresses measured on the flanges are rather below their average stresses (due to variations in the width), and also for the large cross-section of the curved, it will be found that the latter transmit very considerable stresses in a perfectly correct manner.

The same conclusions apply to the curved flanges of the two-branch panel points (Fig. 12), in which the gusset plate edge stresses are little below those of the curved flanges, which transmit very considerable stresses very gradually. This curve shows, on the other hand, that the stresses are very low in the top butt strap despite the top radius of the angle joint of the exterior columns and the top girder. This led to the stresses being measured in the gusset plate along the bisecting line of the joint. Maximum deformations will be noted there at a certain distance from the upper, radiused butt strap—compressions at right angles to the diagonal, and extensions along the diagonal. This goes to prove that the upper rounded portion hasnot sufficient radius and that, from the standpoint of the better transmission of the stresses, this radius should have been increased so as to give the angle plate the typical shape of a curved member.

The above remark is interesting, because certain people have sometimes expressed their doubts to the writer as to the suitability of radiusing the outside angle. Theoretically, it seems rather as though the amount of radius was insufficient. In practice, a reasonable measure of radius was adhered to having regard to appearance and cheapness and with, so it seems, satisfactory results from the strength point of view.

Fig. 13 shows the stressing of a triple-branch panel point. The result is just as satisfactory as the previous ones, although the joint appears rather large in this case, due to its shape being standardised for all the steelwork. The loading stress is applied through a bracket and so puts an incident moment on the panel point. The distribution of this couple between the two other members is clearly shown by the stress diagrams taken.

The same remarks also apply to Fig. 14 referring to the test of a quadruple joint. In this plate are also two diagrams showing the transverse variation of the stresses in the curved flanges, on the two faces. Generally speaking, the curve is parabolic in shape, and the average stress is attained roughly towards one-quarter the width of the flanges. It differs less from the maximum on the axis than it does from the

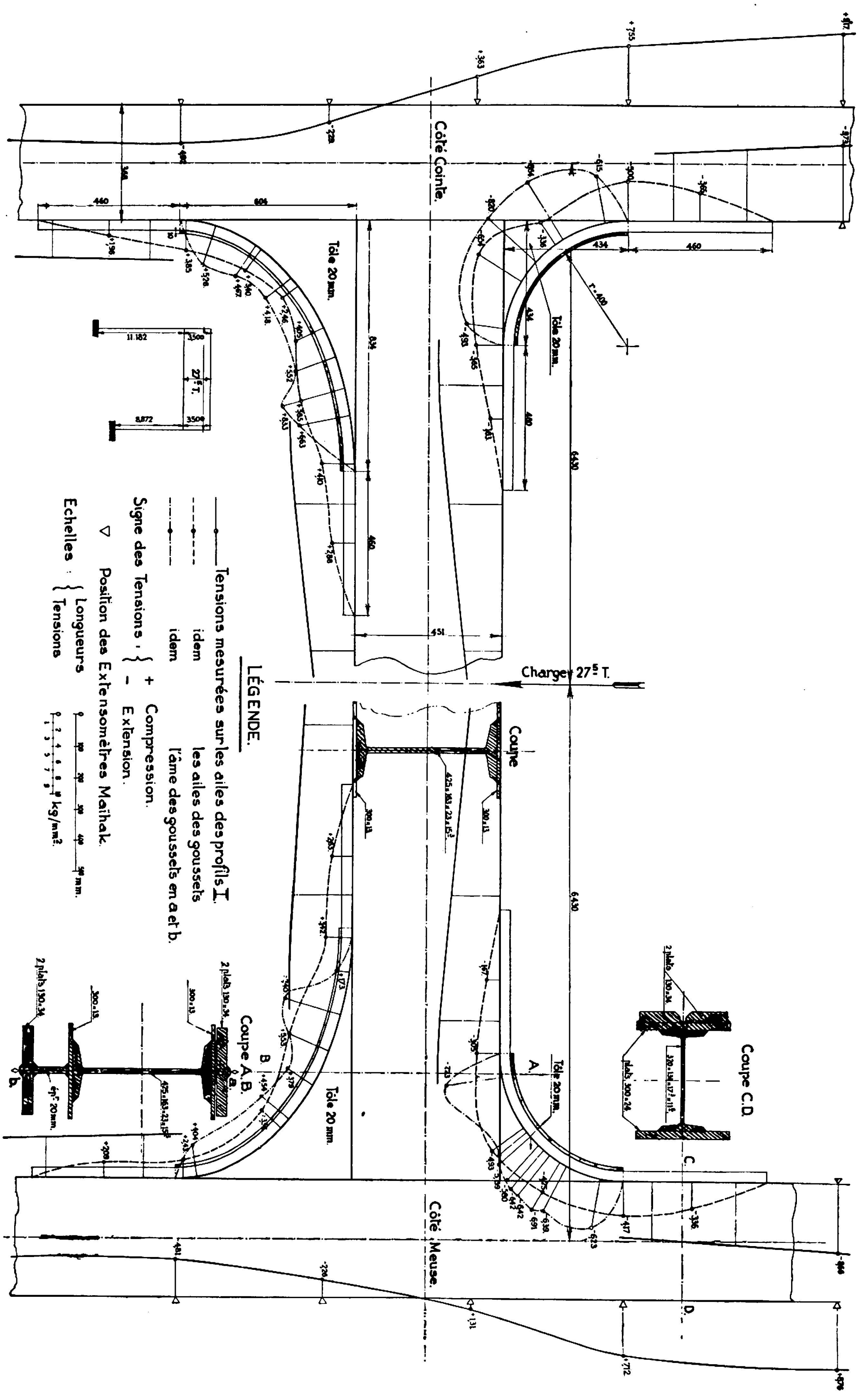

Fig. 11.

minimums at the edges. It was preceded by a comparison with the calculation which will be discussed further on.

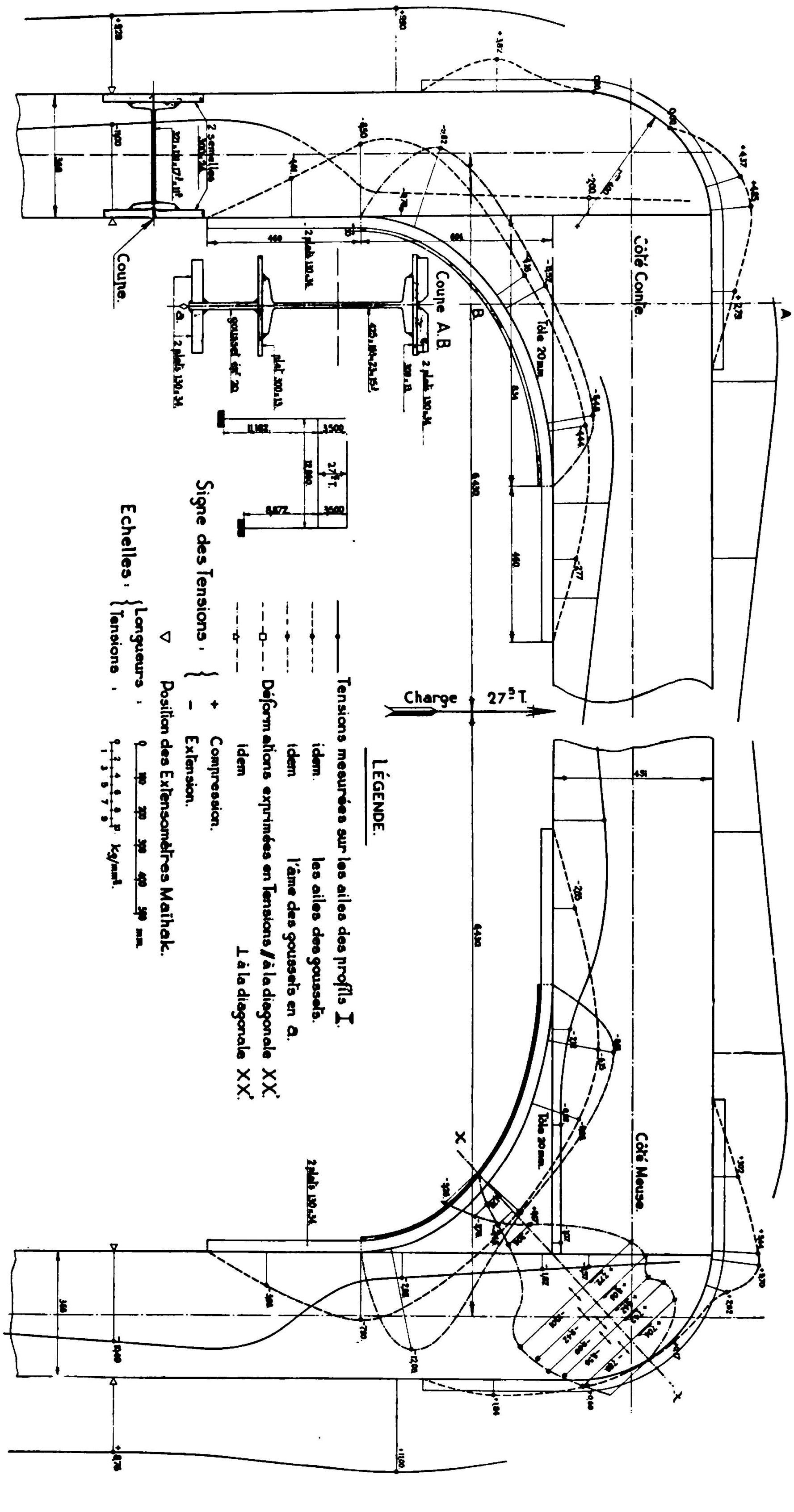

Fig. 12.

The following conclusions emerge from these tests:

(1) The curves showing the variations of stress taken on the outside fibres of the

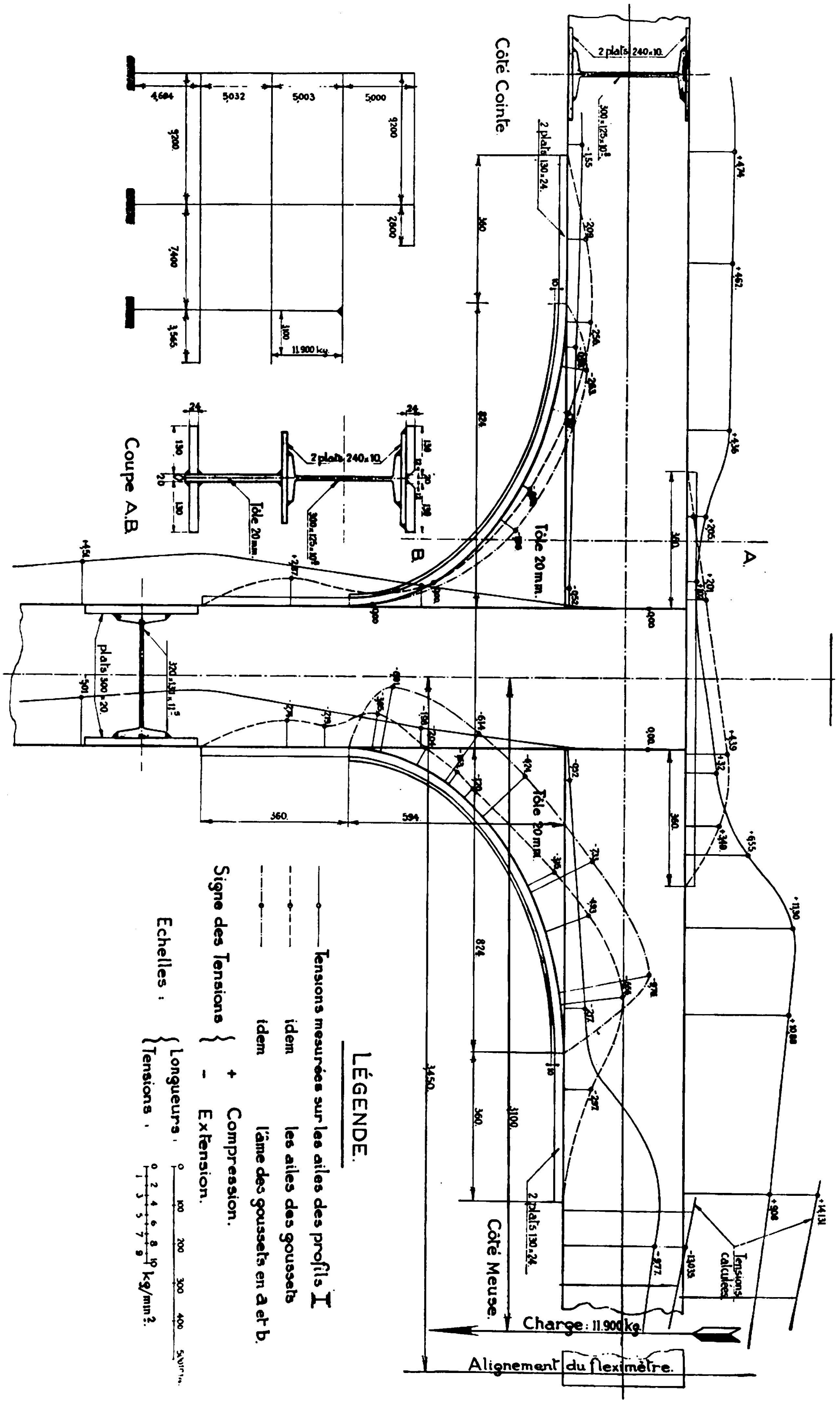

Fig. 13.

curved gusset plates are similar in shape to those obtained in the tests previously mentioned on flat models.

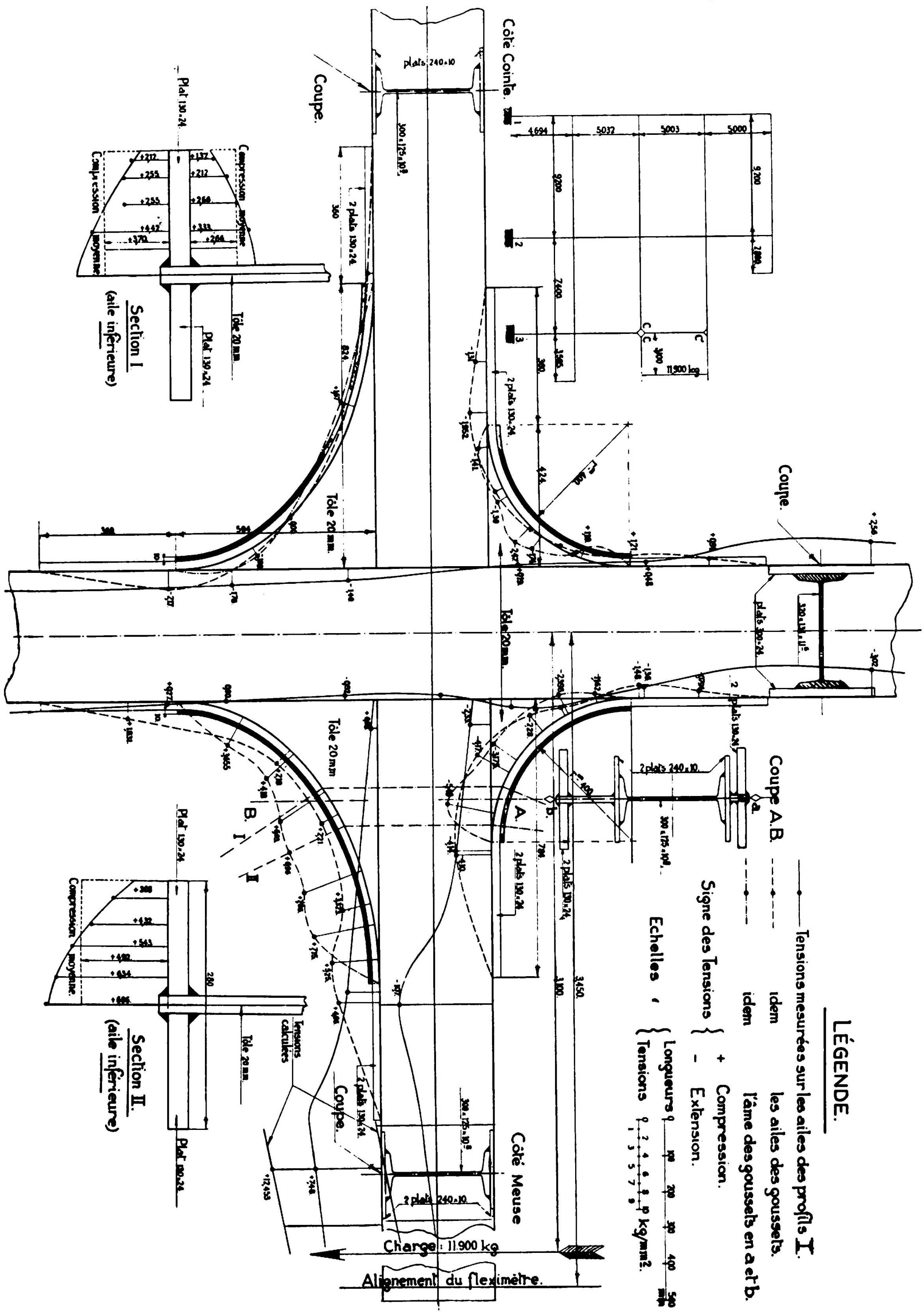

Fig. 14.

(2) Except for local variations, the maximum stresses are set up at a certain distance from the point where the curved joint bears upon the member which undergoes the biggest bending moment.

(3) Gusset plates of elliptic shape ensure a certain uniformity of the stresses in the curved flanges, in accordance with the intention which led to their adoption. These stresses are also more uniform than those recorded at the edges of the gusset plates.
(4) The curved flanges transmit considerable stresses.
(5) The transmission of the stresses in the curved wings from the joint to the plane flanges of the girders ans columns is very gradual and regular, thus enabling the welds—calculated on the principle of equal strength—to properly perform their function.
(6) The stresses in curved flanges vary in the transverse direction, the maximum coming on the axis, and the minimums at the edges; the means come roughly midway between the two.

At this report is mainly given over to the detailed study of panel points, their general effect on the strengthening of the structure, as shown by the general investigation of the buildings, has only been lightly touched on hitherto, by showing that in cases where it was possible to calculate the stresses with certainty, they are generally lower than the measured stresses. We shall deal farther on with this very important effect of well constructed rigid panel points, which has been manifested so markedly by a diminution in the deformations as compared with the calculated values.

VI. Welded Panel Points and Rivetted Panel Points in the Steelwork of the Thermodynamics Laboratory of Val-Benôit, Liége.

In 1933, the New Buildings Committee of the Liége University at Val-Benôit put out for tender a building to house the Thermodynamics Laboratory. It was of the same type as the Civil Engineering building, i. e., continuous, entirely welded, and of steel 58/65 kg/mm^2. It had several improvements over the previous building, of which only those relating to the panel points will be mentioned here. The dimensions and shapes of these joints are very similar to those of the joints described in Section V. They are also made of 42/50 Steel. The main difference was the substitution of butt, vee, or double vee welds for the corner welds, except at the points where the curved flanges join the columns, where this was not entirely possible. The object of this arrangement was to push continuity to its extreme limits in view of reinforcing the strength of the joints and if possible eventually reducing their dimensions.

Fig. 15 shows the typical panel point thus designed. As in the case of the Lanaye bridge, the gusset plates are welded to the columns in the shops. The welded joint for assembling the girders to the panel points are oblique. The amount of welding is appreciably reduced compared with the panel points described in the previous section.

These joints were not actually constructed. Due to circumstances that have nothing to do with the subject-matter of this report, the firm undertaking the work asked to be allowed to substitute rivetted joints for the welded joints. After thorough examination, this permission was granted, and Fig. 16 shows the type of rivetted joint adopted and actually constructed by utilising the elements of the framework and the welded joints already supplied or prepared, and completed by a few butt straps (Fig. 16). The assembly joint was arranged in roughly the same way as in the rivetted structure described in Section III, and the machining, erecting and assembling operations on the site were carried out in the same way without difficulty.

No tests were made on this structure, as the tonnage was low and the work of erection had been delayed. This and similar types of panel points are therefore only mentioned here to show the genesis of this type of welded joints as applied to structures of the type described, and to show easy it is to change over from welding to

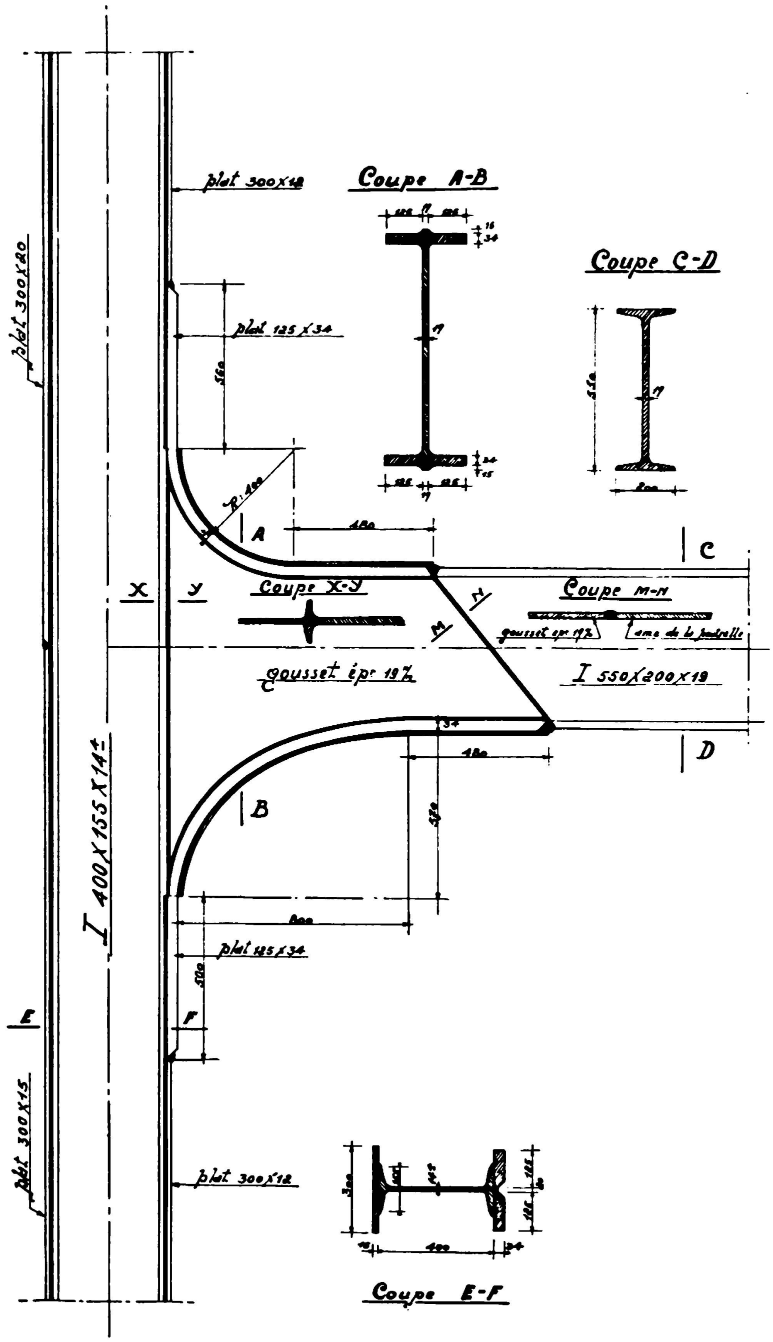

Fig. 15.

rivetting without any change in shape. This is due to the particular type of assemblage joint being adapted to the proper transmission of the stresses, and to the fact that the principle of these joints is independent of the use of welding. Besides, they were originally designed for rivetting and were so made. But there is no doubt that this type of panel point lends itself admirably to welding, since welding prevents tensile

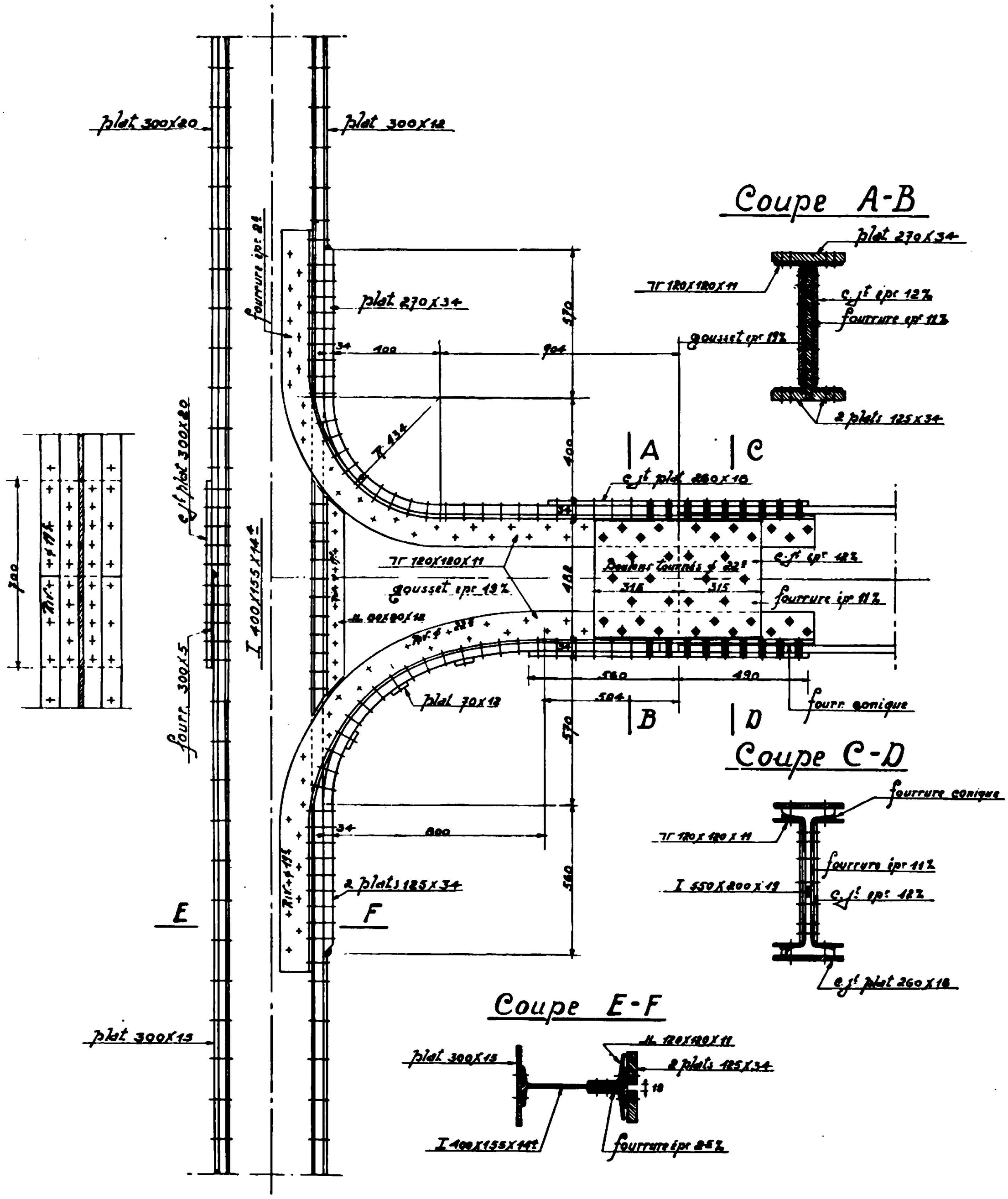

Fig. 16.

stresses being set up at the rivets used for attaching the curved flanges to the curved angle irons.

VII. Other Examples and Investigations of Rigid Panel Points.

All the panel points described above have a definite relationship; they form a family, the genesis, evolution and tests of which have just been described. There are numerous other application of these joints in Belgium and abroad, but it will suffice to give a brief review. The *Vierendeel* road bridges are now very widely adopted in Belgium. At first, they were built with welded joints and rivitted assemblages of the Lanaye type. In particular, this system has been applied to the Muideswing-bridge at Ghent[3], [13], except that the assemblage points were reduced to the extent permitted by Lanaye's tests.

Fresh progress was achieved in 1933–34 by complete welding, similar to the method adopted for the structure of the Civil Engineering Institute. This progress is due principally to *A. Spoliansky*, who was responsible for designing the Herenthals bridges[3], [14]. The panel points are substantially the same as those of the Muide bridge, but are of reduced dimensions in terms of *Vierendeel's* rules, and circular in shape. A large number of other *Vierendeel* welded road bridges have been built in Belgium for the Administration of Roads and Bridges, and by different engineering firms. All of these are typical of the Lanaye and Hérenthals bridges[3], [15], both as regards the panel points and the modifications and improvements that have gradually been made in them. As regards these latter, the constructors have specially endeavoured to weld the tangential connections of the panel points by reducing or eliminating the gaps in the gusset plates existing in the panel points of the Lanaye bridge (see Section IV.). Among other methods, what may be called-dummy joints have been welded to the booms and uprights, and to these the actual joints were attached by ordinary welds[16] (Fig. 17). The gap in the gusset plate still exists. Other designers have adopted the *Vierendeel* type of rivetted panel points, without tangential connection. It seems as if this engineer had long retained a preference for this type of panel point, even in the welded form. Fig. 18 shows the standard type of rivetted panel points designed by this engineer for the Hérenthals bridges. For *Vierendeel* bridges of the rivetted type, this type of projecting panel points is fairly easy to assemble and does not detract from the appearance of the finished structure[17]. This particular form has also formed the basis for designing the panel points of the numerous railway bridges built in Belgium during the past few years[18]. These panel points appear to have been evolved more from practical considerations of assembly than from any endeavour to adapt them more specially to the internal stresses. Safety is ensured by very wide dimensions, characterised by the considerable spread of the joints on the booms, more so than on the uprights. This form is the opposite of that of the elliptic panel points of the type described in Sections V. and VI., and in this sense it is irrational. Moreover, the discontinuity resulting through the non-tangential connection does not satisfy the ideal stress transmission diagram. It is not defective from the viewpoint of local overstressing, this trouble being avoided by the extra-large dimensions. It is certain that the internal stresses may be transmitted without excessive concentrations, owing to the sonsiderable elongation of the joints, certain parts of which, at the ends, are necessarily and apparently

---

13 *M. Storrer:* Calcul des poutres Vierendeel du pont tourant du Muide.
A. Spoliansky: Construction du pont tournant du Muide. — L'ossature Métallique, 1933.

14 *A. Spoliansky:* Pont C d'Hérenthals sur le Canal Albert. L'Ossature Métallique, 1934.

15 *A. Braeckman* and *A. Van Gaver:* Ponts de Schooten sur le Canal Albert. L'Ossature Métallique, 1934. Le Lancement du Pont de Bocholt. Ossature Métallique, 1935. Le Pont de Lanaeken-Smeermaas. Ossature Métallique, 1936.

16 *A. Vierendeel:* Cours de stabilité des constructions, Vol. IV, 1935, p. 378.

17 *A. Vierendeel:* Cours de stabilité des constructions, Vol. IV., 1935, p. 276.
P. G. G. Hauser: The Design and Applications of the Vierendeel Truss. International Congress for Steel Development, 1934.

18 Les ponts-rails d'Hérenthals et de Malines à poutres Vierendeel. L'Ossature Métaliique, 1934. Publications of the Int. Ass. for Bridge and Struct. Engrg., Nr. 3, 1935.

inactive. In the welded panel points of this type, the extra length of the gussets must serve the same purpose as the welds connecting the curved sides to the booms and uprights in the Lanaye bridge type if joint. For welded structures, however, this

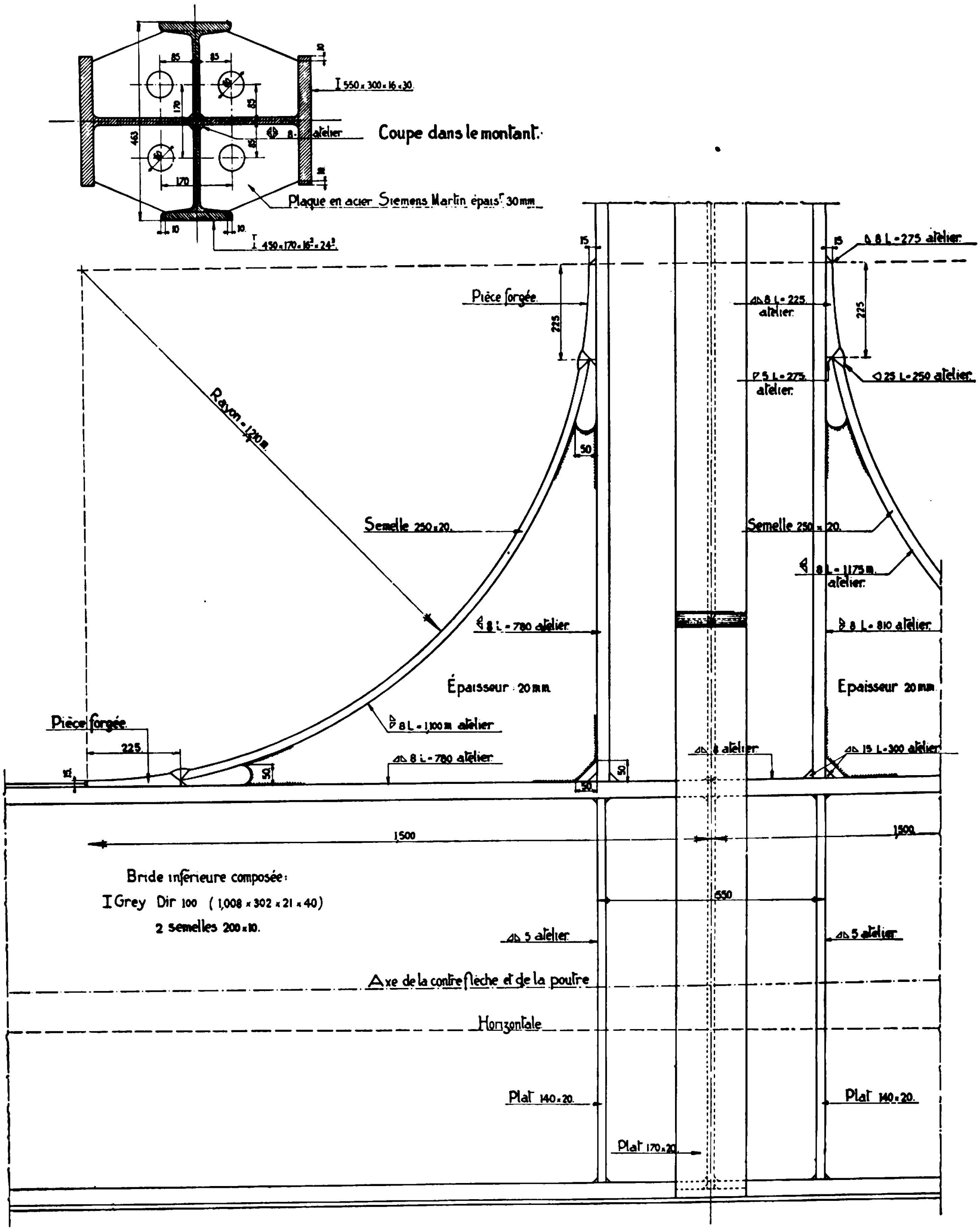

Fig. 17.

latter type apparently deserves and obtains the preference. It is the one the Dutch engineers have adopted for the *Vierendeel* welded road bridge at Nuth[19]. The Japanese engineers have also adopted it for the *Vierendeel* lift bridge at *Mantetsu-Kawasaki-Futo.*

[19] Publications of the Int. Assocn. for Bridge and Struct. Engrg., Nr. 3, 1935.

In Section IV. we mentioned the welded joints of the *Vierendeel* trusses in the marshalling depot at Stendal railway station (Germany)[11]. The distinguishing features of these joints are the exceptional precautions taken in their design for welding purposes — a design involving actual metallurgical operations. For this reason, these joints are rather special in character and, up to now, unique, and rather remote from the functional viewpoint mainly considered in this report. For rivetted bridges, it seems possible to design panel points with tangential connections which give satisfaction in practice, but which differ from the type which appeared most suitable

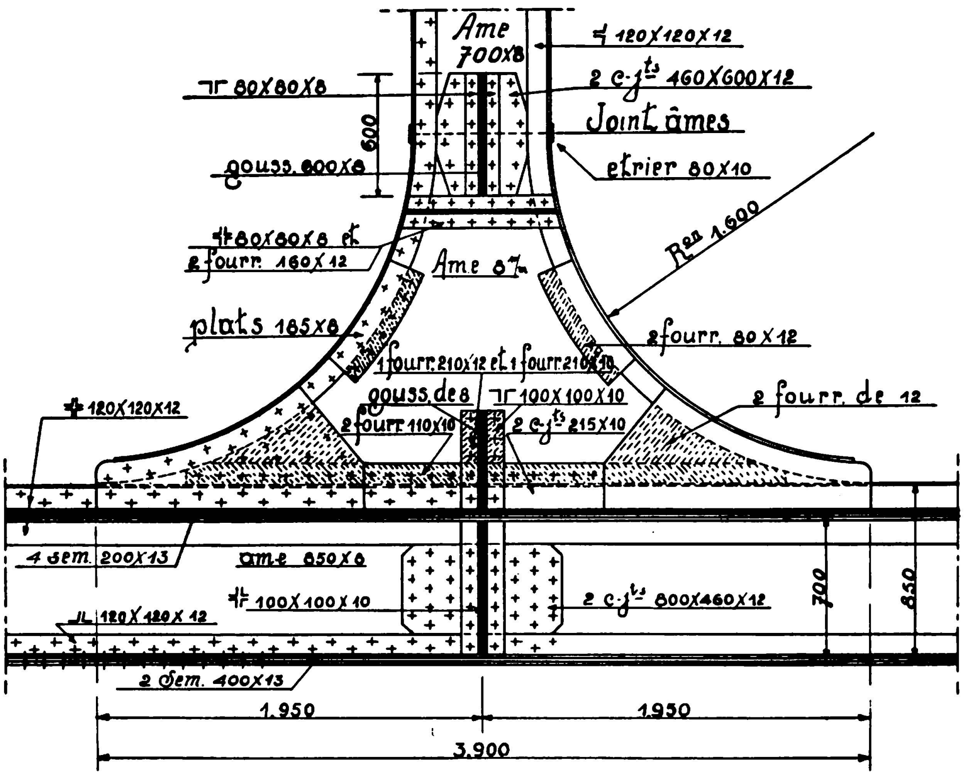

Fig. 18.

for the structures described in Section III. *Vierendeel* calls this joint the "Tervueren type". This reminder of the oldest type of *Vierendeel* bridge joint is a well deserved compliment to the protagonist of this system. It should be noted, however, that it was the tests on flat models (Section II.) and the work preceding the design of the structure described in Section III. which led the present writer to resurrect this type of joint in a modern form and apply it to structures differing from the *Vierendeel* truss, and that he was not aware at that time of the type of joint used in the Tervueren bridge.

For the last of the *Vierendeel* type of rivetted railway bridges shortly to be built across the Meuse at Val-Benôit (Liege), the panel points have tangential connections, and they are similar in form to the ones previously studied. However, they are of fairly large dimensions (Fig. 19).

These joints have been applied in several other continuous frame structures in Belgium, principally by the author's pupils, apart from the rivetted types described in Section III. and the welded or rivetted type of joints described in Sections V. and VI.

One of the best known applications was in connection with the new laboratory buildings of Ghent University (1934—35), now completed, while another remarkable scheme was drawn up for the "Service scientifique" at Brussels (1935). *A. Spoliansky*

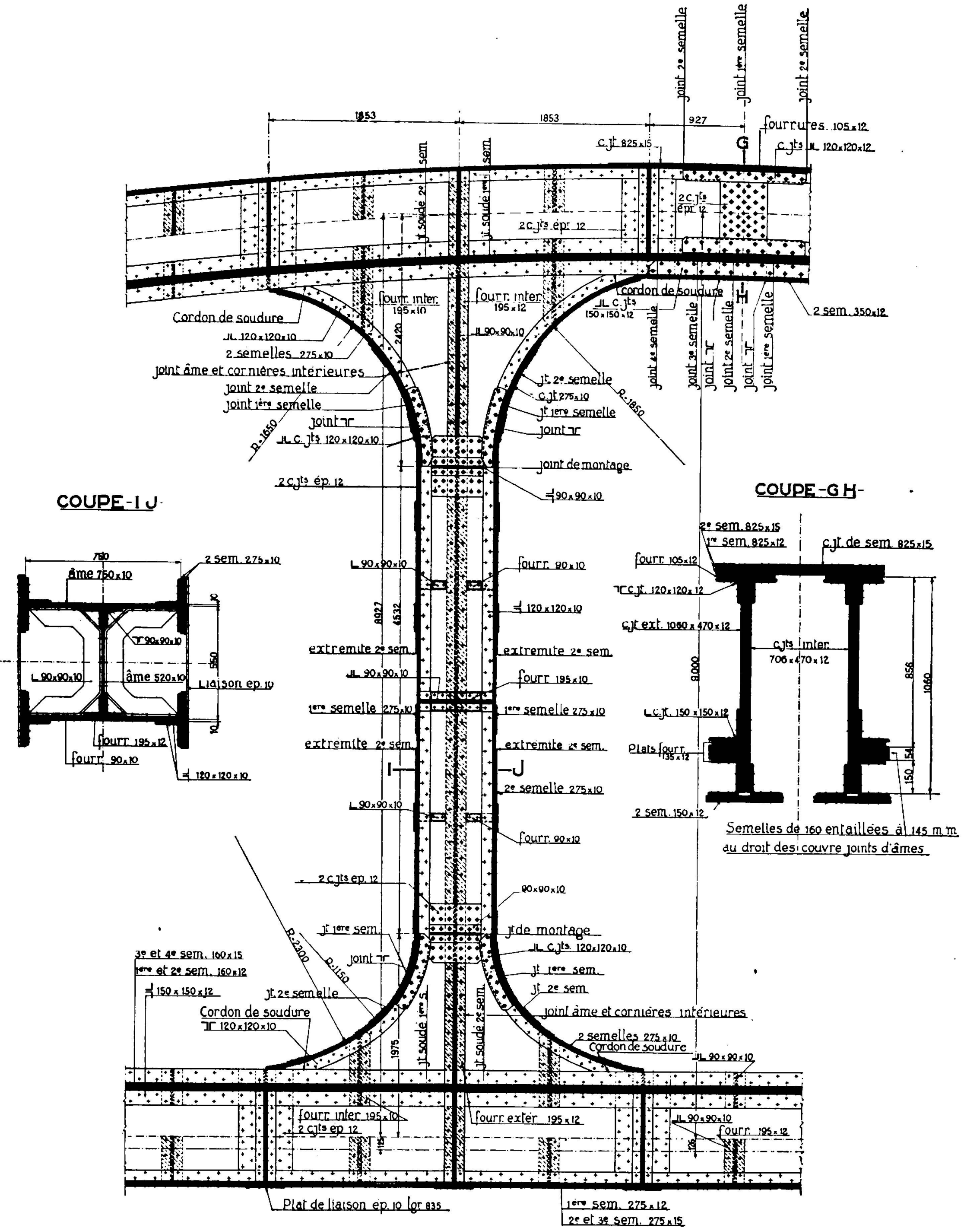

Fig. 19.

was responsible for both schemes, and it is interesting to note that, in both cases, they were competitive schemes or counter-schemes which, owing to considerable commercial and technical advantages, left all the competing schemes far behind. These results are perfectly clear and concord with the fact that, when the contracts for the building described in Sections III., V. and VI. were awarded, all the alternative schemes submitted with different panel points or systems merely remained on paper.

In Germany, the steelwork of the new railway station at Duisburg have circular welded and rivetted joints of similar appearance[20]. It is possible that these joints have also been applied in a similar way in various countries, mainly due to the development of welded structures, but the author has no knowledge of them.

The joints of the steelwork for Ghent University have a top elliptical gusset of very reduced dimensions, whereas the bottom circular gusset is of fairly large size (Fig. 20). These shapes are particularly advantageous from the architectural standpoint.

As regards the testing of joints, the late Prof. *H. Dustin*, assisted by Prof. *Gysen*, has carried out dynamic tests on joints of the *Vierendeel* bridge type, in collaboration with the American Institute of Steel Construction[21]. The particular model was a full-scale model of a two-branch joint without gusset. These first results do not conflict with the present writer's, but, on the other hand, they confirm the concentrations of stress at the inner angle (see Section II.) and the inactivity of the parts adjoining the outer angle, corresponding to a stress transmission by the web and high stresses in the latter (see Section V., Fig. 12). These tests, carried out on a big scale, are only in their initial stages. It is impossible to gather whether the programme includes the testing of panel points having three or four junctions.

An important paper was published in 1934 by *C. R. Joung* and *K. B. Jackson* of Toronto University[22]. This was also a different kind of investigation, comprising tests on models of quadruple joints of sections of girders and columns. These tests were made without measuring the stresses, but the deformations up to rupture were observed in terms of the loads applied. The connections were not the same as the type outlined in this paper, but were simple joints for structures of apparently moderate spans. The authors very wisely select the degrees of restraint achieved as the characteristic result of their experiments. The joints do not directly reinforce the girders. The optimum degree of restraint is 0.75, giving equality of the restraining moments and the moments in the middle of the bay for a complete uniform loading. As we shall see later on, the same result is achieved by the joints studied in this report, but in a different way, in that this type of joint has sufficient strengthening effect to allow of an imaginary diminution in the span of the prismatic portion of the girders. Although the Bureau of Standards and *Young* and *Jackson's* researches differ somewhat from the present researches, there is a certain amount of connection between them, and they at least prove that the question is being actively studied in various countries. Its interest has been stressed by *F. Bleich.*[23] As stated in the introduction, this question is closely related to the study of rigid frame structures on which research has also been done in Great Britain.

---

[20] *Krabbe* and *Ziertmann:* Die Empfangshalle des neuen Empfangsgebäudes auf Bahnhof Duisburg. Die Bautechnik, 9. 8. 35.

[21] Pregress Report Nr. 1 on Stress Distribution on Steel Rigid Frames. National Bureau of Standards, Washington, 1936.

[22] The relative Rigidity of Welded and Riveted Connections. Bulletin 143.

[23] *F. Bleich:* La théorie et la recherche expérimentale en construction métallique. L'Ossature Métallique, 1934.

Dr. Ing. *Harry Gottfeldt*[24] also published an article which confirms in a general way the views of the present writer regarding curved joints.

A still more recent paper by *Kayser* and *Herzog*[25] deals in detail with tests on a double-junction panel point. The results are quite compatible with those in Fig. 12, and confirm the observations in Section V. regarding the shape of double joints. Moreover, it is interesting to note that the measured stresses are distinctly lower than the calculated stresses. This particular model was, however, a very simple one, tested in the laboratory under simple and well defined conditions.

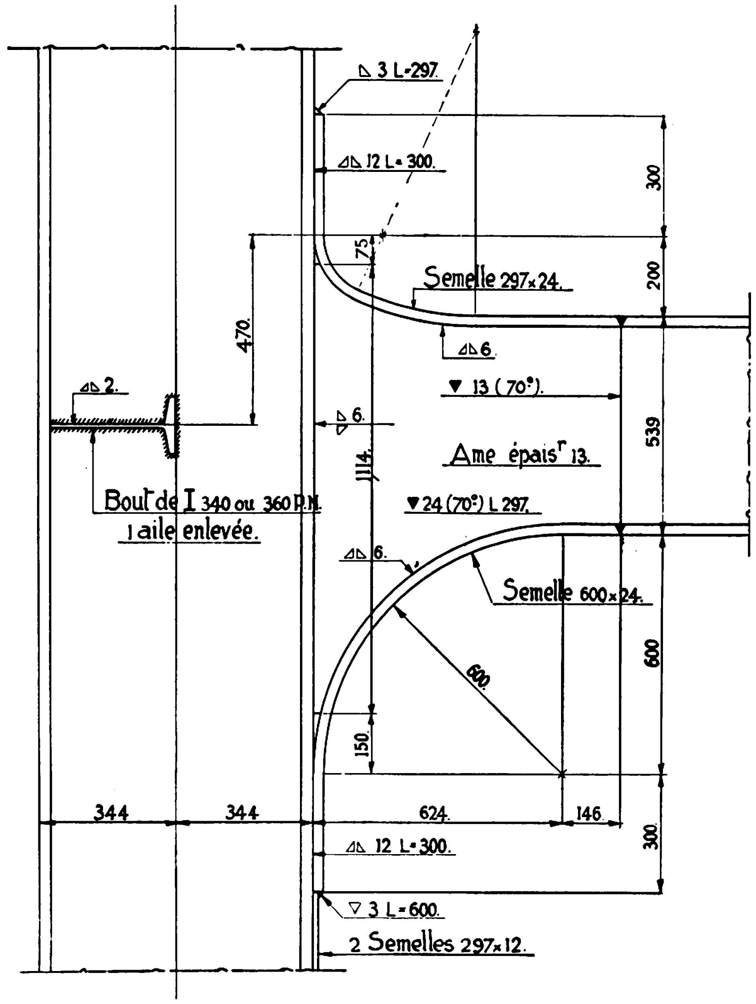

Fig. 20.

VIII. General Features and Calculation of Rigid Panel Points.

These panel points of joints have to ensure, as perfectly as possible, rigid connection between the members to be assembled, i. e., they have to ensure the identity of any deformations at the points where they meet, and transmit the corresponding internal stresses without excessive tensions or local deformations. This function must be

[24] *H. Gottfeldt:* Einige Bemerkungen über geschweißte Brückenknoten. Der Bauingenieur, 1934, p. 200.

[25] *Kayser* and *Herzog:* Die Untersuchung zweiachsig beanspruchter Konstruktionsglieder mit Hilfe des Reißlackverfahrens. Die Bautechnik, 29th May, 1936, p. 314.

performed with a certainty at least equivalent to that of the other elements of the structure, which means that the joints must not be the weak points of the structure, in which breakages may take place below the loads at which the strength of the girders and columns or booms and uprights would be exhausted. Furthermore, as the panel points and joints are less accessible to calculation and more exposed to defects and bad work, it is necessary to insist that the panel points be designed like the strong elements of the structure, in which breakage only exceptionally occurs. It is wise and prudent to apply to the elements which are more complex and more exposed by their nature, a higher coefficient of safety than is adopted for rolled girders or composite girders, whose ultimate strength can be more exactly computed. This rule alone can ensure sufficient uniformity of safety.

This complete definition reveals at once the imperfections in gussetless joints, such as those of the flat model No. 1 (Section II.), of the tests of the National Bureau of Standards, and joints of the type studied by *Young* and *Jackson*, in all of which breakage takes place in the joints and point to local deformations. Only strong double gussets can guarantee an appreciable permanent deformation in the girders before the joints break. In a paper on this subject, *D. Rosenthal*[26] also comes to the conclusion that, if the strength of *Vierendeel* trusses is to agree sufficiently with the calculated figures, it will not do to assemble the uprights to the booms without gussets, but double gussets must be provided for. Single triangular gussets of small dimensions, such as those tested by Mr. *Rosenthal*, are scarcely sufficient to ensure proper concordance between the measured stresses and the calculated stresses.

All the author's tests on curved panel points show that these joints substantially reinforce structures. Not only have the stresses found been very moderate, but a general decrease in the measured stresses in terms of the calculated stresses has always been found throughout the structure. This point has been dealt with in previous sections of this paper, as well as in earlier published works[1, 2, 10, 27]. In the case of metal frame structures, it has been possible to demonstrate partially by calculation, that these reductions of stress were certainly due to the reinforcing effect of the panel points. For these particular structures, measurements have shown, and calculations partially justified the fact, that similar reductions take place in the transverse deformations (deflection) and angular deformations (rotation of the joints). In multiple metal frame structures, this means that the trusses are supported almost perfectly in the columns[1, 2, 8]. Owing to the very moderate dimensions of the joints in these structures, the results of the complete examinations are in good theoretical agreement with the calculated results. The differences ranging from 20 to 10 p. c. are the smallest the author has ever found in actual examinations. The agreement between the measured diagrams and the calculated diagrams is striking. For the *Vierendeel* type bridge at Lanaye, the differences are greater and the theoretical agreement with the calculated figures less apparent. The very large panel points must have a very big influence on these variations; in addition to which all the other elements capable of increasing the uncertainty of the calculations,

---

[26] *D. Rosenthal* and *Charles:* Calcul du joint soudé dans les pièces flèchies. Applications.
D. Rosenthal: Etude expérimentale sur modèle reduit d'une poutre Vierendeel soudée. Association belge pour l'étude, l'essai et l'emploi des matériaux, Nr. 1 bis, 1932.

[27] *F. Campus:* Les charpentes métalliques continues. 2ème Congrès national des Sciences. Brussels, 1935.

while strengthening the work as a whole, must intervene, the railroad distributing the loads and making the structure more rigid, considerable wind bracing, etc. At any rate, the tensions are appreciably below those indicated by the calculation, and the deformations are very low.

That this is very largely due to the reinforcing effect of the joints seems to be verified pretty conclusively by *G. Verplancken*[28]. Experiments on celluloid models made by means of the *Magnel* micro-influencimeter have demonstrated the stress reducing effect of large gussets, amounting to up to 10 p. c. for certain elements. The deflections are also reduced in the proportion of 3 to 1 relatively to the calculated figures. On the other hand, gussetless trusses give stresses which are higher than the calculated values (independently of the excess stresses at the angles), and deflections that are higher than the calculated figures. This paper stresses the advantages of curved gussets, shows that their dimensions could be reduced considerably as compared with those in the old *Vierendeel* bridges, and indicates the suitability of gussets of elliptic section. There is all the more justification for adopting these conclusions in practice, since the author's tests have shown that numerous other elements intervene in well designed *Vierendeel* bridges to increase their strength and rigidity.

It has been proved, then, that the panel points of the type described, even when of very moderate dimensions, and provided they are well designed: (a) achieve their object in every respect, (b) constitute the strong points of the structure, and (c) generally strengthen the structures in terms of the calculated figures. These qualities agree with the principle underlying their design, i. e.. the proper and rational transmission of the internal forces by the assemblage. This concept has not been derived from the old rules applicable to the strength of materials nor from the usual design arrangements, nor is it inspired by the use of welding. In the author's case it is derived from the physical notion which may be formed of the working of an assemblage of this kind, i. e., the notion of stress trajectories, illustrated by experience and knowledge in the field of elastometry. This conception is not confined to the joints of *Vierendeel* trusses or multiple metal frame structures; it may be applied, *ipso facto*, to all the rigid joints, particularly of triangulated girders. This the author stated at the meeting of the Belgian Association for the Study, Testing and Use of Materials[2] held on May 11th, 1932, thus anticipating a similar opinion expressed in 1934 by *H. Gottfeldt*[24] with regard to welded panel points.

In the proper transmission of the internal stresses by the panel points, the author attaches the greatest importance to the curved flanges edging the gussets and connected tangentially to the sides of the straight bars. The tests outlined in Sections II. to V. have demonstrated the advantage of the curved form, but that circular curves do not constitute a rational shape. Elliptical curves are superior to the foregoing owing to the greater regularity of the stresses throughout the flanges, and it has been shown that these latter transmit considerable stresses without excessive tensions or over-tensions. The tangential connection of the flanges is obviously essential, especially if the spread of the joints is to be reduced to a minmum.

[28] *G. Verplancken:* L'influence des goussets dans les poutres Vierendeel à hauteur constant. *G. Magnel:* Calcul pratique des poutres Vierendeel.

The examples mentioned in Section VII. prove that, both in Belgium and abroad, this arrangement seems finally to have met with general favour.

In order fully to assess the value of these panel points, it remains for us to outline the principle underlying their calculation, for they are calculable, with the sole proviso that the strength must usually be expected to be higher than what the calculation shows it to be. This is sound practice and a wise conception of safety, seeing that, in modern construction, the biggest hazards of the work lie in the joints.

The first determination concerns the size of the joints. In the case of multiple frame structures, each girder will be assumed to carry its maximum loads, allowing for the effects of continuity. More often it will be possible, by sufficient approximation, to regard the girder under maximum load as if it were perfectly fixed at the ends. The reinforcing effect of the joints more than compensates for their slight rotation from the point of view of the moment at fixed end. Referring the moments to the girder (considered as prismatic throughout its length), the actual coefficient of fixation due to the joints, is more than unity[1]. For purposes of calculation, it may be taken as equal to unity. On the girders, the points adjoining the joints are determined at which the bending moment is equal (and opposite to) the maximum bending moment set up in the middle portion of the bay. These two points limit the prismatic portion, calculated to withstand these equal bending moments at the ends and in the middle part. This achieves the maximum of economy for the prismatic form of girder and corresponds with *Young* and *Jackson's* coefficient of restraint of 0.75[22], with the added advantage that the span of the prismatic part is smaller than the spacing of the columns. The panel points proper extend from the points thus determined up to the columns.

For the uprights of *Vierendeel* trusses, the problem is not so well defined. As a matter of fact, the points of inflection on the uprights are quasi-invariable, and, hence, the prismatic portion may be almost little. The uprights would be variable in section from the region of inflection towards each chord, and would form a gradual transition with the joints. This particular type of upright is sometimes constructed of reinforced concrete. For purposes of calculation, the uprights are usually considered to have the same moment of inertie and no allowance is made for the increased rigidity of the uprights due to the panel points, so that a certain suitable section of prismatic uprights is taken for all these members. The points farthest away from the points of inflection in which the maximum permissible stresses are attained will be determined. These points will determine the starting points of the joints on the uprights, allowing for the fact that the joints are generally all the same height. The same method would be followed for the members of rigid triangulated lattices.

The determination of the spreads on the columns of multiple frameworks or on the chords of *Vierendeel* trusses may be made by the same consideration: the panel points reinforce the prismatic parts starting from the points where the calculated stresses reach the maximum permissible. This method is more suitable for the highly stressed columns of the multiplestage frameworks, in which compression predominates. It is less determinative for the booms of *Vierendeel* trusses, where it would probably be better to adopt variable sections between the theoretical points of inflection and the panel points. In fact, the prismatic portions of the booms will still be determined by generally considering suitable moments of inertia, and the panel points will be limited to the points where the maximum permissible stresses are attained. More exactly, the designer will tend to operate by successive trials, seeing that a problem of economy is raised. If the spread of the joints on the booms is reduced, the prismatic portions must be strengthened; while, conversely, if the section of the prismatic portions is reduced, the size of the joints must be increased. This means, then, that, in *Vierendeel* trusses, the general dimensioning of the assemblage points is inseparable from the dimensioning of the members. The fact that Prof. *Vierendeel* has always preferred joints of fairly large dimensions proves that, apart from considerations of prudence, joints of this kind are not economically unfavourable. In short, for the concrete case of a *Vierendeel* truss, the engineer will find in the elements of the problem the determination of the limiting dimensions for the joints, influenced eventually by a personal factor. A precise optimum is definable for multiple-stage metal frameworks, where bending plays a moderate part in the maximum stressing of the columns. The shape of the joints will be governed by these limiting dimensions, and perhaps by certain special conditions, say, architectural or constructional (multiple frame structures), and by certain personal factors. From the engineering point of view, the writer would merely recall the rule of the increasing curvature towards the members which have the smallest bending moment. It was this principle which led to the design of elliptical gussets for triple panel points. Other forms may be adopted as necessity demands. The parabola may be suitable; the connection need not always be absolutely tangential.

For double panel points, the circular curve is the best. Besides, this particular joints ought logically to assume the shape of curved members. For quadruple joints and triple ridged joints different forms of curves may be selected, depending on the usual distribution of the moments. Generally speaking, only one or a few types will be selected for the entire structure. This type, or these types, must then be suitable for all the panel points, and not be designed and verified for a single one.

When once a type of joint has been decided upon, it may be verified by calculation, using the formulas of *Résal* or *Vierendeel.* These methods are fairly well known, at least in French-speaking countries. They will be found in the treatises published by these writers. *Vierendeel*, however, merely deals with the case of the symmetrical joint, which it will be easy to generalise.

In *Résal's* method, only the normal (average) stresses in the diverging flanges, which are presumed to be uniform, will be considered. The normal stresses in the web of the gusset plate are neglected. Calling ω and ω' the sections of the two flanges, α and α' the angles which these flanges make with the lengthwise axis of the piece, and h the distance between the axes of the two flanges measured at right angles to the longitudinal axis of the piece, we get:

$$\sigma_m = \pm \frac{M}{h\,\omega \cos\alpha} + \frac{N}{\Omega_2}, \qquad \sigma'_m = \mp \frac{M}{h\,\omega' \cos\alpha'} + \frac{N}{\Omega_2}$$

by writing

$$\Omega_2 = \omega \cos\alpha + \omega' \cos\alpha'$$

Vierendeel's formula takes into account the stresses in the web of the gusset. The following formulas are a generalisation of this for the dissymmetrical panel point:

$$\sigma_m = \pm \frac{M\,v}{I_a + v^2\omega\cos\alpha + v'^2\omega'\cos\alpha'} + \frac{N}{\Omega_2};\ \sigma'_m = \mp \frac{M\,v'}{I_a + v^2\omega\cos\alpha + v'^2\omega'\cos\alpha'} + \frac{N}{\Omega_2}$$

by writing

$$\Omega_2 = \omega_n + \omega \cos \alpha + \omega' \cos \alpha'$$

I_a is the moment of inertia in the web and all the parts of section y attached, other than the curved booms, relatively to the central axis of the reduced section Ω_2, the sections of the curved booms being assumed to be concentrated in their central axes; v and v′ are the distances from this central axis to the axes of the booms (Fig. 21).

Fig. 9 shows, for the reduced-scale model of the Lanaye bridge joint, the stresses calculated by *Résal's* and *Vierendeel's* formulas, compared with the measured stresses. The big differences are partly due to the fact that the stresses measured near the edges of the curved flanges may be lower than the average stresses. *Vierendeel's* formula gives the least exaggerated results. In Fig. 14, referring to the test of a quadruple panel point of the metal framework of the Civil Engineering Institute at Liége, will be found the stresses taken throughout the flanges in Sections I. and II This has enabled a more thorough comparison to be made with the calculated values

Section I., elliptical flange

Average measured stress $+\dfrac{3.70 + 2.66}{2} = +3.18$ kg/mm²

Maximum measured stress + 5.80 kg/mm² in terms of the longitudinal diagram
+ 6.30 kg/mm² in terms of the transverse diagram

Average stress as calculated by *Résal's* formula	+ 8.95 kg/mm²
Average stress as calculated by *Vierendeel's* formula:	+ 6.234 kg/mm²

Section II., elliptical flange

Average stress measured on outside face of flange	+ 4.92 kg/mm²
Maximum measured stress	+ 6.96 kg/mm²
Stress as calculated by *Résal's* formula	+ 9.53 kg/mm²
Stress as calculated by *Vierendeel's* formula	+ 6.70 kg/mm²

Vierendeel's formula may be regarded as the most approximative, and best meeting the conditions of safety.

Fig. 21 compares the measured and calculated stresses in Sections I. and II. The only object of this diagram is to show that the formulas available ensure a sufficient margin of safety.

The shearing stresses may be easily studied by *Vierendeel's* or *Résal's* method; they are reduced by the obliquity of the booms, as expressed by *Résal's* notion of reduced shearing stress. In both methods, the web is supposed to take up the whole of the shearing stress. We therefore get, according to *Résal*:

$$\int_{v'}^{v} \vartheta \, d\omega = T - \frac{M}{h} (\mathrm{tg}\,\alpha - \mathrm{tg}\,\alpha') = T_r$$

Adopting *Vierendeel's* method, we write:

$$\int_{v'}^{v} \vartheta \, d\omega = T - \omega \sigma_m \sin \alpha - \omega' \sigma_m \sin \alpha'$$

allowing for the conventional signs.

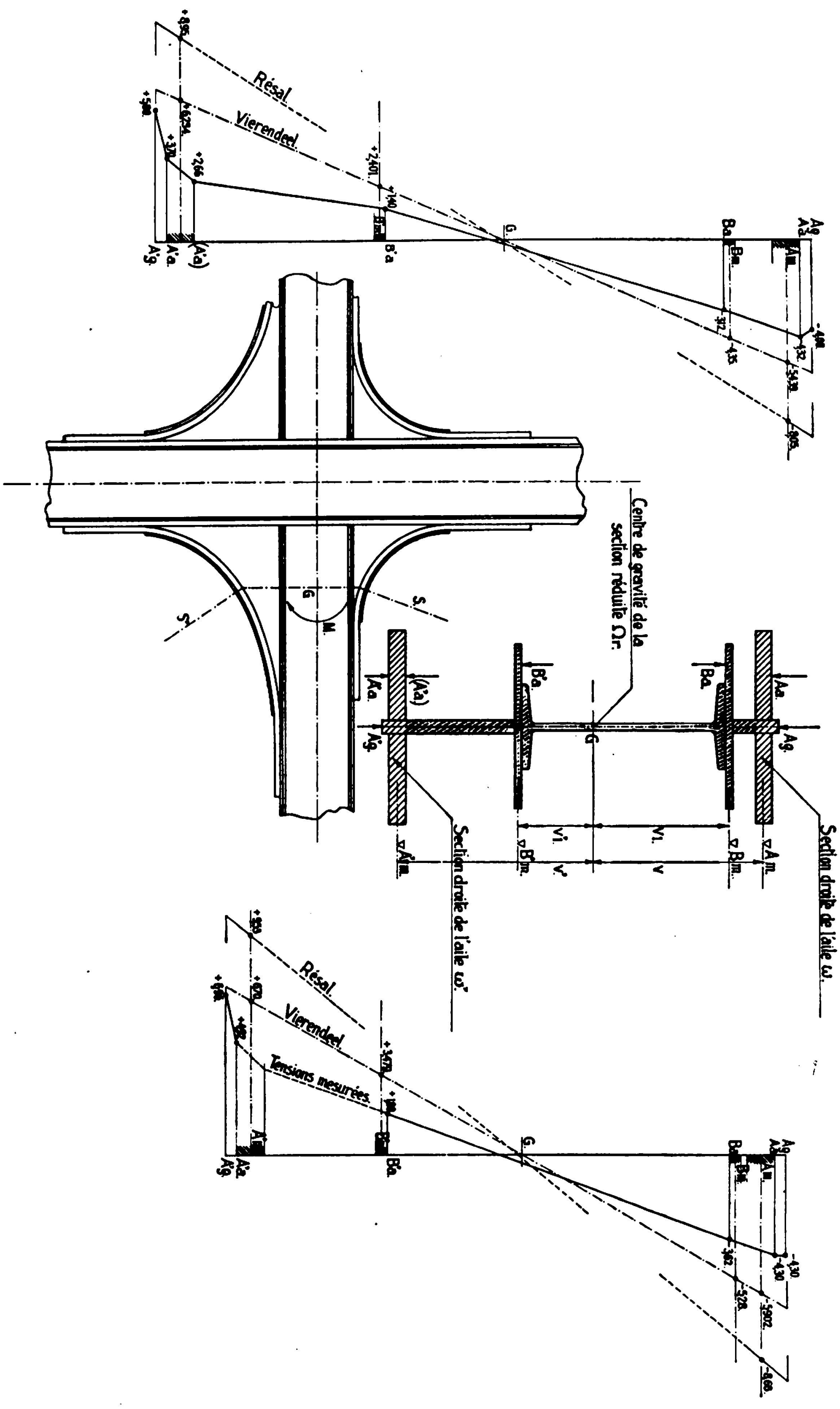

Fig. 21.

The distribution of the tangential stresses in the web and the contiguous parts takes place by analogy with what is admitted for parts with parallel chords. Where the web does not vary in thickness, and is thin compared with the flanges, the tangential stress distribution may be assumed to be uniform.

It is also necessary carefully to consider the effects at right angles to the flanges which take place where these join the gussets, due to the curvature. The mathematical expression for these actions, per unit of length, is $\frac{\omega \sigma_m}{R}$

where ω is the section of the flange

σ_m the average tension (stress) in the flange

R the radius of curvature of the flange.

The strength of the gusset per unit length of edge, and the strength of the means of assemblage (rivets or welding) per unit length, which we may call $(e\,\varrho)$, must be higher than or equal to $\frac{\omega \sigma_m}{R}$

Hence, the thickness of the gusset and its joint at the wings are governed by

$$(e\,\rho) \geqq \frac{\omega\,\sigma_m}{R}$$

The test on the model joint for the Lanaye bridge (Section IV.) showed[2] good agreement between the measured values of $(e\,\varrho)$ and the values deduced from *Vierendeel's* formulas.

This formula may also be considered in the form

$$R \geqq \frac{\omega\,\sigma_m}{(e\,\rho)}$$

which determines a lower limit for the radius of curvature in terms of the thickness of the gusset or the method of assembling. This formula might also be used for checking the curvature of the joints to see whether it is satisfactory. As a first approximation we may call σ_m the maximum permissible stress, and this gives a highly exaggerated value for the limit of R. When σ_m has been calculated by *Vierendeel's* formula, the limit of R might perhaps be reduced, allowing, of course, for the transverse bending of the flanges as studied below.

This leads to a calculation by successive approximations which will finally enable the engineer to arrive at the lowest permissible dimensions for the panel points.

In this way, it is possible to check:

(a) The average normal stresses in the flanges,

(b) The normal stresses set up by the flanges on the gussets and their attachments,

(c) The normal and tangential stresses in the gussets,

(d) The attachment of the gussets to the several members, allowing for the bending moments, and the normal and shearing stresses.

As regards the attachment of the curved flanges to the gussets, if the σ_m of the flanges are fairly constant, there is no need to consider any shearing strength. We have seen that this is set up fairly appreciably in elliptic panel points. But if σ_m varies (circular curves), it will be easy to deduce from it the shearing stress to be taken up by the attachments. Of course, at the ends of the curved flanges, on the straight length connected to straight members, σ_m becomes nil towards the edges, and the attachment is calculated by the shearing stress theory. Experience shows that the stresses vary uniformly, especially where welding is employed. However, the writer looks upon these calculations as a means of verifying the stresses and strains subsequently. He recommends engineers to proceed as far as possible on the principle of equal strength when dealing with the attachments.

At any rate, we have demonstrated that there is not a single element of these joints which is not calculable and calculated. A joint of this kind is an important detail demanding a thorough calculation of all its elements. The above experience has clearly shown that these calculations ensure a high margin of safety.

The above exposé is sufficient for practical requirements, subject of course to the discussion of the transverse distribution of the stresses in curved flanges as dealt with below. From the theoretical standpoint, however, it is interesting to carry the analysis a stage further.

In his treatise, Prof. *Vierendeel* states that, at the point where the gusset joins the booms, and α tends towards $\frac{\pi}{2}$, only the web of the gusset is involved in the transmission of the stresses on to the boom. This is tantamount to regarding the panel point as a simple reinforcement of the upright, assembled in the ordinary way to the boom. In the section at the root, the gusset only withstands the normal and shearing stresses and the bending moment (Fig. 22)

$$\sigma_{max} = \frac{N.}{eH} + \frac{6M}{eH^2}$$

$$\vartheta_{max} = \frac{2}{3}\frac{T}{eH}$$

Such a conception does not call for curved panel points. A triangular gusset would have the advantage of giving a constant α and the stresses would be transmitted by the flanges from end to end, provided suitable attachments were provided at each end, where all the trouble lies. The cancellation of the stress transmitted by the flange at the end of a curved flange near the boom permits of the adoption of Prof. *Vierendeel's* ordinary projecting type of flange. But there is more than that. *Vierendeel's* method is not compatible with the tangential connection of the flange, as the normal maximum stress of the gusset at its root has to be set up at the point where the said gusset ceases and where, therefore, the stress is zero or nearly so. This shows that even *Vierendeel's* usual arrangement, with gussets projecting on the booms, cannot accord with his theory, since the dimensions at the ends of the gussets near the booms are too small to enable the maximum stresses in question to develop there. It is not possible for stresses of this kind to develop at right angles to the boom at these parts of the gusset which do not project well on to the boom, and it is clear that the internal stresses must be transmitted, whether the gussets are tangentially connected or not, differently from what they would be in terms of the method of calculation mentioned above. As already stated, it is clear that, in these projecting panel points, the ends near the booms take very little share in transmitting the stresses, and a satisfactory result can only be achieved by making the spread H very large.

This conception is derived from the classic theory of the strength of materials, and is not suggested by the physical character of the transmission of internal stresses, i. e., by the concept of stress trajectories. We have no authority, in fact, for particularising to the extent of regarding the gusset as belonging to the upright; for it belongs to the boom as well. The curved joint forms the entire connecting zone of the various members, which must allow the stress lines to join up properly without obstacles, constrictions, or excessive dead zones. We shall now consider in this spirit a triple-branch joint with tangential connections.

We may proceed as follows for the case of simple bending. The stresses σ_m of the curved flanges are calculated progressively from the upright towards the booms, in the elastic extension of the upright. The diagram showing the variation of σ_m with the sections is then plotted. Generally speaking, σ_m will diminish in proportion as we get away from the point of origin at the upright.

The same process will be followed from the starting point at each boom to the upright. To facilitate the argument, we will assume the joint to be symmetrical. For a certain point on the curved booms, the same values of σ_m will be found by operat-

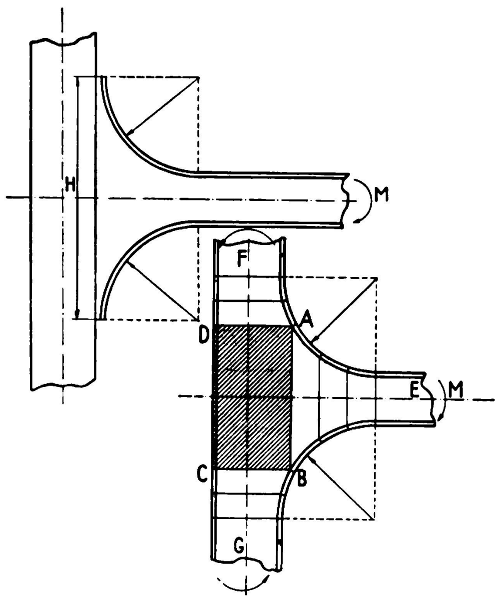

Fig. 22

ing on the uprights as are found when operating on the booms. This then gives the following approximate elastic representation of the functioning of the panel point (Fig. 22):

(1) A zone E A B belonging to the upright, with isostatic lines of polygonal shape;
(2) Two zones F D A and G C B, pertaining to the booms, also with isostatic lines of polygonal shape;
(3) A central zone A B C D forming a disc on the edges of which known internal forces act and which may be studied by various known methods[29], assuming that the section (panel) F A E B G C D is formed by a simple sheet of plating (Tervueren type of panel point).

Actually, if the straight flanges of the uprights and booms are maintained throughout the joint, the gussets only forming corner strengtheners, the zone A B C D will be complex in section and difficult to calculate, except by neglecting the ribs which the flanges form. In joints of the type adopted for the Val-Benôit Institute of Chemistry and Metallurgy building (Section III.) the flat plate of the gusset will be undisturbed except by the V-stiffeners.

In the same system we may consider the effects of the various normal stresses N, allowing for their excentricities with reference to the sections, and, finally, the

[29] *S. Timoshenko:* Theory of Elasticity, French Edition, published by Beranger, 1936.

various shearing stresses. Superposing all these effects, and allowing for the normal stresses exerted by the flanges on the gussets, we may plot an approximate diagram of the stress lines. Without going to that extent, the author believes there are few structural elements for which such a detailed and such a physical idea of the distribution of the stresses may be formed.

For unsymmetrical panel points the problem would be much more complicated, but the idea remains. The extent and shape of the panel A B C D depend on the sections and the rigidities of the members connected, on the size and shape of the gussets, and on the dimensions of the curved flanges. Allowing for the large normal stresses in the booms or columns, the effect of the bending moments is generally greater on the uprights or trusses. Consequently the chord A B is usually near to the point where the gusset joins the boom or the column. This is what was found on the reduced-scale model of the Lanaye bridge joint while the check calculations were being made. However, A B can never be equal to the total spread H of the panel point, and it is just the difference between H and A B which must be applied by way of correction to Prof. *Vierendeel's* conception outlined and discussed above. Stated briefly, A B forms the point of connection of the upright, and it is less than the spread H. A D and B C form the points of connection of the booms. A B C D is a common panel, we might almost say a nodal panel, and is naturally of considerable importance. It is the least known part of the panel point, the one which ought to form the subject of early investigations. However, the author's tests tend to show that, in a well designed joint with curved flanges, these flanges have the effect of stressing the nodal panel very moderately.

The problem presents itself in a similar form for panel points having four symmetrical branches. The joint will be studied in the four directions. The curve, Fig. 22, will be repeated more or less symmetrically depending on the distribution of the moments between the four branches.

In the case of double (two-branch) panel points, if the outside angle is not radiused, the curve will assume the shape of Fig. 23. The most sensible shape would be that of a curved piece (same illustration), which would be calculated by the conventional formulas for a piece of large curvature. But admittedly it would often give rise to practical difficulties with regard to the longitudinal ties, etc., not to mention the actual construction. A middle course would be a rounded portion of small radius at the external right angle.

A method similar to Prof. *Vierendeel's* has been recommended in America[30] for calculating the sides of angle irons subjected to transverse bending. From the brief indications given in the article, and in the absence of other information, the writer understands it to cover circular transverse sections terminating normally at the sides and regarded as isostatic. The article mentions the case of a flat model of a right angle rounded on the inside (angle iron). The centre of the circular section comes at the point of intersection of the straight edge and the tangent to the radiused part (Fig. 24). If the method were applied to an panel point having two symmetrical curves (3 branches), the centre would be on the axis, at the point where the two tangents intersect. In the case of double, dissymmetrical curves, the determination of the centre would be arbitrary. Briefly, this methods leads to a representation

---

[30] *Inge Lyse:* Current Work at Lehigh University. Engineering New Record, April 25, 1935.

similar to that outlined above (Fig. 22), but doubtlessly comes closer to actual facts by substituting curved isostatic lines for polygons. The author is not aware whether formulas have been drawn up, but presumably the complications inherent in this method are not offset by their advantages compared to the use of the *Résal-Vierendeel* formulas as Figs. 22 and 23.

One important point remains to be dealt with: the variation of the stresses in the transverse direction of the curved flanges. The principle has been briefly outlined in a previous paper, which unfortunately contains a printer's error. On the last page (24) of this paper[2], first column, 19th line from the bottom, read: »minimum vers les bords, ce qui doit encore atténuer« ("minimum towards the edges, which must diminish") instead of »maximum vers les bords, ce qui doit encore accentuer« ("maximum towards the edges, which must still further accentuate"). It is essential for selecting the transverse dimensions of the flanges in terms of curvature, and it leads to fairly narrow and thick flanges rather than wide and thin ones.

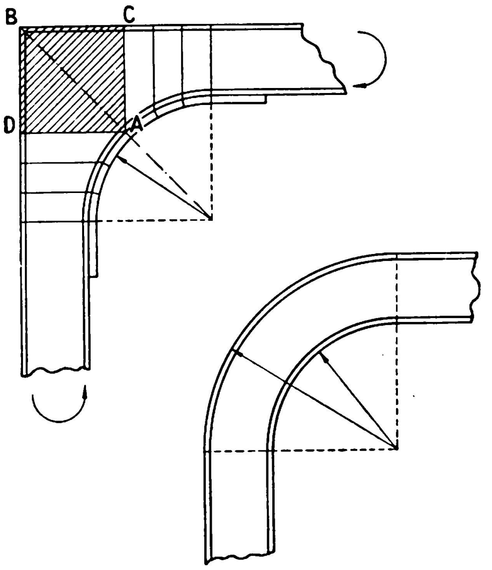

Fig. 23.

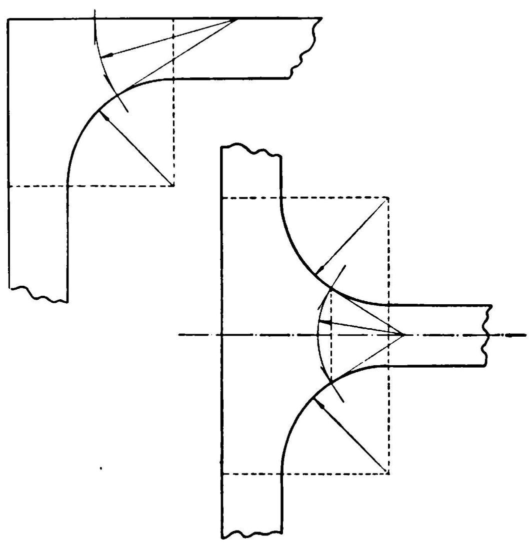

Fig. 24.

An analytical study of this problem has been made by *H. Bleich*[31] for curved pieces with concentric flanges. As regards the transverse distribution of the stresses in the flanges, the solution is admissible for panel points with curved flanges. Whereas *H. Bleich's* calculations of the bending stresses in curved members are based on the effective width of the flanges in terms of maximum stress, it is preferable to adhere to the concept of the average stress σ_m for the total and actual cross-section of the flanges when calculating joint stresses by the *Résal-Vierendeel* formulas.

H. Bleich's calculations show that the stress variations in the transverse direction depends on the parameter $\frac{b^2}{Rd}$,

where b is the free half-width of the flange,
d is the thickness of the flange,
R is the radius of curvature of the flange on the axis.

[31] *H. Bleich:* Spannungsverteilung in den Gurtungen gekrümmter Stäbe mit T- und I-förmigem Querschnitt. Der Stahlbau, 6th Jan., 1933.

The higher the ratio $\frac{b^2}{Rd}$, the more pronounced the variation. Using *Bleich's* notations

$$\sigma'_{max} = \frac{\sigma_{mean}}{\nu}.$$

The maximum transverse bending stress at the point of restraint is expressed by

$$\sigma'_{max} = \mu\,\sigma_{max} = \frac{\mu}{\nu}\cdot\sigma_{mean}$$

ν and μ are functions of $\frac{b^2}{Rd}$ for which the following table is given after *H. Bleich*:

$\frac{b^2}{Rd} =$	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.10
$\nu =$	0.950	0.917	0.878	0.838	0.800	0.762	0.726	0.693	0.663
$\mu =$	0.836	1.056	1.238	1.382	1.495	1.577	1.636	1.677	1.703
$\frac{b^2}{Rd} =$	1.2	1.3	1.4	1.5	2.0				
$\nu =$	0.636	0.611	0.589	0.569	0.495				
$\mu =$	1.721	1.728	1.732	1.732	1.707				

In the panel points previously described, in the regions where the average stresses in the flanges are highest, $\frac{b^2}{Rd}$ is generally round about 0.5, and even lower. In elliptical flanges or the small circular gussets of dissymmetrical flanges (Sections III., V. and VI.), $\frac{b^2}{Rd}$ may be as high as approx. 1.20, but the average stresses are low.

However, this theory is extremely interesting, especially from the point of view of the limitation of the transverse bending stresses in the flange, which may reach a multiple of σ_m. For instance:

$$\text{for } \frac{b^2}{Rd} = 1.20 \qquad \sigma_{max} = \frac{1.721}{0.663}\,\sigma_m = \sim 2.60\,\sigma_m.$$

This consideration, still more than that of the normal stresses exerted by the flanges on the gusset, must lead to the adoption of values of R which are not too low, or, more precisely, to suitably and reciprocally dimensioning R, d, b and e, the thickness of the gusset.

Actually, the problem is more complicated and *H. Bleich's* calculations must be modified. In the first place, *Kayser* and *Herzog's* experiments[25] seem to show that, in the problem we are considering, the measured values are also lower than the calculated figures. Again, in the actual panel points, the curved flanges are integral with the straight flanges at their ends. This modifies the stressing by impeding the transverse bending at the very regions where σ_m is maximum. The variation of curvature and, finally, the eventual use of small stiffening gussets on the flanges, also disturbs the simple type of section on which *H. Bleich's* hypotheses are based. However, it is essential to consider this theory, and full safety is ensured by using the figures given in the table above.

Nor are the stresses uniformly distributed in the transverse direction on the flanges of welded prismatic members. Otherwise it is hard to account for the very considerable differences found between the stresses measures on the straight flanges and the stresses calculated. (Figs. 13 and 14 Section V.)

This very long account will demonstrate the tremendous amount of work devoted in Belgium to investigating rigid panel points of the type described and will, it is hoped, refute the radical and badly informed criticisms of Dr. Ing. *Krabbe* regarding the joints of the Lanaye bridge.[12] Few constructional engineering jobs in Belgium have been carried through with so much care and forethought. This particular work engaged the most careful attention of a large number of engineers of the Bridges and Highways Department, of Prof. *Vierendeel*, of the Société Métallurgique d'Enghien St. Eloi and its technical staff; and, finally, the writer was called in to carry out the tests described and report upon them.

This paper abundantly proves that there are no precarious attachments in these panel points, that it was designed with the object of ensuring proper transmission of the stresses and achieves this purpose; and that, although the agreement between measured stresses and the calculated stresses is not perfect and does not quite come up to theoretical expectations, it is more than sufficient for its purpose, and always on the safe side. It is a big mistake to think that structures generally agree in detail with the calculations of engineers. The method of thorough examination shows that the actual stressing sometimes differs from the presumed stressing. Up to the present the writer has not come across structures which conform better to the theoretical hypotheses than the metal frameworks with multiple rigid frames including panel points of the type studied. And, after all, this is the highest praise that can be given to them.

Vocabulary for illustration texts.

French	English
A	
à droite de la membrure	right of boom
à gauche de la membrure	left of boom
aile droite	right flange
aile gauche	left flange
aile inférieure	lower flange
alignement du fleximètre	mounting of flexometers
âme	web
âme de la poutrelle	web of beam
assemblages rivés	riveted connections
atelier	workshop
avec	with
axe de la contre flèche et de la poutre	axis of camber and beam
axe de la membrure	axis of boom
B	
boulons	bolts
boulons tournés	turned bolts
bout de I 340 ou 360 PN 1 aile enlevée	ends of I 34 or I 36 one flange cut off
bride inférieure composée	lower boom composed of

C	
centre de gravité de la section réduite	centre of gravity of reduced section
charge	loading
colonne . . . stanchion	stanchion
comparaison des modèles	comparison of Models
compression	compression
compression moyenne	average compression
cordon de soudure	weld
côté	side
coupe	section
coupe dans le montant	section of post
couvre-joint ... épaisseur (c. jt. ép.)	cover plate ... thick
D	
déformations exprimées en tensions	deformation in terms of tension
déformée de la membrure inférieure	deformation of lower boom
désignation des instruments	type and position of instruments
E	
échelles	scales
épaisseur	thickness
étage inférieur	lower floor
étage supérieur	upper floor
étrier	stirrup
extension	tension
extension simple	simple elongation
extension simple déduite des tensions mesurées et calculées dans les montants	tension from measured and calculated stresses of posts
extensomètre	extensometer
extrémité $2^{ème}$ semelle	end of 2^{nd} flange plate
F	
ferme	frame
flexion composée	composite bending
fourrure conique	packing plate
fourrure épaisseur ...	packing plate ... thick
fourrure extérieure	outer packing plate
fourrure intérieure	inner packing plate
G	
gousset	gusset corner stiffening
gousset épaisseur 19 mm	gusset 19 mm thick
H	
horizontale	level
J	
Institut du Génie Civil	Institute of Civil Engineering
joint âmes	joint of web
joint âmes et cornières intérieures	joint of web plate and inner angles
joint de montage	erection joint
joints semelles	flange plate joints
joint soudé $1^{ère}$ sem.	welded joint of 1^{st} flange plate
L	
légende	text
liaison	batting
lignes de référence	reference lines
lisse inférieure	lower flange
lisse supérieure	upper flange
longueurs	lengths

M	
Meuse	Meuse
milieu de la poutre	middle of the beam
milieu	centre
mise en charge	loading
montant	post
N	
noeud-type de la Charpente de l'Institut du Génie Civil	panel-point specimen of steel structure for Institute of Civil Engineering
Noeud E	panel point E
Noed à 4 branches	panel point with 4 branches
nouveau Laboratoire Technique	new Technical Laboratory
P	
par la formule de ... plat	according to ... 's formula flat steel
pièce forgée	forged piece
plaque en acier Siemens-Martin épais ...	Siemens-Martin steel plates ... thick
plat de liaison	batten plate
plats fourrure	packing plates
pont route	road bridge
pont du Val-Benoît à Liège	Val-Benoît Bridge in Liege
R	
raidisseurs	stiffeners
rayon	radius
renforcements	reinforcement
rivets d'atelier	shop rivets
rivets de montage	field rivets
S	
section	section
section droite de l'aile	normal section of flange
semelle	flange plates
semelles de ... entaillées à ... au droit des couvre-joints d'âmes	flange plates of ... and ... respectively on web cover plates
serre-joints	clamp
signe des tensions	sign of stresses
soudure	weld seam
sur les ailes des profils I	at the flanges of I-sections
sur les ailes des goussets	at the flanges of corner stiffening
sur l'âme des goussets en a et b	at the gusset of corner stiffening at a and b
T	
tensions	stresses
tensions calculées	calculated stresses
tensions mesurées	measured stresses
tôle	plate
tôle épaisseur ...	thickness of plate
U	
Université de Gand	University of Ghent

Summary.

This paper described an investigation, carried out by the writer, on rigid panel points in continuous steel structures, such as multiple frame buildings and bridges of the *Vierendeel* type. The feature of these panel points is that rigid angular reinforcements are used, formed by means of curved gussets with bent flanges. The study is based on previous tests on metal models, tests carried out on actual structures, calculations and designs of structures and the results obtained when they were built, both as regards the frameworks of buildings, and bridges.

The technical features of these panel points are described, and the main results are summarised and compared, in certain cases, with the calculated results.

In conclusion, the theoretical principles underlying the design of these panel points are defined and discussed. Certain criticisms of these joints are also discussed. Numerous references are given with regard to the matter dealt with, and especially references to recent tests or tests still being carried out. None of these references or tests show the superiority of other types of panel points over the ones which have been investigated by the writer.