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IV b

Wide-span bridges.

Weitgespannte Brücken.

Ponts de grande portée.

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IVb 1

Present-day Tendencies in Large-sized Reinforced Concrete Constructions.

Neuere Gesichtspunkte für den Bau großer Eisenbeton-Bauwerke.

Tendances actuelles dans les grands ouvrages en béton armé.

S. Boussiron,
Paris.

The ambitions in the conception of large structures of reinforced concrete have been faithfully supported, if not provoked by the progress in the manufacture of cement and by the studies on its better utilization with given aggregates.

The resistances to crushing of 400 to 450 kg/cm^2 which it is possible to obtain on the building yard without having recurrence to any exeptional cares (the constancy of which could not be assured) permit the adoption of a working coefficient of 100 kg/cm^2 in round figures for reinforced concrete and of 150 kg/cm^2 for concrete with lateral reinforcements up to 1.10 %. This coefficient can even have as a limit 0.6 of the resistance to crushing, or 240 kg/cm^2 if the lateral reinforcing is done up to 3.6 %.

But more than ever is it necessary to say here that the solution of a great problem is not the amplification of an average one. The adoption of such stresses disengages the action of diverse phenomena, the study of which must be more deeply concentrated on.

The influence of the permanent load in large spans demands a reduction of all the sections to their proper limits; but this must be accompanied by a careful verification of the degree of stability of these sections with regard to an increase of the loads, or, a displacement of the pressure line. Calculation methods are therefore necessary which are not only reliable to disclose all stresses, but are also quick, enabling the author of the project to perceive early enough the difficulties of the arrangements made.

Finally, the construction of large structures can only be considered with practical and safe solutions for the scaffoldings, which are the most important position of expenditure.

In the following, we are indicating the tendencies which can be drawn from what has been done in France in this line in the course of the last few years.

PART ONE.

Arrangements and Calculations.

We are limiting our treatise to arched bridges. In fact, the arches constitute the only solution to which reinforced concrete is economically suitable as soon as spans exceeding 100 m are concerned. Any other solution would only be an adaption to steel constructions or to suspension bridges, and would have the disadvantage of causing the designer to solve such tensile force and tensile joint problems, which, without being irrealisable, are far from representing a judicious application of reinforced concrete.

The study of these arched structures actually marks a definite tendency towards a more scientific determination of their characteristics: shape, rise-span ratio and stresses.

Until now the general shape adopted for arched bridges was that of masonry bridges, perpetuated since the origin of these structures. No systematic research has been undertaken in order to determine the influence of the shape and other characteristics of the arch on the stresses produced in the sections nor of their repercussion on the size even of the sections.

The first research in this direction was made on the occasion of the construction of the Fin-d'Oise Bridge[1]; subsequently this research was completed by various studies[2] which accurately fix the scientific conditions for the design of these structures.

As the full application of these studies has only just been made on the occasion of the construction of the last realized large arched bridge (Roche-Guyon Bridge over the Seine) we consider it best to expose the method of efficient determination of an arch, by description of the research work that has been carried through in order to establish the characteristics of this structure, viz:

1) Study on the influence of the shape of arch (Variation of the moments of inertia of cross sections),
2) Selection of the rise-span ratio,
3) Selection of shape of sections,
4) Selection of working stresses of concrete.

We are then going to show that the limiting span of arched bridges can be deducted therefrom and are comparing the type adopted with other types of arches.

We are finally giving with some detail the mode of precise calculation which has served to determine the stresses in the arch studied.

In a second chapter, we are exposing on the other hand a few considerations on ordinary and special three-hinged arches.

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[1] See "Génie Civil" of February 1st 1930.

[2] *Vallette:* "Génie Civil" of May 9th 1931 and 2nd volume of Reports of the International Association for Bridge and Structural Engineering. Chalos, same work.

1st CHAPTER.

Statically indeterminate systems.

I. Variation of the moments of inertia.

Let us examine the curve of limiting values of the maximum moments which are produced in an arch of constant equivalent inertia and section (curve 1, Fig. 1) the centre of gravity of its pressure line being at $^1/_3$ of the rise (fig. 2, type I).

If keeping the same moment of inertia at the apex, by means of appropriate variations of inertia, the maximum moment at the quarter span points is increased, that at the springings automatically diminishes. The curves of limiting

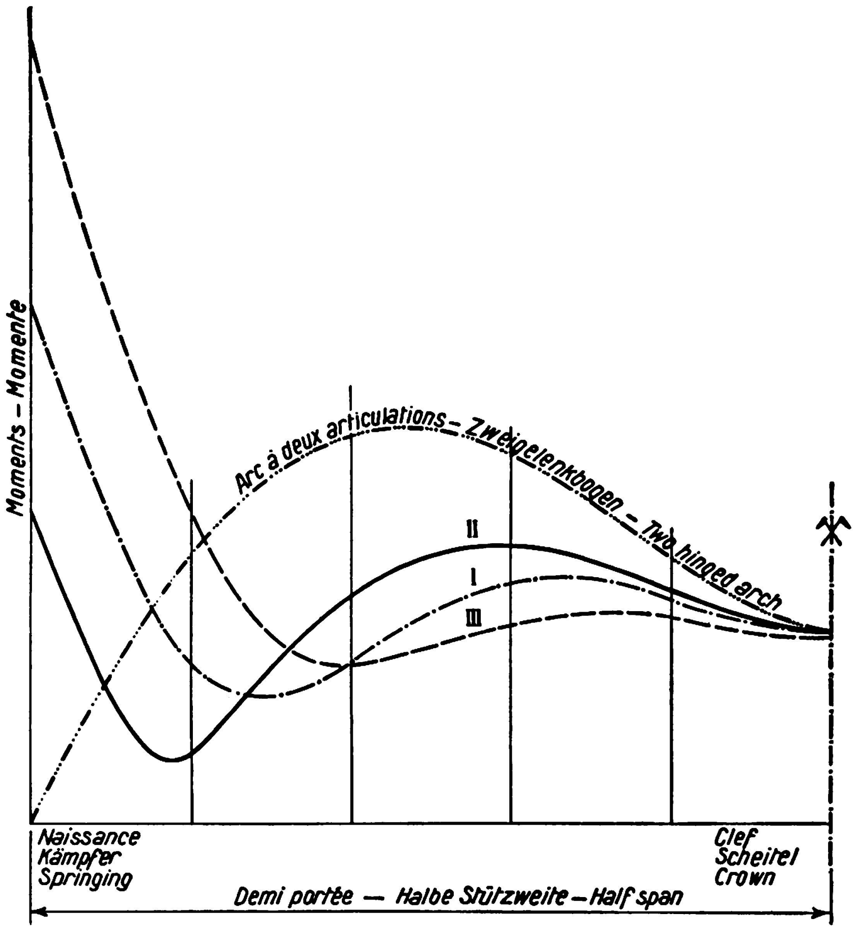

Fig. 1.

Enveloping curves of moments for three types of arches, I, II, III, with equal spans, equal span-rise ratio; and equal moment of inertia at the crown, as well as for two-hinged arches with constant reduced section of equal moment of inertia at the crown as for the above arches.

values of the moments show the course of that marked II in Fig. 1. They correspond to arches with equivalent moment of inertia decreasing from apex to springings, having the centre of gravity of the pressure line in the lower two thirds of the rise (Fig. 2, Type II) and which tend at the limit towards the two-hinged arch (Fig. 1) for which the moment at the quarter span points attains the highest maximum.

On the other hand, if the variations of inertia diminish the moment at the quarter span points, it is noticed that the moment at the springings increases. The curves of limiting values such as III (Fig. 1) are pertaining to arches with equivalent moment of inertia increasing from apex to springings, having the centre of gravity of the pressure line in the upper third of the rise (Fig. 2, Type III) and tending at the limit towards two cantilevers, connected by a hinge at the top.

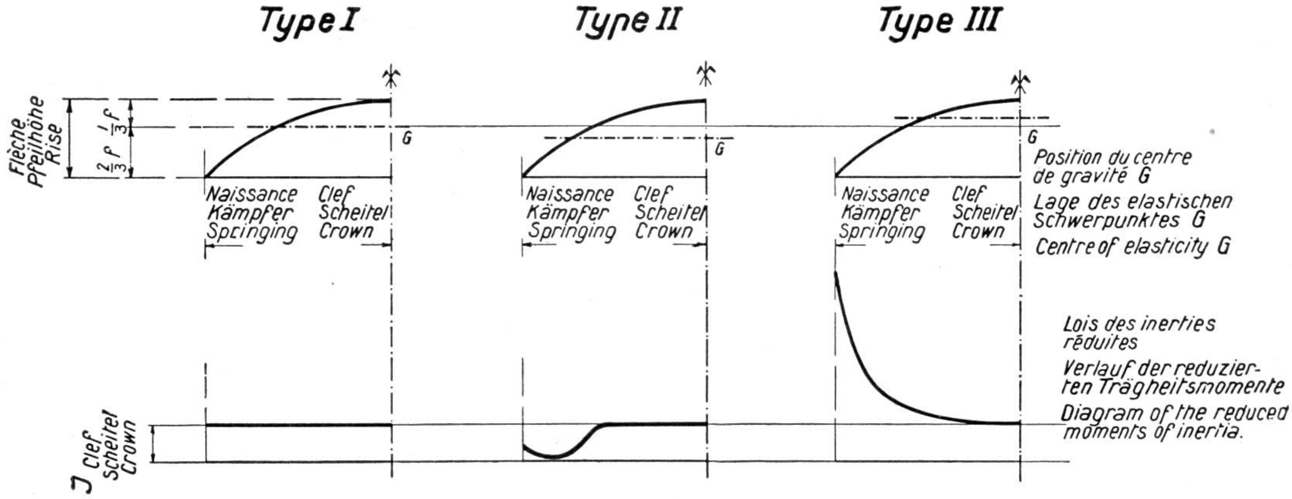

Fig. 2.

Position of the main axis of inertia, passing through point G of the middle fibre; diagram of reduced moments of inertia for arch types I, II and III.

It is obvious that between the two extreme cases, the smallest of these maximum moments will be obtained if the variation of the moments of inertia is such that they are equal at the springings and at the quarter span points. This research has led to the particular type of arch which has already been utilized

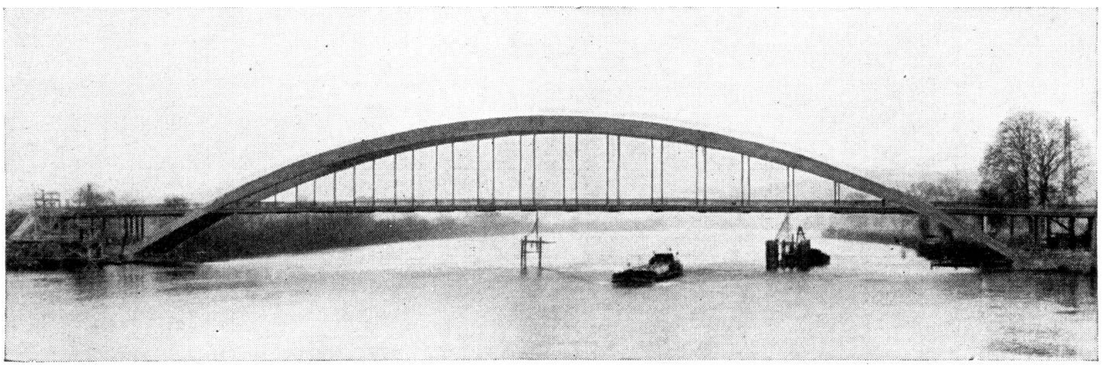

Fig. 3.

Arch-bridge of 161,0 m span over the River Seine at la Roche-Guyon.

in France in 1929 for the construction of the Conflans-Fin-D'Oise Bridge with a span of 126 m and which has just been applied in a still more interesting manner in the bridge constructed by us of 161 m span, across the Seine at La Roche-Guyon (Fig. 3 and 4).

In view of the extent to which we have carried the study of this structure, we shall take it as a basis of comparison with the different conceptions.

In order to do this in a clearer way, it is first of all necessary to determine the selection on the other characteristics of this arch.

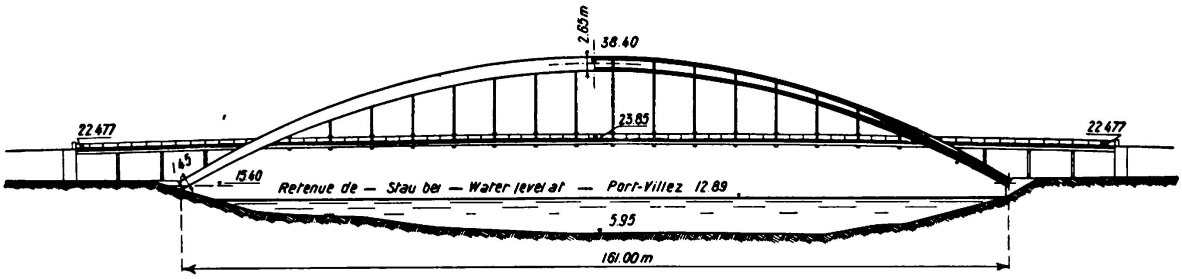

Fig. 4.

Bridge at la Roche-Guyon. Elevation and longitudinal section.

II. Selection of the rise-span ratio.

In order to determine the conditions of this selection curves B (Fig. 5) have been established which show the variation of the average section as a function of the rise-span ratio. On examination of curve 80^k — 161 — II which corres-

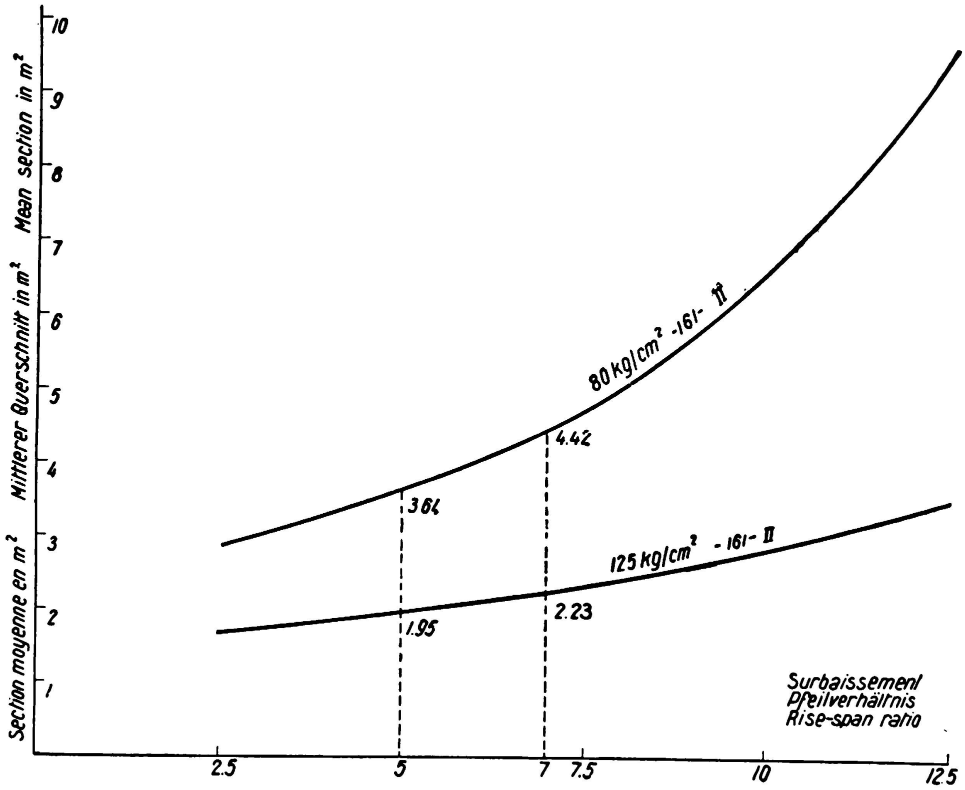

Fig. 5.

Curves B.

Changes of mean sections in relation to the span-rise ratio for arch type II of 161 m span and stresses of 80 and 125 kg/cm² respectively.

ponds to the arch at La Roche-Guyon with a span of 161 m and a maximum working stress of 80 kg/cm², shows that if the ratio rises from $\frac{1}{7}$ th to $\frac{1}{5}$ th the average section decreases from 4.42 m² to 3.64 m², which gives a ratio of 1.21 between the two areas.

With an increased working stress, the variations of section are far from being as rapid. Curve 125^k — 161 — II shows that for the same structure with a working stress of 125 kg/cm² the average section would vary from 1.95 m² to 2.23 m² in passing from a rise-span ratio of $\frac{1}{5}$th to that of $\frac{1}{7}$th (see Fig. 5). The ratio of these average sections is reduced to $\frac{2.23}{1.95} = 1.14$. In fact, in taking into account the respective developments of the two arches, the ratio of the volumes of materials in only $\frac{1.10 \times 1.95}{1.054 \times 2.23} = 1.09$.

Wind effects are more intense on an arch with a large development and may necessitate extra material. This increase will diminish still further the ratio of 1.09 found above.

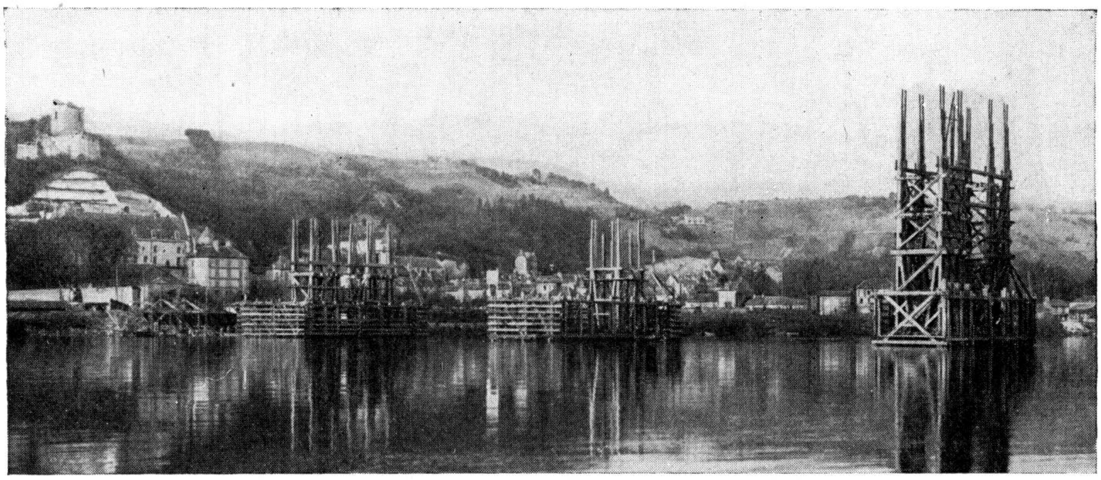

Fig. 6.

Bridge at la Roche-Guyon. View of steep right-hand bank.

Finally, in this particular case, the determining factor in the choice of the rise-span ratio will be the increase of the thrust on the abutments which is nearly proportional to it. Without considerable repercussion on the average section, the degree of reduction of the thrust will be adapted to the facility of abutment foundation, in accordance with construction conditions and aesthetic reasons. At Roche-Guyon, we have adopted a rise-span ratio of $\frac{1}{7}$ th which correctly proportiones the height of arch above the decking with the landscape, dominated by the cliffs on the right bank (Fig. 6).

The limitation of the rise is besides in concordance with the desire of restricting the height of the scaffolding above the flooring.

These conclusions only apply to the types of arch for which the variation of the moments of inertia has been judiciously studied; for other types, the rise-span ratio, according to the working stress, may be of considerable influence to the sections.

III. Selection of the shape of section.

The analysis of the influence of the shape of the section on the working stress would show the necessity of choosing a large section made up of relatively narrow members. (See "Génie Civil", 9th May, 1931. *M. Vallette.*)

In the case of an arch with suspended roadway the span of the cross beams should not be too greatly increased.

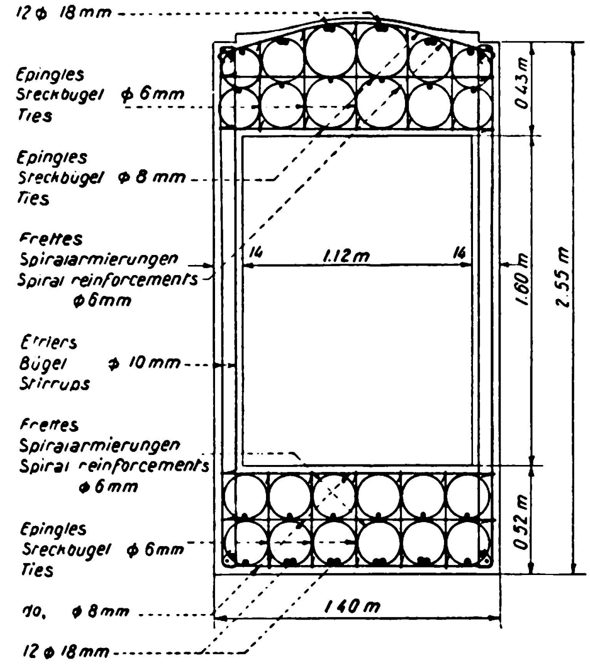

Fig. 7.
Cross section trough crown.

For this reason a breadth of 1.40 m was adopted for the whole of the arch above the level of the roadway. Then, in order to have the smallest possible ratio $\frac{h}{l}$ since a large, low arch is considered (See "Génie Civil", 9th May 1931.

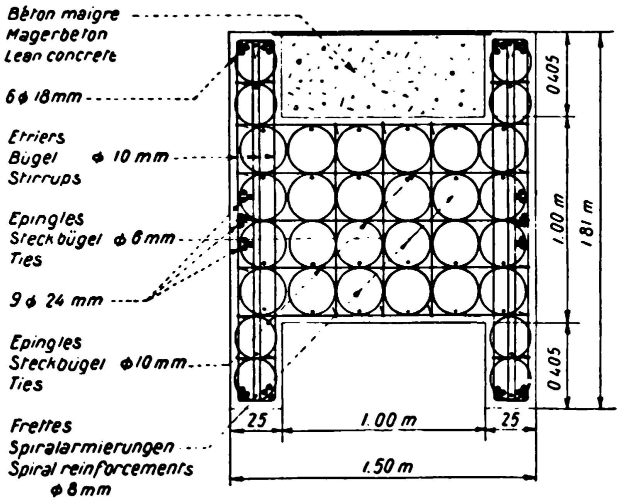

Fig. 8.
Cross section through quarter-span,

M. Vallette), the breadth of the arch was progressively increased up to 3.00 m and its height decreased to 1.45 m or less than $\frac{1}{110}$ th of the span. (See Figs. 7, 8 and 9.)

IV. Selection of the working stress.

The choice of suitable working stress is of great importance, as is shown by the full curve II (Fig. 10) which gives the variations of the average section as a function of the stress for an arch of 161.00 m span with a rise-span ratio of $\frac{1}{7}$ th carrying the required live load and differences of temperature.

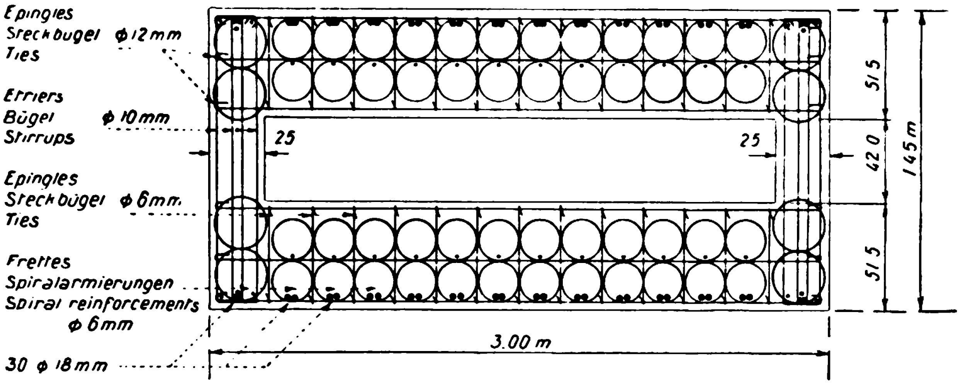

Fig. 9.
Cross section at springing.

If we had not analyzed these variations, we might have been tempted in order to avoid lateral reinforcement of the concrete to choose a current working stress. From the curve it will be seen that for a limit of working stress of 80 kg/cm² the mass of the arch would have been twice that necessary for the arch which was finally chosen having a working stress of 125 kg/cm². Curve 161 — 7 — II; curves I and III are relative to other laws of variation of the moments of inertia which will be mentioned later in the comparative study.

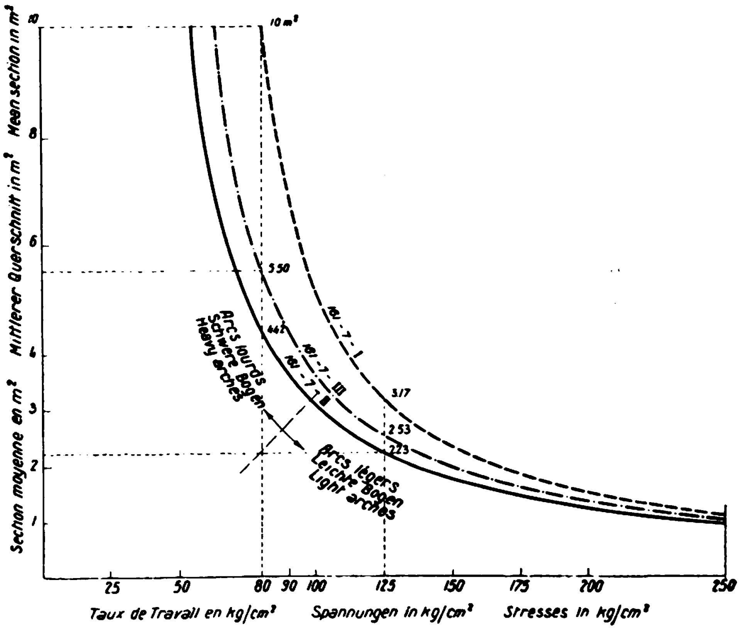

Fig. 10.
Curves C.
Changes of mean sections in relation to stresses for three types of arches of 161 m span and rise-span ratio of 1 : 7.

This choice was governed by the necessity of obtaining the smallest arch capable of retaining sufficient security against sudden increase of stress through accidental causes. For a very small decrease of section, this increase might be considerable in the horizontal branch of the hyperbola which corresponds to "light" arches. For this reason the stress was limited to 125 kg/cm². Its stability was proved by checking the working stresses of several sections to ensure that the figure of 12 kg/cm² for the tension reinforcement and 180 kg/cm² for the concrete were not exceeded when the live load was doubled.

On each of the curves a point can be found where for a decrease $d\Omega$ of the section, there is a corresponding increase dR of the working stress which is such that $\frac{d\Omega}{dR}$ = constant.

The last condition defines the stability of an arch by the value of the tangent to the curve at the point corresponding to the desired working stress (or by a multiple of this tangent when the scale of the Ω and the scale of the R are different and consequently the curve is deformed). These considerations, and the statements previously made, have led to the approximate relationship.

$$l = \frac{\varepsilon R^n}{e^\alpha}$$

V. Limit span of arches.

The constant α is determined by the type of arch and for the same rise span ratio $\frac{1}{e}$, the constants ε and n, approximatively determined, give the arches practically the same stability and consequently the same character, heavy or light. Thus, the formula above gives a practical means of choosing the working stress of the concrete as a function of the span and the rise. As already seen, this choice is of fundamental importance in the design of an arch which is to be stable and economical.

By means of this formula, the limit l of the span of arch can be determined immediately.

We have traced in figure 11, for increasing spans curves, giving the value, as a function of the working stress R, of the variations of the average section of arches type II with a rise of $^1/_5$th of the span.

This figure also shows curves of equal stability for light arches. The curve marked 1 separates the heavy arches from the light arches. With the scales chosen, it corresponds to the value $\frac{d\Omega}{dR} = 0.005$. For "La Roche-Guyon", we have taken $\frac{d\Omega}{dR} = 0.0025$, or 0.5 on the scale of the curve.

From the above equation a curve has been plotted for a stability of 0.4 and is shown by a dotted line. This equation has the advantage of slightly increasing the stability of very large spans.

It will be seen that the curves are drawn on the assumption that, for all spans, each arch carries a load of 6600 kg/m length in addition to its own dead weight: 2000 kg/m for the live load and 4600 kg/m for the floor, the hangers, wind bracing and all other accessories. This corresponds to a free width of 8.00 m. The curves mentioned above are such that the average section of the arches is almost proportional to the live load, the latter being itself proportional to the width.

This results from the general formula (2) established by *M. Vallette* (2nd volume of Publications of the association, Zürich 1934).

$$\Omega_0 = pl \frac{C_5 e\lambda + C_6 \frac{e}{\lambda} + \frac{C_8}{\lambda'} + \frac{C_1 l}{2a^2 h} + \frac{C_2 h e^2}{2l}}{R - C_4 \lambda l e + C_7 \frac{a^2}{\lambda}\left(\frac{h}{l}\right)^2 e^2 - C_3 \frac{h}{l} \times \frac{e}{2}}$$

The application of this formula to arches of type II (type La Roche-Guyon) after determination of the coefficients C, gives for the section at springings the formula (3)

$$\Omega_0 = pl \frac{0{,}124 k_1 e\lambda + 0{,}0376 \frac{e}{\lambda} + 0{,}329 \times \frac{1}{\lambda'} + 5{,}95 + 0{,}00163 e^2}{R - 0{,}191 \lambda l e + t^o e \left(0{,}0025 \frac{e}{\lambda} - 0{,}603\right)}$$

Arches of type I with constant equivalent moment of inertia and section would give (4)

$$\Omega_0 = pl \frac{0{,}121 k_1 e\lambda + 0{,}04 \frac{e}{\lambda} + 0{,}35 \times \frac{1}{\lambda'} + 3{,}57 + 0{,}005 e^2}{R - 0{,}28 \lambda l e + t^o e \left(0{,}0127 \frac{e}{\lambda} - 1{,}54\right)}$$

Arches of type III, evolved by *M. Chalos*, Ingénieur des Ponts et Chaussées, Chef du Service Central d'Etudes Techniques du Ministère des Travaux Publics[3], but improved by the selection of hollow sections, of perceptibly reduced constant area, would give (5)

$$\Omega = pl \frac{0{,}125 k_1 e\lambda + 0{,}038 \frac{e}{\lambda} + 0{,}33 \times \frac{1}{\lambda'} + 4{,}85 + 0{,}002 e^2}{R - 0{,}248 \lambda l e + t^o e \left(0{,}0037 \frac{e}{\lambda} - 0{,}61\right)}$$

In these formulae k_1 represents the ratio of the weight of the floor and the accessories to the live load. Even for a variation of this ratio from 2 to 3, which is the extreme limit, the repercussion of this variation on the formula would be insignificant. It should be borne in mind that the formulae 3, 4 and 5 only apply to road bridges. For railway bridges with the same parameters $\frac{h}{e}$ and a^2, they are modified by respective coefficients[4].

For example for a span of 800.00 m with a free width of 16.00 m figure 11 shows that the section of an arch with a stability 0.8 would be:

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[3] See 2nd volume Publications of the Association, Zurich 1934.

[4] h = height of section, r = radius of gyration = ah Ω = area of section J = Ω a^2 h^2.

$$5.90 \times 2 = 11.80 \text{ m}^2$$

with a stress of 252 kg/cm².

Even prior to the application in actual practice of the new methods from which an increase in the strength of concrete is to be expected, the current methods permit the conception of such a span. Since the working stress of spirally reinforced concrete can be raised if desired by increasing the percentage of reinforcement, the only limit is 0.6 of the crushing strength of concrete without reinforcement.

With careful workmanship a minimum strength of 420 kg/cm² at 90 days can be guaranteed which corresponds to the working stress above of 252 kg/cm².

This possibility as far as it is due to the quality of the material must not, however, eclipse the difficulties of the problem with regard to execution, the essential element of which is the scaffolding. This question will be treated later on.

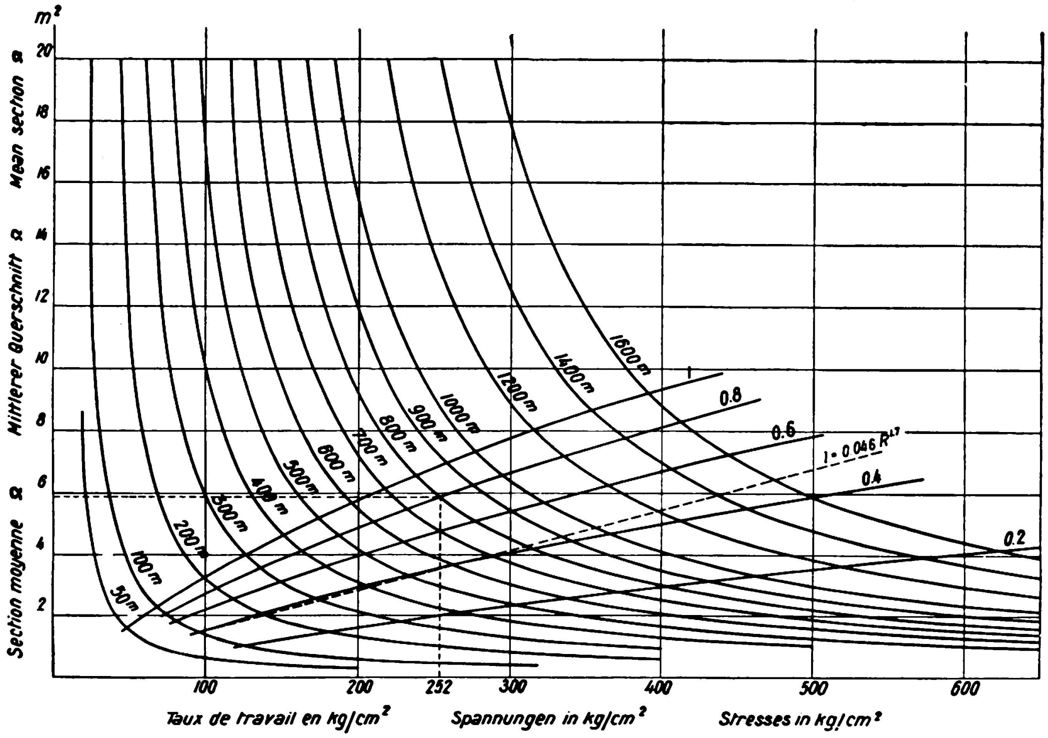

Fig. 11.

Curves C.

Changes of mean sections in relation to stresses and stability curves for arches with rise-span ratio 1 : 5 for different spans. Assumed live load 2 tn/m and change of temperature ± 25°.

VI. Comparison with other types of arches.

Curve II on Fig. 12 indicates the type of arch used, and this is compared with two other clearly defined types, namely:

A parabolic fixed arch with a constant equivalent section (I) mentioned in most text books on strength of materials.

An interesting type of arch whose equivalent moment of inertia increases from the crown to the springings according to the rule:

$$J' = \frac{J_{crown}}{1 - \frac{K-1}{K} m^{\gamma}}$$

evolved by *M. Chalos,* who has prepared tables for its rapid calculation. (International Association of Bridge and Structural Engineering, 2nd volume of Publications, Zurich 1934) m designates the parameter $\frac{x}{a}$, ratio of the abscissa to the half span of the arch, and k is the ratio of the equivalent moments of inertia at the springings and at the crown.

It is to be noticed that for K = 1, the formula applies to arch I.

It was assumed that the arches of La Roche-Guyon were constructed to conform to these types under the same conditions of live load and temperature. In order to reduce their average section to a minimum, we made use of the

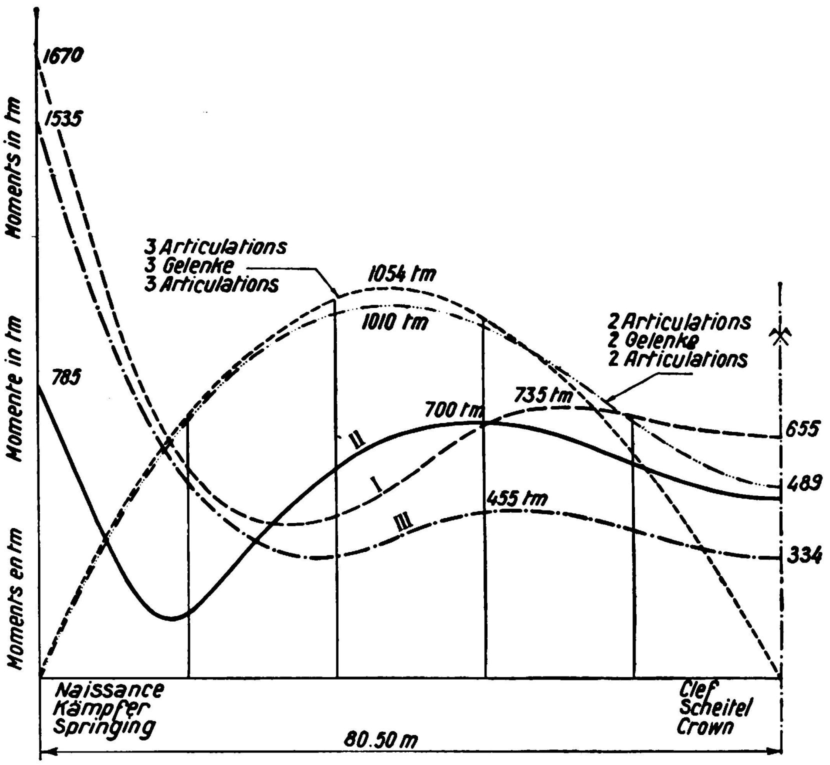

Fig. 12.

Curves of extreme values of moments for the bridge at la Roche-Guyon for five alternative types of arches.

studies of *M. Vallette* as regards shape of cross section. They, therefore, are rectangular in shape, 1.40 m wide for the part above the floor and gradually increasing down to the springings. This increase is governed by the necessity of obtaining a moment of inertia at springings, which in the case of these two types is more than five times as large as the La Roche-Guyon type and necessitates the use of a wide section.

Figure 13 shows the curves of the equivalent moments of inertia J' and of the equivalent sections Ω' which were used in the calculations mentioned above for the three types of arch.

Assuming, as in the case of the arch of La Roche-Guyon that the axis of the arch coincides with the pressure line for dead weight, not taking into account the effects of shrinkage, it was possible, due to the laws of similitude, to trace for these arches the curves which give the variations of the average section as a function of the stress (Fig. 10, curves I, II and III).

The type of arch for which the moments are equal is preferable for arches of average stability. In practice this is found to be the best type, as heavy arches are not economical and very light arches are not stable. Thus, for the working stress of 125 kg/cm^2 used at La Roche-Guyon, giving an average section of 2.23 m^2, a figure of 3.17 m^2 would be necessary for an arch of constant equivalent section, and 2.53 m^2 for an arch of the type recommended by *M. Chalos* i. e. $K = 5$ and $\gamma = 2$.

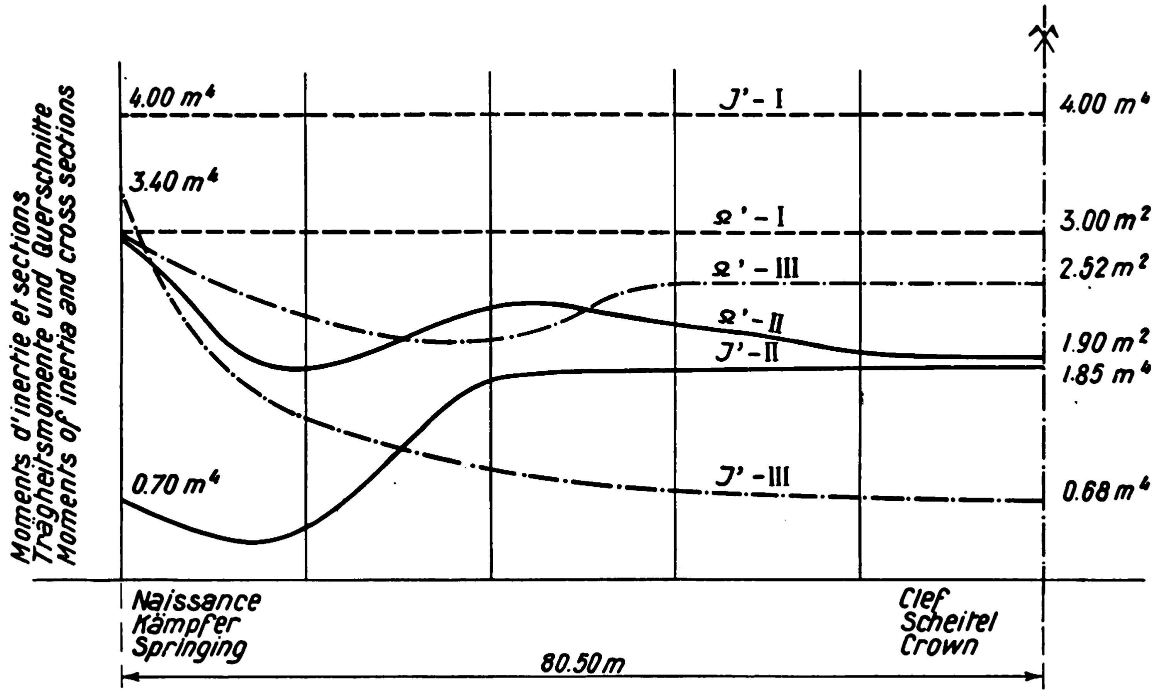

Fig. 13.

Reduced moments of inertia and reduced section for all points of the arch for the La Roche-Guyon bridge for types I, II and III. These values served to plot curve C of fig. 10, of the extreme moments of fig, 12.

We emphasise that in order to obtain this result, the last arch was modified so that the strength should be equal and the stresses amount to 125 kg/cm^2 throughout.

With this object any rule for the areas was chosen and only the rule for the actual moments of inertia was strictly observed

$$J = \frac{J_{crown}}{\left(1 - \frac{4}{5}\, m^2\right) \cos \alpha}$$

This is indeed the only preponderant rule, determining the distribution of the stresses; the areas of the sections are of influence only through their average.

The advantage of the type which we have adopted, already great in decreasing the bulk of concrete, is incontestable with regard to the abutments. It shows the most reduced moments at the springings, as is indicated by figure 12 in which the envelopes of the maximum moments have been plotted. To the advantage with regard to the moments, a further one, concerning the normal force is added. The abutment reactions are as follows:

Type II La Roche-Guyon	M = 785 tm
	N = 1850 t
Type III of *M. Chalos* with optional rule for areas:	M = 1535 tm
	N = 2060 t
Type I with constant equivalent section	M = 1670 tm
	N = 2200 t

It can be argued that a two-hinged arch would have avoided all moments at the abutments. But if a normal stress of 2000 tons for each arch is reached, hinges become either difficult to carry out in spirally reinforced concrete, as they

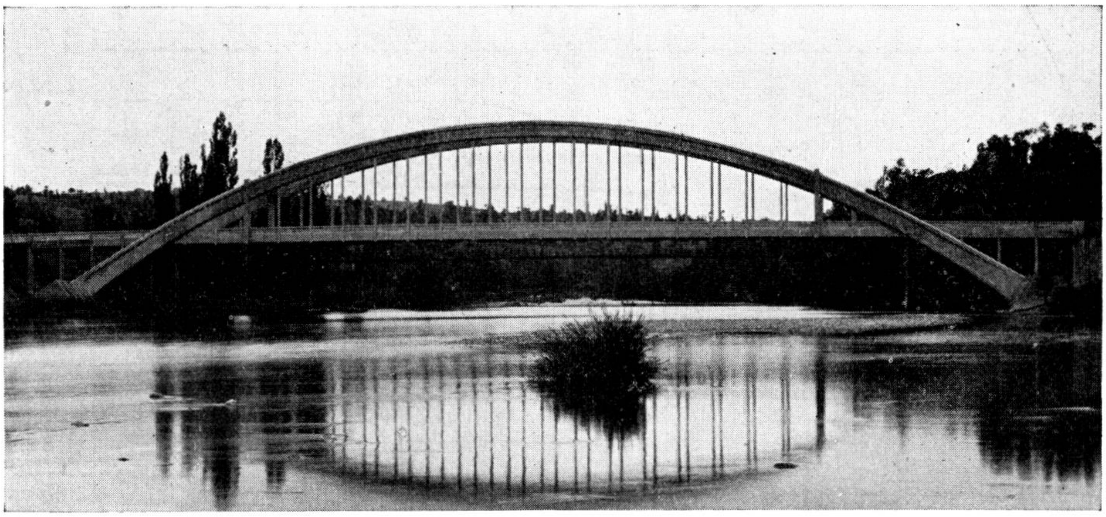

Fig. 14.

Bridge at Bas-en-Basset.

must reach widths of the order of 5.20 m per arch, or costly, if cast steel pieces are used. Further, the average section is larger than in the case of the fixed arch of the type which was used. As a prove, we have plotted the envelope of the maximum moments for a two-hinged arch applied to the conditions of La Roche-Guyon (Fig. 12).

As already seen, it is possible to obtain arches having nearly the same stress in any section, no matter how the moment of inertia varies, i. e. it is always possible to get an arch of equal strength of the reinforced concrete, by varying shape and area of the sections. But among all these types of arch of constant stress, there are two, which give the minimum bulk for the work: one is the arch whose moment of inertia increases from the crown to the springings according to the *M. Chalos* theory, the other is that whose moment of inertia decreases in the same direction of the type used at La Roche-Guyon. According to the actual problem that has to be realized, the advantage will be on one or the other side, depending on the risespan ratio and the ratio of dead weight to live load. For type III, of course K and γ have to be chosen in the best possible way.

M. Chalos' rule is particularly apt for bridges with roadway above, for which the diminution at the crown is disirable not only with regard to the free height, but also concerning aesthetics of appearance, resembling the beautiful masonry bridges enlarged at springings (Fig. 14). Obviously the foundation soil must be capable of bearing the moments, working on piers and abutments.

As examples of the structures built according to this rule, we are mentioning:

The Bridge over the Loire at Bas-en-Basset with a span of 112.00 m erected by the "Société de Constructions Industrielles et de Travaux d'art" (Fig. 14). The axis of the arch is a parabola of 4th degree. The hight of section at the crown is 1.90 m and increases steadily towards springings where it amounts to 3.275 m. The favourable adaption of this type of bridge to the character of the landscape deserves mention. The aspect of scenery is obstructed only to a very small extent by construction members for the double arch and the roadway.

Fig. 15 shows the bridge over the Lignon.

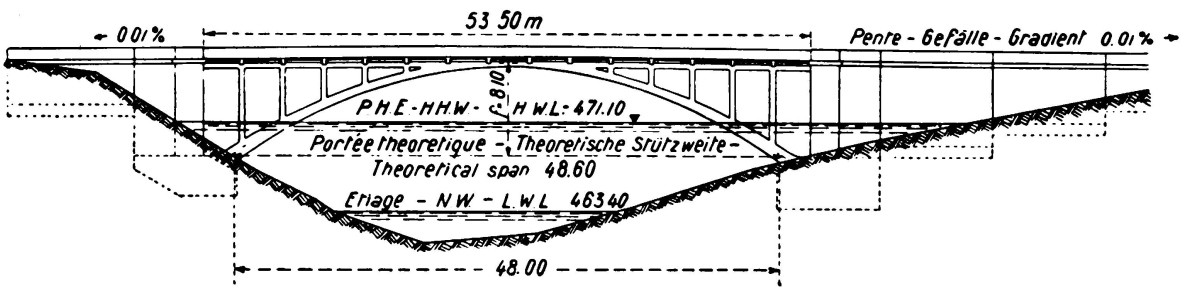

Fig. 15.

Bridge over the River Lignon.

VI. Calculation.

With restriction to that value of the coefficient of stability, below which it would not be advisable to go, — whereby Fig. 10 and 11 may be used to determine its rate —, the light forms are the only ones to be considered for bridges of large span. Contrary to this advantage, they require more accurate calculation than the massive forms, where weight itself helps to stabilise the stresses, but the results justify the extra work. Besides, the rules of general similitude[5] permit to carry out this study only once for all arches of a certain type; stresses and sections of an arch of equal shape but of different span, rise-span ratio and strength of material being deducted therefrom by simple proportion.

We have verified this on the occasion of the La Roche-Guyon Bridge.

A model type for the shape of arch adopted, having been established several

[5] *M. Vallette:* "Génie Civil" of 9th May 1931.

years ago, has furnished, by application of the rules of similitude the following values:

At the springings	M	= — 796 tm
	N	= 1842 t
On the haunches, at 32.00 m from the crown	M	= + 710 tm
	N	= 1646 t
At the crown	M	= + 520 tm
	N	= 1620 t

To bear out these results we have also made the direct calculation of the arch of La Roche-Guyon by using the most accurate methodes.

The formula for stability and the methods of graphic calculation described by *M. Vallette* in the "Annales des Ponts et Chaussées" (VI 1925) have been used.

The graphical method is the only one by which the distribution of stress can be determined accurately and also shows any rapid variations in the moment of inertia at given points and thus eliminates any possibility of error. The degree of accuracy is extremely high and correction to the curve of thrust can be made to within 5 %.

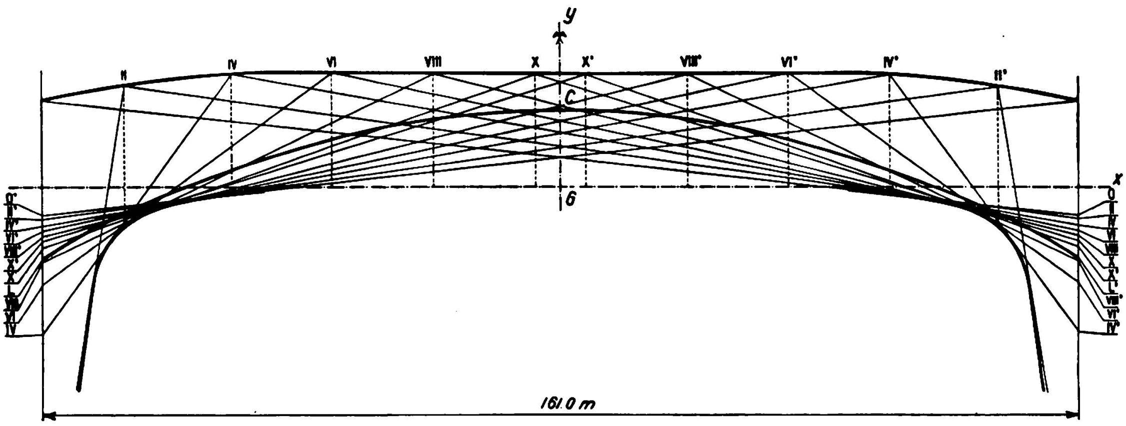

Fig. 16.

Reaction force line and enveloping curve to reaction forces at springing for te arch of the La Roche-Guyon bridge.

In spite of opinions to the contrary, it can be used throughout the whole span of an arch. It is unnecessary to make calculations for obtaining the effect of a load placed near the springings. It is only necessary to adapt this procedure at these points by taking different origins for the abscissae by which a more accurate value may be obtained for certain constants.

The moments, thrusts and shears have been determined directly by means of the graph giving the reactions, each peak being taken to the right of a hanger. The reactions for pairs of hangers are given as well as the curve of the intersections, and the curves of the reactions at the supports (Fig. 16). This proceeding is so sensitive that it is impossible to make an error on the position of the reactions. These succeed each other in order to form the complete curve without overlapping, and cut out, on the verticals of the abutments, segments which increase in size and so conform to recognised design.

It was assumed that the maximum stresses were given at every point by the maximum moment combined with the normal thrust which corresponds to it. This theory bears out the actual fact.

The forces obtained were:

At the springings	M	=	− 785 tm
	N	=	1842 t
On the haunches at 32.00 m from the crown	M	=	+ 702 tm
	N	=	1641 t
At the crown	M	=	+ 490 tm
	N	=	1615 t

If we compare them with the values previously obtained by the rule of similitude, we notice that those were entirely sufficient for the accurate determination of the arch and that the exclusive use of this procedure would have been perfectly justified, even with the minute differences in shape (funicular polygon for dead weight) which always exist between two arches.

The only want of precision of the calculation for the determination of the action of the structure could, therefore, be attributed to the value of the coefficient of elasticity of concrete. But before proceeding for La Roche-Guyon, to the compensation of the further forces from residual shrinkage dead loads of the roadway, we ascertained the value of this important factor. In this connection, we had made some very interesting experiences at the bridge over the Qued Chiffa of the normal gauge railroad from Algiers to Oran.

Each arch of vibrated reinforced concrete was shaped like a rectangular caisson, 5.00 m wide. The type adopted was that for which the moment of inertia decreases from the crown to the springings. This was rendered necessary by the large rise-span ratio, the considerable effects of the live load (locomotives), and the variations of temperature which are of considerable importance in Algeria.

In order to work with precision, we lowered the centering first, as we had noticed in similar cases, that the precision of the observations had been influenced by the fact that the yield of the compression of the timber maintains the support of the arches on the centering at quarter span points. The height by which the arches would have to be raised in order to free them from this support is considerably greater than that which corresponds to the desired compression for the lowering of the centering and the compensation and this would create considerable bending moments in the vicinity of the crown along the freed part of the arch.

The arch having been freed, it was necessary only to operate the jacks in order to obtain the theoretical deformations and to arrive at a position of the arch where the curve of pressure of the loads in action almost coincides with the neutral axis. The arch is then without secondary stress. This condition is called the "neutral state" of the arch and is obtained when, after the centering has been entirely lowered, the arch is brought back by the jacks to the position which it occupies in the first place on the centering (less the lovering) $\int N \frac{ds}{E\Omega} \times \frac{dy}{ds}$, corresponding to the compression which is negligible in this case), the opening of the joint being without rotation.

Having ascertained the trust Qe, which in the neutral state of the arch is centred on the neutral axis, and the real shortening of the arch, definite data is available for determining the coefficient of elasticity E. This coefficient has been found equal to 2.1×10^6 t/m² for the first arch and to 2.3×10^6 t/m² for the second arch. The information was carried ont 18 days after the completion of concreting work for each arch.

This applies to concrete with 400 kg of cement. We were, therefore, right in adopting at La Roche-Guyon where the ratio of mixture is the same, the value of 2.2×10^6.

It will be noticed in the second part of this report, relating to the construction of structures, that this value has been very closely corroborated by precise observations made during the lowering of the centering of successive elements and of the complete arch, as well as during the compensations.

2nd. CHAPTER.

Three-hinged Arches.

The calculation of these arches is very simple, since the reaction are determined without recourse to elastic deformations and as temperature and shrinkage have no influence.

As a means of comparison, we have traced in figure 12 the curve of the maximum moments for an arch of the same shape as that of La Roche-Guyon, but with three hinges.

The area of the Mdx, representing the size of the average moment is larger than that of types II and III, but smaller than that of the arches I and IV. The uneven distribution of the moments, passing from O to a maximum, does not correspond to a good utilization of the material. The advantage which this type may have, due to the suppression of the moments at the abutments, is opposed by the use of expensive hinges which, in our opinion are not apt for large spans because of the high local stresses in the concrete. If for the chosen coefficient of stability, working stresses of more than 125 kg/cm² are permissible and judicious, it will be difficult to use other than carefully constructed cast steel hinges in order to keep within strict limits the indeterminateness of the point of application of the reactions in the surfaces in contact.

For large rise-span ratios, it will be necessary to give full attention to the displacements of this point. If the popular arrangement of rolling a convex surface on a concave one of larger radius or on a plane one, this displacement may attain under the influence of shrinkage, in addition to that of temperature a dimension which is no larger negligible. Furthermore, the influence of the slow compressions of concrete has to be considered, on which Engineer *Freysinnet* has drawn the attention of the designers, several years ago, and which is the object of careful studies in order to determine its laws.

For the three-hinged arches the above mentioned phenomena acquire greater importance because of the unrestricted possibility of rotation. Thus, for a shrinkage of 0.022 mm/m which is to be expected after lowering the centering the fixed arch of La Roche-Guyon settles 0.0548 m whereas a three-hinged arch of the same span and the same rise would settle 0.067 m. The same values would result from a drop of temperature of 20 degrees.

If some particular circumstances were responsible for the construction of three-hinged arches of large span and of large rise-span ratio, we would judge it advisable to reserve the possibility of replacing after some time of service the jacks, used for lowering the centering in order to re-establish the arches in their original position after the effects of shrinkage or of slow compressions of the concrete.

However, it is just to recognize that up to spans of 100 m, the three-hinged type of arch has been able to furnish some interesting solutions. One of the latest applications is that over the Meuse with two bridges of a span of 97.00 m and a rise of 9.00 m according to the plans of the Société Charles *Rabut* et Cie (Fig. 17 and 18).

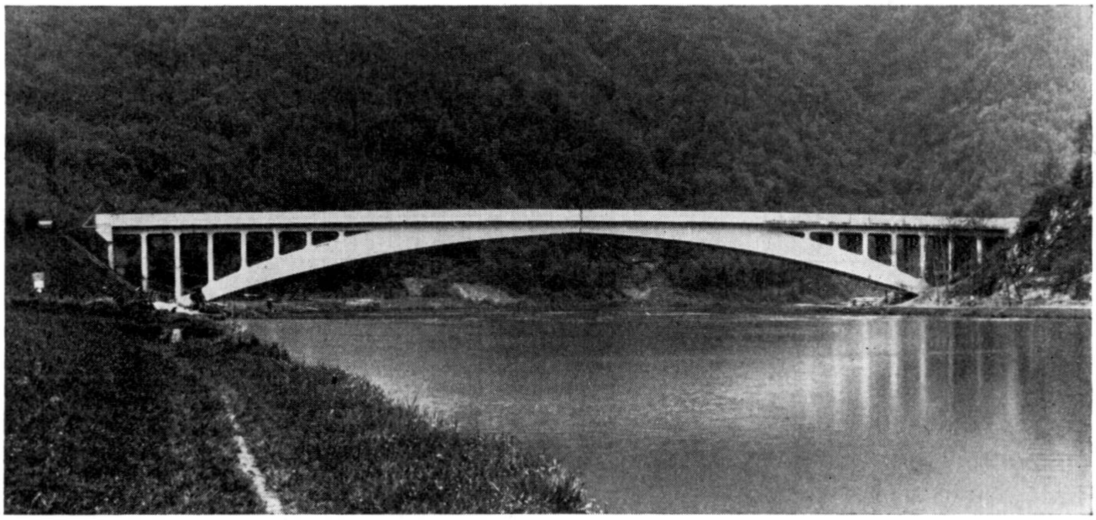

Fig. 17.
Three-hinged arch bridge of 97 m span over the River Meuse at Laifour.

The author of the project has made use of the advantage offered by the arrangement of the roadway above. The roadway platform serves as compression slab on both sides of the crown up to the point from which, with respect both

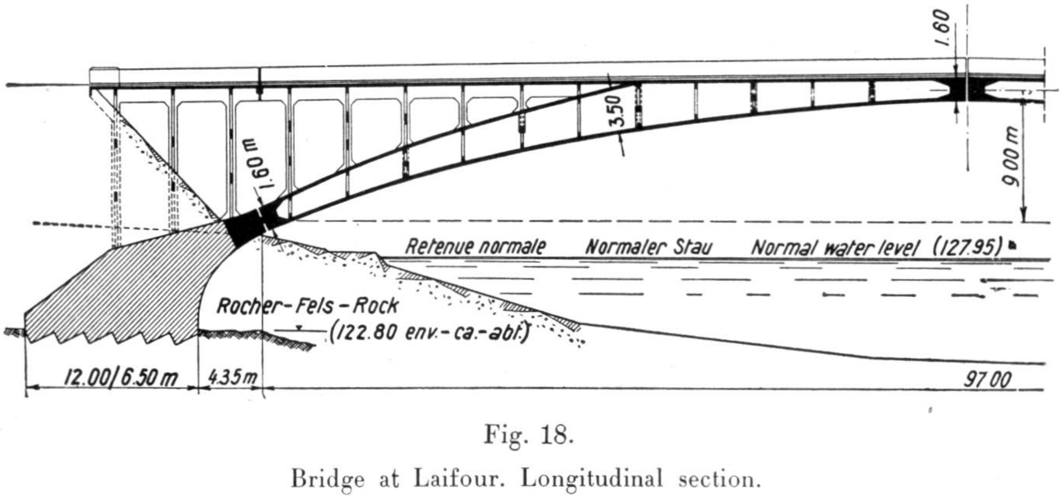

Fig. 18.
Bridge at Laifour. Longitudinal section.

to economy and appearance, the use of hollow webs and a limitation of their height is desirable. From thence, the arches have their own compression slab following the curve.

The author of the project has thus been able, in his opinion, to increase the stability of the arch of which the height at the quarter span joints amounts to 3.50 m, viz. approximately $^1/_{28}{}^{th}$ of the span. He has thereby avoided the indeterminateness of the position of the reactions within the width of arch of 0.42 m which it has been necessary to provide at the hinges.

An additional rise of 0.15 m has been provided, in anticipation of the further settlement under the influence of the phenomena discussed above.

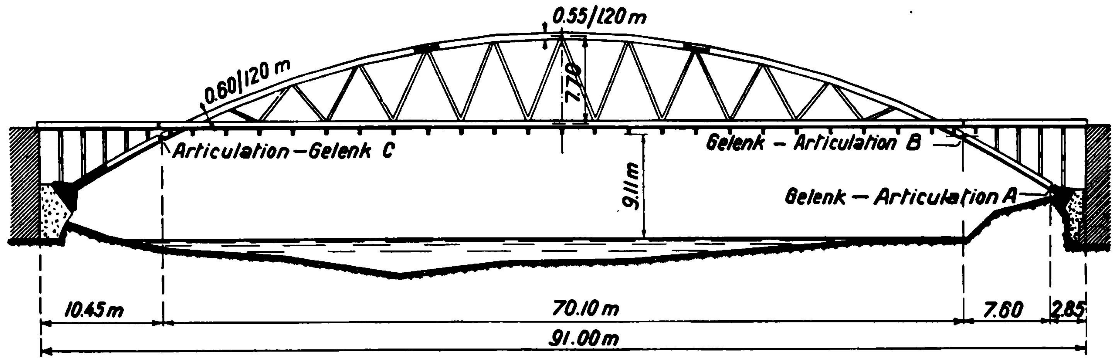

Fig. 19.

Bridge of 91 m span over River Lot at Port-d'Agrès. Three hinges under decking.

It is interesting also, to mention for the three-hinged arch system an arrangement which tends to bridge over large spans by means of parabolic lattice girders. This is made possible by the decrease and even the elimination of the stress in the tension boom, and particularly by the inclination of the reactions by means of an inclined socketed stanchion, placed underneath the roadway (Fig. 19). The 3 hinges are at A.B.C.

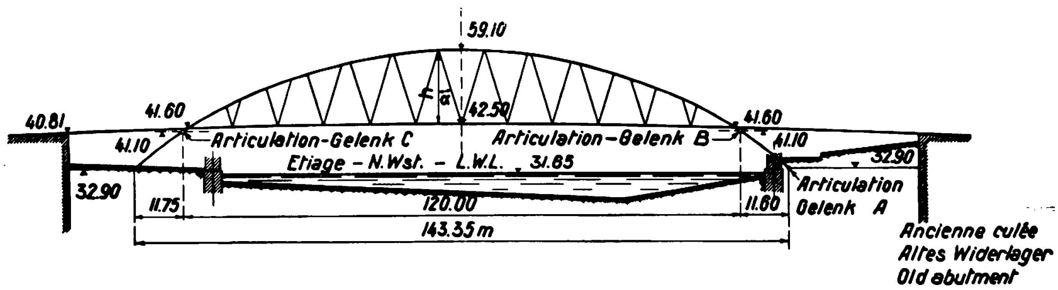

Fig. 20.

Bridge of 143.00 m span over River Lot at Castelmoron. Three hinges under decking.

The socketed stanchion AB fixes exactly the direction of the reaction on one side and consequently also that on the opposite side. The parabolical girder is treated exactly like a girder with vertical reaction, only with the difference that the stress in the tension boom is replaced by a thrust on the abutments. The lattice-work, the stresses of which are determined by the vertical component of the reactions, is exactly the same.

We have made the first application of this arrangement in 1925 for the construction of the bridge of 91.00 m span over the Lot at Port-d'Agres, the general arrangement of which is shown in Fig. 19. (Génie Civil of 18th Febr., 1928.)

The firma Christiani & Nielsen has recently carried out an even more important application with a span of 143.00 m over the same river at Gastelmoron. (Fig. 20 and 21) with the difference that the suspension struts are bare.

The suspension of the floor at the upper boom, by means of suspension struts, produces tensile stresses in the latter, which are curtailed by compressive stresses caused by the live load in its most unfavourable position. The constructors write that at the bridge of Castelmoron, the tension has always been preponderant so that never any of the suspension struts has ceased to work. At the bridge of Port-D'Agrès, compression subsisted in almost all suspension struts, but of such a small amount that all fear of lateral bending was excluded. The preponderance of tension must forcibly disappear with the increase of span because the compression due to the live loads is proportional to the span, whereas the tension due to the dead weight of the floor is practically constant.

The covering of long suspension struts in order to give them compressive strength, and the necessary arrangements to avoid their lateral bending, involve such complications, which demonstrate clearly, that the parabolic girders are not the most economical solution for large spans.

Due to a particular fact, they are prohibitive beyond a certain limit. As soon as the suspension struts are of great length and form a small angle between themselves, which is easily possible because the base of the triangulation is limited by the span of the longitudinal girders, the expansions and contractions of these bars under the influence of the live load provoke deformations which cannot be followed by top boom and the floor. The moments which result therefrom are of such an order that for spans of 150 m, if not below, preference will be given to the arch which is stable in itself without triangular connection with the roadway.

Recognizing the high degree which the internal hyperstatic character of the system may attain, the designers of the Bridge of Castelmoron have taken care that one of the two suspension struts of each knot may become inactive by the preponderance of compression. The utilization of the effect of the inclination of the remaining bars is interesting if the appearance is not hurt. The moments which are produced in an arch with straight hangers are reduced by the moments $\frac{Ph}{tga}$ produced by the horizontal component (Fig. 20).

PART TWO.

Construction.

The construction of large arched bridges is governed more and more by the study of the methods of execution, among which the scaffolding ranks first, for which without endangering the safety, economical arrangements have to be conceived if the selection of reinforced concrete is to be a judicious solution.

In a first chapter we are giving some details of recent scaffoldings and are describing at some length, due to its novelty that which has been used at La Roche-Guyon.

In a second chapter, we are explaining more briefly the methods of execution recently adopted.

1st. CHAPTER.

Scaffolding.

At all times, bridge designers have tended to treat as a work of art the scaffolding itself on which the elements of construction are placed prior to their acquiring the bearing capacity by their mutual reactions.

Examples are numerous of bridges in masonry where much science and art has been spent on the scaffolding.

In the same way this has to apply for reinforced concrete which has succeded masonry in the solution of larger problems.

In spite of the size of the structure, the Annals of Construction will never separate from the Bridge of Plougastel the reminiscence of the scaffolding which has served to its erection. Engineers will always find there one of the best examples for the application of means offered by nature and of the advantages

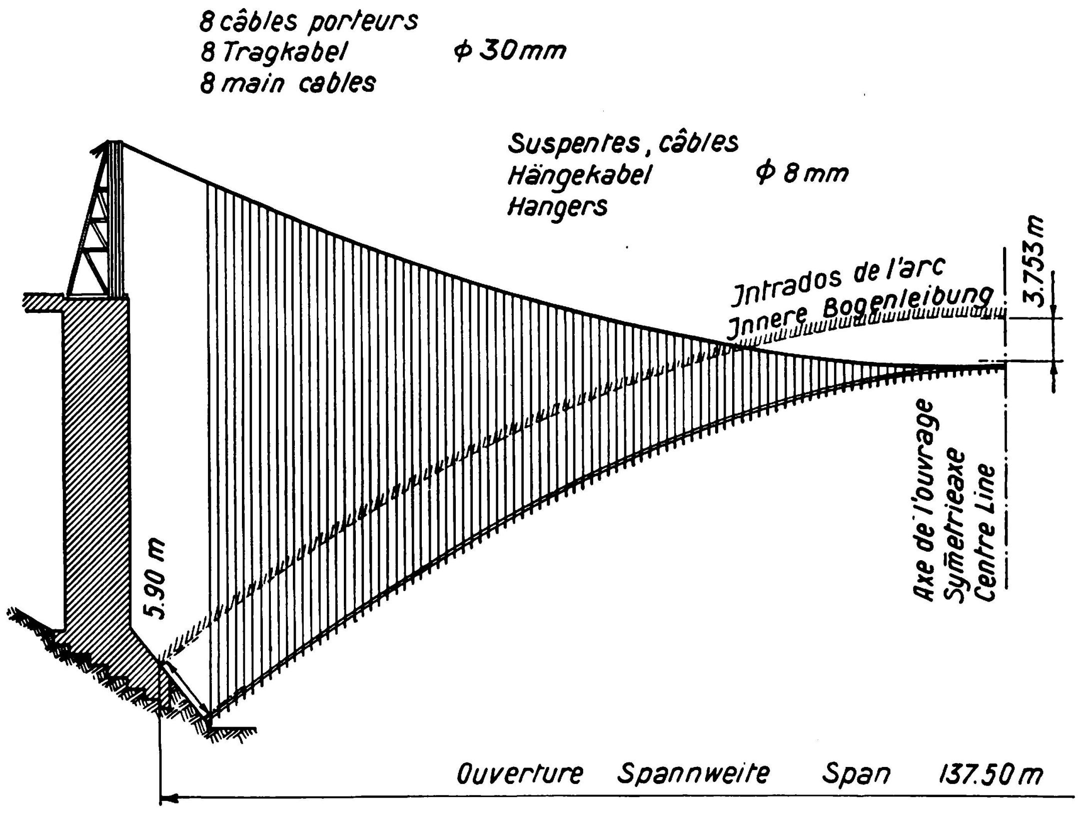

Fig. 21.

La Caille Bridge of 137.50 m span. Erection of false arch work with cables.

which a skilful designer may draw even from elements otherwise hostile to him. Those who have had to study the same project have been impressed by the movement of the tides in this estuary and by the apparent impossibility of supports between the two shores of about 600 m distance. Nevertheless, under the domination of the chief with which docility has the flood not lent itself to carry three times the scaffolding of 150 m span with rising tide, and then to place it again on its supports as the tide went down.

Powerful and expensive installations where necessary indeed, for the utilization of such large natural forces, but they were justified, as they could be used three times. Difficult and delicate manoeuvres had to be carried out and it was necessary to foresee everything in order to work successfully during the interval strictly determined by the tides. But the Enterprise *Limousin*, applying the *Freyssinet* methods, had proved already several times that it did not hesitate to face the most delicate problems of construction.

The situation of the Bridge of Usses (Haute-Savoie), called bridge of the Caille, also deserved that the special arrangements for its scaffolding were studied. The ravine about 150 m deep, did not offer any possibility for the support of the piles of the scaffolding, which necessitated a span of 140 m.

The arrangements studied by Engineer Caquot are a valuable precedent of the solution of constructing scaffoldings over free openings by the aid of suspended cables (Figs. 22 and 23).

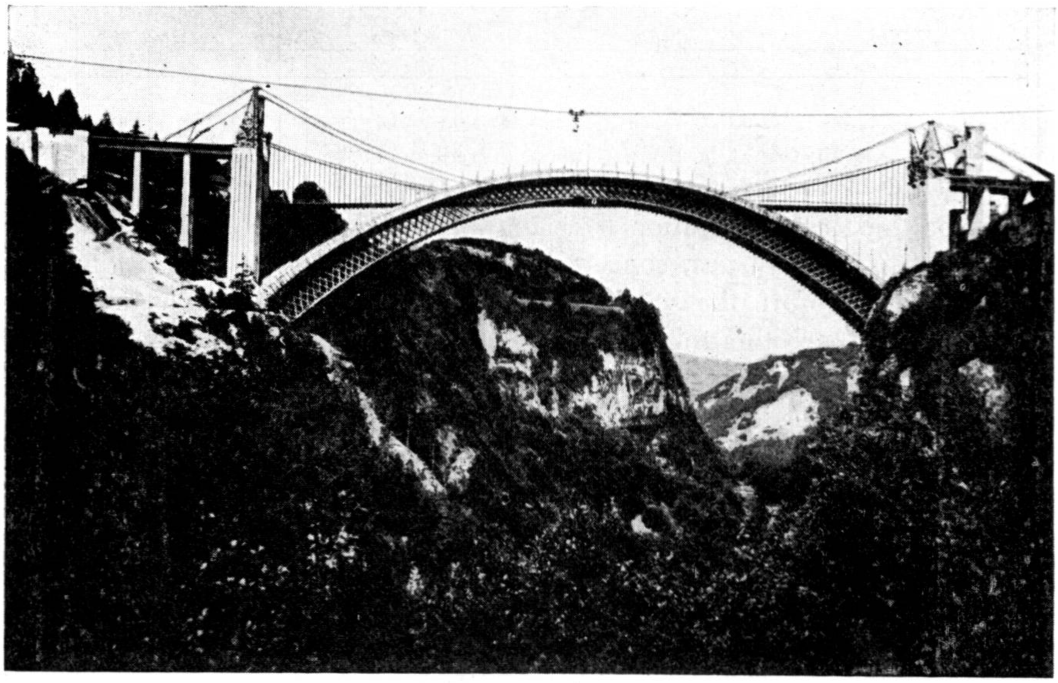

Fig. 22.
La Caille Bridge.

Following the assembly of the first elements of the supporting structure, the strengthening of the centering is made to the intended degree for supporting the first layer of concrete, similar as for a wooden bridge. The necessary rigidity is obtained by multiple framework connecting the booms.

In this solution, there is a judicious application of the suspension bridge in the construction of centerings for large structures; we have the impression that it will always present a favourable solution for large spans which are justly aimed at by reinforced concrete.

The study of the scaffolding of the bridge at La Roche-Guyon has led us to the same conclusions. Even the span of 161 m is the largest which has been bridged over by a type of arch with suspended roadway. This type will always be considered for spanning large rivers where it will rarely be possible to place below the roadway the rise of a large arch.

It would not have been judicous to renounce to supports in the river bed. However, we found the best possible solution by providing only for three piles in the river, spaced at 43.00 m from center to center.

We intended first to have these piles support only the weights of the roadway, of the scaffolding and of the centering, which, after completion of the assembly had to support by themselves the weight of the concrete arch (Figs. 24 and 25).

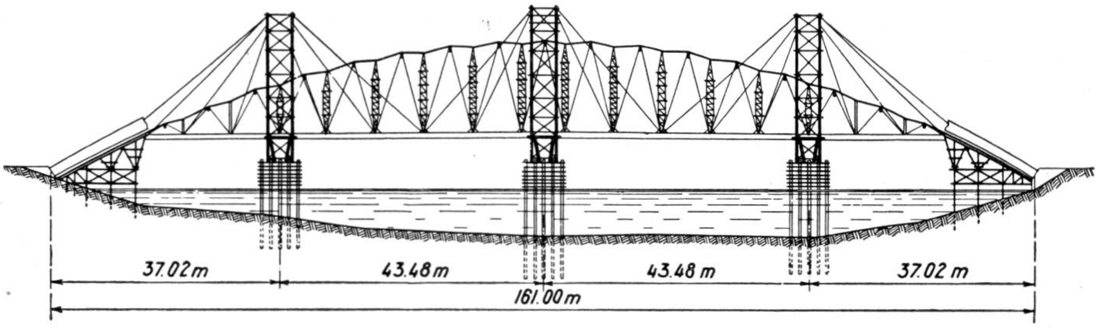

Fig. 23.

Bridge at la Roche-Guyon. Arrangement of staging.

However, for the first application over the largest navigable river of France, we did not want to employ our conceptions without previous verification and we have calculated therefore the scaffolding for supporting the whole dead weight, if by any circumstance this might have been necessary.

The sequence of operations is the following:

Erection of the piles;

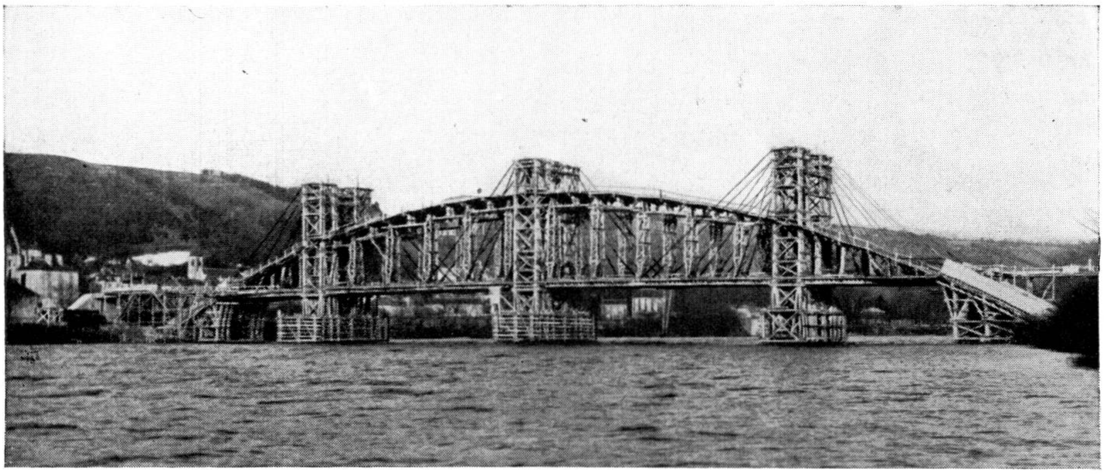

Fig. 24.

Bridge at la Roche-Guyon. View of false arch work.

Advancing the horizontal platform below the reinforced concrete roadway, to be constructed later, simultaneously to both sides of the piles, by using inclined ropes;

Assembly of the centering on this platform.

Previously, the lower parts of the arches up to and above the roadway had been executed by means of closely spaced supports of low height over the slope.

For light arches, whose average heigh of section is hardly $^{1}/_{80}{}^{th}$ of the span, it is essential to survey very accurately the neutral axis and to ascertain that there will be no deviation of it while carrying out the concrete work.

It was not sufficient to fill out with cement mortar the joints of the timbers, as has been done by Mr. *Caquot* at the Bridge of Usses; it was still necessary to be previously assured of the compression of timbers and of the tightening of the elements of assembly. This compression has been exercised by jacks at the crown.

The dead weight of the roadway and that of the centering to which the former is related allowed to apply a thrust of 280 t. We have given this thrust a maximum of 170 t in order to preserve a safe margin of stability.

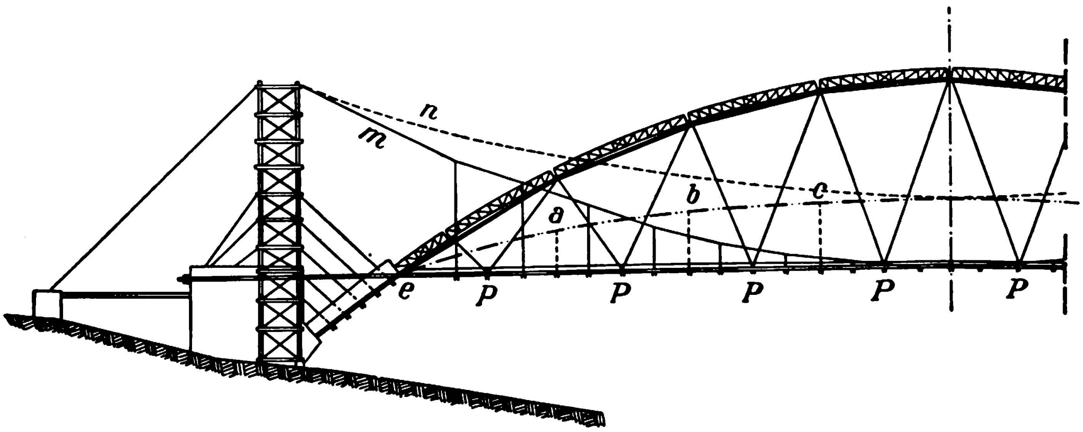

Fig. 25.
Erection of false arch work with cables.

The effect of this pressure is beneficial in several ways:

a) First, it produces compression in the timbers below the arches which surmount the tension, resulting from moments due to the considerable wind loads. We have thus been able to calculate the centering as a girder of 130 m span, between the intersections of the roadway with the arches, below which the full reinforced concrete soffit of the arch acts as wind bracing.

   The frame members of this girder were formed by ties crossing at 45 degrees. This saved us the trouble of calculating the piles for horizontal thrusts and enabled us to construct them lighter or even to omit them.

b) The braces connecting the centering with the roadway are stressed by tension, according to the weight of the roadway which the imposed thrust tends to lift. Fig. 22 shows that these tension members form with the centering and the platform a parabolic beam of 130.00 m span. They are made of round steel in order to expose the smallest possible surface to the wind; their inaptitude to resist to the least compressive stress demands that they be always stretched and that the initial stress be always larger than the compressive stress due to uneven concreting operations. Most frequently, the fraction of the weight of the roadway, which is to be carried will suffice; if not, additional ballast will be needed.

For reasons of precaution, due to very first application of this method, we havė erected additional stays spaced 8.00 m apart which are capable of supporting and transmitting to the ropes, attached to the piles, the whole load, if this should be necessary. The previously applied compression permitted us to reduce the number of stays and thus to diminish the surface exposed to the wind. The line of the timbers is broken and forms projecting angles in the middle of the interval between stays. The previously applied compression caused forces directed upwards which are larger than the weight of arch of 5000 kg/m length.

The concave angle above the stays establishes a uniform compressive stress which necessitates strengthening of these stays. This is preferrable to doubling the number of stays with all their framework which would have offered a large surface to the wind, making it too difficult to establish stability with the horizontal forces, considering the height of 40 m above the point of fixation.

By this arrangement, the span of the timbers has been reduced to 4.00 m instead of 8.00 m and this has enabled us to construct these elements of current sections without using composed girders which, regarding the wind, would have had the same inconvenience as the augmentation of the stays.

Our assumptions have been well confirmed. The whole composition of centering, roadway and braces acted together as a girder of 130 m span. The maximum deformation observed during concrete operations has been 8 mm and yet it was unnecessary to concrete the arch by fragments, uniformly distributed over the length; the concrete-work has been done in a continuous manner, beginning at the two extremities and working up towards the crown. It is of importance to mention that we proceeded by successive elements, as will be seen further on.

Another advantage of the placing of jacks at the crown of the centering was that of lowering the latter by simple releasing of these jacks.

The results obtained in this first application show that it would be possible, as we had intended first, to reduce for large spans the number of piles and even to omit them completely, if the situation of the site permits to do so.

As we have pointed out, the Bridge of Caille is already an example of this reduction, whereby a cable has been used for the erection. If this method necessitates the erection of too high piles on the river banks, the centering can be put in place also with a smaller rise, as shown by the dotted line of Fig. 26, either by means of a platform suspended on a cable "m" or by means of a special cable "n". The centering is then lifted by compression, applied by jacks at the extremities "e". During this lifting operation, made possible by the hinges a, b, c the weight of the roadway gives the necessary stability for the maintenance of the regularity of the centering. It is sufficient to regulate the movement of these hinges a, b, c in proportion to the length of the ordinates of the centering. We do not intend to discuss this question further, which refers to mechanics and for which there are several practical solutions available.

At a bridge with roadway above, the regulation of the movement of the same points could be obtained by anchorages in the soil of the valley or in the bottom of the river bed.

During assembly operations, the compression of the centering will always be maintained sufficient by increasing the weights P, if necessary, to compensate the wind loads.

After completed assembly, the addition of the strengthening braces of the centering will establish rigidity instead of the hinges a, b, c.

The construction at La Roche-Guyon has proved that these ideas may be realized with all necessary security. We are convinced that large, rigid centerings can be constructed economically, opening large perspectives for the construction of large reinforced concrete structures.

2nd CHAPTER.

Methods of execution.

We are dealing here only with those methods which are important for the construction of large spans and which are in relation to the scaffolding, but we are not touching the improvements of the methods for the putting in place of reinforcement bars and concrete, which apply generally to all other structures.

For reasons of economy as well as for safety, the centerings should be loaded progressively, in a manner, improving their stability by the strength of the elements carried out first. For a long time, if not since the very beginning, masonry bridges have always been constructed by succesive layers. The low permissible stresses of this material enable the neglect of the increase of stress in the lower layers, due to the weight of the upper ones.

The bridge of Caille with its span of 140 m is a splendid example of this same process, because it means very much even to design a centering of this span for a load of nearly 13 t/m length, and it was advisable to include in the bearing system the lower boom of the rectangular section of the girder, forming the first ring. In order to exclude any increase of stress, the designers have applied a very ingenious idea of chief engineer Baticle. The concrete was poured between reinforced concrete key-stones, 0.18 m thick, prepared in advance, the mass of which was about one third of the total quantity. As in every layer only the poured concrete is subject to shrinkage, whereas the key-stones were pre-shrunk, it was sufficient to distribute them in the 3 layers, such as to provoke angular deformations by the shortening of the concrete rings, releasing the lower intrados. This method requiring an exact calculation has proved very efficient; no perturbation whatsoever has been noticed in this mass of concrete which was not reinforced.

Mr. *Freyssinet* has used the same method of concreting by layers at the Bridge of Plougastel. Compensation has been effected by jacks at the crown, after lowering of the centering of each arch. An estimate of the distribution of load between the scaffolding and the first layer of concrete has served to determine the contractions to be applied.

All those who are studying still larger problems, such as spans of 300.00 m or more, which produce stresses in the reinforced concrete, which can be faced easily at the present state of the manufacture and application of cement, will have to limit the cost of scaffolding, if this system is to be able to compete successfully with other solutions, particularly with suspension bridges. Without ignoring the merits of the specialists of this construction, it will be recognized that reinforced concrete represents the most judicious solution if the condition of the site allows an easy taking care of the thrust. According to our own opinion, this will only be possible by dividing the execution of large arches into separate elements for which the weight per linear meter be reduced to that amount, compatible with the assurance of cohesion between the successive elements and their own stiffness, prior to the joining.

By this we mean not only the division in separate layers, but also in elements obtained by vertical joints. This latter case is particularly apt for the splitting up of booms of large width, for plate-webbed arches as well as for trussed arches, under the supposition that the hight of section justifies the latter system.

By working with separate layers, the procedure is such, as to apply pressure onto them by means of jacks, simultaneously with the progressive lowering of the centering (first layer 1, then 1 and 2, then 1, 2 and 3 etc.). In this way, additional forces are introduced which can be compensated after complete lowering of the centering as it was done at the Bridge of La Roche-Guyon.

Construction by way of vertical separations does not require these compensations. At Roche-Guyon, we have used successfully methods of measurement (fixed marks and jacks) for controlling the influence of concreting by layers and the results of the compensations made.

This was necessary in view of the general application of this procedure for large spans; it was far from being superfluous as light arches with section of small height were concerned for which a displacement of the pressure line would have occasioned a considerable increase of the moreover high working stresses.

At every partial lowering of the centering, the forces produced by the jacks and the movements of the arch have been observed. At the final lowering of the centering the proper equilibrium of the forces of the free arch was established, by introduction at the crown of a thrust conventiently placed out of centre in order to obtain no displacement of the pressure line. In this way, "the neutral state" of the arch (without any appreciable bending moment) was reached i. e., the state for which all internal moments, due to the mutual influence of the separately loaded vaulting rings are exactly compensated; we have noticed a perfect concordance with the calculations and besides, with what degree of precision this procedure can be executed, because, once the neutral state being reached, the application of an additional thrust of only 1 t at the centre of gravity of the crown section is sufficient to produce an opening of the joint of 14.4 mm and a rise of the crown of 29 mm.

Finally, in all these investigations the coefficient of elasticity was determined without ambiguity; it was found to be from 2.10 to 2.2×10^6 t/m^2 which confirms the results obtained at Qued Chiffa.

For the Bridge at Castelmoron, mentioned above, Inspector General Mr. *Mesnager*, Consulting Engineer of Messrs. Christiani & Nielsen has reached the release of the centering in another way.

The concreting of the parabolic girder has been preceded by the placing, on the centering, of separate elements of a core, prepared in advance. For the section at the crown of 1.00 m $\times$ 1.20 m, the core section amounted to 0.55 m $\times$ 0.80 m.

We conceive that the designers have not been preoccupied by the increase of stress which has been inflicted on this element by the weight of the concrete, because the lateral reinforcing has given it additional strength. There is no inconvenience of the core of a cross section being stressed higher, since it is contracted by the material which envelopes it; furthermore, the increase of stress does not correspond to the whole weight of the concrete, as the centering and the core were acting together.

Conclusions.

I. — *With regard to the strength of material:*

Even at the present state of the manufacture of cements and the current methods of mixing concrete, by observing the rules and regulations governing granulometry and vibration, it is possible to consider spans up to 800 m for reinforced concrete bridges.

II. — *With regard to calculations:*

The present methods are precise enough to obtain for the whole length of the arch the maximum ligthness, compatible with the stability for which a diagram similar to that of Fig. 10 will give the size. It will be easy to verify the stability by multyplying the live load by an increasing coefficient: for example 2, and by ascertaining that the pressure line remains inside of the core, whereby the reinforcements are stressed only to the permissible limit.

The degree of indeterminateness of the coefficient of elasticity is low by the use of vibrated concrete of good granulometric composition.

The calculations are very rapid, due to the rules of similitude, and the tentative tests at the beginning are abolished, since a simple rule allows the determination of the average section for any working coefficient adopted.

III. — *With regard to construction:*

The realization of large spans finds its greatest difficulty in the study of the scaffoldings and their assembly. The relative expenditures can be kept in a non-prohibitive limit by a reduction of weight, to be applied to the scaffolding. This is obtained by the concreting in layers, for which the centering is lowered successively by the means of jacks. Verification has been made of the possibility of rigorously compensating, for a plate-webbed arch, the influence of a layer of arch on the one below. This compensation is not necessary by concreting in sections, devided by vertical joints.

IV. — The comparison of the arch, whose moments of inertia decrease from crown to springings with arches, whose moments of inertia are constant and with arches, whose moments of inertia increase from crown to springings, is to the advantage of the first mentioned: especially for large rise-span ratios, due to the small average section and the reduced thrust on the abutments.

The application of statically determined systems for large spans is not favourable, because of the difficulties of the hinges. With regard to a particular arrangement, made possible by the position of the hinges, we may add that for spans beyond 150 m, it is no longer advisable to compensate the bending moments by a triangular framework between the arches and the roadway; spans of this size should be bridged over by arches stable in themselves, either plate-webbed or trussed.

V. — It is economical to arrive at high strengths by mean of lateral reinforcement. This principle may be applied to its extreme limit for large spans; the longitudinal reinforcement is judiciously supressed or simply reduced to tie rods for the spiral reinforcements and the stirrups.

At the Bridge of La Roche-Guyon, their percentage weight has already been reduced to 0.5, that of the spiral reinforcements and the stirrups together being 1.3.

This reduction will dissipate all fears as to the increased compressive stress of the reinforcement, due to shrinkage and to the influence of the slow compressions of concrete, not fully known up to now.

A capital advantage of the lateral reinforcement is the high uniformity of strength.

Summary.

The first part of the Author's paper refers to the calculation of wide span reinforced concrete bridges. Various factors are to be considered for statically indeterminate arches, such as: the veriation of the moment of inertia, the shape of cross section, the span-rise ratio and the working stress. The Author studies the limits of spans and gives a comparison between different arches and the bridge of Roche-Guyon of which the results of calculation are stated. He describes further a number of three-hinged arch bridges recently built in France.

The second part of the paper deals with the execution and the false arch work of bridges, in particular the false arch work employed for the Roche-Guyon bridge. He gives a short description of systems recently employed.

IVb 2

Elimination of Bending Tensile Stresses in R. C. Bridges[1].

Ausschaltung der Biegezugspannungen bei Balken- und Stabbogenbrücken.

Compensation des efforts de traction engendrés par la flexion.

Dr. Ing. Fr. Dischinger,
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I. Historical Survey.

Since the beginning of the present century numerous scientists have devoted their attention to the subject of pre-stressing the reinforcing bars in reinforced concrete. The object of this idea was to attain

a) a reduction or the entire elimination of tensile stresses in concrete and therewith a reduction or the elimination of hair cracks detrimental to the life of the concrete. A reinforced concrete body subjected to compression only has, like natural stone, an almost unlimited life. This fact is proved to us by the old Roman constructions, in which both the stone itself and the mortar used as binder have withstood all the influence of time and weather up to the present day.

b) An increase in the permissible stresses. As is well known, the latter are very much less in the reinforcement of reinforced concrete than in all-steel constructions for the reason that reinforced concrete is a compound body in which the stresses have first to be transmitted to the steel by grip and shear. An increase of the stresses in the steel admissible for the combination of steel and concrete up to the limit of admissible stressing for all-steel constructions is not practicable owing to the excessive grip and also because of the numerous and rather large hair cracks that would thereby be caused. On the contrary, on this account the experienced reinforced concrete designer endeavours whenever possible to carry out his constructions in St. 37 instead of St. 52, the lesser stressing to which the former is subjected giving much greater safety against hair cracks.

The problem of a practical and economically satisfactory solution of this question is becoming increasingly important as the spans of our bridges and halls grow steadily greater, for the tensile stresses in the concrete become considerably higher in consequence of the rubbing of the tensile reinforcement. By pre-stressing the reinforcement compressive pre-stresses can be imparted

[1] Howe and Toreign Patents applied for.

to the concrete so that the bending tensile stresses are compensated and none or only very few remain.

Koenen, one of the earliest reinforced concrete specialists, was the first to recognise the value of pre-stressing. He carried out a number of tests along these lines with reinforced concrete beams and realised that the pre-stressed reinforcement bars embedded in the concrete again lose a considerable portion of the pre-stresses imparted to them owing to shrinkage of the concrete and the contractions thereby induced. The difficulty arising from this fact caused the principle of pre-stressing to be abandoned as impracticable for quite a considerable time.

In another domain of reinforced concrete, hewever — the two-hinged arch with tie, which is extensively used both in bridge and hall building — pre-stressing has been adopted to a very considerable extent. The author's own process was first put into practice in 1928, in the construction of the 68 m span Saale Bridge at Alsleben by Messrs. Dyckerhoff & Widmann A.G.[2]. As is well known, very considerable additional bending moments are set up in these two-hinged arches with ties on account of the contraction of the arch and the elongation of the tie. In respect to the contraction of the arch we shall first examine that due to compressive forces and shrinkage. The plastic influence of creep will be dealt with later on. By pre-stressing the tie with the aid of hydraulic jacks it is possible to produce similarity between the axis of the pre-stressed system, loaded by its own dead weight, and the projected line of the system. In other words, that the span-rise ratio l/f is identical in both systems, so that the additional bending moments due to deformations are eliminated. The tie must be shortened in just the same manner as the arch, i. e. we must stretch the tie not only to the extent of the elongation it has undergone owing to tensile forces, but also to the extent that the arch has shortened. To enable us to do this, a joint of the width of the contraction of the arch must be left open in the reinforced concrete decking, and only closed when the tie is stretched. By applying this pre-stressing process and removing the tie from the cross section of the reinforced concrete decking, not only the additional bending moments of the arch, but also the tensile stresses of the reinforced concrete decking can be eliminated. After stretching the anchors can then be concreted, so that they now form, in conjunction with the reinforced concrete decking, a common tie in respect to horizontal thrust caused by live load. Of course this horizontal thrust and live load produce tensile stresses in the reinforced concrete decking, but these are slight and can be eliminated as well by increasing the straining force of the anchors on closing the joint to such an extent that the compresssive pre-stresses there by produced in the reinforced concrete decking compensate the subsequent tensile stresses due to the horizontal thrust caused by traffic.

The process described has been widely adopted in recent years in the construction of arched aircraft hangars, particularly those with spans of over 100 m.

[2] *Fr. Dischinger:* „Beseitigung der zusätzlichen Biegungsmomente im Zweiggelenkbogen mit Zugband" (Eliminating additional bending moments in double hinged arch with tie member). Volume I, "Publiactions" of the I. A. B. St. E. — Do. „Beton und Eisen", 1932. — Do. "Science et Industrie", 1932.

For this purpose the ties, constructed of thick round steel bars, were placed in a trench and after straining were concreted to protect them against rusting, The ties must be strained during decentering.

As the pre-stressed system must be similar to the original when the additional bending moments arising from contraction of the arch under compression and from the elongation of the tie are entirely eliminated, and as the contraction of the arch is given by the ratio of the stress to the modulus of elasticity, only very slight lowering at the spindles is necessary when decentering. For instance, a tensile stress of the arch amounting to $\sigma = 60$ kg/cm² and a modulus of elasticity of the concrete of $E_b = 210000$ kg/cm² only involve a lowering of the crown of 1/3500. Owin to the fact that decentering is effected solely by stretching and pre-stressing the tie, and only to a very insignificant degree by lowering the falsework, the considerable bending stresses set up by the usual procedure of decentering the arch by means of spindles are almost totally eliminated.

The lowering of the crown by 1/3500 mentioned above, naturally refers only to elastic lowering during decentering. In this connection the factor of preep must also be considered. Creeping sets in immediately after decentering, and we shall discuss its influence in detail at a later stage. It, too, causes substantial contractions in the axis of the arch and consequently lowering of the crown and deformation of the system. The effects of these contractions of the arch on the flow of forces are, however, fundamentally different from those of shrinkage or of the elastic contractions of the arch. To my knowledge Dr. *Mehmel* was the first to make mention of this fact when the effects of creep were being discussed in connection with the large-span aircraft hangars mentioned above. Creeping is a purely plastic process. In the case of an arch formed exactly on the funicular curve, when the influences of the tie elongation and the elastic contraction of the arch have been eliminated, the latter will also represent a funicular curve after plastic deformation. Plastic deformation thus does not set up any bending moments, and the system acts as if it had been concreted in this form. The lowering of the crown caused by creeping is now only percetible in a slight increase of arch thrust, corresponding to the reduced rise. From this we see that creeping exerts an influence quite different from that of elastic contraction of the arch. The latter produced a reduction of the rise and thus additional bending moments in the arch, which must be eliminated by stretching the tie, while creeping causes a slight increase of arch thrust without inducing additional bending moments. When stretching the tie, therefore, we do not need to make allowance for creep, nor must we eliminate its influence by stretching the tie a single time, as this one operation would be of an elastic nature and would consequently set up bending moments. We would further mention the fact that creeping can induce other and quite different influences if the arch is not formed in the funicular line. The unequal distribution of compressive stresses over the cross section thus caused would induce various degrees of creeping for the various fibres, thus setting up bending of the system and in consequence additional bending moments which are difficult to deduce.

The above process, described in its application to an arch bridge constituting the special case of a strut frame, can of course be adopted for any other form of strut frame with curved compression boom and straight tension boom. In this connection the further development of the idea has been greatly facilitated by *U. Finsterwalder* who, when projects were invited for the Drei-Rosen-Brücke, a bridge in Basle (design Dyckerhoff & Widmann A.G.), Proposed a pre-stressed slab bridge. His project, as will be seen in Fig. 1, comprised arches cantilevering on both sides of the piers, which were encastered in the bedrock. The arch thrust of these overhung arches was to be taken up by pre-stressea straight-line cables connecting crown to crown. These high-tensile steel cables were to be pre-stressed by means of hydraulic jacks while the flasework was being released. This form of bridge asproposed by *U. Finsterwalder* is also a strut frame system with straight tension boom; the cantilever arch can be regarded as the inversion of a two-hinge arch bridge. The panels stiffening the arch extend to the level of the decking, i. e. of the tie, so that these strut frame

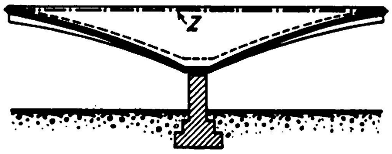

Fig. 1.

systems may also be regarded as beams cantilevering on both sides of the supports. Besides this project for the Drei-Rosen Bridge in Basle, *U. Finsterwalder* has also completed two other designs which have not yet become known in engineering circles, but which propagate the application of the above principle of pre-stressed strut frames to girder bridges.

II. Pre-stressed Girder Bridges.

Although in pre-stressed arches plastic deformations of the concrete due to creeping do not affect the pre-stressing, such is by no means the case with regard to girders. Creeping causes the mass of concrete to contract longitudinally just as do shrinking and compressive stresses, so that the stress drop in the pre-stressed bars is considerably increased by the effect of creep. For this reason *Freyssinet* proposed that for pre-stressing bars special steels of very high yield limits with correspondingly high stressing should be employed. In this manner he succeeds in greatly reducing the drop in stressing in relation to the original pre-stressing. *Freyssinet,* in the same manner as *Koenen,* uses straight bars for pre-stressing and the shuttering as abutments; the concreting is only done after the bars have been stressed. When the concrete has hardened the tensile force is transferred from the formwork to the hardened concrete. *Freyssinet* further provides for the pre-stressing of the stirrups, thus obtaining a concrete that is under compressive pre-stressing acting in all directions and can easily take up the bending stresses and shear occurring in the system. *Freyssinet's* process is certainly of extreme importance for factory-made products, especially in conjunction with his incidental proposals for improving the concrete by reducing creep[3].

[3] *Freyssinet:* Une revolution dans les techniques de Beton, Paris Libraire de l'enseignement technique Leon, Eyrelles, Editeur, 1936.

In large-span bridges and halls I find the application of the scheme involves certain difficulties — on the one hand because the pre-stressing forces are then so great that extremely strong formwork (of steel, for instance) would be necessary to act as abutment; and on the other hand, because straight pre-stressing causes equal moments to be exerted on the beams for each cross section, while the dead weight moments flow in curves. Difficulties are also to be expected at first when steels of very high yield limits are employed.

For this reason the author has worked along another line as regards the pre-stressing of large-span bridges, elaborating the ideas of the German patent DRP 535 440. The object of his endeavours is to effect pre-stressing even with normal St. 52 with elimination of the drop in stressing due to creep and shrinkage. In this manner freely supported reinforced concrete girder bridges of up to 80 m span, and through beams of up to 150 m span can be constructed in accordance with present-day stressing regulations and eliminating bending tensile stresses.

Before proceeding to describe these pre-stressed constructions, I shall touch upon the interesting subject of the sizes of spans achieved up to the present time. Today the maximum practical span for freely supported reinforced concrete girder bridges is about 30 m. Above this figure the dead weight of the structure increases extremely rapidly with the span, firstly because of the increasing depth of the beams, and secondly owing to the greater thickness of the web necessary to accomodate the large number of reinforcement bars. This rapid increase in dead weight quickly brings the structure to the limit above which reinforced concrete can no longer compete with steel, owing to the relative lightness of the latter in bridge construction. Owing to the greatly decreasing bending moments occuring in through beams or Gerber bridges, substantially greater spans can be attained with these types. The longest through beam in existence at the present time, the bridge over the Rio de Peixe in Brazil, has a span of 68 m. From a constructional and statical point of view it is of course possible to attain spans of up to 100 m. In this connection I would mention Prof. *Mörsch*'s[4] project for the Drei-Rosen Bridge in Basle, with spans of 56 — 106 — 56 m and my own unpublished project, submitted by the firms Dyckerhoff & Widmann, Wayss & Freytag, and Christiani & Nielsen, for a road bridge over the Süderelbe at Hamburg with spans of 64.5 — 103 — 102 — 103 — 64.5 m. The costs of these large-span reinforced concrete girder bridges were not very much higher than those of the steel bridges, but still the difference was sufficient to turn the scales in favour of steel.

Reinforced concrete girder bridges can only be improved in a competitive sense by pre-stressing; i. e. by the possibility of erecting pre-stressed structures whose dead weight is greatly reduced. The pre-stressing itself must meet the following demands:

a) The drop in the stressing of the above-mentioned anchors in consequence of the subsequent contraction of the concrete owing to shrinking and creep must be eliminated as completely as possible.

b) The possibility must be created of measuring the stresses in these pre-

[4] See „Beton und Eisen", 1931, Issue 13, p. 14.

stressed anchors at any time and adjusting them if necessary by means of a suitable stressing device.

c) It must be possible to carry out the pre-stressing operation with very simple resources and, with a view to attaining cheapness and simple, rapid construction, with round bars.

d) The tensile stresses in the concrete must, if possible, be entirely eliminated or at least reduced to such an extent that hair cracks cannot occur.

e) The pre-stressed bars must be of such a shape that they transmit the greater portion of the dead weight to the supports, thus relieving the concrete of its high shear and enabling web thicknesses of 30—40 cm to be used even for the largest spans.

f) Pre-stressing must be effected in such a manner that the cross sections of the reinforced concrete are utilised as much as possible on both edges and right up to the admissible maximum degree of stressing for dead and live loads.

g) A still more complete solution than that given in f) is obtained if we succeed in pre-stressing bridges in such a manner that only centric compression forces are exerted in the beams by dead weight, so that the girder system functions as a centrically loaded member. This would have the advantage that under dead weight only elastic but no plastic contractions occur in the beams, so that under dead weight such a bridge would not undergo deflection. Deflections would thus only occur under live load, and then only of an elastic nature, since the live loads would not be permanent.

To fulfil the requirements a) and b) we must remove the main reinforcing bars from the cross section of the concrete, as in the case of German patent DRP 535440, for only by doing so are we in a position to eliminate the drop in stressing to a degree corresponding to the tension or thrust set up, these are eliminated by measuring the stressing in the anchoring and adjusting same.

To fulfil the requirement c), pre-stressing must be effected with thick round bars and not with riveted steel sections. Unfortunately, these round bars are not available in the necessary lengths, but in resistance welding we possess today an absolutely reliable means of manufacturing, independently and at site, any desired lengths of round bars by welding shorter bars together. Resistance welding is certainly the most reliable method, and if, after welding, a fair length of the round bar on each side of the joint is made red hot, the joint can be subjected to a certain amount of upsetting and the cross section of the weld thereby enlarged. In this way, too, any self-stresses that may have been set up by the heavy drop in temperature can be obviated at the same time.

To fulfil the requirements from d) to g), the anchors must be constructed as hanging trusses. The form of these trusses must be suited as nearly as possible to the line of dead weight moments, i. e. the distances of the suspension boom from the neutral axis must be proportional to the magnitudes of the dead weight moments. If at the same time the cross section of the reinforced concrete is cleverly designed we can succeed in obtaining a construction in which, under live load corresponding to the requirement f), similar extreme fibre stresses are set up without bending tensile stresses, or in which, in accordance with the requirement g), the dead weight moments and the shear forces caused by dead weight are almost completely eliminated, so that the pre-stressed beam acts under dead weight loading as a centrically loaded column. Let us now examine two girder bridges of very large span in which these ideas have been put into practice.

1) *Gerber girder bridge with spans of 98.5 — 110 — 125 — 110 — 98.5.*

The bridge is shown in Fig. 1. All the suspended girders have a span of 70.0 m. The cantilevering girders, however, have a span of 110 m with 27.5 m cantilevers, giving a central bay of 125 m. The depth of the girders is assumed to be 5 m throughout. The depth-span ratio of the girders is thus 1/25 in the central bay and 1/22 for the cantilevering girders. Even in comparison with steel bridges these are extremely small girder depths and in spite of their great slenderness the deflections undergone by them under live loads keep well within permissible limits. The structure was calculated as a first-class bridge with an 8.5 m wide decking and two footpaths each 2.0 m wide. The dead weigth of the suspension girders, including the necessary cross beams and the weight of the pre-stressing bars removed from the section of the concrete, amounts to 29.65 tons, while the cantilevering girder, which also had to be provited with a lower compression flange on account of the varying live load, comes to 35.10 tons.

The unvarying live load on the suspension girder totals, when calculated according to German regulations, $p_1 = 8.5\ (0.525 - 0.70) + 40.5 = 5.87$ t/m. To these must be added the points loads, composed of a steam roller of 24 tons and two lorries of 12 tons. From this, deducting the unvarying live load, is obtained a substitute load of $P_1 = 27.5$ tons.

For the cantilevering girder with 110 m span the corresponding values came to $p_2 = 5.52$ t/m, $P_2 = 29.7$ tons.

For the cantilever arms, for which the total span of 125 m is taken according to German loading prescriptions, the corresponding values are $p_3 = 5.4$ t/m and $P_3 = 30.0$ tons.

In order to simplify calculation we shall assume — against regulations — that when determining the negative moments of the cantilevering girder the point load P can act at both ends simultaneously. The dead weight and live load moments occurring for this loading are listed in Table 1 and entered in Fig. 3; the dead weight moments are given on the right and the live load moments and maximum moments M_{max} and M_{min} on the left.

Table 1.

	M_g	$+M_p$	$-M_p$	M_{max}	M_{min}
Middle Suspension Girder .	+18200	+4080	—	+18280	+18200 tm
Middle Cantilevering Girder	+ 9600	+9160	—8075	+18760	+ 1525 tm
Above Support	—45200	—	—8075	—45200	—53275 tm

Figs. 4 and 5 show the suspension and the cantilevering girders with the anchors removed from the concrete cross section and placed between the webs.

Fig. 2

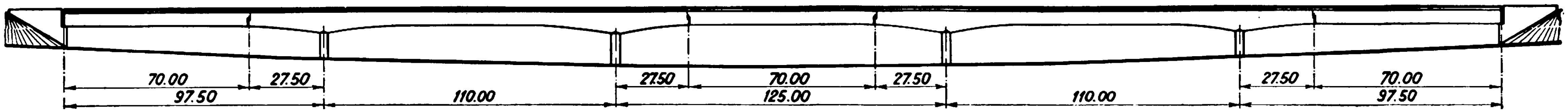

Fig. 3

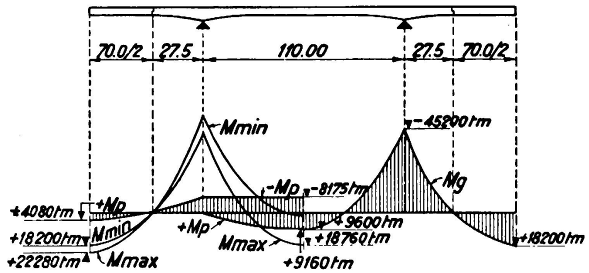

Fig. 4

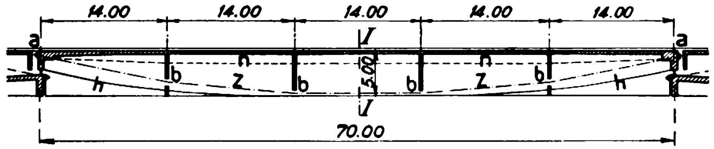

Fig. 7

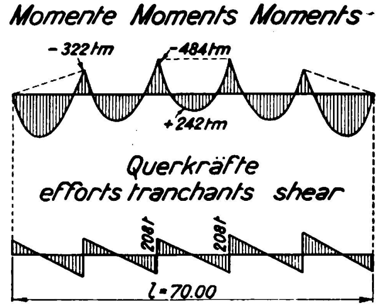

Fig. 5

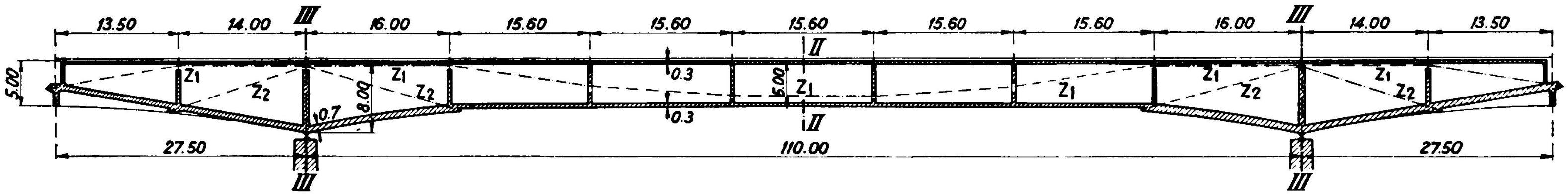

Fig. 6c III-III

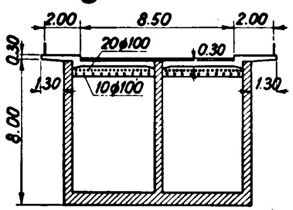

Fig. 6a I-I

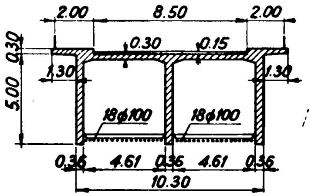

Fig. 6b II-II

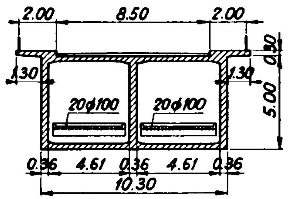

Fig. 10

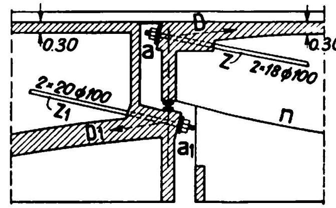

Fig. 2. Pre-stressed continuous hinged girder bridge.
Fig. 3. Bending moments due to dead weight and live loads.
Fig. 4. Longitudinal section of suspended girder.
Fig. 5. Longitudinal section of girder with cantilever arms.

Fig. 6. 6a. Cross section I—I through suspended girder.
6b. Cross section II—II through cantilever girder.
6c. Cross section III—III through cantilever girder.
Fig. 7. The moments and shear forces due to dead weight after pre-stressing.
Fig. 10. Detail of hinge.

Figs. 6a — 6c illustrate the shape of cross section in the middle of the suspension girder, in the middle of the cantilevering girder, and in the middle of the support. To begin with, we shall now discuss the effect of pre-stressing on the dead weight loading.

a) *Internal forces due to dead weight loading.*

The suspension boom of the suspended girder is composed of 36 bars of 100 mm $\varnothing$ = 2820 cm². The bars of the suspension boom are anchored at the ends of the suspended girder (a) stiffened decking slab. The ordinates of the suspension boom were so arranged that the total dead weight is transmitted by the boom to the supports, and for this purpose the reinforced concrete beam is supported on roller bearings or rockers at the bending-points of the suspension boom, so that longitudinal displacement can take place. Consequently the reinforced concrete beam is no longer continuously self-supporting for dead weight over a length of 70 m, but only over the distance between two cross beams. The latter are arranged at intervals of 14 m and in the form of through beams over five bays. In this manner the dead weight moments are reduced to about 1/40 and the shear forces drop to about one-fifth (see Fig. 7). The stretching of the suspension boom is effected by hydraulic jacks. The latter can be applied either at the anchor positions (a) to stretch the suspension bars longitudinally — for which reason, as already stated, they must be supported in a manner enabling them to be displaced longitudinally — or the jacks may also be applied at the bending-points of the suspension boom (b). In the latter case the stretching process is then effected downwards by the jacks to an extent corresponding to the elongation of the anchors caused thereby. This latter manner of stretching is only practicable as a general rule for suspended girders, i. e. for girders supported freely. Longitudinal stretching, however, is much more suitable for cantilevering girders. Fig. 8 shows the points of support, allowing longitudinal displacement, of the hanging truss in relation to the cross beams, Fig. 8a illustrating a roller bearing, Fig. 8b a rocker bearing and Fig. 8c a rocker bearing with interchangeable and longitudinally adjustable rocker, provided in case stretching is effected transversely to the direction of the tensile bars instead of longitudinally, by increasing the distances between the cross beams. The bends of the suspension bars are pulled down by the anchors (c) in Fig. 8c and also by means of hydraulic jacks, the latters being supported on the reinforced concrete beam with the assistance of auxiliary I-sections. In Fig. 9 the stretching device for the round anchors is shown in the form of hydraulic ring jacks.

The suspension bars are stressed while the bridge is being decentered. As a matter of fact the process is the reverse, for the straining of the anchors transfers the dead weight from the falsework to the hanging truss, unloading the falsework. Lowering the falsework by means of spindles is only necessary to the extent to which the wood has expanded on being unloaded. To transmit the dead weigth of the bridge to the suspension boom, the anchors must be strained up to 1900 kg/cm², giving for the reinforced concrete beam compression pre-stressing of 5350 tons. As the suspension boom is anchored in the decking slab, which has been strengthened for this purpose, the neutral axis

Fig. 8a

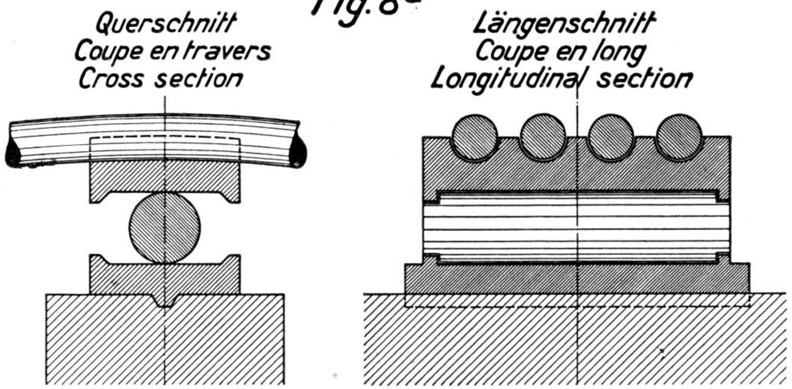

Fig. 8b

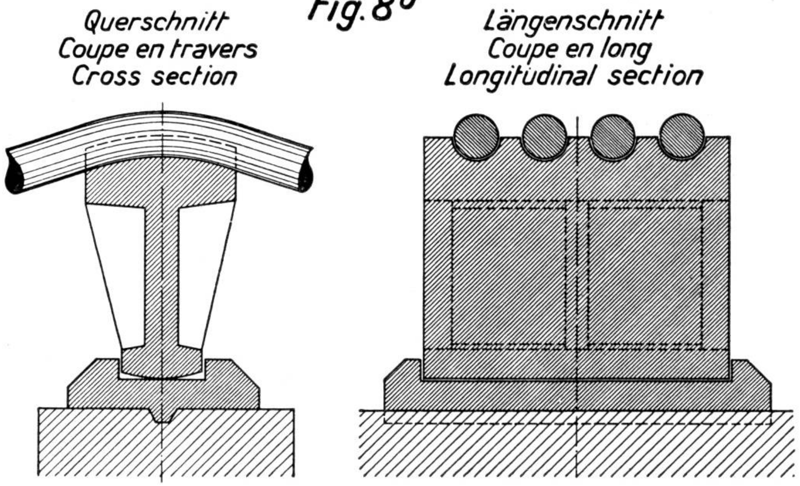

Fig. 8c

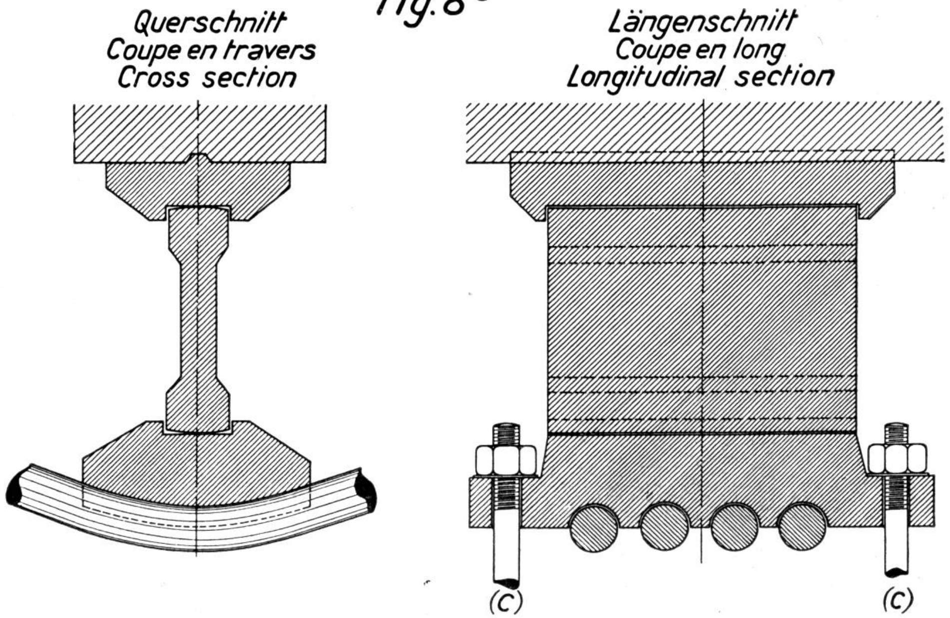

Fig. 9

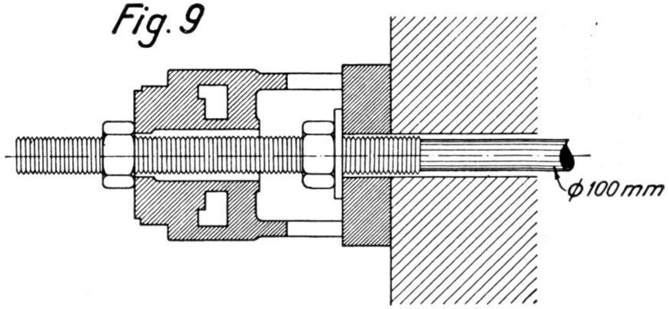

Fig. 8. 8a. Roller bearing for the support of ties.
8b. Rocker bearing for the support of ties.
8c. Rocker bearing for ties with removable rocker.

Fig. 9. Pre-stressing release arrangement.

must be very far up if a comparatively great degree of eccentricity of the compressive force is to be avoided. To do so, the lower part of the central rib is cut oïf, as shown in Fig. 4, and the lower portions of the outer webs prevented from contributing to the static action by means of transverse joints. The static action of the web surfaces below the line (h) in Fig. 4 is eliminated. But even if this should not be done, a portion of the webs would not be operative, because according to the strict theory of the disc, the law of proportionality cannot apply in the vicinity of the point of attack of the compressive force. Clearer static conditions, however, may be attained by eliminating the activity of these lower portions of the web. The suspended girder now assumes the static form of a Pauli girder. The advantage of this is that the beam under dead weight is only loaded by centric compressive forces. The position of the neutral axis (n) is shown in Fig. 4. From it we see that the distances between the suspension boom (Z) and the neutral axis are proportional to the dead weight moments, and consequently only centric compression forces act on the beam, as has already been stated. In order to obtain a greater lever arm of internal forces for the section of the support, with its very great negative dead weight moments, the co-operation of the decking slab was eliminated with the assistance of transverse joints. Thus the decking slab is not statically active; the fact that these portions of the cross section are not shaded (Fig. 5c) denotes this.

Fig. 10 shows the point of hinging. The dead weight of the suspended girder is transmitted to the supports of the cantilevering girder by the suspension boom (Z). This support reaction now resolves itself in its turn into a compressive and a tensile force. The compressive force is taken up by the 60 cm thick concrete compression slab of the cantilevering girder, the tensile force by the hanging truss (Z_1). The suspension boom (Z_1) consists of 40 bars of 100 mm $\varnothing = 3140$ cm^2 and extends over the whole length of the cantilevering girder, i. e. $110 + 2 \cdot 27.5 = 165$ m. But this boom alone is not strong enough to take up at its support the whole dead weight moment of 45200 t/m. For this reason it has been strengthened over a length of 30 m, at the places where the greatest negative moments occur, by the addition of an auxiliary boom (Z_2) consisting of 20 bars of 100 mm $\varnothing = 1570$ cm^2. The main boom (Z_1) only transmits the loads to the cross beams adjacent to the support, and these pass the loads on to the auxiliary boom (Z_2), which in turn transfers them to the main point of support. As the auxiliary boom is only half as thick as the main boom, it has been set at a greater incline, thus enabling it to take up the vertical loads of the boom (Z_1).

b) *The internal forces due to live load.*

As the pre-stressed reinforced concrete beam, exempt from bending tensile stresses, only undergoes deflections under live load, it is far more rigid than the hanging truss. Consequently a shear of no more than 411 tons is set up under live load in the boom (Z) of the suspended girder. Out of the total live load moment of 4080 t/m, therefore, only 1400 t/m is carried by hanging truss formed by the reinforced concrete beam and the tension boom, the remaining part, 2680 t/m, being carried by the beam alone. Thus the beam supports 65.5 % and the truss 34.5 %. In the cantilevering girders the portion carried by the truss is several times smaller. This is due to the fact that the truss cuts the neutral axis several times and is consequently only affected by the deflections of the beam to a very small extent. From these elucidations we see that on the whole the live loads must be transmitted by the pre-stressed beam. Reinforced concrete beams are very well suited to this purpose, since in consequence of the great compressive pre-stressing they are only able to take over the live load moments without undergoing bending tensile stresses by re-arrangement of the compressive pre-stresses. As the positive and negative moments in the middle of the bay are approximately of equal magnitude in the cantilevering girder, a lower compression slab had to be provided in this case also; this slab is of the same thickness as the decking slab. Thus the problem of reinforced concrete beam bridges without bending tensile stresses is solved by pre-stressing effected with hanging trusses.

c) *The effects of temperature changes.*

The influence of various changes of temperature on the reinforced concrete beam and hanging truss causes additional bending moments owing to the relative elongation or contraction of the hanging truss in relation to the beam. As the booms lie inside the beams, these temperature differences are very slight. German regulations provide for a temperature difference of $\pm 5^0$ C. The resultant influences on the stresses in concrete and steel are given in the following Table 2.

d) *The effects of creep and shrinkage.*

Pre-stressing of the hanging truss to enable it to take over the dead weight loads may produce compressive pre-stresses of over 50 kg/cm^2 in the reinforced concrete beams. Under the influence of these compressive pre-stresses the concrete undergoes an elastic contraction which is, however, without effect on the pre-stressing, since we could make allowance for them by shortening the anchors accordingly. On the other hand, special measures must be taken to counteract the contractions of the reinforced concrete beam caused by shrinkage and creep, since these influences only become apparent after decentering, i. e. after the anchors have been strained, and thus effect a considerable drop in stressing in the pre-stressed steel. These contractions of the concrete extend over a comparatively long period, particularly if poor mixtures — such with low fineness modulus of the aggregate — have been employed. The amount of creep is also to a great extent dependent upon the age of the concrete when first subjected to stressing, i. e. when the structure is decentered, and on the relative humidity of the air. This drop in stressing causes a partial re-arrangement of the dead weight loads of the hanging truss in its relation to the reinforced concrete beam. This detrimental effect can of course be reduced by keeping the creep of the concrete as low as possible. This can be done by employing rich mixtures, a good granular composition of the aggregate, by allowing the concrete to harden as long as possible before removing the formwork, by using high-quality cements and by thorough watering of the concrete (cf. also *Freyssinet*'s suggestions under 2) over a long period.

In spite of these precautions, however, there will always be sufficient shrinkage and creep to cause a very substantial drop in the stressing of the hanging trusses. Creeping and shrinking of, for instance, $40 \cdot 10^{-5}$ after decentering produces a drop in stress of 430 kg/cm² in the anchoring of the suspension girders and of about 700 kg/cm² in that of the cantilevering girders. For pre-stressing of 2000 kg/cm² in the hanging trusses this would mean that between one-third and one-fifth of the total dead weigth loads would be transmitted from the hanging trusses to the reinforced concrete beam. The pre-stressing might now be increased correspondingly when decentering is effected. But this would create opposed moments in the reinforced concrete beams. So these measures do not achieve the desired end. Even if the reinforced concrete beam could withstand such loading, the girder would undergo plastic deformations in consequence of these great moments. The right way is to employ straining devices of such a kind that they can be put into operation again to counteract any contraction that the beam has undergone owing to shrinkage and creep, the object being to obviate the drop in stressing that has already taken place and thus to raise the pre-stressing once more to the height originally calculated. The pre-stressing forces can be measured in various ways. We can measure them

1) with gauged hydraulic jacks,
2) with tensometers, applied directly at anchoring,
3) by measuring the deflection of the free-hanging, cantilevering truss between the points of bending.

As we have provided in the above project, by the form of the hanging truss and the shape of cross section, that only centric compressive forces are set up in the beam by dead weight load, so that the beam does not undergo deflection after decentering, we have still another way of regulating the amount of strain in hanging trusses.

4) As soon as the reinforced concrete beam shows a deflection of, for instance, 1 cm owing to shrinkage and creep, this is the sign of a certain drop in stressing which has a connection with the bending moments in the beam. We can eliminate these deflections by putting the hydraulic jacks in operation once more, now, however, we raise the beam not only as far as the neutral axis but beyond it, till it shows a negative deflection of 1 cm. In the course of time the beam will sink again in consequence of shrinkage and creep, and we repeat this subsequent stressing at ever-increasing intervals until creeping and shrinking have entirely disappeared. The constant variation of the deflection round about the neutral axis, serves the purpose of subjecting the reinforced concrete beam to plastic contraction only, but not to plastic bending, so that from the deflection of the reinforced concrete beam itself we can always deduce the magnitude of the forces in the hanging trusses. At the position of the neutral axis the actual stresses in the steel therefore correspond to those obtained by calculation.

As regards the cantilevering girders, the hydraulic jacks can be allowed to remain at the straining points (a_1) even after the bridge has been put into service, so that the drop in stressing caused subsequently by creep and shrinkage can be eliminated at any time. In the suspension girders, however, the straining points (a) are too high, and for this reason both initial and subsequent straining must in this case be effected at the bending-points of the hanging truss (b) by enlarging the distances with regard to the cross beams. This is effected by lengthening the interchangeable rocker in Fig. 8c. In the following Example 2 of a through girder, another method of re-stressing is shown, enabling re-stressing to be effected by longitudinal stretching without its being necessary to block traffic. In Table 2 appended the stresses resulting in the concrete have been given for cross sections I, II and III, in which the greatest bending moments occur.

Table 2.	Section I		Section II		Section III	
	σ_o	σ_u	σ_o	σ_u	σ_o	σ_u
Dead weight	— 51,3	— 51,0	— 46,7	— 46,7	— 53,5	— 53,5
Travic $+ M_p$	— 19,0	+ 26,7	— 42,3	+ 43,6	— 2,4	+ 0,3
$- M_p$	—	—	+ 37,2	— 39,8	+ 36,4	— 16,3
Temperature	± 1,3	± 7,4	± 0,1	± 3,6	± 10,1	± 1,6
σ_{max} for simple . . .	— 50,0	— 16,9	— 9,4	+ 0,5	— 7,0	— 51,6
σ_{min} live load . . .	— 71,6	— 58,4	— 89,1	— 90,1	— 66,0	— 71,4
σ_{max} for double . . .	— 50,0	+ 9,8	+ 27,8	+ 44,1	+ 29,4	— 51,3
σ_{min} live load . . .	— 90,6	— 58,4	— 131,4	— 129,9	— 68,4	— 88,7

For the simple live load there are no tensile stresses set up in the concrete, with the exception of the insignificant value of + 0.5 kg/cm² in Section II. It is only under double live load that such bending tensile stresses appear in the reinforced concrete beams as make the formation of hair cracks a possibility. The reinforcement of the actual reinforced concrete cross sections is so dimensioned that the reinforcing bars can take up the tensile forces emanating from the double live load. This, however, by no means exhausts the carrying capacity of the reinforced concrete structure. For as hair cracks appear a substantial reduction of the moments of inertia and the modulus of elasticity take place, and consequently the live loads are now taken up to an ever-increasing extent by the hanging truss construction, whilst the reinforced concrete beam is relieved of its load. Rupture will occur when both the steel of the hanging truss and, in the same manner, the reinforcing bars of the beam have reached the yield limit, i. e. the rupture point. Proceeding from the yield limit, calculation shows that the suspension girder ruptures at eightfold, the cantilevering girder at fivefold live load. The safety of pre-stressed bridges is therefore extremely high, and for the following reasons:

a) because the anchoring is only stressed up to 2100 kg, though 2400 kg would be admissible, including additional forces. The latter, caused by wind and braking force, however, are taken up in the case of very low stresses by the cross sections of the reinforced concrete.
b) Though bending tensile stresses are not set up under single live load, yet for safety's sake strong reinforcing bars have been provided for the beams, thus raising the safety coefficient.

c) The dead weight of these solid bridges is considerably higher than that of steel bridges, and consequently they are much less sensitive to increased live loads.

It is also interesting to study the deflections set up in these very slender bridges by live loading. They are calculated for a span 1 = 70 m at 1/3160, for the cantilevering girder, which is substantially more slender (span 1 = 110 m) at 1/1100 and for the 125 m central bay at 1/1000.

It would have been more economical, and above all of advantage from a statical point of view, if the slenderness ratio of the cantilevering girder of 110 m unsupported length had been taken a little lower, say at $^1/_{20}$, corresponding to a girder depth of 5.5 m. This would have reduced the deflection of the cantilevering girder and the 125 m central span to app. 1/1500. At the same time, however, there would have been considerably smaller variations in the stresses caussed by live load for the respective cross sections in Table 2, so that under double live load the safety factor against hair cracks would still have been quite adequate.

The most simple manner of protecting the anchoring from rust is to give it an asphalt coating and then wrap sacking round the steel. The anchor bars may also be encased in concrete, the slab being only connected to the rest of the reinforced steel structure at the points of anchorage and hanging free, as did the suspension boom in the previous example, between the points of bending. Concreting is best done after the bridge has been previously loaded, thus preventing the setting up of tensile stresses in the reinforced concrete slab, due to shrinking. When creeping and shrinking are entirely accounted for, the anchor bars can be so concreted that the new reinforced concrete slab is connected to the webs. The whole then becomes a uniform, monolithic reinforced concrete structure acting under live loads in a manner different from that of hanging truss structures — a fact that must be taken into account in calculation.

2) *Continuous girder bridges with spans of 100 — 150 — 100 m.*

For calculation we take the same cross section of bridge as in Example 1, and for the central bay of 150 m span the same girder depth of 5.0 m. For the end bays of 100 m span, which are very unfavourably loaded, a girder depth of 6.25 m is provided. The considerably greater moment of inertia of the end bays causes a substantial unloading of the central span. For the ordinary through girder the dead weight and live load moments are shown in Fig. 12. In addition to the externally statically indeterminate self-stress moments now come the internally statically indeterminate forces of the hanging truss. In order to keep the calculation from becoming too involved, and furthermore to attain clear static action, we must proceed with as few hanging trusses as possible. From these and other considerations we see the necessity of a hanging truss extending over the whole length of the girder, anchored at the ends of the beams and so designed that it is able to transmit all the dead weight loads to the supports. Just as in Example 1, the hanging truss must be displaceable longitudinally in relation to the cross beams, i. e. borne on roller or rocker bearings. Its shape must be such that the distances between truss and neutral axis are proportional to the dead weight moments.

Owing to the great difference between the dead weight moments at the supports and those in the bay, the girder depths for the support cross sections would be several times greater (see Fig. 13a). Thus for practical reasons this principle of a simple hanging truss extending over the whole length of the beam must be discarded and auxiliary booms (Z_2) provided for the portions of greatest negative moments, as in Fig. 13b. The thickness of these booms should be such that $M_g = Z_1 f_1 + Z_2 f_2$, the object being that the total dead weight moments are carried by the suspension boom. As Fig. 13 shows, this type of pre-stressing forms à combination of hanging truss and strut frame, for not only is the suspension boom subjected to bending, but also the beam, which acts as a compression element, is bent in respect to its neutral axis. By the pre-stressing as shown in Figs. 13a and 13b we have now succeded in eliminating all but centric compressive forces from the through beam under dead weight, apart from the slight bending moments for the transference of the loads from one cross beam to the other. The construction of the continuous bridge and the form of the hanging truss in Fig. 14 were executed in this manner. The bridge cross sections at mid-span are depicted in Figs. 15a and 15c, and the cross section of the support in Fig. 15b.

Since the hanging truss Z_1 extends over the whole length of the beam, very long round bars are necessary. The latter can of course be made at site by welding shorter pieces together, but they are extremely difficult to lay on account of their great weight. It is therefore advisable, instead of splicing, to use steel rockers (see Fig. 16) to connect these long bars at one or more places. This form of connection does not affect the forces in the booms. As the connections are in the interior of the reinforced concrete beam, the hydraulic jacks can remain in their position of application until creep and shrinkage has entirely abated and thus the final re-stressing effected. The influence of live load on the stressing of the continuous suspension boom is also very slight in the case of the through bridge, because this suspension boom intersects the neutral axis a number of times. The same applies, however, for the auxiliary boom (Z_2), because over the short distance this boom can hardly undergo bending from the live load in the reinforced concrete beam, in particular owing to the strong haunches. For the suspension girder (Z_1) 40 bars of 100 mm ∅ are necessary, and for the auxiliary boom 20 bars of 100 mm ∅. In spite of the much longer spans under consideration in Example 2, therefore, the same cross sections of steel would be sufficient. This fact is due to the favourable influence of continuity. To lower the position of the neutral axis in the range of negative moments and thereby to obtain good lever arms for the suspension booms, particularly at the cross sections of the supports, the activity of the decking slab would have to be eliminated as in Example 1 by means of transverse joints; these would be situated in the region between cross beams 6 and 10. The decking slab in this region is designed as shown in Fig. 17. The transition to complete elimination of the decking slab action must of course be effected gradually. This can be achieved in quite a simple manner by graduating the transverse joints.

In contrast to the *Gerber* girders, in the through girder the auxiliary boom (Z_2) was not anchored in the lower compression slab, but in a separate, intermediary slab in the vicinity of the neutral axis (see Fig. 18), thereby eliminating all

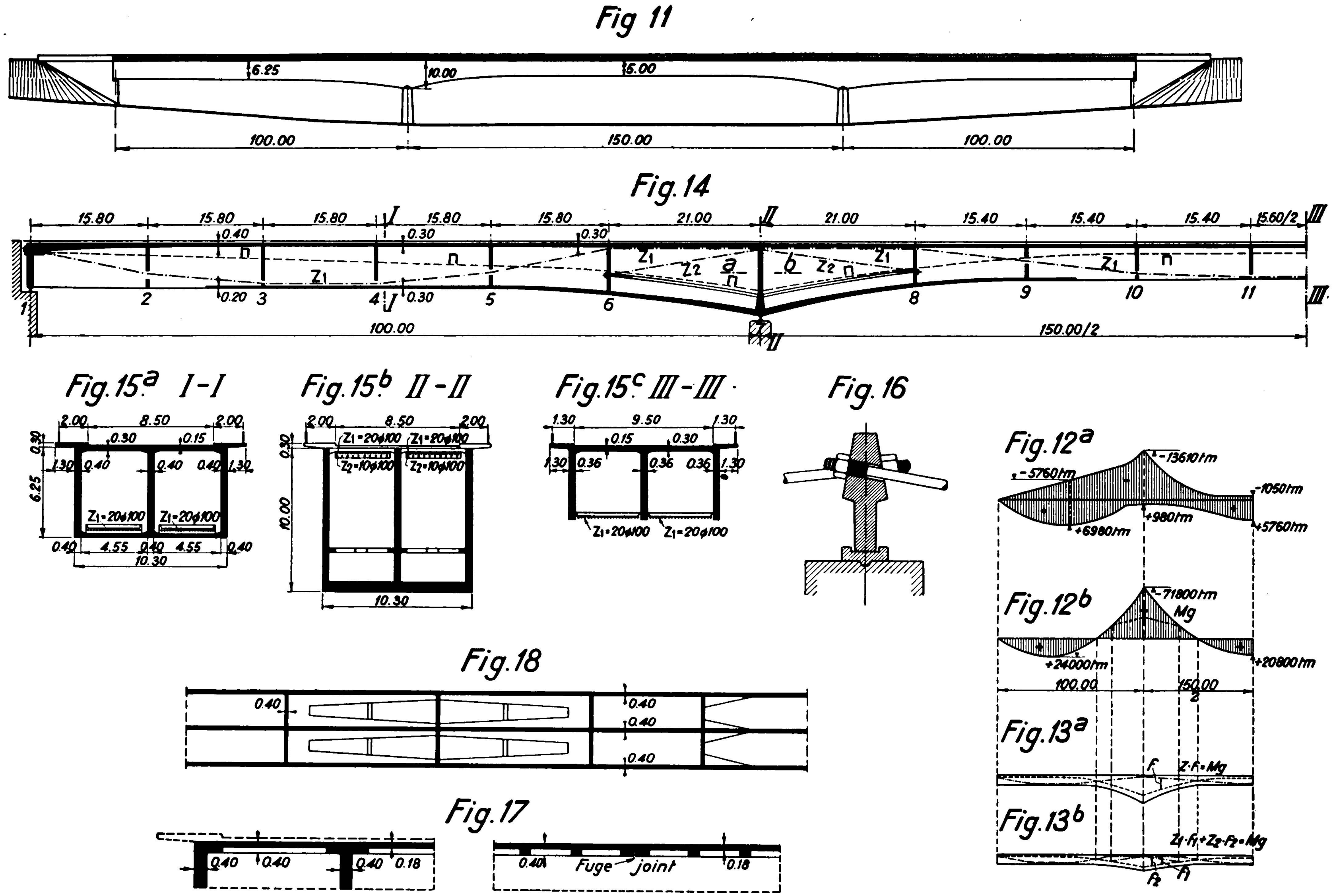

Fig. 11. Pre-stressed continuous girder bridge.
Fig. 12. 12a. Moments due to live load.
12b. Moments due to dead weight.
Fig. 14. Longitudinal section.

Fig. 15. 15a. Cross section I—I.
15b. Cross section II—II at support.
15c. Cross section III—III.
Fig. 17. Construction of bridge decking in the zone of negative moments due to dead weight.
Fig. 18. Intermediate slab for anchoring of auxiliary tie Z_2 (section a—b)

secondary stresses caused by eccentric application of forces. This solution also has the advantage that the high webs are given intermediate stiffening above the supports. Towards the latter the force of this slab is gradually transmitted to the webs, so that as the supports are approached the slab can be narrowed down, as shown in Fig. 18. The highest stresses in the two suspension booms again amount to 2100 kg/cm². The resultant stresses in the concrete of the respective cross section 4, 7 and 12 are given in Table 3.

Table 3.	Section 4		Section 7		Section 12	
	σ_o	σ_u	σ_o	σ_u	σ_o	σ_u
Dead weight	— 43,5	— 43,5	— 62,0	— 62,0	— 57,0	— 57,0
Traffic $+M_p$	— 22,9	+ 23,4	+ 28,5	— 18,0	— 27,5	+ 47,5
$-M_p$	+ 19,9	— 24,5	— 5,2	+ 3,9	+ 6,7	— 14,8
Temperature	± 0,7	± 4,0	± 4,3	± 1,7	± 2,1	± 10,1
σ_{max}	— 22,9	— 16,9	— 29,2	— 56,4	— 48,5	+ 0,6
σ_{min}	— 67,1	— 72,0	— 71,5	— 81,7	— 86,0	— 81,9

The distribution of stresses in these pre-stressed bridges is, as can be seen in Tables 2 and 3, the same as that with which we are familiar in arch bridges. In the latter the compressive stresses of arch action are superimposed by bending forces due to live load, temperature changes and shrinkage. In pre-stressed girder bridges the compressive force taking the place of arch action is artificially produced with hydraulic jacks by means of trusses. The only difference between the two systems is that the pre-stressed girder bridge possesses a greater degree of safety when the live load is increased. Arches with high piers have a safety coefficient of about n = 2.5 in respect to increase of live load. For flat bridges these values lie between n = 3 for solid arches and n = 6 for heavily reinforced hollow arches. The safety is higher for pre-stressed bridges because the suspension boom assists the beam under bending stress the more, the nearer we approach rupture. This also applies for the cross sections with varying live load moments, in which, for example, the suspension boom lies on the opposite side of the neutral axis for negative live load moments, for in the state of rupture the neutral axis is displaced to a considerable degree towards the edge, so that the suspension boom loses its lever arm in respect to the neutral axis and thus the negative moments set up by n-times the live load are counteracted by the dead weight moments, since the latter are no longer taken up by the suspension boom.

Comparison between the system of pre-stressed plate girder bridges and the arches discussed at the commencement of this article reveals an astonishing similarity in the action of the two. In the arch bridges pre-stressing eliminates bending moments under dead weight, which is due to the fact that by shortening the tie and suspension rods we have succeeded in creating similarity with the geometrical original when the system is under dead weight. It is only a little smaller, owing to contraction caused by compressive stresses. In the preceding paragraphs we have attained exactly the same end for girder bridges as well. Under dead weight the beam did not undergo bending, nor was it deformed after decentering because it was only loaded by centric compressive forces, just as in the case of the arch bridge. Only a shortening of the axis of the beam occurred, owing to its centric compressive forces and to shrinkage and creep.

Thus we have now found the basic principle for the hydraulic pre-stressing of reinforced concrete girder systems, and in the following paragraphs we shall outline the application of this principle for other types of bridges also. As there is not sufficient space available in this article, however, I shall confine myself to a very brief description and leave the details for another occasion.

In conclusion, the approximate dimensions of the two projects discussed may be quoted. For the suspended girder of the *Gerber* bridge about 0.9 m^3/m^2 is necessary; for this bridge as a whole an average of 1.23 m^3/m^2 and 370 kg/m^2 round bars. The through bridge with a span of 150 m needs the same quantities of concrete, but 400 kg/m^2 round bars; Prof. *Mörsch*'s project for the Drei-Rosen Bridge in Basle, with spans of 56—106—56 m required 1.63 m^3/m^2 concrete and 350 kg/m^2 round bars[5].

III. Pre-stressed Suspension Bridges and Bridges of the Bowstring Girder Type.

We shall first discuss suspension bridges with eliminated horizontal thrust, considering the stiffening girder under compression to be of reinforced concrete instead of steel and seeing what advantages and disadvantages a composite system of this kind possesses. As is well known, the deflections undergone by suspension bridges are very great. The maximum deflection depends on the ratio between cable tension due to live load, and the modulus of elasticity of the cable. For increasing dead weight of the stiffening girder the cable must be stronger, which causes the tension in it due to live load, and with it the deflection of the bridge, to be reduced. When the stiffening girder is executed in reinforced concrete double the dead weight of the bridge must be reckoned with, and consequently the deflections caused by live load are reduced by about one-half. Degree of rigidity is naturally not decisive in itself; the economic aspect of the bridge is of far greater importance. The cost of the cable will be doubled. This increase in cost, however, is offset by the saving that can be effected by using reinforced concrete, which is cheaper than steel, for the stiffening beam and the decking. For spans of up to 200 m I find that there is no doubt as to the greater economy of such composity-bridges. For longer spans, such as the 315 m span at Cologne-Mülheim, composite construction could only be considered for admissible stresses in the concrete of from 130 to 140 kg/cm^2. And as present-day high-grade cements attained strengths of about 600 kg/cm^2 there need be no hesitation whatever. Owing to the great moments of inertia of reinforced concrete hollow structures, slight girder depths for the stiffening beams are sufficient. One of these suspension bridges, having spans of

[5] The *first* pre-stressed bridge to be built on this system is at present in course of construction.

60 — 200 — 60 m, is shown in Fig. 19. The stiffening girder has a depth of only 3 m in the middle bay, i. e. 1/67 of the span. At Cologne-Mülheim the depth of the stiffening girder is 6.0 m, or 1/52.5 of the span. In spite of the fact that the stiffening girder is much more slender, the deflection under live load for the suspension bridge in Fig. 19 is only 1/725, while in the Cologne-Mülheim bridge this value comes to 1/400. In this connection it should be remembered that at Cologne-Mülheim the stiffening beam is designed as a *Gerber* girder with two hinges and is therefore operative in transmitting live loads to the suppports. In the suspension bridge shown in Fig. 19, on the other hand, there are three hinges in the centre bay, so that the whole dead weight of this bay has to be carried by the suspension cable alone. In the end bays the depth of the girder is 4.0 m because, as Fig. 20 shows, here greater bending moments occur. The form of cross section is illustrated in Fig. 21. The dead weight of the bridge amounts to 52.5 t/m in the centre bay, 63.5 t/m in the end bays, including the high-grade steel cable. Life load was calculated as 8.5 t/m. For a rise-span ratio of 1/9 of the cable in the centre bay, the horizontal thrust due to dead weight amounts to 11 800 tons and for live load to 1900 tons. The permissible cable tension of $\sigma_c = 5000$ kg/cm^2 is accordingly divided up into 4310 kg/cm^2 for dead weight and the very small figure of 690 kg/cm^2 for live load. This small live load stress has a direct bearing on the slight deflection mentioned above. The stresses in the concrete of the stiffening beam are the following:

	Centre Bay	End Bays
Due to Dead Weight	— 67.3	— 64.0
Due to Live Load	— 24.5	— 24.9
σ_{min}	— 91.8	— 88.9

Since the stresses emanating from live load as extremely small in relation to the compressive stresses due to dead weight, hair cracks need not be feared in the stiffening beam until the live load has been increased several times its original value.

The influence of expansion in the cable and contraction (compression) in the concrete, as well as the estimated influence of creep and shrinkage, on deflection, is best obviated (see Fig. 22) by cambering the suspended girder in the centre bay in the manner customary in steel construction. The same applies to the erection of the cable. However, the effect of creep and shrinkage is difficult to estimate mathematically, besides which it goes on over a considerable period of time. In order to eliminate these influences with certainty, therefore, the suspension rods must be shortened with the assistance of hydraulic jacks (H) to an extent corresponding to the progression of creep. The exact amount of shortening necessary can be determined at any stage owing to the fact that under dead weight the stiffening girder may be taken as being free from bending. It is free from bending when no deflection whatever occurs in it. The arrangement of the hydraulic jacks (H) for shortening the suspension rods can be seen in Fig. 20. These jacks are left at their place of application even after the bridge has been put into service, and remain there until such

Fig. 19

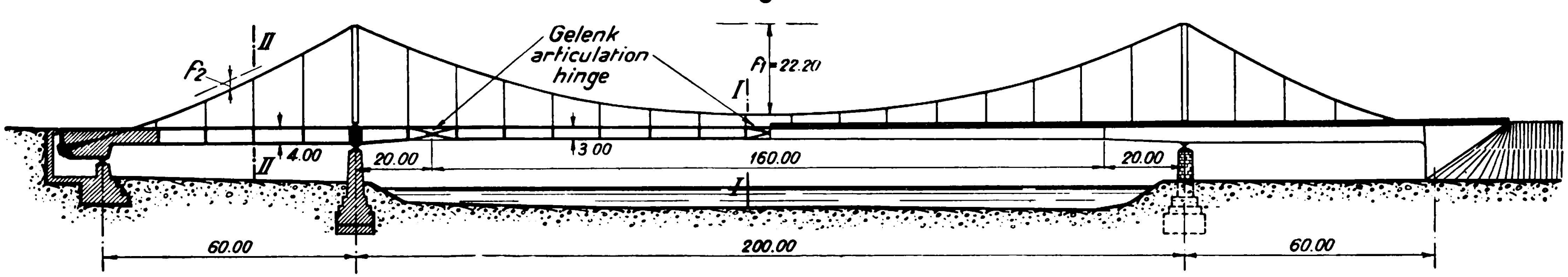

Fig. 20

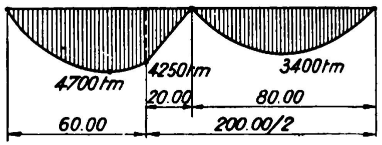

Fig. 21 a

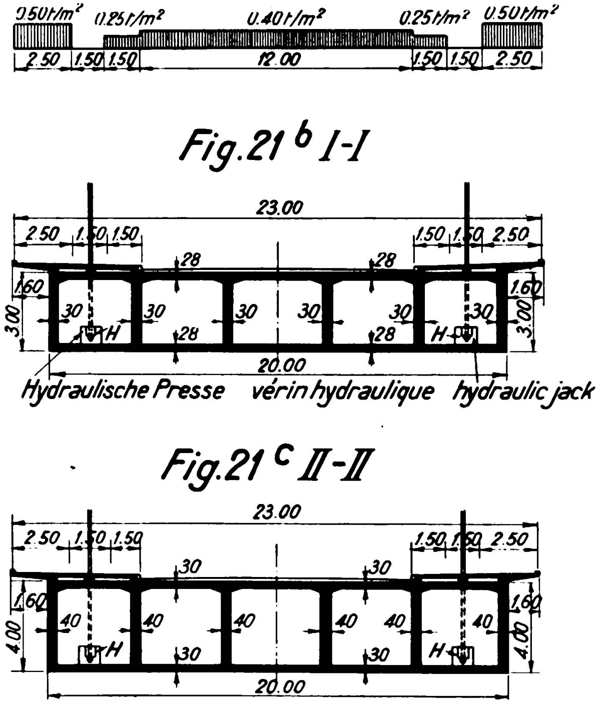

Fig. 22

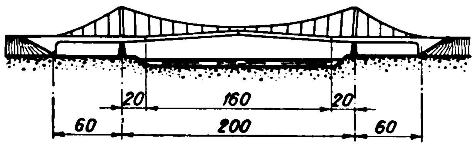

Fig. 23a

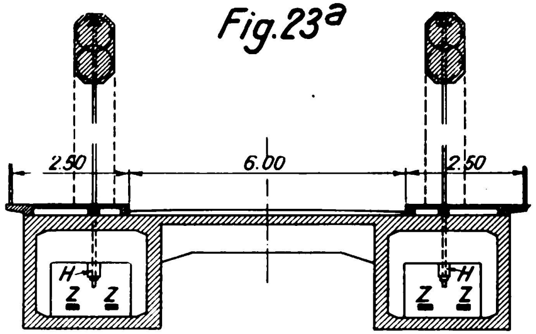

Fig. 23b

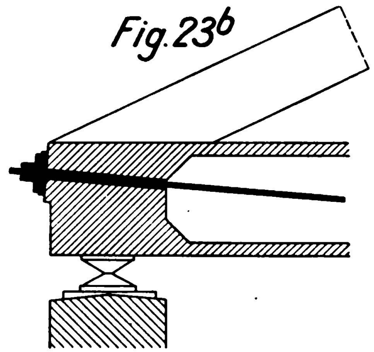

Fig. 19. Longitudinal section and elevation.
Fig. 20. Diagrams of moments due to live loads.
Fig. 21. Sections, arrangement of live loads.
Fig. 22. Erection scheme.
Fig. 23. Pre-stressed girder bridge with stiffening arch.

time as creep and shrinkage are completely over. By subsequently closing the centre joint the stiffening girder could also be made to transmit a portion of the live load, as is customarily done in steel construction, and the deflection of the bridge due to live load still further reduced. However, this is not necessary, for the deflections of these suspension bridges in composite steel and concrete are in themselves extremely slight.

In conclusion mention should be made of pre-stressed bowstring beams in reinforced concrete. Pre-stressing should be effected in such a manner that the stiffening girder under dead weight loading is again entirely free from bending moments. Consequently we must see to it that after decentering the system resembles the geometrical original projected. The bowstring girder under compression contracts by the amount $\frac{\sigma}{E}$. Now, however, we must also shorten the stiffening girder and suspension rods to the same extent by pre-stressing. For this purpose it is necessary first of all to leave an open joint in the stiffening beam, which is only closed when the tie is strained. Fig. 23 shows the cross section of a bowstring type bridge of 100 m span for a rise-span ratio of $b/1 = 1/7$. The bowstring is spirally reinforced, the object being to keep the depth of section as small as possible. The stiffening beam is composed of two hollow sections which have to take up the bending moments. The dead weight of the bridge is 27 tons, the live load 6.0 t/m. This yields a horizontal thrust of $H_g = 2380$ t, $H_{g+p} = 2910$ t. Accordingly, for the tie $2910/2.1 = 1380$ cm^2 is necessary. However, we go up to 2000 cm^2. By straining the tie we first close the joint, already mentioned, left open in the stiffening beam; the tie is then further stressed up to 2400 kg/cm^2, producing in the stiffening girder a compressive stress of $2000 \cdot 2.4 - 2380 = 2420$ tons. This compressive force in the stiffening girder corresponds to a compressive force of 38 kg/cm^2. When the live load is unfavourably placed a tensile stress of 25 kg/cm^2 is produced in the stiffening girder, so that the latter is also free from bending stresses under live load. Owing to this high compressive pre-stressing the stiffening girder is able to take over about $2^1/_2$ times the live load before hair cracks appear.

A brief account of the effect of creep and shrinkage must now be given. The stiffening girder is shortened thereby, and a drop in stressing, which can be measured exactly on the sag, takes place in the freely hanging tension boom, which runs the whole length of the bridge. We obviate this drop in stress by re-stressing with the hydraulic jacks, which are permanently in place. The effect of creep is very much greater in the bowstring girder, so that the arch is consequently flattened somewhat. The resulting deflection of the stiffening girder must be counteracted by shortening the suspension rods in the same manner as for the suspension bridge discussed above.

We have thus demonstrated that in nearly all reinforced concrete structures it is possible, by hydraulic pre-stressing of the tensile elements, to obtain similarity between the system under dead weight and the geometrical original concrete. Reinforced concrete beams and arch bridges possess an almost unlimited life when subjected to compression only. This is also true, though in a much less degree, of steel tensile bars under stressing that does not increase to any great extent. Tensile bars made of steel can easily be renewed in time to come; indeed, in girder bridges this operation can even be carried out without putting the bridge out of service.

The process of pre-stressing reinforced concrete girder systems, as described above, may also be applied to other types of structure, and particularly in hall construction. By employing it, halls of up to 100 m span can be constructed. I shall refer to this point again in a later publication.

Now that it is apparent how simply the drop in stress caused by creep and shrinkage can be eliminated, there need be no further doubts as to the advisibility of using cables of high-grade steel for pre-stressing instead of St. 35. These cables have the advantage that fulfil requirements with much less weight and smaller cross sections. The anti-rusting property of the cables is extremely good, and they can be stretched before erection.

Summary.

Proceeding from systems of pre-stressed arched and strut frames with pre-stressed ties, it is shown that in the case of girder bridges, suspension bridges and those of the bowstring type, it is also possible to eliminate bending stresses in the reinforced concrete and furthermore to exclude all but centric compression forces occurring even in girder bridges under dead weight. The main point of the problem is so to choose the method of pre-stressing that, after decentering, the system loaded only with its dead weight is similar to the geometrical original projected; in other words, that in the system under dead weight loading no deformations of any importance are caused, although contractions due to compression stresses do occur.

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IVb 3

Wide-span Reinforced Concrete Arch Bridges.

Weitgespannte Eisenbeton-Bogenbrücken.

Arcs à grande portée, en béton armé.

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I. General remarks. Properties of materials. Working stresses. Shape of cross section.

The construction of long span arched bridges in reinforced concrete, and the possibility of further increasing their span, is governed by a variety of considerations. In the first place great importance attaches to the properties of the cement, the strengths obtained in the concrete and the behaviour of the latter (as regards shrinkage and plastic deformation, etc.) after the arch is completed. The form of cross sections adopted for the arch, the relation between the arch and the decking construction from a structural point of view, the ratio of rise to span, the deformation undergone by the arch, the manner of construction and removal of shuttering, and the false arch work, are all matters that demand attention.

Each of these factors is subject to some limit which is used as a theoretical basis for the design of the arch to guarantee the latter against failure. The various factors all become considerably more important than is the case in bridges of medium span and this makes it necessary that all the operative influences should, like the bearing value of the ground, be rigorously checked. Up to a certain point the permissible stresses set a limit to the increase in span, and the answers to the various questions that arise in the construction of long-spanned bridges in reinforced concrete must, therefore, be gathered from investigations of theory, of practice, and of testing technique.

In this paper new suggestions are put forward for the stricter calculation and execution of long span reinforced concrete bridges, and a description is given of the author's design for a bridge of 400 m span for which a new method of construction is proposed. In addition only a few of the effects mentioned above as being relevant to the design and construction of long span bridges are discussed in greater detail. The question of economic comparison with steel arch bridges of large spans is not dealt with.

a) Properties of materials.

One thing is certain: if reinforced concrete arch bridges are to be built with still greater spans this can be done only through the use of concrete of considerably higher strength than hitherto, and this in turn implies high-quality cement. It may have been possible, up to now, to equalize the extreme fibre stresses by the adoption of special procedures for relieving the arches, and by this to ensure a better distribution of stress over the arch as a whole; future again looses its importance, because the dead weight of the bridge itself considerably increased, but if so its tensile strength will have to be increased also.

With very large spans, however, this increased tensile strength of the concrete again looses its importance, because the dead weight of the bridge itself considerably exceeds the live load and if the rise is great enough it becomes possible, by adopting a suitable design, to obtain a purely "compression arch".

It is important also to endeavour that the consistency of the concrete should be kept as uniform as possible, although it is impossible to avoid climatic effects on concrete of varying age in the arch. In this paper uniformly worked concrete will be assumed for the purpose of the mathematical investigation of the arch.

By the use of special cements it is already possible to obtain compression strengths in the concrete as high as 600 kg/cm²; in the Traneberg bridge, for instance, the concrete was made with 400 kg/m³ Portland cement and this gave a compressive strength of 620 kg/cm². "Ciment Fondu", which is a rapid hardening cement, appears particularly well adapted to the purpose: in France, by adding 300 kg of cement to a mixture of 1200 liters of sand and gravel, this gave an elastic modulus of 350,000 kg/cm² after 7 days and 450,000 kg/cm² after 28 days (Lossier, Génie Civil, 1923/II, p. 205). These cements have a shrinkage figure of 0.4 mm/m after 30 days and 0.5 mm/m after 6 months, which, however, is greater than the corresponding figure for ordinary Portland cements.

According to No. 227 of Research Work in the Field of Engineering (Forschungsarbeiten auf dem Gebiete des Ingenieurwesens, Beton und Eisen, 1923, p. 4), Graf obtained the following results for elastic moduli of concrete:

Strength at 28 days:	300	400	500	600 kg/cm²
E_b	360,000	418,000	440,000	463,000 ,,

It is clear from these figures that high cube strengths may be associated with high values for elasticity, and the application of the latter to the construction of arches will be mentioned later.

b) Permissible stresses.

On the basis of the foregoing, the permissible stress in the concrete may be increased to 200 kg/cm² assuming that a suitable proportion of high-strength cement or special cement is used. Naturally, however, if stresses of the order of 150 to 200 kg/cm² are to be allowed in the concrete, the repercussions of these high values on the remaining properties of the concrete and particularly on the elastic modulus must first be ascertained. The detailed calculations for the design of an arch of 400 m span which are given here in Section VIII indicate the possibility of working to a permissible stress of 160 kg/cm², and for spans of less than 400 m — unless the arch is particularly flat — an even lower value will suffice. Dischinger has succeeded in designing a three-hinged arch of 260 m span, and the exceptionally low rise ratio of 1/15.4, with a permissible stress in the concrete of 150 kg/cm² (Bautechnik, 1934, p. 658). Freyssinet, in designing an arch of 1000 m span, went up to a permissible stress of 280 kg/cm² — a value which, for the near future at least, appears rather high, but 200 kg/cm² might be justifiable even at present.

c) Form of cross-section.

It is clear that the hollow form of cross section for the arch — or in very long spans possibly an open frame arch — are the only types calling for consideration, since the heavy extreme fibre stresses are confined to the top and bottom slabs of the cross section. Whether Freyssinet's "béton traité" involves a qualification of this statement is a question which can be decided only when more detailed information in regard to it is available.

In the construction of arch bridges of long span for which the design follows the pressure line, it is possible to reduce or partially to equalize the maximum stresses under consideration of the three following points:

1) By establishing an axis deviating from the pressure line but keeping the rise unchanged.

2) By consideration of the deformation theory as means for calculation, whereby the rise may undergo a slight change.

3) By special modi operandi, involving the use of hydraulic jacks, when striking the false arch work.

Point 1) is to be chiefly considered for small bridges, and either point 2) or points 1) and 2) together in the case of long span arches.

II. Reduction of maximum stress in arch bridges by adjusting the axis of the arch (elastic theory).

An adjustment of the axis of the arch to take account of the elastic compression suffered by the latter may easily lead to nil values being obtained for the incremental bending moments at the springing and crown, but, if so, it will also lead to greater moments than exist in an unadjusted arch at a distance of about $^1/_6$ l in the case of a two-hinged arch or at distances of $^1/_{12}$ and $^1/_3$ l in the case of an encastré arch.

In an fixed arch the compression of the arch axis causes an additional horizontal thrust Δ H, and additional moments which are given by

$$M_x = - H\eta - \Delta H(y - \eta) + \Delta M$$

the latter being particularly important in the case of flat and stiff arches. If the arch is assumed to be cut at one end it is possible to establish factors K for the correction η of the arch axis, by calculating the horizontal displacement of the free end of the arch due to the loading $\left(g + \frac{p}{2}\right)$, change of temperature, shrinkage and plastic deformation.

The correction η of the ordinates y ot the arch axis can be calculated from a function $\eta = KF(x)$ wherein F(x) represents the equation for the thrust line. For parabolic axes this is a function of the second degree; for arches conforming in shape to the line of thrust it is a function of the fourth or higher degrees or may even be an angular hyperbolic function. The additional moments to be added at the springing and crown remain zero at all these cases, if the correction is chosen: $\eta = 0$.

The maximum values of the additional moments can be reduced if $\eta = \alpha K F(x)$, wherein $\alpha < 1$. Small moments will then be set up in the crown and springing and this leads to a better distribution of the additional moments.

The problem is to determine the amount of adjustment which will cause the additional moments — including those due to the least favourable position of the live load — to be a minimum. This problem is of an indeterminate character since there is a free choice of both α and $F(x)$.

In the case of an encastré arch the adjustment expressed by this function is subject to the condition that the adjusted axis intersects the original axis at the level of the elastic centre. It is impossible, however, to compensate the moments completely at all points of such an arch, and this fact has its analogy where the striking of the false arch work is carried out with the aid of hydraulic jacks.

The solution $\eta = K F(x)$ is due to Campus (International Congress on Reinforced Concrete, Liége 1930, p. 163)[1] and reference may also be made to Chwalla in Mitteilungen des Hauptvereins Deutscher Ingenieure in der Č.S.R., 1935. A different solution has been put forward by M. Ritter (International Congress on Bridge Construction, Zürich 1926), whereby the axis of the arch is determined according to the line of thrust due to dead load with the addition of virtual loads acting upwards, and the moments and normal thrust are calculated by the complementary force method of Mörsch. The values for these virtual loads are found from the pre-determined position of the axis of the arch at the springings and crown. The additional loads occur between the zero points of the summated influence lines appertaining to two symmetrically placed point loads, or in the case of flat arches Ritter makes use of distributed loads. In consequence of the virtual loads the elastic centre of gravity comes a little higher than it otherwise would.

Other methods are described in the literature here cited[1]. Generally speaking, it may be said that the value of η is arbitrary, and that the degree of improvement that can be attained, in pursuit of economy in material, depends on the greater or lesser approximation to the ideal.

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III. Closer calculation of arches and deformation theories.

If the permissible stresses in arch bridges of long span are to be fixed as high as is envisaged here, it follows that especially rigorous methods of calculation must be applied — taking account, for instance, of the real values for the elastic properties of the concrete. This requirement is in no way inconsistent with the fact that the calculated stresses are mere approximations and not mathematical quantities, for the method of dimensioning to be adopted is one of the relevant factors in the problem, and by more rigorous scrutiny of experiments carried out on large bridges it will be possible to build up a progressively clearer understanding of the conditions actually obtaining in arches of this kind.

Known methods of calculation will, therefore, be discussed, together with new investigations on the part of the author.

1) *Calculation of arched bridges by the exponential law.*

The quest for a closer understanding of reinforced concrete arch bridges through the Bach-Schüle exponential law, by applying this in conjunction with elastic theory, leads to calculations of some complexity. For small spans, as Dr. M. Ritter has shown (Schweizerische Bauzeitung 1907/I. p. 25), there is no occasion to apply the exponential law as variations in the extreme fibre stress amount to 2.5% at the most, and are on the safe side $\left(\frac{f}{l} = \frac{1}{10}\right)$.

As yet no numerical evaluations have become known for large spans, but it may be anticipated that their results would show large deviations from those obtained by the ordinary methods of calculation for fixed arches.

The equation for elastic deformation $\varepsilon = \alpha \cdot \sigma^m$ (taking m = 1.1—1.14) results in greatly increased values for elastic deformation at the crown of an fixed arch, and this fact is important from the point of view of more rigorous investigation. Straub (Proc. Am. Soc. Civ. Eng., Jan. 1930), considering small spans with an excessively high value of m = 1.3, obtained somewhat large deviations of the deformation angles and deflections by comparison with m = 1. With m = 1.3, however, the sum of the angular changes worked out at practically zero, as is true when m = 1. The horizontal displacements of the ends of the arch resulting from the compression of the latter were considerably larger with m = 1.3 than with m = 1. (Straub: Trans. Am. Soc. Civ. Eng., 1931, p. 665.)

With the arch fully loaded the line of thrust, taking m = 1.3, comes closer to the axis of the arch (assumed by Straub as a parabola in all cases). Non-uniformly distributed loads have a greater effect on the shortening of the axis with low values of m than with high values.

Straub's treatment is given for a generalised form of arch of rectangular cross section. In view of the parabolic shape of axis assumed in his illustrative examples, the conclusions reached are valid only for flat arches.

2) *Assumption of an elastic modulus which is uniform over the cross section but varies along the axis of the arch.*

A method which is simpler to apply than the exponential law and is more practical even for large spans — though not quite so accurate — is to assume the validity of Hooke's law with a variable elastic modulus in successive elements of the arch. Such a variation can be justified by the length of time that elapses in the concreting of the arch and, therefore, in the different age of the concrete as between the springing and the crown. Again, the measurements made by Prof. Dr. Roš on the Baden-Wettingen Bridge (Schweiz. Bauzeitung, 1929/I, 2nd March) disclosed variations in the elastic modulus across the bridge: in this pure arch structure the value for E_c was 343,000 kg/cm² at the springing and 284,000 kg/cm² at the crown, but it was not found possible to establish a law governing the variation. The elastic modulus determined from the mean of the extreme fibre stresses was smaller at quarter span of the arch than at either the springing or the crown. The eccentricities of the line of thrust as measured were throughout smaller than as calculated.

In the Hundwiler arch E was 541,000 kg/cm²; at the quarter span points the values found from the stresses at the intrados were 725,000 and 624,000 kg/cm² respectively, comparing with 362,000 kg/cm² at 9 weeks as determined in the laboratory. (Schweiz. Bauztg. 1929/II. 10th Aug.).

It is not clear how far these differing results of measurements may be attributable merely to incidental deviations. At present the measured results at the quarter span points vary too much to admit of systematisation, though the measurements of elongations at these points are fairly consistent. It would be valuable to obtain a check on the determination of E by applying the exponential law to the measured deformations.

The simplest and best grounded assumption for the variation of the elastic modulus — a uniform quality of concrete being assumed — is to relate this quantity to the age of the concrete and to the length of time needed for building the arch. In this way the elastic modulus at the springing E_K works out higher than that at the crown E_S and the change can be taken as linear over half the length of the arch. E_K and E_S can be determined from preliminary experiments.

For an encastré arch the consequence of this assumption is to raise the elastic centre of gravity. The author has obtained the following values of moments for the centre of gravity of sections in an arch of 400 m span with 1:4 rise (shaped to the line of thrust):

With permanent load of 1 ton per m over whole span:

At springing, + 381.56 tm, as against
+ 374.7 tm, when E is constant
$\Delta = +1.8\%$.

At crown, + 119.76 tm, as against
+ 134.7 tm, when E is constant
$\Delta = -11.2\%$.

With load of 1 t/m over half the span:

At left-hand springing, — 2210.00 tm, as against
— 2092.6 tm, when E is constant
$\Delta = +5.8\%$.

At right hand springing, + 2549.56 tm, as against
+ 2467.3 tm, when E ist constant
$\Delta = +5.2\%$.

At crown, + 59.88 tm, as against
+ 67.35 tm, when E is constant
$\Delta = -11.2\%$.

The ordinates of the influence lines for the crown when the arch is cut at the crown, work out as follows:

$X = 0.942$ t (as against 0.923 t)
$Y = 0.500$ t (as against 0.500 t)
$Z = 50.800$ tm (as against 53.400 tm)

corresponding to

$E_K = 470{,}000$ kg/cm²
$E_S = 350{,}000$ „
$E_m = 410{,}000$ „

This shows that, under consideration of a variable E, the moments at springing become greater, and those at the crown smaller, than for a constant value of E.

3) *Variation of the elastic modulus in an arch girder of hollow cross section.*

The construction of large arch bridges in reinforced concrete proceeds by first concreting the lower face, possibly with parts of the walls, over the whole of the span, and the remaining portions of the cross section which then follow are built in the same sequence from the springing towards the centre. Owing to the intervening lapse of time the elastic modulus will, therefore, vary over the depth of the cross section, being greater towards the bottom and smaller towards the top.

If methods of de-centering which have the effect of reducing the maximum stresses are applied to the arch, due regard must be taken of these different elastic moduli, since, when the arch is closed, the weight of the decking and its supports as well as the stresses due to temperature, shrinkage, plasticity and live load will all be imposed on the arch itself.

The relevant calculation will now be given in reference to an encastré arch. E_1 and E_2 represent the mean values of the elastic moduli at the intrados and extrados respectively and the transition from the one to the other is assumed to follow a straight-line law. The hollow cross section has a total depths 2v and is symmetrical about the horizontal axis.

Writing $K_1 = \frac{E_1}{E_2} + 1, \qquad K_2 = \frac{E_1}{E_2} - 1, \qquad K = \frac{E_1 - E_2}{E_1 + E_2};$

we have for the angular change γ

$$\tan\gamma = \frac{ds}{2vE_1}\left[\frac{N}{F}K_2 + \frac{Mv}{I}\cdot K_1\right]$$

$$\Delta\, dx = \frac{ds\cdot\cos\varphi\, K_1}{2E_1}\left[\frac{N}{F} + \frac{Mv}{I}\cdot K\right]$$

$$\Delta\, dy = \frac{ds\cdot\sin\varphi\, K_1}{2E_1}\left[\frac{N}{F} + \frac{Mv}{I}\cdot K\right]$$

and the three unknowns are girder by setting $Q_o = \sum_x^{\frac{1}{2}} G$

$$H = \frac{\int\frac{M_o y}{I}\cdot ds - K\int\frac{M_o v\, ds\cos\varphi}{I} + \int\frac{Q_o\, ds\sin\varphi\cos\varphi}{F} - K\int\frac{Q_o y\, ds\sin\varphi}{Fv} + \frac{2E_1\omega t}{K_1}}{\int\frac{y^2 ds}{I} + \int\frac{ds\cos^2\varphi}{F} - K\left[\int\frac{v\, y\, ds\cos\varphi}{I} + \int\frac{y\, ds\cos\varphi}{Fv}\right]}$$

$$V = \frac{\int\frac{M_o x}{I}\cdot ds - K\int\frac{M_o v\, ds\sin\varphi}{I} + \int\frac{Q_o\, ds\sin^2\varphi}{F} - K\int\frac{Q_o x\cdot ds\sin\varphi}{Fv}}{\int\frac{x^2 ds}{I} + \int\frac{ds\sin^2\varphi}{F} - K\left[\int\frac{v\, x\cdot ds\sin\varphi}{I} + \int\frac{x\cdot ds\sin\varphi}{Fv}\right]}$$

$$M = -\frac{\int\frac{M_o\, ds}{I} - K\int\frac{Q_o\, ds\sin\varphi}{Fv}}{\int\frac{ds}{I}} + HK\frac{\left[\int\frac{v\, ds}{I}\cos\varphi - \int\frac{ds\cos\varphi}{Fv}\right]}{\int\frac{ds}{I}}.$$

In the same way account can be taken of a variation in E in every panel from the springing to the crown, if K remains with the $\int$, and if the term $+ 2\omega t\int\frac{E_1\, ds}{K_1}$ expresses the effect of temperature.

These values allow the moments, normal thrusts and stresses in the arch to be calculated.

4) *Deformation theory of the arch under varying E and I.*

A further refinement in the calculation of the arch consists in allowing for variations both of E and of I. This is now given for the first time, constant values for E and I having been assumed in all previous publications on the subject. Here only the final results will be stated and not their derivation, which will be published elsewhere.

It is necessary first to make some assumption as to the mode of variation either of E alone or of E and I together.

If $E_{\varphi o}$ and $I_{\varphi o}$ denote respectively the elastic modulus and the moment of inertia at the springing, E and I the corresponding quantities at the crown, then the transition for an intermediate point x, y, φ can be calculated according to a parabolic law, and at any given point in the arch:

$$E_\varphi I_\varphi = E I \left[\frac{E_{\varphi o} I_{\varphi o}}{E I} - \frac{4}{l}\left(\frac{E_{\varphi o} I_{\varphi o}}{E I} - 1\right) x + \frac{4}{l^2}\left(\frac{E_{\varphi o} I_{\varphi o}}{E I} - 1\right) x^2\right]$$
$$= E I [A - Bx + Dx^2].$$

The axis of the arch is assumed to be a parabola and the origin of the coordinates is taken to be at the left hand springing.

The following is the differential equation for the displacement η of the arch:

$$\eta'' = -\frac{H\eta}{E_\varphi I_\varphi} - \frac{H}{E_\varphi I_\varphi} F(x) \qquad \text{with } \frac{H}{EI} = c^2 \text{ this becomes}$$

$$\eta'' + \frac{c^2 \eta}{(A - Bx + Dx^2)} + \frac{c^2 F(x)}{(A - Bx + Dx^2)} = 0 \tag{1}$$

whence F(x) for a fixed loading on any type of arch (three-hinged, two-hinged or fixed may be expressed as follows:

$$F(x) = m + nx + kx^2.$$

The homogeneous equation belongs to the type known as a hyper-geometric differential equation, and as it entails calculations with complex quantities an exponential series has been introduced.

The solution to differential equation (1) is:

$$\eta = -\left(m - \frac{2Ak}{c^2 + 2D}\right) - \left(n + \frac{2Bk}{c^2 + 2D}\right) x - \frac{c^2 k}{c^2 + 2D} x^2 + c_1 \eta_1 + c_2 \eta_2. \tag{2}$$

the values η_1 and η_2 being expressible by rapidly convergent exponential series thus:

$$\eta_1 = 1 - a_2 \xi^2 + a_4 \xi^4 - a_6 \xi^6 \ldots$$
$$\eta_2 = \xi - a_3 \xi^3 + a_5 \xi^5 - a_7 \xi^7 \ldots$$

wherein ξ is of the form

$$\xi = rx - r_1 = x\sqrt{D} - \frac{B}{2\sqrt{D}}.$$

Another possible statement is by means of Fourier series, the unknown quantity H being calculated from an equation of least work:

$$g\int \eta\, dx = \frac{1}{EI}\int \frac{M_x^2\, ds}{A - Bx + Dx^2} + \frac{1}{EF_m}\int \frac{N_x^2\, ds}{A - Bx + Dx^2}. \tag{3}$$

H, however, can also be determined from the horizontal displacements of the springings, and this method of calculation can be applied to all forms of arch.

In equation (3), M_x^2 is of the form

$$M_x^2 = H^2 (S + S_1 x + S_2 x^2 + S_3 x^3 + S_4 x^4).$$

In the case of a load completely covering the span, the very important second term on the right hand side of equation (3) becomes

$$\frac{H\Phi}{EF_m} \cdot \frac{2\,lv}{\varepsilon^2}\left[\left(a - \frac{1}{2}\right)\ln\frac{v_2}{v_1} + 4v\sqrt{1+16v^2} + \frac{a^2}{8\,(u_1 - u_2)}\ln\frac{(v_1^2 - u_1)(v_2^2 - u_2)}{(v_1^2 - u_2)(v_2^2 - u_1)}\right]$$

where $v_{1,2} = \pm 4\,v + \sqrt{1+16v^2}$; $\varepsilon = \frac{E_{\varphi 0}}{E} - 1$; $a = \frac{16v^2}{\varepsilon^2} - 1$; $v = \frac{f}{l}$.

and u_1, u_2 are the roots of the equation

$$u^2 + 2u\,(1 + 2a) + 1 = 0$$

Φ is the plane of the H-line.

Matters are considerably simplified if the first term on the right hand side of equation (3) is absent, M_x being then related to the un-deformed axis and so becoming zero.

5) *Deformation theory of an fixed arch with axis following the line of thrust.*

So far a parabolic arch axis has been assumed for the purpose of stating the theory of deformation. Since, however, the arch becomes deformed according to the line of thrust even over short spans, rigorous calculation is necessary in respect of all loads and influences arising subsequent to its closure. This calculation, also, is now set forth for the first time.

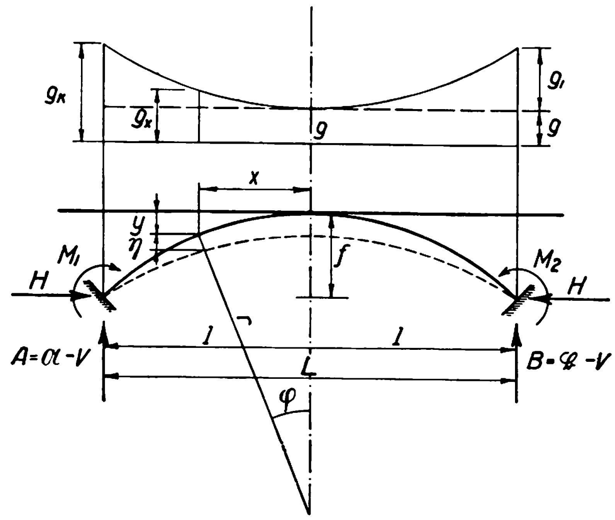

Fig. 1.
left abutement: 𝔄 + V instead of 𝔄 — V.

Let the following be assumed as the equation to the arch axis:

$$y = \frac{f}{m-1}(\cosh \alpha x - 1) = fv\,(\cosh \alpha x - 1) \tag{1}$$

wherein $m = \frac{g_x}{g} = \cosh \varkappa$; $\varkappa = \text{arc}\cosh m$; $\varkappa = \alpha\,l$; $\alpha = \frac{\varkappa}{l}$.

The loading curve is determined by the loads g; $g_{\varkappa}$ at the crown and springings respectively (Fig. 1)[1], and a load g_x is assumed to fix the law of transition for any given point x, y, φ referred to an origin of co-ordinates at the springing: $g_1 = g_k - g$

$$g_x = (g - g_1 v) + g_1 v \cosh \alpha x = g \cosh \alpha x \qquad (2)$$

In a bridge with a large rise the difference between the loads g_x and g can be very large, and for this reason the calculation given below is strongly to be recommended.

The moment is generally

$$M_x = \mathfrak{M}_x + V(l - x) - H[f - (y + \eta)] + M_1 \qquad (3)$$

where $$H\,F(x) = \mathfrak{M}_x + V(l - x) + M_1 - H(f - y) + H \cdot \frac{2\,I}{r\,F_m} \qquad (4)$$

$r = \frac{l^2}{8f}$ and $c^2 = \frac{H}{E\,I}$ the differential equation is

$$\eta'' + c^2 \eta + c^2 F(x) = O \qquad (5)$$

and the solution becomes

$$\eta = A \sin c\,x + B \cos c\,x - F(x) + \frac{1}{c^2} F''(x) - R \cosh \alpha x \qquad (6)$$

whereby $$R = \frac{\alpha^4 \left(f\,v - \frac{g}{\alpha^2 H}\right)}{c^2 (\alpha^2 + c^2)}$$

Both F(x) and F''(x) are hyperbolic functions. This calculation, while somewhat tedious, involves no great difficulty, and it is one which may well be adopted in the case of a long-span bridge. To facilitate integration it has been deemed preferable to fix the origin of coordinates at the crown.

The moment $\mathfrak{M}_x$ becomes

$$\mathfrak{M}_x = \frac{g}{\alpha^2}(\cosh \alpha\,l - \cosh \alpha\,x) \qquad (7)$$

The restraint moment is:

$$M_1 = H \left[B \cos c\,l + f(l + v) - \frac{2I}{r\,F_m} - \frac{\cosh \alpha\,l}{\alpha^2 + c^2}\left(\frac{g}{H} + f\,v\,c^2\right)\right] \qquad (8)$$

and the moment M_x is given by

$$M_x = H \left[A \sin c\,x + B \cos c\,x - \frac{2\,I}{r\,F_m} + \left(f\,v - \frac{g}{\alpha^2 H}\right) \frac{\alpha^2}{\alpha^2 + c^2} \cdot \cosh \alpha\,x\right] \qquad (9)$$

Under symmetrical loading V = O.

The horizontal thrust H may again be found by trial from the work equation

$$\int_0^l g_x\,\eta\,d\,x = \frac{1}{E\,I}\int_0^l M_x^2\,d\,s + \frac{1}{E\,F}\int N_x^2\,d\,s \qquad (10)$$

By means of calculations according to equations (3) to (5) it is possible to ascertain the deformations and statically indeterminate quantities more closely than hitherto.

IV. Stability of arches against buckling.

In flat three-hinged arches of varying cross section the resistance to buckling can be calculated according to the method of Dischinger (Bautechnik, 1924, p. 739) which is particularly applicable where the moment of inertia in the neighbourhood of the crown or springing is not uniform. In arches of this type the limiting span is smaller than in arches with a larger rise, because of the heavy horizontal thrust developed and the difficulty of adequately anchoring this in the ground; the most suitable final state will, therefore, be an encastré type of arch. As is well known, however, there is some advantage, from the point of view of more uniform distribution of stress, in first building the arch as a three-hinged structure and later converting it permanently to the encastré type. For this purpose, and also for application to arches with a large rise, it is recommended to use the three-hinged arch of constant cross section with an adjusted line of thrust, the hinges being subsequently locked. This method applies especially to very large spans wherein the live load is small compared with the dead load; it is impracticable only for very flat arches of constant cross section.

Since a relatively small thickness of the arch may at first appear to be sufficient, it is important before finally determining this to examine the resistance against buckling, having due regard to the operations it is proposed to carry out with the arch as such, before its resisting moment is increased by the completion of the decking.

The author's formula, as follows, may then be used as a first assumption for the thickness of a rectangular hollow reinforced section:

$$\rho^3 \{B[1-(1-2\gamma)^3] + r\rho l(1-2\gamma)^3 + 3\beta^2 \alpha Bn\} = \frac{Ns(1+4v^2)}{8 E_b A l}$$

from which $\rho = \frac{h}{l}$ is to be determined. Here h denotes the overall height of the hollow cross section of width B. If $2fe = \alpha B h$, the reinforcement percentage is $\alpha = \frac{2fe}{Bh}$; $\beta = \frac{h'}{h}$, h' being the spacing of the steel bars. γh is the thickness of upper and lower slabs and of the side wall; r is the number of walls occurring in a width B; s is the required factor of safety against buckling; N is the thrust at the springing, $v = \frac{f}{l}$; $A = \frac{2+k^2}{8-k^2}$ and in the case of a parabolic arch k is found approximately from the buckling formula $k = \frac{1}{2(1+4v^2)}$.

In the case of arches on which a decking slab is later to be superimposed, a factor of safety of $S < 3$ may be chosen — say $S = 2$ to 2.5 — provided that acceptable experimental values for E have been obtained and that the three-hinged arch is later to be converted to an fixed arch.

If the elastic deformation of the three-hinged arch is taken into account for the purpose of exact calculation, the degree of resistance to buckling under uniformly distributed loading can be found by the method of Fritsche (Bautechnik, 1925, p. 465), which is valid for a flat parabolic arch axis.

The buckling load H_K due to the horizontal force at the crown is

$$H_K = \frac{4\varkappa^2 EI}{l^2}$$

wherein $\varkappa$ can be calculated from the equation $\vartheta = \gamma v^2$

$$\tan\varkappa + 3\vartheta \frac{[\varkappa^2(2\varkappa^2+1) + 16(\sec\varkappa - 1)]}{\varkappa[\varkappa^2(6-7\vartheta) - 120\vartheta]} = 0.$$

In the case of an fixed arch, $\varkappa$ in the equation for H_K may be found from

$$\tan\varkappa - \frac{\varkappa(12+7\vartheta)}{12+\vartheta(6\varkappa^2-12)} = 0.$$

Freyssinet recommends that the crown thickness of an encastré arch should be taken as 1/80 l, having regard to the possibility of buckling in the plane of the bearing walls, and Mesnager has proposed 1/100 l. Maillart made the thickness of the full arch in the Landquart bridge in Klosters equal to 1/115 l at the crown and 1/88 l at the springing (Bauingenieur, 1931, Nọ. 10).

As regards safety against buckling, a hollow form of arch is naturally superior to a solid form. With a large rise it is safe to make the thickness of the arch even smaller than the values given above, and this applies of course to long-spanned bridges.

Safety against buckling must also be checked for the arch as finally completed, assuming unfavourable combinations of live loads.

A more rigorous treatment of buckling conditions the publication of which is pending) can be derived from the author's solutions in the theory of deformtion given here under III—4, 5.

The problem of buckling is also dealt with by F. Bleich in his Theorie und Berechnung der eisernen Brücken (Theory and Calculation of Steel Bridges), p. 213; Fritsche in Bautechnik, 1925, p. 484; E. Gaber in Bautechnik, 1934, p. 646; and F. Dischinger in Bautechnik, 1934, p. 739. There is need for further investigations of the problem of buckling with E changing in time.

V. Shrinkage and plastic deformation (creep) of the arch.

In arch bridges of large span an important part is also played by the plastic deformation of the concrete under load — known as flow or creep — because this effect is associated with a sinking of the axis of the arch which gives rise to parasitic stresses.

The significance of shrinkage and plastic flow may be understood from the publication by C. C. Fishburn and J. L. Nagle describing experiments carried out on the Arlington Memorial Bridge (Research Paper R. P. 609, U. S. Standards Journal of Research, Vol. 11, Nov. 1933). In this bridge, an fixed arch of 57.24 m span, the movement at the crown due to this cause was at the end of a year 68 % greater than the temperature effect.

This is a reason in favour of adopting high-strength cements, particularly ciment fondu. The physics of shrinkage and creep has hitherto not been completely explained, though ample numerical data are available on shrinkage and its consequences. Since the time relationships of the contractions due to shrinkage and creep are extremely similar, it would appear that both these properties of concrete derive from a single physical principle, whereof creep constitutes the general case and shrinkage a special case corresponding to the load $P = 0$.

Straub, in his paper (Trans. Am. Soc. Civ. Eng., 1931), has put forward a theory of arches under plastic deformation in which account is taken of the time t and according to which the plastic deformation ε_p is governed by the law $\varepsilon_p = k\,\sigma^p t^q$. Here $p = 2$, $q = 0.15$ is taken for concrete two weeks old, and $p = 1.25$, $q = 0.4$ for 1 : 2 : 4 concrete after 4 months' hardening. It would be more correct to put p equal to m in the exponential law. For an arch, however, the mathematical developments of this are much more complicated than those of the deformation theory; they are of scientific interest and could be made use of in loading tests. A further point in this connection is the assumption of a superposition, which does not in fact arise: that is to say the angular changes suffered by different points in the arch in consequence of elastic and plastic deformations are added together.

For the present, then, it is better to abstain from introducing the time element into the theory of arches and to frame the calculations on experimentally determined laws for the increase of deformation with time. As regards the amount of the plastic strain, ε_p, it will be possible, according to the anticipated lapse of time in constructing and closing the arch and completing the bridge, to form some impression of the stage likely to be reached in the development of plastic strain at each of these stages of the work, and, therefore, of the balance of plastic effects still to be expected after the construction has been finished.

After a certain lapse of time these deformations come to an end, but at present no accurate information is available regarding the precise time that the creep condition is terminated. The researches of Gehler and Amos, very fully recorded in N. 78 of the German Commission for Reinforced Concrete (Deutscher Ausschuß für Eisenbeton) indicate that the termination occurs after one year, whereas according to Whitney (Journ. Am. Concrete Inst., March 1932), Davis and Glanville, creep does not cease till after four or five years — only very small contractions occurring in the last two years.

According to Gehler and Amos as quoted above, at the end of three months some test specimens reinforced on one side only of the cross section opened out, through plastic deformation alone, by 142% of the amount due to shrinkage when the concrete was compressed to 40 kg/cm², and by as much as 408% of that amount when compressed to 120 kg/cm². After a year the corresponding values were 158 % and 365 %. It was hoped to arrive at an accurate numerical determination of the magnitude of the shrinkage and creep, but difficulties compression at the end of a year — the 28-day strength being 296 kg/cm². So far as could be determined from experiments made by assuming values of E and n varying in time, measurements being taken in the cracked tension zone of the concrete, the creep value after 150 days under a concrete strength of

40 kg/cm² amounts to 118% of the shrinkage value, and under a stress of 120 kg/cm² to 270%. After 270 days the corresponding values are 138% and 300%.

If the shrinkage measurement in reinforced concrete at the end of a year be taken as 0.2 mm/m, the creep measurement under a stress of 40 kg/cm² will be about 0.28 mm/m, and under 120 kg/cm² will be 0.6 mm/m.

In bridges of large span these relatively high values are, of course, important; the greater part if not the whole of them must, therefore, be eliminated by the adoption of suitable means of construction. Whatever remains outstanding will take effect after closing the arch. The magnitude of this remainder depends on the time of striking the centres, and therefore, on the span and construction time. From 2/3 to 4/5 of the main effect can always be eliminated.

No danger to the permanence of the arch is present, as the effect eventually comes to an end and the elastic modulus of the concrete increases.

Freyssinet has given the following limiting values for the amount of shrinkage:

With 350 kg cement per m³: $\varepsilon_s = 4$ to $6 . 10^{-4}$
With 400 kg cement per m³: $\varepsilon_s = 5$ to $7 . 10^{-4}$
With 450 kg cement per m³: $\varepsilon_s = 6$ to $8 . 10^{-4}$

(Génie Civil, 1921/II, p. 126) and he proposes to use the values $\varepsilon_s = 0.4$, 0.5 and 0.6 mm/m for reduction of stress in application to his method of construction.

In reference to long span bridges there is still a need for really searching experiments on the amount of shrinkage and plastic deformation occurring in concrete made with different admixtures of high-strength and other special cements.

The stresses caused by shrinkage and plastic deformation can be calculated according to the methods described by M. Ritter[1] or in the author's book.[2]

VI. Methods of construction and de-centering.

The Spangenberg-Melan system of construction and the Freyssinet system of de-centering are well known. The difficulty with the first mentioned is the great mass of material for pre-loading, that has to be supported in case of long span arches, therefore the limits for the scope of this method lies at about 180 m. Fig. 2 shows a proposal by Melan for the suspension of the centering.

With the Freyssinet system it is impossible completely to balance the extreme fibre stresses on each face of a cross section as the ends of the arch have to be assumed as fixed from the start. Considerable reductions in the surface stresses can, however, be attained, as in the following examples.

Villeneuve-sur-Lot bridge: 31% top of crown, 30% underside of springing,
St. Pierre de Vauvray bridge: 25% top of crown, 29% underside of springing,
St. Bernand bridge: 25% top of crown, 43% underside of springing.

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[1] Dr. *M. Ritter:* Wärme- und Schwindspannungen in eingespannten Gewölben (Temperature- and Shrinkage Stresses in Encastré Arches). Schweiz, Bauztg., Vol. 95, March 1930.

[2] Dr. *A. Hawranek:* Nebenspannungen von Eisenberon-Bogenbrücken (Secondary Stresses in Reinforced Concrete Arch Bridges). W. Ernst and Sohn, Berlin, 1919.

In these bridges, therefore, the maximum stresses were only 57.5 and 76.9 kg/cm²; in the Plougastel bridge the maximum was 75 kg/cm² and at La Roche Guyon 80 kg/cm².

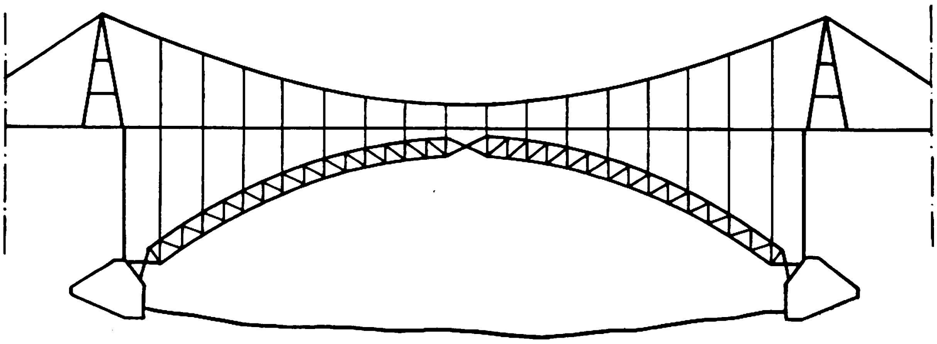

Fig. 2.

The system thus provides a means for increasing the attainable span, and it allows of building a 500 m span without the permissible stress having to exceed 159 kg/cm². Fig. 3 shows a design by Freyssinet (later abandoned) for suspending the shuttering for an arch of 350 m span from wire ropes.

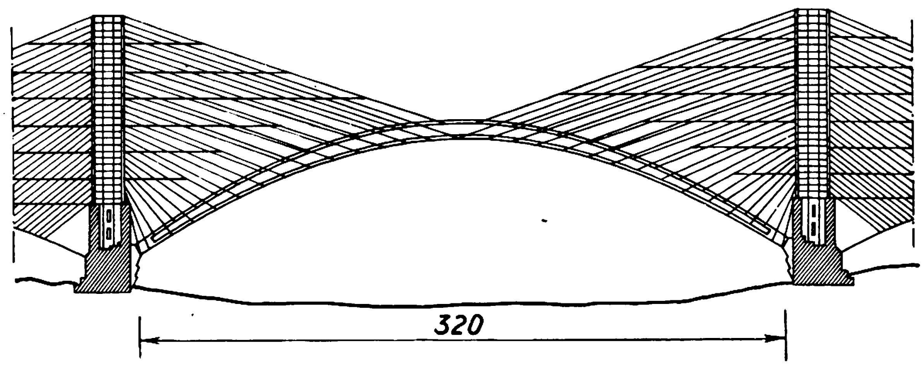

Fig. 3.

Another method of adjusting the position of the arch axis during construction is by the vertical action of hydraulic presses mounted on firm supports, as suggested by Lossier for an arch of 460 m over the Rance (Beton und Eisen, 1931, p. 370). This form of steel centering is shown in Fig. 4.

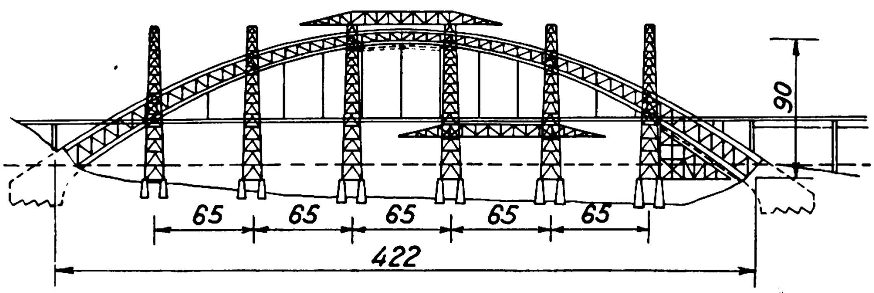

Fig. 4.

Another approach to the problem is that proposed by Dr. Fritz[3] whereby the undesirable stresses that arise in arched beams and arches are avoided by first building a three-hinged arch on fixed supports and later converting this

[3] Dr. *B. Fritz:* Vereinfachte Bestimmung des Einflusses der Systemverformung beim Dreigelenkbogen (Simplified Manner of Ascertaining the Influence of Axial Deformation of a Three-hinged Arch). (Bauingenieur, 1935, Nos. 15/16; Schweiz. Bauztg., 1935/II, p. 277).

to an fixed arch. In the final structure the line of thrust and the axis of the arch are brought practically to coincide by giving a slight excess of height to the two halves of the three-hinged arch before they are subjected to the pressure of the centering, the effect of arch shortening due to dead and live load, the shrinkage effects and the spreading of the supports.

Hinges are built into the skewbacks for later removal after the shrinkage gaps in the arch and crown have been filled with concrete. When the superstructure is complete the centering is struck; shrinkage is compensated by an additional imposed load of Δp_s and likewise the displacement of the supports by a load Δp_w; finally the hinges are taken out of action by the insertion of appropriately shaped voussoir stones.

If the amount of shrinkage allowed for in the calculation does not actually occur, bending moments due to $g+\frac{p}{2}$ will arise in the arch. In the case of large spans the additional loads Δp_s and Δp_w would reach excessive values, and it is necessary, therefore, to await the occurrence of shrinkage and of displacement of the supports before closing the arch, whereupon the artificial loading required will be only $\frac{p}{2}$.

The waiting period is of relatively long duration, and the displacements of the supports are not completed until the full load has been imposed. To mitigate these objections the hinges may be fixed eccentrically at points corresponding to those through which the line of thrust in the abutments and the crown is to pass.

A similar method has been worked out by Dischinger (Bauing., 1935. Nos. 12—14).

These systems are available as a basis for further development of arch bridge construction, and one more will be described in Section VIII.

VII. Centering.

The type and cost of the falsework plays a decisive part in the construction of long span bridges, representing as it does a significant part of the total cost of a reinforced concrete arch bridge. Not only the cost but also the nature of the material used for the falsework is of importance. Hitherto it has been the practice to use timber even for bridges up to 187 m span (as in the Elorn Bridge near Plougastel), and this material has been proposed even for still greater spans. Centering has been applied according to the methods usual in small spans, supported on poles arranged to suit the rise of the arch. Alternatively, special arrangements have been followed, such as the nailed segments and framing used by Freyssinet.

In building the Traneberg bridge at Stockholm, which has a span of 181 m, solid-webbed steel arches were used as centering; these were formed of high-tensile steel with a normal permissible stress of 1800 kg/cm² which in this instance was (quite justifiably) increased by 35% to 2430 kg/cm². This centering involved about 1000 tons of steelwork, used twice over by lateral displacement to form the twin arch ribs. Up to the present this represents the only instance of the use of steel centering in a reinforced concrete arch bridge of large span, as distinguished from certain American examples of relatively small span.

There can be no doubt that for fairly large spans (say up to 250 m) the use of timber must be confined to bridges of low rise built either over moderately shallow streams with a firm bottom, or over solid ground, for under any other conditions the weight of the concrete would be excessive. This remains true even if measures are adopted to lighten the structure, on the plan followed with advantage by Freyssinet in the Roche-Guyon bridge of 161 m span — for it will scarcely be possible to adopt that method in application to still greater spans and to particularly high rises of arch: Freyssinet's method was to subject the lower slab of the box-shaped cross section to pressure applied by jacks, thereby ensuring a temporary connection between this slab and the centering so that any tendency of the arch to buckle would be avoided and so that no additional load would be imposed on the centering as the result of subsequent stages in the construction of the arch.

Timber falsework is exposed to heavy wind stresses. Its weight is considerable and a great deal of labour is involved in its erection. In a strong current, or in deep water, it becomes expensive because if the span is a wide one several foundations for the falsework have to be formed and afterwards removed. This is so even if use is made of framed timber arches or of nailed solid-webbed girders on the Lembke system. If the height is considerable the spread of the supports will have to be developed laterally in order to give the necessary stability.

Hence the use of steel — for preference high tensile steel — will become essential for the centering if spans exceeding 200 to 250 m are required. This may take the form either of high tensile rolled steel or of steel cables, and in view of the merely temporary usage the permissible stress in St. 52 may be increased to 2500 kg/cm^2 or that in steel cables to 7000 kg/cm^2. The use of steel has the great advantage of being independent of all those various questions of bearing pressures at the joints, which have to be considered in the case of timber framing.

Of course steel centering, like any other, should be so designed as to pick up only the unavoidable minimum of the weight of the arch and so as to make the latter self-supporting as soon as possible.

A proposal put forward in reference to a design by the present author is reproduced in Section VIII.

If the use of steel arches as centering for spans even exceeding 200 m should prove feasible at all, it will be necessary to allow for say 4000 tons of steel in the case of 400 m span, even assuming that the centering is to be moved sideways for concreting twin arches in succession. The erection of such a structure, and still more this lateral displacement, will present difficulties. In the case of a flat arch the weight of steel will be greater still, and in any case additional stiffening will be required to resist buckling in either of the main dimensions, and wind loads.

It will be necessary, therefore, to replace the steel arch by a suspended construction or by some suitable combination of a supporting with a suspended structure. The relative advantage of any one such solution compared with another must be decided by cost. The sort of suspension, that may be envisaged, is an anchored cable, having a rise of $^1/_{10}$ to $^1/_{15}$ of the span, from which the suspenders and the shuttering would be hung: this, however, proved uneconomical when examined by complete calculations having reference to the design of a bridge of 400 m span with cable construction of the same span.

In this instance, however, it was found that economy could be attained by combining the use of a substructure projecting 88 m from each of the abutments with a suspended construction to carry the shuttering in the intervening 224 m. This is shown in Fig. 5, and it should be emphasized that the arch in question had a rise of $^1/_4$ of the span. The cable can thus be anchored directly into the abutment of the arch so that no special anchoring block is needed. Moreover, by taking the cable below the crown of the arch the height of the pylons is reduced, and the central portion between the intersections of the cable with the arch can be utilised for stiffening, besides having the effect of reducing the total length of cable.

The deformations that occur in the cable when the arch is concreted are, like the temperature effects, accurately calculable, and may be compensated as the work progresses by means of turnbuckles in the suspenders; alternatively they may be corrected by the action of hydraulic jacks under the cable saddles on the towers.

These conditions entail no difficulty in concreting, for similar deformations occur if steel arch centering is used, and if joints are left open in the concrete the shrinkage effects may be facilitated.

Finally, Fig. 6 shows a proposal for a stiffened suspension construction applied to smaller spans. This would take the form of a two-hinged or rigid frame, the top members of which could later be incorporated as main longitudinal girders in the stiffened decking of the bridge. Such frames offer the further advantage that they could be used to support the shuttering and to provide a horizontal passageway for the supply of concrete and other material. Moreover they would have a considerable effect in reducing the deformations.

The elastic-theory and the deformation-theory of such a suspension bridge with a rigid frame has been given by the author[1].

It will be clear from the above that the combined use of concrete and steel is not only promising, if greater spans than hitherto are to be obtained, but is both necessary and economical.

VIII. A proposed new method of construction for long span arch bridges in reinforced concrete.

Design for such a bridge of 400 m span.

In order to reduce the costs of shuttering and concreting in an arch of this magnitude, the proposal is made that it should be built in the form of

[1] Dr. *A. Hawranek:* Hängebrücke mit einem Zweigelenkrahmen als Versteifungsträger (Suspension Bridge with two-hinged Frame Stiffening girden). (Deformation Theory of this System.) Stahlbau 1935.

Dr. *A. Hawranek:* Verformungstheorie dieses Systems. "Publications" of the Internat. Association for Bridge- and Structural Engineering. Vol. III, 1935.

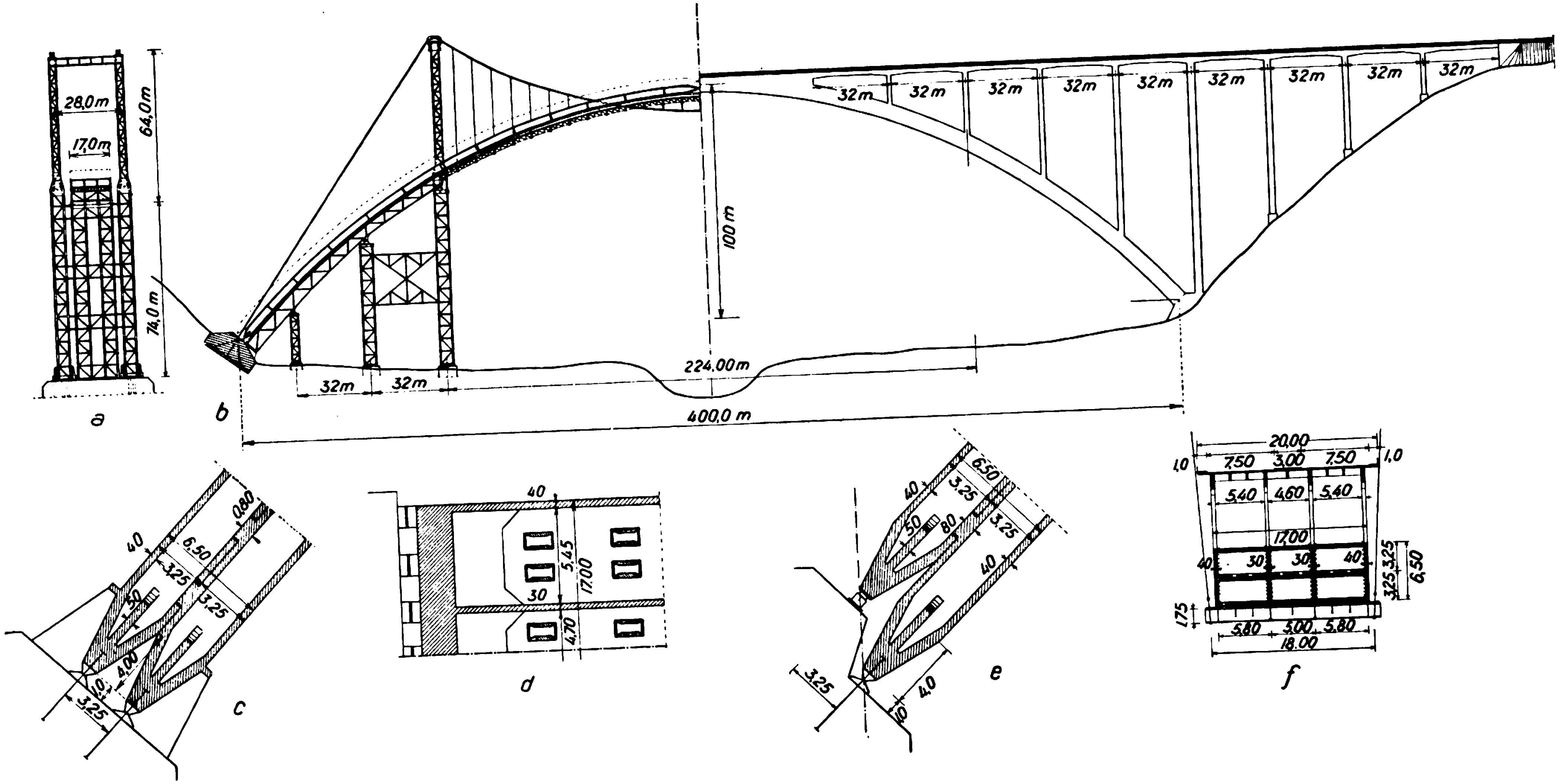
28,0m
17,0m
64,0m
74,0 m
a
b
32m
32m
100 m
224,00m
400,0 m
c
d
e
f
20,00
18,00

Fig. 5.

two parallel superimposed arches. The first of these would be concreted on centering; the centering would then be struck and the first arch used as a support for building the second; finally both arches would be connected together so as to work as a single encastré arch.

If each of the two arches were first formed as a three-hinged arch it would be free to adjust itself to the effect of axial contraction, and also — to an extent governed by the time of closing the arch — to the effects of shrinkage and plastic strain. As an example, an arch of 400 m span has been designed, having the same rise of 100 m as the design published by Prof. Dischinger in Bauingenieur, 1935, Nos. 11—14. The support of the decking has also

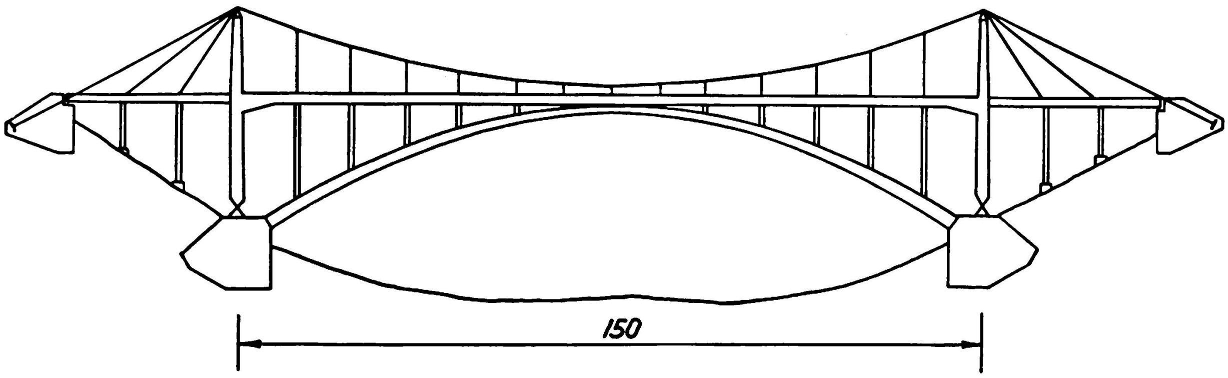

Fig. 6.

been arranged as in that example so as to make the results of the calculations fully comparable (Fig. 5). The proposed new form of falswork will be described presently.

1) *Lower arch.*

The first arch B must first be imagined as formed under its own weight or under a loading calculated to allow the maximum possible reduction of stress, and a constant height and shape of cross section will be assumed. The cross section F is symmetrical about each of the principal directions. The arch rests upon steel hinges in the springings, and provision is made for the temporary presence of hydraulic jacks in the crown. The arch centering is given an excess height in accordance with its own deformation, the elastic contraction of the arch axis, the shrinkage effect, and the plastic strain in the arch. When the arch has been completed it is lifted off the centering by the action of the jacks, thereby sustaining the elastic axial compression due to its own weight.

Alternatively, if the centering is of the suspended type, it may be struck by the direct use of the turnbuckles in the suspension bars, or if it is supported from below it may be operated by vertical jacks.

Temperature effects. It will not always be possible to ensure that closure of the arch takes place at a moment, when the temperature happens to be 10^0 C, the average for Central Europe. If, then, the temperature effects in the fixed arch are to correspond to equal maximum positive and maximum negative variations in either direction, the difference between the mean temperature and the temperature obtaining at the time the centering is struck must be compensated by lifting or dropping the crown accordingly.

This will be successfully accomplished only if the three-hinged arch can be allowed to continue to act as such until some occasion when the mean temperature is actually attained, the closure of the arch being effected at that moment. Premature closure will alter the system, making it statically indeterminate like a two-hinged or an encastré arch, and causing a difference in the maximum values of temperature variation as between the positive and negative directions, so that compensation of stress will be more difficult to secure.

In practice only slight changes in pressure will be undergone by jacks inserted axially at the crown. The necessary alteration in the height of the axis of the three-hinged arch to correspond with these temperature effects may effectively be made by a vertical adjustment of the height of the centering, and if the latter is of the suspended type the turnbuckles will provide a simple means of doing so.

Heat of setting in concrete will probably be apparent only in those portions of the arch, which have most recently been concreted, but no certain means has yet been devised for taking account of this.

Given proper attention to all the measures here indicated in connection with the various effects, the lower arch will be the more favourably stressed of the two.

After plates have been inserted close to the jacks, symmetrically about the axis of the arch, and some of the load has been removed with these in position, the three-hinged arch thereby freed from the centering will itself become available to serve as shuttering for the second arch to be built above.

2) *Upper arch.*

The upper arch, of exactly similar dimensions, can now be concreted and bedded on top of the lower one. The necessary corrections must be applied in the same way, taking account, as before, of the age of the component portions, the relevant amount of shrinkage, and the temperatures in question.

Should it happen that the axis of the upper arch, on completion, fails to run parallel to that of the lower arch, contact between the two arches can be perfected by making suitable slight alterations in the pressures respectively applied in them.

The question whether, assuming opposed pressures in the haunches of the two arches, the pressure exerted by the upper jacks would be increased or that exerted by the lower jacks decreased if the distance between the arch axes were greater at the haunches than at the crown or springings, is one that depends on the degree of success attained in compensating the maximum stresses with the arch in its final condition.

When these processes of assimilation have been completed the two arches are connected by the casting of concrete in those portions which key into one another (Fig. 5d) so that they will work together. In this way a single arch of double thickness is constituted in which the stresses are fairly well distributed and the bending moments correspondingly small.

3) *The bonded double arch.*

By reason of the jacks still existing within the bonded double arch, the latter is susceptible to stress-adjustments, as required, in accordance with the loading later to be imposed. For this purpose the double hinges in the skewbacks continue to be useful as their arrangement confers rigidity on the ends of the arch.

Another possibility would be to build in jacks instead of hinges at the springings of the upper arch, so as to enable later adjustment of the line of thrust in the combined arch.

The decking and spandrels can now be concreted, a gap being left over the crown of the arch. With the aid of the jacks it becomes possible to level out the stresses very completely, taking due account of the loads arising after the completion of the bonded double arch. The relevant factors are the weight of the decking and spandrels, the live load, such further shrinkage effects as have not already been allowed for, further plastic deformation, wind and temperature effects.

Since two or more superimposed rows of jacks are available in the crown it is possible to reduce the maximum stresses by the application of different pre-calculated pressures in the several rows. If, in addition, jacks are provided in the springings, the line of thrust can be still more completely controlled.

The closure of the arch and the concreting of the crown, the skewbacks and the gap in the decking can then follow. At the same time the reinforcing bars necessary for the encastré action of the arch will be embedded in concrete at the places where the action of the hinges is to be discontinued.

The question of whether the full arch should be closed immediately on completion of the two component arches, or not until the decking has been built, must depend on the span and rise and the available forces of the jacks and mainly on the relation between the dead and live loads.

4) *The bond of the double arch.*

Since the two separately constructed arches are later to form a single whole it is equally necessary to prevent their mutual displacement along the line of contact and to ensure that this contact will hold good for ever.

To attain the first of these objects the extrados of the first arch is provided with several rows of transverse reinforced projections, dovetailed in section, which are concreted together with the arch itself (Fig. 5 c, d). Corresponding gaps are left in the bottom slab of the upper arch, and the intervening space is filled with concrete only after both arches have been fully adjusted. This operation is made possible by the provision of manholes in the cross walls and in the top slab of the upper arch, so providing access and even making it possible to remove the inside shuttering.

There is no difficulty in forming these cross-pieces to act as dowels, as they are made on the extrados of the lower arch, and the same applies to the gaps left in the intrados of the upper arch which, since the operation is accomplished while the lower slab is being concreted, is easily accessible.

The vertical bonding of the arch is effected by radial anchor bolts placed close to the inside cross ribs and passing through both of the component arches.

The connection may also be ensured by means of the longitudinal walls themselves.

5) *Position of the skewback hinges.*

The skewback hinges may be arranged in a gap perpendicular to the axis of the arch. The spans of the two component arches will then be slightly different and their respective rises will also differ appreciably (Fig. 5 c).

Alternatively the hinges in each springing may be arranged vertically one above the other, in which case each of the component arches will have the same span (Fig. 5 e) and will receive almost the same stresses due to its own weight, so that the same procedure may be followed as regards its completion (apart from measures necessitated by shrinkage and creep.

Another possibility would be to arrange the skewback hinges, and perhaps also the crown hinge, eccentrically from the beginning.

6) *Corbelled hinges.*

For large spans there is great advantage in the use of projecting temporary hinges. Such projections from the abutments give the possibility of thickening the arch at its ends so as to make it better able to whitstand wind and temperature stresses after its closure; moreover, in a narrow bridge, stability can be increased by widening the corbel. The projecting portions can be built on fixed scaffolding which is relatively low. The arch itself may be made thinner, and the span of the three-hinged arch in the first phase of the construction is reduced. Finally, the saving in weight enables the dimensions of the falsework for the three-hinged arch to be made smaller.

The hinges are thus situated at points where the surface stresses in the final the fixed arch will be lower than either at the springing or at the crown, and where the full permissible stresses cannot be utilised: hence the additional reinforcing bars required when the hinges are taken out of action and the structure is converted to an encastré arch may be reduced in cross section. There is a saving in this respect also, and the operation of embedding the additional bars in the arch is rendered easier.

Finally, the steel hinges are reduced in weight in accordance with the lighter thrust at the springing resulting from the smaller span.

The advantage of corbelled hinges is particularly well marked in the case of bridges with a large rise, partly because in such cases the distance that the hinges can be brought forward is relatively great.

Corbelled hinges are equally effective as permanent hinges or as temporary hinges in such arches.

The many advantages offered by the system which is here proposed, and the variety of the adjustments it makes possible by suitable arrangement of the details, commend it as a means for the construction of long span arch bridges in reinforced concrete.

Summary.

The paper deals chiefly with the properties of materials as required in the construction of long-span arch bridges. It is ascertained that at the present time an admissible stress limit of 200 kg/cm^2 is attainable in the concrete. The question of reducing maximum stresses in arch bridges by means of correcting the axis of the arch is discussed and new methods for the stricter calculation of arches on the deformation theory investigated. New general formulae are elaborated for the variability of the modulus of elasticity E in a hollow arch section, the deformation theory for arch bridges with variable E and I, and the deformation theory for an encastred arch with a funicular axis curve for effective loads. In every case the solutions to the differential equations and the formulae for the deformations and moments are published here for the first time. Then the influence of shrinkage and creep in concrete arch bridges is considered.

For the false archwork of large concrete arch bridges a steel construction, parts of which are suspended, is proposed and a new method of erecting an arch of 400 m span given. This method consists in constructing an arch of half the final thickness, which, on removal of the formwork, serves to carry a second, parallel arch which is superimposed upon it. When completed both arches are connected to give united action. By the combined use of provisory hinges and hydraulic jacks it is possible to eliminate the effect of shrinkage and creep in concrete. Finally, the advantage of provisional corbelled hinges is discussed.

No comparison is drawn between reinforced concrete and steel arch bridges.

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IVb 4

The Arches of the Traneberg Bridge in Stockholm.

Die Gewölbe der Tranebergsbrücke in Stockholm.

Les voûtes du pont de Traneberg à Stockholm.

S. Kasarnowsky,
Ingenieur, Erster Konstrukteur der Brückenbauabteilung der Hafenverwaltung, Stockholm.

The arch bridge over the Tranebergssund in Stockholm built during 1932 to 1934 combining railway and road bridge is at present the widest arch bridge in the world having a span of 181 m (fig. 1).

Dr. ing. *Dischinger* introduced as criterion for arch bridges the radius of curvature of a parabola $\frac{l^2}{8f}$ passing through springing and crown, which he termed "the degree of boldness" of an arch.

It would be better to define this "degree of boldness" by introducing the actual radius of curvature at the crown as this value multiplied with the specific weight of the bridge material is almost identical with the normal stress in the crown due to dead weight. This radius of curvature at the crown of the Tranebergs bridge with a rise of 26,2 m measures 183 m and surpasses by about 7 m the corresponding radius of the new Mosel bridge at Koblenz and by about 50 m the radius at the crown of the Plougastel bridge at Brest.

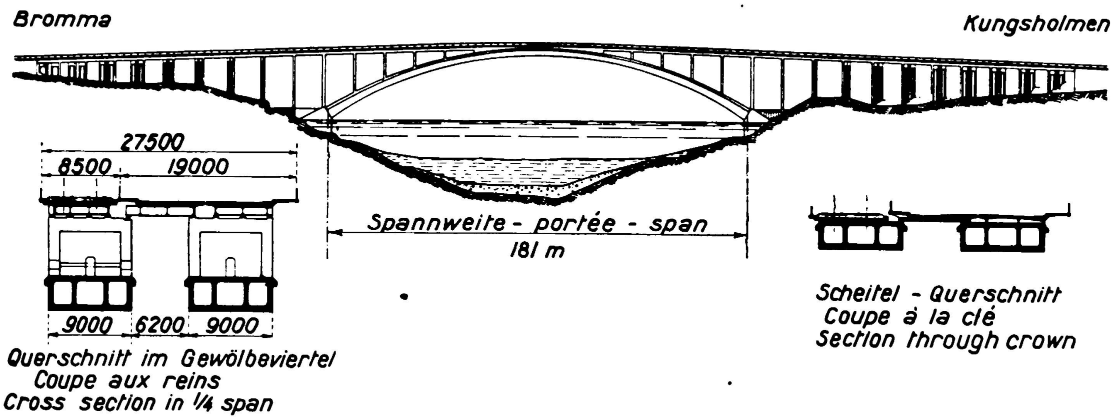


Fig. 1.

Bridge decking.

This bridge possesses two distinct tracks independent from each other. One decking is for road traffic only with a width of 19 m of which 12 m are for the road proper. The two side walks measure 2,5 m and 2 m respectively. Apart from this there are two bycicle tracks each of 1,25 m. The other decking with a width of 8,5 m is used entirely by a double track suburban railway. The tracks being 3,5 m apart.

The design of the road decking was guided by the importance of not introducing too much weight on to the arches. The decking is composed of 10 welded longitudinal girders supporting a reinforced concrete slab 22 cm thick. The span of these girders measures 13 m and is the same as for the approach spans.

Conditions of loading.

Live load: for footpaths and bycicle tracks 0,4 t/m², causeway 4 loading strips according to fig. 2a.

Suburban railway two trains according to fig. 2b.

The combination of these surcharges is identical to a loading factor of 7,5 t/m for one arch rib.

Wind pressure: 0,125 t/m²; *Temperature* ± 16° C.

Shrinkage: corresponding to —10° C.

Fig. 2a.

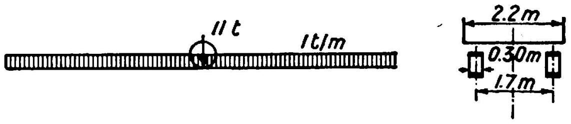
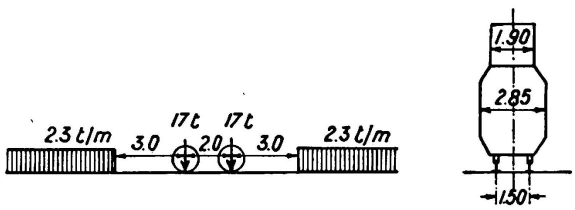


Fig. 2b.



Permissible stresses and materials.

A permissible stress of 100 kg/cm² was allowed for normal loading composed of dead weight, live load, ± 8° C temperature and shrinkage. The maximum permissible stress of 120 kg/cm² was allowed for exceptional loading, being in addition to the normal loading, the influences due to ± 8° C temperature and wind pressure. The properties of concrete used for arch and arch foundations are shown in the following table:

Portland Cement kg/m³	Water Cement Ratio	Aggregates: Cement: Sand: fine shingle (7 to 30 mm): coarse shingle (30 to 60 mm)	Mean values of strength of test cubes kg/cm²			
			7 days	28 days	90 days	1 year
400	0,54	1 : 2,20 : 1,11 : 1,12	274	464	497	478
365	0,54	1 : 2,54 : 1,24 : 1,24	258	451	488	485

The thickness h_0 of the crown was ruled by the following points:

1) *Safety against buckling in the plane of the arch.*

On account of the comparatively high compressive stresses (of 70 kg/cm² for dead weigth only) it was found necessary to arrange fo a stiff arch giving the necessary safety against buckling in the plane of the arch. The lower limit of slenderness for a concrete post where buckling is likely to occur is known to be about 55. The free buckling length for an encastré arch of this type is about $^1/_3$ of the span l and the radius of gyration about 0,37 of the thickness h_o at the crown, hence, the ratio of slenderness for the arch

$$\frac{0{,}33\, l}{0{,}37\, h_o} = 0{,}91 \frac{l}{h_o},$$

giving a least required thickness at the crown of

$$0{,}91 \frac{l}{h_o} = 55 \text{ or } h_o = \frac{l}{60} \tag{1}$$

2) *Additional stresses due to bending of the arch.*

It is known that arches of wide spans in particular produce, due to deflection of the arch, additional stresses, which reduce the factor of safety considerably if the arches are slender[1].

If X stands for horizontal thrust due to dead weight and live load and if Δ indicates the deflection of the arch produced by live load, the additional bending moment due to deformation is approximately

$$c \times \Delta \cdot X \tag{2}$$

The coefficient c has the value 0,7 for the crown and $\sim$ 1,0 for the springing. (The deflection in $^1/_4$ of the span is decisive for the springing.)

The calculation of actual deflection can be based on the deflection of the undeformed system if multiplied with a factor Γ derived from the following formula

$$\Gamma = \frac{\sigma_k}{\sigma_k - \sigma_n} \tag{3}$$

In this formula σ_k represents the buckling stress according to Euler out of equation N° 1 and σ_n is the stress due to dead weight and live load. With *Young*'s modulus of E = 210000 kg/cm² we receive for the Tranebergs bridge the values σ_k = 960 kg/cm² and σ_n = 76 kg/cm², hence:

$$\Gamma = \frac{690}{690 - 76} = 1{,}12.$$

The most important deflections for arches of the type of the Tranebergs bridge can be calculated by using the following expressions:

Crown $$\Delta = 0{,}000093 \left(\frac{p\, l^4}{J_o\, E}\right) \Gamma \tag{4}$$

$^1/_4$-span $$\Delta = 0{,}000122 \left(\frac{p\, l^4}{J_o\, E}\right) \Gamma \tag{5}$$

(p = live load per meter, J_o = moment of inertia of crown).

[1] See Kasarnowsky: Stahlbau 1931, No. 6.

For the Tranebergs bridge the above formulae with a live load of p = 7,5 t/m supply values of deformations which are 2,6 cm for the crown and 3,3 cm for $^1/_4$ span (see table). The horizontal thrust due to dead weight and live load for this bridge amounts to 8588 + 782 = 9370 t, hence the additional moments according to equation N⁰ 2 are as follows:

$$\text{Crown } 0{,}7 \cdot 2{,}6 \cdot 9370 \cdot 0{,}01 = 170 \text{ tm}$$
$$\text{Springing } 1{,}0 \cdot 3{,}3 \cdot 9370 \cdot 0{,}01 = 310 \text{ tm}$$

The additional stresses are not more than 1,6 kg/cm^2 or 1,6 % of the permissible stresses. Sufficient stiffness for the arch results from the thickness of the crown if based on formula (1) and the actual permissible stresses.

Arch construction.

The arch consists of two ribs 15,2 m apart. These ribs are of hollow section with two internal longitudinal partition walls. For a length of 54 m, symmetrical to the crown the thickness of the arch remains constant 3 m, and increases up to 5 m at the springing. The extrados of the arch has a continuous projection

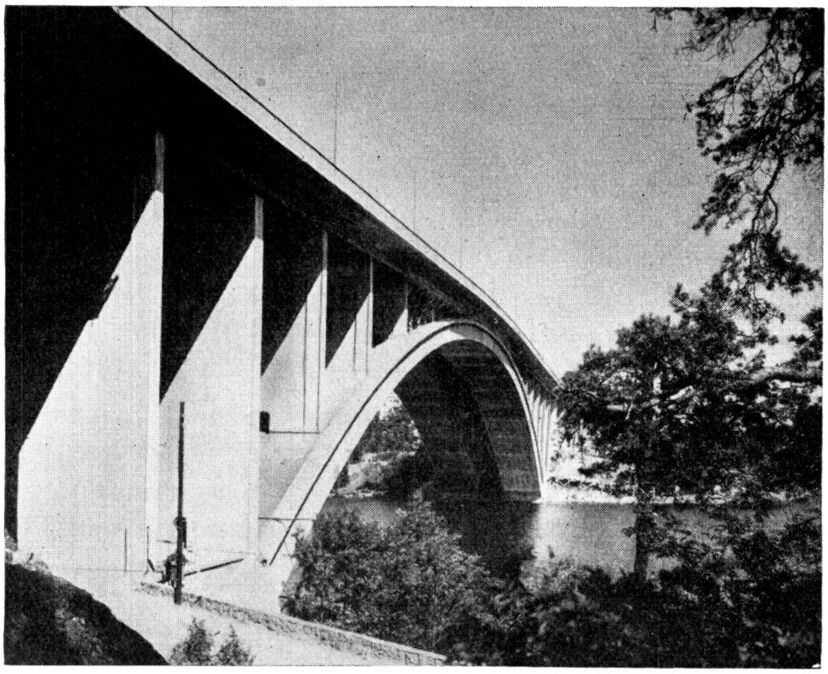


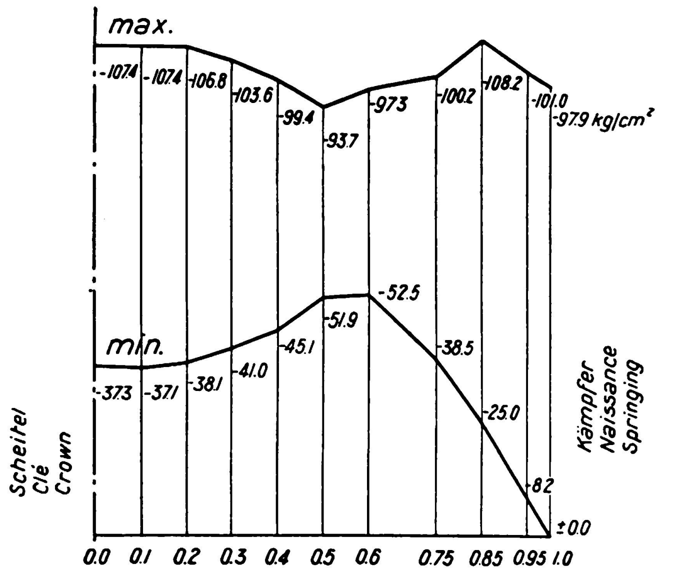
Fig. 4.

to give a more slender appearance to the whole arch (fig. 4). The width of the arch rib is constant, measuring 9 m. The following table contains the most important data of the cross section in crown, $^1/_4$ span and springing:

Cross Section	Thickness of arch m	Cross sectional area m^2	Moment of Inertia m^4	Modulus of section m^3
Crown	3,00	12,85	15,52	10,30
$^1/_4$ span	3,16	13,18	17,99	11,30
Springing	5,00	22,05	69,93	28,00

The distribution of minimum and maximum stresses in the longitudinal direction of the arch is shown in fig. 3. It will be seen from this figure that no tension stresses are produced in the arch even for the most unfavourable conditions. The highest stress is found in a distance of 77 m from the crown amounting to 108,2 kg/cm² and is composed as follows

Dead weight	64,3	kg/cm²
Influence due to lateral excentricity on account of dead weight	1,0	,,
Live load	17,8	,,
Influence due to lateral excentricity on account of live load	1,1	,,
Temperature — 16° C	9,9	,,
Shrinkage (— 10° C)	6,2	,,
Wind	7,9	,,
Total	108,2	kg/cm²



Verteilung der max. und min. Spannungen im Gewölbe. (Exceptionneller Belastungsfall)

Répartition des tensions max. et min. dans l'arc. (Cas de charge exceptionnel)

Distribution of stress minima and maxima in arch (Exceptional case of loading)

Fig. 3.

The execution of each arch rib was done in two sections (fig. 5) and each section was composed of 10 parts pro half-span, with joints 1,2 m wide. The filling-in of concrete was done simultaneously at two corresponding places to avoid unsymmetrical loading of the arch formwork.

The concrete was prepared in a concrete factory and brought to site in special trucks with rotating drums of 1,25 m³ capacity. From these trucks the concrete was poured into buckets and hoisted by cables to the site of concreting.

The total volume of concrete required for these arches was 2740 m^3 without foundations. The arches were reinforced with 62 kg of steel per m^3. From this amount of steel 45 % were used for longitudinal and 55 % for cross reinforcement. The steel was of building steel quality, St. 50, with a lower yield limit of minimum 30 kg/mm^2 and an elongation of minimum 20 % of the standard length of testing specimens.

Arch foundations.

On both shores sound granite rock was found at the surface forming a natural load carrying bed for the foundations. The design for the foundations was carried out in such a way that the extreme fibre stresses were not more than 30 kg/cm^2. In each foundation a hollow space was formed for the passing of water pipe lines of 1 m in diameter. The execution of the foundations was done in open dry pits, protected against the influx of water by circular cofferdams built for a maximum depth of water of 8 m.

False-arch work (fig. 5).

Four plated fixed steel arch ribs were forming the false-arch work for the construction of this bridge. These ribs were placed under the vertical walls of the arches and had a span of 172 m with a rise of 25,25 m. The cross section of this steel forms was constant throughout and consists of a web plate 2400 · 18 mm and four unequal angles 100 · 200 · 18 mm and two flange plates of 800 · 24 mm. The web plates were stiffened with two continuous channel sections NP 26. The material used for the false-arch work is high grade steel St. 52 with a lower yield limit of minimum 36 kg/mm^2 and an elongation of 20 % of the normal standard length of test pieces. Vertical full loading produced a maximum stress of 2210 kg/cm^2 in the false-arch work. (The dimensions of these arches were based on the required safety against buckling in the plane of the arches.)

The steel requirements for the false-arch work were as follows:

Steel St. 52 (plated arches)	660 t
Steel St. 44 (bracings)	195 t
Cast steel and rollers	27 t
Steel St. 37 Tracks for shifting the false arches .	66 t
Total	948 t.

The erection of the false-arch work was done with the help of a floating gantry. See fig. 6.

Execution.

After the pouring of concrete was complete, 18 hydraulic jacks each acting with a pressure of 330 tons were inserted at the crown. With these hydraulic presses the arch was lifted at the crown by 17 cm and removed from the false-arch work over a length of 20 m, thus opening the crown by 11 cm at the extrados and 10 cm at the intrados. The horizontal thrust under these conditions was measured and found to be 6000 t which was 575 t more than the horizontal thrust calculated under the assumption that the centre line of the arch coincides with the pressure line. After this 16 presses were applied at the springing, two for each bearing of the steel arch ribs. With this arrangement the false-arch work was lowered, thus releasing the concrete arch completely. The whole false-arch work was set on rollers and moved sideways 15,2 m into a new position for constructing the second arch rib.

The stresses in the arches were regulated according to the procedure of *Freyssinet* using 22 presses. A negative bending moment of 1590 tm and an excess of horizontal thrust 375 t was introduced at the crown. This enabled finally to keep the gap at the crown uniform 4 cm in width at the intrados and extrados. After filling the gap in the crown with mortar of 750 kg cement per m³ the presses were removed and concrete was placed into their former positions.

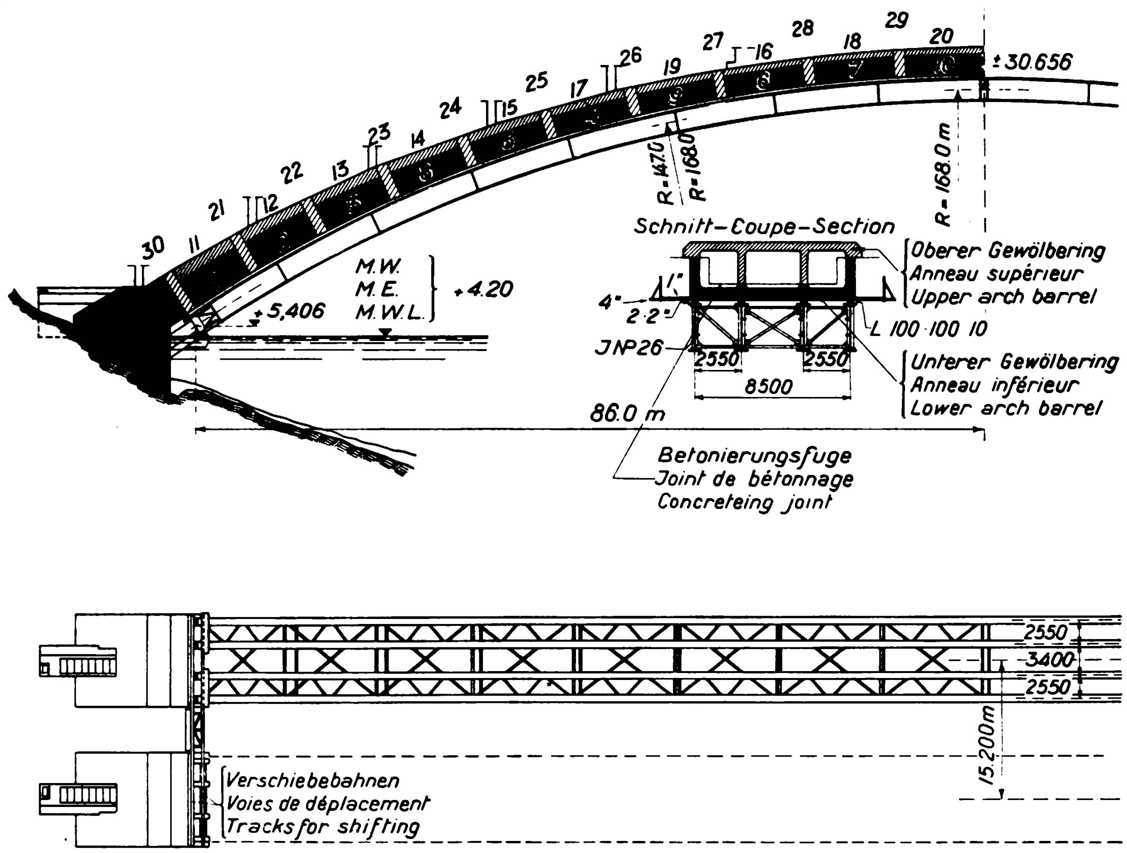


Fig. 5, 6.

Test loading of the arches.

The static test was carried out with a superload of sand for the causeways and loaded waggons on the railway tracks. The total superload was 8,45 t/m, or 13 % more than required by calculation. The sagging at the crown was measured to 28,7 mm of which about 10 mm were permanent deformations, similarly in $^1/_4$ span a sagging of 29,7 mm was found with a permanent deformation 7 mm. At the same time stresses were measured at the springing with a *Huggenberger* deformeter giving a maximum stress of 17,7 kg/cm², which coincides with the theoretically calculated stress for this place, using *Young*'s modulus of E = 300 000 kg/cm².

The dynamic testing was carried out with two 33,5 t tram car bogies, one on each track passing with a speed of from 15,9 to 43,8 km/h. The deformations at the crown were measured with a Stoppani oscillograph. The figures 7b c d show the diagrams of deformations in a form similar to influence lines. The maximum deflection (independent from speed) was measured to 1,7 mm which coincides with calculated results based on *Young*'s modulus of $E = 570\,000\,kg/cm^2$. Finally horizontal and vertical oscillations of the arches were measured using for this purpose an astatic pendulum constructed by the author. The horizontal oscillations of the arches gave with an own frequency of 4 seconds of the instrument a frequency of 1,3 Hertz. Fig. 7e. The vertical oscillations of the arches were established by four men jumping rhytmically up and down causing this way a frequency of 2,0 Hertz. Fig. 7.

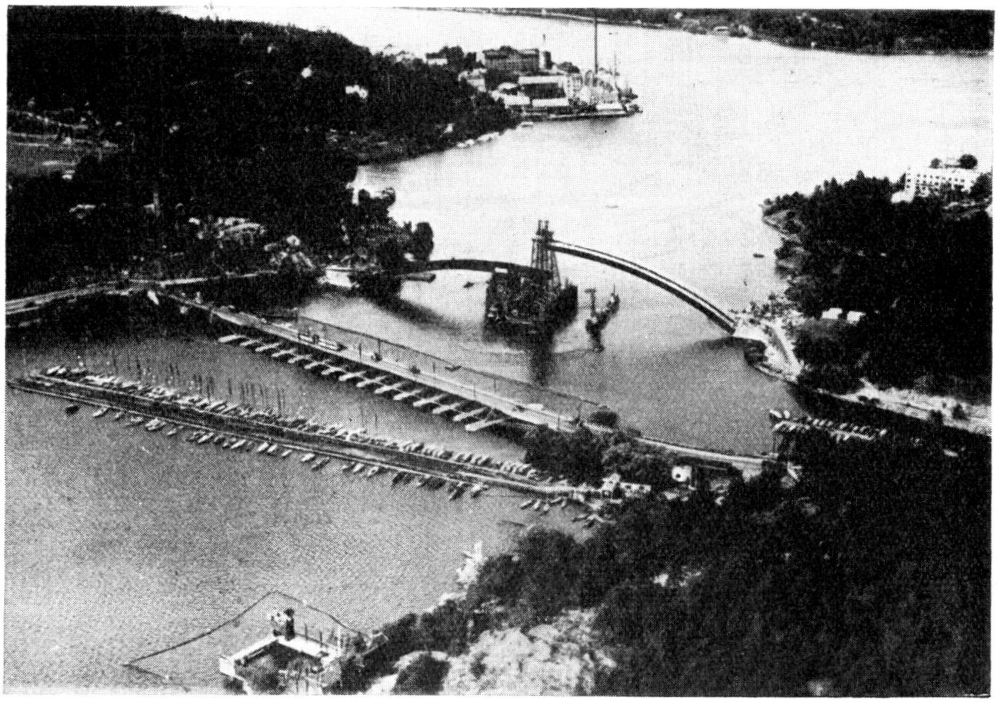


Fig. 7.

Temperature.

For the purpose of measuring the temperature in the concrete of the arches, thermostats were used at the crown and at the springing. Simultaneously were measured the temperature of the concrete, the temperature of the air in the hollow of the arches, the temperature outside and the vertical movements of the crown. To every centigrade corresponds according to calculation (based on a dilatation coefficient of 0,000010) a movement of the crown of 3,4 mm which figure was found to be correct according to tests.

These observations will still be carried out over a period of years with the purpose to establich finally the exact value of shrinkage of concrete in arches. With the recorded movements of the crown during the years 1934, 1935 it would be possible to calculate the shrinkage, corresponding to a decrease in temperature of -5^0 C for the southern and -3^0 C for the northern arch rib respectively.

The total costs for arch foundations, arches and false-arch work amount to 1.633.00 Kr. distributed as follows:

Foundations	4 nos. 1858 m³ . . .	Kr.	255 000	or 15,6 %
Arches	2 nos. 5840 m³ . . .	Kr.	634 000	or 38,9 %
False-arch work and bearing brackets		Kr.	744 000	or 45,5 %
	Total	Kr.	1633 000	or 100 %.

For the purpose of comparison the costs of a corresponding arch bridge in steel are given herewith:

Foundations	Kr.	145 000
Superstructure and wind brazings 2100 tons à 850 Kr .	Kr.	1785 000
Total	Kr.	1930 000.

The costs of the road decking are assumed to be the same in both cases. This comparison shows that the concrete bridge in this case is more economical than a steel bridge for the same purpose.

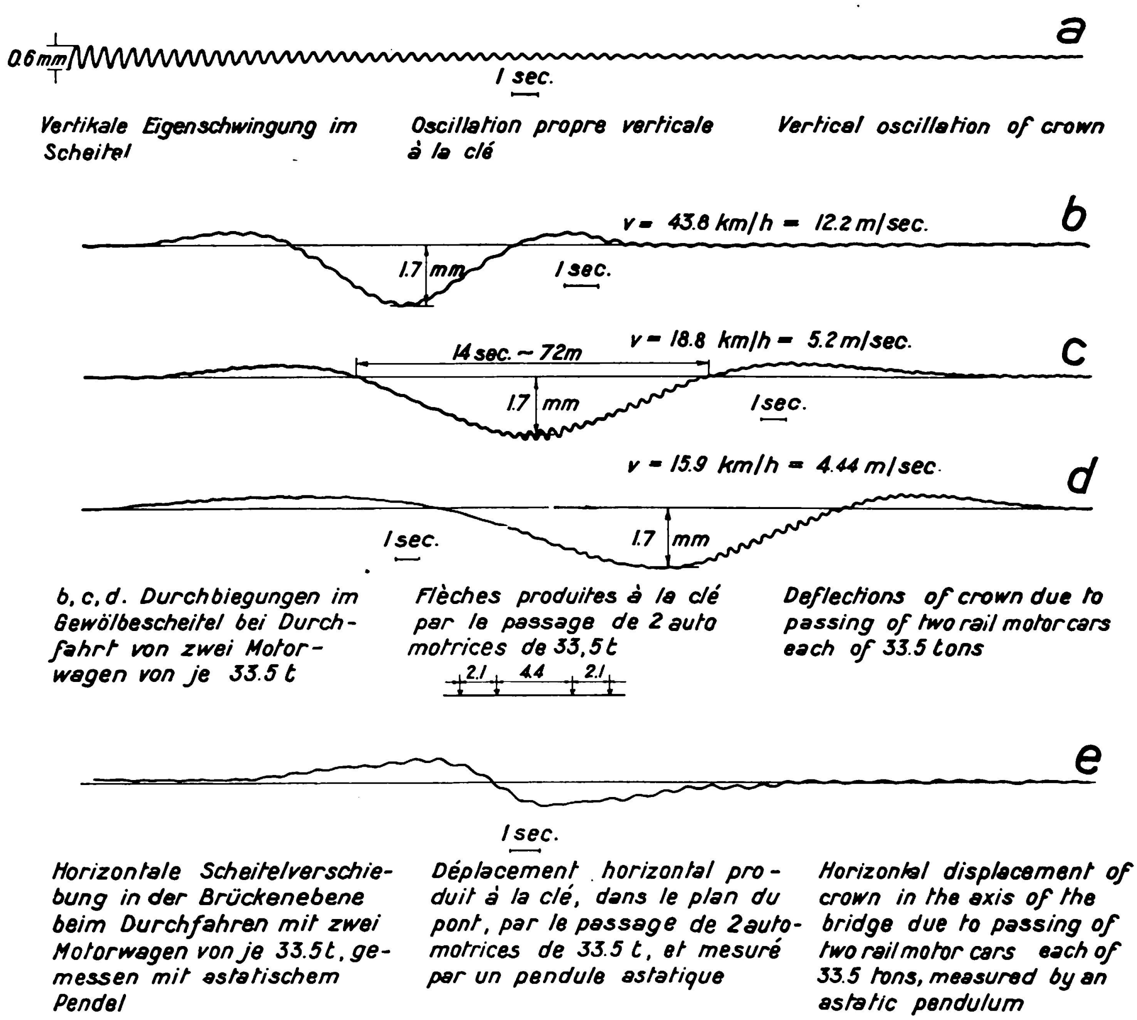


Fig. 8.

Summary.

The experience made with the construction of the Tranebergs bridge allows for the following recommendations for designing arch bridges of wide spans.

The arch can be designed with a constant cross section between quarter-span and crown with a thickness of $^1/_{60}$ of the span. The thickness of the arch at the springing can be 1,4 to 1,8 of the thickness in the crown. Free standing arch ribs should have a width of $^1/_{28}$ to $^1/_{30}$ of the span to establish sufficient safety against side buckling.

The unwanted stresses caused by the compression of the axis of the arch due to dead weight as well as a portion of the stresses due to temperature and shrinkage can be eliminated with the procedure established by *Freyssinet.*

While designing false-arch work special care should be taken to give this structure sufficient stiffness, as otherwise the removal and shifting causes great difficulties.

The construction of the false-arch work can be carried out in steel or timber.

Finally it may be mentioned that in case of competition between concrete and steel, the number of arches, which can be carried out with one false-arch work plays a decisive part. The higher the number of such arches the more economical the construction. The execution of one single arch construction can only be economical under particularly favourable circumstances.

IVb 5

The effect of Braking Forces on Solid Bridges.

Die Wirkung der Bremskräfte bei den massiven Brücken.

L'influence des forces de freinage dans les ponts massifs.

Dr. Ing., Dr. techn. h. c. E. Mörsch,
Professor an der Technischen Hochschule Stuttgart.

The consideration of braking forces is not only necessary for railway bridges but has been prescribed for motor car roads as well. The traction and initial forces due to acceleration of engines (locos) act in the same way as braking forces. If the braking forces received no or only insufficient consideration in statical calculation it was for the lack of a proper method. A clear conception of the problem can only exist if the braking forces are *not* treated separately from the load by which they are produced. In the following a procedure will be shown illustrating how the influence lines for core moments can be altered to render them suitable for the inclusion of the additional influences due to braking forces. The alteration to influence lines varies according to the type of superstructure over the arches.

The braking forces are assumed to act, according to regulations, in the plane of the decking or along the top of rails. But, following the laws of dynamics, the braking forces produced at the decking and transmitted into the vehicle must act in the centre of gravity of the vehicle if they are to help in reducing the speed. By transferring this braking force from the point of action into the

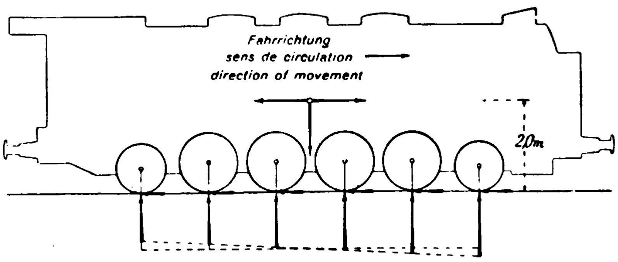


Fig. 1.
The forces acting on a locomotive with put-on brakes,

centre of gravity it is necessary for equilibrium to apply this force twice but reverse in sense, thus creating this way an additional couple. In reality the weight of the vehicle develops a tendency to be pushed forward and thus causes for unequal distribution on to the axles. Naturally sligltly different moments would result from the influence lines for altered values of the axle loads. The regulations, however, allow this fine point to be neglected by placing the action of braking forces level with the bridge decking.

Strictly speaking, the braking forces would also be dependent on the gradient of the decking. The vertical axle loads on which the usual calculation of structures are based already embody a certain amount of braking forces if standing on an incline, to which they are not in a right angled position. A train, for instance cannot stand on an incline unless with locked brakes. The law of friction says that the angle formed between the resultant due to axle load and friction, and the vertical to the decking, at most be equal to the angle ρ of friction, and as is known, the coefficient of friction is expressed by $\mu = \operatorname{tg} \rho$.

According to Fig. 2 the decking has an inclination φ to the horizontal and if the brakes are put on for a train moving down the braking forces S are developed. These, combined with the axle load P, give the resultant R which forms

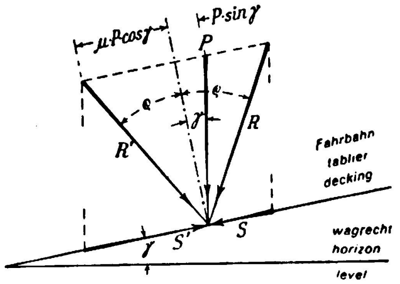


Fig. 2.

an angle ρ with the vertical to the decking. With this extreme position for R the value of the braking force is determined, which will have to be added to the vertical loads P:

$$S = P \cdot \cos\gamma \cdot \operatorname{tg}\rho - P \cdot \sin\gamma.$$

For small values of γ this formula reduces to:

$$S = P\,(\mu - \sin\gamma).$$

If, however, a train is moving up the incline with its brakes on, the resultant R′ has a reversed inclination and the braking force to be added to the vertical load P can be expressed as follows:

$$S' = P \cos\gamma \cdot \operatorname{tg}\rho + P \sin\gamma$$

or correspondingly as above

$$S' = P\,(\mu + \sin\gamma).$$

The regulations for the German State Railways prescribe for $\mu = \frac{1}{7}$.

The axle loads of a vehicle with its brakes full on act under an oblique angle to the decking, accordingly on to the superstructure of an arch also. Since in general for the longitudinal direction of the bridge no distribution of loads through decking and superstructure is required (also according to DIN 1075 § 6) it *must consequently apply also for axle pressures somewhat inclined due to the braking action.* With this inference we receive the action of braking forces on to the axis of an arch as given in Fig. 3a and b for the two directions of a moving vehicle.

The oblique but parallel forces could be divided into two components at those places where they intersect with the axis of the arch. But with these forces P shifted into the axis of the arch the use of the influence lines for care points would become intricate, as for such a procedure the distances between the forces would alter. To avoid these difficulties we proceed as follows:

From Fig. 3c it can be seen that it is of no importance for the external equilibrium conditions of a three hinged arch if the two forces P and S act in their original position or are moved parallel to any point of their resultant. This applies also when determining the moments M_x and M_k and M_o of two

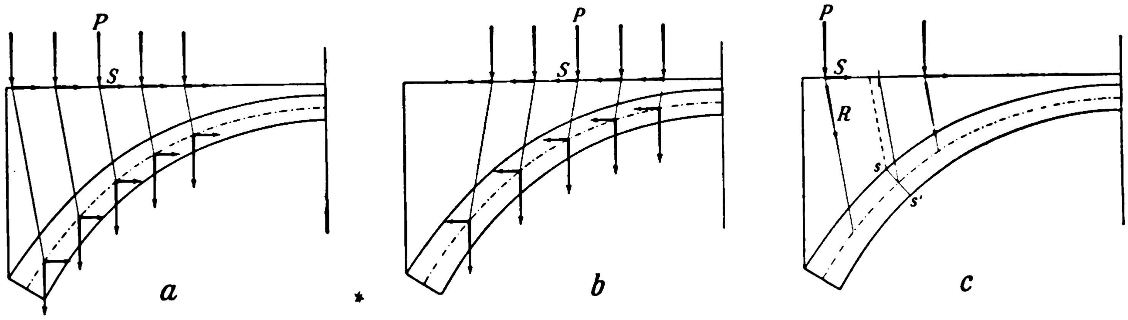


Fig. 3.
Compound action of axle loads and braking forces on arches with solid and open spandril construction with longitudinal partition walls.

hinged and fixed arches. If the *loads* P are left unchanged in their original position to the decking their action can be determined with the normal types of influence lines and the unaltered axle loads. It will only be necessary to find the *alterations* to bending moments, reactions etc. caused by the forces S.

It can be assumed as for the calculation with vertical loads only, that the oblique resultants R from P and S act directly on to the axis of the arch. The dividing line for the forces P and S left or right of an arch section s—s' is therefore no longer established by the vertical through the centre of the axis in section s—s', but that point of the decking through which passes a parallel line to the resultants starting from the intersection with the axis of the arch. This limit can be improved by a parallel through the upper edge of section s—s'. The position of the point of division changes with the direction of the braking forces.

For a unit force S = 1 moving over the decking, its particular action in the form of moments, reactions etc., can be determined and the calculated amount plotted as ordinates from a horizontal base line, for each particular momentary position of the moving force S. By this procedure we only receive an influence line for those braking forces for which the contributary amounts are identical with those of the axle loads P, in the same way as given by the normal influence lines. Since the braking forces always represent a definite portion of the axle loads, their contribution to the influence line ordinate can be added to the vertical axle loads, so that now only the *improved* or *resulting influence lines* for the axle loads require to be worked out. The position of the load division points for the resulting influence lines can be greatly changed in respect es the ordinary influence lines, but more accurate results are received than for separate influence lines with the same train loads positions giving the extreme values with normal influence lines. Very often the difference is not considerable. Despite the above mentioned dislocation of the point of loading division for the loads left and right of the section, we are permitted to use the normal influence line as used for vertical loads to define the improved influence line. For statically determinate systems the abrupt stepping on the influence line only moves with the section for which the influence line is drawn. For statically indeterminate structures in which all influence lines can be interpreted and produced by funicular curves or funicular polygons, it would be necessary to lengthen the normal influence line by a tangent up to the dislocated positions of the loading division through which the new ordinates will be defined. For practical purposes, the influence lines for sections near the crown do not show any change of shape, because the dislocation is only small. This applies also to places where the curves are of small curvature. For the examples treated in the following, alterations to the influence lines were only required as shown in Fig. 17 and 19 for the influence lines M_k for the springing and abutment of fixed arches with curves with strong curvature.

According to the German Regulations braking forces do not require to be increased by an impact coefficient, whilst this is demanded for axle loads. This difference can be considered while working out the improved influence lines. If, for example, an impact coefficient of $\varphi = 1.1$ is chosen, and the braking forces be $\mu = \frac{1}{7}$ of the axle loads, then the ordinates of a normal influence line have to be altered by $\frac{\mu}{\varphi} = \frac{1}{7,7}$ of the corresponding ordinates of the influence line due to braking forces only.

If the decking has a steep angle φ of inclination, then the ordinates of the influence line due to braking forces only require to be multiplied by $\left(\frac{\mu}{\varphi} \mp \sin\gamma\right)$ before being added to the ordinates of the main influence lines, the sign (—) stands for moving down and the sign + for moving up on the incline of the decking. The resultant, composed of φ. P and the corresponding braking force forms an angle with the vertical to the decking which is sufficiently accurately expressed by the term $1 : \frac{\mu}{\varphi}$.

Since the ordinates of the influence lines complemented by the action of braking forces are composed of the contribution of P and S, forming the oblique resultant R, we receive exactly the same ordinates again by dividing the resulting axle pressures into the components P and S, in a plane parallel to the decking but at a lower level. The resultant influence line established in this way will be found to have shifted horizontally only to such an extent as the points of attack for P and S have moved. The computation of the influence lines leads to the same results.

Up to now we have chiefly been concerned with conditions for railway bridges, which are also to form the basis of the following examples. But for road bridges the DIN 1072 prescribe braking forces of $^1/_{20}$ of the weight of a uniformly distributed human crowd over the whole length of the superstructure, or at least for each track 0.3 of the regulation motor lorry loads. The contribution of both influences to the action of braking forces can be determined with the separate influence lines for braking forces for care moments etc., since the points for loading division change unnoticeably little for the small value of $\mu = {}^1/_{20}$. The stibulation that the whole length of superstructure has to be taken into account for the braking forces overlooks the fact that for fixed arches the influence lines for M_k due to braking forces have positive and negative areas. It does not involve much work for road bridges either to draw conclusive influence lines and to consider hereby the braking force equal to ${}^1/_{20}$ of the total human crowd and 0.3 of the load of regulation lorries.

The braking forces for simply supported girders.

The braking force $S = 1$ pertaining to a certain axle load creates an additional bending moment $M_x = -\frac{z}{l} \cdot x$ if acting to the right of section x, shown in Fig 4. If $S = 1$ is acting to the left of section x an additional bending moment $M_x = -\frac{z}{l} \cdot x + h$ is produced.

The influence line M_x, drawn hatched, is for braking forces only. The vertical off-set from the upper to the lower horizontal line lies below the border (zero

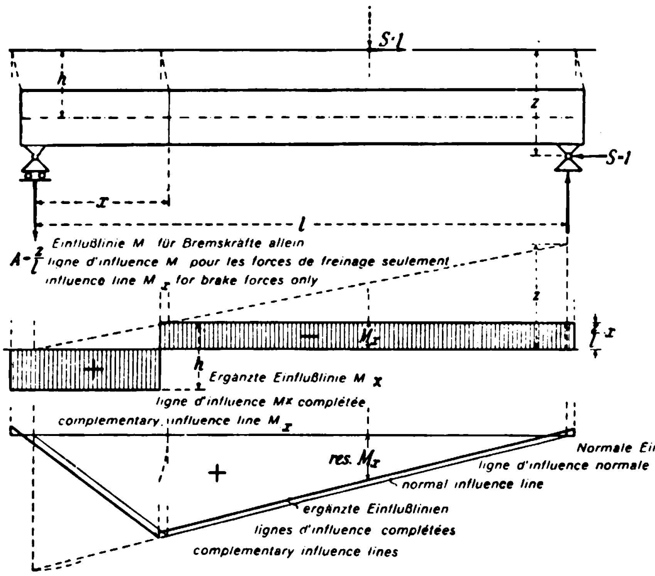


Fig. 4.

Influence line for bending moment M_x due to braking forces for a simply supported beam.

point of the influence line for section x) of the forces with applied brakes left and right of section x, in other words below that point of the decking which is defined by the oblique line under the angle $\frac{\mu}{\varphi}$ to the vertical, starting from the top of section x of the beam.

The final influence line M_x is shown in bold lines and already contains the contributions due to braking forces if the computation is made for axle loads multiplied by the impact coefficient φ.

In addition to the values of M_x there also exists an axial compressive force: $N_x = \mu \sum_0^x P$ for section X. Should this braking force be directed towards the movable bearing A then the opposite sign has to be applied, since the compressive force for section x changes into tension.

The braking force $S = 1$ produces for every position a constant shear force for the whole length of the beam, of $Q_x = -\frac{z}{l}$, thus causing the normal straight-lined influence lines for shear to move up or down for a constant amount of $\frac{\mu}{\varphi} \cdot \frac{z}{l}$, according to the direction of the braking action of the applied forces. The offset in the influence line for shear lies at the same place as that for the influence for moment M_x.

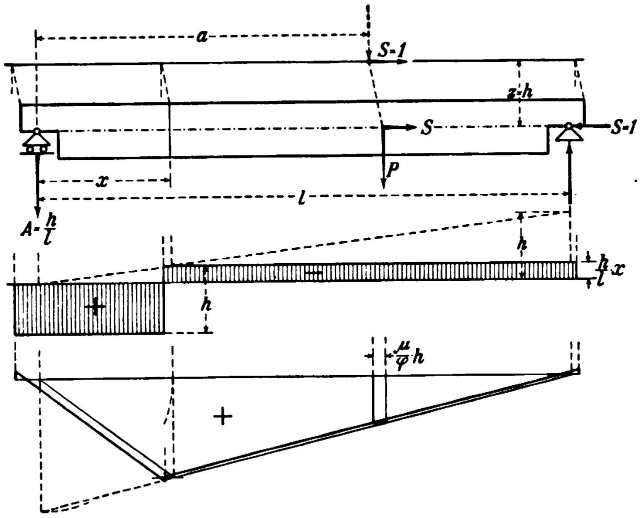


Fig. 5.
Influence line M_x for a simply supported beam with centres of bearings level with the axis of the beam.

If, as shown in Fig. 5, the beam has its bearing centres level with the axis of the beam, then $z = h$, and the steps of the influence lines M_x for braking forces only, have the heights shown in Fig. 5. Alterations to the normal influence line M_x due to the influences of braking forces, cause the two lines to move horizontally for an amount of $\frac{\mu \cdot h}{\varphi}$, so that they cross each other at the same vertical distance below the horizontal axis. The computation for the triangle of the slightly shifted influence line, but not considering the small offset at the point of the triangle, supplies the same values for bending moments as for stationary axle loads, in which case the influence of the braking forces is expressed by anaxial force in the beam of $N_x = \pm \mu \sum_0^x P$.

The same shape of the final influence line for M_x including the effects of the braking forces, is obtained if the line of action of the braked axle load, standing at a, is produced to cross the axis of the beam, and at this place the force is divided into the vertical component P and the braking force S. For this position the force S is without influence, therefore it is only the load P, shifted into the axis of the beam, which for $P = 1$ produces the ordinary, normal influence line M_x. If the ordinate of the moment is platted directly under the load standing on the decking, we find the influence line for M_x moved to the left by $\frac{\mu}{\varphi} \cdot h$, in which case this influence line includes braking forces also.

The braking forces for continuous beams.

If the beam (Fig. 6) rolsts on elastically fixed columns and has one end immovable we can accept the beam as being supported in points along its axis.

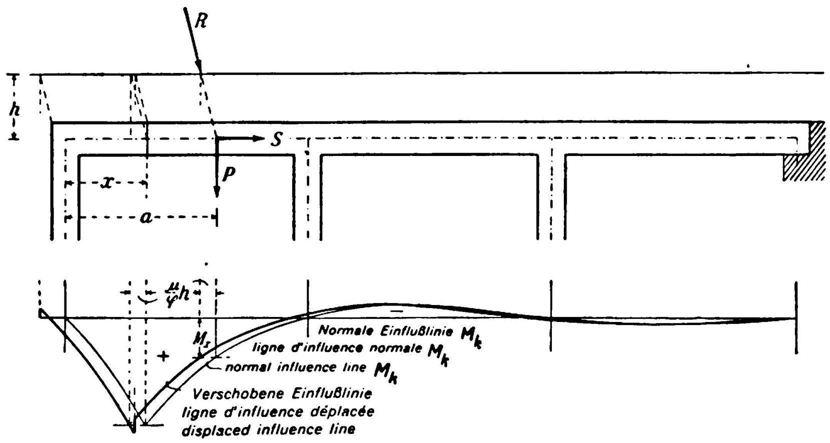


Fig. 6.

Influence line M_x for a continuous beam with centres of bearings level wilh the axis of the beam.

Based on the conditions laid down for the simply supported beam, it can be easily seen that the final influence line M_x is moved sideways by $\frac{\mu}{\varphi} \cdot h$ compared with the original influence line for M_x, giving the same values of moments as for stationary loads.

As regards *shear forces,* the same considerations apply as made for the resultant R of forces acting in the axis of the beam. We therefore receive an influence line for Q_x moved by $\frac{m}{\varphi} \cdot h$ in the opposite direction to the movement of the train compared with the normal influence line for Q_x. The offset between the upper and lower curve lies under that point of the decking which is cut out by a line drawn under the angle $\frac{\mu}{\varphi}$ from the top of section x. The computation of this influence line will hardly give different values than for stationary load.

If the beam supported on elastically fixed supports has no fixed bearing, then the influence of the braking forces for moments and shear will be quite considerable. The influence of every braking force can be obtained with the influence lines for moments and shear forces, provided the influence lines are based on a force = 1 acting in the axis of the beam. The additional influences of braking forces can therefore simply be added to the ultimate values for moments and shear forces derived from stationary loadings. But the ordinary influence line can be improved by the complementary ordinates for braking forces, in which case we find moved the positions for the zero points of influence lines slightly moved.

For the *freely supported continuous beam,* not supported, however, in points of its axis, the consideration made in connection with Fig. 6 at hirst holds good, i. e. the force R is shifted into the axis of the beam, for which position the normal influence lines for moments and shear can be employed as for forces P acting there. But for this arrangement the forces S acting along the axis of the beam demand an additional distribution of bending moments and shear forces produced by the action of the couple S (z — h) on to the fixed bearing (Fig. 2) causing a moment in the beam at this place. As explained above the action of this couple can be considered together with the ultimate values for M_x and Q_x received from stationary loads, or the influence lines for M_x and Q_x can be complemented directly for the action of braking forces. The additional contribution can be taken directly from the bending moment and shear force diagrams. The braking forces also cause a longitudinal force in the beam.

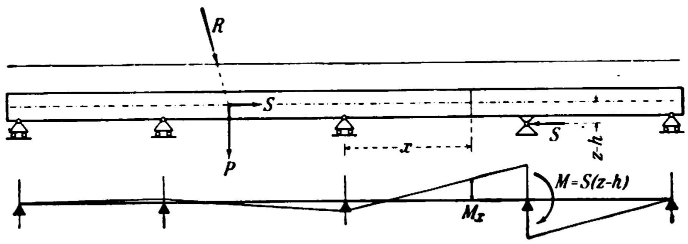


Fig. 7.

Diagram of additional bending moments due to a braking force S placed into the axis of the beam, for e freely supported continuous beam.

The braking forces for simple frames.

For fixed and two-hinged portal frames with horizontal brace and posts of equal height, the oblique force R of the braked axle loads are divided into components P and S at the intersection point with the axis of the brace. Any force S = 1 acting in the axis of the brace produces the same reactions and in consequence thereof a constant amount of additional moments for posts and brace. The braking forces out of any loading position can be collected in one force S, acting in axis of the brace, for which the additional stressing can be calculated. The influence lines for core moments can be complemented directly by contributions due to braking forces.

Braking forces acting on three-hinged arches.

The braking forces acting on such arches influence the stresses in the arch itself and in the abutments, in other words the influence lines for core moments, and the shear and normal forces as well are undergoing changes.

For the sake of distinctness the decking is given an exaggerated inclination in following figures.

Influence lines M_k for moments round the core of the arch.

Based on the considerations made for Fig. 3c, we allow in Fig. 8 the forces P and S to act in the decking and determine the moment M_k produced by a moving

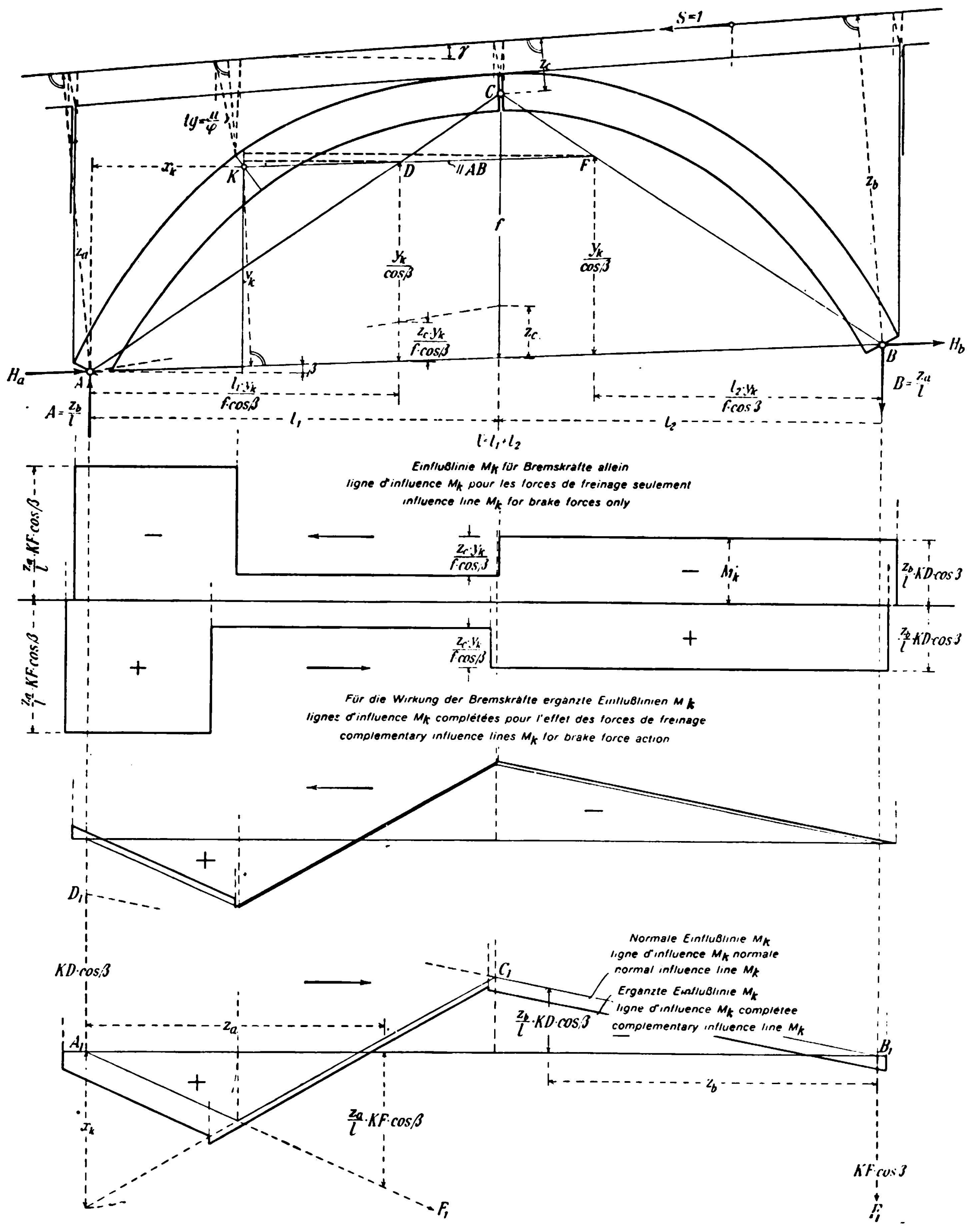


Fig. 8.

Influence lines M_k for braking forces for a three hinged arch with solid or hollow spandril construction with longitudinal partition walls.

load S = 1. For a section in the left half of the span we receive for S = 1 acting on the right half:

$$M_k = A \cdot x_k - H_a \cdot y_k = \frac{z_b}{l} \cdot x_k - \frac{z_b \cdot l_1 \cdot y_k}{l \cdot f \cdot \cos\beta} = -\frac{z_b}{l}\left(\frac{l_1 \cdot y_k}{f \cdot \cos\beta} - x_k\right).$$

hence $M_k = -\frac{z_b}{l} \cdot KD \cdot \cos\beta$.

$KD\cos\beta$ represents the horizontal distance between point D and the vertical through point K. The line KD is parallel to AB.

But if the force S = 1 is situated above the line KC and employing the proper terms for A and H_a the following expression for M_k is found:

$$M_k = \frac{z_b}{l} \cdot x_k - \frac{z_b \cdot l_1 \cdot y_k}{l \cdot f \cdot \cos\beta} + \frac{z_c \cdot y_k}{f \cdot \cos\beta} = -\frac{z_b}{l}\left(\frac{l_1 \cdot y_k}{f \cdot \cos\beta} - x_k\right) + \frac{z_c \cdot y_k}{f \cdot \cos\beta}$$

$$M_k = -\frac{z_b}{l} \cdot KD \cdot \cos\beta + \frac{z_c \cdot y_k}{f \cdot \cos\beta}.$$

The term $\frac{z_c\, y_k}{f \cdot \cos\beta}$ signifies the offset between K and C in the influence lines for M_k for braking forces only, compared with the corresponding influence lines between B and C.

For S = 1 acting over the portion AK of the arch and starting the calculation from the right hand side we find:

$$M_k = -B(l - x_k) + H_b \cdot y_k = -\frac{z_a}{l}\left(l - x_k - \frac{l_2 \cdot y_k}{f \cdot \cos\beta}\right) = -\frac{z_a}{l} \cdot KF \cdot \cos\beta$$

wherein $KF\cos\beta$ represents the horizontal distance of point F from the vertical through k.

The total influence line for point K for braking forces only is therefore composed of the three offsets shown in Fig. 8. The sign of this influence line naturally changes if the movement of the rolling loads reverses to the right. In Fig. 8 we have also shown the influence line M_k for normal loads, containing already the contribution of braking action for a rolling load = 1. The ordinates of the influence lines for braking forces only for up or down movement respectively were multiplied by $\left(\frac{\mu}{\varphi} \mp \sin\gamma\right)$ and added to the normal influence lines. The results are shown in bold lines, having offsets exactly at the same places as the influence lines for braking forces only. These offsets lie under such points of the decking as are established by lines drawn through the top of section K and the top of the crown at the separation of road metalling and concrete filling under an angle $1 : \frac{\mu}{\varphi}$ to the vertical to the decking. The position of the offsets changes with the sense of direction of the braking forces.

An unsymmetrical three-hinged arch with open spandril construction is shown in Fig. 9. For a construction of this kind all braking forces for loads acting on the open spandril work are transmitted either to point R or L (of the arch) where the decking construction connects up with the arch. Columns or cross walls are by far too elastic to transmit directly the action of braking forces to the arches. The section containing the core point K belongs to the left half of the arch, situated below the open spandril portion. The braking force S is again acts in the decking of the bridge. For S = 1, acting only on the right half of the span, nothing is changed in respect to A, H_a and M_k and is the same as for Fig. 8. The same applies for the positions of S = 1 if situated within the portion KC of the arch, since such forces are only transmitted at L, which is to the right of the section under consideration. The influence line for braking

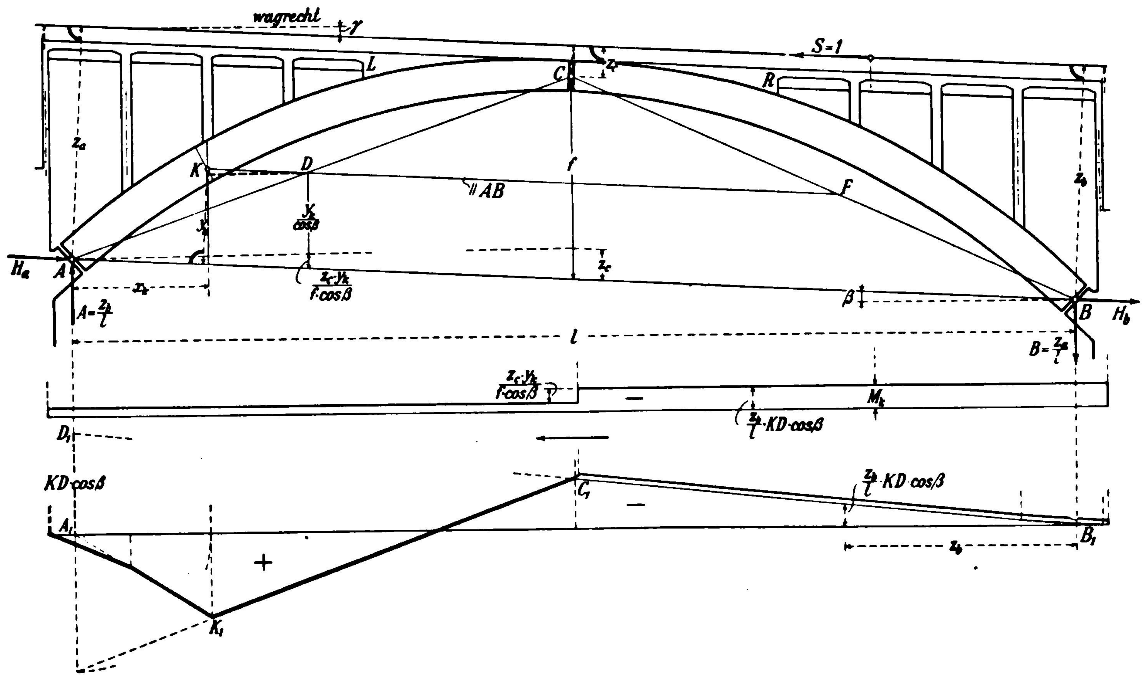


Fig. 9.

Influence lines M_k for braking forces for a three hinged arch with open spandril construction.

forces only is therefore composed of two portions only, see Fig. 9. In this figure, in light and bold lines respectively, normal and composite influence lines for M_k are also shown. If the section of the arch containing point K lies between L and C, then all equations given for M_k apply with reference to Fig. 8, and therefore the stepped influence line M_k for braking forces only has the same shape as for arches with solid spandrils.

It suffices for practical purposes of stress calculation and dimensioning to complement the normal influence lines for M_k for braking action simply by increasing the positive and negative triangles accordingly.

Influence lines M_k for abutments and intermediate piers.

Provided the braking forces have their origin in axle loads placed over the arch, the core moments in the abutment or intermediate piers are determined by the forces passing through the hinges at the springing. These forces are independent of the type of superstructure, therefore the influence lines given by Fig. 10 and 11 are applicable for arches with solid and open spandril construction.

With the denominations of Fig. 10 we receive for S = 1 acting to the left on the left half of the arch:

$$B = \frac{z_a}{l} \quad \text{und} \quad H_b = \frac{z_a \cdot l_2}{l \cdot f \cdot \cos\beta}$$

and hence for a section of the right hand abutment:

$$M_k = B \cdot x_k - H_b \cdot y_k = -\frac{z_a}{l}\left(\frac{l_2 \cdot y_k}{f \cdot \cos\beta} - x_k\right) = -\frac{z_a}{l} \cdot KD \cdot \cos\beta$$

The point D is defined by a line through K, parallel to AB and the line BC,

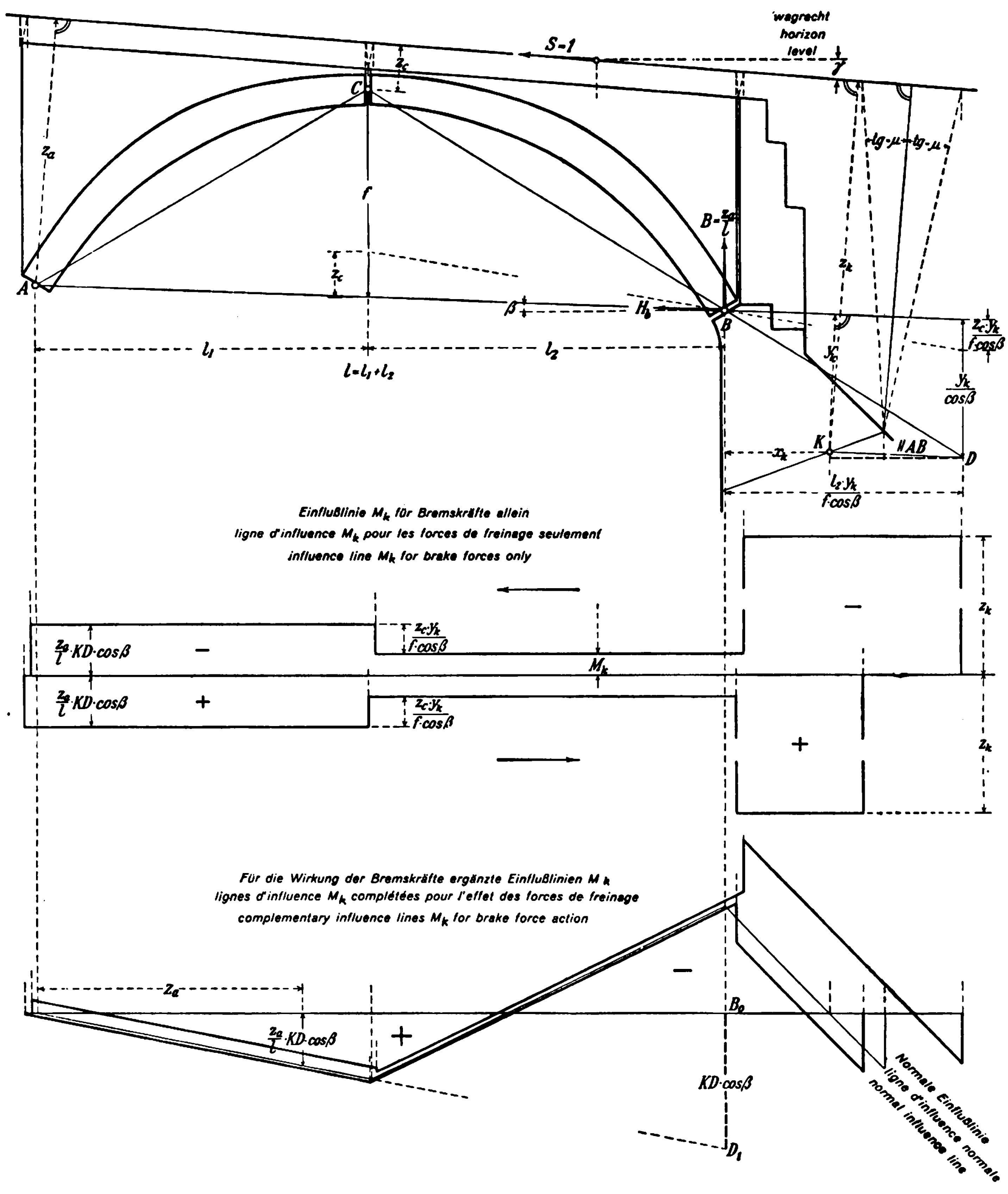


Fig. 10.

Influence lines for braking forces for a section through an abutment.

the term $KD\cos\beta$ marks the horizontal distance of D from the vertical through K.

For S = 1 acting on the right half of the arch we receive:

$$B = \frac{z_a}{l} \quad \text{and} \quad H_b = \frac{z_a \cdot l_2}{l \cdot f \cdot \cos\beta} - \frac{z_c}{f \cdot \cos\beta} \quad \text{and hence}$$

$$M_k = -\frac{z_a}{l}\left(\frac{l_2 \cdot y_k}{f \cdot \cos\beta} - x_k\right) + \frac{z_c \cdot y_k}{f \cdot \cos\beta} = -\frac{z_a}{l} \cdot KD \cdot \cos\beta + \frac{z_c \cdot y_k}{f \cdot \cos\beta}$$

If the force S = 1 acts directly over the abutment, we receive, with the considerations made for Fig. 30, $M_k = -1 \cdot z_k$. The braking forces exert in fact an unfavourable action on to the abutments, since they cause an increase in earth

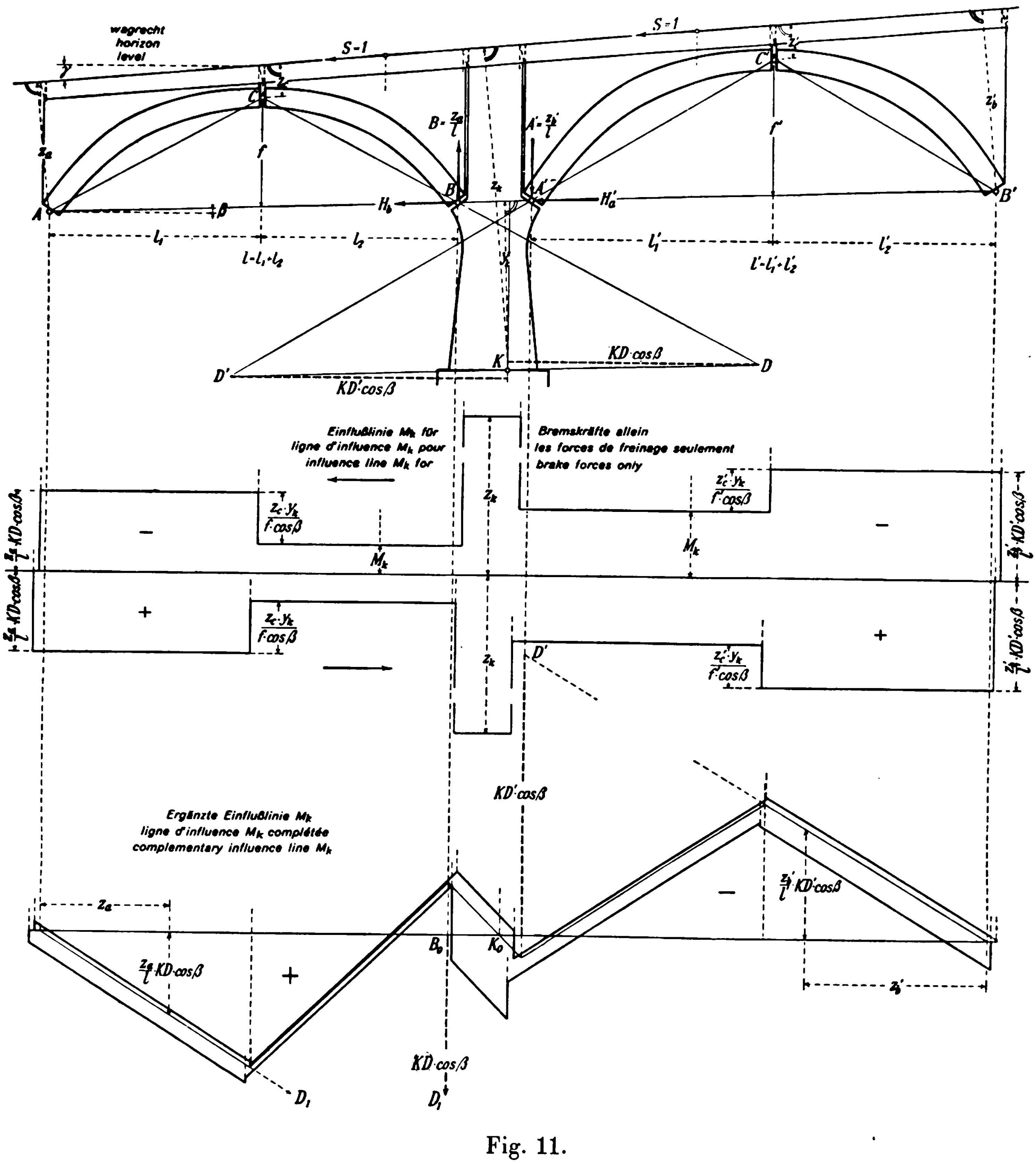


Fig. 11.

Influence line M_k for braking forces for a section through an indermediate pier.

pressure on the retaining wall in front of the dilatation joint. Fig. 10 shows the stepped influence line M_k for braking forces only and the complemented normal influence containing the contribution due to braking forces. These influence lines are for the section passing through K in the abutments. According to DIN 1075 the calculation for abutments and piers shall be made for braking forces not increased by an impact factor, hence in this case $\varphi = 1$. For movement on the down and up-gradient, the ordinates of the influence line composed of straight lines require to be decreased or increased by $(\mu \pm \sin \gamma)$ — times the amount of the ordinates of the stepped influence line for braking forces. The influence lines M_k of Fig. 11 for a section of an intermediate pier are easily derived from Fig. 10 by assuming the intermediate pier to take once the place of the right hand bearing of the left span l and once that of the left hand bearing of the right hand span l'.

Influence lines for normal and shear forces for the hinges of the arch.

The ordinates of this stepped influence line for braking forces only are determined in the easiest way by graphical construction for unsymmetrical arches; for this purpose the force $S = 1$ is placed on the left and right half of the arch and for these positions the reactions at the springing hinges can be determined from the triangle of forces. The reactions will then be divided into two components, one vertical and the other horizontal in respect to the joint of the hinge. The type of spandril construction has no influence on the shape of the influence line.

The polygon of forces (1) in Fig. 12 applies for the case of force S standing over the right half of the arch and the force polygon (2) in case the force S is situated over the left half of the arch. The components N_c and Q_c of the force acting at the hinge of the crown can be determined by the reaction at the springing of the unloaded side of the arch. The normal influence line for forces in the hinges are complemented for braking action (see Fig. 12) by adding $\left(\frac{\mu}{\varphi} \pm \sin \gamma\right)$ — times the value of the ordinates of the stepped influence lines. The joint of the hinges shall, according to DIN 1075 be right angled to the dead-weight pressure line of the arch due to dead weight.

For symmetrical three-hinged arches with horizontal decking it is easy to determine the ordinates of the stepped influence line.

Braking forces on two-hinged arches.

First the influence line for horizontal thrust due to braking forces has to be worked out, this line established, all other influence lines for core points can be derived from it.

Influence line for arch thrust H_a.

The form of an arch deviates very considerably from the pressure line produced by braking forces. It is therefore permissible to neglect the deformations caused by normal forces N_x, which in this case are composed of tensile and compressive forces, hence we can write for the arch thrust:

$$H_a = \frac{\int \frac{M_O \cdot y \cdot ds}{J}}{\int \frac{y^2 \cdot ds}{J}}$$

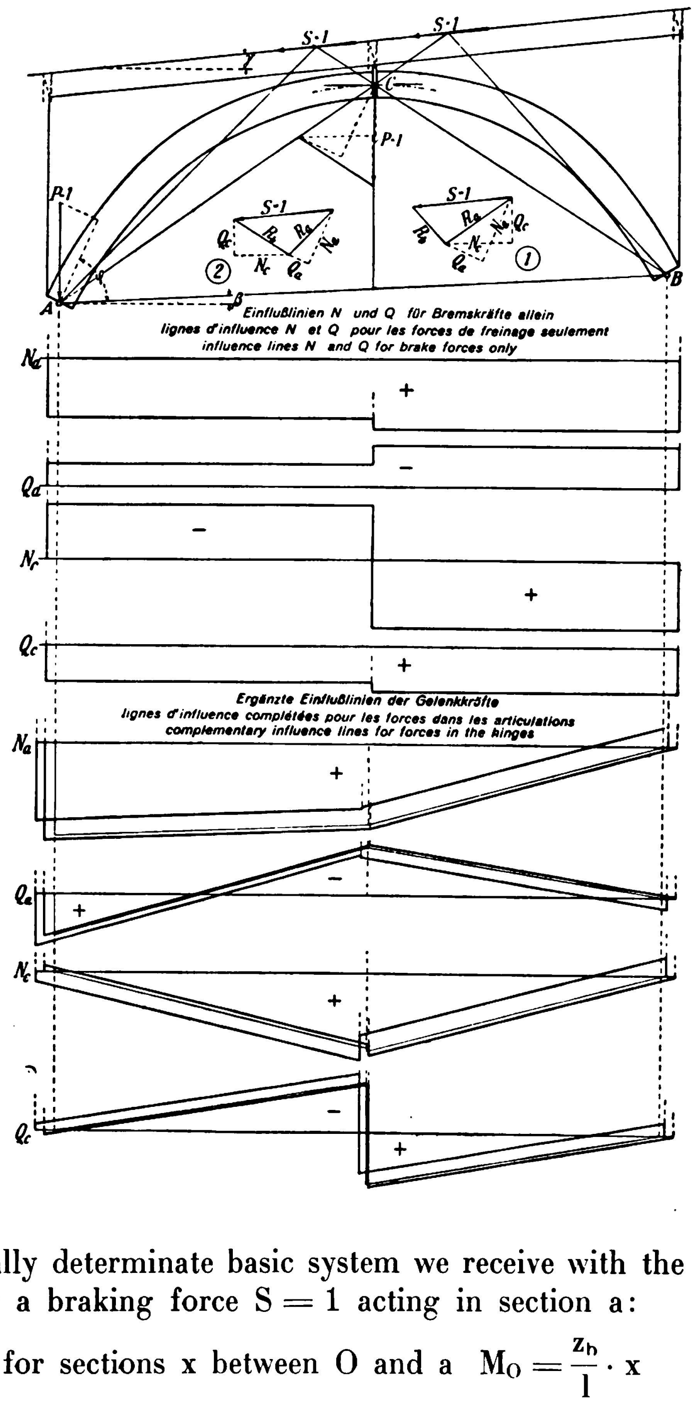


Fig. 12.
Influence lines for normal and shear forces due to braking forces, for the hinged joints of a three-hinged arch.

For the statically determinate basic system we receive with the designation given in Fig. 13 for a braking force $S = 1$ acting in section a:

for sections x between O and a $M_O = \frac{z_h}{l} \cdot x$

for sections x between a and l $M_O = \frac{z_h}{l} \cdot x - z$

hence with these terms the following expressions are obtained:

$$\int \frac{M_0 \cdot y \cdot ds}{J} = \int_0^a \frac{z_b}{l} \cdot \frac{x \cdot y \cdot ds}{J} + \int_a^l \left(\frac{z_b}{l} \cdot x - z\right) \cdot \frac{y \cdot ds}{J} = \frac{z_b}{l} \int_0^l \frac{y \cdot ds}{J} \cdot x - \int_a^l \frac{y \cdot ds}{J} \cdot z$$

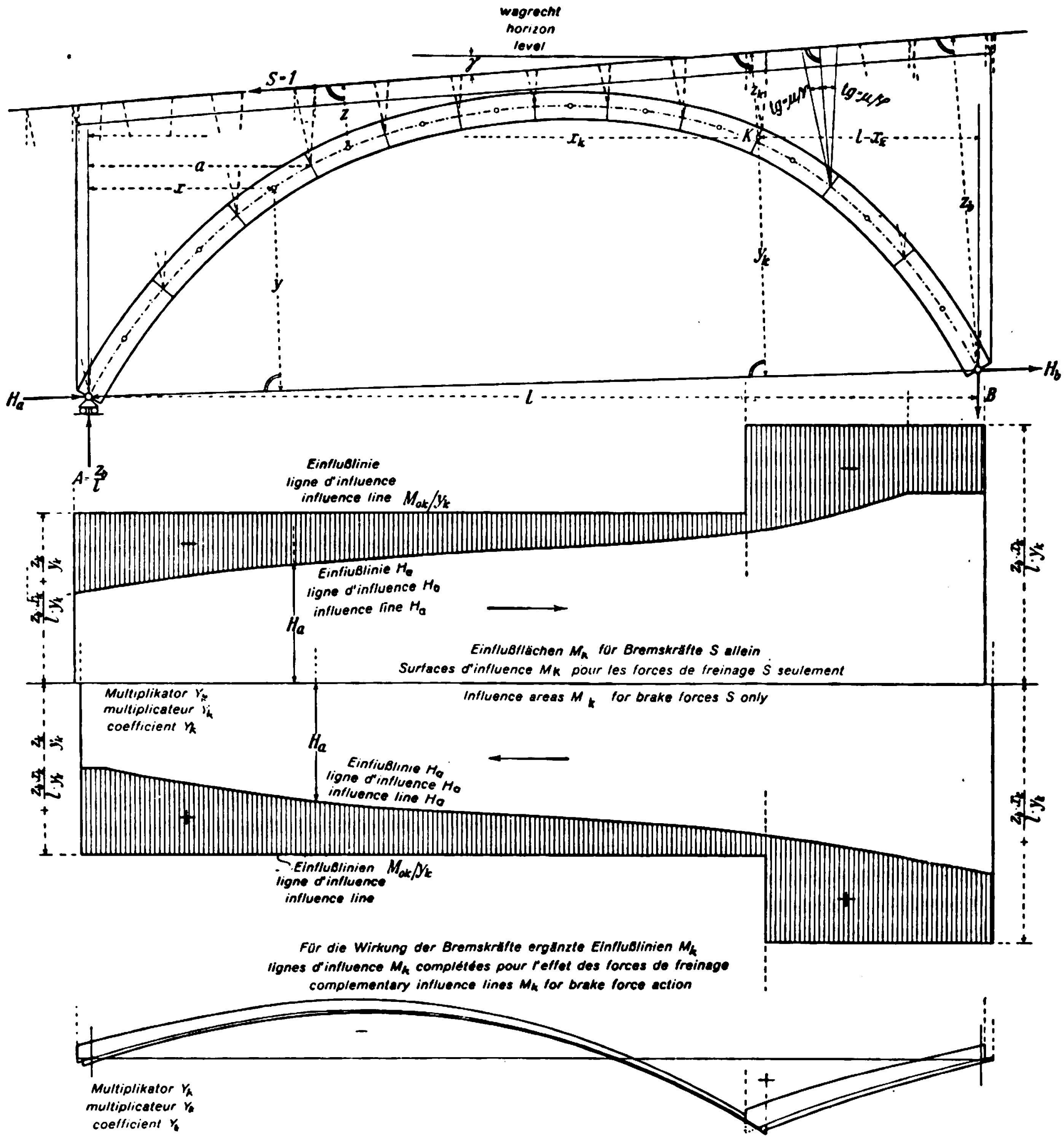


Fig. 13.

Influence lines H_a and M_k for braking forces for a two-hinged arch with solid spandril or spandril construction consisting of longitudinal partition walls.

For finite portions s of the arch, and introducing $w_y = \frac{y \cdot s}{J}$ for the elastic weights, we receive:

$$H_a = \frac{\frac{z_b}{l} \sum_0^l x \cdot w_y - \sum_a^l z \cdot w_y}{\sum y \cdot w_y}.$$

The summations of the nominator are quite easy to calculate, particularly the second summation is best done by starting at the right end by pushing the section a forward to the left step by step, from element to element. The ordinates of the influence line H_a thus received, are plotted from a horizontal line at those particular points where the line from the axis point of the section a under an angle $\frac{\mu}{\varphi}$ to the normal of the decking crosses the horizontal base line.

According to the change in the direction of the braking forces a slight displacement takes place between the ordinates and the influence line.

The influence line for a core moment is given by the following relation

$$M_k = M_{Ok} - H_a \cdot y_k = y_k \left(\frac{M_{Ok}}{y_k} - H_a\right)$$

in the form of the difference between the area of the influence lines H_a and M_{Ok}/y_k.

The computation of the hatched influence line in Fig. 13 has to be done by multiplying the results by y_k. The influence line M_{Ok} for braking forces only is a stepped line, since for forces $S = 1$ to the right of point R we receive

$$M_{Ok} = A \cdot x_k = \frac{z_b}{l} \cdot x_k,$$

and for $S = 1$ left of K the influence line for braking forces is expressed by

$$M_{Ok} = A \cdot x_k - z_k = \frac{z_b}{l} \cdot x_k - z_k$$

In Fig. 13, bottom, the normal influence line $\frac{M_k}{y_k}$ for a section belonging to the right half of the arch is shown in these lines. The bold lines however are a composition of the normal influence line $\frac{M_k}{J_k}$ by adding $\left(\frac{\mu}{\varphi} \pm \sin\gamma\right)$ — times the value of the ordinates of influence lines for braking forces only.

For the normal influence line H_a it is of no practical avail if the points of attack for $P = 1$ coincide with those for $S = 1$, in other words are no longer placed directly vertically above the dividing points. Vertical offsets are formed directly under point K between the two curves. The location of these offsets coincides with the stepping in the influence line M_{Ok} for braking forces.

For symmetrical arches and those unsymmetrical arches in which the line through the hinges at the springing is parallel to the decking, the ordinates of the influence lines H_a are complementary for symmetrically placed elements (dividing points) in respect to l. It therefore suffices to calculate the ordinates only for one half of the bridge.

In connection with Fig. 3c it was found that the resultant out of axle load and braking force can be divided into its components again at any point of its line of action and that these components produce the same compound action on any point of a statically indeterminate system as if the resultant itself were acting. On account of this, in the present case, the oblique axle loads can also be applied in a plane parallel to the decking for instance a parallel plane tangent to the crown, where these forces will be divided into the components P and S.

If the calculation as described is carried out for H_a and M_k due to braking forces S only and finally the composite influence line is formed for core moments, we find that the influence lines thus produced are shifted slightly sideways by the line of action of the braked forces when crossing the two parallel planes. The computation of the influence lines thus produced furnishes the same limit values.

For arches with open spandril construction it is not possible to choose arbitrarily the position of this reference plane, since the braking forces for an open arch superstructure are transmitted trough the decking to the points L and R, where they act directly on to the arch, the decking being rigidly anchored to the arch at these points. To receive the correct moments M_o for the statically determinate basic system it is necessary to transfer the braked axle loads to the central plane of the decking and this applies also for the portion in which the decking is resting directly on the arch. In this plane the forces are divided into the components P and S, so that for the force S = 1 the axle distances of the loads have not to be changed while computing the final influence line.

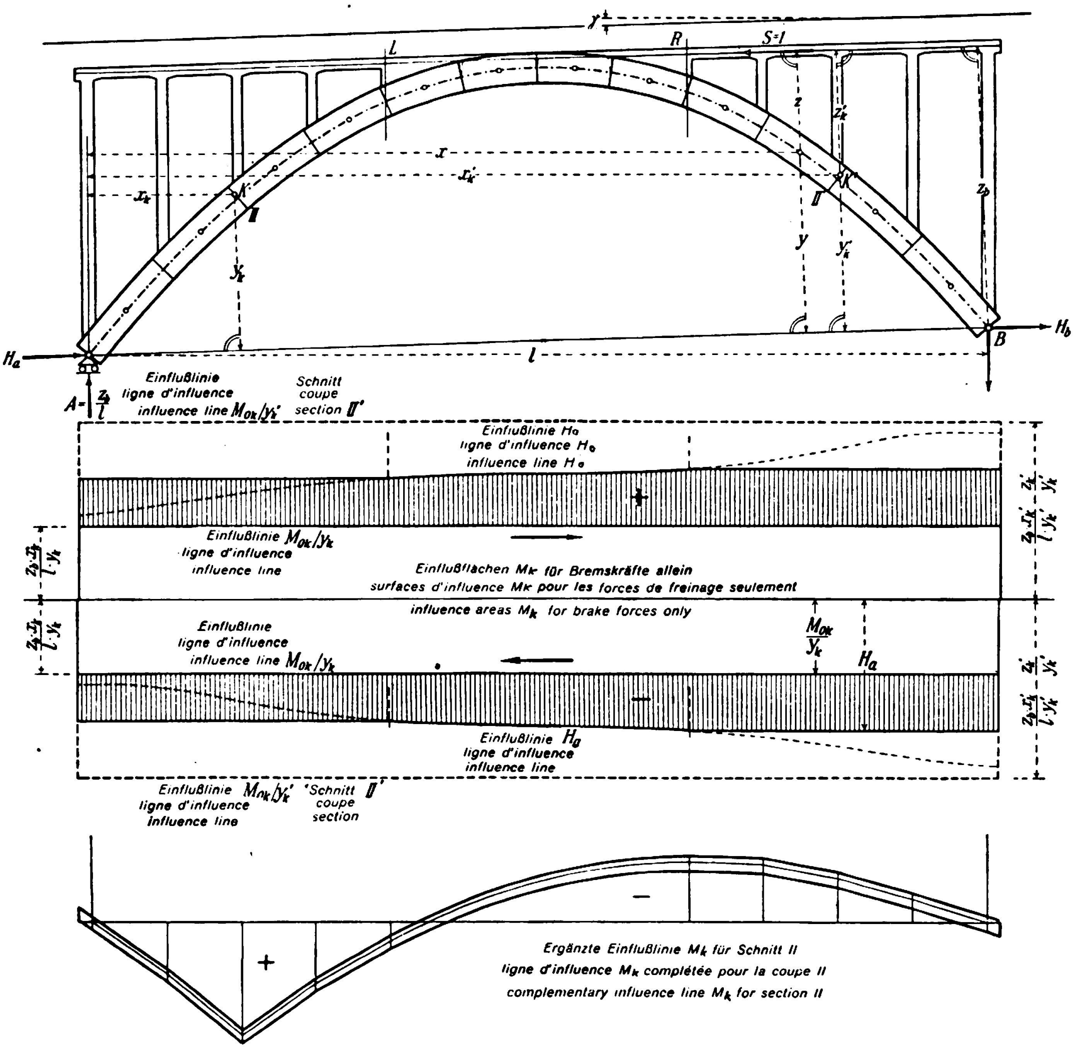


Fig. 14.

Influence lines H_a and M_k for braking forces for a two-hinged arch with open spandrils.

This plane has to be maintained for the same reason when working out the influence line M_k for abutments and piers. In Fig. 14 an arch with oblique symmetry with open spandril construction is shown, where the decking is anchored to the arch at L and R. It is quite obvious and easy to realise that the integration of the nominator in the formula for H_a does not change the forces $S = 1$ if acting between L and R, and remains identical for this stretch as for arches with solid spandril construction. For any other position of S acting on the open spandril sections the integrations of the nominator are calculated as if the force S were acting in L or R. This is the reason for the influence line H_a having two horizontal sections from L to A and from R to B respectively.

The influence line M_{Ok} for braking forces only, assumes a similar shape for arch sections situated between L and R as in Fig. 13. But if the section containing the core point R has its position under the open spandril, between A and L, then for any position of the force $S = 1$ the moment $M_{Ok} = A \cdot x_k$. The influence area for these positions is therefore a rectangle with a height of $\frac{z_b}{l} \cdot x_k$. The same applies if the section of the arch is situated between R and B, only in this case the rectangle of the influence line has a height of $\frac{z_b}{l} \cdot x_k - z_k$. The hatched area in Fig. 14 represents the influence line area M_k for section II for braking forces only, being the difference between two areas represented by

$$M_k = y_k \left(\frac{M_{Ok}}{y_k} - H_a\right)$$

For computation the factor y_k has to be considered.

Influence lines M_k for abutments and intermediate piers.

The force $S = 1$ acting to the left and placed in the middle plane of the decking, produces in the case of open spandril work, in a section for left abutment a core moment at K as expressed by:

$$M_k = A \cdot x_k - H_a \cdot y_k = \frac{z_b}{l} \cdot x_k - H_a \cdot y_k.$$

With this it is easier to calculate the ordinates for the influence line M_k as is done by forming the difference between two influence line areas, since the ordinates of the influence line for the abutment are $M_k = - z_k$; in other words equal to the normal distance from K to the plane passing through the centre of the decking. This plane, as previously mentioned, is also the plane of reference for influence lines for the abutments (Fig. 15).

For a section through the right abutment we receive

$$M'_k = B \cdot x'_k - H_b \cdot y'_k = \frac{z_a}{l} \cdot x'_k - H_b \cdot y'_k.$$

Based on these formulae are shown in Fig. 15 the calculated influence lines M_k and M'_k for braking forces only. The extreme ends left and right of these influence lines lie under those points of the central plane through the decking which are cut out by lines under the angle μ to the normal of the decking passing through the outer edges of the respective sections of the abutments. The complemented influence lines are shown also.

The influence line M_k for a section through an intermediate pier is based on the information given by Fig. 15. The intermediate piers require only to be considered as taking alternatively the place of the abutments of the adjoining

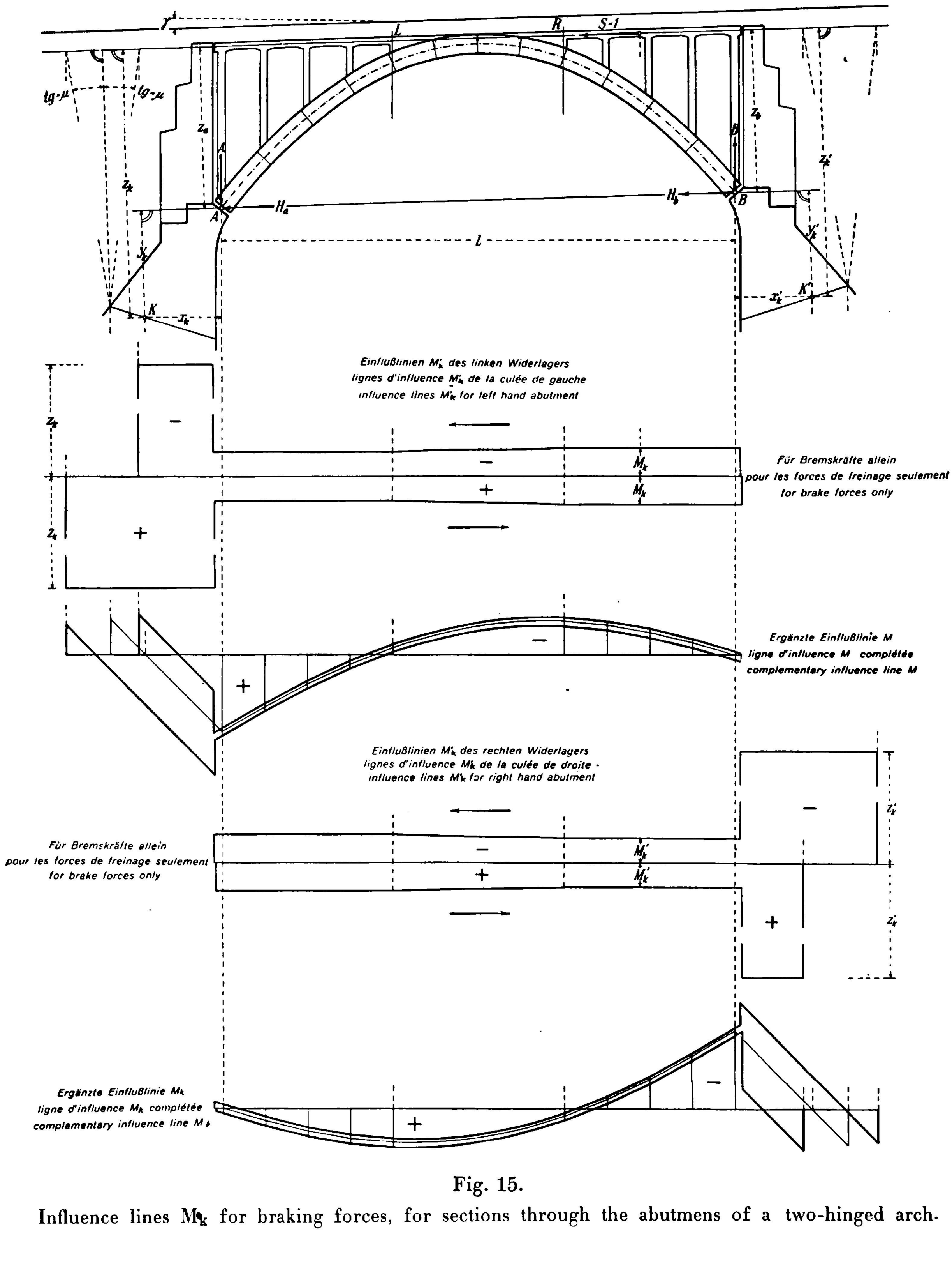


Fig. 15.

Influence lines M_k for braking forces, for sections through the abutmens of a two-hinged arch.

spans. The condition is only (as already assumed above for the abutments) that the intermediate pier is comparatively low and of a squat nature, for which elastic deformations can be neglected.

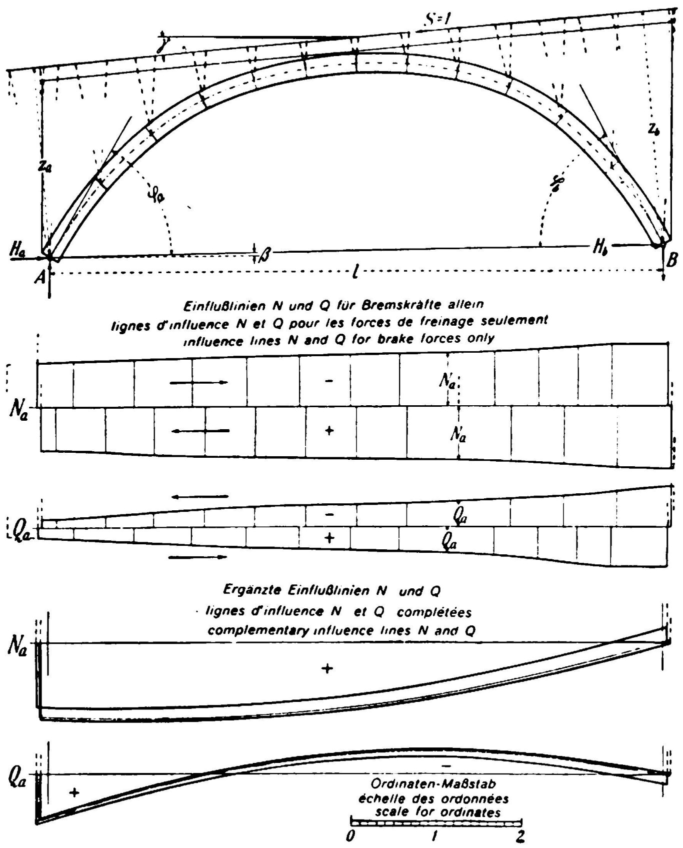


Fig. 16.

Influence lines for the normal and shear forces in the left hand hinged joint of a two-hinged arch, due to braking forces.

Influence lines for normal and shear forces for the hinge at the springing.

For a two-hinged arch as given by Fig. 16, the angles φ_a and φ_b are those which are formed by the reactions at the springing and the line connecting the springings. The plane tangent to the bearings at those places shall stand at right angles to the reactions.

The influence line for the normal force N_a acting on the left hinge due to a braking force $S = 1$ pointing to the left, is based on the following equation

$$N_a = A \cdot \sin(\varphi_a + \beta) + H_a \cdot \cos\varphi_a = \frac{z_b}{l} \cdot \sin(\varphi_a + \beta) + H_a \cdot \cos\varphi_a.$$

For the influence line for shear force the equation applies:

$$Q_a = A \cdot \cos(\varphi_a + \beta) - H_a \cdot \sin\varphi_a = \frac{z_b}{l} \cdot \cos(\varphi_a + \beta) - H_a \cdot \sin\varphi_a.$$

The influence lines in Fig. 16 for N_a and Q_a have been calculated on the basis of these formulae.

The braking forces for fixed arches.

For the same reasons as for the two-hinged arch the influence on deformations due to normal forces N_x can be dispensed with in the formulae for reactions. The investigation is based on a basic system formed by an arched cantilever arm with the fixed end at the right hand side. To this statically determinate system apply the wellknown formulae (*Mörsch* 2nd vol. part 3)

$$H = \frac{\sum M_O \cdot w_y}{\sum y \cdot w_y} \qquad V = \frac{\sum M_O \cdot w_x}{\sum x \cdot w_x} \quad \cdot \quad M = \frac{-\sum M_O \cdot w}{\sum w}$$

provided the components of the reactions at the free end of the cantilever have been moved into the elastic centre of the arch. The formulae are valid for any positions and directions of forces acting in the plane of the arch. A braking force acting at a and pointing to the left (Fig. 17) produces in the cantilever arch of the basic system bending moments $M_o = - z$, in the region from a to B only. To establish the influence lines H, V, M, for braking forces it is only necessary to form for progressive values of a the summations:

$$\sum M_O \cdot w_y = -\sum_a^{-\frac{1}{2}} z \cdot w_y \qquad \sum M_O \cdot w_x = -\sum_a^{-\frac{1}{2}} z \cdot w_x \qquad -\sum M_O \cdot w = \sum_a^{-\frac{1}{2}} M_O \cdot w$$

The easiest way to from these sums is by calculation, starting at the right end of the cantilever arm for the force $S = 1$ acting in turn above every section of two adjoining elements of the arch. For all these progressive positions the sum of the static moments of the elastic weights

$$w_y = s \cdot \frac{y}{J}, \quad w_x = s \cdot \frac{x}{J} \text{ and } w = \frac{s}{J}$$

are formed to the right of the position of the force in respect to the decking, according to the type of superstructure above the arch proper. For every progressive step of the force S only one term has to be added to the previous sum. The elastic weights w, w_y and w_x and the sums of the denominator are already known from the preceding arch calculation. The ordinates of the influence lines have to be plotted under the abscissae a, i. e. below those points of the decking which are defined by the dividing line starting from the arch axis under the inclination $\frac{\mu}{\varphi}$ to the vertical. According to the sense of direction of the braking forces not only the sign changes but the influence lines also receive a slight horizontal movement in respect to each other.

The influence lines established by calculation for the components H, V and M of the reaction at the springing, for a symmetrical fixed arch with horizontal decking, are given in Fig. 17. The left ordinate of the influence line H is equal $= 1$ and correspondingly for the influence line for M this ordinate has the value $= y_o + z_s$. The analytical computation in tabulated form shows that a large number of figures are the same as those for the normal influence lines. The ordinates of the influence line for H are complementary to 1 for sections placed symmetrically.

By means of the ordinates of the influence lines for H, V and M the influence lines for core moments for any section of the arch due to braking forces can be produced by using the formula:

$$M_k = M_{Ok} + M - H \cdot y_k - V \cdot x_k$$

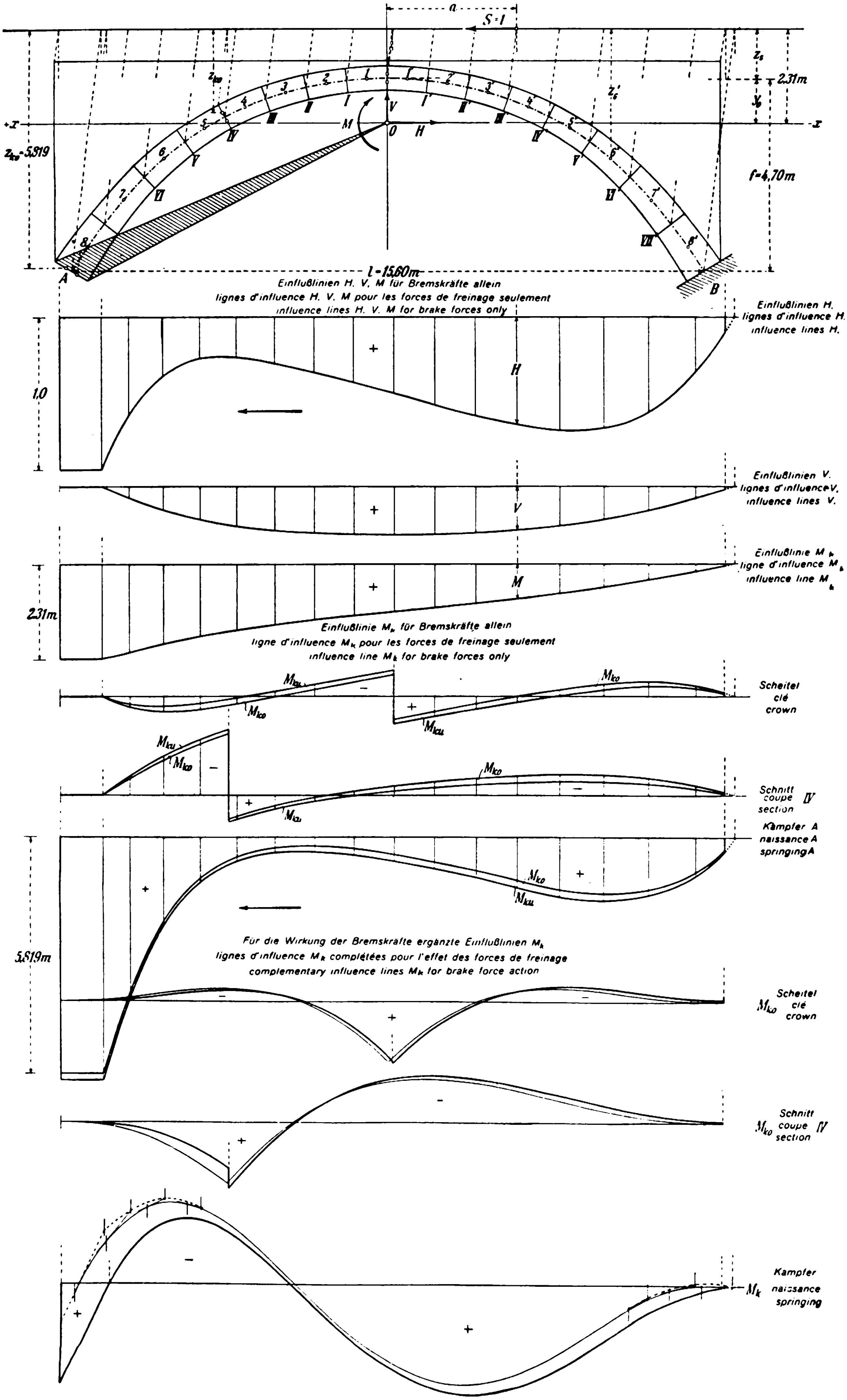
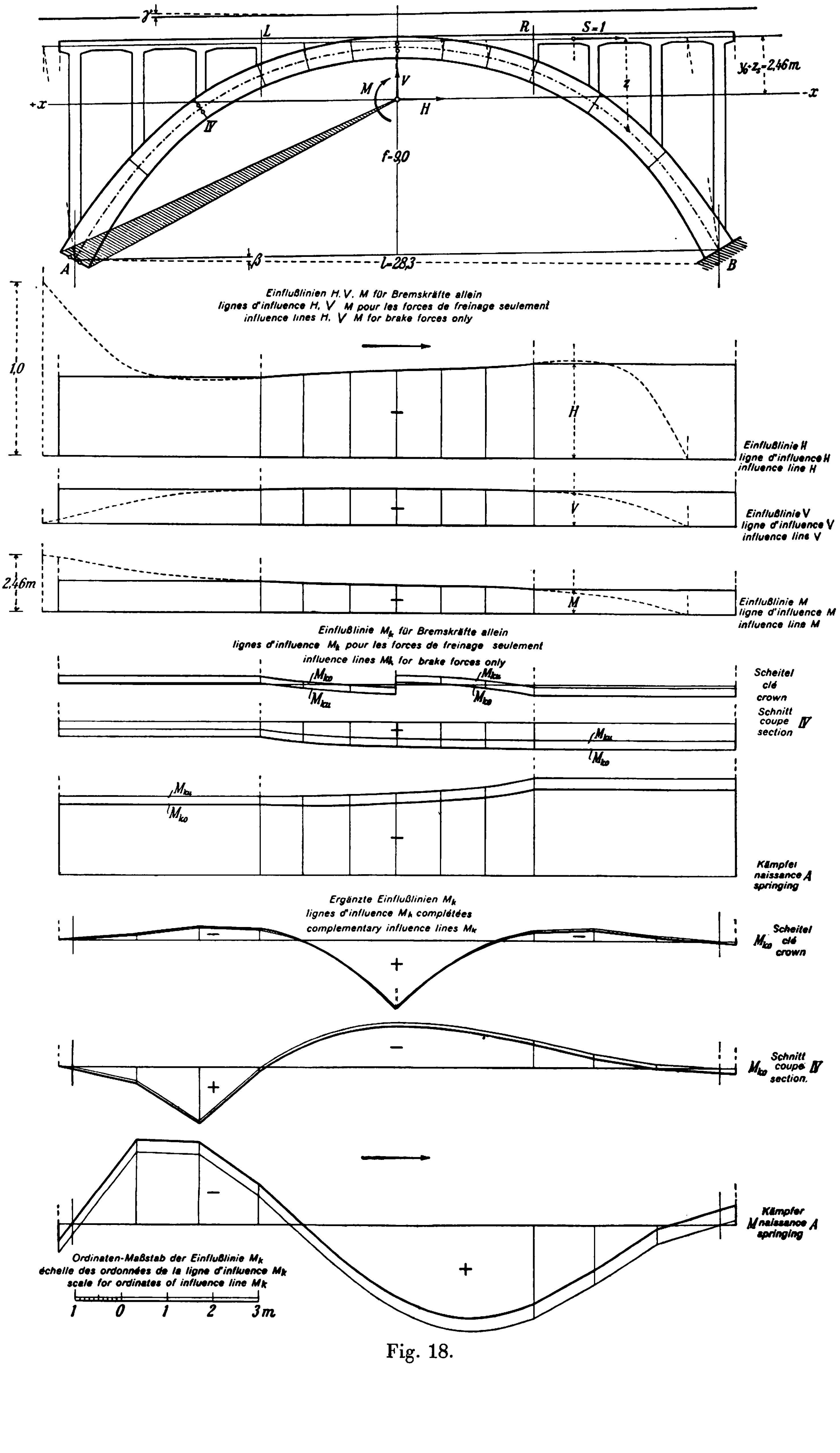

l = 15,60m
f = 4,70m
2,31m
Einflußlinien H. V. M für Bremskräfte allein
lignes d'influence H. V. M pour les forces de freinage seulement
influence lines H. V. M for brake forces only
Einflußlinien H.
lignes d'influence H.
influence lines H.
Einflußlinien V.
lignes d'influence V.
influence lines V.
Einflußlinie M_k
ligne d'influence M_k
influence line M_k
Einflußlinie M_k für Bremskräfte allein
ligne d'influence M_k pour les forces de freinage seulement
influence line M_k for brake forces only
Scheitel
clé
crown
Schnitt
coupe IV
section
Kämpfer A
naissance A
springing A
Für die Wirkung der Bremskräfte ergänzte Einflußlinien M_k
lignes d'influence M_k complétées pour l'effet des forces de freinage
complementary influence lines M_k for brake force action
Kampfer
naissance
springing

Fig. 17.



Einflußlinien H. V. M für Bremskräfte allein
lignes d'influence H. V M pour les forces de freinage seulement
influence lines H. V M for brake forces only
Einflußlinie H
ligne d'influence H
influence line H
Einflußlinie V
ligne d'influence V
influence line V
Einflußlinie M
ligne d'influence M
influence line M
Einflußlinie Mk für Bremskräfte allein
lignes d'influence Mk pour les forces de freinage seulement
influence lines Mk for brake forces only
Scheitel
clé
crown
Schnitt
coupe
section
Kämpfer
naissance
springing
Ergänzte Einflußlinien Mk
lignes d'influence Mk complétées
complementary influence lines Mk
Ordinaten-Maßstab der Einflußlinie Mk
échelle des ordonnées de la ligne d'influence Mk
scale for ordinates of influence line Mk
1 0 1 2 3m

Fig. 18.

To achieve this it is best to start at the right end, applying the force $S = 1$ in turn at every division point, and to tabulate the results in the same way as for normal influence lines: $M_{0k} = - z_k$ for a force S pointing left if acting left of the arch-section containing the point K, but if acting right of this section then $M_{0k} = 0$. In this manner the influence lines M_k seen in Fig. 17 for braking forces only, for the crown, springing and section IV were established. Further are shown the final influence lines for these sections containing the effects of braking forces. The scale for the ordinates of influence lines shown in light lines is double the scale of the drawing for the arch. The bold lines are for complemented influence lines containing $\frac{\mu}{\varphi}$ times the values of the ordinates of the influence lines M_k for braking forces only.

It was also necessary to correct the ends of the normal influence line for the springing, for the vertical component of axle load belonging to S comes to stand much outside the respective division line with the arch axis, the tangents to the division points at the curve had therefore to be produced up to the position of the corresponding loads P. The curve shown in dotted line represents the correction to the normal influence line M_k. The corrections at other places to the normal influence line are so minute that it is better to ignore them altogether.

It may also be mentioned that the usual basic system of cantilever arm fixed at the left end also proves very useful in calculating with braking forces.

The equations for the components H, V and M of the arch reaction for braking forces $S = 1$ remain unaltered, even for unsymmetrical arches and arches with oblique symmetry. The x-axis is inclined in such cases, and the ordinates z are represented to be the normal distance of the arch elements from the decking.

The arch shown in Fig. 18 has an open spandril construction as the arch of Fig. 14. The inclined braked axle loads require also in this case to be applied in the middle plane of the decking where they are divided into the components P and S. The forces S are transmitted to the arch only at L and R. On account of this, the ordinates of the influence lines for H, V and M under the open spandrils remain constant, equal to the ordinates of point L and R respectively. The flow of the influence lines between L and R is similar to that of the influence lines for solid spandril construction, only that in this case the reference plane for the forces S lies at a lower level.

The influence lines have been given for braking forces pointing to the right.

The influence lines shown dotted are those which would apply where the spandril construction is solid.

Influence lines M_k for the abutments of fixed arches.

Using the denominations given in Fig. 19 the core moments in a section through the left abutment for braking forces $S = 1$ are expressed by:

$$M_k = M + H \cdot y_k - V_k,$$

wherein the sign of the components has been considered.

For a section through the right hand abutment we receive:

$$M'_k = z'_k - M - H \cdot y'_k - V \cdot x'_k.$$

It was on these two formulae that the ordinates of the influence lines for braking forces for the respective section trough the abutments were calculated. The ordinates of the influence lines below the abutments are z_k and z'_k respecti-

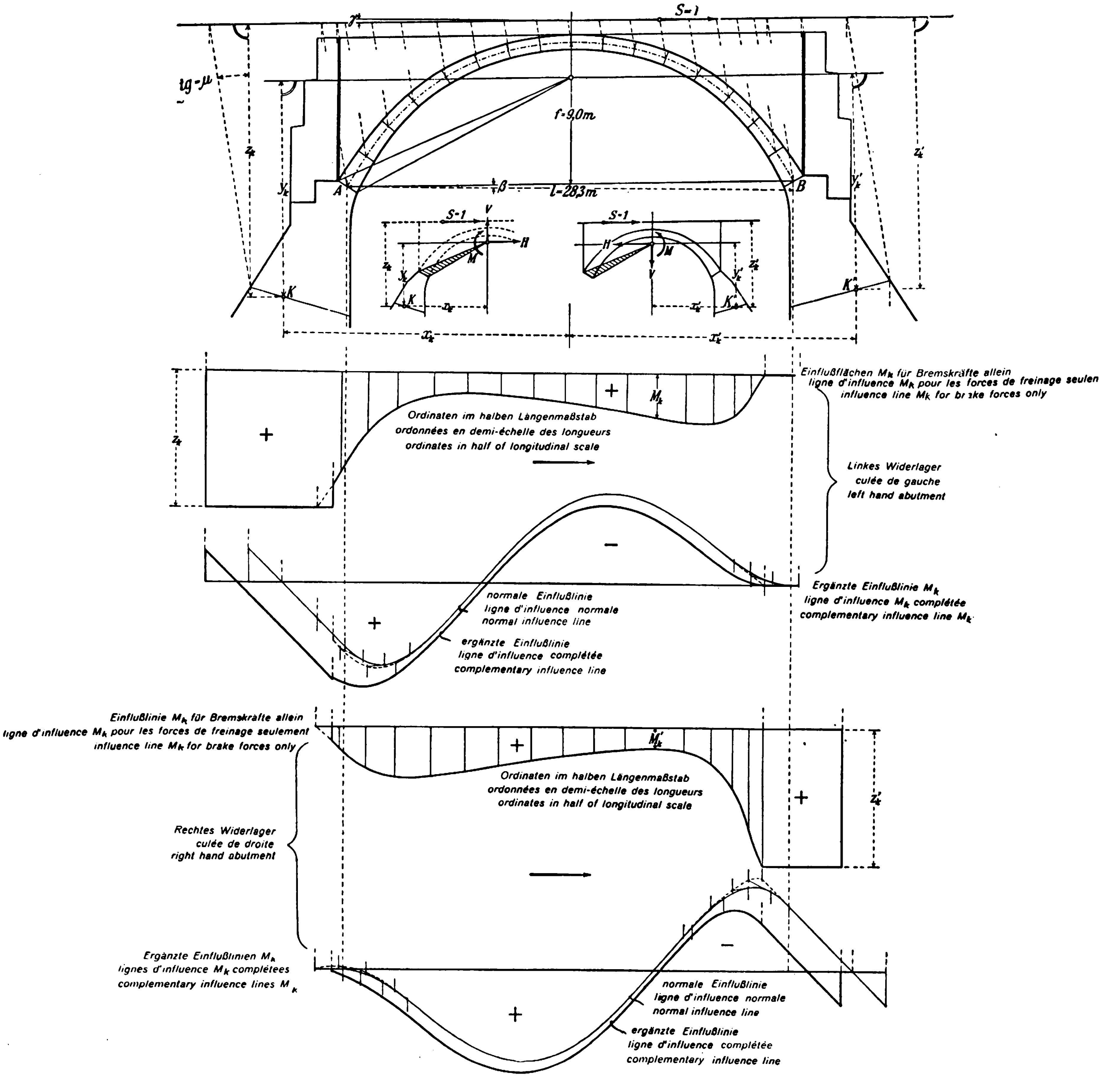


Fig. 19.

Influence lines M_k due to braking forces for sections through the abutments of an fixed arch of oblique symmetry.

vely. By adding μ-times the value of the ordinates for braking forces only to the ordinates of the ordinary influence lines we receive the final influence lines. It is however necessary, as in the case of Fig. 17, to correct the ordinary influence lines for the portions under the abutments on account of the massive construction, previous to adding the values of the ordinates for braking forces only.

In a similar manner the influence lines for sections trough an intermediate pier are received by considering the intermediate pier as taking in turn the position of end abutments for each adjoining span. It is only necessary for the pier to be of a stout nature, such that elastic deformations can be neglected.

Summary.

A study is made of the influences of braking forces on various types of beams and arches. The braking force is considered to be a part of the axle load. The effects of braking forces on moments, core-moments, reactions, are given in the form of influence lines; finally these influences are combined with those resulting from the effects of vertical axle loads, thus supplying by computation with the normal (standard) train of loads the ultimate values, including those resulting from the action of braking forces. The shape of the influence line for braking forces only varies with the type of spandril construction over the arch proper; it is different for solid and open spandril work.

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IVb 6

Present-day Tendencies in Large-sized Reinforced Concrete Constructions.

Neuere Gesichtspunkte für den Bau großer Eisenbeton-Bauwerke.

Tendances actuelles dans les grands ouvrages en béton armé.

Dr. Ing. C. Parvopassu,
Professeur à l'Ecole Royale d'Ingénieurs de Padoue.

Introduction.

The theme of the task, with the outlining of which before this gathering of distinguished engineers I have been honoured is an exacting one, since it calls for an extensive knowledge of one of the very fields in which modern structural engineering has proved most fruitful in difficult achievement.

Indeed it is a task impossible of satisfactory accomplishment without profound knowledge of and close familiarity with one of the most complicated and advanced fields of science and structural practice, and I can do no more than hope to express myself not unworthily by drawing upon the records of the study I have pursued in thirty and more years of keenest devotion, — the study of large structures in plain and reinforced concrete — both by theoretical and experimental teaching, and on site.

The claims, advantages and scope of reinforced concrete.

The use of reinforced concrete for the construction of works of any notable size dates back less than fifty years ago, namely to the time when it became possible to overcome those misgivings and criticisms to which every new idea gives rise, even among men of recognised competence. This was due to the progress made in the industrial production of slow-setting cements (both natural and artificial), an exacter understanding of the behaviour of the latter in the concrete, and the attainment of an accurate knowledge of those physical properties which enable concrete and steel to collaborate effectively in resisting the complicated stresses due to tension, compression and thrust that arise in structures exposed to bending, shear and torsion, and also due to the direct forces of compression and tension.

The unaccustomed boldness of what was being done gave rise to the impression, at first, of a disregard for those criteria of prudence which are normally and rightly given precedence over speed of construction and over economy in cost. This, however, was true only in appearance, and in a short time great progress had been realised all over the world in building large works; most notably road bridges and viaducts.

These developments of the new method of construction also became conspicuous in Italy already in the beginning of the century, for structures of medium and large size, and may justly be regarded as forming an advance guard in the attainment of various exceptional features. Italian technicians in this field are now an active and industrious body well equipped to accomplish even bolder and more remarkable works by turning the high qualities of Italian materials ot the fullest account, employing medium and high strength cements to bond together the excellent stone and the steel bars used as reinforcements in the concrete. Reinforced concrete is now accepted as offering the best qualities for combining solidity, beauty, compactness and gracefulness with speed of execution and ease of maintenance.

It is argued that the strength and permanence of cemented construction must be dependent upon the mixing of the concrete, of the arrangement of the steel reinforcements, and the maintenance of the latter in their exact positions while the concrete is being poured. This fact, however, has only the same importance as have the many delicate questions which arise in connection with steelwork construction regarding the special qualities of the steel used for rolled plate and sections, rivets, bolts, suspension cables, and bearings, and as to the proper execution of riveting or welding. In either case experience, conscientiousness and honesty have to be assumed as normal attributes of the constructor concerned; suitable individuals of proved technical capacity have to be chosen to fill each role in the planning and direction of the work whether great or small, and it is necessary to employ specialised workmen of a type who love their job and are imbued with the team spirit. Another essential, of course, is careful and prudent checking of all material supplied during the construction of the work.

Having taken these precautions, no fear need be entertained in choosing reinforced concrete as the material to be used in those works of ever increasing size which modern civilisation demands from the technician. Its marked economic advantage may be taken as certain, and it remains only to express the hope that the architectural engineers will succeed in endowing the cemented form of construction with a style which harmonises with the laws of aesthetics.

What is meant by large works in reinforced concrete are mainly those connected with road work, in the form of bridges, viaducts, retaining walls and foundations for these, but the term of course covers also building construction — especially earthquake-proof buildings — of large dimensions, such as columns, beams and cantilevers, large roofs and cupolas, skyscrapers, silos, towers, bell towers, transmission line masts and aerials, as well as hydraulic and maritime structures such as high multiple-arch dams, reservoirs aqueducts, intakes, pipe lines of large diameter, harbours, dry docks, breakwaters and jetties, and many other structures which are ancillary to industrial plants on an extensive scale.

It may now be claimed for this vast field of work that the methods of design, the quality of the materials and the procedures followed on the site have reached such a stage of development and show such promise of further improvement as to justify the expectation, in no distant future, of works of surprising size and beauty.

The evolution of large works in reinforced concrete.

A rapid survey of the course of development of major ,works in reinforced concrete during the past thirty years may be of value for a better understanding of the present trends in their construction and design, because as already stated, the boldness of what was being undertaken even at the beginning of this period is such as to compel the admiration of engineers of today and to justify the most sanguine hopes for tomorrow.

The construction of the Pinzano bridge over the Tagliamento dates from 1906. This structure comprises three great arches each of 48 m span and 24 m rise which contain a total of 1800 m^3 of cement concrete and are provided with self-supporting reinforcement of rolled steel sections in the form of triangulated trusses. The arches are fixed at the ends and are hinged at the crown only, but actually the axis is so shaped — having been designed in such a way as to deviate little from the shape of a possible line of thrust — that the calculation could be made on the assumption that they were statically determinate like three-hinged arches. The roadway decking, which is carried on flat spandrel arches, has a width of 6 m.

The Calvene bridge on the Astico was built in 1907. This is a fixed arch of 35 m span and 2 m rise, which is made monolithic with the abutments and with the decking. This was shortly afterwards followed by a similar bridge over the Ourthe at Liége which has a span of 55 m and a rise of 3.25 m.

The three years from 1909 to 1911 are dominated by the remarkable Risorgimento Bridge over the Tiber in Rome, which is a single flat arch of 100 m span with 10 m rise, again monolithic with the roadway slab and with the two abutments. Each of the latter projects 24 m from the bank and is of cellular construction, as is the arch itself. This bridge, which is noted for its imposing yet slender character, carries a roadway 20 m wide. Great care was exercised, and special measures adopted, in order to make every part of the structure as sound as possible. Thus the reinforcing bars are of a special convex-concave section and have suitably spaced transverse projections inside and out, bonding with projections formed on the longitudinal ribs of the arch itself, to lessen the effects of shrinkage during the hardening of the concrete and also to eliminate any excessive stresses in the latter due to the variations in temperature to which the structure is periodically exposed.

April 1910 was the opening date of the Auckland Bridge in New Zealand, a single great arch of 98 m span with 56 m rise, built in at each end, supporting a roadway 12 m wide. Another similar structure built at nearly the same date is the bridge of 98 m span but only 20 m rise over Lavimer and Atherton Avenues in Pittsburgh, U.S.A. The same period of years witnessed the construction of large high buildings in reinforced concrete for use as living quarters, offices and stores in New York and other cities of the United States, and in Europe; also very tall industrial chimneys from which water reservoirs of considerable size are suspended at various levels; great siphons such as that near Albeida for the Aragon-Catalonia canal in Spain which has an internal diameter of 4 m a length of 75 m and a maximum effective pressure of three atmospheres; large balconies, roofs and cupolas for stadia, public halls and theatres.

The Centenary Palace at Breslau calls for special mention as being one of the first buildings in which the outstanding possibilities of the application of reinforced concrete have been exemplified both from a technical and an architectural point of view. This notable building (erected to mark the centenary of the victory over Napoleon) is of circular plan 100 m in diameter with a reinforced concrete skeleton. A central cupola 42 m high and 65 m diameter at the springing is supported on a ring which in turn is carried on the frontal arches of four huge apses. The 32 great meridianal ribs of the cupola are built into this ring and are continued through it into the ground as external counterforts covering over the apses. The total area of the large hall is 5500 sq.m and it provides accomodation for 6000 seated and 1000 standing spectators.

The large overhead reservoirs for aqueducts began to appear between 1915 and 1925. One of the largest, which deserves to be mentioned here, is the one built in connection with the municipal aqueduct at Padua in 1924 which has a covered basin 20 m in diameter holding 2000 cu.m at a height of more than 40 m above the street level. Here the concrete work has been bonded with a masonry sheathing and crown which give a monumental tower-like appearance, and the base of the tower has been made to include a votive chapel in memory of the victims of the air raids over Padua during the Great War, 1915—1918.

The same period, and the succeeding decade, witnessed a noteworthy development of large building structures — especially in Germany — on the part of architects aiming at the creation of a new and appropriate style, though not, in the present writer's opinion, with success.

At the same time the construction of important bridges and viaducts both for ordinary roads and for railways was rapidly increasing, and many remarkable examples of these were built in Italy. The following types may be distinguished:

a) Straight girder "through" or "deck" spans, reaching a maximum single opening of 140 m in the case of the footbridge of Ivry near Paris.

b) Unrelieved arch spans with or without hinges. The maximum is reached in the Plougastel Bridge over the Elorn near Brest, built to the daring design of Freyssinet in 1928—9, wherein each of the three arches of cellular section covers an opening of 186 m and high-strength cement has been employed for making concrete cast under vibration. The results were so encouraging that a similar bridge was constructed over the Tranebergsund channel near Stockholm having two twin arch ribs of cellular structure, 181 m span and 26.20 m rise, 6.20 m apart internally. The decking has a total width of 27.50 m including a double track railway and an ordinary road 19 m wide. The maximum stress allowed at the crown of the arch is 120 kg/cm^2, to be resisted by concrete containing 300—400 kg of cement per cu.m.

(Smaller spans, up to 90 m, have been achieved in a fine series of Italian works such as the bridge over the Adda for the Milan-Bergamo "Motor Road" and the bridges over the Brenta at Primolano, over the Savio at Monte Castello, over the Piave at Belluno, over the Isonzo at Plava, for the canal over the Brembo, and others.)

c) Arches in which the thrust is eliminated by means of a suspended tie which serves to carry the roadway slab. The greatest span of this type is found in the bridge over the Oise at Conflans fin d'Oise in France, which has a lightened arch and a suspended intermediate decking which acts as a tie, covering an opening of 126 m.

(Smaller spans, up to more than 90 m exist in the Lot Bridge at Port d'Agrès which has a suspended lattice, and in the Oned Mellègue Bridge in Tunisia which is of triangulated construction. The considerable span of 74 m is attained by the San Bernardino ad Intra Bridge in Italy, carrying a railway with road underneath.)

d) Cellular structures in which a very thin arch carries the intrados during construction and is afterwards made solid with the permanent arch drum, with the upper decking (for which it serves partly as shuttering), and with the supports on piers and abutments. An example of this is given by the Graubünden Bridge in the Grisons, which has a considerable span and a daringly small rise.

Current tendencies.

The present trend, as already stated, is in the direction of very considerably increasing the spans both of straight girders on two or more supports and of arches with or without elimination of thrust.

As regards arches, it appeared only a few years ago that the clear opening must necessarily be limited to a few hundred metres, but today it is held feasible by high authorities to reach a span of well over a thousand metres. A proposal to build an arch of 1000 m span was brought before the Liége Congress by Freyssinet in 1930, and a span of 1400 m, which corresponds to more than seven times the existing maximum of 186 m, has been indicated by H. Lossier as a possibility for a heavily reinforced concrete arch — though at the same time the adoption of steel suspension bridges appears preferable to him in the case of spans exceeding 800 m. With straight girders on two or more supports Lossier considers that clear spans of 500 m are possible, corresponding to about four times the present maximum of 126 m.

Lossier also favours the use of mixed structures in which rustless steel would be adopted for those members which receive only tensile forces and reinforced concrete for those subject to compression and bending. The concrete members would be pre-cast, and when they were erected the joints would be made by autogenous welding applied to portions of the steel left projecting from the concrete, to be subsequently covered in high-strength rapid-hardening cement mortar. Mixed construction of this type is suggested as being particularly well suited for suspension bridges, spans of the order of 5000 m being contemplated. This would be about five times the maximum hitherto obtained, which is a suspension bridge entirely of steel spanning 1077 m — the George Washington Bridge over the Hudsonriver at New York. The latter, however, is already in course of being exceeded by the 1270 m span of the bridge of similar type now being built at Golden Gate on San Francisco Bay in California.

It is to be expected that considerable developments are to be awaited in this direction.

Referring generally to straight and arched spans, the types of bridge which in these respective categories appear preferable to the author are the *continuous girder* and the *hinged arch with thrust partly relieved.* It is agreed that in these two forms it should be possible to attain the limits of span indicated above, notwithstanding those imperfections which must inevitably attend constructional operations and those uncertainties which must enter into the calculations through the presence of defects and unascertainable conditions in the reinforced concrete. The limits put forward may well appear to be exaggerated, in the sense that one may doubt the real necessity for trying to justify the very heavy cost of overcoming the exceptional difficulties of carrying out work of such dimensions; but from the purely technical point of view they need cause no misgivings.

As things are now, when disastrous failures do occur they are the result almost invariably of errors in execution, attributable 90% to the shuttering being stripped prematurely or unreasonably and 10% to accidental defects in either the concrete or the steel reinforcement (wherein very high values of specific resistance may be obtained). So far as the calculations are concerned it is possible to eliminate practically all uncertainty by making sure that the hypotheses correspond with reality; this means interpreting theory in the light of common sense and bringing a maximum of perspicacity to bear on the study of secondary stresses, for generally it is through these that fractures occur in the compressive zones where least expected, and in the tensile zones even where abundant steel reinforcement is present in a small section of concrete.

Standards of construction and methods of design.

Stone, sand, cement and reinforcing steel must be strictly selected and their properties accurately controlled. On the site there must be good supervision of the mixing and placing of the concrete, of its protection while setting and hardening, and of the removal of shuttering from the various parts of the construction. Observance of these essentials will almost certainly ensure that the work is in accordance with plan, and that it will be fully able to bear the maximum stresses (whether static or dynamic) likely to be imposed on it in service. In the opinion of the author the compressive stresses in the concrete may reach 250 or 300 kg/cm^2 and the tensile stresses in the steel bars values ten times as great.

At the present time cements of very high strengths are being produced in all industrial countries. With gravel, stone and sand suitably chosen as regards quality and size of grain, and with the admixture of cement and water properly controlled, it is possible normally to make concrete which after a few weeks will give a cube compression strength of 400, 500, 600 or even more kg/cm^2 and likewise a considerable tensile strength. If the concrete be cast under vibration these results will be even more easily obtained. Concrete of this character has excellent elastic properties extending to the highest compressive loads imposed on it; its cycles of deformation give rapid adjustment and with well arranged steel reinforcement it has an ample capacity for undergoing expansion without damage. In all countries such reinforcement is available in the form of long bars of rustless steel offering a resistance to tensile fracture of 52 to 56 kg/mm², with an elastic limit of over 30 kg/mm² and a yield point of over 40 kg/mm², with an elongation of the order of 30—35% at fracture and a constriction value of 60—70%.

Such materials as these, used with the precautions indicated above, provide further remarkable possibilities for of reducing the percentage of steel in the cross section and increasing the "specific ligthtness" (ratio of unit working stress to weight) of the deposited concrete, thus increasing the principal dimensions that can be given to the load bearing members. The rigidity of the latter is favoured by the high values of the moduli of elasticity of the concrete and the steel, and this in turn implies that the deformations due to dead and live load will be small.

In large reinforced concrete structures the slenderness of the members presents a striking contrast to the effective load bearing capacity of the whole. Where this is the case, the methods of calculation tend necessarily to be based directly and entirely on the theoretical principles and experimental results reached through recent research into the equilibrium of rigid bodies or systems, and the elastic behaviour of deformable bodies or systems, in which the connections are adequate or excessive, and which change their condition of stress and strain according to more or less regular laws when subjected to static or dynamic loads or to progressive or periodical variations in temperature.

Among the systems of this kind particular mention may be made of long spans of lattice or cellular construction with or without hinges, in the form of straight girders, ordinary arches, or arches having the thrust eliminated by a tie bar; also multiple frames, continuous arches made monolithic with the supports, and slabs — flat, cylindrical, conical, spherical, ellipsoidal, paraboloidal, etc. — as applied to form ceilings, walls, vaults, blocks, and hinged or fixed bearings.

Researches on the physical and mechanical properties of hydraulic cements, coarse and fine aggregates, concrete, metal reinforcing bars, and reinforced concrete as such, are the basis of all others, and remain capable of further useful development even today. Such work may well be directed towards the study of granulimetry, proportioning of cement and water, and percentage of steel reinforcement in the cross section. Another problem is to determine the best methods of casting the concrete so as to ensure that it shall be dense, shall undergo little deformation through shrinkage while setting and hardening, and shall have a high modulus of elasticity, high compressive strength, good adhesion to the steel, and ability to cooperate with the latter without sustaining damage under high tensile stresses. On account of their importance is becoming more and more usual to associate researches of this kind with others on the elastic behaviour of finished structures under heavy moving loads, comparisons in this respect being made between works that have been in service since first constructed some tens of years ago with the cement and steel then available, and other works of recent date, bolder in type and size, built with the better materials available today.

The theoretical methods for forecasting the conditions of stress and strain have been improved in a way which facilitates the design and checking of reinforced concrete structures, and this in turn has lead to further developments in the most complicated and difficult branches of the theory of elasticity.

For structures which are not homogeneous, still less isotropic, the difficulties to be overcome in formulating a satisfactory theoretical system as regards the physical and mechanical properties of their constituent materials, are still very great (particularly when account has to be taken of structures containing hyperstatic connections), and progress must necessarily be slow. On the other hand the urge to overcome these difficulties is both strong and productive.

The lack of homogeneity and isotropy of the concrete, which leads to uncertainties in applying the theory, can partly be corrected by combining a suitable choice of the cement and aggregate with attention to the granulometric composition of the stone or gravel and sand and the admixture of cement and water, which determines the density of the concrete. The essential requirement is to grade the gravel or stone in such a way as to minimise the voids and therefore the amount of cement-sand mortar necessary to fill them, give adequate bond between the particles, and eliminate porosity. If these conditions are secured there will also be a reduction in the ratio between the volume of cement mortar and the volume of the resistant nuclei und the mass of concrete itself. Consequently there will be an increase in the breaking strength and mean elastic modulus in compression and tension, which characteristics will tend to approximate more closely to the values they reach in the hard strong nuclei themselves. (The latter, of course, are of a different nature from the more or less plastic cement mortar which serves to bind them together.)

So far as compressive stresses are concerned the resulting concrete will then be practically equivalent to a homogeneous and isotropic mass, metal reinforcement being included mainly with a view to resisting tensile stresses. An increase in compressive strength can be obtained by means of spiral hooping. The necessary resistance to tension can be supplied by rods arranged in the appropriate directions, namely one direction in the case of thin beams, two at right angles in the case of flat or curved thin slabs, three at right angles (perhaps coinciding with the probable isostatic lines) in the case of three dimensional structures.

The high internal temperatures that are apt to arise while the concrete is setting, followed by natural cooling under conditions which cannot be defined, may result in non-uniform thermal expansions and contractions. These in turn may result in the initial internal stresses remaining and causing very heavy latent compressions or tensions in the concrete or steel reinforcement.

Partial correctives to these effects may be found in the use of low-heat cements as already specified in the official specifications of certain countries, and employing these in smaller quantities relatively to the specific strengths which the concrete is required to develop. Large masses may further be divided into blocks, by leaving joints to be closed after seasoning.

Another cause of ill-defined latent stresses being established in the concrete, and in the reinforcement embedded therein, is the shrinkage of the concrete during its curing and hardening process — a phenomenon attributable mainly to the elimination of the excess mixing water which has not entered into combination. In non-reinforced concrete these stresses may amount to more than 20 to 30 kg/cm^2 in tension, alternating with much higher values in compression. In reinforced concrete, where the shrinkage is less pronounced as the result of the presence of the reinforcement, the stresses may correspond to an order of magnitude of 5 to 15 kg/cm^2, and become greater if the percentage of steel in the section is increased; they are balanced by compressive stresses in the steel of between 100 and 20 times as great, and the latter become smaller if the percentage of steel is decreased.

These effects, again, may be partially corrected by reducing the percentage of steel to a minimum or by introducing suitable contraction joints; also by using concrete in which the cement content is not excessive and in which the quantity of water used is the minimum strictly necessary to enable the concrete to flow into the forms and between the steel bars. Finally the concrete should be kept damp by suitable sprinkling throughout the curing period and until the shuttering is struck.

Summary.

The author briefly outlines the development of the engineering of major works in reinforced concrete and proceeds to discuss present tendencies in their design and construction, giving examples of the attendant difficulties and doubtful points and of how these may be overcome or mitigated.

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