

Zeitschrift: IABSE congress report = Rapport du congrès AIPC = IVBH
Kongressbericht

Band: 2 (1936)

Artikel: Use of high-grade steel in reinforced concrete

Autor: Gehler, W.

DOI: <https://doi.org/10.5169/seals-3164>

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IIc 2

Use of High-Grade Steel in Reinforced Concrete.

Anwendung des hochwertigen Stahles im Eisenbetonbau.

Application de l'acier à haute résistance dans le béton armé.

Dr. Ing. W. Gehler,

ord. Professor an der Technischen Hochschule,
Direktor beim Staatl. Versuchs- und Materialprüfungsamt, Dresden.

Whilst the safety of steel structures in *high-grade steel* can easily be determined as against structures made of ordinary *commercial steel*¹, this question leads to intricate problems in the case of reinforced concrete constructions on account of the compound action of steel and concrete, which can only be solved by exhaustive research investigations. A considerable contribution to this is furnished by the large series of tests carried out by the German Commission for Reinforced Concrete (Deutscher Ausschluß für Eisenbeton), in particular also the tests undertaken at Dresden. The knowledge attained by such tests will be elucidated in the following:

A. High-grade steel as used in steel structures.

To classify an important element of building steel with its given property characteristics and at the same time to compare steel structures and reinforced concrete structures, it is advisable to start by elucidating our present-day conception about the application of high-grade building steel in structural engineering and bridge building.

1. The high-quality steel St. 52 for steady or more or less steady loading, i. e. for structural steel work and road bridges in steel.

Whilst the minimum value of tensile strength e. g. $\sigma_B = 52 \text{ kg/mm}^2$ or 37 kg/mm^2 is generally used to classify a certain kind of steel (e. g. St 52 or St 37), the ratio of yield limit forms the actual basis of the permissible strength of the two kinds of steel.

$$\sigma_{adm 52} : \sigma_{adm 37} = \sigma_{s 52} : \sigma_{s 37} = 36 : 24 = 3 : 2 \quad (1)$$

hence the permissible stressing for $\sigma_{adm 37} = 1400 \text{ kg/cm}^2$

and correspondingly

$$\sigma_{adm 52} = 2100 \text{ kg/cm}^2 \quad (2)$$

¹ Commercial building steel is of mild-steel quality with a minimum tensile strength of kg/mm^2 , a maximum tensile strength of 50 kg/mm^2 and a minimum elongation before rupture of 18 % for the long standard test bar. This kind of steel must suffice for cold bending of 180° round a steel pin of $D = 2a$. (For commercial round bars for concrete these figures are not yet guaranteed).

To make full use of the advantages offered by 50 % higher stressing for St 52 is unfortunately not possible for two sub-fields of static investigation, since Young's modulus $E = 2\,100\,000 \text{ kg/cm}^2$ is practically constant for all kinds of steel.

a) Although the deflection f is not in general restricted by regulations, the disadvantage still exists, that the deflection increases in proportion to the stresses for a constant beam section; for instance a simply supported beam with the depth h ($M = \sigma \cdot W = \sigma \cdot \frac{2J}{h}$) the deflection f has the value:

$$f = \frac{5}{48} \cdot \frac{Ml^2}{EJ} = \frac{5}{24} \cdot \frac{l^2}{h} \cdot \frac{\sigma}{E} \quad (3)$$

which proves very unfavourable, particularly in structural steel work.

b) Since the buckling load within the Euler-range $P_k = \frac{\pi^2 EJ}{s_k^2}$ is practically the same for a given length s_k for all kinds of steel, no advantage is attained for slender bars (with $s_k : i < 100$) made of *high-grade steel*.

The chief advantages of high-grade steel lie in the saving in dead weight, particularly for wide spans (e. g. about 26 % for the bridge over the Little Belt, span $l = 200 \text{ m}$), and with it the possibility of being able to execute structures standing on weak subsoil (coal handling bridges of wide spans) and finally the reduction of weight and customs duty in case of exporting such steel.

As a simple characteristic for the quality of steels for statically stressed structures, the elongation δ_B before rupture takes place has been introduced, since this characteristic, similar to the behaviour in a cold bending test, is an indicator for the *toughness for steel worked cold in the shop as well as on site*. From the stress-strain-lines of the tensile test Fig. 1 (see also Table I) it follows that

Table I

Kind of Steel	Ultimate strength σ_B kg/mm ²	Yield limit σ_s kg/mm ²	Elongation δ_B %	Constriction %	Quality coefficient $\sigma_B \cdot \delta_B$ (kg/cm ²)	Deformation energy		$\frac{A_B}{\sigma_B \cdot \delta_B}$
						Rupture energy A_B kgcm/cm ²	Energy capacity A_{ges} kg cm/cm ²	
St.37 (min)	42,8	31,0	18	59,7	770	490	650	0,637
St.38 (max)	42,8	31,0	30	59,7	1294	860	1180	1,11
St.48	56,8	33,9	21	48,7	1193	760	1000	0,637
St.52	56,0	38,2	26,5	59,5	1484	910	1280	0,614
St.52	56,4	42,5	27	56,0	1523	940	1290	0,617
								mean abt. ² /s

for the usual qualities of steel the area of the specific deformation energy (up to point B of the tensile strength) has a mean value of

$$A_B = \frac{2}{3} \sigma_B \cdot \delta_B \quad (4)$$

The area of the circumscribed rectangle is expressed by the term

$$A = \sigma_B \cdot \delta_B, \quad (5)$$

and represents the so-called "work of rupture" which can be regarded as a *practical quality coefficient for the toughness of steel*. According to table I this value fluctuates between 800 and 1500 kg/cm²;².

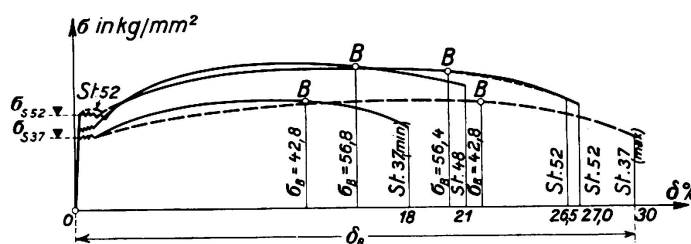


Fig. 1.
Stress-strain lines
for various steels.

II. The high-quality steel St. 52 in riveted steel bridges, under rail tracks, with frequently changing loads.

Whilst dynamic influences on road bridges are compensated by introducing ample rolling loads and by multiplication of the forces in the members produced by these loads, by an impact coefficient φ , which depends on the span l (where $\varphi = 1.4 - 0.0015 l$);³, the investigation only requires to be made for static loads, but for railway bridges, riveted or welded, the fatigue strength has to be considered⁴.

The safety of such bridges is therefore based on the *static basis of calculation*; the fatigue, however, is laid down by fatigue tests through the number n of loading repetitions, while in bridges the fatigue is characterized by the number of passing trains. The *fatigue strength* depends to a very great extent on the nature of loading, e. g. alternating loads and surge loads (oscillation of loads without change of direction), or on the ratio of the extreme values of stresses in the members:

$$\xi = S_{\min} : S_{\max} \quad (6)$$

As found by tests, the fatigue strength σ_D for riveted connections for St 52 and St 37 no longer gives the same ratio as the yield stresses (see Eq. 1) and with it the permissible stresses $\sigma_{D adm}$ are altered too. For St 52 $\sigma_{D adm} = 1800 \text{ kg/cm}^2$, however, for St 37 1400 kg/cm^2 again applies. It is essential to note that the *yield strength has to be ruled out in judging the safety of a structure*, its place being taken by the *cohesion or separating strength*. The *safety can only be conceived after fatigue tests have been carried out*.

III. Welded connections, in structural work, railway-and road bridges of building steel, require, based on tests, to have their permissible stresses differently reduced in respect to the various types of welds and the various "Form

² See W. Gehler: The development and importance of high-grade steels in steel structures and reinforced concrete, World Engineering Congress Tokio 1929, Paper Nr. 218. — „Die Entwicklung und Bedeutung der hochwertigen Baustähle im Eisenbau und Eisenbetonbau.“

³ See W. Gehler: Hand book for Civil Engineers, Edition V, Vol. II, p. 375. Berlin 1928. Julius Springer.

⁴ See W. Gehler: Contribution to „Discussions“ ad IIIb.

coefficients α'' ; for butt and fillet welds (end and side fillets) and if it concerns welding of ordinary or high-class workmanship.

IV. The two outstanding features of the progress made in the last decade i. e. the introduction of high-grade building steels and electric welding in steel structures, both enabling a reduction of costs of about 15 % (for wide-spans even more), have induced us to study the various questions regarding the safety of steel structures as indicated by Pars. I to III. Fundamentally the same questions arise with high-grade steels in reinforced concrete construction, only that the conditions are altered on account of the compound action of steel and concrete.

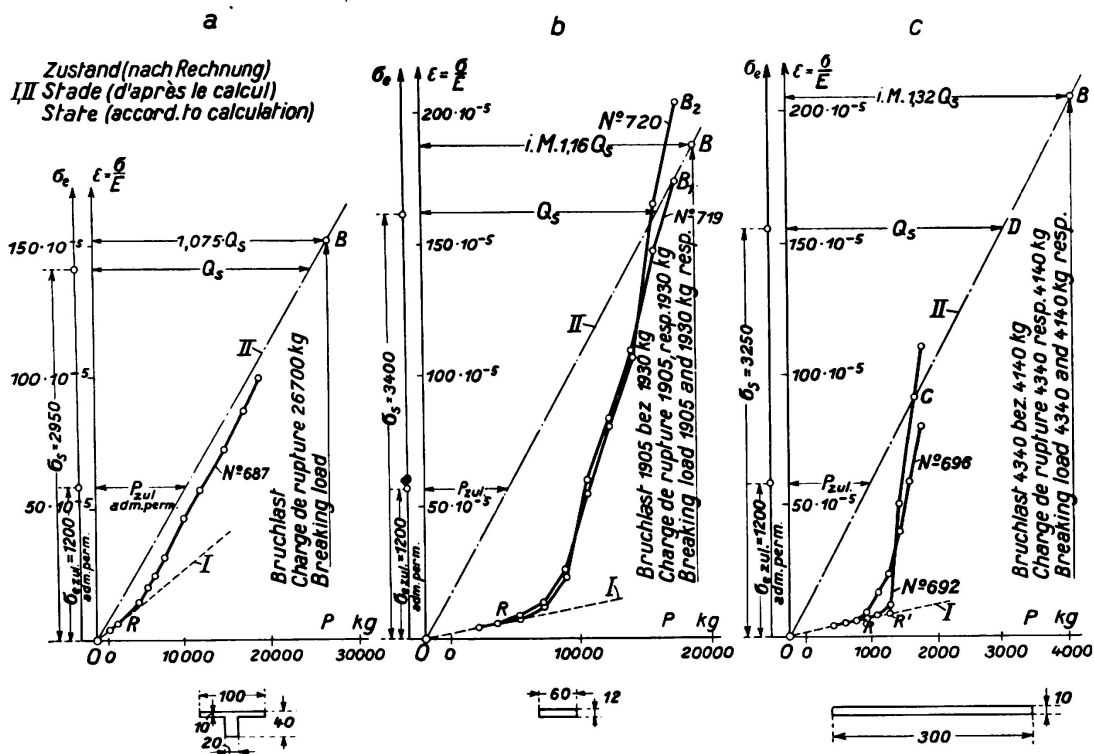


Fig. 2a—c.

Load-elongation diagram for steel for:

a T-beam.

b Slabs.

c Slabs cross-wise forced supported all sides.

B. The ratios expressing the safety of structural elements in reinforced concrete against failure and the formation of cracks.

1) The safety against rupture based on the measured load-steel elongation lines ($q - \epsilon_s$ -lines).

In Fig. 2a—c are shown, for a T-beam, a slab-strib, and a slab supported on all four sides, the measured load-steel elongation lines (full lines) in comparison to the usual lines (dotted) received by calculation⁵. (The determination of the stresses from the measured elongations was based on the total elongations.)

⁵ Dissertation by Walter Heide: The Dresden tests on crosswise reinforced concrete slabs, in comparison to the usual mode of calculation. „Die Dresdener Versuche mit kreuzweise bewehrten Eisenbetonplatten im Vergleich mit der üblichen Berechnung.“ Chair: Prof. Gehler, Inst. of Tech. Dresden, 1933, pp. 12 and 28.

a) *T-beams*. Fig. 2a (Dresden tests⁵ 1928, Issue 66 p. 65, No. 687, reinforced with St. 37 dimensioned for $M = \frac{ql^2}{8}$). *Test results and calculations coincide for state I*. The tensile area, being small, has only little influence on the rising of the curve. The values received from the usual mode of calculation for state II coincide with those already measured for cases of small stages of loading. *The stress in steel max σ_e calculated from the ultimate strength (rupture strength) is only 7,5 % above the yield limit $\sigma_s = 2950 \text{ kg/cm}^2$ (compare also D III):*

$$\max \sigma_e = 3170 = 1,075 \sigma_s \quad \text{or} \quad = \frac{\max \sigma_e - \sigma_s}{\sigma_s} = 7,5$$

The safety against rupture is therefore:

$$\nu_B = \frac{q_B}{q_{adm}} = \frac{\sigma_s}{\sigma_{adm}} = \frac{2950}{1200} = 2,5$$

To state the safety against rupture the expression $\nu_B = \sigma_s : \sigma_{adm}$ is used with advantage; it is based on the yield limit. The expression

$$\nu'_B = \frac{\max \sigma_e}{\sigma_{e adm}} = \frac{3170}{1200} = 2,63$$

should, however, not be used (as it will also be seen from explanations given under D III).

b) *Slab-strip* (with rectangular section) (Dresden slab tests, 1932, Issue 70, pp. 179 and 180, No. 719 and 720, span 3.0 m, reinforced with St 37, designed for $M = \frac{ql^2}{8}$, Fig. 2b). For state I good agreement exists between calculation and test results. The line OR rises under a steep incline, since the tensile area of concrete is large in this case and therefore the steel reinforcement is to a great extent relieved. The values measured for steel elongation are very small until the appearance of the first cracks (see point R).

From point R a great deviation takes place between the line OR for measured and OB for calculated elongation. From this point steel alone takes over all the work of tension, and the steel elongations increase with the increase of loading. As the conclusion of the test (on attaining the yield limit $\sigma_s = 3400 \text{ kg/cm}^2$) the two lines OB and OB₁ (or OB₂) join almost at one point, which again allows the safety against rupture to be based on the yield limit of steel, as follows:

$$\nu_B = \frac{q_B}{q_{adm}} = \frac{\sigma_s}{\sigma_{adm}} = \frac{3400}{1200} = 2,8$$

c) *Crosswise reinforced slab, supported on all sides*.

(Dresden slab tests 1932, Issue 70, pp. 52 and 100, No. 692 and 696⁶, $l_x = l_y = 3.0 \text{ m}$ reinforced with St 37, designed for $M = \frac{1}{27,4} ql^2$ Fig. 2c).

The values calculated compare well with those measured, for state I. Fundamentally the same remarks apply as for the slab-strip (see above under b). After the appearance of cracks (see point R) the steel-elongation increases com-

⁶ See Preliminary Publication of Ist Congress of I.A.B. St.E. Paris 1932, p. 205 and 237.

paratively rapidly. The lines for calculated and measured values cross each other in point C. For rupture (in point B) we receive:

$$v_B = \frac{q_B}{q_{adm}} = \frac{4200}{990} = 4,2$$

Since the two lines do not meet in point D of the yield limit, *the value*

$$\frac{\sigma_s}{\sigma_{adm}} = \frac{3250}{1200} = 2,7$$

of the yield limit can therefore not be decisive for the safety.

Conclusion: For slabs (with rectangular cross-section) and T-beams the safety against rupture is expressed by the term:

$$v_B = \frac{q_B}{q_{adm}} = \frac{\sigma_s}{\sigma_{adm}} \quad (7)$$

This relation, however, does not apply for slabs crosswise reinforced, supported on all sides, but here the ratio ultimate load to working load has to be used:

$$v_B = \frac{q_B}{q_{adm}} \quad (8)$$

2. The safety against cracking.

If q_R indicates, for uniformly distributed load, that particular stage of loading for which the first crack becomes visible, and if q_{zul} stands for the working load (or permissible load) then the safety against cracking is given by the ratio:

$$R = \frac{q_R}{q_{adm}} \quad (9a)$$

Loading ratio. For point loads the values q_R and q_{adm} are replaced by the bending moments M_R and M_{adm} or by the stresses σ_{eR} and $\sigma_{e adm}$ which are proportional to the moments M_R and M_{adm} therefore the following relation applies:

$$v_R = \frac{q_R}{q_{adm}} = \frac{M_R}{M_{adm}} = \frac{\overline{\sigma_{ek}}}{\sigma_{e adm}} \quad (9b)$$

If, however, the steel-elongation ϵ_R is measured, in other words if the cracking point stress $\sigma_{eR} = E \cdot \epsilon_R$ is established by tests, and hence the relation

$$v'_R = \frac{\sigma_{ek}}{\sigma_{e adm}} \quad (10)$$

(stressing ratio) is formed, then the question arises whether the term v'_R is equal in value to the safety against cracking v_R . (Acc. Eq. 9b). Evidently this is only true if the load steel elongation line, or the load steel stressing line OA in Fig. 3 increases in linear proportion up to the value of ultimate load. In this case the point R coincides with R', and comes to fall on line OA, and for $\sigma_{eR} = \overline{\sigma_{eR}}$. The equation 10 changes into Eq. 9b. This result is to be expected and is sufficiently accurate for T-beams according to Fig. 2a, as proved by Fig. 4. In Fig. 4 the values v_R and v'_R (according to Eqs. 9 and 10) have been plotted in relation to the cracking point stress σ_{eR} , for which the values for σ_{eR} have been measured by means of the steel elongations. For T-beams, according to the Dresden tests of 1935, the values for v_R and v'_R coincide in a satisfactory

manner as shown by lines CD and EF. This was not the case with earlier tests, (1928, Issue 66) [an indication that a specially developed technique was

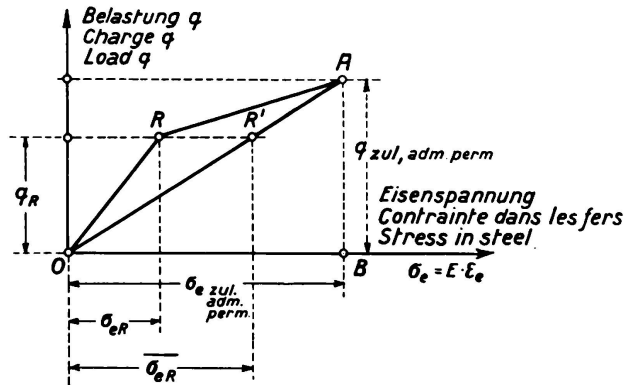


Fig. 3.

Load-stress diagram of steel.

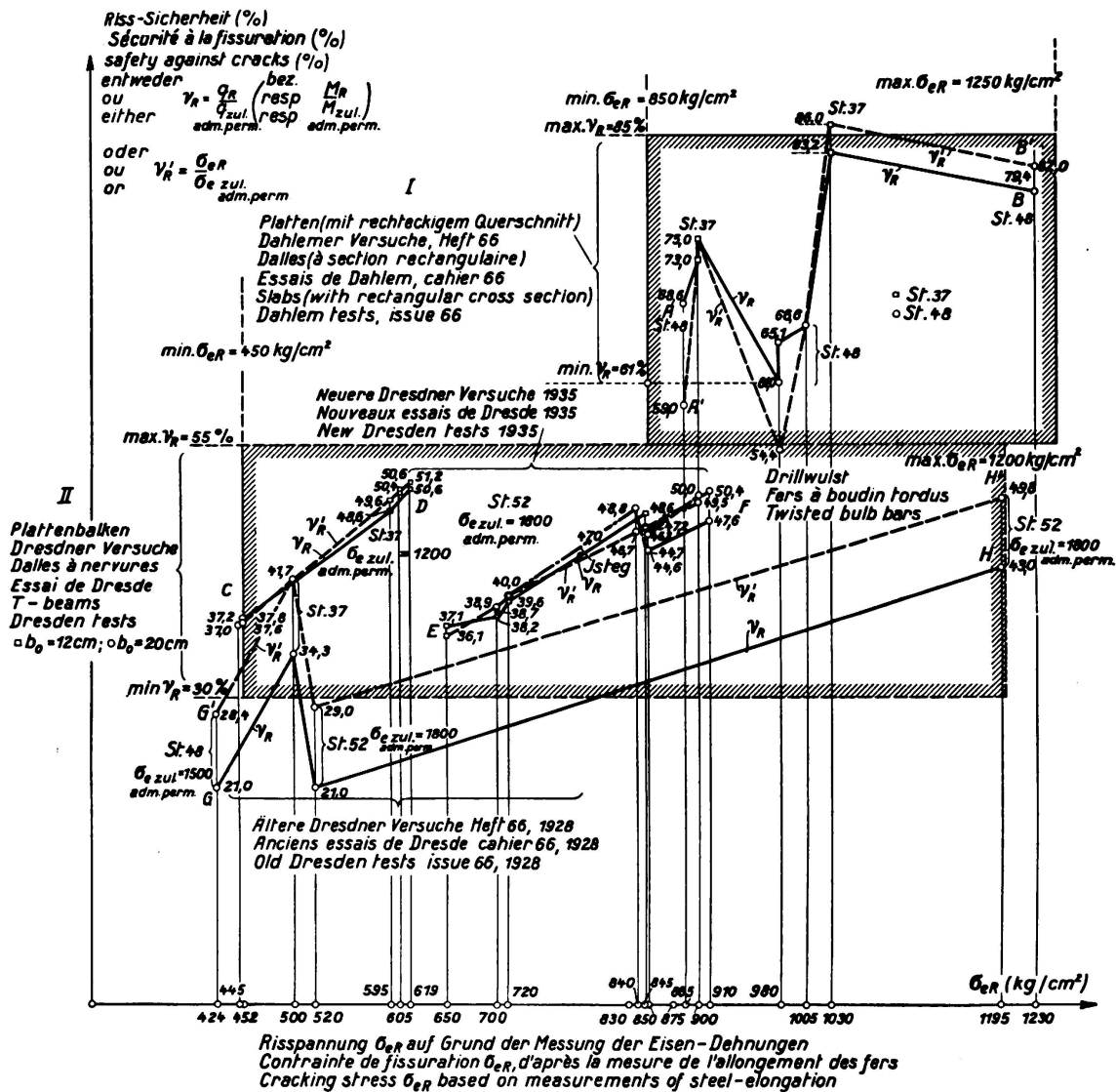


Fig. 4.

Dependence of safety against cracks v_R (scale of loads) or v'_R (scale of elongations) from the cracking stress σ_{eR} for slabs and T-beams.

required to investigate the safety against cracking]. For slabs with rectangular section (see lines AB and A'B') for which such new investigations have not yet been carried out, this question has to remain open until the Dresden tests now in progress have been concluded. *It is therefore advisable to base such slabs only on the loading ratio:*

$$v_R = \frac{q_R}{q_{adm}} \quad (9)$$

but the stressing ratio $v'_R = \frac{\sigma_{eR}}{\sigma_{eadm}}$ (Eq. 10) applies also for T-beams, not only the ratio according to equation 9.

C. The safety against cracking for slabs and T-beams in concrete reinforced with high-grade steel.

I. The quantities to be measured from tests are:

1) steel elongation ϵ_{eR} at the moment when the first crack appears, and the resulting cracking point therefrom:

$$\sigma_{eR} = E \cdot \epsilon_{eR},$$

2) depth of cracks:

t_l for $\sigma_{eadm} = 1200 \text{ kg/cm}^2$ for St 37

t_l for $\sigma_{eadm} = 1800 \text{ kg/cm}^2$ for St 52 and special steels.

3) width of cracks for various stages of loading, especially for:

b_R for σ_{eadm}

b'_R at the yield limit σ_s .

The procedure for the tests of 1935 was as follows:

a) The width of the first through cracks was measured for every test beam at the place of the centre of gravity of the reinforcement, by means of a microscope with attached ocular micrometer. Two of the cracks of each test beam were photographed 23 times enlarged (See Fig. 5).

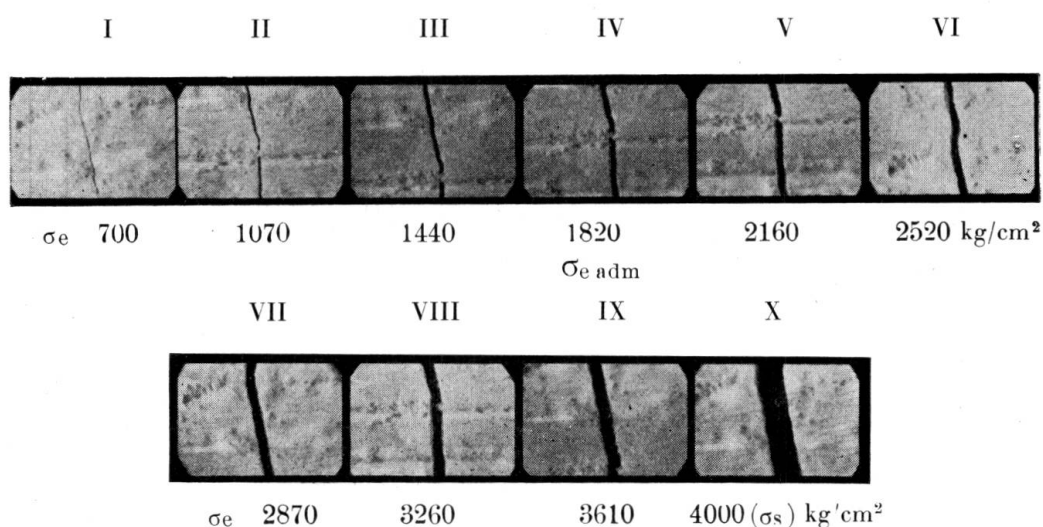


Fig. 5.

Measuring of crack-widths with microscope, enlargement 11.5 times, Dresden tests 1935—36.

b) At the moment when the test beams had attained the calculated permissible stressing value, alcohol was injected to moisten the cracks, after which a coloured liquid was injected. After completion of the test the cracks were opened to see how far the liquid had entered. The depth measured is called "depth of crack t ".

II. The dependence of the cracking point stress σ_{eR} on shape of cross section, type of supporting, quality of concrete and percentage of reinforcement.

1) *The quantities on which the safety against cracking depends.*

a) As regards the shape of cross section, for uni-axial stress conditions (e. g. beams over two and more supports) distinction must be made between the following points:

α) Slabs with rectangular section (Issue 66)⁷,

β) T-beams with broad flanges, and T-beams with narrow web (Dresden tests 1935),

γ) different forms of cross sections (e. g. factory-made reinforced concrete building elements, Issue 75)⁸.

b) *The supporting* on all sides of crosswise reinforced slabs leads to duo-axial stress conditions, which are favourable in respect to the safety of cracks. (Dresden slab tests, Issue 70)⁹.

c) The quality of concrete is best indicated by the cube strength after 90 days⁹

$$W_{b90} = 1,15 W_{b28} \quad (11)$$

and the corresponding tensile strength⁹ is expressed by

$$K_z = 0,09 W_b. \quad (12)$$

d) *The percentage of reinforcement* is expressed as usual by¹⁰

$$\mu = \frac{F_e}{b \cdot h} \quad (13)$$

F_e = cross sectional area of steel,

b = width of compressive area of concrete,

h = effective height of rectangular section or T-beam.

2) *The cracking point stress σ_{eR} for St 37 and St 48 for slabs (with rectangular section) in dependence on the cube strength W_{b90} and the percentage of reinforcement μ , as attained from Dahlem tests 1928, are shown in Fig. 6.*

⁷ See Issue 66, German Commission for Reinforced Concrete (D.A.f.E.B.) *H. Burchartz* and *L. Krüger*: Dahlem Tests with steel reinforced beams Part I, p. 31, Berlin 1931. Publ. W. Ernst and Son. „Dahlemer Versuche mit stahlbewehrten Balken.“

⁸ See Issue 75 of D.A.f.E.B. *W. Gehler* and *H. Amos*: Tests with factory-made reinforced concrete building elements, p. 42. Berlin 1934. W. Ernst and Son. „Versuche mit fabrikmäßig hergestellten Eisenbetonbauteilen.“ reinforced slabs, p. 119, Berlin 1932. W. Ernst and Son. „Versuche mit kreuzweise bewehrten

⁹ See Issue 70. D.A.f.E.B.: *W. Gehler*, *H. Amos* and *M. Bergsträsser*: Tests with crosswise Platten.“

¹⁰ See *W. Gehler*: Explanations on the Regulations relating to Reinforced Concrete 1932. 5th edition, pp. 33, 300 and 302, Berlin 1933. W. Ernst and Son. „Erläuterungen zu den Eisenbetonbestimmungen 1932.“

The corresponding values of safety against cracking are given in Fig. 4. In spite of the irregular positions when plotting the test results due to the difficulty of noticing in time the appearance of cracks, and only decreased in course of time by improving the technique of measuring, the following results are found:

a) The safety against cracking (Fig. 4) as well as the cracking point stress σ_{eR} has higher values for slabs of rectangular sections reinforced in one direction

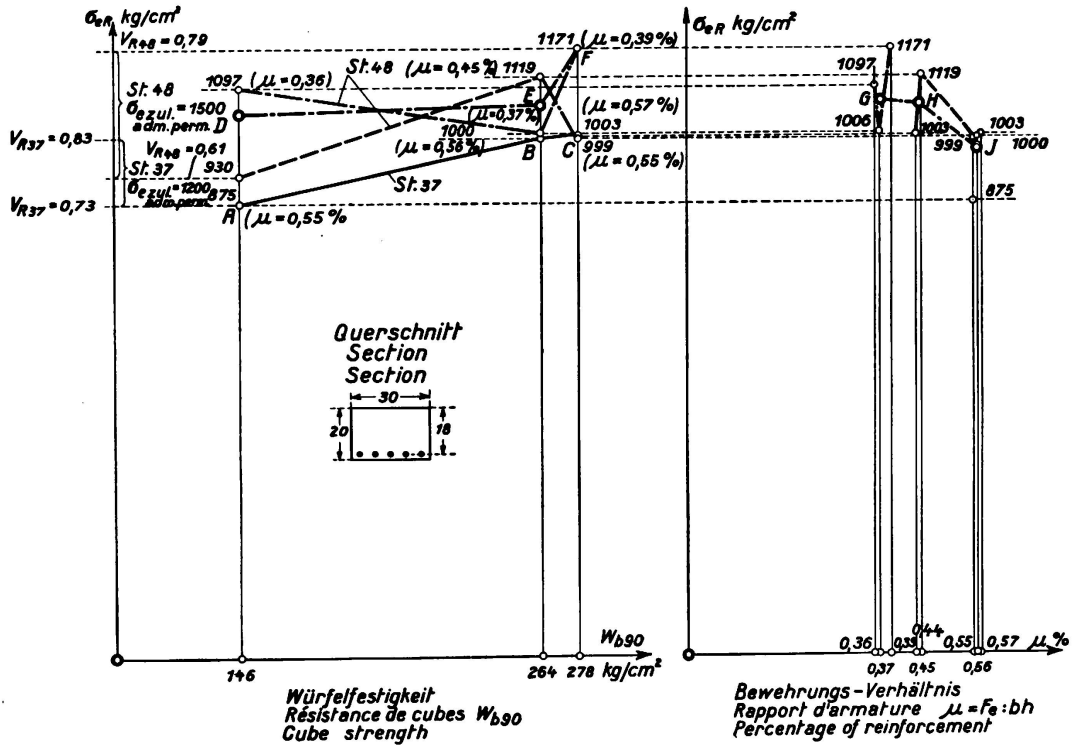


Fig. 6.

Cracking point stress σ_{eR} of St 37 and St 48 for slabs in dependence on the cube strength W_{b90} and the percentage of reinforcement μ .

($h = 16$ to 18 cm, $d = 18$ to 20 cm, $b = 30$ cm) than for T-beams (see Fig. 6). According to the lines ABC and DEF (mean values) the following results are received for such slabs:

for St 37 $\sigma_{eR} = 875$ to 1000 kg/cm²,

for St 48 $\sigma_{eR} = 930$ to 1175 kg/cm²,

the yield stress for these cases was $\sigma_{s37} = 3000$ kg/cm² and $\sigma_{s48} = 3900$ kg/cm² with elongations at rupture of 34% and 28% respectively. The safety against cracking assumes the following values, according to Fig. 4:

for St 37 $v_R = \frac{q_R}{q_{adm}} = 0.73$ to 0.83 , or a mean of 0.78 ,

for St 48 $v_R = \frac{q_R}{q_{adm}} = 0.61$ to 0.79 , or a mean of 0.70 .

Based on this the mean value for safety against cracking for slabs can be put down as:

$$v_R = \frac{q_R}{q_{adm}} = \frac{3}{4} \quad (16)$$

b) With increasing cube strength the cracking point stress σ_{eR} increases also (see lines ABC and DEF).

c) but decreases with increasing percentage of reinforcement $\mu = \frac{F_e}{bh}$ (see line GHJ).

d) For the arbitrarily selected permissible stress $\sigma_{e adm} = 1500 \text{ kg/cm}^2$ for St 48 the safety against cracking is almost that of St 37 with $\sigma_{e adm} = 1200 \text{ kg/cm}^2$ (see Eq. 15).

3) The safety against cracking v_R for crosswise reinforced, rectangular slab, supported on all sides, has been extensively studied on the basis of the Dresden slabs tests of 1932¹¹. The surprisingly high values v_R for square slabs, supported on all sides were:

$$v_R = \frac{q_R}{q_{adm}} 1.36 \text{ to } 2.05 \text{ or a mean of } 1.8 \quad (17)$$

for St 37 with $\sigma_{e adm} = 1200 \text{ kg/cm}^2$ and a corresponding stressing of steel of $\sigma_{eR} = 1630 \text{ kg/cm}^2$ to 2460 kg/cm^2 or a mean of 2160 kg/cm^2 . (18)

The loading-deflection line in Fig. 2c shows the appearance of the first cracks at a point marked R, which lies level with the permissible loading q_{adm} . For the static effect of slabs the fracture point R' is decisive. This point is the intersection point of the two lines $\overline{OR'}$ and $\overline{CR'}$, and shows itself distinctly in the load-deflection line. This point has a similar significance as the limit of proportionality in the stress-strain diagram for building steel (Fig. 1). This fact is of fundamental importance, as it allows reinforced concrete slabs to be calculated as isotropic slabs, up to the loading of $q_R = q_{adm}$, and it is therefore admissible to base the safety against cracking on equation 10 as well as on equation 9.

For slabs supported at the four corners (advance-tests for mushroom-slabs) the following results were found for square and rectangular slabs ($l_x:l_y = 2:1$)

$$v_R = 1.38 \text{ to } 1.40 \quad (19)$$

hence for $\sigma_{e adm} = 1200 \text{ kg/cm}^2$ St 37:

$$\sigma_{eR} = 1650 \text{ to } 1680 \text{ kg/cm}^2 \quad (20)$$

4) The cracking point stress for St 37 and St 52 for T-beams in relation to the cube strength W_{b90} (Dresden Tests of 1928, issue 66 and of 1935/36)¹² (see Fig. 7 and 4).

a) For St 37 with low grade concrete ($W_{b28} = 104 \text{ kg/cm}^2$ and 145 kg/cm^2) and $\mu = 0.34\%$ was found:

$$\sigma_{eR} = 590 \text{ to } 615, \text{ or a mean of } 600 \text{ kg/cm}^2 \text{ and } v_R = 0.4 \text{ to } 0.5 \quad (21)$$

(see line CD in Fig. 4). These results apply for webs of $b_o = 20 \text{ cm}$ and $b_o = 12 \text{ cm}$, and again prove the correctness of the well-known values of the

¹¹ See Footnote 9.

¹² See Issue 66, D.A.f.E.B., W. Gehler and H. Amos, 2nd Part, Berlin 1931, W. Ernst and Son.

c) The Dresden tests of 1927 (Issue 66) for T-beams with $b_0 = 20$ cm used St 52 and a special quality cement with $W_{b28} = 374$ kgkg/cm² and were designed for a reinforcement with $\sigma_{e adm} = 2400$ kg/cm² giving a percentage $\mu = 0.165$ %. For this case the cracking point stress had a value of

$$\sigma_{eR} = 1195 \text{ kg/cm}^2 \text{ and}$$

$$\nu_R = \frac{1195}{2400} = \text{about } \frac{1}{2}. \quad (24)$$

(see point P in Fig. 7 and H in Fig. 4). The higher the value of W_{b28} and the smaller the percentage μ , the higher the cracking point stress σ_{eR} .

5) The cracking point stress σ_{eR} of St 37 and St 52 for T-beams in relation to the percentage μ of reinforcement (See Fig. 8).

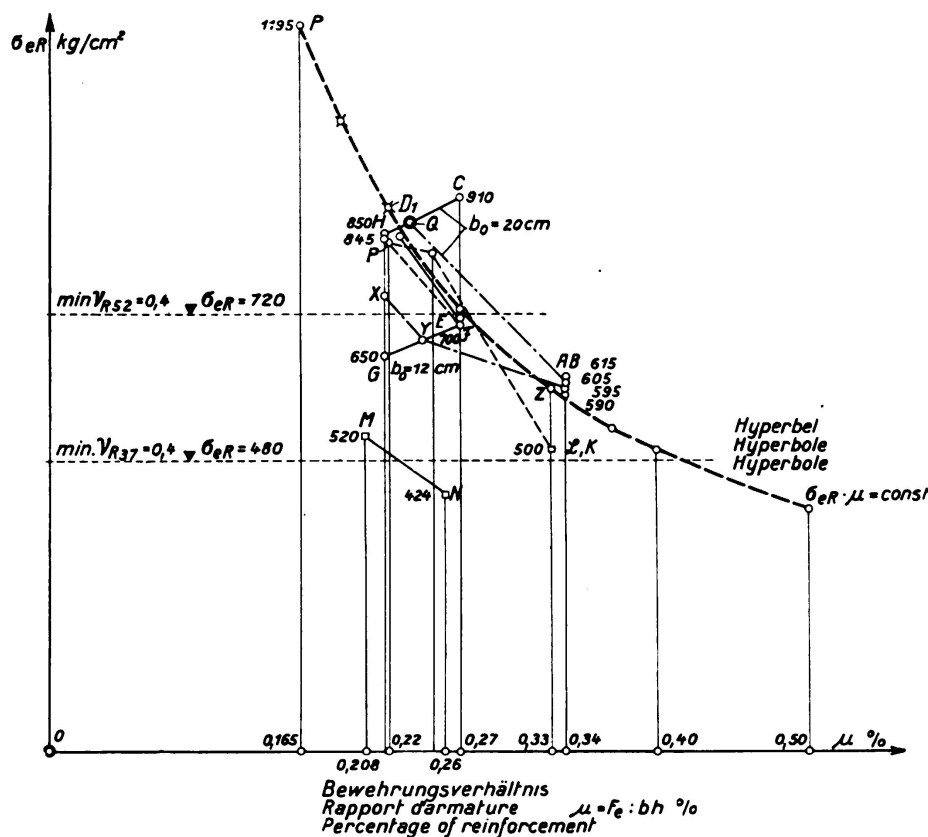


Fig. 8.

Cracking point stress σ_{eR} of St 37 and St 52 and special steels in relation to the percentage of reinforcement μ for T-beams with width of web $b_0 = 20$ and 12 cm respectively.

For the points A to P in Fig. 7 the abscissae give the μ -values and the ordinates the values of σ_{eR} , and in spite of the irregular positions of these points we find lines distinctly falling to the right e. g. line PLK, and MN. Establishing further the gravity centres X, Y and Z, it will be noticed that presumably these falling lines will not be straight. It follows, therefore, that the smaller the cross sectional area of reinforcement the conditions remaining otherwise unchanged, (under consideration of the required safety against rupture), the higher the value for cracking point stressing σ_{eR} . If the yield limit σ_s is

b) Also in this case the σ_{eR} values decrease with increasing μ (see lines TU and RS).

III. Attempt to establish a function for the cracking point stress in relation to the quality of concrete, shape of section and percentage of reinforcement.

1) The tests (Fig. 4 to 9) reveal that:

a) σ_{eR} increases proportionally to the quality W_B of concrete and also with the tensile strength $\sigma_{bz} = 0.09 W_b$.

b) $\sigma_{eR'}$ however, decreases with increasing values of μ .

c) These two conditions can be expressed by the following function:

$$\sigma_{eR} \cdot \mu = (0.09 W_b) \cdot C \quad (26)$$

Considering that $\mu = \frac{F_e}{F_b}$ and that the term on the left side contains $(\sigma_{eR} \cdot F_e) = Z_e$ (tensile force in steel) which accordingly demands on the right side the tensile force in concrete of $Z_b = \sigma_{bz} \cdot F_{bz} = (0.09 \cdot W_b) \cdot F_{bz}$ we can express 26 in the following manner:

$$k \cdot \sigma_{eR} \left(\frac{F_e}{F_b} \right) = 0.09 \cdot W_b \left(\frac{F_{bz}}{F_b} \right) \quad (27)$$

or:

$$k \cdot \sigma_{eR} \cdot F_e = (0.09 \cdot W_b) \cdot F_{bz} \quad (28a)$$

$$k \cdot Z_e = Z_b \quad (28)$$

In this equation Z_b represents the fractured tensile zone = depth t of crack x width of rib b_o and k is a coefficient or percentage still to be determined.

The equation 28 established entirely by statistical interpretation of test results allows for the following physical conception. The tensile area of concrete F_{bz} cracks at the very moment the stressing in steel has the value σ_{eR} , due to sudden exhausting of the tensile strength in concrete σ_{bz} . Therefore the tensile force

$$Z_b = \sigma_{bz} \cdot F_{bz} = (0.09 W_b) \cdot F_{bz}$$

previously held by concrete will be handed over to the steel as an additional stressing. The magnitude of the tensile force Z_b in concrete can be expressed as a certain fraction (in %) of the tensile force in steel $Z_e = \sigma_{eR} \cdot F_e$ acting at that very moment in such a way that

$$Z_b = k \cdot Z_e.$$

On the left side of the basic equation 28 (steel side) the reinforcing ratio $\frac{F_e}{F_b} = \mu$ enters into account, which we shall call "*Form coefficient of the sectional area of steel*" (Formziffer des Eisenquerschnittes), correspondingly on the right side (concrete side) a new ratio appears:

$$\frac{F_{bz}}{F_b} = \alpha, \quad (29)$$

This new ratio will be termed "*Form coefficient of the concrete tensile area*" (Formziffer der Betonzugzone). With these explanations equations 27 can now be written:

$$k \cdot \sigma_{eR} \cdot \mu = (0.09 W_b) \cdot \alpha \quad (30)$$

2) a) Now only the factor k is left for determination. In compressed sections of reinforced concrete ($F = F_b + 15 F_c$), the concrete area is only capable of taking $\frac{1}{n}$ of the stresses of steel; this is expressed by:

$$n = \frac{E}{E_b} = \frac{2\,100\,000}{140\,000} = 15 \quad \text{or}$$

$$\text{for } E_b = 210\,000 \text{ kg/cm}^2 \text{ is } n = 10.$$

But for the tensile area of concrete (under consideration) we shall have to introduce a corresponding coefficient:

$$n_z = \frac{E}{E_{bz}} = \frac{2\,100\,000}{250\,000} = 8,4 \quad (31)$$

(modulus of elasticity E_{bz} for tension according to issue 66).

b) The formation of a separating crack in the tensile area not only depends on the elastic behaviour, but also on the *brittleness of concrete*. As is well known, the tensile strength does not keep pace with the increasing compressive strength of concrete, therefore it seems advisable to introduce for each of the *three most common qualities of concrete with minimum cube strengths of $W_{b28} = 120, 160$ and 225 kg/cm^2* a separate coefficient s expressing the *brittleness*.

Hence according to equation 31 we receive

$$k = \frac{s}{n_z} = \frac{s}{8,4} \quad (32)$$

c) In Fig. 10 the tensile strength K_{bz} of un-reinforced concrete beams (chiefly for $55 \cdot 15 \cdot 10 \text{ cm}^3$ loaded with two point loads) is plotted in relation to W_{b90} (see e. g. line DE and FG). For the three qualities of concrete mentioned (see points A, B and C) we may assume the values:

$$K_{bz} = 20, 30, \text{ and } 40 \text{ kg/cm}^2 \text{ respectively.} \quad (33)$$

The values of the tensile strength of concrete prisms ($75 \cdot 20 \cdot 26 \text{ cm}^3$) which are shown as well, remain far behind, for increasing cube strength, chiefly on account of the great difficulty of establishing a fully centric tensile action (see lines D'E', F'G' and H'J'). These values are not useful for strength consideration.

Substituting in Eq. 28, in place of $(0.09 W_b)$, the tensile strength K_{bz} sought, we see that K_{bz} can be calculated without difficulty from *the measured depth of cracks t* simply by assuming a fixed value of k (or for s) for each of the three qualities of concrete. By choosing for s the following values, which are easy to remember,

$$\left. \begin{array}{ll} s = \frac{1}{3} \text{ hence } k = \frac{s}{8,4} = \frac{4}{100} & \text{for low-grade concrete} \\ & (W_{b28} = 120 \text{ to } 160 \text{ kg/cm}^2) \\ s = \frac{2}{3} \text{ „ } k = \frac{8}{100} & \text{for medium grade-concrete} \\ & (W_{b28} = 160 \text{ to } 225 \text{ kg/cm}^2) \\ s = 1 \text{ „ } k = \frac{12}{100} & \text{for high-grade concrete} \\ & (W_{b28} = 225 \text{ kg/cm}^2) \end{array} \right\} \quad (34)$$

we receive the following mean values from our tests for these three ranges:

$$K'_{bz} = 18.29 \text{ and } 39 \text{ kg/cm}^2.$$

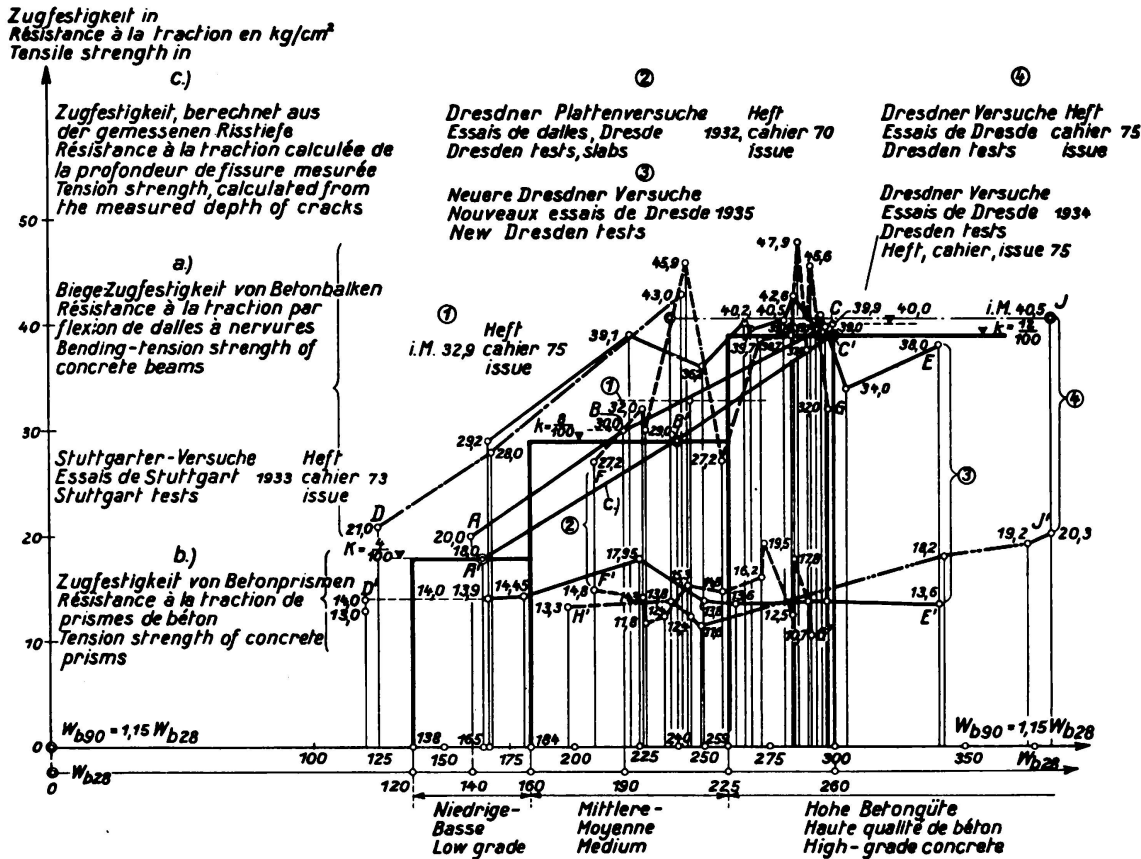


Fig. 10.

Tension strength of concrete in relation to cube strength (Dresden Tests 1928 to 1936).

The points A', B' and C' represent these values, and have the same incline as for the points, A, B and C. Hence according to Eq. 30 the relation expressing the cracking point stress for the three ranges can be written:

$$\sigma_{eR} = \frac{9}{4} W_b \cdot \frac{F_{bz}}{F_e}, \quad \sigma_{eR} = \frac{9}{8} W_b \cdot \frac{F_{bz}}{F_e} \quad \text{and} \quad \sigma_{eR} = \frac{9}{12} W_b \cdot \frac{F_{bz}}{F_e}, \quad (35)$$

wherein $F_{bz} = b_o \cdot t$ stands for the fractured tensile area.

3) Examples.

a) For point B₁ in Fig. 7 (St 37) with $b_o = 12 \text{ cm}$, $F_e = 12.72 \text{ cm}^2$, $W_{b90} = 167 \text{ kg/cm}^2$ and a load corresponding to the permissible stress $\sigma_e = 1200 \text{ kg/cm}^2$ shall be calculated the depth of the crack. With Eq. 34 for low-grade concrete and $k = \frac{4}{100}$ from Eq. 28 we receive a depth of crack of:

$$t = \frac{F_{bz}}{b_o} = \frac{1}{b_o} \cdot \frac{k \cdot \sigma_{eR} \cdot F_e}{0.09 \cdot W_b} = \frac{1}{12} \cdot \frac{4}{100} \cdot \frac{1200 \cdot 12.72}{0.09 \cdot 167} = 3.4 \text{ cm}, \quad (36a)$$

while $t_1 = 3.5 \text{ cm}$ was measured in the test.

b) For point D in Fig. 7 (St 52, with $b_o = 20$ cm, $F_e = 8.15$ cm², $W_b = 150$ kg/cm²) and again for a load corresponding to the permissible stress, in this case $\sigma_{e adm} = 1800$ kg/cm², shall be calculated the depth of crack. Since in this case $W_{b28} = W_{b90} : 1.15 = 217$ kg/cm², hence for medium grade concrete we receive with $k = \frac{8}{100}$ (according to Eq. 34) from Eq. 28a:

$$t = \frac{1}{20} \cdot \frac{8}{100} \cdot \frac{1800 \cdot 8.15}{0.09 \cdot 250} = 2.6 \text{ cm}, \quad (36b)$$

the crack measured was $t_1 = 3$ cm.

c) For point E in Fig. 7, for a load which for $\sigma_{e adm} = 1800$ kg/cm² produced a depth of crack $t_1 = 3.00$ cm (based on $W_{b90} = 305$ kg/cm², $b_o = 20$ cm and $F_e = 8.17$ cm²) the stress shall be calculated at the moment of the appearance of the first cracks. Assuming that the depth of cracks is more or less proportional to the stress, and that the safety against cracks averages about $v_R = 0.5$ then the depth for the first crack of $t = 0.5 \cdot 3.0 = 1.5$ cm has to be assumed. Hence according to Eq. 34 and for $k = \frac{12}{100}$ (high-grade concrete) we receive for the stress sought:

$$\sigma_{eR} = \frac{0.09 \cdot W_b \cdot b_o \cdot t}{k \cdot F_e} = \frac{9}{12} \cdot \frac{305 \cdot 20 \cdot 1.5}{8.17} = 840 \text{ kg/cm}^2$$

instead of $\sigma_{eR} = 850$ kg/cm² as obtained from the test.

4) By giving the equations 28 and 30 the following form:

$$\sigma_{eR} = \frac{1}{k} \cdot 0.09 \cdot W_b \cdot \frac{F_{bz}}{F_e} = \frac{1}{k} \cdot 0.09 \cdot W_b \cdot \frac{\alpha}{\mu}, \quad (37)$$

the conclusion below can be derived:

a) The cracking point stress σ_{eR} and with it safety against cracking $v'_R = \frac{\sigma_{eR}}{\sigma_{e adm}}$ is, under otherwise identical circumstances, proportional to the cube strength W_b and also, since $F_{bz} = b_o \cdot t$, proportional to the width of the rib b_o ,

b) but inversely proportional to the section of steel F_e or the percentage of reinforcement. As, under similar circumstances

$$\sigma_{eR} \cdot \mu = \text{const.} \quad (38)$$

the $\sigma_{eR} - \mu$ — lines in Figs. 6, 8 and 9 are parts of a quadratic hyperbola, which is shown dotted in Fig. 8.

c) The new "form coefficient" for concrete sections (See Eq. 29):

$$\alpha = \frac{F_{bz}}{F_b}$$

corresponds to the "form coefficient" of the sectional area of steel, which is known as the ratio of reinforcement $\mu = \frac{F_e}{F_b}$ (mostly expressed in %) wherein $F_b = b \cdot h$.

IV. The importance of the shape of cross section in respect to the safety against cracking.

For the Dresden tests with factory made reinforced concrete building elements (1934, Issue 75, German Commission on Reinforced Concrete) the safety against cracks for very slender beams ($1:h = 5.82:0.81 = 32$) according to Eq. 9b was fixed by the ratio $v_R = M_R:M_{adm}$. Based on a safety factor of 3, the permissible bending moment M_{adm} was stipulated to be $M_{adm} = \frac{1}{3} M_B$. In addition to this, the stress σ_{e1} had been calculated based on the moment M_{adm} . With those assumption the cracking point stress can be expressed approximately by

$$\sigma_{eR} = v_R \cdot \sigma_{e1} \quad (39)$$

It is now possible, based on Eq. 36a, to calculate for the eight sections of Fig. 11 the cracking depth t , and to state by means of the ratio $t:e$ (e = concrete cover measured from bottom edge to centre of bars) a *quality factor* for

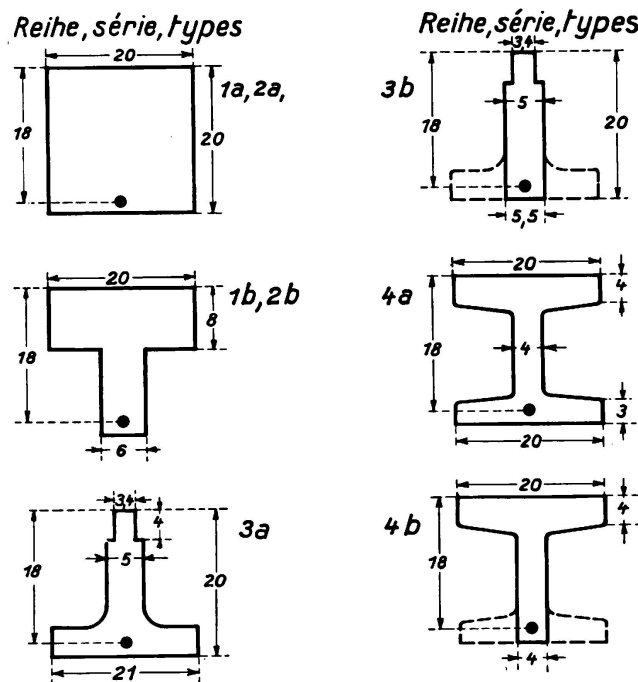


Fig. 11.

Shape of cross sections and safety against cracks.

the safety against cracks for each particular type of section. For these cases as shown in Table II the elastic ratios $n_z = \frac{E}{E_{bz}}$ (Eq. 31) were worked out, based on the values E_{bz} received from tests for the various qualities of concrete. It has to be pointed out, however, that the cross sectional form 1a, with $W_{b28} = 198 \text{ kg/cm}^2$ was for medium quality concrete with a brittleness coefficient of $s = \frac{2}{3}$ (Eq. 34); the other forms 1b to 4b are for high quality concrete with $s = 1$. Based on a constant cross sectional area of steel $F_e = 2.55 \text{ cm}^2$

were calculated the cracking depth t according to Eq. 36a and the *quality factor of the safety against cracks* $e : t$ (concrete cover over bars $l = 1.9$ cm).

Table II.

Type of section	b_o	W_b	$v_R \cdot \sigma_{e adm} = \sigma_{eR}$	$n_z = E : E_{bz}$	s	$\frac{l}{k} = \frac{n_z}{s}$	t according to Eq. 36 cm	$e : t$ ($e=1.9$ cm)
1 a	20	198	965	11,05	$\frac{2}{8}$	16,6	0,41	4,6
1 b	6	237	998	9,46	1	9,46	2,10	0,9
2 a	20	367	1440	7,14	1	7,14	0,78	2,4
2 b	6	384	1270	7,14	1	7,14	2,18	0,9
3 a	21	394	875	7,14	1	7,14	0,42	4,5
3 b	5,5	377	680	7,14	1	7,14	1,30	1,5
4 a	20	374	980	7,27	1	7,27	0,51	3,7
4 b	4	342	785	7,50	1	7,50	2,16	0,9

From table II it can be seen that the quality factor of the safety against cracking $e : t > 2$ not only occurs for the two rectangular sections of types 1a and 2a but also for type 3a (reversed T-beam section) and type 4a (I-shaped section). The most unfavourable quality factor of the safety against cracks ($e : t = 0.9 < 1$) stands for T-beam sections of types 1b, 2b and 4b, the section type 3b with small width of rib $b_o = 5.5$ cm ranges in between the other quality classes¹³.

The foregoing investigations might permit the conclusion to be drawn that the sections e to h proposed by Fig. 12 may give increased safety against cracking for wide span girder bridges.

V. *The permissible width of cracks* was laid down empirically by the Dresden tests of 1936, experience made in practice having shown that T-beams dimensioned according to regulation with $\sigma_{e adm} = 1200$ kg/cm² for St 37 had proved safe against the danger of rust. The width of cracks photographed in 23 times enlargement and measured at the level of the reinforcement, are shown in table III.

Table III

Measured width of cracks b_R , at $\sigma_{e adm}$, in $\frac{1}{1000}$ mm.

Kind of steel	St 37	St 52	Isteg	Twisted bulb bar
Nos. of beams	2 + 3	4 + 4	4	4
width of ribs $b_o = 20$ cm	70 to 70 or mean 70	40 to 130 or mean 90	80 to 110 or mean 94	75 to 120 or mean 89
$b_o = 12$ cm	25 to 60 or mean 41	10 to 70 or mean 35	—	—

¹³ As regards the carrying capacity for the various types of section another order naturally applies, as will be seen also from Issue 75.

From the width of the cracks measured for a stressing of $\sigma_{e adm}$, the distance between the cracks and the number of cracks on a testing specimen, it is possible by using the law of proportionality $\frac{\Delta l_1}{l_1} = \frac{\sigma_e}{E}$ to arrive at the stressing of steel (elongation $\Delta l_1 = b_R$, observed length, or measuring length) and to draw further conclusions (e. g. according to Fig. 17, see E 2). From table III it follows that:

- 1) The permissible width of cracks can be accepted as about:

$$b_{R adm} = \frac{125}{1000} \text{ mm} = \frac{1}{8} \text{ mm} \quad (40)$$

- 2) The striking difference in the greater crack widths for $b_o = 20$ compared with $b_o = 12$ forms a confirmation of the physical conception (see under VI 1)

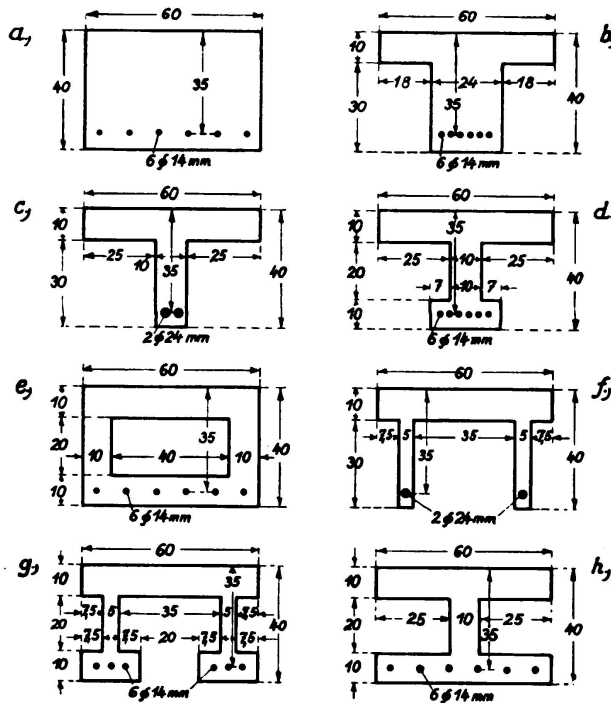


Fig. 12.

Shape of cross sections for future tests.

which has its expression in our Ev. 28. In both cases at the moment of first appearance of cracks, for equal depth of cracks (e. g. $t = 3$ cm) the ratio between the released concrete tensile forces is the same as the ratios of the areas $b_o \cdot t$; hence

$$Z_{b20} : Z_{b12} = 20 t : 12 t = 5 : 3 = 1.7,$$

according to table III for measured width of cracks we receive $W_{20} : w_{12} = 70 : 41 = 1.7$ or the same value as above. The greater the suddenly released tensile force Z_b , the wider the crack opens. If according to Eq. 40 the permissible width of cracks is limited to $b_{R adm} = \frac{1}{8} \text{ mm}$, then this limit prevents

the constriction of the gross section of steel F_e going too far (we had $\mu = 0.34$ to 0.22%). These intricate relations, however, can only be solved by further experiments.

VI. *Summary of conclusions in respect to the safety against cracking, based on the Dresden tests of 1928 and 1935.*

1) *Physical conception.* At the moment of the appearance of the first cracks the part section $F_{bz} = t \cdot b_0$ and with it the corresponding tensile force are eliminated. The amount of tensile force in the section is governed by Eq. 28 and is expressed as a fraction of the tensile force in steel $Z = \sigma_{eR} \cdot F_c$ and amounts to 4, 8 or 12 % according to the quality of concrete.

It may be mentioned that this additional amount of tensile force only occurs at the place of rupture, but not at unfractured places.

2) The tests show that with regard to safety against cracking the *cross sectional shape of the reinforcement* diminishes in importance. But contrary to this, the *percentage of reinforcement* μ is of a decisive nature. The *smaller the cross section of steel* in comparison to the section of concrete and the width b_0 of the concrete tensile area, *the greater the safety against cracking*. A restriction is only given by the fixing of the *permissible width of cracks* (according to Eq. 40) which is limited to $b_{Radm} = \frac{1}{8}$ mm. The wider the rib the wider the crack (F_c remaining constant).

3) Safety against cracking increases considerably with *increasing quality of concrete*. But since the brittleness becomes more pronounced with using cement of high compression strength (or the ratio between tensile strength Z and compressive strength D becoming smaller), this increase in compressive strength can only show itself in a very restricted limited manner in respect to the safety against cracking with the brands of cement in use nowadays.

4) As regards *the shape of cross section*, it is to be expected that the *application of I-shaped and box-shaped cross sections* to structures of wide spans proves favourable in respect to the safety against cracking and the carrying capacity. On account of this possibility the German Commission for Reinforced Concrete suggested that tests be carried out with such sections, using high-grade concrete of about $W_b = 450 \text{ kg/cm}^2$ and high-quality steel (dimension of test pieces to be about half the actual sizes required) (See Fig. 12).

5) In view of the higher safety against cracking (see Eq. 16 and 18) of slabs with rectangular section ($v_R = \frac{3}{4}$) compared with T-beams ($v_R = 0.4$ to 0.5), it is advisable to employ *high-quality building steel* for the reinforcement of slabs. At the same time the cross sectional area of steel F_c should be kept down as much as possible, i. e. as far as is permitted by the crack-width b_{Radm} . Such tests with slabs are urgently required.

6) As regards *safety against cracking for statically stressed T-beams with reinforcement* of St 52, the stipulation for permissible stresses: $\sigma_{eadm} = 1800 \text{ kg/cm}^2$ is quite justified, as also follows from comparison tests with St 37 ($\sigma_{eadm} = 1200 \text{ kg/cm}^2$).

D. The safety against rupture of slabs, and T-beams reinforced with high-grade steel.

I. The line: carrying-capacity percentage of reinforcement.

1) *The calculated carrying capacity of rectangular sections reinforced with different percentages of St 37 and St 52 respectively.*

The carrying capacity of rectangular sections has to be calculated for bending, according to the German Regulations for reinforced concrete under the following assumptions:

- a) The tensile area of the concrete is not taken into account (so-called calculation according to state II).
- b) The ratio of the moduli of elasticity between steel and concrete to be $n = E : E_b = 15$.

Further the permissible stresses to be as under:

- c) For concrete to have a safety factor of 3,

$$(\nu_B = 3), \text{ also } \sigma_{b \text{ adm}} = \frac{1}{3} W_b$$

(W_b = cube strength).

- d) Steel reinforcement to have a safety factor of 2,

$$(\nu_e = 2), \text{ also } \sigma_{e \text{ adm}} = \frac{1}{2} \sigma_s$$

(σ_s = yield point stress of steel).

In fig. 13 are given the calculated results for rectangular sections, reinforced with St 37 and St 52 respectively, in dependence on the percentage of reinforcement

$$\mu = \frac{F_e}{b \cdot h}$$

The ordinates have the values:

$$y = \frac{M}{bh^2} \text{ (in kg/cm)}^2, \quad (41)$$

(wherein $\sigma_B = \frac{M}{W_i}$ represents the rupture point stress and $W_i = \alpha \cdot b h^2$ the modulus of section¹⁴. Based on the above the two following ranges can be distinguished:

- a) *The range of lightly reinforced sections* (rupture due to exceeding the yield point stress σ_s of steel).
- b) *The range of heavily reinforced sections* (rupture due to exceeding the bending compressive stress of concrete).

The tests carried out by the German Commission for Reinforced Concrete particularly the Dresden tests with high-grade steel, showed that the calculated results for the range of lightly reinforced sections coincide well with test results. *The yield point stress of steel was decisive for rupture in these cases* (see

¹⁴ F. v. Emperger: Austrian Reinforced Concrete Standards of 1935. „Die Normen für Eisenbeton 1935 in Österreich.“ Beton und Eisen 1935, Vol. 34, Issue 16, p. 254.

section B and Eq. 7). For the second range of heavily reinforced sections it was found that the actual carrying capacity was considerably greater than the calculated values.

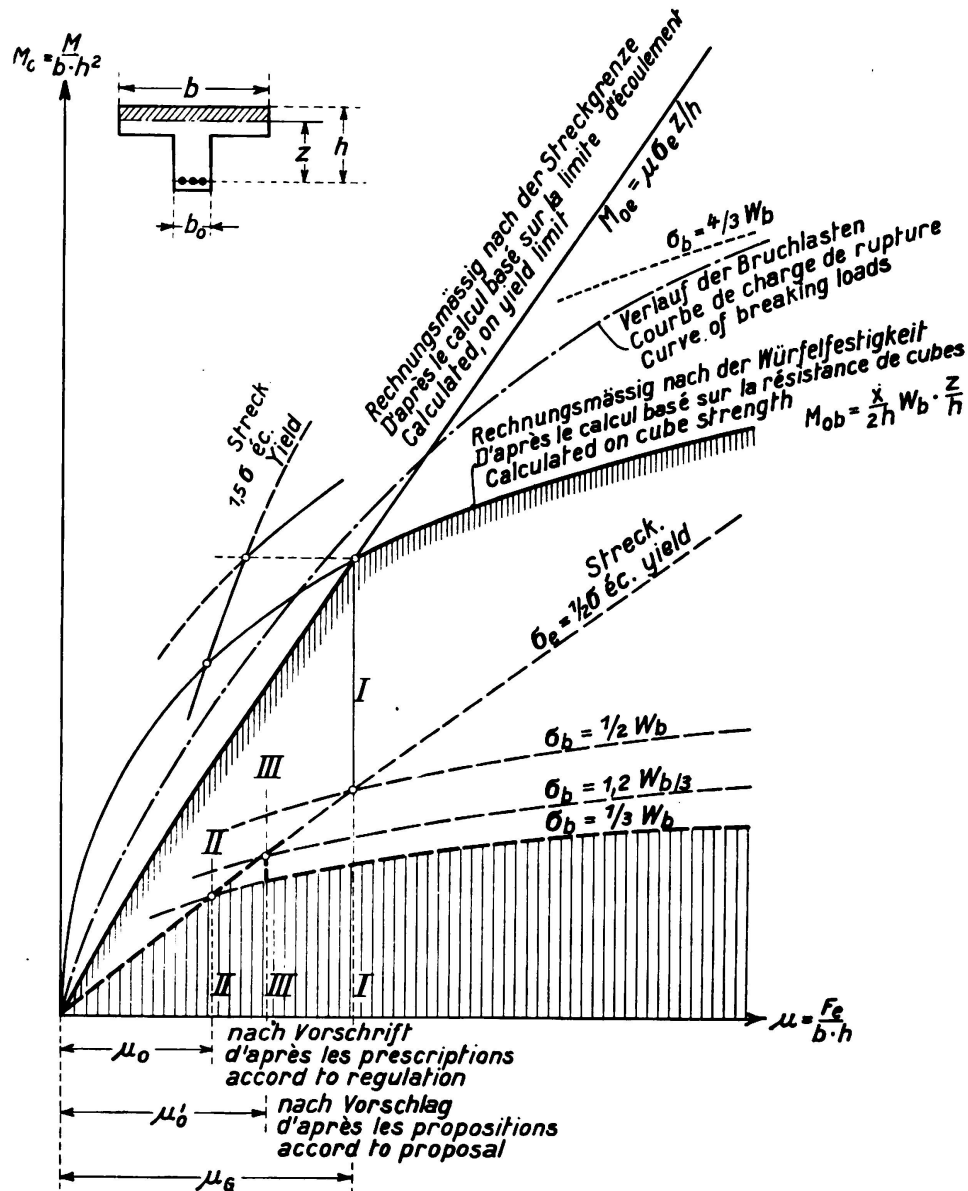


Fig. 13.

The carrying capacity of T-beams in dependence on the percentage of reinforcement (according to Emperger and Haberkalt).

The purpose of the Dresden tests of 1935—1936 was the following:

I. To find by experiment that limit of the percentage of reinforcement which divides the first from the second range.

II. To establish by how much the compression stress of concrete is exceeded. At the state of rupture for tests, compared with the carrying capacity as found by calculation based on cube strength.

II. The importance of tests for the methods of calculation.

Determination of limits.

The question of the carrying capacity of reinforced concrete slabs and T-beams in relation to the percentage of reinforcement was first brought to discussion by the Austrian Commission for Reinforced Concrete under the guidance of *F. Gebauer*. The problem was studied to the extent that a proposal made by *F. v. Emperger* and *C. Haberkalt* should be embodied in the Austrian Regulations (see Fig. 13). This proposal states that the limit between the two ranges — where the yield limit or the cube strength is decisive — should be raised (see point III) against the prescription of the present regulations (see point II). This raise corresponds to an increase of 20 % of the usual permissible stress. But since the permissible stress has been maintained the line for carrying capacity shows an offset. This solution of the problem is not quite satisfactory since it is only based on rectangular sections, and since cases may occur in which the calculated carrying capacity drops on account of additional bars. At places where high compressive stresses in concrete occur, it is customary to provide haunches in the vertical or horizontal direction, also bars are provided in the compressive zone if necessary, but full use can never be made of such bars, apart from being a great hindrance while concreting and reducing the compound action of steel and concrete. Since haunches are mostly not wanted for the sake of appearance, particularly if concrete is in competition with steel structures, it still remains to find a satisfactory solution of this problem¹⁵.

a) For the first range, for which the yield point stress of steel is of a decisive nature, the Dresden tests of 1936 carried out with rectangular beams (Fig. 14) and reinforced with St 37 or Isteg-steel respectively, yielded the following results: The carrying capacity line is almost a straight line, for which the ordinates average only 12,5 % higher than the values calculated. Therefore a welcome margin of safety is established. The yield point stress again proves to be decisive for safety in case of the first range. There is therefore no need to change the customary mode of calculation. The Dresden tests (Fig. 14) have given the following limits for the percentage of reinforcement, dividing the first range for which the yield point stress of steel is decisive from the second range which is based on the cube strength of concrete:

for St 37 with $\sigma_s = 2800 \text{ kg/cm}^2$ and $W_b = 110 \text{ kg/cm}^2$ $\mu_G = 1.82 \%$

for Isteg-steel $\sigma_s = 4100 \text{ kg/cm}^2$ and $W_b = 110 \text{ kg/cm}^2$ $\mu_G = 0.72 \%$

for Isteg-steel $\sigma_s = 4100 \text{ kg/cm}^2$ and $W_b = 150 \text{ kg/cm}^2$ $\mu_G = 0.95 \%$

The lines CD for St 37 and EF for Isteg-steel as found by tests for the second range lie considerably higher than the carrying capacities calculated and shown by line AB. It may be mentioned that for a percentage of reinforcement of 1.6 % two of the four points near J are for $W_b = 110 \text{ kg/cm}^2$ and two for $W_b = 150 \text{ kg/cm}^2$; in other words for these cases the carrying capacity depends on the cube strength.

¹⁵ See also *R. Saliger-Vienna*: Experiments on the nature of concrete and with concrete with compressive reinforcement (Versuche über zielsichere Betonbildung und an druckbewehrten Balken) Beton und Eisen 1935, Nr. 1, p. 12.

b) New test series to determine the limit percentage of reinforcement μ_G for slabs and T-beams with different kinds of reinforcement are being carried out.

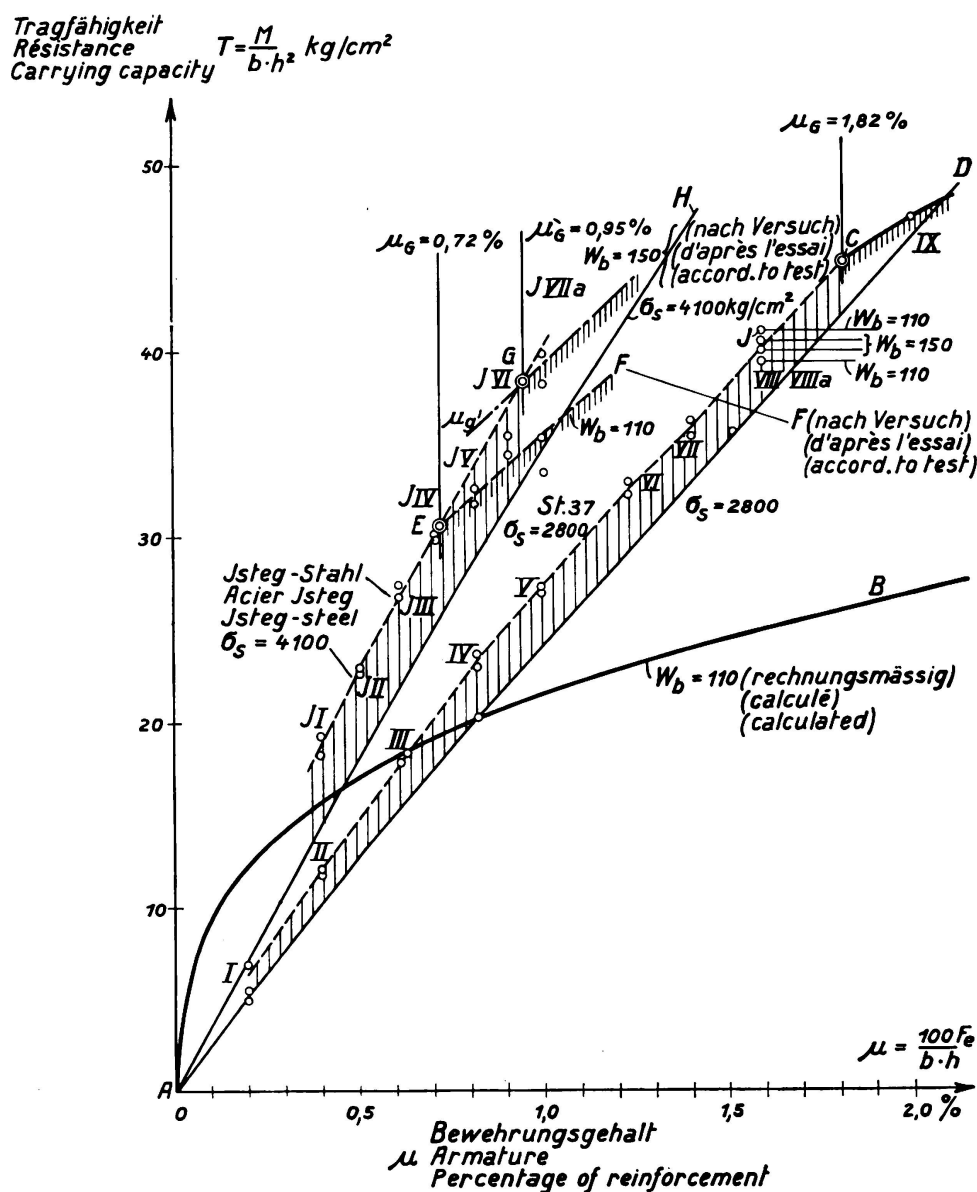


Fig. 14.

The carrying capacity of rectangular beams in dependence on the percentage of reinforcement, according to Dresden tests.

In connection with the Dresden tests a remarkable method of calculation to determine the limit reinforcement has been developed by Dr. E. Friedrich, Scientific Collaborator at the Dresden Test House¹⁶. A very clear arrangement is obtained if the ordinates represent the so-called carrying capacity $T = \frac{m \cdot h}{J_i}$ and the abscissae are given the values $\frac{l}{s} = \frac{h}{x}$. If the state of cracking of the

¹⁶ See: Contribution to discussion (Diskussionsbeitrag) by E. Friedrich-Dresden.

compression zone of concrete be based on the cube strength $\sigma_p = 0.75 W_b$ and not on the usual state IIb with triangular stress-distribution, but on a new state IIc with rectangular stress distribution (under consideration of the plastic deformability of concrete), the limiting value can be expressed by the formula

$$s'_g = \frac{x}{h} = \frac{3}{2} - \frac{1}{2} \cdot \sqrt{\frac{3(1+3k)}{3+k}} \quad (49)$$

The coefficient k is expressed by the term

$$k = \frac{\sigma_s}{n \cdot \sigma_p} \quad (50)$$

and hence the limit of reinforcement is obtained from:

$$\mu_G = s'_g \cdot \frac{\sigma_{b \text{ adm}}}{\sigma_{e \text{ adm}}} \quad (51)$$

The comparison with results received from beams with rectangular section (Dresden tests 1936) is quite satisfactory.

c) To make use of the carrying capacity in this second range for which the cube strength is the decisive factor for rupture, proposals to replace the exceptional permissible increase of stresses for frames and haunches (full rectangular sections § 29 Table IV and No. 5, b, β and δ) can only be made after completion of the Dresden tests now in progress for slabs and T-beams, reinforced with St 37, St 52 and other high-grade steels.

The result of these considerations is that for the first range of lightly reinforced beams no change in the usual mode of calculation is required. But it will be permissible in future to extend this range to the reinforcing limit μ_G , which still requires to be calculated and proved by tests which have not yet been concluded. It implies that for the second range beyond this limit a new method of calculation will be required for the compressive stresses in concrete if reinforcement on the compression side and haunches are to be avoided.

III. Objections have been raised against the acceptance of the yield limit σ_s as basis of the safety against rupture ($v_B = \sigma_s : \sigma_{e \text{ adm}}$; Eq. 7) for lightly structural elements of reinforced concrete. The objection was that on account of the plastic deformability of concrete in rupture tests have given higher calculated values than yield limit i. e.

$$\max \sigma_e > \sigma_s,$$

or in other words that the safety margin could still be utilised¹⁷. These "excess values"

$$\beta = \frac{\max \sigma_e - \sigma_s}{\sigma_s} \quad (42)$$

¹⁷ Compare W. Gehler: International Congress for the Testure of Materials Zurich 1931 (Paper: Strength, Elasticity and Shrinkage of Reinforced concrete pp. 1079 to 1087 [Festigkeit, Elastizität und Schwinden von Eisenbeton] where the nature of plasticity of concrete in comparison to building steel was studied very exhaustively). Congress publication, Zurich 1932, published by I.V.M.

are shown in Fig. 15 in relation to the cube strength W_{b90} , as they were found by recent Dresden tests carried out with particular care. With increasing quality of concrete an increase in the value β can fundamentally be noticed; this varies

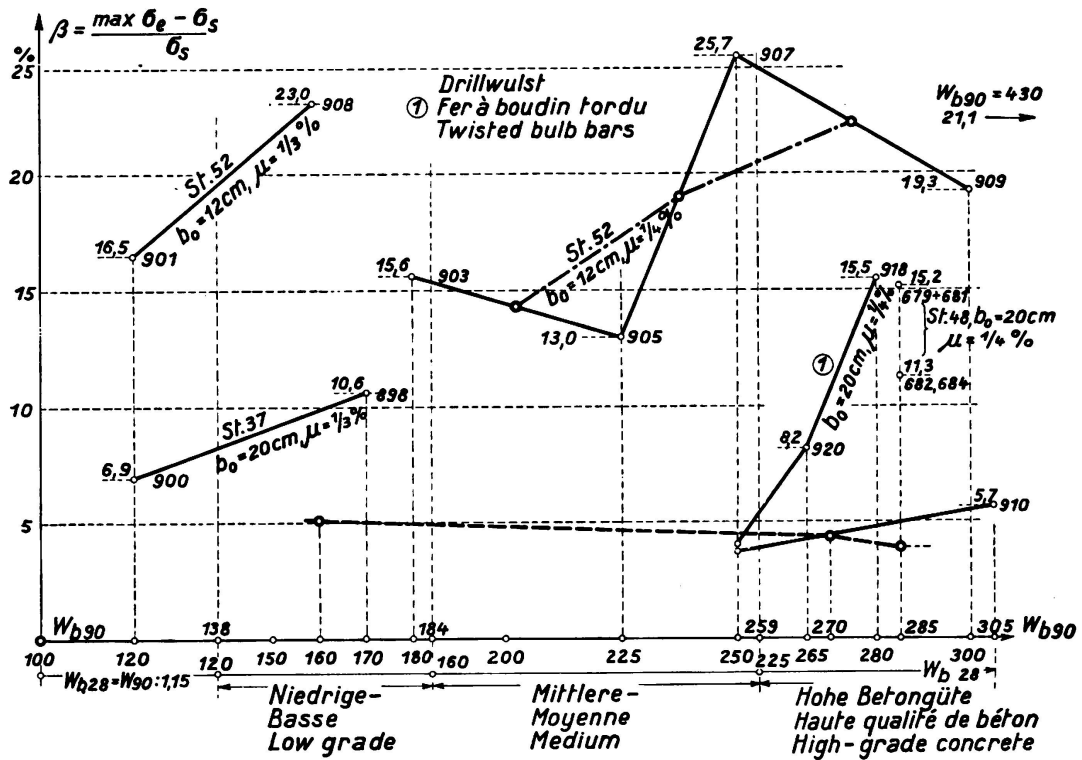


Fig. 15.

The value of the plastic deformability β in relation to the quality of concrete W_{b90} (Dresden tests 1936 for T-beams).

for T-beams between 4 % and 26 % (in the case of Fig. 14 β had an average value of 12.5 %). It was, however, not possible to find a law. *It is therefore advisable not to make use of this varying margin of safety, but to base as usual the safety against rupture on the yield limit for lightly reinforced concrete beams.*

IV. The degree of safety against rupture as found by the Dresden tests with T-beams is compiled in table IV.

Table IV

Type of steel	St 37	St 52	Isteg	Twisted bulb steel.
σ_s mean	2610 to 2935 2790 = abt. 2800	3840 to 4445 3980 = abt. 4000	4035 to 4425 4110 = abt. 4100	4000 to 4390 4200
$\sigma_{e adm}$	1200	1800	1800	1800
$\nu_B = \sigma_s : \sigma_{e adm}$	2.33	2.22	2.28	2.33

The required minimum factor of safety of 2 against rupture is therefore sufficiently maintained for statical stressing for St 37 and 1200 kg/cm² and for high-grade steel reinforcement with 1800 kg/cm².

E. The application of high-grade steel as anti-shrinkage reinforcement in concrete road construction.

Stuttgart tests carried out by Prof. Mörsch have clearly proved that by providing reinforcement the degree of shrinkage of concrete is reduced to about half, which indicates that the use of reinforcement to counteract shrinkage in concrete roads is quite justified. The question arises whether for the same cross-sectional area of steel, Steel St. 37, or high-grade steel, e. g. the well-known types of building steel fabric should be given preference on the strength of the Dresden test results concerning safety against rupture.

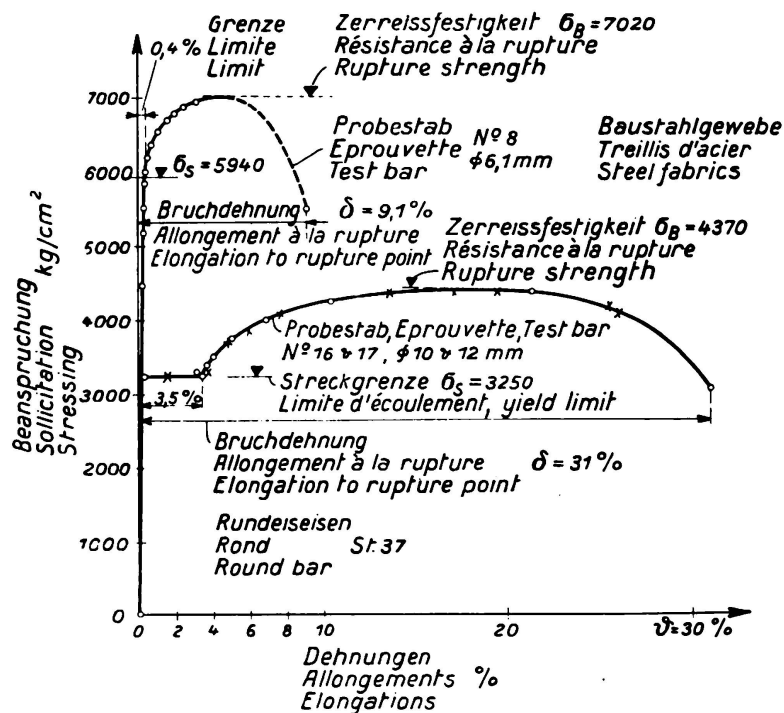


Fig. 16.

Stress-strain diagrams for steel fabrics and round bars of St 37.

1) Comparison between the stress-strain line of building steel fabric and round bars of St. 37 (Fig. 16) shows that yielding of the building steel fabric cannot be observed whilst this is clearly noticeable with St 37. According to DIN 1602 the 0.2 % limit of permanent elongation has to be accepted as yield limit, which as proved by the 0.4 % limit of the total elongations, therefore we receive for building steel fabric $\sigma_s = 5940$ kg/cm² for $\sigma_B = 7020$ kg/cm² with a rupture point elongation $\delta = 9.1$ %.

2) Concrete roads are also subject to deformations arising from normal traffic loads. Dresden tests of 1934, carried out with concrete strips reinforced with building steel fabric, allow a distinction to be drawn between the three follow-

and the elongation of steel for the same length:

$$\varepsilon_1 = 1.375 \text{ mm} : 950 \text{ mm} = 0.145 \text{ ‰}.$$

In Fig. 17 OP'S' represents the stress-strain line for St 37 and line OPS the corresponding line for building steel fabric (compare also Fig. 16), their inter-



Fig. 18.

Slab reinforced with building steel fabric (Dresden tests 1934).

section points H' and H with the vertical line at a distance $\delta = \varepsilon_2 = 0.145 \text{ ‰}$ from the axis of ordinates yields the stress values $\sigma'_2 = 2700 \text{ kg/cm}^2$ and $\sigma_2 = 3050 \text{ kg/cm}^2$.

The upper limit for the hair crack width $b_R = \frac{1}{8} \text{ mm}$ therefore lies 13 ‰ higher on stress scale than for St 37.

3rd range: Slabs with fine cracks. The upper limit for the width of "fine cracks" may in this case be taken as double $b_{R \text{ adm}} = \frac{1}{8} \text{ mm}$, hence $b_3 = \frac{1}{4} \text{ mm}$. For this range the protection against rust according to experience still exists although not fully. The upper limit $b_3 = \frac{1}{4} \text{ mm}$ is attained for an elongation of steel of $\varepsilon_3 = 2 \varepsilon_2 = 2 \cdot 0.145 \text{ ‰} = 0.29 \text{ ‰}$. With this value as abscissa we receive in Fig. 17 the points F' and F of the two diagrams for which the coordinates are: $\sigma'_3 = \sigma_{s37} = 3070 \text{ kg/cm}^2$ and $\sigma_3 = 5500 \text{ kg/cm}^2$.

The upper limit ($b_3 = \frac{1}{4} \text{ mm}$) for fine cracks is reached for St 37 already at the yield limit $\sigma_s = 3000 \text{ kg/cm}^2$, but lies for building steel fabric at a stressing of $\sigma_e = 5500 \text{ kg/cm}^2$, a value 80 ‰ higher. The safety against rust, depending on the crack-width b_R , is higher for building steel fabric than it is for St 37 (assuming same sectional area of steel).

F. Oscillation-stressing of structural elements of concrete reinforced with high-grade steel.

For the purpose of drawing conclusions in respect to the degree of safety from available bending test results of structural concrete elements reinforced with building steels with high yield limit, the following procedure is recommended:

1) The so-called "working safety" is expressed by the term

$$\nu = \frac{w_v}{w_R} \quad (43)$$

Herein w_v represents the maximum amplitude, as found by fatigue tests, a value which can just be produced infinitely often¹⁸ while w_R represents the highest conceivable amplitude which is taken as the basis for static calculation. If σ_o stands for the upper and σ_u for the lower stress limit of an oscillation test (in the DIN standard No 4001 called upper stress and lower stress) we receive

$$w_v = \sigma_o - \sigma_u \quad (44)$$

According to the static calculation, under consideration of impact, the permissible stress shall not be exceeded, therefore this stress forms the upper limit of stressing, whilst the stressing due to permanent load only represents the lower limit in the static calculation, hence

$$w_R = \sigma_{adm} - \sigma_g \quad (45)$$

2) To determine the amplitude for the static calculation a certain particularly unfavourable case is used which can be expressed by the two following assumptions:

a) $\sigma_p : \sigma_g = 2 : 1$.

b) As impact coefficient, according to DIN 1075 the maximum possible value $\varphi = 1.4$ is chosen. For this extreme case we receive

$$\sigma_{adm} = \sigma_g + \varphi \cdot \sigma_p = \sigma_g + 1.4 (2 \cdot \sigma_g) = 3.8 \sigma_g \quad (46)$$

and

$$w_R = \sigma_{adm} - \sigma_g = \sigma_{adm} \left(1 - \frac{1}{3.8}\right) = 0.737 \sigma_{adm} \quad (47)$$

With the expressions (44) and (47) the "working safety" (Eq. 43) assumes the following form

$$\nu = \frac{\sigma_o - \sigma_u}{\sigma_{adm} - \sigma_g} = \frac{\sigma_o - \sigma_u}{0.737 \sigma_{adm}} \quad (48)$$

3) The proposed degree of "working safety" is $\nu = 2$. This means that for fatigue rupture the amplitude is double the amplitude on which the static calculation is based. From the comparison with the rules (γ — procedure) laid down in "the Basis for Calculating Railway Bridges in Steel (B.E.) of the German State Railways 1934" we learn that the above determination of the degree of safety offers a higher degree of safety than demanded for steel railway bridges (especially if the unfavourable influences of riveted and welded connections are considered).

4) The Stuttgart fatigue tests carried out with Isteg steel on this procedure are compiled in table V which gives the basic values for slabs and the values

¹⁸ According to the numerous fatigue tests with building steels it can be assumed for the range under consideration that the amplitude is practically independent of the static pre-stressing (mean-stressing).

for w_v , w_R and v which were calculated with equations (44), (47) and (48) (Fig. 19).

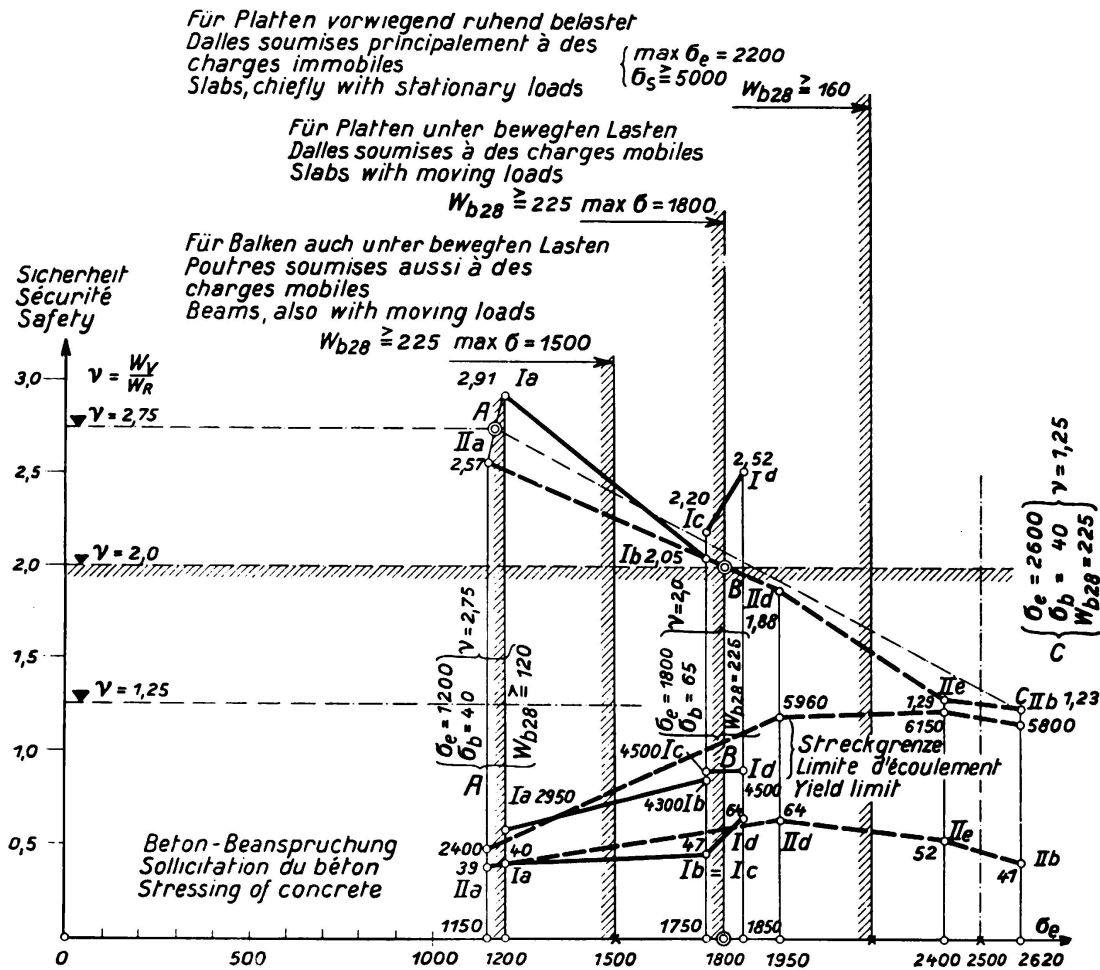


Fig. 19.

Results of Stuttgart fatigue tests on slabs with Isteg — and steel fabric reinforcement (I and II respectively).

Results: The high-grade steel St 60 in combination with concrete of $W_{b28} = 120 \text{ kg/cm}^2$ yields a degree of safety $v = 2.05$ compared with $v = 2.9$ in case steel St 37 is used. The degree of safety obtained for two slabs reinforced with Isteg-steel giving $v = 2.5$ and 2.2 lies between the above mentioned values. By using high-quality concrete with $W_{b28} = 225 \text{ kg/cm}^2$ an increase in safety from 2.2 to 2.5 is noticed over concrete of ordinary quality.

5) The results of the Stuttgart fatigue tests of 1934 for slabs reinforced with Isteg steel were examined in the same way (series a' to d').

Results: A slab with St 37 of ordinary concrete showed a somewhat smaller degree of safety namely $v = 2.57$. The two slabs reinforced with building steel fabric of the series b' and e', which were designed for the extraordinary high stressing of steel of $\sigma_e = 2620 \text{ kg/cm}^2$ and 2400 kg/cm^2 respectively, show too small a degree of safety, namely $v = 1.23$ and 1.29 , which is considerably below $v = 2$. But the slab of series d' reinforced with building steel fabric and designed for $\sigma_e = 1950 \text{ kg/cm}^2$ with high-grade concrete

$W_{b28} = 225 \text{ kg/cm}^2$ gave a degree of safety of $\nu = 1.88$. Assuming a more or less straight-line proportionality, the degree of safety for $\sigma_e = 1800 \text{ kg/cm}^2$ and $W_{b28} = 225 \text{ kg/cm}^2$ works out at

$$\nu = 1.88 \cdot \frac{1950}{1800} = 2.03.$$

6) The examination of the Stuttgart fatigue test results have proved that the permissible stress of steel $\sigma_e = 1800 \text{ kg/cm}^2$ in slabs is properly chosen and suitable also in the case of moving loads; provided, however, that the concrete possesses at least a cube strength of $W_{b28} = 225 \text{ kg/cm}^2$. With this latter stipulation concerning the quality of concrete, according to experience gathered, sufficient safety against cracking can be assumed. Since corresponding tests for beams subjected to moving loads have not yet been, carried out we propose that the permissible stress for beams shall be left at $\sigma_e = 1500 \text{ kg/cm}^2$.

Table V.

Type of reinforcement and nature of concrete	Test series	Thickness of slab α (cm)	Basis of dimensioning $\frac{\sigma_b}{\sigma_e}$	quality of concrete W_{b28}		quality of steel		Plasticity factor $\beta' = \frac{\max \sigma_e}{\sigma_s}$	w_V	w_R	Working safety $\nu = \tan \frac{w_V}{w_R} = \frac{w_V}{w_R}$
				ideal	actual	σ_s	$\max \sigma_e$				
Isteg + high-grade concrete	d	11,5	64/1850	200	260	4500	5800	1,29	3440	1365	2,52
Isteg + concrete	c	14,2	47/1750	120	118	4500	5600	1,24	2840	1290	2,20
St 60 + concrete	b	13,7	47/1750	120	123	4300	4920	1,14	2640	1290	2,05
St 37 + concrete	a	14,1	40/1200	120	123	2950	3440	1,16	2570	884	2,91
Building steel fabric + high-grade concrete	d'	13,8	64/1950	210	239	5900	7120	1,21	2700	1435	1,88 ¹⁹
do. + concrete	b'	17,4	41/2620	160	219	5800	8160	1,41	2360	1920	1,23
do. + concrete	e'	10,8	52/2400	180	195	6150	7740	1,26	2280	1770	1,29
St 37 + concrete	a'	14,0	39/1150	130	115	2400	3200	1,33	2180	847	2,57

$$^{19} 1,88 \cdot \frac{1950}{1800} = 2,03.$$

G. The permissible stresses of reinforcements with high yield limit for slabs and T-beams of reinforced concrete.

1) The permissible stresses given in table VI were laid down on January 14th 1935 by the German Commission for Reinforced Concrete and subsequently embodied in governmental regulations. Some knowledge gained since from tests and some further explanations on the foregoing are added herewith:

a) Based on the Stuttgart tests with slabs of concrete of $W_{b28} \geq 225 \text{ kg/cm}^2$ and reinforced with St 52 or corresponding high-grade quality steels, it is permitted for slabs (2nd line, 6th row) to increase $\sigma_{e \text{ adm}} = 1500 \text{ kg/cm}^2$ to 1800 kg/cm^2 ; this also in case of moving loads.

Table VI.

Table of permissible stresses
for reinforcement with high yield limit for slabs and T-beams in reinforced concrete.

1	2	3	4	5	6	7	8
No.	Type of steel	Min. yield limit ²⁰	Min. rupture point elongation	Min. cube strength of concrete	$\sigma_{e \text{ zul}}$		Range of validity
					for slabs	for T-beams	
		kg/cm ²	%	kg/cm ²	kg/cm ²	kg/cm ²	
1	St 52	3600	20	120 225	1500 1500	1200 1500	Also for moving loads ²²
2	St 52	3600	20	120 160 225	1500 1800 1800	1200 1200 1500 ²³ 1800 ²⁴	For chiefly stationary loads and only for Building constructions without atmospheric influences.
3	Special steel ²¹	3600	14 ²⁵	120 160 225	1200 1800 1800	1200 1200 1500 ²³ 1800 ²⁴	
4	Special steel ²¹	5000	14 ²⁶	120 160 225	1200 2200 2200	1200 1200 1500 ²³ 1800 ²⁴	

²⁰ *Yield limit*, According to the R. C. Regulations § 7 the properties of steel require to be proved. For steels having no distinct yield limit, till such time as the question receives final settlement by tests which are being carried out, it is permissible instead of the 0.2 % limit of the permanent elongation as stipulated by DIN 1602 to take as yield limit the 0.4 % limit of the total elongation.

²¹ *Reinforcement of special steels* for special arrangements requires permission.

²² Corresponds to existing regulations.

²³ If the cross section of one single bar is $> 3.14 \text{ cm}^2$ (for twisted steels the compound section is decisive).

²⁴ If the cross section of one single bar is $\leq 3.14 \text{ cm}^2$ (otherwise as in 23).

²⁵ For slabs steels with a minimum rupture point elongation of 10 % are also permissible.

²⁶ For slabs steels with a minimum rupture point elongation of 8 % are also permissible.

b) The restrictive stipulations given in Footnotes 4 and 5 with the fixed limit-value $F_e = 3.14 \text{ cm}^2$ should be based on our Eq. 28 and 40 — replaced by a more conclusively founded regulation; this however is only possible after the necessary tests have been carried out.

2) The critical examination of the safety against rupture of reinforced concrete beams (see under D II and Figs. 13 and 14) lead to the following results.

a) The usual mode of calculation need not be altered for structures coming under the first range of lightly reinforced beams for which the yield limit of the steel is decisive (normal case).

b) Should the reinforcing limit μ_G reach the point separating the two ranges as fixed by the Dresden tests for St. 37 and high-grade steel, it is permissible to extend the first range up to this limiting value and the usual method of calculation can again be employed.

c) Beyond this limit, i. e. in the second range for which the compression strength of concrete is decisive (rare case) a new method making increased use of the material properties would require to be laid down. The employment of compression reinforcement and haunches could then mostly be avoided and the appearance of structures would be improved.

Our tests with high-grade steels in reinforced concrete construction led to the realisation, that by maintaining the safety against cracking and rupture a considerable increase of permissible stresses, and a better exploitation of the properties of material, is justified (see table VI). Further it led to a critical examination of safety and finally to the incitement to supplement the usual mode of calculation and with it to improve designing. By these investigations based on test research an old gap in our knowledge of the nature of reinforced concrete has been filled.

Summary.

The question of employing high-quality steel in reinforced concrete was brought to a certain conclusion through the new Dresden tests carried out for the German Commission on Reinforced Concrete. The width of cracks were photographed with a 23-times enlargement, and the depth of cracks was measured too.

Safety against cracks, i. e. the ratio of loading at the moment of cracking to the working load has the value 1.8 for crosswise reinforced slabs supported on all sides, and for slabs supported on all 4 corners (Advance tests for mushroom slabs) the value 1.4 for slabs reinforced in one direction only 0.75 and for T-beams 0.5. The employment of high grade steel as reinforcement is therefore most suited for slabs. The safety against cracks increases also with increasing *quality of concrete*, but only to a very small degree on account of the increased brittleness of high quality cement. The smaller the percentage of reinforcement compared with the cross section of concrete and the width of the webs, the greater the safety against cracks and therefore the smaller the *depth of cracks*. Since on the other hand the width of cracks increases with the increased width

of web, a limit is given to the increase of web-width by stipulating a permissible width of cracks for working loads (e. g. $\frac{1}{8}$ mm).

There merely statistic facts lead to the *physical conception* that with the appearance of a crack the cracked portion of the cross section (depth of crack t x width of web b_0) is put out of action and with it also the tensile force belonging to this area. The magnitude of this force can be 4.8 or 12 % of the tensile force in steel at the moment of cracking for low, medium or high grade concrete respectively. Increased degrees of safety against cracks can be expected for I and box-type cross section.

T-beams reinforced with St 52 and subjected chiefly to stationary loads for a permissible stress of steel of 1800 kg/cm² offer the same safety against cracks as if designed for St 37 with a permissible stress of 1200 kg/cm².

Based on the determination of cracking-point loads, on the 14th Jan. 1935 a table showing the permissible stresses by using steel of high yield point, was arranged by the German Commission on reinforced concrete (Table VI), and further it gave rise to the following results.

For the first range of lightly reinforced beams, for which the yield point of steel is the decisive factor (normal case) nothing requires to be changed in the usual mode of calculation. As soon as the dividing limit which separates the two ranges are brought to conclusion by the Dresden test which are being carried out at present, then the first range can be extended up to this limit and with it the validity of the usual mode of calculation. For the second range for which the compressive strength of concrete is decisive (rare case) a new procedure can be introduced making extended use of the properties of materials with the purpose to eliminate in designing the application of haunches and Compression reinforcement. This would allow to improve the appearance of concrete constructions.

For moving loads the Dresden tests revealed that for slabs reinforced with St 52 a permissible stress of 1300 kg/cm² is applicable if the concrete has least a cube strength of 225 kg/cm². For T-beams the conditions remain the same as before, the permissible stress to be 1500 kg/cm².

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