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The Tensile Strength of Stressed Parts in Reinforced Concrete

Zugfestigkeit des Betons in Eisenbetonkonstruktionen.

Sur la résistance des pièces tendues dans les constructions en béton armé.

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All those who have carried out experiments agree that, given the same class of metal section, tensile strength increases in reinforced structures and cracks are reduced in proportion as the number of bars used is increased, and therefore the diameter of these latter is reduced.

Meanwhile, even though this fact appears to have been definitely laid down and confirmed by experiment, the interpretation given to it by various authors appears less definite.

It is no use referring to circumstances which are evident, such for instance, as the fact that the diameter of the steel bars decreases the ratio between the circumference of their cross section and their area, however, if the conditions of adhesion between steel and concrete are improved — especially when the experimental determination made extends to cases in which girders are subjected to ordinary bending stresses in which in theory the adhesion would not even have any reason for coming into play.

As a matter of fact, that reference acquires some value, even a very definite and clear value as we shall soon see, only if, when trying to analyse what exactly takes place in a structure which has been concreted to increase its tensile strength, we set aside all those very elementary and often contradictory conceptions to which we have to resort when making static calculations.

Every one knows that in those calculations we set aside all participation of the concrete in resisting tensile stress, admitting that it is borne solely and entirely by the steel, if those calculations aim only at confirming the strength of the material: on the other hand, however, we rely entirely on its participation and admit that the internal stresses are distributed between the steel and concrete according to the respective moduli of elasticity every time we set about calculating deformation, no matter whether this is of direct interest to us or intended to be used when determining certain unknown hyperstatical quantities.

The truth is — and we are well aware of it — that in practice, neither the one nor the other case will arise: or to be more precise, the two hypotheses occur only exceptionally and then for certain metal sections; they pass then gradually from one to the other limit-case across a whole scale of intermediate static conditions in which the concrete is subjected to a part, but a part only, of the stress which really belongs to it.

It would be idle to try and introduce this partial participation of the concrete in the strength of the structure into those calculations, because such participation changes considerably according to circumstances, and in each particular case according to each part of the structure, because the degree of homogeneity of the concrete and its degree of adhesion to the steel vary, and above all because the number and position of certain very small, almost imperceptible cracks may be due to so many different causes.

It has been said that no reinforced concrete structure exists in which, after careful investigation, some of these imperceptible cracks will not be found, and as a rule they are the result of internal stresses set up in the structure when the concrete contracts, or resulting from variations of temperature.

In any case, where such cracks do occur, the resulting tensile stresses will all have to be borne by the steel. But in the adjacent zones, where the steel is surrounded by a sound, compact and thoroughly adhesive mass of concrete, the latter, forced to follow the course of distortion, will take an active part in the resistance and relieve the steel of a certain part of the strain which, in relation to the cracks, it will bear.

Now it is just in these alternating changes when the stresses pass to and fro between steel and concrete — changes that are unexpected and cannot be foreseen by static calculations — that tangential stresses are set up in the mass of the concrete, and these tangential stresses have nothing to do with those stresses which are connected with any shearing tension that may be present.

It is just these stresses which, if they exceed the limits of resistance of the material, may lead to the spread of the existing cracks or form new ones.

Meanwhile the problem to be solved is that of inducing a more rapid and effective participation of the sound mass of concrete in resisting stress in the structure — while limiting as far as possible the zones of low resistance — and preventing the tangential stresses from exceeding those limits or from reducing stability of the whole system.

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But there is another point in connection with the theoretical presentation of facts which deserves to be closely and critically examined.

It is a well-known fact that one of the fundamental postulates on which the usual static theory of reinforced concrete structures rests is that the distribution of internal stresses does not depend on the particular methods of application of the external stress.

In view of the stresses belonging to a given section of the structure, it will be admitted that *De Saint Venant* was right when he said that the law according to which the internal strain is in the section itself is the only law and a very definite law, no matter how the forces which determine that strain are applied.

Now in reality the position is such that this method of application exercises an influence which must be taken into account, and which in this special case of reinforced concrete girders may even become very marked on account of the varying conditions under which, with reference to the loads actually applied, the concrete mass and the respective metal framework are placed.

As a matter of fact, cases in which external stresses, when being set up, are distributed between steel and concrete in such proportions as to result in distortion along the respective surfaces in contact with each other are so rare as to justify the hypothesis of the maintenance of plane sections.

It is much more likely that cases will occur in which the stress on a certain girder will affect the metal framework across the braces which are suitably arranged to connect the various steel parts. In that case the metal framework, subjected to distortion by the action of the stress, will also force the mass of concrete of which it forms part to become distorted and induce it to participate more or less actively in resisting the stress. But it is obvious that this transmission of stress from steel to concrete cannot take place except as a result of adhesion and the setting up of a system of tangential strain which is not justified by the stresses alone but by the particular method in which they are applied.

The opposite is likely to occur more frequently: the external forces which set up stress are generally applied in the form of superficial pressure bearing on the mass of concrete. Then it is the concrete which, subjected to deformation, causes the enclosed steel to become distorted and subjected to a fraction of internal stress, relieving the overstressed concrete to a greater degree than theory would seem to warrant. Once again the transmission of stress from concrete to steel cannot take place without a certain system of tangential strain which the stresses alone cannot account for being set up, und its cause must be sought exclusively in the fact that the state of equilibrium is not such as is laid down in theory.

This maintains its whole value of limit-theory which is to be confirmed in those sections of the girder that are sufficiently distant from the points of application of the external stresses. This therefore means that, under the ordinary conditions of loading girders in an ordinary reinforced concrete structure, the theory would never be rigorously confirmed.

It is not permissible, therefore, to disregard in practice the fact that under the conditions of load specified above, the internal tension in the concrete in the neighbourhood of the points of application of the load may assume, and actually do assume, higher values than those theoretically laid down, and these are indeed so much higher and extend more widely over the girders in proportion as the passage of stresses between concrete and steel proceeds more slowly.

And thus it is that — in other circumstances and in quite a new form — the same problem arises once more: namely, the problem of whether, and in what manner, this alternating passage of stresses can be accelerated and made more effective — without the resultant tangential tensions exceeding the limits of the resistance of the materials — and in this way the zones in which the abnormal distribution of the stresses occurs are reduced and the differences between such distribution and that laid down mathematically become less marked.

* * *

And thus, by going back to well-known and very elementary methods of calculation and adapting them to the present case, a problem of this kind might be brought nearer to solution.

Let us suppose, in order to crystallise thought, that a small steel disc with a diameter that we shall call $2\,r$, has one of its perpendicular cross sections stressed by a normal tension σ_f only, and in an adjacent section at a distance d_z from the preceding one, by a similar unique tension $\sigma_f + d\sigma_f$ (Fig. 1).

The equilibrium of the portion of steel between these two sections evidently requires that a tangential tension be exerted on its lateral cylindrical surface (this is possible owing to the adhesion of the concrete). This average sole tension τ must comply with the following formula, namely:

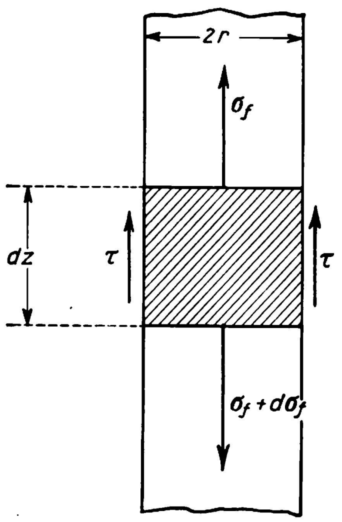
$$d\,\sigma_f \cdot \pi r^2 = \tau \cdot 2\,\pi r \cdot d_z$$

from which it is seen that:

$$\frac{d\,\sigma_f}{d_z} = 2\,\frac{\tau}{r} \tag{1}$$

And now let uns consider the cylindrical layer of concrete which envelops that steel.

Fig. 1.



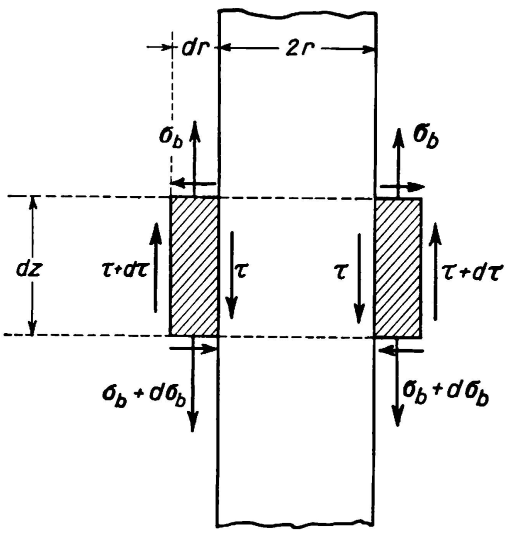


Fig. 2.

Assuming that dx is the minimum thickness of the layer, σ_b the normal sole tension to which it is subjected in relation to the first of the perpendicular cross sections considered, $\sigma_b + d\sigma_b$ the corresponding tension on the other perpendicular cross section, situated according to our hypothesis at a distance of d_z from the first one (Fig. 2).

The same considerations with regard to the equilibrium that we have just applied in the case of the steel when referring to this cylindrical concrete layer, lead us to a further formula which is the following:

$$d\sigma_b\,[\pi(r+dr)^2 - \pi r^2] = (\tau + d\tau)\cdot 2\,\pi\,(r + dr)\cdot dz - \tau \cdot 2\,\pi r \cdot dz.$$

Here we have naturally indicated by $\tau + d\tau$ the average sole coefficient of the tangential tension that the portions of concrete which surround the layer in question exercise on its external surface.

If we disregard the exceedingly reduced limits of a higher order than the second, this equation will take the following form:

$$d\sigma_b \cdot 2\,\pi r \cdot dr = \tau \cdot 2\,\pi\,dr \cdot dz + d\tau \cdot 2\,\pi r \cdot dz$$

or also:

$$\frac{d\sigma_b}{dz} = \frac{\tau}{r} + \frac{d\tau}{dr} \tag{2}$$

But if the cylindrical layer of concrete is to adhere absolutely to the concreted iron, it will be necessary to admit that on the surfaces which are in contact with each other, the deformations of the two materials are identical.

And thus, given E_f as the normal modulus of elasticity of the iron, and E_b that of the concrete, we should have:

$$\frac{\sigma_f}{E_f} = \frac{\sigma_b}{E_b} \quad \text{and} \quad \frac{d\sigma_f}{E_f} = \frac{d\sigma_b}{E_b}$$

In these circumstances, the ratio deduced from the coexistence of the two formulas of equilibrium shown above will be:

$$\frac{d\tau}{dr} = \frac{2E_b - E_f}{E_f} \cdot \frac{\tau}{r} \tag{3}$$

in which the coefficient:

$$\frac{2E_b - E_f}{E_f}$$

is always a negative one.

If, as happens in practice, it be admitted that

$$E_f = 10\,E_b,$$

that coefficient will have the value of:

$$-\frac{4}{5}$$

In any case it is possible to state in a general way that the tangential tensions in concrete decrease fairly rapidly as soon as the distance from the surface of the steel is increased. The rate of decrease will be accelerated in proportion as the ratio $\frac{\tau}{r}$ between the maximum intensity attained by these tangential stresses on the above mentioned surface and its radius is greater.

But in connection with the first of the equilibrium equations given above, it must be remembered that the rapidity with which the σ_f vary (and accordingly the σ_b also) related to z, also depends on the coefficient of the ratio $\frac{\tau}{r}$.

In this way we are led to conclude that two conditions must occur if the transmission of the stresses from the steel to the concrete (or vice-versa) are to take place either longitudinally or transversally in a very limited zone. These conditions are:

(1) a high coefficient of τ which means satisfactory adhesion between the two materials;

(2) a low coefficient of r which means distribution of the metal section in a number of iron bars of small diameter.

The first of these is self-evident, while the second is a direct reminder of those experimental results to which allusion was made in the early part of this paper, and which enabled us to specify the double advantage to be gained by the use of iron parts of small diameter, an advantage which could be realized, according to circumstances, in that, given equal maximum tangential tension, the transmission of the stresses from steel to concrete (or vice-versa) can be effected in a zone with a maximum of limitation either longitudinal or transversal, or by the fact that where other circumstances are equal, this transmission of stresses will set up less marked tangential tensions.

Summary.

The Author describes the incompleteness of assumptions on which the calculation of reinforced concrete sections is based. He examines the transmission of tangential stresses and shows how this can be improved. The calculation offers a proof to the well known fact that a large number of thin reinforcing rods is preferable to a small number of heavy rods.