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Increasing the Tensile Strength and Avoidance Formation of Cracks in Concrete.

Erhöhung der Zugfestigkeit und Verminderung der Rißbildung des Betons.

La résistance à la traction et la fissuration du béton.

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Introduction.

It has long been the aim of material-research workers to find means of avoiding cracks in concrete and of keeping unavoidable cracks within safe limits. This aim has become even more important with the development in the practical application of concrete and reinforced concrete and the efforts made to attain higher stress limits. In this connection emphasis must always be laid on the close relation between the formation of cracks and the tensile strength of the concrete, and on the necessity of increasing the latter. The object of the following report is to review this field of inquiry in the light of the knowledge and experience at present available.

I. Tensile strength of concrete.

a) Measuring the tensile strength of concrete.

The tensile strength of concrete can be measured either directly by means of tensile tests, or indirectly by means of bending tests. Tensile tests are but seldom employed, since they can only be carried out with the help of expensive testing apparatus and test pieces which are difficult to make, besides which they require much more care in execution than do bending tests. In addition to which comes the fact that bending tests mostly give a better reproduction of the stressing to which the concrete is actually subjected than do tensile tests.

In both forms of test the result is dependent upon the cross section of the pieces, i. e. on the size of the latter, since as a rule the larger the piece the smaller the strengths obtained (1 p. 84). The reason for this is primarily to be sought in the self-stresses which are set up as, for instance, the test piece dries out (1 p. 87) (cf. Ic 8). In bending tests, moreover, the arrangement of the load has to be observed. Two single loads situated at a certain distance apart give on an average a smaller rupture-point bending stress than one single load, because in the case of two single loads the greatest amount of stressing is distributed over the whole section between them and is thus more likely to hit on the weakest places in the concrete (1 p. 93). The rupture-point stresses calculated from tensile and bending tests carried out on the same concrete and

with the customary assumptions (uniformly distributed stress for tensile tests, stress of linear proportion for bending tests) do not coincide; the rupture-point bending stress is even greater than the tensile strength. The principal reason for this is that in the tensile test the greatest stress between the points of attack of the loads is set up simultaneously at all points of a cross section, whereas in the bending test it appears at first only in the extreme fibres (1 p. 93). Moreover, there exists no proportionality between stresses and strains in the tensile zone of the concrete even under slight loading, so that the distribution of stresses does not correspond to the assumptions made in the calculation of the stresses (4 p. 39) (39 p. 73).

b) Relation between tensile, bending and compressive strengths.

Up to the present no law has been discovered to govern the relation between the tensile, bending and compressive strengths of concrete and to allow reliable conclusions to be drawn between one strength and the other. The reports available as to relative values differ largely from each other.

Graf (1 p. 92) found that between the compressive strength, calculated on 30 cm cubes, and the tensile strength of bodies with a cross section of 400 cm², $K_d : K_z =$ from 8 to 17. *Guttmann* (3) found $K_d : K_z =$ 14 to 28 for cubes and tensile pieces of 100 cm² cross section. In both cases the difference between the relative values is therefore the same amount, while its absolute amount is obviously influenced by the dimensions of the test pieces.

Graf states that for the relation of the compressive to the bending strength (2 p. 83) $K_d : K_b =$ from 4 to 12; he bases his figures on a large number of tests.

The variations in the relation between bending and tensile strength are correspondingly large. *Graf* (2 p. 91) found that with 400 cm² cross section under tension $K_b : K_z =$ 1.6 to 2.9, the maximum value being 3.5 for centrifugally cast concrete, while *Guttmann* (3) gives $K_b : K_z =$ 2.3 to 4.2 for 100 cm² cross section and bend stressing caused by one single load. *Dutron* (5) observed that $K_b : K_z =$ 1.3 to 2.0 with the same cross section under tension but with bend stressing from two single loads. The influence of the loading arrangement becomes clear from the last two sets of figures, as referred to under Ia.

When considering the variations, however, it must be remembered that mixtures were compared which differ in several factors at the same time. If the number of variables were to be limited, there would probably be more chance of finding some regulated relationship. Hummel's observations on the relations between bending and compressive strengths give him the equation $K_b = K_d^x$ (6 p. 15). From other tests it can be concluded with the same degree of probability that there is a corresponding relation $K_z = K_d^y$ between tensile and compressive strengths. These two equations inform us that tensile and bending strengths increase as compressive strength increases, though not in the same proportion as the latter, but the more slowly the greater the compressive strengths become. As was to be expected from the above-mentioned variations of the ratios $\frac{K_d}{K_b}$ and $\frac{K_d}{K_z}$, x and y are not constant values holding

good for all cases. It is even probable that x varies between 0.55 and 0.70 and y between 0.45 and 0.60 (see Table I). Nevertheless, the exponents x

Table 1.

Influence of the composition of concrete on its tensile and bending strength,

Group	Tests by	Aggregates	Fuller sand curve as per Fig.1	Grading of sand 0-0,2 %	Sand to total aggregates in %	Cement per m ³ of concrete kg/m ³	Water-cement ratio W	Bending (tensile) strength $K_b = K_d$ kg/cm ²	Compressive strength K_d kg/cm ²	$x(y)$ for $K_b = K_d^x$ $K_d = K_b^y$	Consistency
1	2	3	4	5	6	7	8	9	10	11	12
1	Bach and Graf (13) p.42	Natural sand+gravel	below B	2	58	~240	0,82	(12)	138	(0,513)	very soft
			"	2	55	~320	0,61	(17)	201	(0,534)	
			"	2	57	~430	0,50	(23)	264	(0,560)	
2	Graf (11) p.40	Natural sand+gravel	A	—	43	264	0,63	41	278	0,659	soft
			B	22	60	257	0,77	36	183	0,687	Spread
			below C	40	71	254	0,98	18	133	0,620	
			C	44	80	250	1,17	13	81	0,586	
			A	—	43	308	0,52	50	301	0,686	~51cm
			B	17	59	297	0,65	44	242	0,689	
			below C	37	71	294	0,81	29	170	0,656	
			C	44	79	296	0,96	16	119	0,581	
			A	—	40	353	0,51	48	362	0,657	
			B	12	58	343	0,58	46	256	0,690	
			below C	34	70	351	0,67	37	231	0,654	
			C	44	79	367	0,68	24	166	0,621	
3	Hertel (15)	Natural sand+gravel									soft
			below B	9	38	306	0,70	29	240	0,616	
			" B	7	46	302	0,70	37	250	0,653	
			above C	15	95	300	1,00	14	90	0,584	
		Natural sand+chips (Squat basalt chips)	C	11	37	310	0,72	31	195	0,686	
			B	13	41	307	0,72	30	238	0,619	
			above A	8	50	303	0,72	36	244	0,651	
4	Gutman (3)	Natural sand+gravel	below B	8	43	278	0,60	(18)	354	(0,484)	soft
			"	8	54	275	0,60	(21)	353	(0,520)	
		Crushed sand+chips (basalt)	below B	8	43	301	0,68	(16)	303	(0,486)	
			"	8	54	300	0,68	(21)	305	(0,533)	
5	Bach and Graf (13) 28	Natural sand+gravel	below B	2	55	~320	0,61	(17)	201	(0,534)	very soft
		Natural sand+chips (basalt)	B	6	56	~350	0,77	(21)	197	(0,573)	
		Crushed sand+gravel (basalt)	below B	13	55	~320	0,90	(17)	157	(0,558)	
		Crushed sand+chips (basalt)	B	17	56	~350	1,05	(16)	124	(0,572)	
6	Graf (17) p.5+6	Natural sand									
		Natural sand+gravel	above C	5	40	301	0,69	40	265	0,661	less soft
		Natural sand+chips	"	6	40	300	0,71	38	193	0,691	Spread
		Squat basalt chips	"	5	40	299	0,71	41	227	0,685	~40 cm
		Natural sand+chips Basalt chips flat	"	5	40	297	0,76	32	207	0,650	liquid
		Natural sand+gravel	"	5	40	308	0,93	24	109	0,678	Spread
		Natural sand+chips	"	5	40	295	0,87	25	155	0,638	~67 cm
		Squat basalt chips	"	5	40	295	0,87	25	155	0,638	
		Basalt chips flat	"	5	40	295	0,87	25	155	0,638	
7	Dutrun (5)	Natural sand+gravel				346	0,50	(20)	384	(0,507)	soft
		Natural sand+chips (Porphyrous)				350	0,55	(21)	335	(0,526)	
		Natural sand+chips Furnace slag				355	0,60	(25)	350	(0,553)	
		Crushed sand+chips (Porphyrous)				365	0,67	(24)	273	(0,569)	
		Crushed sand+chips Furnace slag				360	0,74	(22)	247	(0,561)	
8	Walz (16)	Natural sand+gravel	A	2	42	254	0,54	57,5	330	0,698	(1) moist
			"	2	42	250	0,64	51,0	280	0,698	(2)
			A	2	42	345	0,46	63,5	445	0,681	(1) moist
			"	2	42	343	0,50	59,5	395	0,684	(2) moist

and y enable a more reliable judgment to be formed as to the action of certain measures on the relation of bending or tensile strength to compressive strength than do the simple ratio values between these strengths, because the exponents obviously cope very satisfactorily with the variation in the ratio values, which is dependent upon the compressive strength figure.

c) Influences on the tensile strength of concrete.

If it becomes a question of raising the tensile strength of concrete all factors and possibilities must be investigated that might have an influence on the properties of the material, such as: — the cement, nature of stone, shape and mesh of aggregates, cement-aggregate-water mixture, manner of working up, external conditions during setting and afterwards — e. g. temperature and humidity — and age and loading.

1) Cement.

As the strength of concrete is produced by the binding properties of the cement, these properties are responsible in the first instance for the tensile strength of concrete. This consideration would seem to be contradictory to the fact that the classification of different grades of cement according to the standard tests for tensile strength in general use until a short time ago, is quite a different thing from that yielded by tensile tests on concrete made of these cements: In other words, the fact that cement showing greater tensile strength in the standard tests does not always produce the stronger concrete, as for instance in (3). Conclusions have been drawn from these, and from similar observations made during compression tests to the effect that the traditional standard system of testing with uniformly granulated sand and low water-cement ratio does not represent an adequate standard of evaluation for the binding properties of cements made up into concrete; new methods of testing, with sand of mixed granule sizes and a higher water-cement ratio, were therefore developed. (7) to (10) Experiments made by the Forschungsgesellschaft für das deutsche Straßenwesen (German Institute for Road Research) have revealed fair coincidence between the relations of the bending strengths of various cements as given by the new testing method, and the bending strengths of concrete made with these cements. This supplies proof that the bending strength of concrete can be increased by the employment of a cement that the new testing methods have proved to be superior. Furthermore, it has now become possible to investigate the reasons why certain cements are superior to others. Judging by the knowledge which we possess today as regards the action of cements, however, it is hardly to be expected that the quality of cement in respect of the tensile strength of concrete can be increased to any great extent above present-day standards.

The relation between the bending and the compressive strength of concrete also varies, according to the new testing method, between widely-differing limits and as a rule works out the more unfavourably as regards bending strength, the greater the compressive strength (10). The relation between these two strengths in concrete is thus influenced by the specific properties of the cement also.

To what extent the consistency of the paste is important in determining the

amount of water necessary for the attaining of a certain degree of workability of the concrete, and how far the tendency to shrink and the time taken in setting affect its tensile strength, will be discussed later on in another connection (cf. S. 3 and 8).

2) Quantity of water in concrete.

For the same aggregates and consistency of the concrete, its tensile or bending strength increases with the amount of cement it contains (11 p. 48) (12). This increase can be followed in Table 1, group 1, and also in group 2, by comparing the values belonging to the same Fuller curve. It can be concluded from the alteration of the exponents y in group 1 that the relation $\frac{K_z}{K_d}$ becomes more favourable as the cement content increases. It probably approaches the specific value for the respective brand of cement. In group 2, on the other hand, x is partly constant. The reason for this would seem to lie in the higher content, graduated according to the amount of cement, in the granulation 0 to 0.2 mm, whereby the cement is not merely used as a filler, even in the case of lean mixtures. But as the quantity of cement increases, so do the internal stresses produced by the drying out process (cf. S. 8), since the sections dry more slowly (25 p. 34) so that the strengths may drop for a time (12) in spite of a higher cement content.

3) Quantity of water in concrete.

The quantity of water in fresh concrete influences the tensile and bending strength in just the same manner as it does the compressive strength. As the water-cement ratio

$$w = \frac{\text{weight of water}}{\text{weight of cement}}, \text{ increases,}$$

the tensile and bending strength drops, being dependent upon w ; *Graf* puts this relation at approximately $\frac{1}{w^2}$ (2 p. 86). Owing to the relation mentioned under 2b) between the strengths, the loss of strength as w increases is of course comparatively smaller for tensile and bending strength than for compressive strength. To reduce the quantity of water necessary for the attaining of a certain workability of concrete, the use of liquid aggregates (14) may be found advantageous — in addition, of course, to the choice of a suitable brand of cement (cf. S. 1) and proper granulation (cf. S. 4).

4) Granulation of the aggregates.

As the granulation of the aggregates affects primarily the quantity of water required in concrete, and as its influence on the tensile or bending strength, on the one hand, and on the compressive strength, on the other, is of the same nature, it is to be expected that the rules for granulation laid down with regard to compressive strength are also conducive to the production of high tensile or bending strengths. Fig. 1 illustrates the extreme Fuller curve values at present recognised in Germany for reinforced concrete mixtures. Comparing group 2, Table 1, with these, it will be found that the Fuller curves lying within the zone marked 'very good' do actually give the best bending strengths. Considering

groups 2—4 and keeping an eye on the exponents x and y separately while doing so, it becomes clear that the most suitable percentage of sand in relation to the whole of the aggregates for concrete worked soft is between 50 and 60, also when the material is naturally graded. In group 3 it is conspicuous that sand whose granulation is unfavourable according to Fig. 1, influenced the bending strength less unfavourably than it did the compressive strength as long as it was not present in excessive quantities. It is possible that this result was affected by the fact that the finely granulated mortar dried more slowly. *Pfletschinger* (16) has found that for bending strength it is important to have the coarse aggregates well graded (> 7 mm), while this does not matter so much for compressive strength.

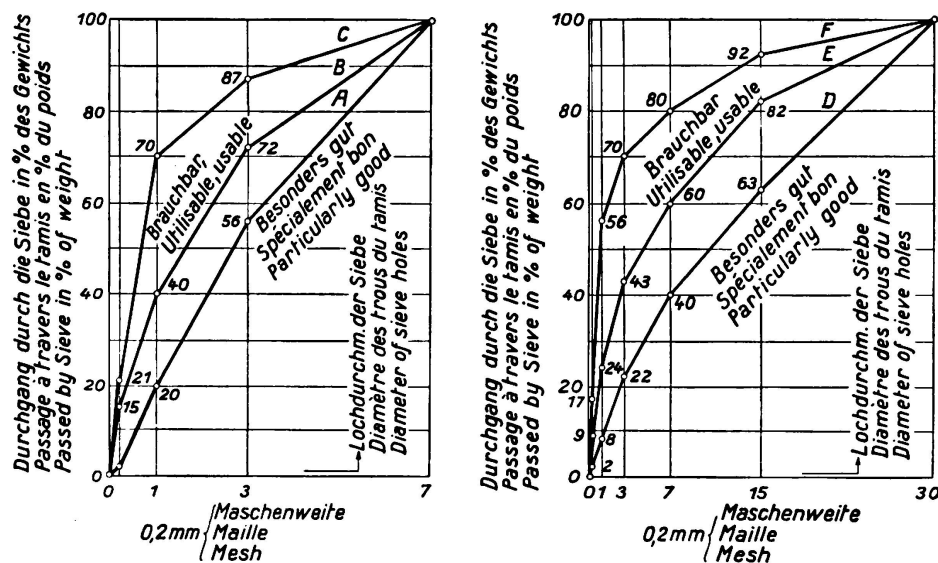


Fig. 1.

Ultimate Fuller Curves acc. to Germain Commission for Reinforced Concrete.

Fuller Curves for Sand only, Fuller Curves for all Aggregates.

5) Shape of granules and surface character of aggregates.

The shape of the granules and the surface character of the aggregates determine the quantity of water required for fresh concrete — given a certain cement content and size of granules — for the attainment of a certain workability. In order to keep down the water content, it is better to have granules of as round and cubical a shape as possible (Length: breadth: thickness between 1:1:1 and 1:0.6:0.2, as laid down in the general specifications for surfacing on the German arterial roads), and not an over-rough surface. This applies particularly for soft and liquid concrete. Moreover, the adhesion of the cement to the stones and the cohesion between mortar and coarse aggregates is dependent upon the surface character of these aggregates. This factor would seem to matter more for tensile and bending strength than for compressive strength, for when the concrete is subjected to tensile or bend stressing the granules of the aggregate cannot mutually support each other. Aggregates with rough and irregular granule surfaces can therefore have an advantageous effect on the tensile and bending strength provided that the unfavourable effect of increased

water content does not outweigh its good influence. Accordingly, it can be seen from groups 5—7, Table I, that the use of broken instead of naturally graded aggregates improves the relation of the tensile or bending strength to the compressive strength (increased values of x and y); it will also be observed, however, that by this procedure the absolute values of tensile and bending strengths are by no means always favourably affected. In group 6, for example, in the case of liquid concrete, there is even a perceptible lowering of the strengths owing to the broken aggregates. Groups 4, 5 and 7 show that as regards tensile and bending strength crushed sand and natural sand are of the same quality, but that crushed sand has quite a bad effect on the compressive strength. There is thus no cause to prefer crushed sand to natural sand in respect of tensile or bending strength.

6) Type of stone in aggregates.

The tensile strength of the stone used in concrete aggregates is generally greater than the strength of concrete usually attained till now. If it is remembered, however, that the tensile strength at present attainable in comparatively old concrete can be estimated at app. 55 kg/cm² on the basis of the bending strengths observed (2 p. 90), and that there are types of stone, suitable for use as concrete aggregates, but with a smaller tensile strength than this, then it becomes clear that the tensile strength of the stone must be considered if especially high qualities are to be attained (12 footnote 12). The bending strength of the stone is hardly of much importance for the strength of the concrete, for in the tensile and bending tests only particularly longshaped pieces can be destroyed by bending.

The surface character of the aggregates, the importance of which has already been discussed, depends upon the type of stone and — in the case of broken aggregates — the manner in which it was crushed. Nothing has yet been said of the extent to which the stone's capacity for absorbing water has to be considered, whether with the object of improving the water-cement ratio (provided that the aggregates were not wetted beforehand) (16), of reducing the rapidity of drying out, or perhaps even of increasing adhesion of cement to stone.

Aggregates composed of types of stone or artificial material which — as for instance blast-furnace slag or cement clinkers — react chemically in conjunction with the cement and thus produce stronger binding, can be employed to advantage.

A few instances of the influence of the surface character of various types of stone on the tensile and bending strength of concrete are given by *Dutron* (group 7, Table I) and *Guttman* (3).

Finally, the type of stone has also to be considered for its effect on self-stresses (cf. S. 8).

7) Casting concrete.

The more compact concrete is made in casting, the greater is its tensile and bending strength likely to be. Thus *Graf* has repeatedly recorded bending strengths of up to 80 kg/cm² in machine-rammed concrete products — in individual cases as much as 120 kg/cm² (2 p. 90). Greater differences in the performance of rammed concrete, however, are only possible with moist

mixtures. Vibration has a particularly favourable effect in the case of the latter, not only because it affords greater compactness, but also because of the smaller water-cement ratio required. Group 8, Table I, shows the advantages of vibrating over ramming, and it should be noted that the amount of ramming work done was exceptionally great. The higher strengths given for the same cement content are those for vibrated, the lower those for rammed concrete. The relation of the bending to the compressive strength remained the same for both methods of compacting. Mortar which was sprayed on attained high tensile strength.

8) Moisture and temperature.

The tensile and bending strength of concrete is largely dependent on the effects of moisture and temperature. For as soon as the moisture or temperature is unevenly distributed over the section of the concrete body, stresses are set up in the concrete itself although there may be no forces acting from outside. These self-stresses form a pre-stressing of the concrete and cause the strength calculated from rupture loads to work out smaller than the real strength.

Differences in the degree of moisture in concrete, causing self-stressing, are set up when, for instance, moist concrete dries out or dry concrete is moistened, since the change in the degree of moisture and the consequent shrinking or swelling of the concrete takes place gradually, proceeding from the surface towards the interior of the body. Now if, for example, the surface region is drier than the core of the section, the former is restrained from shrinking to the extent corresponding to its degree of moisture, so that tensile stresses are set up in the surface region and balanced out by compressive stresses in the core (13 p. 106). When the concrete is moistened the condition of stressing is reversed.

Fig. 2, based on experiments carried out by *Graf* (19 Fig. 4), gives some insight into the actions set up by differences in degree of moisture during the drying process. Here bodies of various sizes of cross section but under the same conditions of storing were observed. On the assumption that in the smaller bodies, practically, there are no moisture differences within the section, the bold lines show to what extent the concrete on the surface of the larger bodies would have contracted if it had not been expanded by self-stresses and prevented by moisture from the interior of the section from drying out as quickly as the concrete of the smaller bodies. It will also be noted from the smaller axial contraction of the large bodies (dotted line), how much more slowly these latter dry out than the smaller bodies; in this connection it should also be considered that compressive self-stresses increase the axial contraction above the amount caused by shrinkage alone. And, finally, a comparison between the surface and the axial contraction of the large bodies (dotted and bold line) shows to what a great extent the differences in contraction to be expected from the differences in degree of moisture are cancelled out by self-stresses. The latter become correspondingly greater as the differences in degree of moisture in the section increase and the more the cement and aggregates tend to shrink or swell. The differences in degree of moisture are dependent upon the relation between surface and section of the body, upon the character of the concrete

pores which determine the rapidity with which the drying out process proceeds towards the centre (20 Pt. I) (21) and upon the rapidity with which the surface region dries out; this speed is higher, the greater the difference between the moisture content of the concrete and that of its surroundings (22 p. 136). To retard drying, the concrete is best coated with an isolating substance (3) (23)

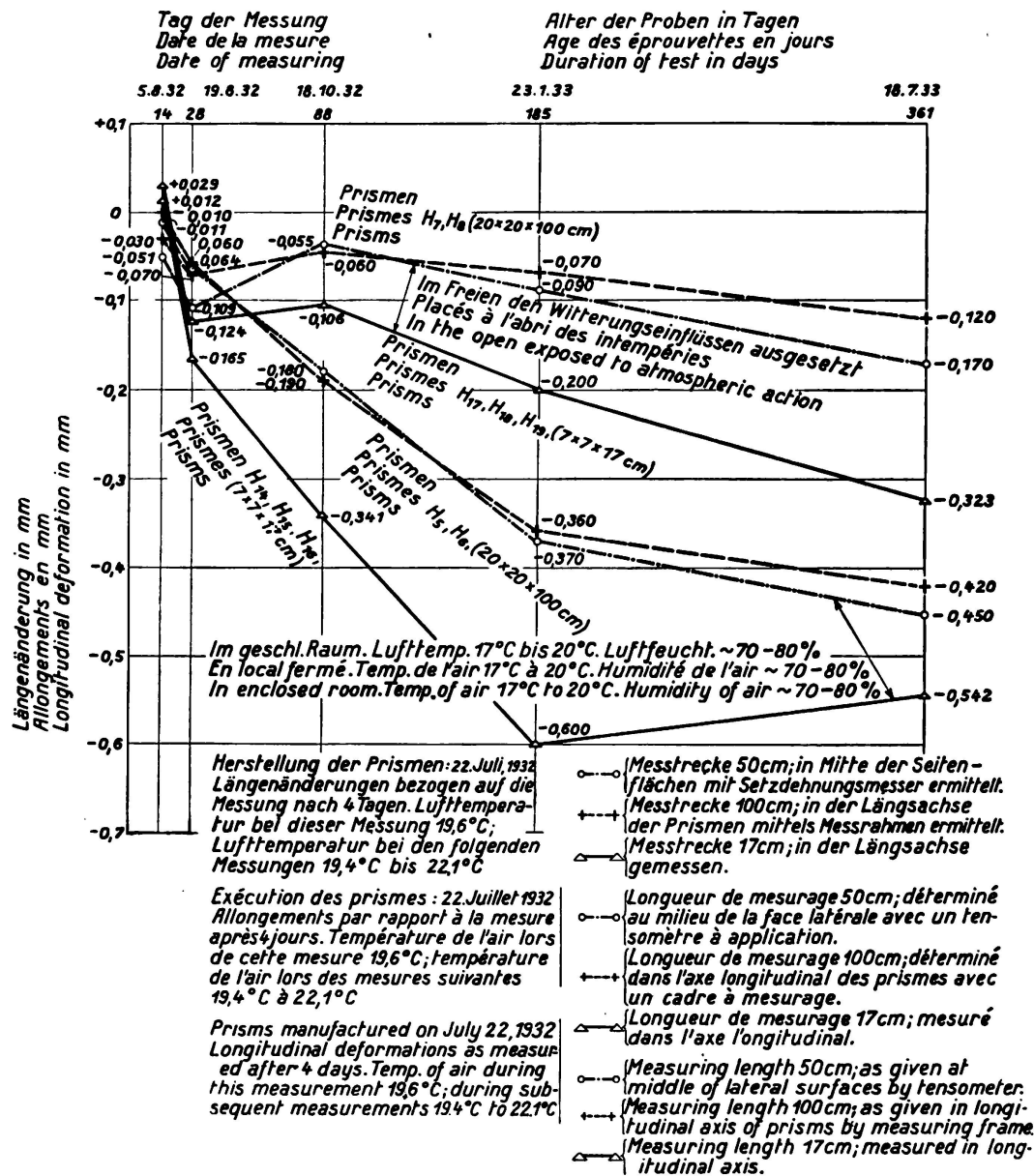


Fig. 2.

Shrinking and Swelling of various Sizes of Bodies.

(39 p. 139). The self-stresses, on the other hand, decline as the degree of elasticity decreases and as the creeping of the concrete increases (24). Furthermore, the rapidity of hardening has to be taken into account. The quicker the concrete hardens, the sooner does the magnitude of the self-stresses fall behind that of the real strength, whereas the self-stresses increase in quicker-drying concrete because the degree of elasticity grows more rapidly and creeping

stops sooner than in slow-drying concrete. These numerous influences, some of which have mutually contrary action, result in differences in storing conditions having variously marked effects on the tensile and bending strengths of concretes of various compositions (1 p. 90 and 94) (12 Table 12).

Although a decrease of tensile strength is always to be expected when the degree of moisture alters, *Graf* found that, on moistening concrete which had been dried for a more or less long period, the bending strength increased (26 Table 9). The reason for this can be found in the fact that the self-stresses act throughout the whole section during the tensile test, so that tensile stresses in the core decrease the tensile strength when the surface region is moistened, whereas in the bending test only the self-stresses in the surface region are decisive. The latter, however, are at first compressive stresses when moistening takes place, and therefore increase bending strength. It is worth noting that in these tests the tensile stresses in the core acting vertically to the direction of compression decrease the compressive strength.

In addition to the self-stresses described, long storage under water can also decrease the tensile and bending strength of concrete (1 p. 90) (26 Table 3).

Warming and cooling produce self-stresses of the same kind as do moistening and drying; the magnitude of these stresses naturally depends upon the differences of temperature in the section (23).

On summing up these considerations it will be found that the most unfavourable effect on the tensile and bending strength is produced by abrupt changes of storing conditions. On the other hand, a beneficial action can be created if the surface of the concrete is kept moist for a considerable period and then allowed to dry as slowly as possible (4 p. 49).

9) Age.

The tensile and bending strength of concrete grow with its age in accordance with the rapidity with which the cement hardens. This fact, however, is partially or wholly cancelled out by the effects of storage as elucidated in S. 8, so that, according to circumstances, over a considerable period the tensile and bending strength can be observed to remain constant (27 p. 51) or even to decrease (12 Table 12) (26 Table 3), even though the compressive strength keeps on increasing. In this connection it should be noted that in general concrete dries very slowly (1 p. 89), much more slowly than it absorbs water (28 Fig. 24) (22 p. 140).

If, after the storage conditions have been altered, the tensile and bending strength again increase with progressive equalisation of the degree of moisture, owing to the corresponding decrease of self-stresses, this increase is often greater within a certain period than is the simultaneous increase in compressive strength. At the end of this period the exponents x and y are therefore often greater than at the beginning (cf. Table 2) and approach the values set up by a hardening process involving no self-stresses (cf. the two last lines of the Table). This Table shows further that concrete containing more water hardens at a different rate than does concrete containing little water, so that the self-stresses increase and decrease at different rates in accordance with the water-cement ratio.

Table 2.

Influence of curing and age on the tensile strength of concrete¹.

Storage	Age	$W_1 = 0,53$			$W_2 = 0,61$		
		K_{z1} kg/cm ²	K_{d1} kg/cm ²	γ_1	K_{z2} kg/cm ²	K_{d2} kg/cm ²	γ_2
1	2	3	4	5	6	7	8
7 days moist then dry	28 days	12,4	225	0,466	12,0	191	0,474
	45 days	13,7	253	0,472	11,8	209	0,463
	6 month	19,5	337	0,511	15,3	297	0,480
	1 year	23,7	371	0,536	23,1	329	0,543
Continually moist	45 days	19,0	224	0,545	17,0	201	0,534

10) Alternate loading.

Tensile and bending strength is decreased by frequently repeated loading and unloading. In terms of surge-load strength, the permanent bending strength works out at about half the bending strength as determined in the usual manner (29 p. 117). In this connection it should be noted that frequent changes of temperature also act as alternate loading.

d) Means of increasing the tensile strength of concrete.

Summing up the preceding considerations, the following means and measures can be recommended for increasing the tensile strength of concrete:

1) First of all, a suitable brand of cement should be selected. The cement should produce maximum bending strength as determined by the testing process with soft mortar of various granule sizes; it should also shrink as little as possible and enable the concrete to be worked easily even when the water content is small. Slow-hardening cements should be given preference provided they attain sufficient strength and allow of good curing of the concrete.

2) Aggregates should be used whose tensile strength is greater than that of the desired tensile strength of the concrete. The best materials in this connection are those which show little shrinkage, vigorous creep and a low coefficient of elasticity. It is advantageous to use rough-surfaced granules provided that the amount of water required is not too greatly increased. Attention must be paid to this point when using crushed stone sand and stone chips. When applying the stone, care should be taken that no fissured pieces are let through.

3) For grading the aggregates the same general principles hold good as those elaborated for the production of concrete with a maximum of compressive strength. It would, however, seem advisable to use at least 50% sand (granulation < 7 mm, calculated on the total weight of the aggregates, even for naturally graded materials. For coarse aggregates care should also be taken that the granules are well graded.

5) The quantity of water in concrete, i. e. water-cement ratio, should be kept as low as possible. It may thus be practicable to use liquid substances for wetting.

6) Concrete should be compacted as much as possible. Provided the concrete is thick enough, vibration will therefore be of advantage.

7) Concrete should be kept moist as long as possible at the beginning and dried as slowly as is practicable. Repeated and particularly abrupt changes of the moisture and temperature conditions to which the concrete is exposed should be avoided, especially while the concrete is still fresh.

II. Ductility of concrete.

a) Ductility of concrete under loads of brief duration.

Tests made to ascertain the ductility of concrete under tensile and bend stressing yielded the following results:— Large test pieces undergo for the same stressing somewhat greater elongations as smaller bodies (30) (31).

Greater rupture elongations are recorded for bending tests than for tensile tests (25 p. 39); the reason will be found under 1 a. The modulus of elongation $\alpha = \frac{1}{E}$, calculated on the whole or elastic deformations, coincides for small

stressing in tension and in compression. For greater but equal stresses α becomes a little larger for tensile than for compressive loading (30 p. 50). The modulus of elongation α increases with the loading of the concrete.

In the case of concrete composed of the same materials (brand of cement, type of stone in the aggregate), the modulus α decreases for the same stressing, the greater the strength of the concrete (30 p. 50) (24) (31). By changing the brand of cement or the type of stone, thus varying the deformability of the aggregates, concrete of various degrees of ductility can be obtained for the same strength. In Table 3 three ratio values taken from *Hummel's* investigations (24) are quoted, which show to what extent the ductility of concrete can be influenced in this manner. It is noteworthy that the greater ductility of concrete becomes the more pronounced, the more the stressing approaches the rupture stress point.

Table 3.

Ratio Values of Coefficients of Elongation $\alpha = \frac{1}{E}$ after Hummel.²⁴

Concrete distinguished by:	K_d kg/cm ²	K_b kg/cm ²	15	25	$\sigma_{bz} = 35$ kg/cm ²	40	45	K_b
Type of stone in aggregates	555	48	1	1	1	1	1	1
	510	49	1.06	1.02	1.04	1.07	1.22	1.35
	479	48	1.35	1.32	1.33	1.34	1.42	1.53
Cement	532	48	1	1	1	1	1	1
	544	48	1.0	0.98	1.0	1.02	1.05	1.25
	500	47	1.0	1.08	1.24	1.55	—	1.95

The ductility of concrete is evidently influenced by self-stresses, according as these are set up by the storage conditions. However, the experiments carried out (24) (30) (32) cannot be adequately compared to yield general conclusions.

To what extent the coefficient of elongation α is affected for tensile and bend stressing of the concrete by frequently repeated loading and unloading below the permanent strength limit, does not become apparent.

b) Ductility of concrete under lasting stationary tensile loading (creeping properties).

The creeping of concrete under tensile stress has hitherto been but little investigated. Only one test is known — reported by *Glanville* (33) — in which the degree of creepage for concrete, tested at an age of one month, was the same for tensile stressing as for compressive loading. In the test reported it amounted 0.1 mm/m after six months under a stress of 10 kg/cm². As creepage increases in direct ratio to stress, it will become greater under loading in the vicinity of the ultimate tensile strength than does the rupture-point elongation in the loading test of short duration, i. e. app. 0.0045 mm/m per kg/cm² tensile strength (4 p. 51). If it is permitted to generalise as regards the observations made on the creep ratio for tensile and compressive stressing, then investigations carried out into the creeping of concrete under compressive stress [according to a report on tests made by *Davis, Glanville, etc.* (34)] show that the ductility of concrete under loading of long duration may be very much greater than the ductility recorded in short-period tests, and that it can also be influenced to a much greater extent by the composition and treatment of the concrete.

c) Importance of the ductility of concrete in the formation of cracks.

Distinction must be made between rupture-point elongation and the coefficient of elongation α .

The magnitude of the rupture-point elongation is of no importance in all cases where the carrying capacity of the structure is exhausted as soon as cracks appear. Here the only thing that matters is that the tensile strength of the concrete is sufficient to take up the stresses with safety. In all other cases the danger of cracking becomes less, the greater the rupture-point elongation of the concrete. In this connection it is not always insignificant whether the greater rupture-point elongation corresponds to a greater or a smaller tensile strength. Take, for example, a concrete roadway slab which is expanded by friction forces while drying. The following facts will be observed:— The magnitude of the friction forces is limited. As soon as its limit is reached, the slab begins to slip on its foundation and does not undergo further expansion. The greater the forces necessary to expand the slab to rupture-point, i. e. the greater the tensile strength of the slab, the greater is the likelihood of the slab slipping and the avoidance of cracks.

The magnitude of the coefficient of elongation α has an indirect influence on the formation of cracks. The greater α is, i. e. the more ductile the concrete is, the smaller will be the stresses set up when deformation of the concrete caused by changes in temperature or degree of moisture is restrained (cf. I 8

also). However, the smaller these stresses are, the less will be the danger of their exceeding the tensile strength either alone or in conjunction with the stresses set up by loads.

The creeping power of concrete acts in the same sense as the coefficient of elongation α . Creeping primarily decreases the shrinkage stresses, which develop very slowly and whose action covers a long period (23) (24) (34).

Summing up, it will be seen that maximum attainable ductility of the concrete combined with greatest possible tensile strength is desirable if the possibility of crack formation is to be reduced. This statement, however, requires some qualification. As greater ductility of the concrete under tensile and compressive stressing also corresponds in general to a greater malleability under compressive stressing, such great deformations can take place in structural members subjected to bending as the deformability of the concrete increases — particularly because of creeping — that a far-reaching change occurs in the distribution of stresses as calculated in the usual manner, and the degree of stability and safety against cracking is reduced (34) (35).

III. Formation of cracks in reinforced concrete.

a) General.

In reinforced concrete construction the admissible elongations of steel, $\epsilon_e \geq 0.6$ mm/m correspond to admissible stresses on the steel of $\sigma_e \geq 1200$ kg/cm², while the maximum rupture-point expansion of concrete for tensile tests attained up to the present amounts to about 0.2 mm/m (36 p. 3) — for bending tests not more than 0.3 mm/m (24). For this reason cracks generally occur in reinforced concrete structural members well below the safe load limit. Experience has shown that these cracks do not endanger the actual life of the structure as long as they do not become so wide that the steel is exposed to destructive influences (11, Report by Krüger) (37) (38). Measures for reducing the possibility of crack formation must therefore aim at

- 1) restricting the actual occurrence, i. e. taking care that expansions which are too great for the steel to withstand are confined to as small a portion of the structure as possible, and
- 2) preventing the gaping of unavoidable cracks.

b) Initial stresses in reinforced concrete.

As is well known, self-stresses are set up in the concrete of reinforced structural members resting on movable supports by the slippage resistance of the steel reinforcement as the concrete shrinks and swells. When shrinking occurs these self-stresses are tensile, while swelling produces compressive stresses, corresponding in the steel to compressive and tensile stresses respectively. These stresses in concrete and steel are together designated initial stresses because they are already present in structures not yet subjected to loading.

The magnitude of these initial stresses is difficult to obtain from tests, for the following reasons (39 p. 127) (40) (33): The contractions of the concrete in shrinking are transmitted to the steel reinforcement by the slippage resistance of the latter. The resistance itself is set up by the friction of the steel against the concrete and by gripping forces caused by the special constriction of

the concrete (36 p. 32). Slippage resistance develops but gradually as the concrete hardens. It is therefore likely that in the first stages of hardening movements are possible between concrete and steel without the creation of stresses. Subsequently the shrinkage contraction of the concrete is transmitted from the end of the test piece to the reinforcing steel by the slippage resistance, whereby the relative movements gradually decrease until, in the middle region of the body, they disappear entirely. In like manner the initial stresses increase from 0 at the end to their maximum value at the middle zone of inertia. The law governing this increase, and consequently the length of the zone of inertia also — which does not appear at all in short bodies — are unknown. As soon as stresses are set up in concrete, it begins to creep. It is therefore impossible to arrive at the initial stresses in concrete from the difference in contraction of reinforced concrete as against that of shrinking concrete. The elongation of the concrete caused by the reinforcing bars, as against the contraction in the case of unrestrained shrinkage, is, on the contrary considerably greater owing to creeping than the elastic elongation due to initial stress (25 p. 36). The magnitude of the initial stress can thus only be measured by measuring the malleability of the steel in the zone of inertia. A test of this kind was carried out by *Glanville* (36 p. 53). The result, however, cannot be generalized owing to the size of the test piece. Initial stresses are not evenly distributed over the concrete section. In close proximity to the steel, creeping is greatly assisted by slippage resistance. This influence of the steel decreases as the distance from it increases, at first rapidly, then more slowly. For this reason test pieces showed a distinct curvature of the end surfaces (25 p. 37).

It is at any rate true of the zone of inertia that the contraction of the concrete, besides that of the steel reinforcement caused by the contraction of the concrete, in the case of unrestrained shrinkage δ reduced by the elastic elongation $\frac{\sigma_b}{E_b}$, and the creepage κ of the concrete, must be equal to the contraction of the steel $\frac{\sigma_e}{E_e}$. Here the creepage is a function of the temporal course of shrinkage and of the magnitude of the stress σ_b . Moreover, there must be equilibrium in the section. From the two equations: —

$$\delta - \kappa - \frac{\sigma_b}{E_b} = \frac{\sigma_e}{E_e} \quad \text{and} \quad \sigma_b F_b = \sigma_e F_e$$

we obtain with $\mu = \frac{F_e}{F_b}$

$$\sigma_b = \frac{(\delta - \kappa)}{\left(\frac{E_e}{E_b} + \frac{1}{\mu}\right)} \cdot E_e.$$

The initial stress in concrete therefore becomes the greater, the more the concrete shrinks and the higher the coefficient of elasticity of the concrete and the percentage of reinforcement are; whereas it decreases, the more the concrete creeps. In this connection it is mainly a question of the creeping that takes place during the initial stages of hardening; this initial creeping may be considerable when the hardening process is sufficiently slow, even though the

creepage when the concrete is older remains small, as is desired when allowing for permanent deformation under loads.

The danger of cracking is increased by initial tensile stresses in concrete. If the formation of cracks due to initial stresses is to be reduced, concrete of little shrinkage, high ductility and slow rate of drying should therefore be used, and care taken that a slow, steady drying rate is maintained (cf. I 8). In other words, the same precautions have to be taken as have already been recommended for unreinforced concrete. In addition, the percentage of reinforcement should be kept as low as possible. For this reason, too, weld splicing of the steel bars is more practical than overlapping or turn-buckles.

From the observations made concerning the fading of the action of steel on concrete in the proximity of the reinforcing bars, it can be concluded that for the same cross-sectional area of concrete and steel the deformations of the concrete, set up by the steel, cover an increasing part of the sectional surface, the more evenly the section of the steel is distributed over the section of the concrete, i. e. the more numerous the bars with a correspondingly smaller sectional area of the individual bar. The range of elongations in a concrete section is therefore reduced by better distribution of the steel section, and it is probable that the initial stresses are also decreased thereby.

Initial stresses, too, are most likely the reason why in high-webbed beams with a strongly reinforced tensile zone — and especially in I-shaped beams, the cracks appear first in the portion of the web situated above the reinforcement. As explained above, during the drying process the concrete has been expanded in the proximity of the steel bars, so that the contraction of the reinforced zone has kept much lower than for unrestrained shrinkage. Owing to cohesion the concrete in the web must now expand just as much above the reinforced zone, and this is also only possible by creeping. Here, however, there is no slippage resistance from the reinforcement to encourage creeping, so that the amount of creepage becomes smaller as the distance from the reinforced zone increases and a larger portion of the elongation to which the concrete is subjected produces stresses. The initial stresses in the web above the reinforced zone are thus greater than in the zone itself. This phenomenon is particularly marked in I-shaped sections, because between the wider tensile zone and the narrow web greater moisture differences and therefore greater shrinkage differences can easily occur. To protect the web against cracks caused in this manner it is advisable to attain good distribution of the reinforcement near the surface — a measure which has already been frequently been employed (e. g. 41), even if only in consideration of the fact that above the reinforced zone, too, the concrete is subjected to high tensile load-stresses (42).

c) *Development and consequences of cracks in reinforced concrete.*

Emperger (36) recently made an exhaustive investigation into the occurrences that take place in the neighbourhood of a crack. His observations yielded fundamental coincidence with the occurrences apparent from, or indirectly indicated by, the results of tests carried out with the object of determining initial stresses (cf. III b). Instead of creeping under the protracted influence of loading, when cracking takes place, plastic elongation is caused by the effect

of slippage resistance in the direct proximity of the reinforcement; this is greater than the normal elongation (36 p. 18), and decreases rapidly as the distance from the reinforcement increases (25 p. 40).

The tensile force of the steel at the point of cracking must again be transmitted to the concrete by the slippage resistance. First of all there will be an area, situated just beside the crack, in which the concrete has loosed itself from the steel owing to its inability to follow the elongations and cross-sectional constriction of the steel (zone of separation); then follows a zone in which the concrete undergoes plastic elongation in progressively increasing layers and where the slippage resistance, which rapidly attains its maximum value, operates (plastic zone). Here the concrete assumes part of the tensile force from the steel, until finally the elongations of the latter no longer exceed the elastic ductility of the concrete (elastic zone). (36 Fig. 20) (25 p. 53). As the stressing of the steel increases and the distance between the cracks decreases, the two last-mentioned zones vanish successively, so that eventually the steel slips along its whole length between two cracks.

It can be concluded from the occurrences in the proximity of a crack that the co-operation of the concrete in the tensile zone is afforded correspondingly longer, the greater the slippage resistance of the reinforcement and the plastic deformability of the concrete. As far as the concrete is concerned, the slippage resistance increases with the strength of the concrete, though to a lesser degree (25 p. 56), whereas the plastic deformability increases as the strength of the concrete decreases (36 p. 73 (30 p. 50)). Slippage resistance can be more easily influenced by the type of reinforcing bar used, e. g. steel with as rough a surface as possible, or rods of special cross section, such as bulb or twisted bars, whose shape affords good connection with the concrete (25 p. 58) (36 p. 73). The co-operation of the concrete can also be made more effective by causing a greater cross-sectional area of the concrete to be affected by the reinforcement, e. g. by better distribution of the cross-sectional surface of the steel (25 p. 41) or by the arrangement of reinforcement on all sides (cross reinforcement, wire mesh, expanded metal). In this connection it should be noted that cross reinforcement (stirrups) placed between the main reinforcement and the surface of the concrete favours the formation of cracks.

More active co-operation of the concrete in the tensile zone has the following effects:— The mean value of the stress to which the steel is subjected under a certain loading is reduced, and with it the elongation of the tensile area (25 p. 51). Further, the distance between the cracks becomes smaller, since the unloading of the concrete from the first crack only operates over a small distance (25 p. 48). However, the more numerous the cracks are, the less they will gape (25 p. 50). The reason for this is the fact that the expansion of the steel along a certain length in which cracks have occurred, is chiefly dependent on the elongation of the steel in the cross sections of the cracks, since there the steel is stressed most. Now, the more numerous the cracks, the smaller will be the fraction of the total elongation of the steel pertaining to the separate cracks.

As the stressing of the steel increases, the cracks gape more widely; unless, of course, new cracks appear, before the slippage resistance between the existing cracks has not been overcome. As soon as the slippage resistance is exhausted,

the width of the cracks ceases to be dependent upon their number (25 p. 51). On the contrary, some of the cracks may gape exceptionally wide, while others may close again somewhat. Thus tests with frequently repeated loading and unloading (36 p. 114) (43) revealed that, in the case of round steel reinforcement, bars whose slippage resistance has been overcome by permanent loading, the greatest width of crack increased considerably with repetition of loading, although the number of cracks remained the same. On the other hand, where Isteg steel was used and slipping in the concrete was impossible, this width remained unchanged. When such permanent loading is applied the number of cracks at first increases but soon enters a state of inertia. New cracks can also be formed under permanent, stationary loading as a consequence of creeping, since the neutral axis is displaced towards the side under tension and the stressing of the tensile area thereby increases (33) (34).

The gaping of cracks is greatly affected by the fact that the steel has suddenly to take over the force formerly carried by the concrete up to the moment of cracking (36 p. 44) (44). The bars are therefore subjected to additional elongation at the spot where cracking occurs — the only place where they can expand without restraint. The cracks will thus gape the wider, the smaller the relation between the cross section of the reinforcement and that of the ruptured concrete, and the greater the former tensile force of the concrete, or, speaking generally, the greater the width of the tensile area (44) and the tensile strength of the concrete. When the crack forms, the concrete springs back in consequence of the unloading. This movement is smaller in the proximity of the reinforcement, owing to slippage resistance, than at a distance from it where the elastic elongation was also greater. The crack therefore gapes somewhat more at the surface of the body than near the steel bars (36 p. 48), just as it begins at the surface and progresses in the direction of the reinforcement (39 p. 117).

It is a fact of particular importance that the cracks remain thin under dead weight and therefore close again as far as they can when traffic load has been removed (45). The permanent width of crack seems to be chiefly dependent upon the permanent elongations of the concrete above the reinforced area. In the crack itself, too, permanent elongations of the steel occur, since the bars are prevented by slippage resistance from springing back completely (36 p. 73). There is as yet no information available as to the width of crack that can be allowed to remain without ever endangering the member. Investigations with centrifugal concrete, i. e. with concrete of a high degree of compactness (37), gave a permissible permanent crack-width of up to 0.3 mm, and a temporary width of 0.5 mm.

Gehler (44) concludes from the crack-widths measured in T-beams at a stressing of steel of 1200 kg/cm², that a width of $\frac{1}{8}$ mm is permissible. *Graf* (43) comes to a similar result. The object of such definite conclusions is to form a standard by which to judge the results of tests as to the admissibility of increased stressing in the steel, independent of the quality of cement and the type and arrangement of the reinforcement. It is, however, doubtful whether, with the knowledge and experience at present available, the width of cracks can be restricted to a certain limit (cf. 44 Table III).

Our discussions on the gaping of cracks show that the degree of safety against

cracking, calculated as the relation of the loading at which the crack occurs to the permissible loading, or as the relation of the stressing of steel, calculated in the usual manner without considering the co-operation of the concrete, to the permissible stressing of steel (44), in general only attains a magnitude of practical utility when it is greater than 1. As long as it keeps below 1, an increase does afford advantages if, in consequence of greater safety against cracking, cracks subsequently created under dead weights gape less than cracks which appeared prematurely.

The safety against cracking, as has been laid down in the foregoing, increases with the bending strength and ductility of concrete, the efficiency in type and arrangement of the reinforcement, and the difference between the calculated stressing of the steel and the real stressing due to the co-operation of the concrete and finally, inversely as the reinforcement ration $\mu = \frac{F_e}{bh}$ (25 p. 24) (45). It is therefore greater for slabs with crosswise reinforcement, supported on all sides, than for mono-axially stressed slabs, and for slabs than for T-beams; in the case of the latter, it increases with the width of the tensile area of the concrete (44).

d) Measures for the prevention of cracking in concrete.

From Sections b) and c), III, a summary reveals the fact that if the formation of cracks in reinforced concrete is to be reduced, concrete that shrinks little, is very ductile and hardens as slowly as is practically possible should be employed, and further that the drying process should take place slowly and steadily. As long as there is any possibility of preventing cracking under safe loads, it is a question of attaining a maximum degree of tensile strength in the concrete. If cracks are unavoidable, however, as for instance in T-beams under customary conditions, then the ductility of the concrete becomes a more important factor than its tensile strength.

Particular care should be taken that in the tensile area the co-operation between concrete and steel is sustained up to the highest possible point of elongation of the steel. For this reason the cross section of the latter should be distributed over as large an area as possible. Rough-surfaced steel bars should be given preference, together with such as possess high slippage resistance, like bulbed or twisted rods. (Naturally these bars must also possess the other properties required in reinforcing material, and their shape should not be such as to cause splitting of the concrete.) Transverse reinforcement, rigidly connected with the tensile reinforcing at suitable places, also offers advantages.

Besides these generally applicable measures, it is possible in special cases to cause such great compressive pre-stresses in the concrete by pre-stressing the reinforcement, that no tensile stresses whatever occur in the concrete under safe loads. This possibility was investigated quite a long time ago (25 p. 44), but it remained for *Freyssinet* (cf. work just mentioned) to succeed in exploiting it after he had recognized the significance of creeping in the amount of pre-stressing required and found new ways of reducing volume deformations in concrete (20). His views and elucidations on the subject of the extent to which the properties of concrete can be influenced will probably contribute also to the clearing up of the questions treated in the present report.

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- (45) *Abeles*: Über die Verwendung hochwertiger Baustoffe im Eisenbetonbau (The use of high-quality materials in reinforced concrete construction). Beton und Eisen 1935, Heft 8—9.

Summary.

The present paper is primarily an exploitation of the copious literature dealing with the theme under discussion. It may be summarized as follows: —

The tensile strength of concrete is mainly dependent upon the tensile strength of the respective cement used, calculated as bending strength on the more recently introduced testing methods, with plastic mortars of mixed granule sizes. In general, the same measures should be taken for attaining good composition as for producing a concrete of maximum compressive strength. Broken aggregates improve the relation of the tensile to the compressive strength, but in general do not produce greater tensile strength than do naturally graded aggregates when the concrete is soft and liquid. This is due to the greater quantity of water needed. Tensile strength can be influenced to a greater extent by the properties of the stone aggregates than compressive strength can. Slow drying out of the concrete is a particularly important factor.

In evaluating the separate influences it proved practicable to follow the relation of tensile to compressive strength with the aid of exponents of compressive strength which allow for the fact that this relation depends upon the magnitude of the compressive strength.

Since the ductility of concrete is an important factor in the formation of cracks, the effect of this property was also investigated. It generally decreases as the strength increases, but can be influenced by the type of cement and type of stone aggregates employed. Creeping has a beneficial effect on tensile strength, since it reduces undesired shrinkage stresses.

Tensile strength is only of supreme importance as regards the prevention of cracks in concrete when there is a possibility of completely avoiding the formation of cracks under safe loads. It is only in this case, too, that high degree of safety against cracks, i. e. the relation of the cracking-point load to the admissible load, is of practical importance. Where cracks cannot be avoided, the ductility of the concrete is more important than its tensile strength, and slight gaping of cracks more important than if these cracks develop only after great loads have been applied.

For the rest, it is a question of dividing up the reinforcement as much as possible and of using types of reinforcement which develop great slippage resistance in the concrete. When treating the subject of crack formation it was proved that the compound action of concrete and steel when initial stresses are set up coincides in principle with the occurrences in the proximity of a crack, and that the initial stresses are probably smaller than has hitherto been assumed, owing to the creeping of the concrete.