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Influence of the Plasticity of Materials and of Variable Loads, on the Stability and Life of Structures.

Einfluß der Plastizität der Baustoffe und der veränderlichen Lasten auf die Stabilität und die Dauerhaftigkeit der Bauwerke.

Rôle de la plasticité des matériaux et des efforts variables dans la stabilité et la durée des constructions.

L. P. Brice, Paris.

In the present paper we will examine the conditions required of a structure to render it stable and durable under prolonged statical and dynamical stresses, for which purpose we refer to some simple experimental facts derived from the deformation of material used in reinforced concrete.

The importance of considerable deformation prior to rupture, has not been ignored by engineers. They have stated that only those metals should be used in structures which yield sufficient elongation before fracture. However, the resistance of the classic materials is strictly limited to the study of elastic deformation and does not allow for the possibility, or the consequence of, non-elastic deformations. They are however, such, as tend to neutralise stresses by producing in the structure a redistribution of internal stresses in conformity with their possibility of resistance, thus alleviating our difficulties concerning the real conditions as to the redistribution of stress.

Some 15 years ago, *M. A. Caquot* was the first to call attention to these phenomena and proposed to designate them by the word "adaptation" which he defined thus:

A member is durable when its deformations comprise,

1) A permanent deformation within prescribed limits.
2) A reversible deformation within the range of endurance.

The sum of the permanent deformation and the initial stresses, constitute the phenomenon of "adaptation".

The strict application of the laws of the resistance of materials, or of the theory of elasticity, to some simple cases, prove that theoretically, certain structures should be incapable of resisting the stresses which, in fact, they are supporting.

It is thus that the theoretical stress at the edge of a hole pierced in a metallic member, is three-fold the stress in the circumscribing metal. Thus in riveted joints, the average calculated stress is one half the elastic limit. As this average stress is perhaps augmented by about 30 % on account of the rigidity of the joint, the stress at the edge of the hole will considerably exceed the elastic limit and even the breaking load.

Another example is furnished by the numerous slabs or girders in reinforced concrete, rigidly held down at their supports and calculated as if only partially held. The stress in the members thus held down, ought therefore, theoretically, to be well in excess of the elastic limit; nevertheless, experience shows that in numerous cases the structures, calculated by simple methods, adjust themselves to the loads they have to support.

Lastly, a solid concrete beam can resist to shear forces through an arrangement of reinforcements acting in tension only, and adapt itself, all through this phenomena, to the mode of functioning for which the structure is meant.

These examples show that if for the condition of stability and endurance of a system, it is sufficient that it should be perfectly elastic at all points, it is far from being always necessary.

We will look at the problem closer and find what conditions the structures should possess in order that their resistance and their endurance should be maintained under the influences alike of the dead and live load.

Deformations of Materials.

1) *Steel.*

The stress-strain curves of mild steel used in construction indicate a considerable range of elongation following the elastic deformation (Fig. 1).

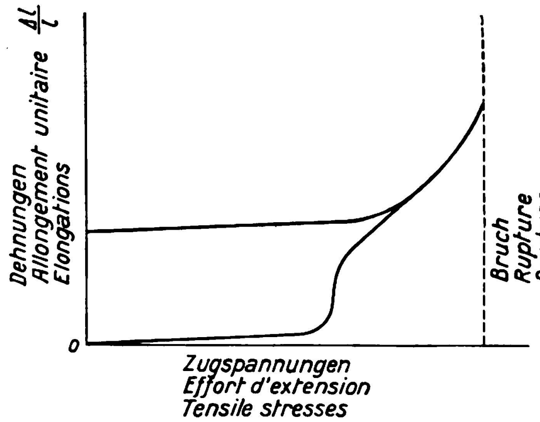

Fig. 1.

Stress-elongation diagram of steel.

If a specimen is submitted to a stress slightly in excess of the elastic stress, the elongation above this stress is greatly augmented by small increases of stress. On decreasing the tension to zero, one finds that the specimen is shortened in accordance with a law of elasticity almost identical to that which represents the initial elastic deformation. The bar thus treated acts as an elastic body between loads of 0 and a new elastic limit which is higher than the elastic limit of the bar not so treated. This principle is frequently used for augmenting the elastic limit of mild steel, and enables it, permitted by Regulations, to withstand higher stresses.

If, by a suitable arrangement a specimen is submitted not only to tension but to alternate stresses of tension and compression, (taking care to avoid premature deformations due to buckling), it is found that the specimen, even after considerable initial deformation, can be submitted to a large number of alternating stresses, provided the maximum and minimum values of such stresses are within pre-determined limits.

Endurance curves can thus be determined which exactly define the limits between which a definite number of alternations can be applied, without causing fracture. If limiting stresses, for example, slightly higher than the elastic limit in tension and in compression are adopted, the number of alternations should be reduced.

If, on the contrary, the limiting values are placed at one third of the actual values, the number of possible alternations are practically infinite.

These limits of endurance are of great importance in machine construction. They indicate, in particular, that if the alternating stresses approach locally to the values of the elastic limit, the fracture of a piece which from preliminary calculations appears remote, becomes dependent upon the number of alternations.

There is the well known case of machine shafting, provided with sudden changes of sectional area, in the neighbourhood of which the elastic stresses can be very high, and the shaft breaks without apparent cause.

Similarly, riveted joints for which the actual stress approximates to the elastic limit at the edge of the rivet hole, are not suited for alternating loads; thus it is that the riveted framework of locomotives, wagons and motors, needs to be very much heavier than welded or pressed frames.

2) *Concrete.*

The stress-strain curve of concrete has been much less studied than that of the metals. Its shape presents a practically straight line from the origin, passing into a curve when a load in the region of half the breaking load is passed.

When a specimen is submitted to stresses well beyond the limit of proportionality, without special precautions, cracks are formed which increase with the load up to the point of complete destruction.

If, on the contrary, care is taken to give lateral support to the concrete, either by bands, hydraulic pressure, or by masses of concrete surrounding the compressed member, it is found that, as with metal, a new state of the material sets in after plastic deformation, which resists variations of stress, within defined limits, as an elastic body (Fig. 3).

The case of concrete is complicated by the fact of its special properties, shrinkage and slow non-reversible deformations under permanent loads.

It has been shown by *M. Freyssinet* that the shortening of a specimen of concrete subjected to a load for a long period, was twice the amount of that obtained when a similar specimen was submitted to a rapidly applied load. All this happens in concrete as if the modulus of elasticity was reduced by about 50 % when the concrete was loaded for a long period. This phenomena plays an important part in equalising the stresses in concrete members under compression.

For example, the flange of a T girder cast some time after the web of the girder, which was already under load, would be able, after a certain time to take a share of the compressive stress which would have caused a gradual shortening of the compressed area of the web.

The distribution of stress in a post under load is modified in the course of time. The stresses in the bars exceed their initial value, when the concrete under the load had not taken up its total shrinkage.

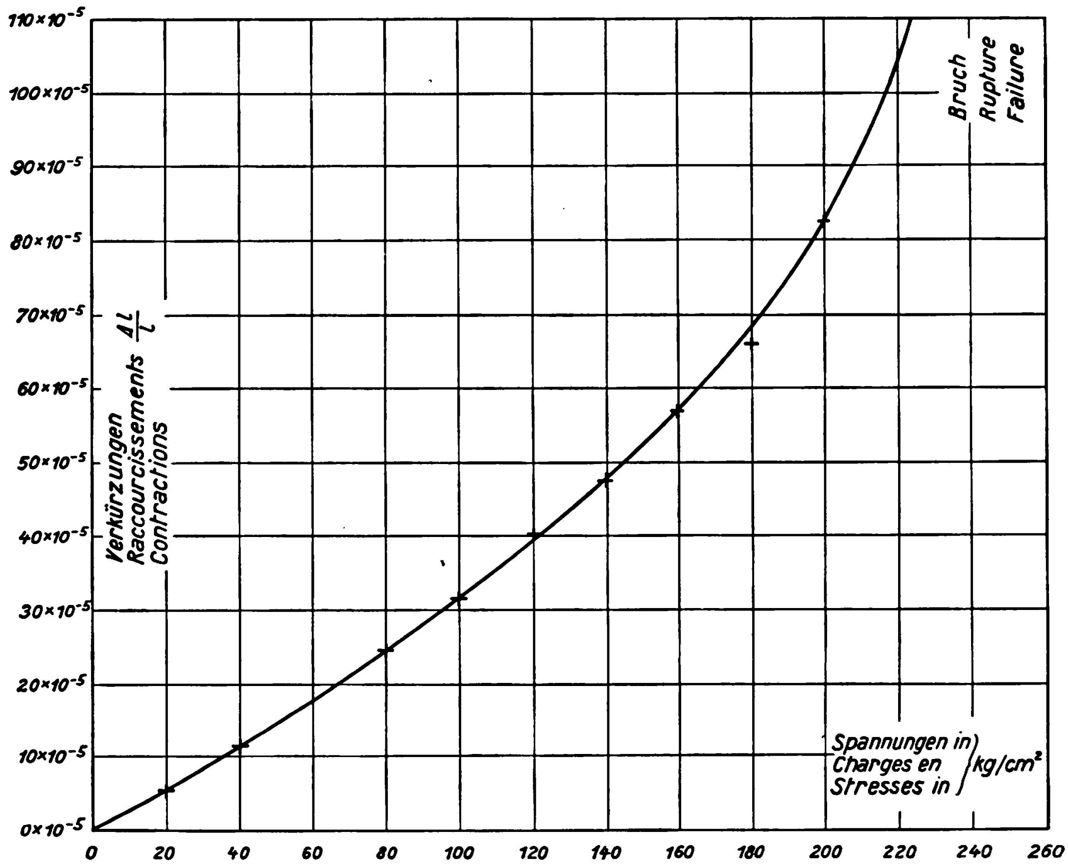

Fig. 2.

Diagram of deformations for cement concrete.

Summarising, a concrete member stressed to about 60 % of its breaking load, will be deformed six to eight times more than the same member under a normal stress of short duration, consistently with the shape of the stress-strain curve and the increase of shrinkage under permanent load.

3) *Bond between concrete and metal.*

In a reinforced concrete structure the bond between the steel bars and the concrete is assured more by the grip of the concrete than by the property of surface adhesion. This is proved by the force necessary to slide a metal bar in its sheath. If it was simply surface adhesion, the force would diminish to a very small value after the initial displacement was effected (Fig. 4).

This action of sliding, transmits the stress from one bar to another through the concrete. It is, in fact, impossible that a bar transmits its load to a neighbouring bar, without there being a relative slip between the two bars and the concrete, which perhaps is of the order of one-two hundredth of the diameter of the bar.

In order that this adhesion to the sheath may not be broken it is very necessary to avoid too great a variation of stress between the concrete and

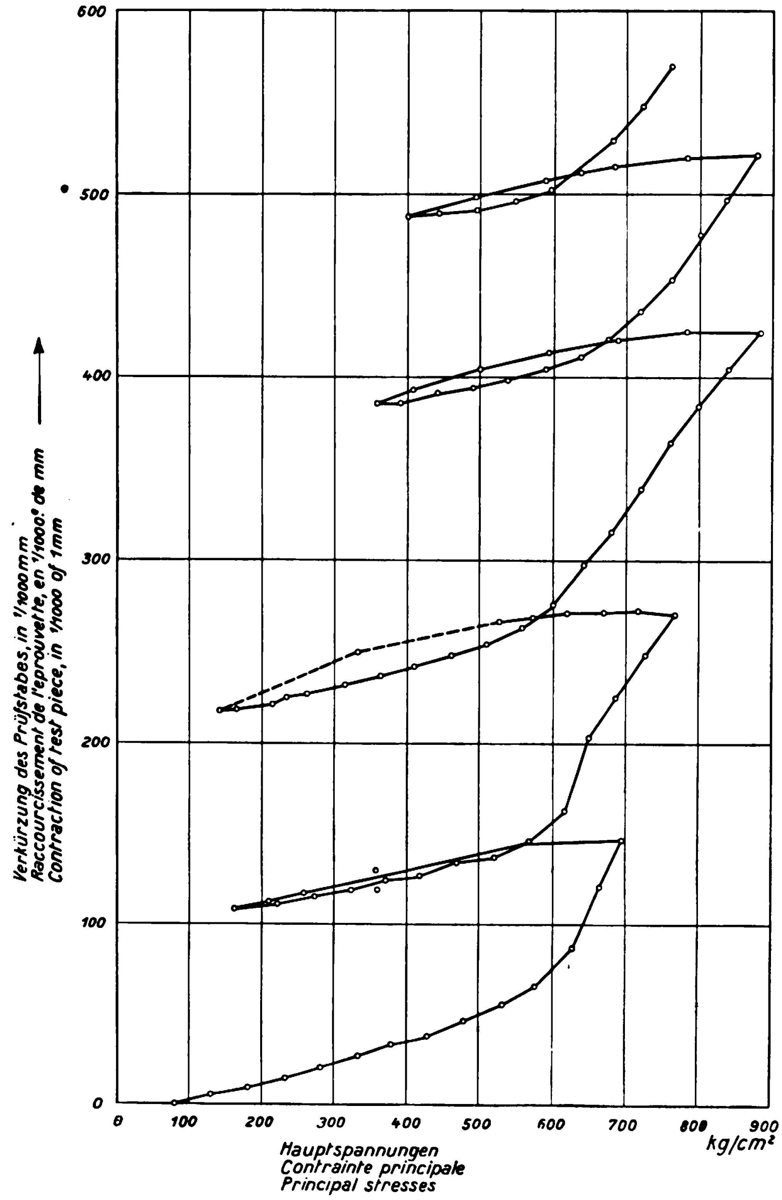

Fig. 3.
Deformation of a test-piece of concrete held by lateral hydraulic pressure.

the bar. There is reason to believe that successive relative displacements of two members may lead eventually to a disintegration of the concrete bed and a consequent slackening of the adhesion. This simple remark enables one to appreciate the utility of hooks for tension bars which, perhaps without providing any increased resistance when the structure is new, become of service if the adhesion is weakened by abnormal or too frequent deformations.

4) *Deformations during bending.*

The non-elastic deformations of a girder during bending may be caused by the non-elastic deformations of either the steel or concrete, or a combination of these two properties.

If the compressed concrete is largely in excess, the deformation of the metal comes solely into play. Experience shows that large deformations are initiated, when the elastic limit of the metal is reached, and can reach a considerable magnitude in a well-designed girder without setting up a significant variation of the stresses that can be sustained.

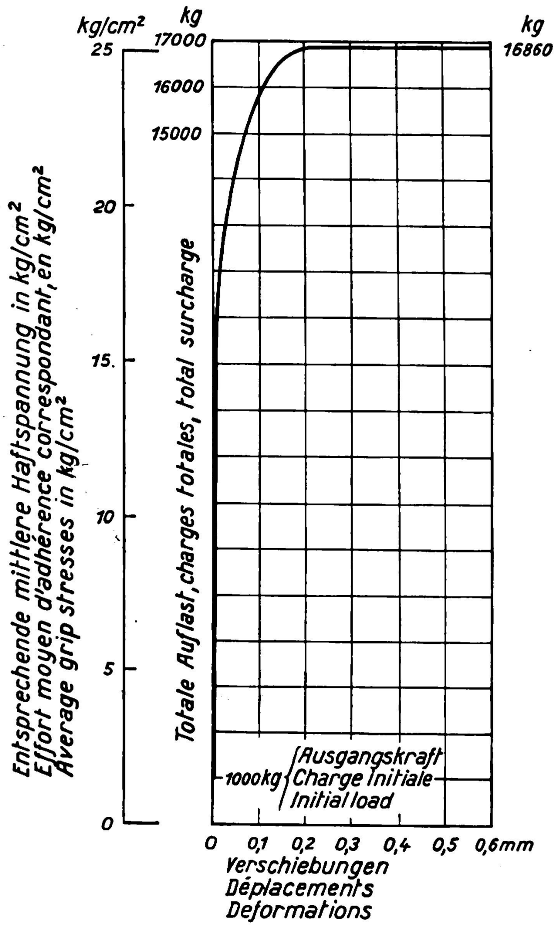

Fig. 4.

In the usual methods of calculation, the bending moment necessary to produce these large deformations is essentially double that which served for the calculation of the girder.

If the load decreases, the girder loses part of its deformation and is again able to act as an elastic body. Its elastic properties are, however, modified and one can take into account the greater ductility of the deformed zones, by making use only of the reduced section, where the stretched concrete is ignored, and in taking a consistent value for the ratio modulus $n = \frac{E_s}{E_c}$.

These results are confirmed by the experiments made by *M. Dumas*, Engineer of Bridges and Roads, on the deformation of girders under repetition loading.

These large deformations do not take place without starting the formation of cracks in the stretched portion of the concrete. Practically, these cracks limit the magnitude of the possible deformation. The admissible size and number of these cracks are extremely variable in conformity with the uses of the structure.

These cracks are often met with in the vicinity of the slab supports where concrete under compression is in excess; if it is a question of floor slabs covered with a timber floor, these cracks are of very little importance, if on the contrary, the slab is covered with an adhesive covering, the appearance of these cracks on the floor is of worse effect. One should have greater latitude with a protected work, than with one subjected to atmospheric conditions; external corrosive agents find an easy means of access through these cracks. Whatever the cause, it appears difficult, without precautions, to lessen the

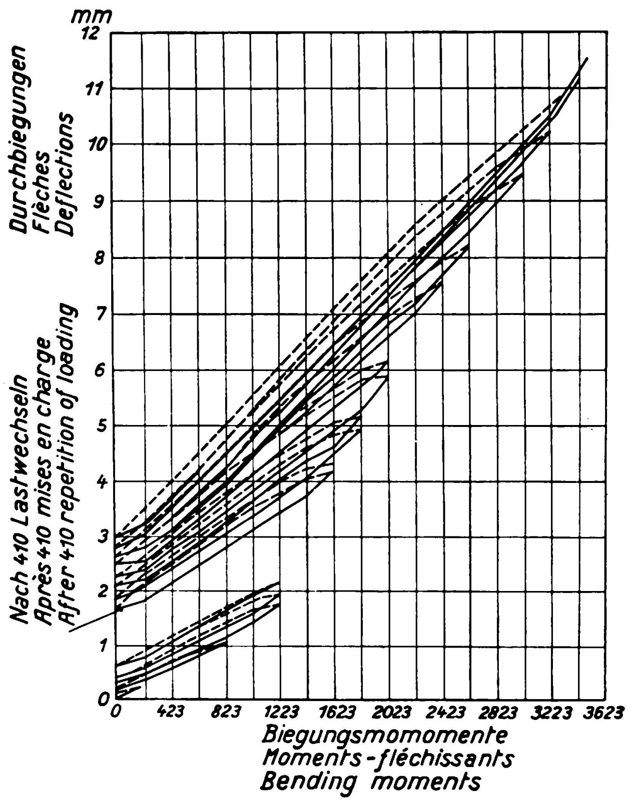

Fig. 5.
Deformation of a beam in banding.

injurious effects of cracks, and the elongation concentrated to a limited zone equal to two or three times that of the normal elastic stress.

In the case of the plastic deformation of concrete, the phenomenon is essentially more complex. The line of deformation of the concrete being a curve, the diagram of distribution of the stresses is deformed and the lever arm of the resistant couple decreased.

However, the phenomenon maintains in its entirety, the same properties as simple compression. The large deformations due to the concrete, contrary to those due to the metal, have only a small tendency to open cracks. But it is necessary to make sure that the concrete is in such a condition that it can withstand, without rupture, the plastic deformation. A limit is still necessary. If the concrete was not specially prepared, it should not exceed a stress corresponding to one half the ultimate breaking load (which in course of time leads to deformation four or five times greater than under the normal pressure). The deformations can be greater, if a suitable hooping ensures the bearing of the core to the high pressures.

When the two phenomena are superimposed, which is the usual case for rectangular beams, we have the best conditions for obtaining maximum deformation with a minimum disturbance.

5) *Plastic deformation due to shear.*

Experiments concerning the resistance to shear of solid beams are scarce. However, the experience made by engineers and some tests which we have been able to study, or to carry out, show that the various systems of reinforcement which can be used (rods parallel to the direction of shear or inclined at an angle ranging from 0 to 45^0) are equal in quality. That proves that the girder adapts itself, but the deformation of cracks at 45^0 is always found, since the tensile resistance of the concrete is exceeded. These cracks are of such frequency that one has become so used to their presence as to consider them perfectly normal. They are, however the apparent sign of this phenomenon of adaptation which one often refuses to allow for bending. There is no reason for making this difference.

Function of Plastic Deformation.

We think to have shown in this way that plastic deformations intervene in all the members of a structure to modify the distribution of the stresses among the several parts, concrete and metal, of a member. They are equivalent, in fact, to the transfer of the action of the external loads from one member to another.

The considerable deformation of overloaded members results in increasing the load on members which are slower to react and are relatively much too rigid.

The plasticity of constituent members therefore, enables a structure to "adapt itself", that is to say, to submit to non-elastic deformations, under the influence of external loads, such that the less resistent members are relieved by those which are more rigid.

This fresh distribution of stresses can only be produced if the two following conditions are fulfilled.

a) The plastic deformations must be produced without compressing the bond of the structure, that is to say, a method of resolving the stresses within the system must be found compatible with the resistance of the members. In particular, if the great deformations are accompanied by cracks, the bond between the two faces of the gap must be assured by steel rods of sufficient sectional area to transmit the stress across the opening.

b) If a member of the structure is deficient in strength, its deformation must result in transmitting a portion of its load to a more resistant member.

In a word, the system must be hyperstatic and stable. Members of too small a resistance will play the part of semi-articulations which will support during their deformation a force which does not exceed that corresponding to the elastic limit of the constituent parts.

The resistance of the new system thus established is different to that of the initial system; it has produced a distribution of forces more conformable to its actual possibilities and in spite of the erroneous conception regarding the theory of elasticity, it functions perfectly well under the influence of the loads. The interest of hyperstatic systems appears here; an error can be compensated by the deformation of some weak member, whilst in a strictly isostatic system, every weakness, even local, leads infallibly to considerable deformation, since nothing is opposed to their development when they are set up by a permanent external cause.

However, the problem becomes more complex when the loads are variable. It is necessary to study what happens when, after fully loading the system, it is unloaded.

The system tends to recover its initial form, so far as the plastic deformations of its members will permit.

This rearrangement will have, in consequence, according to the ratio of live to dead load, a reduction or even a reversal of the stresses in the members which have been subjected to plastic deformation.

Thus, we have seen above that although the elastic deformations can be repeated a large number of times, the plastic deformations can only be applied once, or for a very limited number of times.

To assure the life of the system, it will be necessary that the deformations remain perfectly elastic under every possible variation of the live load, when referred to the total deformation, elastic or not, to which the system was subjected under the most unfavourable disposition of the dead and live load.

The uncertainty of the exact value of all the internal stresses, will make it necessary to limit the elastic deformation, by insisting for example, that the changes of stress due to variable loading in members under plastic deformation, calculated by the usual methods, must not exceed defined values.

Until precise experiments justify figures, or in other words, fix the limits of endurance under alternating stress, it appears that the limits usually adopted — one half of the elastic limit for steel, 28 % of the resistance to compression for concrete — should give ample security. With regard to the adhesion of the bars, one should make sure that good anchorages well be capable of transmitting the stress.

Applications.

Semi-hinges.

A semi-hinge consists in principe of a thin layer of concrete between two groups of reinforcement, arranged as hoops supporting a core of concrete for transmitting the stress to the rest of the structure.

The non-elastic displacements which are produced by shrinkage or application of the load will be absorbed by the core of concrete during its plastic deformation, which may be considerable. It will be sufficient, to ensure durability, that under the application of alternating loads the variations of stress in the core remain within prescribed limits.

Semi-fixed slabs.

It is usual, in construction, to build floors of reinforced concrete of slabs and beams for which the bending moments are not calculated conformably to the theory of elasticity, but more simply; for example, for a member uniformly loaded, the bending moment at the centre of the span is less than that for a girder freely supported, and the moments at the supports are slightly in excess of the value, inherent upon the moment taken at the centre.

Let us see, under these conditions, what will be the actual distribution of the bending moments in a section at the centre and in a section at the supports, while an increasing load is applied to the girder.

At the first application of the load the whole of the girder acts as an elastic body, the distribution of stress conforms to theory; as the strain in the section at the support approaches a plastic strain, the deformations increase under a

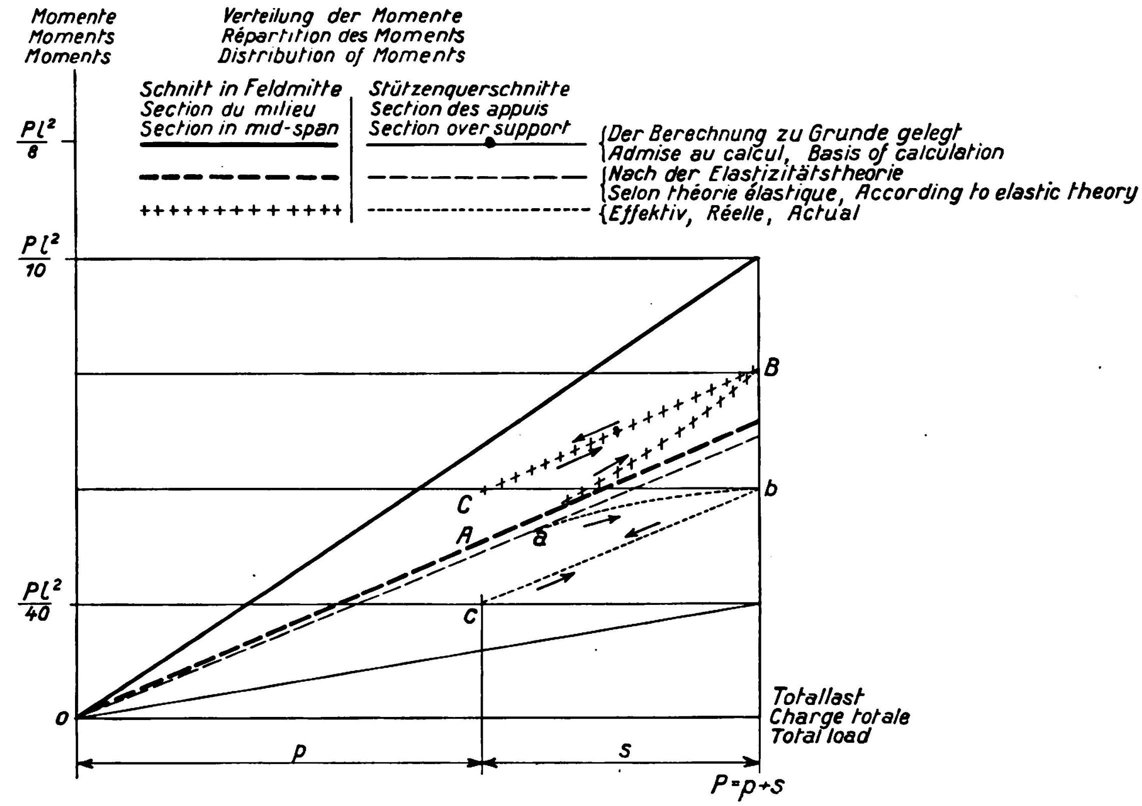

Fig. 6.

Fatigue at the supports and in the mid-span of a semi-fixed beam.

practically constant moment, it produces a rotation about the support, and the stresses in the section at the centre increase more rapidly than at the start (curves AB and ab, Fig. 6).

A given load (p + s) is thus obtained, for which the stress sustained by the section at the supports does not much exceed double that which had been estimated, whilst the stress at the centre is below its estimated value.

If at this moment part of the load is removed, the system acts as an elastic body. The stress diagram for the section at the supports is represented by the line bc, whilst in the central section the stress diminishes according to the line BC.

If the load is reduced to 0, a point is reached at which the stresses in sections at the support are neutralised, then change in sense. But we have seen that, to assure the durability of the structure, it is necessary that under the live load only, the decrease of stress in the section previously submitted to plastic deformations, remains within a certain limit bc.

We are thus able to determine what the maximum ratio should be of live load to dead load so as to assure the durability of the structure.

Let us apply this rationally to an ordinary ribbed slab in which the central section has been calculated with a bending moment equal to $\frac{Pl^2}{10}$

$$P = p + s$$

where p is the dead load,
and s the live load uniformly distributed.

The sections at the supports are supposed calculated with the moment $\frac{Pl^2}{30}$.

Applying the theory of elasticity to a girder of this type and taking note of the different moments of inertia at the centre and over the supports, it is found that the moment over the support is practically equal to the product of the uniform load and $\frac{l^2}{15}$.

If therefore, it is wished that the removal of live load only produces a change of moment in the section over the support, compatible with its resistance, it will be necessary that

$$\frac{sl^2}{15} < (p + s)\frac{l^2}{30}.$$

This inequality shows that s should be less than p.

If we now consider the case of a rectangular beam uniformly loaded, it is not unusual to calculate it with a bending moment equal to $\frac{Pl^2}{15}$ at the centre and over the supports.

Now, the actual moment over supports for such a girder, since its inertia is constant, is equal to $\frac{Pl^2}{12}$.

To assure safety it will therefore be necessary that in the expression

$$\frac{sl^2}{12} < (p + s)\frac{l^2}{15} \quad \text{applies,}$$

out of which follows thats $s < 4$ p.

This simple calculation serves to show why it is possible to construct a floor for carrying small live loads (dwelling house floors for which the live load is less than the weight of the floor itself) using empirical formulae in apparent contradiction to the theory of elasticity, whilst it is impossible to construct from the same formulae, warehouse floor heavily loaded, without discrepancies being soon manifest.

The smaller the ratio of dead load to live load, the greater is the difficulty of maintaining the structure: the case is well known of the girders of swing bridges which are submitted to alternating stresses, and more particularly, railway sleepers of reinforced concrete; in the latter case, the conditions of adhesion present such difficulties, that their use under the running roads is very precarious. At the same time it is not unlikely that the application of special processes, such as putting the steel under such tension as will avoid reversal of stress, will enable the problem to be solved. It has been solved in this manner in the case of posts for electric lines, by M. *Freyssinet,* who has found that a post reinforced with rods severely stretched, is incomparably more resistant and more durable than a similar post reinforced with ordinary rods.

Framework for buildings.

The preceding theories can be applied to the framework for buildings.

It appears to us, moreover, that the almost complete ignorance concerning the distribution of stresses during construction, renders the literal application of formulae derived from the theory of Strength of Materials somewhat illusory. In fact, the construction is generally carried out in such a way that the members are only loaded in succession in an order that is not always foreseen. Further, during the work some accidental loads may intervene to modify to some extent the distribution of stresses.

It appears more suitable to consider for the system, the possibility of non-elastic deformation in the less resistant members under the worst conditions of loading, and to examine what happens when the live load is removed. The deformations produced, remain sufficiently elastic, that it may not be in vain to apply the classic theory of the deformable system, taking account in the calculation of the inertia of the new properties of members plastically deformed. It seems that the current practice which allows the centres of posts to be jointed, may be in a general way, justified by the preceding.

Summary.

This short paper will have sufficed, we hope, to show that in the stability of structures, the action of the dead load and that of variable live loads are essentially different, from the fact that the nature of materials "adapt themselves" under, a constant load, but cannot do so under variable loads.

It is moreover ,this fact, supported by the actual experience of all construction, that has led the authors of the regulations of the „Chambre Syndicale des Constructeurs en Ciment Armé de France" to increase the live loads relatively to the dead load, in a manner which accounted simply and in an effective way for their distinctly unfavourable action.

Large structures have from this point of view a real superiority over the others: the solid slabs and supported floors are marked out for heavy live loads. In large works, bridges or viaducts, it is well to avoid extreme lightness of the decking by the use of cross girders and bearers of small inherent weight; it is, from all points of view, more rational to place the principal girders under the heavy loads.

Summarising, the most durable structure is that in which the variations of stress under the live load have the lowest possible relative values. This result can be obtained either by increasing the dead load or by any means tending to increase the relative value of the stresses due to the permanent load.