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L. P. Brice, Paris.

In the present paper we will examine the conditions required of a structure to render it stable and durable under prolonged statical and dynamical stresses, for which purpose we refer to some simple experimental facts derived from the deformation of material used in reinforced concrete.

The importance of considerable deformation prior to rupture, has not been ignored by engineers. They have stated that only those metals should be used in structures which yield sufficient elongation before fracture. However, the resistance of the classic materials is strictly limited to the study of elastic deformation and does not allow for the possibility, or the consequence of, non-elastic deformations. They are however, such, as tend to neutralise stresses by producing in the structure a redistribution of internal stresses in conformity with their possibility of resistance, thus alleviating our difficulties concerning the real conditions as to the redistribution of stress.

Some 15 years ago, *M. A. Caquot* was the first to call attention to these phenomena and proposed to designate them by the word "adaptation" which he defined thus:

A member is durable when its deformations comprise,

1) A permanent deformation within prescribed limits.
2) A reversible deformation within the range of endurance.

The sum of the permanent deformation and the initial stresses, constitute the phenomenon of "adaptation".

The strict application of the laws of the resistance of materials, or of the theory of elasticity, to some simple cases, prove that theoretically, certain structures should be incapable of resisting the stresses which, in fact, they are supporting.

It is thus that the theoretical stress at the edge of a hole pierced in a metallic member, is three-fold the stress in the circumscribing metal. Thus in riveted joints, the average calculated stress is one half the elastic limit. As this average stress is perhaps augmented by about 30 % on account of the rigidity of the joint, the stress at the edge of the hole will considerably exceed the elastic limit and even the breaking load.

Another example is furnished by the numerous slabs or girders in reinforced concrete, rigidly held down at their supports and calculated as if only partially held. The stress in the members thus held down, ought therefore, theoretically, to be well in excess of the elastic limit; nevertheless, experience shows that in numerous cases the structures, calculated by simple methods, adjust themselves to the loads they have to support.

Lastly, a solid concrete beam can resist to shear forces through an arrangement of reinforcements acting in tension only, and adapt itself, all through this phenomena, to the mode of functioning for which the structure is meant.

These examples show that if for the condition of stability and endurance of a system, it is sufficient that it should be perfectly elastic at all points, it is far from being always necessary.

We will look at the problem closer and find what conditions the structures should possess in order that their resistance and their endurance should be maintained under the influences alike of the dead and live load.

Deformations of Materials.

1) *Steel.*

The stress-strain curves of mild steel used in construction indicate a considerable range of elongation following the elastic deformation (Fig. 1).

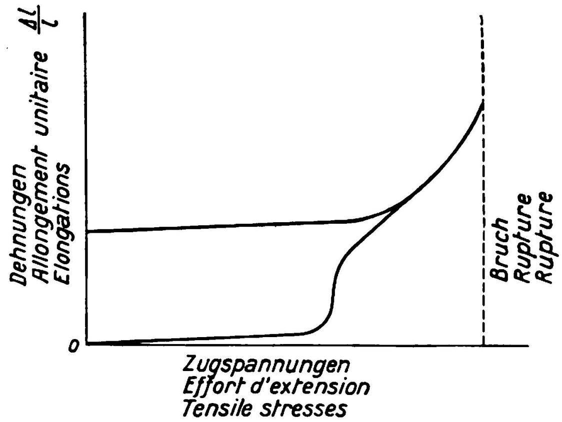

Fig. 1.
Stress-elongation diagram of steel.

If a specimen is submitted to a stress slightly in excess of the elastic stress, the elongation above this stress is greatly augmented by small increases of stress. On decreasing the tension to zero, one finds that the specimen is shortened in accordance with a law of elasticity almost identical to that which represents the initial elastic deformation. The bar thus treated acts as an elastic body between loads of 0 and a new elastic limit which is higher than the elastic limit of the bar not so treated. This principle is frequently used for augmenting the elastic limit of mild steel, and enables it, permitted by Regulations, to withstand higher stresses.

If, by a suitable arrangement a specimen is submitted not only to tension but to alternate stresses of tension and compression, (taking care to avoid premature deformations due to buckling), it is found that the specimen, even after considerable initial deformation, can be submitted to a large number of alternating stresses, provided the maximum and minimum values of such stresses are within pre-determined limits.

Endurance curves can thus be determined which exactly define the limits between which a definite number of alternations can be applied, without causing fracture. If limiting stresses, for example, slightly higher than the elastic limit in tension and in compression are adopted, the number of alternations should be reduced.

If, on the contrary, the limiting values are placed at one third of the actual values, the number of possible alternations are practically infinite.

These limits of endurance are of great importance in machine construction. They indicate, in particular, that if the alternating stresses approach locally to the values of the elastic limit, the fracture of a piece which from preliminary calculations appears remote, becomes dependent upon the number of alternations.

There is the well known case of machine shafting, provided with sudden changes of sectional area, in the neighbourhood of which the elastic stresses can be very high, and the shaft breaks without apparent cause.

Similarly, riveted joints for which the actual stress approximates to the elastic limit at the edge of the rivet hole, are not suited for alternating loads; thus it is that the riveted framework of locomotives, wagons and motors, needs to be very much heavier than welded or pressed frames.

2) *Concrete.*

The stress-strain curve of concrete has been much less studied than that of the metals. Its shape presents a practically straight line from the origin, passing into a curve when a load in the region of half the breaking load is passed.

When a specimen is submitted to stresses well beyond the limit of proportionality, without special precautions, cracks are formed which increase with the load up to the point of complete destruction.

If, on the contrary, care is taken to give lateral support to the concrete, either by bands, hydraulic pressure, or by masses of concrete surrounding the compressed member, it is found that, as with metal, a new state of the material sets in after plastic deformation, which resists variations of stress, within defined limits, as an elastic body (Fig. 3).

The case of concrete is complicated by the fact of its special properties, shrinkage and slow non-reversible deformations under permanent loads.

It has been shown by *M. Freyssinet* that the shortening of a specimen of concrete subjected to a load for a long period, was twice the amount of that obtained when a similar specimen was submitted to a rapidly applied load. All this happens in concrete as if the modulus of elasticity was reduced by about 50 % when the concrete was loaded for a long period. This phenomena plays an important part in equalising the stresses in concrete members under compression.

For example, the flange of a T girder cast some time after the web of the girder, which was already under load, would be able, after a certain time to take a share of the compressive stress which would have caused a gradual shortening of the compressed area of the web.

The distribution of stress in a post under load is modified in the course of time. The stresses in the bars exceed their initial value, when the concrete under the load had not taken up its total shrinkage.

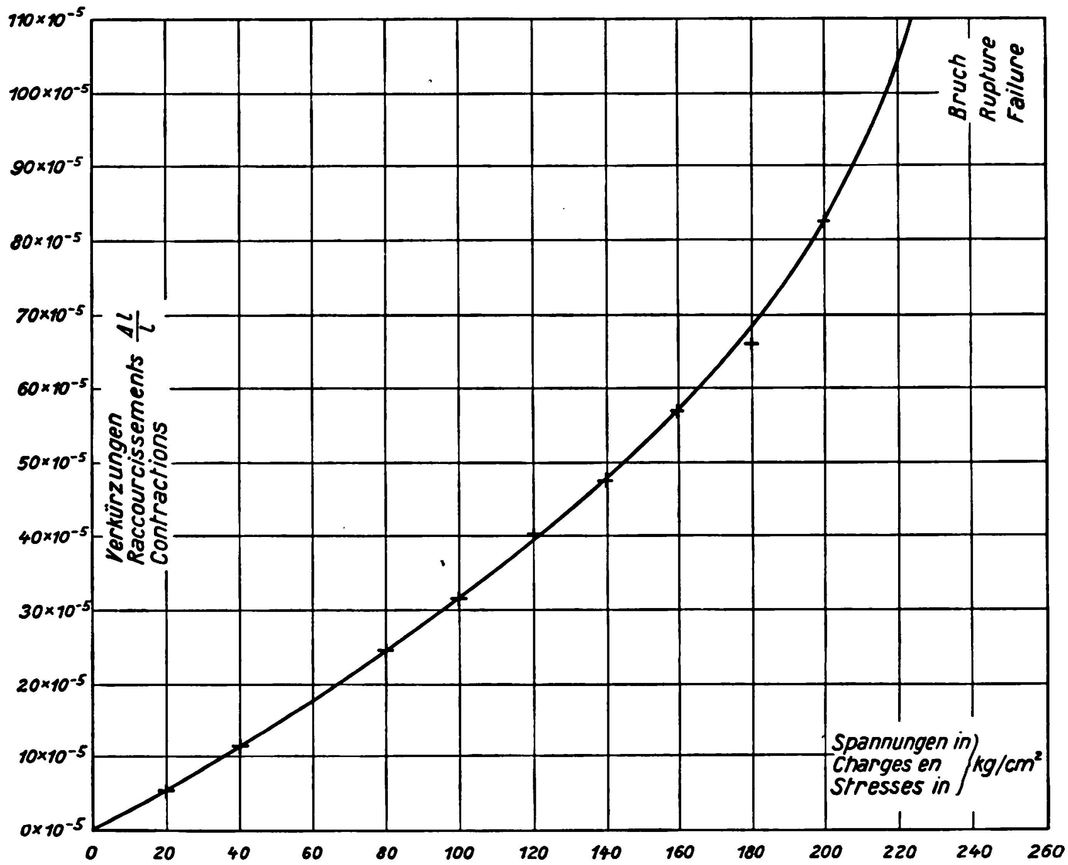

Fig. 2.

Diagram of deformations for cement concrete.

Summarising, a concrete member stressed to about 60 % of its breaking load, will be deformed six to eight times more than the same member under a normal stress of short duration, consistently with the shape of the stress-strain curve and the increase of shrinkage under permanent load.

3) *Bond between concrete and metal.*

In a reinforced concrete structure the bond between the steel bars and the concrete is assured more by the grip of the concrete than by the property of surface adhesion. This is proved by the force necessary to slide a metal bar in its sheath. If it was simply surface adhesion, the force would diminish to a very small value after the initial displacement was effected (Fig. 4).

This action of sliding, transmits the stress from one bar to another through the concrete. It is, in fact, impossible that a bar transmits its load to a neighbouring bar, without there being a relative slip between the two bars and the concrete, which perhaps is of the order of one-two hundredth of the diameter of the bar.

In order that this adhesion to the sheath may not be broken it is very necessary to avoid too great a variation of stress between the concrete and

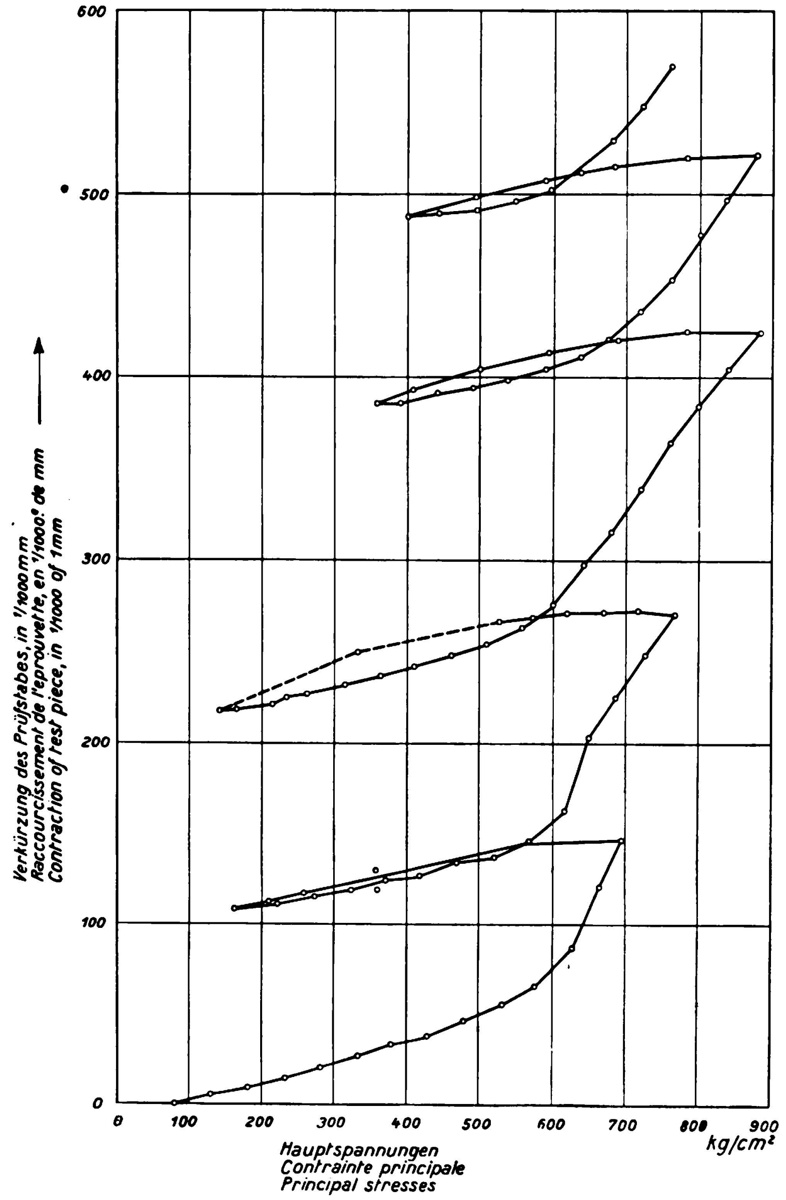

Fig. 3.
Deformation of a test-piece of concrete held by lateral hydraulic pressure.

the bar. There is reason to believe that successive relative displacements of two members may lead eventually to a disintegration of the concrete bed and a consequent slackening of the adhesion. This simple remark enables one to appreciate the utility of hooks for tension bars which, perhaps without providing any increased resistance when the structure is new, become of service if the adhesion is weakened by abnormal or too frequent deformations.

4) *Deformations during bending.*

The non-elastic deformations of a girder during bending may be caused by the non-elastic deformations of either the steel or concrete, or a combination of these two properties.

If the compressed concrete is largely in excess, the deformation of the metal comes solely into play. Experience shows that large deformations are initiated, when the elastic limit of the metal is reached, and can reach a considerable magnitude in a well-designed girder without setting up a significant variation of the stresses that can be sustained.

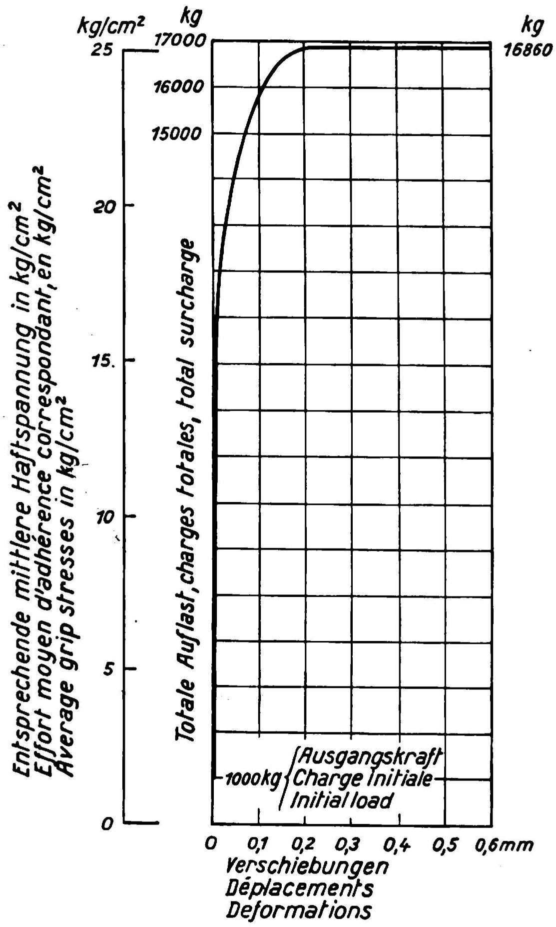

Fig. 4.

In the usual methods of calculation, the bending moment necessary to produce these large deformations is essentially double that which served for the calculation of the girder.

If the load decreases, the girder loses part of its deformation and is again able to act as an elastic body. Its elastic properties are, however, modified and one can take into account the greater ductility of the deformed zones, by making use only of the reduced section, where the stretched concrete is ignored, and in taking a consistent value for the ratio modulus $n = \frac{E_s}{E_c}$.

These results are confirmed by the experiments made by *M. Dumas,* Engineer of Bridges and Roads, on the deformation of girders under repetition loading.

These large deformations do not take place without starting the formation of cracks in the stretched portion of the concrete. Practically, these cracks limit the magnitude of the possible deformation. The admissible size and number of these cracks are extremely variable in conformity with the uses of the structure.

These cracks are often met with in the vicinity of the slab supports where concrete under compression is in excess; if it is a question of floor slabs covered with a timber floor, these cracks are of very little importance, if on the contrary, the slab is covered with an adhesive covering, the appearance of these cracks on the floor is of worse effect. One should have greater latitude with a protected work, than with one subjected to atmospheric conditions; external corrosive agents find an easy means of access through these cracks. Whatever the cause, it appears difficult, without precautions, to lessen the

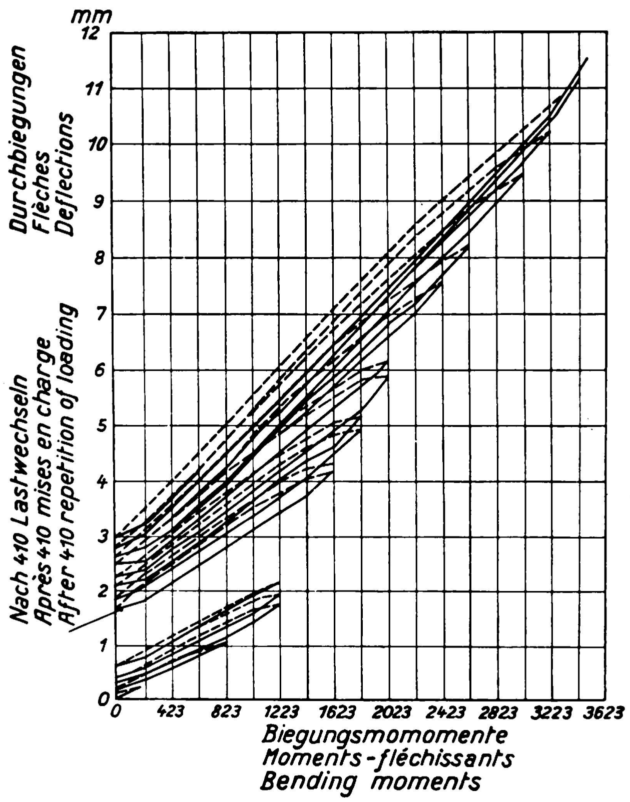

Fig. 5.
Deformation of a beam in banding.

injurious effects of cracks, and the elongation concentrated to a limited zone equal to two or three times that of the normal elastic stress.

In the case of the plastic deformation of concrete, the phenomenon is essentially more complex. The line of deformation of the concrete being a curve, the diagram of distribution of the stresses is deformed and the lever arm of the resistant couple decreased.

However, the phenomenon maintains in its entirety, the same properties as simple compression. The large deformations due to the concrete, contrary to those due to the metal, have only a small tendency to open cracks. But it is necessary to make sure that the concrete is in such a condition that it can withstand, without rupture, the plastic deformation. A limit is still necessary. If the concrete was not specially prepared, it should not exceed a stress corresponding to one half the ultimate breaking load (which in course of time leads to deformation four or five times greater than under the normal pressure). The deformations can be greater, if a suitable hooping ensures the bearing of the core to the high pressures.

When the two phenomena are superimposed, which is the usual case for rectangular beams, we have the best conditions for obtaining maximum deformation with a minimum disturbance.

5) *Plastic deformation due to shear.*

Experiments concerning the resistance to shear of solid beams are scarce. However, the experience made by engineers and some tests which we have been able to study, or to carry out, show that the various systems of reinforcement which can be used (rods parallel to the direction of shear or inclined at an angle ranging from 0 to 45^0) are equal in quality. That proves that the girder adapts itself, but the deformation of cracks at 45^0 is always found, since the tensile resistance of the concrete is exceeded. These cracks are of such frequency that one has become so used to their presence as to consider them perfectly normal. They are, however the apparent sign of this phenomenon of adaptation which one often refuses to allow for bending. There is no reason for making this difference.

Function of Plastic Deformation.

We think to have shown in this way that plastic deformations intervene in all the members of a structure to modify the distribution of the stresses among the several parts, concrete and metal, of a member. They are equivalent, in fact, to the transfer of the action of the external loads from one member to another.

The considerable deformation of overloaded members results in increasing the load on members which are slower to react and are relatively much too rigid.

The plasticity of constituent members therefore, enables a structure to "adapt itself", that is to say, to submit to non-elastic deformations, under the influence of external loads, such that the less resistent members are relieved by those which are more rigid.

This fresh distribution of stresses can only be produced if the two following conditions are fulfilled.

a) The plastic deformations must be produced without compressing the bond of the structure, that is to say, a method of resolving the stresses within the system must be found compatible with the resistance of the members. In particular, if the great deformations are accompanied by cracks, the bond between the two faces of the gap must be assured by steel rods of sufficient sectional area to transmit the stress across the opening.

b) If a member of the structure is deficient in strength, its deformation must result in transmitting a portion of its load to a more resistant member.

In a word, the system must be hyperstatic and stable. Members of too small a resistance will play the part of semi-articulations which will support during their deformation a force which does not exceed that corresponding to the elastic limit of the constituent parts.

The resistance of the new system thus established is different to that of the initial system; it has produced a distribution of forces more conformable to its actual possibilities and in spite of the erroneous conception regarding the theory of elasticity, it functions perfectly well under the influence of the loads. The interest of hyperstatic systems appears here; an error can be compensated by the deformation of some weak member, whilst in a strictly isostatic system, every weakness, even local, leads infallibly to considerable deformation, since nothing is opposed to their development when they are set up by a permanent external cause.

However, the problem becomes more complex when the loads are variable. It is necessary to study what happens when, after fully loading the system, it is unloaded.

The system tends to recover its initial form, so far as the plastic deformations of its members will permit.

This rearrangement will have, in consequence, according to the ratio of live to dead load, a reduction or even a reversal of the stresses in the members which have been subjected to plastic deformation.

Thus, we have seen above that although the elastic deformations can be repeated a large number of times, the plastic deformations can only be applied once, or for a very limited number of times.

To assure the life of the system, it will be necessary that the deformations remain perfectly elastic under every possible variation of the live load, when referred to the total deformation, elastic or not, to which the system was subjected under the most unfavourable disposition of the dead and live load.

The uncertainty of the exact value of all the internal stresses, will make it necessary to limit the elastic deformation, by insisting for example, that the changes of stress due to variable loading in members under plastic deformation, calculated by the usual methods, must not exceed defined values.

Until precise experiments justify figures, or in other words, fix the limits of endurance under alternating stress, it appears that the limits usually adopted — one half of the elastic limit for steel, 28 % of the resistance to compression for concrete — should give ample security. With regard to the adhesion of the bars, one should make sure that good anchorages well be capable of transmitting the stress.

Applications.

Semi-hinges.

A semi-hinge consists in principe of a thin layer of concrete between two groups of reinforcement, arranged as hoops supporting a core of concrete for transmitting the stress to the rest of the structure.

The non-elastic displacements which are produced by shrinkage or application of the load will be absorbed by the core of concrete during its plastic deformation, which may be considerable. It will be sufficient, to ensure durability, that under the application of alternating loads the variations of stress in the core remain within prescribed limits.

Semi-fixed slabs.

It is usual, in construction, to build floors of reinforced concrete of slabs and beams for which the bending moments are not calculated conformably to the theory of elasticity, but more simply; for example, for a member uniformly loaded, the bending moment at the centre of the span is less than that for a girder freely supported, and the moments at the supports are slightly in excess of the value, inherent upon the moment taken at the centre.

Let us see, under these conditions, what will be the actual distribution of the bending moments in a section at the centre and in a section at the supports, while an increasing load is applied to the girder.

At the first application of the load the whole of the girder acts as an elastic body, the distribution of stress conforms to theory; as the strain in the section at the support approaches a plastic strain, the deformations increase under a

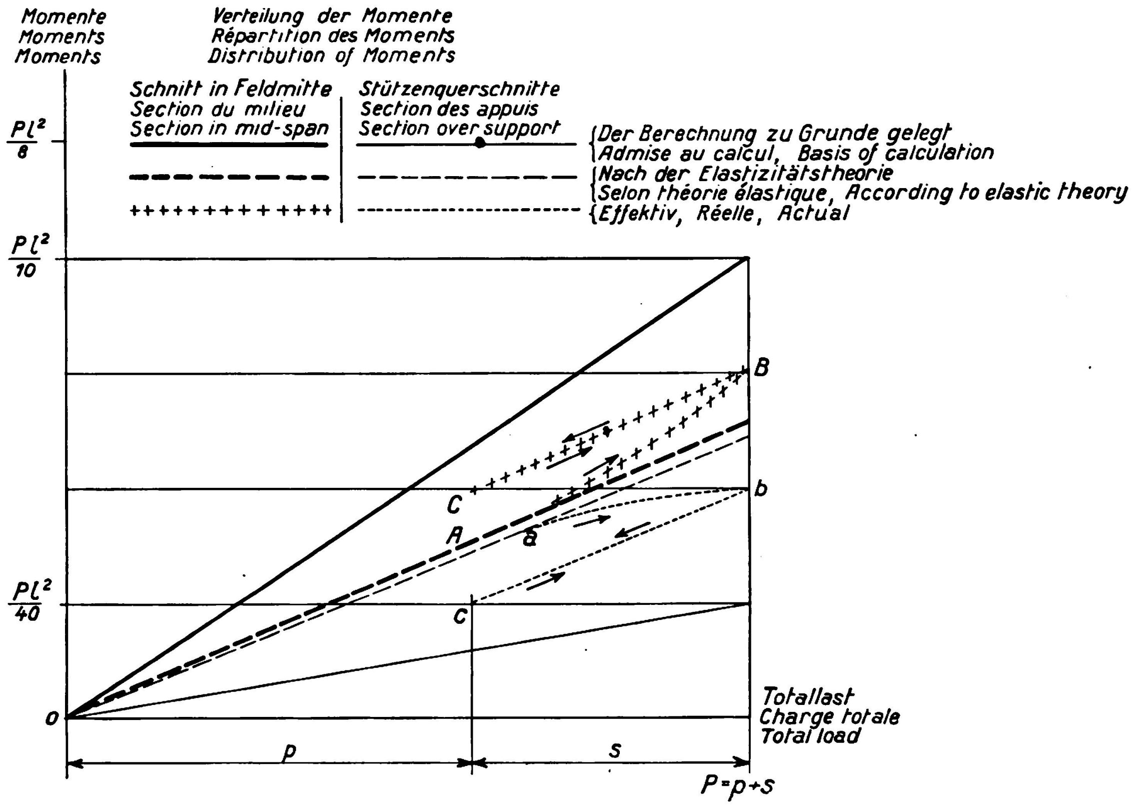

Fig. 6.

Fatigue at the supports and in the mid-span of a semi-fixed beam.

practically constant moment, it produces a rotation about the support, and the stresses in the section at the centre increase more rapidly than at the start (curves AB and ab, Fig. 6).

A given load (p + s) is thus obtained, for which the stress sustained by the section at the supports does not much exceed double that which had been estimated, whilst the stress at the centre is below its estimated value.

If at this moment part of the load is removed, the system acts as an elastic body. The stress diagram for the section at the supports is represented by the line bc, whilst in the central section the stress diminishes according to the line BC.

If the load is reduced to 0, a point is reached at which the stresses in sections at the support are neutralised, then change in sense. But we have seen that, to assure the durability of the structure, it is necessary that under the live load only, the decrease of stress in the section previously submitted to plastic deformations, remains within a certain limit bc.

We are thus able to determine what the maximum ratio should be of live load to dead load so as to assure the durability of the structure.

Let us apply this rationally to an ordinary ribbed slab in which the central section has been calculated with a bending moment equal to $\frac{Pl^2}{10}$

$$P = p + s$$

where p is the dead load,
and s the live load uniformly distributed.

The sections at the supports are supposed calculated with the moment $\frac{Pl^2}{30}$.

Applying the theory of elasticity to a girder of this type and taking note of the different moments of inertia at the centre and over the supports, it is found that the moment over the support is practically equal to the product of the uniform load and $\frac{l^2}{15}$.

If therefore, it is wished that the removal of live load only produces a change of moment in the section over the support, compatible with its resistance, it will be necessary that

$$\frac{sl^2}{15} < (p + s)\frac{l^2}{30}.$$

This inequality shows that s should be less than p.

If we now consider the case of a rectangular beam uniformly loaded, it is not unusual to calculate it with a bending moment equal to $\frac{Pl^2}{15}$ at the centre and over the supports.

Now, the actual moment over supports for such a girder, since its inertia is constant, is equal to $\frac{Pl^2}{12}$.

To assure safety it will therefore be necessary that in the expression

$$\frac{sl^2}{12} < (p + s)\frac{l^2}{15} \quad \text{applies,}$$

out of which follows thats $s < 4$ p.

This simple calculation serves to show why it is possible to construct a floor for carrying small live loads (dwelling house floors for which the live load is less than the weight of the floor itself) using empirical formulae in apparent contradiction to the theory of elasticity, whilst it is impossible to construct from the same formulae, warehouse floor heavily loaded, without discrepancies being soon manifest.

The smaller the ratio of dead load to live load, the greater is the difficulty of maintaining the structure: the case is well known of the girders of swing bridges which are submitted to alternating stresses, and more particularly, railway sleepers of reinforced concrete; in the latter case, the conditions of adhesion present such difficulties, that their use under the running roads is very precarious. At the same time it is not unlikely that the application of special processes, such as putting the steel under such tension as will avoid reversal of stress, will enable the problem to be solved. It has been solved in this manner in the case of posts for electric lines, by M. *Freyssinet,* who has found that a post reinforced with rods severely stretched, is incomparably more resistant and more durable than a similar post reinforced with ordinary rods.

Framework for buildings.

The preceding theories can be applied to the framework for buildings.

It appears to us, moreover, that the almost complete ignorance concerning the distribution of stresses during construction, renders the literal application of formulae derived from the theory of Strength of Materials somewhat illusory. In fact, the construction is generally carried out in such a way that the members are only loaded in succession in an order that is not always foreseen. Further, during the work some accidental loads may intervene to modify to some extent the distribution of stresses.

It appears more suitable to consider for the system, the possibility of non-elastic deformation in the less resistant members under the worst conditions of loading, and to examine what happens when the live load is removed. The deformations produced, remain sufficiently elastic, that it may not be in vain to apply the classic theory of the deformable system, taking account in the calculation of the inertia of the new properties of members plastically deformed. It seems that the current practice which allows the centres of posts to be jointed, may be in a general way, justified by the preceding.

Summary.

This short paper will have sufficed, we hope, to show that in the stability of structures, the action of the dead load and that of variable live loads are essentially different, from the fact that the nature of materials "adapt themselves" under, a constant load, but cannot do so under variable loads.

It is moreover ,this fact, supported by the actual experience of all construction, that has led the authors of the regulations of the „Chambre Syndicale des Constructeurs en Ciment Armé de France" to increase the live loads relatively to the dead load, in a manner which accounted simply and in an effective way for their distinctly unfavourable action.

Large structures have from this point of view a real superiority over the others: the solid slabs and supported floors are marked out for heavy live loads. In large works, bridges or viaducts, it is well to avoid extreme lightness of the decking by the use of cross girders and bearers of small inherent weight; it is, from all points of view, more rational to place the principal girders under the heavy loads.

Summarising, the most durable structure is that in which the variations of stress under the live load have the lowest possible relative values. This result can be obtained either by increasing the dead load or by any means tending to increase the relative value of the stresses due to the permanent load.

IIa 2

The Strength of Concrete and Reinforced Concrete under Sustained and Frequently Repeated Loading.

Festigkeit des Betons und des Eisenbetons bei dauernder und bei oftmals wiederholter Belastung.

La résistance du béton et du béton armé soumis à des efforts permanents et répétés.

O. Graf,
Professor an der Technischen Hochschule Stuttgart.

Investigation of the fatigue strength of concrete is a matter which, for the following reasons, involves lengthy and extensive research: the strength of the concrete depends on its age; it depends also on the treatment which the concrete has undergone (for instance on its water content, its temperature and its previous condition with or without shrinkage). Again, the development of strength of the concrete is influenced by the properties of the cement. The strength of the concrete depends also on the size of the concrete test blocks, and the action of the steel reinforcing bars in carrying the load is governed by the variable resistance of the concrete to deformations. This last factor can vary within very wide limits, being in turn dependent on the duration of loading, on the magnitude of the load, and the degree of moisture of the concrete.

In reference to fatigue effects, account has further to be taken of all those many influences which play a part in ordinary experiments on concrete and reinforced concrete — such as the amount of cement, the water-cement ratio, the grading, and the nature of the aggregate, the manner of preparation, etc. For each of these factors it has to be ascertained whether its effect on the fatigue strength is in any way different from that on the ordinary strength.

Here the expression "fatigue strength" is used in a general sense, but for technical application it becomes necessary to define the exact form in which the term is used — whether in the sense of resistance to tension, compression, alternations between tension and compression, bending, shear, buckling etc. — and also to define the method of loading whether this be stationary, frequently repeated, or in part stationary and in part frequently repeated. All these factors require to be ascertained and to be taken into account.

In the following pages a brief summary will be given of what is already known regarding the fatigue strength of concrete and reinforced concrete, and it shows — as the author has frequently mentioned elsewhere — that many questions still remain open for systematic research.

1) *Fatigue compression strength of concrete.*

The figures quoted below refer to concrete which was more than six months old at the time of testing and which after being at first treated wet had been put in storage.

a) *Fatigue compression strength under stationary loading (stationary fatigue strength).*

No results of experiments on the strength of concrete under long sustained loading are yet available, but a few observations have been made which may serve as a basis for further experiments. These are the results of the experiments described under b) and c) below, from which it may be anticipated that the stationary fatigue strength of the concrete will work out at something over 4/5$^{\text{ths}}$ of the strength ascertained in ordinary compression tests[1].

b) *Fatigue compressive strength under frequently repeated loading (surge load strength).*

In completion of the exploratory experiments by *Joly, Hatt, Ornum*[2] and from *Mehmel*[2], extensive researches have lately been carried out by *Graf* and *Brenner*[3] for the German Committee on Reinforced Concrete.

According to these experiments, the fatigue compressive strength of concrete columns of varying compositions — containing, in particular, varying amounts of cement and varying grading of aggregates — was found to be about 0.6 times the prism strength as determined by the ordinary breaking test. The composition of the concrete was of smaller significance, but as the strength increased the ratio generally decreased a little.

The number of loading repetition for such tests was of about 260 per minute, the total number of loading repetitions applied for obtaining the value of fatigue strength was two millions.

The number of repetitions required to cause fracture increased with increasing frequency of loading repetitions per minute (the frequencies used for the tests varied from 10 to 450 per minute). The fatigue strength was a little higher in value for increased frequencies.

c) *Fatigue compressive strength under the simultaneous effect of stationary loads and frequently repeated loads.*

When stationary loads were added to the frequently repeated loads, the amplitude of stresses due to moving loads (imposed two million times) decreased as the stationary loads increased. This is exemplified in Fig. 1 which shows the

[1] If it is a question of increasing the permissible compressive stress of concrete in this proportion it must be remembered that the magnitude of the deformation of the concrete under long duration of loading come into account also (compare, for instance, *Graf:* "Beton und Eisen" 1934, pages 167 and following. Also *Hummel* in "Zement", 1935, pages 799 and following.

[2] Compare also *Graf:* Die Dauerfestigkeit der Werkstoffe und der Konstruktionselemente. (Julius Springer, Berlin), pages 116 foll.; also *Hatt* and *Mills,* Bulletin 34 of the Purdue University, 1928.

[3] Compare also Publication N° 76 of the German Committée for Reinforced Concrete. A later report will appear in 1936.

following values for the amplitude S in the case of concrete for which the prism strength was 180 kg per sq. cm:

for stationary load	$\sigma_u = 6$ kg per sq. cm;	$S = 109$ kg per sq. cm
,, ,, ,,	$\sigma_u = 118$ kg ,,	$S = 39$ kg ,,
,, ,, ,,	$\sigma_u = 157$ kg ,,	$S = 8$ kg ,,

Here each experiment lasted for at least five days. The figures just quoted also show that with a duration of about five days the resistance to stationary loads approximates to the prism strength as determined by the ordinary breaking test (165 kg per sq. cm being the total load in the fatigue experiment as against 180 kg per sq. cm in the breaking test).

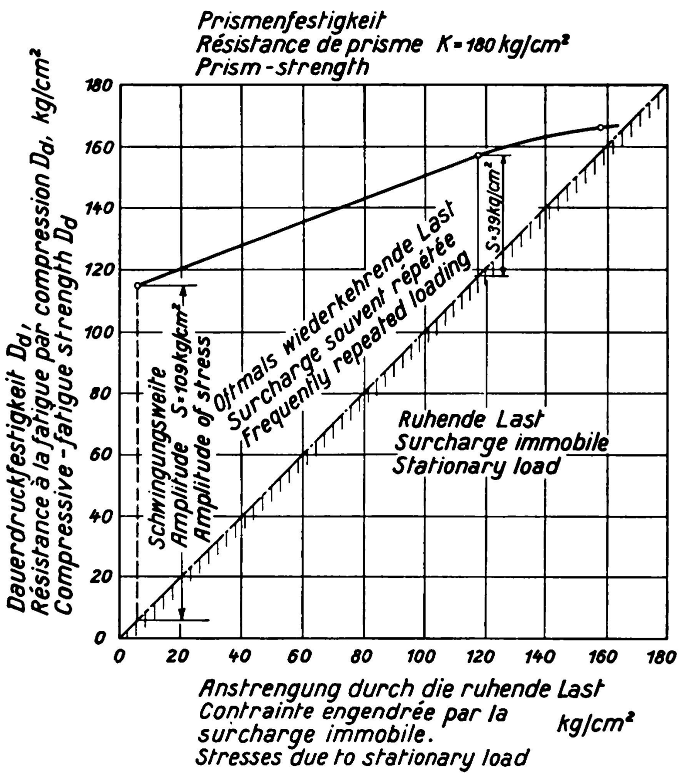

Fig. 1.

Compression fatigue tests with prisms of plain concrete.

d) *General remarks on fatigue strength.*

According to the regulations for concrete and reinforced concrete, the permissible compressive stress in concentrically loaded columns is at most one third of the strength of a concrete test cube 28 days old. If, then, the prism strength is assumed to be at least two-thirds the strength of a cube, the permissible stress in the concrete amounts to one half the prism strength.

This amount of stress is not much less than the resistance of prisms to frequently repeated compressive loading if the increase in strength with age is left out of account. If an appreciable increase in strength with age is assumed to take place, then the assumptions currently made in Germany as to the usual stressing of concrete in columns concentrically loaded are suitable even from the point of view of frequently repeated loadings.

At some future time it may be determined under what conditions an increase in the permissible compressive stress in the concrete may be allowed in cases where stationary loads predominate or are alone decisive.

2) *Fatigue tensile strength of concrete.*

Experiments relating to this matter have been carried out in Karlsruhe, and these showed similar ratios in the case of tensile loading as those given under 1a) and 1b) for compressive loading[4]. These results have not yet been published.

3) *Fatigue bending strength of concrete.*

Clemmer[5] and later *Olden*[5] have investigated the fatigue bending strength of road concrete. The frequency of the load was 40 per minute. The results

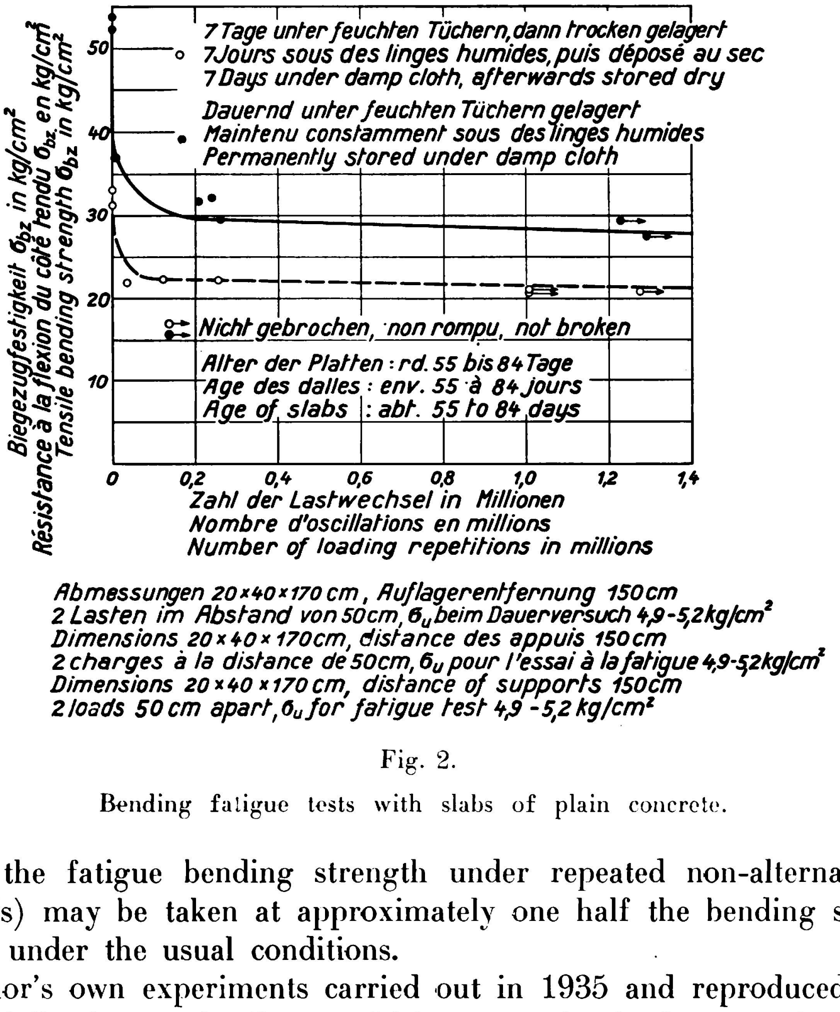

Fig. 2.

Bending fatigue tests with slabs of plain concrete.

show that the fatigue bending strength under repeated non-alternating loads (surge loads) may be taken at approximately one half the bending strength as determined under the usual conditions.

The author's own experiments carried out in 1935 and reproduced in Fig. 2 yielded the following results. Beams which were maintained constantly wet showed fatigue bending strength under repeated non-alterning loads (surge loads) up to 28 kg per sq. cm, the ordinary bending strength being 53 kg per sq. cm, so that the ratio was 0.53 to 1. Beams which after being originally treated wet were subsequently stored dry showed a fatigue bending strength under repeated

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[4] As reported by Prof. Dr. Ing. *Kammüller.*

[5] In the report by *Graf:* Die Dauerfestigkeit der Werkstoffe und der Konstruktionselemente, page 117.

non-alternating loads (surge loads) up to 21 kg per sq. cm only, the ordinary bending strength being 32 kg per sq. cm, so that the ratio was 0.66 to 1.

Further experiments are in hand.

4) *Fatigue compressive strength of reinforced concrete columns.*

The following points call for notice in investigating the fatigue strength of reinforced concrete columns.

a) The elasticity of the steel, and therefore the resistance of the reinforcement to buckling, is not influenced, or, only to an inappreciable extent, by long duration of loading, or by frequently repeated loading.
b) Under loading of long duration the crushing limit is reduced[6].
c) The deformations of concrete are largely dependent on the duration and the magnitude of the load. Hence the participation of the concrete in transmitting the forces that occur in reinforced concrete columns changes according to the duration and magnitude of the loading, and apart from this is dependent on the composition and humidity of the concrete.

No test results have come to the knowledge of the author regarding any experiments about the carrying capacity of reinforced concrete columns under long duration of loading, under frequently repeated loading, or under the simultaneous influences of stationary and frequently repeated loading.

5) *Fatigue bending strength of reinforced concrete slabs.*

The carrying capacity of reinforced concrete slabs constructed in the usual way is determined by the resistance of the steel in the tensile zone.

Under loads which increase gradually and slowly, the yield point of the steel is exceeded in the tensile zone, and the deformations which then occur in the slab under ordinary conditions are so great as to make the latter appear practically unusable. The carrying capacity of slabs under a stationary load is therefore directly dependent on the yield point of the tensile reinforcement. The yield point is found to be somewhat smaller under long duration loading than it appears in the ordinary tensile test (see under 4).

The resistance of the tensile reinforcement when subjected to frequently repeated (non-alternating) loading may be as high as the elastic limit of ordinary round bars, provided the surface remains in good condition[7]. In steels having a high yield point the fatigue tensile strength is lower than the yield point, and in such steels the dependence of the fatigue strength on the surface conditions of the rods is much more marked than is the case in ordinary commercial rods. In simple slabs, for instance, the tensile reinforcement broke down when:

σ_s max. was in excess of .	2900	3100	3300	2830	kg per sq. cm
(amplitude of stress . .	2570	2640	2830	2565	kg ,,)
for an yield point of . .	2970	4280	4500	6150	kg ,,
using a grade of steel . .	St 37	St 60	"Isteg"	common Steel[7]	

[6] See: among others *Siebel* and *Pomp:* Mitteilungen aus dem Kaiser-Wilhelm-Institut für Eisenforschung, Band X, Publication N° 100.

[7] See, for instance, in "Beton und Eisen", 1934, page 169.

According to this and other experiments carried out at Stuttgart it is advisable to assume, for the present, that the resistance of the tensile reinforcement when subject to frequently repeated loading is limited by an amplitude of 2600 kg per sq. cm, presuming in this connection that the surface conditions are sound, is adequate and will remain undisturbed.

It follows, as in the case of steel construction, that those steels which possess a high yield point under stationary loading may be subjected to higher permissible stresses under stationary loads than under moving loads: such steels should, therefore, be used for those structural members which are mainly subject to stationary loads.

The permissible loading on slabs should be governed not only by the carrying capacity, but also by the consideration that cracks in the concrete in the tensile zone may reduce the protection afforded to the reinforcement if the width of these cracks exceeds a limit depending on the prevailing conditions — as for instance, in the case of structures in the open[8]. What width of cracks may be tolerated as a means of reconciling experimental results with experience on old structures has not yet been determined.

6) *Fatigue bending strength of reinforced concrete beams.*

It has been assumed, in the explanation of the conditions governing the fatigue strength of slabs in Section 5, provided that ordinarily the reinforcement possesses adequate anchorage, and that this being so the properties of the concrete do not come into account in other words that the current regulations are fulfilled and that this prescribed minimum strength of the concrete is overstepped. In the case of beams, however, this assumption is in general not valid, because thicker steel bars have to be used in beams than in slabs, with the consequence that the pressure on the concrete from the hooks and so forth is greater. Moreover, the bent bars have to resist appreciably higher radial compressions at the places of bending, which, where the bars are of large diameter and the concrete is of moderate strength, may lead to the concrete being damaged before the tensile forces in the steel bars cause these to reach the yield point[9].

The strength of the concrete should, therefore, be governed by the dimensions of the reinforcement, or at any rate the required strength of concrete should be in relation to the reinforcement, dependent on limits which however have yet to be determined. These conditions become more obvious for frequently repeated loadings than for stationary loads[10].

Most of the fatigue bending experiments on reinforced concrete beams which have hitherto been published have investigated the effect of frequently repeated loading below the value of fatigue strength in relation to the ultimate load, determined in the usual way after completion of fatigue tests.

It was to be expected, from numerous results of other kinds of fatigue tests, that frequently repeated loads, considerably in excess of the permissible load but below the fatigue strength, should have little or no effect on the ordinary

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[8] See "Beton und Eisen", 1935, page 148.

[9] See "Beton und Eisen", 1935, page 147.

[10] The German regulations for reinforced concrete, in their present form, tend to operate in this direction.

breaking load[11]. Hence the fatigue bending strength of reinforced concrete beams can be found only by determining those frequently repeated loadings which do not cause breakage but if increased slightly will cause failure.

It has been suggested, to make the permissible *load* dependent on a maximum permissible width of cracks. If this principle is to be applied to the case

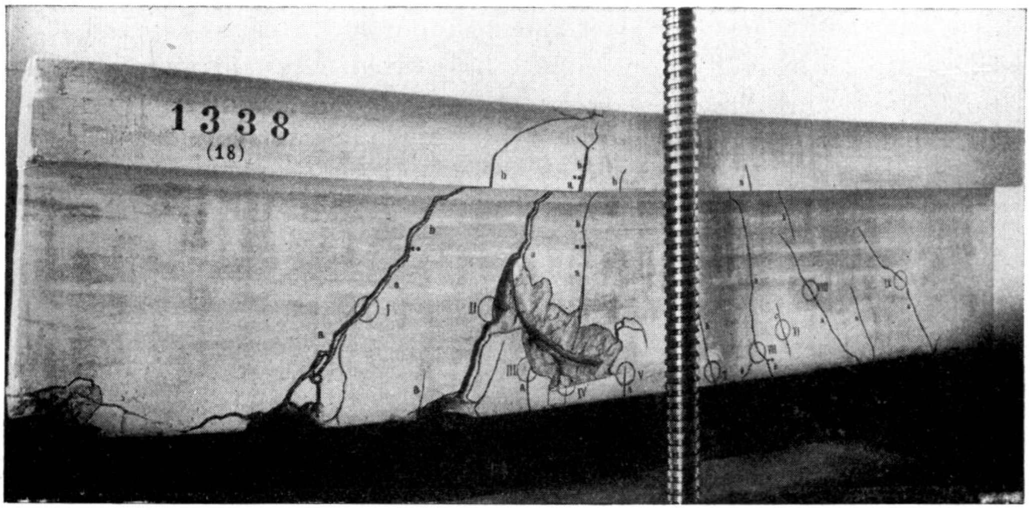

Fig. 3.
Reinforced concrete beam after frequently repeated bending.

of beams it must be specially noted that the width of the cracks is maximum in those regions where the sectional area of the reinforcement undergoes a change, particularly at places of bent up bars, but otherwise under normal conditions, the permissible loads would be dependent on the distance between the cracks. In this connection attention should also be paid to what was said at the end of Section 5.

Summary.

Investigations as hitherto carried out show that the strength of concrete subjected to frequently repeated loading (surge loads) such as compression, tension and bending, is at least half the strength as obtained in ordinary (static) rupture tests. If stationary loads are acting in conjunction with repeated loadings then the limits for live loads which the concrete is capable of withstanding, are reduced. The resistance to stationary loads is to be estimated at $^4/_5$ of the strength produced by ordinary rupture tests.

As regards the fatigue strength of reinforced concrete test results are only available for slabs and beams. Tests have shown that the reinforcing steel in con-

[11] See, for instance, Handbuch für Eisenbetonbau, Vol. 1, 4[th] edition, pages 46 and following, and the references given therein.

crete behaves similarly to steel subjected to fatigue tests. It is advisable to employ steels of high yield limit for the reinforcement of structures chiefly subjected to stationary loads only. The resisting power of concrete of beams which have to undergo frequently repeated loading is mostly exceeded at the places where the bars are bent up or at the location of hooks. This in particular if the arrangement of the reinforcement follows prevailing practice.

In view of these facts the German Commission for Reinforced Concrete (Deutscher Ausschuß für Eisenbeton) has decided to arrange further investigations concerning the resistance against slipping and the anchorage of bars, for reinforced concrete subjected to frequently repeated loading.