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STABILITY OF PLATE GIRDERS SUBJECTED TO BENDING

LA STABILITÉ DE L'AME DES POUTRES SOLLICITÉES A LA FLEXION

DIE STABILITÄT DER STEGBLECHE VON BIEGUNGSTRÄGERN

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Introduction.

In the design of a plate girder not only the stresses but also the elastic stability of the structure should be considered. It is well known [1] that an I-beam bent in the plane of the web may prove to be insufficiently stable and buckle sidewise. The critical value of the load at which such a buckling may occur depends not only on the lateral rigidity of flanges but also on the torsional rigidity of the beam and on the ratio of the span length to the depth of the beam. To eliminate this kind of instability an adequate system of lateral bracing is necessary [2].

Another problem of elastic stability which arises in the design of plate girders is the determination of the thickness of the web and of the spacing of the web stiffeners. It will often be found that the web thickness must be increased beyond that calculated for direct shear, on account of the tendency to buckling. Instead of increasing the thickness of the web, adequate placing of web stiffeners can be used to the same purpose of insuring the stability of the web.

The problems of sidewise buckling of an I-beam and of buckling of the web of a plate girder are discussed in this article.

I. Lateral Stability of a Girder.

1. Notations.

In this discussion the following notations are used :

$2l$ is the span of the plate girder.

h is the depth of the plate girder.

$B_1 = EJ_1$ is the flexural rigidity of the girder in the plane of the web.

$B_2 = EJ_2$ is the flexural rigidity of the girder in the direction perpendicular to the web.

1. L. Prandtl. Kipperscheinungen, Dissertation, Nürnberg, 1899.

2. An example of the failure of girders in consequence of sidewise buckling is given by the bridge disaster near Tarbes, La Revue Technique, November 15, 1897.

Q_{cr} is the total load at which the girder becomes unstable and sidewise buckling begins.
σ_{cr} is the maximum bending stress corresponding to the load Q_{cr}.
C is the torsional rigidity of the girder.

$$\alpha = \frac{C}{B_2}\left(\frac{2l}{h}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\beta = \frac{B_2}{B_1}\left(\frac{h}{2l}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (2)$$

2. Critical Load.

The critical load in all cases can be represented by the equation [1]:

$$Q_{cr} = K\frac{\sqrt{B_2 C}}{(2l)^2} \quad \ldots\ldots\ldots\ldots \quad (3)$$

in wich K is a numerical factor depending on : (*a*) the magnitude of the quantity α given above by eq. (1) (*b*) the kind of loading and (*c*) the manner of fastening at the ends of the beam. It is seen that Q_{cr} can be easily calculated if the factor K is known. It should be noted that a change in the magnitude of C has a comparatively small influence on the critical load, hence in calculation of C we can use approximate formulae. In the case of an I cross section shown in figure 1 we can assume that the torsional rigidity of the beam is approximately equal to the sum of torsional rigidities of the three rectangular bars into which the beam can be subdivided, and take :

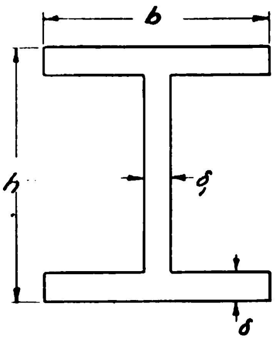

Fig. 1.

$$C = G\left(\frac{2}{3}\, b\, \delta^3 + \frac{1}{3}\, h\, \delta_1^{\,3}\right) \quad \ldots \quad (4)$$

in wich G denotes the modulus of rigidity. In a more general case C can be calculated by the use of the approximate equation of SAINT-VENANT :

$$C = \frac{1}{40}\,\frac{A^4}{J_p}\,G \quad \ldots\ldots\ldots\ldots \quad (5)$$

in which A denotes the cross sectional area and $J_p = \frac{B_1 + B_2}{E}$ is the polar moment of inertia of the cross section.

3. A Uniformly Loaded Beam Simply Supported at the Ends.

In this case it is assumed that the ends can freely rotate with respect to the axis of symmetry of the end cross sections, but are restrained from rotating about the axis coinciding with the axis of the beam. The buckled form of the beam is shown in figure 2. It will be noted that, owing to the type of end fastening, sidewise buckling of the beam is accompanied by twist. This explains why the stability of the beam depends not only on the lateral flexural rigidity B_2, but also on the torsional rigidity C, as shown by equation (3).

The values of the coefficient K in equation (3) calculated for this case (1) are given in table 1 as a function of the quantity α, defined by eq. 1. If the dimen-

1. S. TIMOSHENKO. Zeitschr. F. Math. u. Phys. Vol. 58, 1910.
S. TIMOSHENKO. Annales des Ponts et Chaussées, 1913-IV.

sions of the plate girder are known, the values of C and B_2 can be easily calculated. Then α will be found from eq. (1) and the corresponding value of K obtained from the table. Substituting this value in eq. (3), the critical load Q_{cr} is obtained.

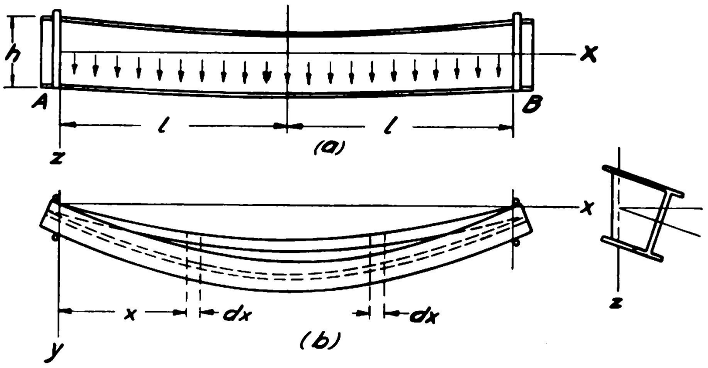

Fig. 2.

Table 1.

Factor K and critical stresses in lb. per sq. in.
for a uniformly loaded beam with simply supported ends.

$\alpha =$	1	1	2	4	6	8	12
K =	143	53.0	42.6	36.3	33.8	32.6	31.5
σ_{cr} =	8520	9950	11300	13600	15600	17300	20300
σ'_{cr} =	5510	6810	8070	10300	12200	13800	16800
σ''_{cr} =	13200	14500	15800	18000	20000	21500	24500
$\alpha =$	16	20	32	50	70	90	100
K =	30.5	30.1	29.4	29.0	28.8	28.6	28.6
σ_{cr} =	23000	25200	31200	38600	45300	51000	53700
σ'_{cr} =	19400	21600	27600	35000	41600	47400	50000
σ''_{cr} =	27200	29400	35300	42600	49200	55100	57600

The corresponding value of the critical stress is

$$\sigma_{cr} = \frac{Q_{cr}\, E h l}{8 B_1}$$

or by using equations (1) and (2).

$$\sigma_{cr} = \frac{KE}{16}\, \beta \sqrt{\alpha} \quad \ldots\ldots\ldots\ldots \quad (6)$$

From this equation the values of σ_{cr} can be easily calculated provided K is known. The third line of the table 1 gives the values of σ_{cr} calculated from equation (6) on the assumption that $\beta = 10^{-4}$ and $E = 30.10^6$ lbs. per sq. in. The critical stresses for a beam with any other value of β and with the modulus E, are obtained by multiplying the corresponding number in table 1 by $10^4 \beta \frac{E_1}{E}$.

It should be noted that the lateral stability of a plate girder varies with the position of the load. The values of K given in the table 1 are calculated on the assumption that the load is distributed along the axis of the beam. When a uniform load is distributed along the upper flange of the beam, the stability decreases and in accordance with this the values of the coefficient K become smaller. The corresponding values of critical stresses, denoted by σ'_{cr}, are given in the fourth line of the table 1. Line 5 of the table 1 furnishes the values of critical stresses σ''_{cr}, when the uniform load is distributed along the lower flange of the beam.

4. Numerical Examples.

Let us consider as a first example a rolled I beam of the following dimensions : Span $2l = 20$ ft., depth $h = 24$ in., flange width $b = 7$ in., thickness of web $\delta_1 = .5$ in., mean thickness of flanges $\delta = \frac{1}{2}(60 + 1.14) = .87$ in., area of cross section $A = 23.3$ sq. in., flexural rigidity $B_1 = 2087E$ lb. sq. in. flexural rigidity $B_2 = 42.7$ E. The torsional rigidity, from eq. 5, is :

$$C = \frac{23{,}3^4\, G}{40\,(2087 + 42{,}7)} = 3{,}46\ G.$$

Then from eqs. (1) and (2) we find :

$$\alpha = 3.24\,;\ \beta = 205.10^{-6}$$

Table 1 gives, by interpolation, for $\alpha = 3.24$,

$$\sigma_{cr} = 11300\ \frac{1}{2}\,(13600\text{-}11300)\ 1.24 = 12700 \text{ lb. per sq. in.}$$

This is the critical stress for $\beta = 10^{-4}$, in our case the critical stress is :

$$\sigma_{cr} = 12700\ \beta\ 10^4 = 26000 \text{ lb. per sq. in.}$$

If, instead of eq. (5), eq. (4) is used in calculating torsional rigidity we find $C = 4.07$ G. Then $\alpha = 3.80$ and the table 1 gives :

$$\sigma_{cr} = 11300 + \frac{1}{2}\,(13600\text{-}11300)\ 1.80 = 13400 \text{ lb. per sq. in.}$$

The critical stress for our case then is :

$$\sigma_{cr} = 13400.\ \beta.\ 10^4 = 27400 \text{ lb. per sq. in.}$$

which is about $5\frac{1}{2}$ per cent greater than the stress obtained previously.

The usual approximate method of calculating critical stresses for laterally unsupported beams consists in considering the upper compressed flange of the beam as a column. On account of the fact that the compressive force in the flange is proportional to the bending moment and follows the parabolic law, the reduced length $L = 0.694.20.12 = 167$ in., instead of the actual length, must be substituted in the column formula. The radius of giration of the cross section of the flange is $r = \frac{7}{2\sqrt{3}} = 2.02$ in. Therefore, $\frac{L}{r} = 83$. For such proportions the Euler's formula gives the critical stress beyond the yield point of ordinary structural steel. Taking the usual straight-line formula we find :

$$\sigma_{cr} = 52500\text{-}220 \frac{L}{r} = 34300 \text{ lb. per sq. in.}$$

which is a stress 32 per cent greater than that obtained above by the use of the equation (3).

As a second example let us consider a plate girder consisting of a web plate, 26 by 3/8 in., and for flange angles, 5 by $3\frac{1}{2}$ by $\frac{1}{2}$ in. The dimensions of the beam are :

$$2l = 480 \text{ in.}, \; h = 26\frac{1}{2} \text{ in.}, \; A = 25{,}75 \text{ sq. in.}$$

$$B_1 = 3000 \text{ E lb. sq. in. } B_2 = 95 \text{ E lb. sq. in.}$$

The torsional rigidity, from eq. (4), is :

$$C = 3.55 \text{ G}.$$

Substituting in to eqs. (1) and (2) we find :

$$\alpha = 4.91 ; \beta = 0.967.\ 10^{-4}.$$

From table 1. by interpolation :

$$\sigma_{cr} = 13600 + \frac{1}{2}(15600\text{-}13600)\ 0.91 = 14500 \text{ lb. per sq. in.}$$

In the case considered :

$$\sigma_{cr} = 14500.\ \beta.\ 10^4 = 14000 \text{ lb. per sq. in.}$$

In order to show the variation of critical stresses with changes in ratios $\frac{2l}{h}$ and $\frac{h}{b}$ the calculations were made for I beams (fig. 1) with $\frac{\delta}{\delta_1} = 2$ and $\frac{b}{\delta} = 10$. The values of critical stresses as functions of the ratio $\frac{2l}{h}$ for three different

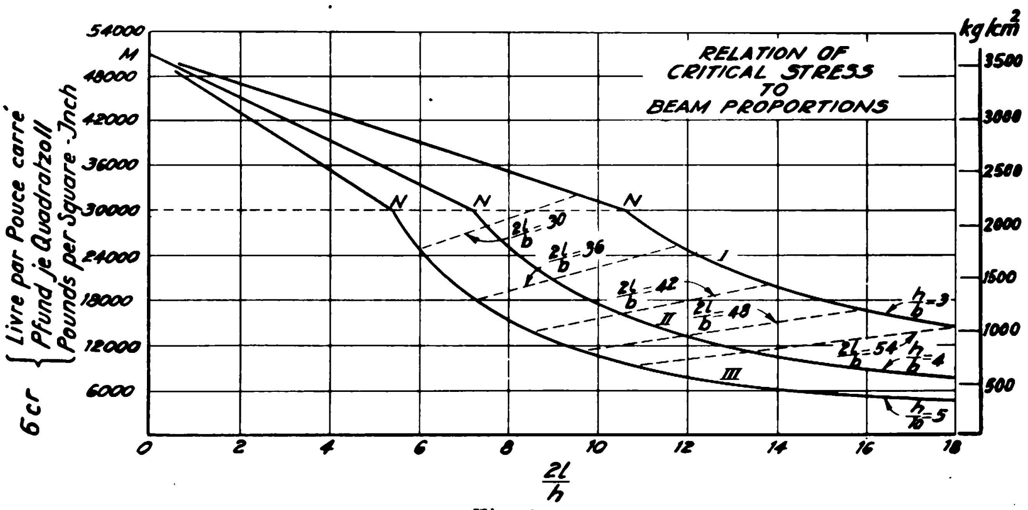

Fig. 3.

Relation of Critical Stress to Beam proportions.

Relation entre l'effort critique et les dimensions de la poutre.

Beziehung zwischen der kritischen Beanspruchung und den Trägerabmessungen.

values of the ratio $\frac{h}{b}$, namely $\frac{h}{b} = 3$; $\frac{h}{b} = 4$ and $\frac{h}{b} = 5$, are shown in fig. 3 by the curves I, II and III. The points corresponding to $\frac{2l}{b}$ = constant, are connected by dotted lines. It will be noted that, for a constant magnitude of the ratio $\frac{2l}{b}$, the critical stress decreases with an increase in the depth of the beam. This fact is not taken into account in usual approximate method, mentioned above, in which the compressed flange is considered as a column. In the table below critical stresses as obtained by the approximate method and from eq. 3 are given for comparison for the case when $\frac{h}{b} = 3$.

$\frac{2l}{b} =$	54	48	42	36
σ_{cr} approximat. =	17800	22700	30300	33500
σ_{cr} exact. =	14400	16800	20000	25100

With an increase in the ratio $\frac{h}{b}$ the discrepancy between the exact and approximate values increases.

5. Stresses beyond the Elastic Limits.

Equation (3) is based on the assumption that the material of the beam follows Hooke's law ; therefore, the critical stresses obtained from table I, or figure 3, represent the true values of these stresses only if they are not greater than the elastic limit of the material. Otherwise, the critical stresses obtained in this manner will be too large. For an approximate calculation of critical stress beyond the elastic limit the straight-line column formula may be used. Assuming, for instance, that the elastic limit for structural steel is 30000 lb. per sq. in., we conclude that curves I, II and III of figure 3 can be used only below the points N. For higher stresses the curves should be replaced by straight lines MN. The crushing compressive stress for the steel is taken in figure 3 equal to 51000 lb. per sq. in.

If the stress-strain curve for the material of the beam beyond the elastic limit is given ; a more accurate determination of critical stresses in the unelastic region is possible. This can be done by taking into account the diminishing of the lateral rigidity B_2 due to the straining of the flanges beyond the elastic limit. Calculations of this kind [1] show that the use of straight lines MN (figure 3) give values on the safe side.

6. Effect of Additional Constraint of Beams.

Any additional constraint of the beam results in an increase of stability, i. e. in an increase of the factor K in equation 3. Take, for instance, a beam with

1. S. Timoshenko. Amer. Soc. of Civil Engineers. Vol. 87, 1924, p. 1247.

built in ends. The deflection curve in the case of sidewise buckling has the shape shown in figure 4 (*a*). The critical value of the uniformly distributed load is given by eq. 3. The values of the factor K and also the critical stresses σ_{cr} calculated on the assumption that $E = 30.10^6$ lb. per sq. in. and $\beta = 10^{-4}$ are given in table 2 below.

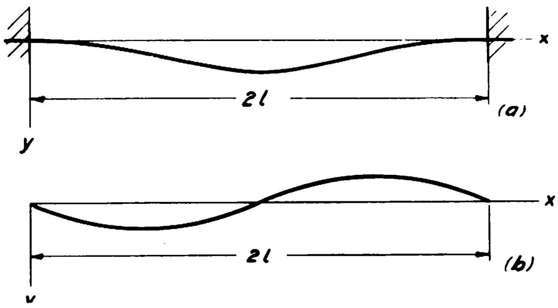

Fig. 4.

Table 2.

Factor K and critical stresses in lb. per sq. in.
($\beta = 10^{-4}$, $E = 30.10^6$) for uniformly loaded beam with fixed ends.

$\alpha =$	1	1	2	4	8	24	32	50	100
$K =$	488	161	119	91.3	73.0	58.0	55.8	53.5	51.2
$\sigma_{cr} =$	19300	20100	21100	22800	25800	35500	39500	47200	64000

In the calculation of the values in this table it was assumed that the load is distributed along the length of the axis of the beam.

When a beam is laterally supported at the middle of the span in such a manner that the middle cross section cannot move laterally and cannot rotate about the axis of the beam, the deflection curve in the case of sidewise buckling has the form shown in figure 4 (*b*). The values of the coefficient K in equation 3 and the values of critical stresses σ_{cr}, σ'_{cr} and σ''_{cr} produced by uniform load distributed 1) along the axis of the beam, 2) along the upper flange and 3) along the lower flange are given in table 3.

Table 3.

Factor K and critical stresses in lbs. per sq. in. ($\beta = 10^{-4}$, $E = 30.10^6$)
for a uniformly loaded beam with lateral support at the middle.

$\alpha =$	.1	1	2	4	8	24	32	50
$K =$	673	221	164	126	101	79.5	76.4	72.8
$\sigma_{cr} =$	39900	41600	43500	47000	53300	72900	81000	96600
$\sigma'_{cr} =$	34800	36500	38400	42000	48300	67800	75900	91500
$\sigma''_{cr} =$	45900	47300	49200	52800	59100	78600	86600	102000

7. Concentrated Loads.

Equation (4) holds also true for a concentrated load, Q_{cr} being then the critical value of this load. If the load is applied at the middle of the span of a simply supported beam, table 4 below gives the values of the factor K and also the critical stresses for the three different loading conditions : 1) load applied at the center of the middle cross section, 2) at the supper flange and 3) at the lower flange.

Table 4.

Factor K and critical stresses in lbs. per sq. in.
($\beta = 10^{-4}$, $E = 30.10^6$) for a load concentrated at the middle of a simply supported beam.

$\alpha =$	.1	1	2	4	6	8	12
$K =$	86.4	31.9	25.6	21.8	20.3	19.6	19.0
$\sigma_{cr} =$	10200	12000	13700	16400	18800	20700	24300
$\sigma'_{cr} =$	6080	7580	9000	11600	13800	15800	19200
$\sigma''_{rc} =$	17300	18800	20300	22800	25100	27000	30500

$\alpha =$	16	20	24	40	60	80	100
$K =$	18.3	18.1	17.9	17 5	17.4	17.2	17.2
$\sigma_{cr} =$	27500	30300	32900	41600	50400	57900	64500
$\sigma'_{cr} =$	22400	25100	27600	36300	45000	52500	59100
$\sigma''_{cr} =$	33600	36300	38700	47400	56100	63600	70200

With the increase of α the factor K approaches the value $K = 16.9$ which we obtain for a beam of a very narrow rectangular cross-section. For such a beam the problem of elastic instability is solved [1] for any position of the concentrated load along the length of the beam. If c is the distance of the load from the nearest support of a simply supported beam, the values of the factor K are given in the table below :

Table 5.

$\frac{c}{2l} =$	.50	.45	.40	.35	.30	.25	.20	.15	.10	.05
$K =$	16.9	17.2	17.8	19.0	21.0	24.1	29.1	37.9	56.0	112

It is seen that the factor K does not change very much if the load remains in the middle third of the span. Hence if we have several loads acting along the middle third of the span we can replace them by one load at the middle and calculate the critical value of this load by using table (4). If concentrated

1. A. Roboroff. Bulletin of the Polytechnical Institute, Kiew, 1911.
A. Dinnik. Bulletin of the Don Polytechnical Institute, 1913.

loads are distributed throughout the span they can be replaced by a uniform load and the critical values of this load will be found by using tables 1, 2 and 3.

II. Stability of the Web of a Plate Girder.

8. Experiments.

The first experiments with buckling of thin webs transmitting shearing and bending stresses were made by Wm. FAIRBAIRN in connection with the construction of the famous Britania and Conway tubular bridges. These classical experiments [1] up to the present time have held great interest for engineers working with thin walled structures. The Britania bridge is of tubular form having a rectangular cross section. The larger tubes have a span of 450 feet and cross sectional dimensions of 27 by 16 feet. At that time this was an unusually large structure and for determining the safe dimensions of the tube and the most favorable distribution of the material it was decided to make experiments with models. After considerable amount of preliminary experimenting it was decided to test large models, one-sixth the lineal dimensions of the intended bridge. The sides of these model tubes consisted of sheets $3' 9''$ deep and only .1 of an inch thick. The first experiments showed that at a comparatively small load undulations in the sides appeared which formed angels of about 45 with the line of the bottom. " It was evident, from these experiments, that the tension throughout the bottom and the compression troughout the top stood in the relation of action and reaction to each other, the diagonal strain in the sides being the medium of communication ". " A diagonal wave of puckering clearly exposed the line of severest strain. It was evident that the sides were exposed to unfair strain from the change of shape consequent on the tendency of the top and bottom to come together, the plates being strong enough, if they could but be kept in shape ; and it was therefore determined, in this experiment, to modify the construction of the sides. This was done by the addition of pillars of angle-iron throughout, of the whole height of the sides, riveted to them, having the effect of stiffening them, and at the same time of keeping the top and bottom in place. They were prototypes of the T-iron pillars used in the large tubes ". The further experiments illustrated the importance of the pillars in the sides, as, with a small addition of metal to the weight of the tube, the top and bottom remained precisely the same as before, while the breaking-weight was increased considerably. From these experiments had been learned that " as the depth of a web increased, the precautions requisite for maintaining the sides in shape become very formidable ". The T-irons, gussets and stiffening plates for this purpose in one of the Britannia Tubes, weigh 215 tons, or upwards of one-third of the whole weight of the sides.

The experimental tubes were submitted to a concentrated load at the middle and the shearing force was constant along the length of the span. In the design of the actual bridge it was taken into account that the maximum

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1. Wm. FAIRBAIRN. Conway and Britannia Tubular Bridges, 1849.
Edwin CLARK. Britannia and Conway Tubular Bridges, London, 1850.

shearing force diminishes towards the middle, and the web was taken 1/2 inch thick in the middle portion and 5/8 inch thick at the ends.

Some experimental work on plate girders was also made at that time [1]. The tickness of the web of the model girder was one-quarter inch throughout; the over-all depth was 10 feet at the center, and 6 feet at the ends, and the distance between the bearings was 66 feet. The girder failed by buckling of the web. Later on the girder was repaired and the vertical web stiffened by the addition of angle-iron pillars at each joint in the vertical plates of the web. In this way the strength of the girder was considerably increased and finally it failed at a larger load by a simultaneous collapse of the top and the bottom.

Further experiments with plate girders were made by a Belgian ingenieur Houbotte [2]. Two plate girders, 1.50 mt. span length; 0.5 cm thickness of the web; and 30 cm and 49 cm depth, were tested. Loaded at the middle both these girders failed by buckling of the web, which had no stiffeners. The girder of large depth failed at smaller load although its section modulus was twice as great as that of the girder with smaller depth.

In more recent time some work with plate girders has been done by Professor W. E. Lilly [3]. A plate-girder of the following dimensions was constructed: Depth 9 1/2 in.; length, 5 ft. 3 in. The flanges were made up of two plates 2 in. by 3/8 in.. and two angle-irons, 1 1/4 in. by 1/4 in. The framework of the girder was made in separate halves, and bolted together to the web. This construction allowed different thicknesses of web to be used in the experiments. A large number of tests were then carried out with different thicknesses of the web and spacing of stiffeners.

Applying the load at the middle, the wave formation in the web was obtained. " It was found that the wave-length of the wave formation is nearly independent of the thickness, if the stiffeners are of great strength compared with the web. The angle of incliclination of the waves depends upon the

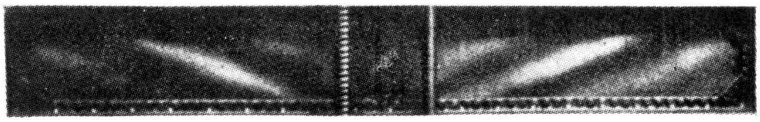

Fig. 5.

distance apart of the stiffeners and the depth of the girder. The stiffeners prevent the formation of the waves, and severe local stresses are set up around the ends of the stiffeners, causing a crumpling up of this part of the web ". Photograph 5 represents the wave formation in the case when the web had been stiffened only at the middle, where the load was applied.

9. Web Thickness and Spacing of Web Stiffeners.

From the experiments made it may be seen that a plate girder can transmit the shearing force to the bearings in two different ways: 1) If the load is not

1. Edwin Clark. Britannia and Conway Tubular Bridges, London, 1850.
2. M. Houbotte. Der Civilingenieur, 1856, vol. 4.
3. W. E. Lilly. The design of plate girders and columns, 1908.
   W. E. Lilly. Engineering, vol. 83, 1907, p. 136.

sufficient to produce wave formation, the web of the girder transmits the shearing force by working in shear. 2) In the case of larger loads, which produce wave formation, one part of the shearing force is transmitted by shearing stresses in the web, as before, and the other part — as in a truss, in which the web plate is working as ties and the stiffeners as struts. The magnitude of the load at which wave formation begins depends on the thickness of the web and on the spacing and dimensions of stiffeners. In the case of a sufficient thickness of the web and a satisfactory stiffening, a plate-girder can carry the total load, for which it is designed, without any buckling in the web. We usually have such proportions in bridges. On the other hand there are constructions with very thin webs which buckle at the very beginning of loading and the total load is practically transmitted as in a truss. We have examples of such girders in aeroplane constructions [1].

Although buckling of the web does not mean an immediate failure of the girder the dimensions in the case of bridges are usually taken so as to eliminate buckling under service condition. The usual procedure is to adopt a certain value for the working stress in shear and on this basis to decide upon the web thickness [2]. Then the spacing of stiffeners is determined so as to enable the web to transmit shearing stresses without buckling.

Observing that in railway girders the total load varies approximately as the span and assuming the ratio of the depth to the span constant, it may be seen that the above procedure would result in nearly the same thickness for all spans. Assuming that this thickness is satisfactory for small bridges it certainly will be insufficient for larger spans and some increase in the thickness for eliminating the possibility of buckling of the web becomes necessary. This is provided for in some specifications. For instance, American Railway Engineering Association specifications [3] require that the thickness of the web shall be not less than $\frac{1}{20}\sqrt{h}$, where h represents the distance between flanges in inches.

Another limitation for thickness is usually obtained from the consideration of corrosion and from the fact that too thin plates, if deep and long, are very awkward to handle. The 3/8 in. thickness is usually considered as the least thickness permissible to provide against corrosion and ascertain a satisfactory handling of material during construction and shipping.

For spacing and dimensioning stiffeners, various specifications give certain rules, which to a large extent are of empirical character. American Specifications mentioned above, require, for instance, that the distance between stiffeners shall not be larger than : 1) six feet, 2) the depth of the web, 3) the distance d given by the formula :

$$d = \frac{t}{40}(12.000 - \tau)$$

1. H. Wagner. Zeitschrift für Flugtechnik u. Motorluftschiffahrt, vol. 20, 1929, p. 200.
H. Rode. Der Eisenbau, vol. 7, 1916, p. 217.
Eng. News, vol. 40, 1899, pp. 154, 399.

2. Sometimes the working stress is obtained by using some variation of Rankine's formula.
American Institute of Steel Construction Specifications, New York, 1928, p. 156.

3. American Railway Engineering Association's Specifications, third edition, 1925.

in which t is the thickness of the web in inches, and τ is the shearing stress in pounds per square inch at the point considered. If the depth of the web between the flange angles is less than 50 times the thickness of the web, intermediate stiffeners may be omitted. Intermediate stiffeners shall be riveted in pairs to the web of the girder. The outstanding leg of each angle shall be not less than 2 inches plus one thirtieth of the depth of the girder, nor more than 16 times its thickness. Sometimes the stiffeners are proportioned to make them to duty as vertical struts in a triangular girder ; in which case it is sufficient to ensure that the stiffeners shall, as struts, be strong enough in the aggregate to take the whole shear force at the section considered [1].

Being the result of a long experience, the rules for determining web thickness and stiffeners spacing usually give satisfactory proportions. At the same time they are flexible enough and leave considerable freedom for individual judgment wich finally results in a variety of dimensions of plate girders designed for the same span and the same load. Comparing, for instance, plate girders with a span 100 ft. and depth 10 ft. we find that the thickness of the web varies from 7/16 in. to 5/8 in [2]. A comparison of two plate girders of 90 ft. span one for an American railway and the other for a British railway [3] shows that in the American type the stiffeners comprise 25 °/₀ and in British 40 °/₀ of the material in both web and stiffeners.

From the above discussion it is seen that the proportioning of plate girders is based to a great extent on empirical rules. To get a theoretical basis for the design it is necessary to go into a study of the elastic stability of thin webs. In discussing buckling of the web we must consider two extrem cases : 1) Near the supports the shearing force is the most important factor and the part of the web between two stiffeners can be considered as a rectangular plate subjected to the action of uniform shear (fig. 6). 2) At the middle of the span the shearing stresses can be neglected in comparison with normal stresses. Then, the part of the web between two stiffeners will be in the condition of pure bending represented in fig. 7. In the next articles these two cases will now be discussed.

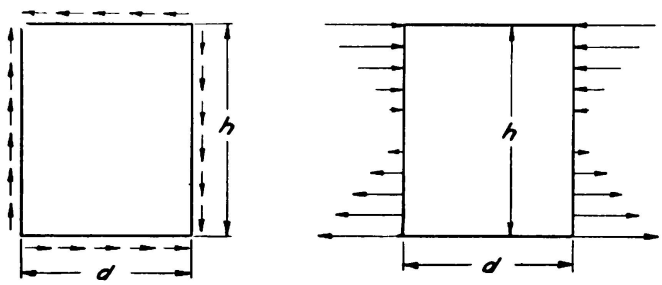

Fig. 6 u. 7.

10. Stability of Rectangular Plates under Pure Shear.

The investigation of the stability of rectangular plates under the action of shearing forces (fig. 6) shows that the critical value of shearing stress at which buckling occurs can be represented in the following form [4] .

1. W. H. Thorpe. Engineering, vol. 78, 1905.
2. H. Rode. Der Eisenbau, vol. 7, 1916, p. 217.
3. H. M. Gibb. Engineering, vol. 90, 1910.
4. S. Timoshenko. Bulletin of the Polytechnical Institute at Kiew, 1910. The translation of this paper in French is in Annales des Ponts et Chaussées, 1913.
S. Timoshenko. Der Eisenbau, vol. 12, 1921, p. 147.

$$\tau_{cr} = K \frac{\pi^2 D}{d^2 t} \quad \ldots \ldots \ldots \ldots \ldots \quad (7)$$

In which K is a numerical factor depending on the manner in which the edges of the plate are fixed and on the ratio $\frac{h}{d}$ between the sides of the rectangle.

$D = \frac{Et^3}{12(1-\mu^2)}$ is the flexural rigidity of the plate.

If h is very large in comparison with d [1] the factor K depends partically only on boundary conditions along the long sides of the rectangle [2]. If the plate is simply supported along these sides, K = 5.35. If the longitudinal sides are clamped, $k = 8.98$.

If the sides of the rectangle are of the same order, the calculation of K becomes more complicated [3]. Several approximate values of this factor for the case of simply supported edges are given in the table below [4] :

Table 6.

Values of the Factor K in eq. 7 for simply supported rectangular plates.

$\frac{h}{d}$ =	1	1.2	1.4	1.5	1.6	1.8	2.0	2.5	3.0
K =	9.42	8.0	7.3	7.1	7.0	6.8	6.6	6.3	6.1

From equation (7) it may be seen that the critical values of shearing stress are proportional to $\left(\frac{t}{d}\right)^2$. Hence stability of a plate rapidly diminishes with the diminishing of the thickness of the plate.

Applying the values of K given in table 6 to steel plates ($E = 30.10^6$ lb. per sq. in., $\mu = .3$), and assuming the distance between the stiffeners equal to 5 ft., the critical values of shearing stress for different depths, h, and different thicknesses, t, are given in table 7 below :

Table 7.

Critical Shearing Stresses, in Pounds per Square Inch, for various values of h and t. $d = 60$ in.

t =	3/8 in.	7/16 in.	1/2 in.	9/16 in.
h = 60 in.	9980	13600	17700	22400
72 in.	8450	11500	15000	19000
84 in.	7730	10500	13700	17400
96 in.	7500	10100	13200	16700
108 in.	7200	9800	12800	16200
120 in.	7000	9500	10400	15700

1. Such conditions existed, for instance, in Britannia Tubular bridge mentioned before.
2. R. V. Southwell and S. W. Skan. Proc. of the Royal Society, London, vol. 105. A, 1924.
R. V. Southwell. Phil. Mag. vol. 48, 1924, p. 540.
3. S. Timoshenko. Der Eisenbau, vol. 12, 1921, p. 147.
4. It is assumed that d in equation (7) denoted the smaller side of the rectangle.

It should be noted that a portion of web between the flanges and two adjacent stiffeners is in a more favorable condition than assumed above. The edges are partially fixed and the critical stresses will be somwhat higher than given in table 7.

11. Stability of Rectangular Plates under Pure Bending

If a rectangular plate is in a condition of pure bending in the plane of the plate (Fig. 7), the critical value of the maximum bending stress is found from the equation :

$$\sigma_{cr} = K \frac{\pi^2 D}{h\ t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

In which K is a numerical factor and the other symbols have the same meaning as in equation (7). For a simply supported plate the values of K are given in the table below [1].

Table 8.

Values of the factor K in eq. 8 for simply supported rectangular plates

$\frac{d}{h}$	.4	.5	.6	.67	.75	.8	.9	1.0	1.5	2	3
K	29.1	25.6	24.1	23.9	24.1	24.4	25.6	25.6	24.1	23.9	24.1

If d is larger than h there is only small variation in K with variation of the ratio $\frac{d}{h}$ and with the increasing of d, K approaches the minimum value 23.9 obtained for the ratio $\frac{d}{h} = .67$. This follows from the fact that a plate with large d buckles in several waves with vertical nodal lines and the ratio of wave length to the depth h approaches the value .67. For instance, in the case of $\frac{d}{h} = 2$ we will have three waves and each wave is in the same condition as the plate with the ratio $\frac{d}{h} = \frac{2}{3}$. When $\frac{d}{h} = 3$ we have four waves with the ratio of waves length to the depth equal to .75.

Taking for K the minimum value 23.9 and assuming $\frac{t}{h} = \frac{1}{100}$ we find, from equation (8), for steel.

$$\sigma_{cr} = 64.800 \text{ lbs. per sq. in.}$$

For any other value of the ration $\frac{t}{h}$ the critical value of stress then is :

$$\sigma_{cr} = 64800.10^4 \frac{t^2}{h^2}$$

Taking, for instance, $t = \frac{1}{2}$ in. $h = 120$ in. we find.

$$\sigma_{cr} = \frac{64800.10^4}{10^2.(24)^2} = 11200 \text{ lbs. per sq. in.}$$

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1. S. Timoshenko. Der Eisenbau, vol. 12, 1921, p. 147.
J. Boobnoff. Theory of Structure of Ships, vol. 2, p. 525, 1914, St. Petersburg.

If, instead of a very large d, we take $d = 1/2\ h$. (vertical stiffeners 5 feet apart) then, from the table, $K = 25.6$ and $\sigma_{cr} = 11.200 \frac{25.6}{23.9} = 12.000$ lbs. per sq. in.

It is seen that for the proportions taken the critical stress is smaller than the stress usually considered as a safe bending stress for plate girders. In calculation of the table 8 it was assumed that the edges of the plate were simply supported; in practice they are rigidly connected with flanges. Therefore, the actual critical stresses will be somewhat greater than the theoretical. Nevertheless, it is probable that in the case of thin webs and large depths some buckling of the web may occur under ordinary loading conditions. This buckling is so small that it can remain unnoticed. It does not represent an immediate danger to the girder. It means only that when the load surpasses its critical value and buckling begins the web does not take longer its share in transmitting compressive bending stresses which causes a certain overstressing of the compression flange.

12. Stiffeners.

Considering a portion of the web between two consecutive stiffeners it was assumed on our previous discussion that the stiffeners have a sufficient flexural rigidity and remain straight during the buckling of the web. If this rigidity is not sufficient the inclined waves of buckled webs run accross the stiffeners and buckling of the web is accompanied by bending of the stiffeners. Such bending was evident, for instance, in some of *Fairbairn's* experiments discussed before (article 8). To determine the necessary flexural rigidity, sufficient to prevent the stiffeners from bending during buckling of webs, let us consider the case represented in figure 8 : A rectangular plate of the length 2 d and the width h simply supported at the edges is submitted to pure shear. To prevent lateral buckling the plate is stiffened by a pillar AB. If the flexural rigidity of the pillar is small its effect on the magnitude of the critical shearing stress will also be small. The waves of the buckled plate will cross the pillar and bending of the pillar will be produced. By subsequent increases of the rigidity of the pillar we finally may arrive at the conditions in which each half of the plate will buckle as a rectangular plate of the dimensions $h \times d$ with simply supported edges and the pillar will remain straight. The corresponding limiting value B of the flexural rigidity of the pillar can be found from the consideration of strain energy of bending of the plate and of the pillar [1].

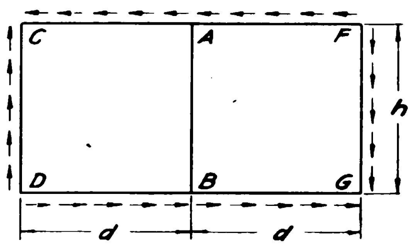

Fig. 8.

Several values of the ratio of this rigidity 2dD of the web, if bent to a cylindrical surface, is given in the table below :

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1. S. Timoshenko. Der Eisenbau, vol. 12, 1921, p. 147.

Table 9.

$\frac{2d}{h} =$	2	1.5	1.25	1
$\frac{B}{2dD} =$	.83	2.9	6.3	15

In calculation of this table it was assumed that only the pillar AB in figure 8 is flexible and the pillars CD and FG are absolutely rigid. If all three pillars are of the same flexibility, the limiting rigidity B must be larger than would be obtained from table 9. Assuming that it is twice as large as the calculated value, we arrive at the following values of the required moment of inertia J of the cross section of stiffeners for various proportions of plate girders and for $d = 5$ ft :

Table 10.

$h =$	60 in.	80 in.	96 in.	120 in.
$\frac{2d}{h} =$	2	1.5	1.25	1
$\frac{B}{2dD} =$	1.7	5.8	12.6	30
t = 3/8 in.	.99 in.^{4}	3.36 in.^{4}	7.30 in.^{4}	17.4 in^4
t = 7/16 in.	1.56 in.^{4}	5.34 in.^{4}	11.6 in.^{4}	27.6 in^4
t = 1/2 in.	2.35 in.^{4}	8.00 in.^{4}	17.4 in.^{4}	41.4 in^4
t = 9/16 in.	3.36 in.^{4}	11.5 in.^{4}	25.0 in.^{4}	59.4 in^4

It is seen that for smaller depths the calculated cross sectional moment of inertia is much smaller than that which is actually used. For larger depths the calculated values of J are approaching the usual proportions. For instance, in the case of $h = 10$ ft. the stiffner, following the american rule[1] consists of two angles $6'' \times 3\frac{1}{2}'' \times 3/8''$. The moment of inertia for this stiffner for t = 9/16 in is J = 62.5 in^4. which is near to the value 59.4 in^4. given in the table above.

In the above discussion we considered a plate submitted to the action of shear, as we have in the web, near the supports, of a plate girder. At the middle of the girder the web stresses are principally bending stresses and from the discussion of article 11 it can be concluded that vertical stiffeners do not increase substantially the stability of the web at this place. A much greater stiffening effect may be achieved by using in the compressed zone of the web a stiffener parallel to the compressed flange of the plate girder.

Conclusions.

The results regarding elastic stability of plates obtained in the previous articles may be used for proportioning in plate girders as follows : In determining the thickness of the web consider not only shearing stresses at

1. See H. A. L. Waddell. Bridge Engineering, New-York, 1916, p. 1670.

supports and the minimum thickness permissible to provide against corrosion but also elastic stability of the web. In discussing buckling of plates under pure bending (article 11) it was shown that vertical stiffeners do not affect substantially the stability of the web and it seems logical to choose the thickness of the web so as to eliminate the possibility of buckling due to bending under service conditions. For this purpose equation (8) can be used. Substituting for σ_{cr} the maximum permissible compressive stress, say 15000 lbs. per sq. in., and taking for K the minimum value 23.9 from table 8 we find for steel ($E = 30.10^6$ and $\mu = .3$) :

$$\frac{h}{t} = 208. \quad \quad (9)$$

Hence to provide against buckling of webs at the middle of girders the ratio of the depth to the thickness of the web must not excede the value (9). It is not necessary to provide in this case for an extra factor of safety, because some additional safety is realized by fixing the edges of the web at the flanges.

When the thickness of the web is decided upon, spacing of stiffners must be determined so as to enable the web to transmit shearing stresses without buckling. For this purpose the results of article 10 may be used. These results can be represented as shown in figure (9). By using equation (7) and table 6 for each value of critical shearing stress a curve is constructed the ordinates of which are the ratios $\frac{h}{d}$ of the depth of the girder to the stiffener spacing and abscissae the corresponding ratios $\frac{d}{t}$ of the stiffener spacing to the thickness of the web. By using such curves the necessary distance d between the stiffeners can be obtained if the critical value of the shearing stress is chosen.

Considering the curve for critical stress $\sigma_{cr} = 20.000$ lbs per sq. in., we see that for large values of the ratio $\frac{h}{d}$, i. e. when the stiffener spacing is small in comparison with the depth of the girder, the ratio $\frac{d}{t}$ approaches 90. When $\frac{h}{d}$ is small then h takes the place of d in the previous discussion and the ratio $\frac{h}{t}$ approaches the same numerical value 90. From this a practical conclusion can be made : Shearing stress 20000 lbs. per sq. in. can be considered as yield point for shearing stress in the case of usual structural steel. Hence if $\frac{h}{t}$ is not larger than 90, the resistance of the web to buckling is not smaller than the resistance to yielding in shear and stiffeners are necessary only at places of application of concentrated loads.

In choosing the critical stress in fig. 9 a certain factor of safety must be introduced because in this case 1) the fixity of the edges of the web at the flanges does not effect much the critical value of shearing stress if $\frac{h}{d} > 2.$ and 2) the buckling changes the condition of the work of the web (see article 9) which results in additional tensile stresses in the web and undesirable overstressing of certain rivets. It is suggested to take .6 of critical stresses as safe stresses [1]. These stresses are given in figure (9) in parenthesis.

Let us apply now the curves of fig. 9 in determining stiffener spacing for girders of the depth 6' and 10'. In the case of $h = 6$ ft., applying equation (9), we conclude that the thickness t can be taken 3/8 in. Assuming working stress equal to 9000 lbs. per sq. in. we find then, from fig. 9, $\frac{d}{t} = 112$; $d = 42$ in. $= 3\frac{1}{2}$ ft. By taking working stress 6000 lbs. per sq. in. we find in the same manner $d = 45$ in. $= 4\frac{1}{2}$ ft.

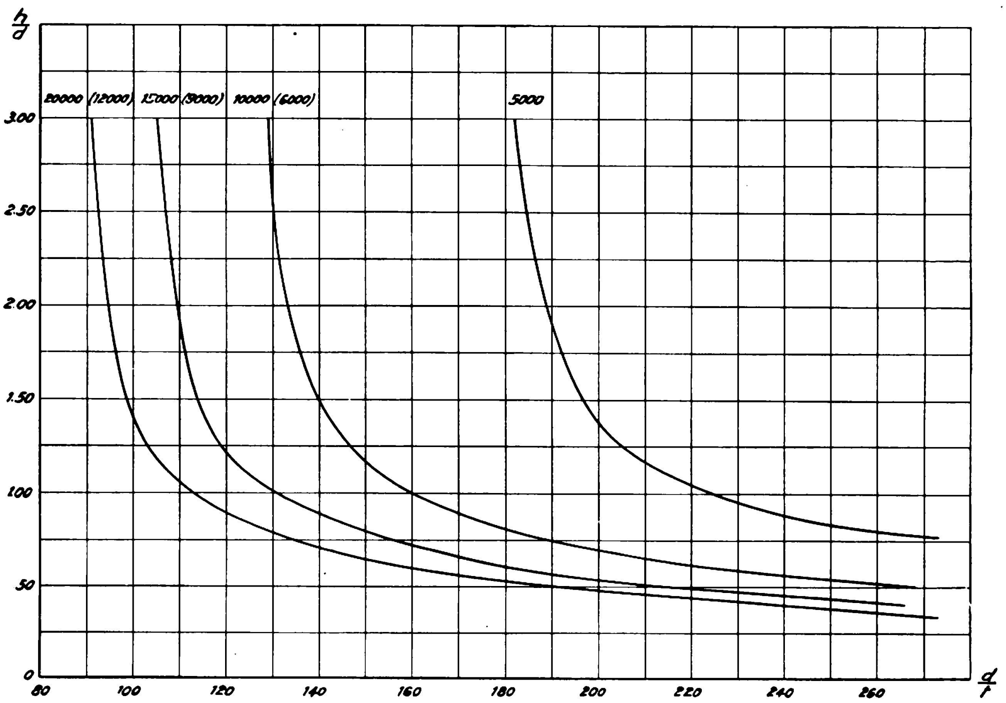

Fig. 9.

Applying to the same case the rules of American specifications (article 9), we find $t = \frac{1}{20}\sqrt{h} = .424$ in. or 7/16 in. The distance between stiffeners for working stress 9000 lbs. per sq. in. $d. = \frac{t}{40}(12000 - \tau) = 75\,t = 33$ in. For working stress 6000 lbs per sq. in. $d = 150\,t = 66$ in.

The suggested manner of proportioning plate girders gives in this case a smaller stickness of the web. The stiffening is also somewhat lighter for the working stress 9000 lbs, per sq. in. For the 6000 lbs, per sq. in. stress the proposed stiffening is somewhat heavier than that obtained by using American specification.

In the case of $h = 10$ ft. we find, from eq. (9), $t = .575$ in. a little more

1. This means the same factor of safety as if using a working stress in shear of 12000 lbs. per sq. in. when yield point in shear is 20000 lbs. per sq. in.

than 9/16 in. and the thickness $t = 5/8$ in. should be used. The American specification for the same depth gives $t = .548$ in. and 9/16 in. thickness should be used. The distance of stiffeners, for $\tau = 9000$ lbs per sq. in., is, from figure 9, $d = 112t = 70$ in. For $\tau = 6000$ lbs. per sq. in. $d = 144t = 90$ in. The American specifications give in this case 42 in. and 72 in. respectively. In this case the suggested manner of proportioning plate girders gives slightly larger stickness of the web, but the stiffening is lighter for both values of working stress. The distance between the stiffeners determined above for the maximum shearing stress can be increased with increasing distance from the supports. In this changing of stiffeners figure 9 also can be used, although the web stresses at intermediate cross sections are different from pure shear and for a more satisfactory solution of the problem the stability of the plate under combined bending and shearing stress should be considered.

In proportioning stiffeners table 10 may be used. From practical considerations the cross-sectional moment of inertia J will be taken, for smaller depths, larger than it is given in the table.

For further improvement of the design of plate girders from the point of view of elastic stability it is desirable: 1) To develop the theory of buckling of rectangular plates with fixed edges. 2) To consider buckling of rectangular plates under combined bending and shearing stress. 3) To investigate in more detail the question of required flexural rigidity of stiffeners.

Experiments with large size model girders will give a chance to check the theory and to investigate such important points as stress distribution in the web and in the rivets after buckling begins.

TRADUCTION,

par M. Gossieaux, Ing., Paris.

Introduction.

Pour le calcul d'une poutre composée, il faut prendre en considération non seulement les charges, mais également la stabilité élastique du système. On sait [1] qu'une poutre en I qui subit un fléchissement dans le plan de l'âme peut accuser une stabilité insuffisante et subir un flambage latéral. La valeur critique de la charge pour laquelle ce flambage peut se produire dépend non seulement de la rigidité latérale des ailes, mais également de la rigidité à la torsion de la poutre, ainsi que du rapport entre la longueur de la portée et la hauteur de cette poutre. Pour éliminer cette cause d'instabilité, il est nécessaire de prévoir un dispositif approprié réalisant un renforcement latéral [2].

Un autre problème lié à la stabilité élastique se pose pour le calcul des poutres composées : c'est la détermination de l'épaisseur de l'âme et de l'écartement des pièces de renforcement de cette âme. On pourra constater souvent qu'il est nécessaire de pousser l'épaisseur de l'âme au delà d'une valeur cor-

1. L. Prandtl. Le phénomène du flambage. Thèse, Nuremberg, 1899.
2. L'accident survenu à un pont situé près de Tarbes fournit un exemple de la rupture d'une poutre à la suite d'un flambage latéral (La Revue Technique, 15 novembre 1897).

3. de faire des recherches plus approfondies sur la rigidité à la flexion qu'il est nécessaire de demander aux éléments de renforcement.

Des essais sur des modèles de poutres de grandes dimensions donneront l'opportunité de contrôler la théorie mise en avant et d'élucider des points importants, tels que la distribution des efforts dans l'âme et dans les rivets lorsque le flambage commence.

Summary.

In dimensioning the webs of plate girders not only must the shearing stress at the supports and the danger from rust be considered, but also the elastic equilibrium of the webs.

If only pure bending stresses come into consideration, vertical stiffening angles will have no essential influence on the buckling of the stiffened web and it is more preferable to choose the thickness of the stiffened web in such a way that buckling is quite impossible under the working load. For instance, in the middle of the beam the proportion should be

$$\frac{h = \text{height of the web}}{t = \text{thickness of the web}} < 208.$$

When the thickness of the stiffened web has been determined, the distance of its stiffening angles are to be dimensioned so that the stiffened web may also be able to withstand shearing stresses without bulging. With $h/t < 90$, there is the same security against bulging and reaching the elastic limit in consequence of shearing stresses, so that stiffenings are only necessary at those places where single loads act.

In selecting the critical stresses, the factor of safety 0.6 must be introduced, since the holding of the edges of the stiffened web in the flange angle will have little influence on the critical shearing stress, as soon as the height of the stiffened web is greater than twice the distance between the stiffening angles, and since at the commencement of bulging additional tensile stresses in the plate and overstraining of individual rivets are caused.

For a stiffened web 6ft. high, the thickness is 3/8 in. With a stress of 9.000 lbs per sq. in. the distance between the stiffening angles is 42 in. with a stress of 6000 lbs. per sq. in. it is 54 in.

In order to throw further light on the question of stability of the stiffened web, it would be desirable to carry out further tests to determine the resistance to buckling of rectangular plates with supported edges, especially when submitted simultaneously to bending and shearing stresses, and also for detailed determination of the required resistance to bending of the stiffening angles.

Résumé.

Pour la détermination des dimensions des tôles de l'âme des poutres composées, il faut tenir compte non seulement des efforts tranchants aux appuis,