THE COMPUTER-AIDED RECONSTRUCTION OF AN EARLY VIOLA DA GAMBA

by Vasileios Chatziioannou

Introduction

Physical modelling can be used to characterise the sound generation mechanism of musical instruments. By modelling the oscillations of the vibrating parts of the instrument, valuable information about its acoustic properties can be established. The advantage of a physics-based approach (as opposed to other modelling approaches, such as additive, subtractive, granular or FM synthesis) is that the model parameters have a direct physical interpretation. Parameters related to the geometry of the instrument can be measured directly and those depending on material properties can be either approximated experimentally or obtained from the relevant literature. Therefore, physical modelling offers a valuable tool to predict the effect of structural modifications during the making of an instrument. Furthermore, it can help determine which properties of the instrument are perceptually significant for the radiated sound.

A computer simulation is based on a numerical description of the laws that govern the oscillations of the instrument. There are several ways to obtain such a description. An extensive study of modelling techniques has been based on the modal decomposition of a vibrating object whose vibrational behaviour can be approximated using a number of oscillating modes. Such a practice is referred to as modal synthesis (MS).[1] Based on this approach, the functional transformation method (FTM) was developed, using transfer functions, still based on the assumption that the solution of the mechanical vibration problem can be approximated by a set of modes.[2] The digital waveguide method (DWG) assumes instead that vibrating objects can be modelled using travelling waves.[3] Due to the lack of interaction between the invoked oscillating modes, the above methods are not particularly suitable for non-linear problems.[4] Therefore, if the focus is shifted from pure sound synthesis to the study of the sound production mechanism, a direct simulation method should be employed.

[1] Jean-Marie Adrien, „The missing link: modal synthesis", in: *Representations of Musical Signals*, Cambridge, MA: MIT Press 1991, 269–298.

[2] Rudolf Rabenstein, „Digital Sound Synthesis of String Instruments with the Functional Transformation Method", in: *Signal Processing* 83 (2003), 1673–1688.

[3] Julius O. Smith, *Music Application of Digital Waveguides*, Stanford: CCRMA 1987.

[4] Vasileios Chatziioannou, *Forward and Inverse Modelling of Single-reed Woodwind Instruments with Application to Digital Sound Synthesis*, PhD Thesis Queen's University Belfast, N. Ireland 2010.

Describing the vibrating system using partial differential equations derived from physical laws, it is possible to obtain a numerical approximation of the solution. The discretisation of the underlying equations is often carried out using the Finite Difference Method which has been widely used in the field of musical acoustics. When modelling complex geometries which cannot be easily approximated by symmetric, rectangular grids, the finite element method can be employed.[5] This method approximates a solution over a discretised domain, making use of a collection of basis functions that have a relatively simple form. The COMSOL software provides the interface for the definition and solution of such problems within various fields of physics, as well as the coupling between them (e.g. acoustic-structure interaction models).

Within the presented project, the vibrating body of a viola da gamba (viol) is simulated, and its frequency response calculated, given the material properties of the instrument. This curve reveals useful information on the vibrating properties of a resonating body and is often considered for the quality testing of bowed string instruments.[6] The effect of changes in both the geometry and the materials during the construction of an instrument can be thus analysed, revealing the trajectories towards which such changes contribute. Furthermore, an eigenfrequency analysis can be carried out in order to visualise the different vibrating modes of the instrument.

Physical modelling

The objective of this paper is to predict the acoustical and vibrational properties of an early viola da gamba, the construction of which is based on illustrations and descriptions by Silvestro Ganassi.[7] A thorough examination of both iconographical sources and preserved instruments has led to a hypothesised geometry for the instrument.[8] A particular feature of this geometry is its asymmetric design, involving a variable thickness at the belly of the instrument and the absence of a bass bar and a sound post. The structural and acoustical properties of such a design can be analysed using a physical modelling approach, based on the finite element method.

5   Gwynne Evans, Jonathan Blackledge and Peter Yardley, *Numerical Methods for Partial Differential Equations*, London: Springer 1999.

6   Jesús Alonso Moral and Erik Jansson, „Input Admittance, Eigenmodes, and Quality of Violins", Speech Transmission Laboratory, Quarterly Progress and Status Report/STL-QPSR 23 (1982), 60–75.

7   Silvestro Ganassi, *Regola Rubertina*, Venedig: l'autore 1542; ders., *Lettione seconda*, Venedig: l'autore 1543.

8   Thilo Hirsch, „An Evidence-based Reconstruction of a Viol after Silvestro Ganassi", Paper presented at the Galpin/CIMCIM Conference ‚Musical Instruments – History, Science and Culture', Oxford, UK, Faculty of Music, 25–29 July 2013. This can be found, with further information including illustrations and sound recordings, at http://www.rimab.ch/content/research-projects/project-early-bowed-instruments (26.8.2017). See also the article by Thilo Hirsch in this volume.

For numerical purposes the viol is discretised using a number of tetrahedral elements, as depicted in Figure 1. The surrounding air domain also has to be discretised in a similar fashion. In order to accurately capture the acoustic radiation in the air, at least ten elements per wavelength need to be considered. Therefore, a frequency dependent mesh has been constructed, suitable for modelling wave propagation up to a fundamental frequency of 700 Hz. For the discretisation of the viol body no such restriction was imposed, since the mesh is already fine enough, in order to account for the complex geometrical shape of the instrument.
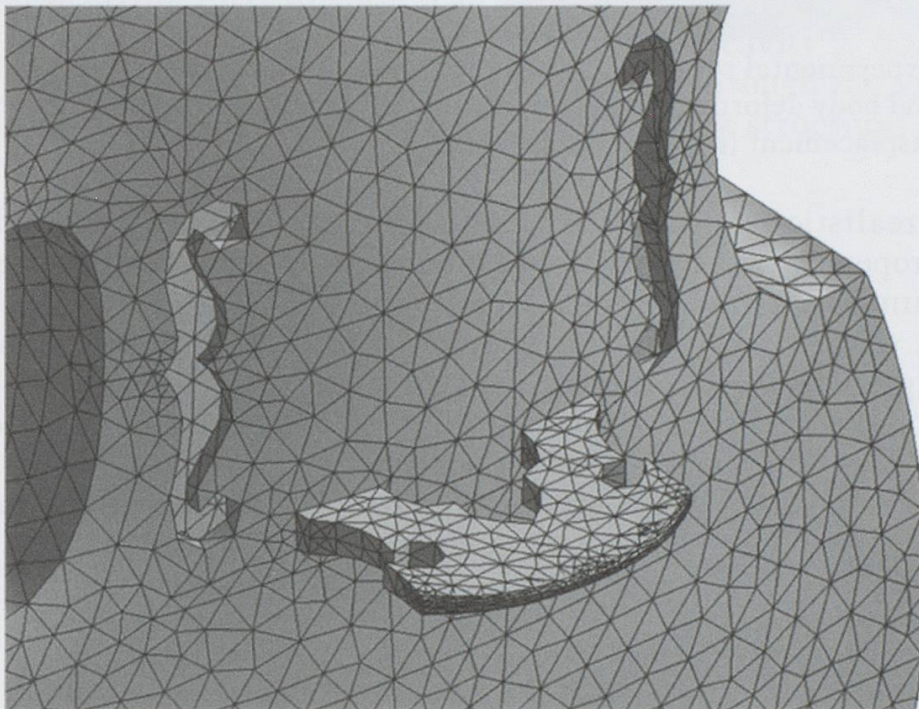

Figure 1:Mesh for the finite element simulation.(detail: surrounding air domain omitted for visualization). 3D-model: René Racz

In accordance with experimental admittance measurements (see Figure 2, left) simulations are carried out in the frequency domain where a unity force is applied to one side of the bridge and the resulting mobility (acceleration) is measured on the other side. At the same time, the ‚acoustic efficiency‘ of the instrument can be evaluated by calculating the sound pressure emitted by the same excitation (in this case measured 0.5m in front of the sound holes). However, before proceeding with the frequency-domain simulations, it is necessary to consider the static load presented to the instrument by the tension of the strings. Knowing the material properties of the (gut) strings and the angles at which they are stretched over the bridge, it is possible to calculate that the resulting vertical force at the bridge (due to the tension of all six strings of the instrument) is approximately 47 N. Figure 2 (right) illustrates the deformed shape of the instrument due to this static load. In order to calculate the vibrational properties of the instrument, a harmonic excitation has to be superimposed on to this deformed state.
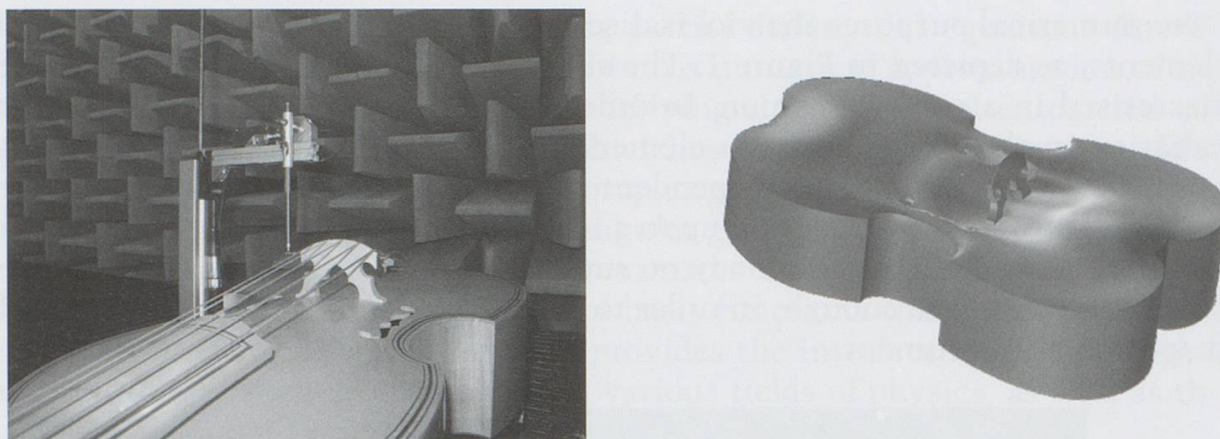
Figure 2: Experimental setup for admittance measurements (left: Photo Thilo Hirsch) and body deformation due to string tension, illustrated using an exaggerated displacement (right: graphics IWK).

Finally, a realistic simulation of the instrument requires knowledge of the material properties of the wood used during construction. The parameter values used for numerical simulations, extracted from several references[9], are given in Table 1, where ρ is the density (given in kg/m^3), E the Young's modulus of elasticity (given in GPa), ν the Poisson's ratio and G the shear modulus (given in GPa). L indicates longitudinal, R radial and T tangential direction and in the case of double subscripts the first letter refers to the direction of the stress and the second to that of the strain. The damping factor was set to 0.04.

Table 1: Material properties for the resonance woods used in the numerical simulations.

	E_R	E_L	E_T	ν_{TR}	ν_{RL}	ν_{TL}	G_{RL}	G_{LT}	G_{RT}	ρ
Red Maple	1.582	11.3	0.757	0.354	0.063	0.044	1.5	0.836	0.034	450
Sitka Spruce	0.866	11.1	0.477	0.255	0.04	0.025	0.71	0.677	0.033	450

---

[9]   See David E. Kretschmann, „Mechanical Properties of Wood", in: *Wood Handbook. Wood as an Engineering Material*, General Technical Report FPL-GTR-190, Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory 2010, 5–1 – 5–44; Christoph Buksnowitz, *Resonance Wood of Picea Abies*, PhD thesis University of Natural Resources and Life Sciences (BOKU), Vienna, 2006; and Neville Fletcher and Thomas Rossing, *The Physics of Musical Instruments*, 2nd edn., New York: Springer 2010.

Simulation results

For the numerical simulations, the body of the viol was surrounded by a spherical air domain of 70 cm radius, with a non-reflecting boundary condition (corresponding to an anechoic chamber). The inclusion of this air domain is necessary for accurate simulations (in contrast to structural simulations that only take into account the vibration of the instrument body); apart from imposing an extra boundary load to the surface of the instrument due to the surrounding fluid, it also allows for predicting the air resonances that are built within the viol, the lowest of which is crucial for the acoustical properties of any bowed-string instrument.[10] The first two internal air resonances are depicted in Figure 3. In the first one, the whole air cavity is oscillating in phase (at a frequency of 123 Hz) and air is radiated through the sound holes, giving sufficient support to low frequency notes, where structural resonances are absent.
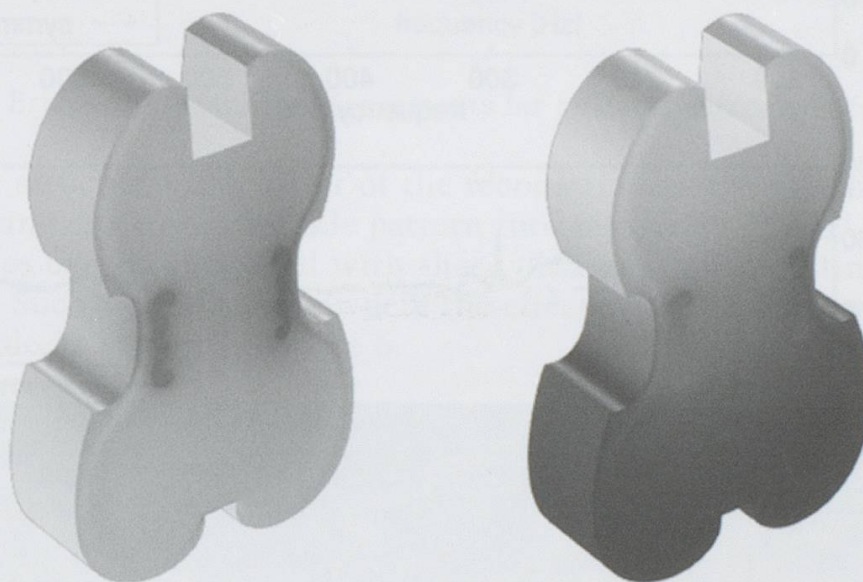

Figure 3:   First two internal air resonances of the viol body, at 123 Hz (left) and 296 Hz (right). Graphics IWK

With the aid of numerical simulations, it is possible to predict the outcome of a quality testing carried out via admittance measurements. The acoustic efficiency of the postulated asymmetric design can be thus compared to an equivalent symmetric geometry, with a constant top-plate thickness of 4 mm. It can be observed (see Figure 4) that even though the bridge mobility is generally larger in the symmetric case, the sound radiated from the instrument is greater in the case of the asymmetric geometry. This can be explained by the

---

[10] Murray Campbell, Clive Greated and Arnold Myers, *Musical Instruments. History, Technology & Performance of Instruments of Western Music*, Oxford: Oxford University Press 2004.

fact that symmetric modes of vibration can result in an acoustic short-circuit effect, reducing the amount of sound radiation to the far field. The asymmetry of the structural modes (usually enforced in modern instruments by the usage of a bass bar and sound post) is enforced here by the variable thickness of the viol body. The effect of the air resonances inside the instrument cavity is illustrated by the sharp peaks (high Q-factor resonances) in the acoustic efficiency plot. They assist in reinforcing the radiation of the viol, especially in those frequency regions, where the bridge admittance retains low values.
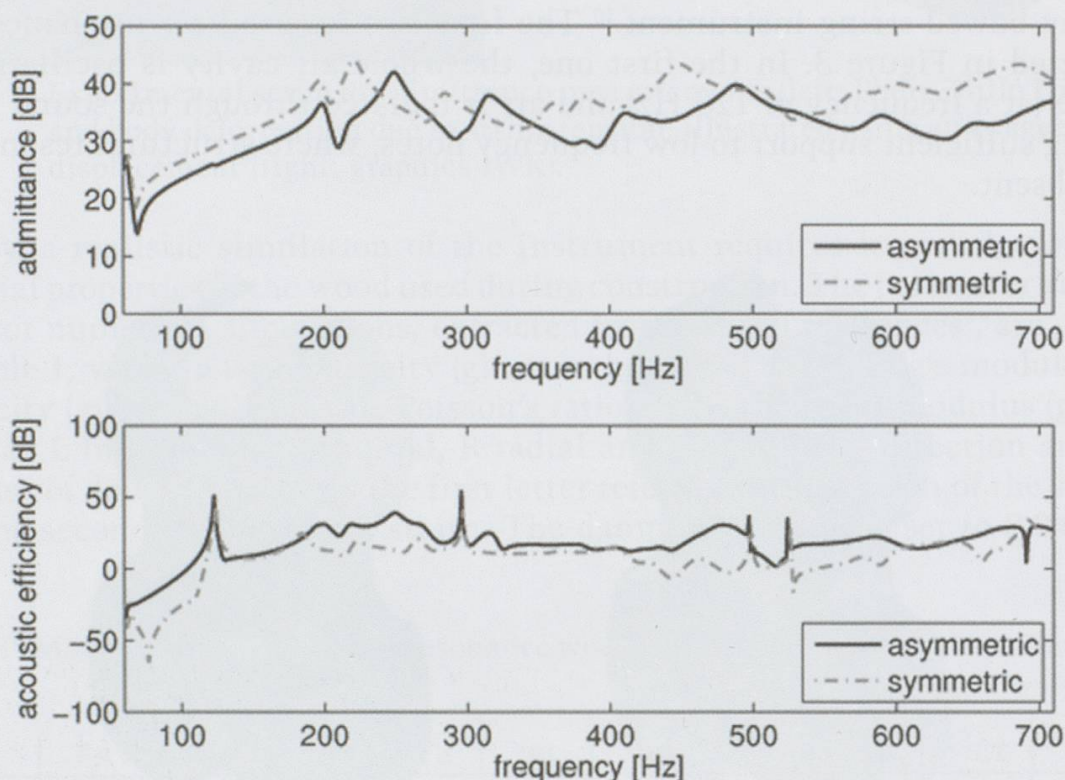

Figure 4:   Simulated bridge admittance (top) and acoustic efficiency measured 0.5 m in front of the sound holes (bottom) for an instrument with an asymmetric (black solid line) and a symmetric (grey dashed line) top plate.

Experimental measurements

Three instruments were constructed based on the previously discussed geometry, two of which were transported to the Institute of Music Acoustics in Vienna for experimental measurements. The admittance of these instruments, measured inside an anechoic chamber, is depicted in Figure 5. Using an impulse hammer for the bridge excitation, allowed for tracking the applied force Fhamm which in practice is not constant. Dividing all frequency-domain data by Fhamm renders them comparable to simulations, using an excitation force of 1 N. Several additional structural resonances are present in the measured curves, as expected due to the complicated nature of the instrument, but the overall trend is similar to that of the simulated curve. Furthermore, both

experimental and simulated admittance curves show that, in the region of the first strong resonances, the system energy is spread along neighbouring peaks, with no admittance peak being distinctly higher. This results in the absence of wolf tones that often appear in bowed string instruments.
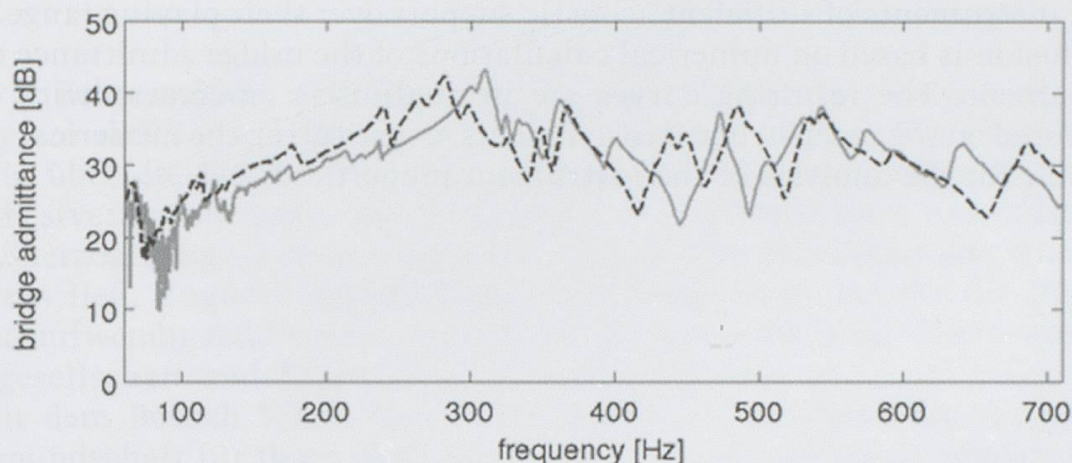

Figure 5:   Bridge admittance measurements for two reconstructed instruments

Finally, the structural behaviour of the reconstructed instruments has been analysed using electronic speckle pattern interferometry (ESPI)[11] and the observed modes can be compared with those obtained from the finite element simulation. Such a comparison, where the effect of the asymmetric design is clearly visible, is shown in Figure 6.
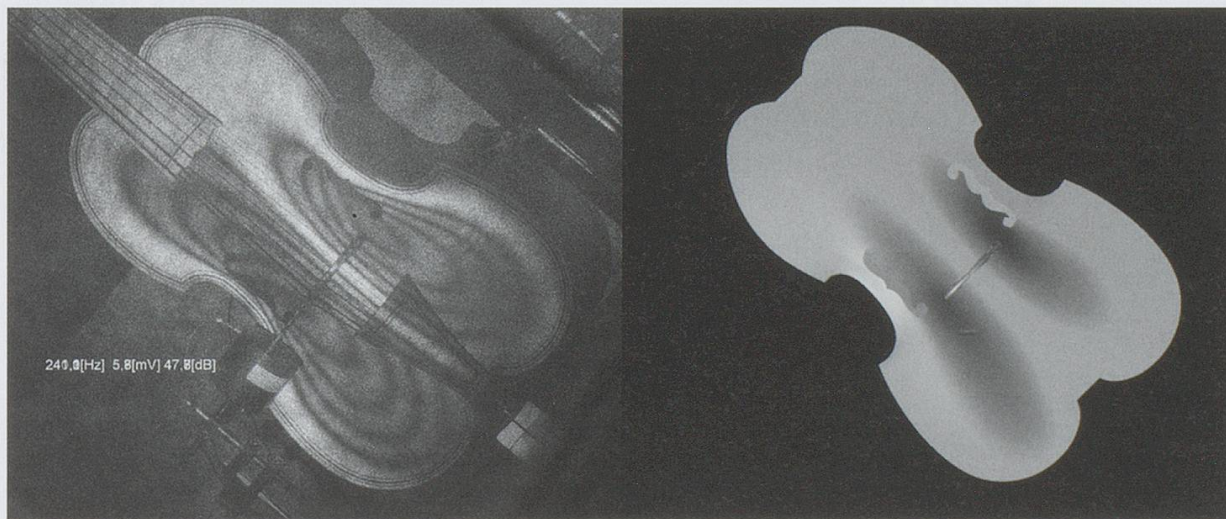

Figure 6::The first significant mode of vibration observed using ESPI (left) and simulation (right). Photo/graphics: IWK

Summary

[11] Thomas Moore and Sarah Zietlow, „Interferometric studies of a piano soundboard", in: *Journal of the Acoustical Society of America* 119 (2006), 1783–1793.

The structural and acoustic behaviour of a viola da gamba by Ganassi is analysed, using physics-based numerical simulations. An asymmetric design of a viol, without a bass bar or a sound post, is assumed and numerical simulations, using the finite element method, are employed to predict the oscillations of the instrument. It turns out that the hypothesised asymmetric design may yield instruments of sufficient acoustic support over their playing range. This conclusion is based on numerical calculations of the bridge admittance of the instrument. The resulting curves are in qualitative agreement with those measured on the reconstructed instruments, constituting the numerical model suitable for the analysis of the instrument properties.[12]