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REVERSIBLE MOTION AND IRREVERSIBLE EVOLUTION: QUANTUM KINETICS AND THE POSTULATE TEI (Time, Energy, and Information)

MOUVEMENT RÉVERSIBLE ET ÉVOLUTION IRRÉVERSIBLE: LA CINÉTIQUE QUANTIQUE ET LE POSTULAT TEI (Temps, Energie et Information)

BY

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ABSTRACT

In Einstein's SR theory, the 2nd postulate on the velocity of light lacks of universality by referring only to one specific fundamental interaction and by restricting the manifold to only four dimensions. One yields a new theory of Special Quantum Kinetics (SQK) by substituting to the former an Universal Postulate TEI about the structural coding of Time as coordinate of the event-space, of Energy as coordinate of the being-space, and Information as the parameter of motion-evolution, the latter recoded into Action (or Entropy when Time is simultaneously recoded into Natural Temperature). SQK recovers not only Newton's Classical Mechanics and Einstein's SR, but also de Broglie's and Schrödinger's Wave Mechanics (and Gibb's thermostatistics by the mentioned recoding). The relations PAM of passion, action and metrics are analyzed, and the fundamental symmetry TANTRA in the time-trajectory manifold is exhibited. In SQK, local determinism, quantum undeterminism and probability are all three necessarily rooted in the structure of differentiability and megethos (physical dimension).

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§ 1. INTRODUCTION SCIENTIFIC REVOLUTIONS AND DEREVOLUTIONS

It is now commonplace, both by the scientists and the philosophers, to consider the theories of Relativity (Special and General) and of Quantum Mechanics as two major scientific revolutions in the physics of this century. These two theories are certainly scientific revolutions, if one retains the sense given to that concept by T. S. Kuhn in his famous essay [1] of a change of paradigm in the practice of a specific scientific community. But more is generally claimed in the present case by such a denomination. One aims at an essential rupture and break of these theories with Classical Mechanics. One insists on the coalescence of space and time into a single continuum, the equivalence between mass and energy, between inertial mass and gravitational mass, etc., in the case of Relativity Theory, on the characteristic features of discreteness, undeterminism and probabilism, the blurring of the distinction between particle and wave, etc., in the case of Quantum Mechanics. This is certainly true regarding our old habits about discursive concepts, i.e. the concepts we have to use in our discourses for the description of the physical world. But, as Einstein did remark a long time ago, we generally use these concepts without remembering of the specific conditions in our practice which prompted us to call them into existence, conditions necessarily submitted to historical drift and change, and we are surprised when new developments in this practice compel us to call them again in question.

On the contrary, at the level of the structural concepts, the author's thesis is that such a rupture and break does not exist, that Relativity and Quantum theories are only two necessary readjustments of the same structural scheme as Newton's scheme of differentiable dynamics, and that consequently they can be submitted to the process of derevolution.

Here some precision about terminology is required. First, the structural concepts are those concepts which code our discursive concepts into a definite structure, in the present case a mathematical one, in order to give the status of a precise theory to the expression of our discourse, here about the physical world and especially about motion, evolution and historical change.

Second, derevolution refers to a kind of scientific revolutions which has escaped completely to Kuhn's attention [2]. Kuhn's analysis of scientific change, indeed, is essentially dualistic. Normal science is opposed to extraordinary science, and a scientific revolution is depicted as the change from a well established paradigma to a new paradigma, incommensurable with the first one. This is very reminiscent of the dialectic opposition between a thesis and an antithesis. The Kuhnian scheme can, and in the author's opinion has to, be completely dialectized with the introduction of the scientific derevolution as the third term of synthesis (in this sense it really functions as a kind of overrevolution): a more general structural frame is proposed, where both the old disthroned and the new disthroning paradigmas are assigned peacefully their respective place. Such was the case, in the 19th century, for the triple: Euclidean geometry, not Euclidean geometrics, and F. Klein's projective geometry [3].

For about ten years now, the claim has been made that such a processus exist for the Relativity Theory and Quantum Mechanics, at least for their elementary forms of Special Relativity (SR) and of Wave Mechanics (WM). First it was announced and presented in a rather schematic and programmatic way in some papers and one book [4]. More recently a structural reinterpretation of these theories has been developed in two simultaneous papers [5]. There is a serious advantage to work only at the level of structural concepts. There is no need to cope with a more or less subtle and difficult analysis of the related discursive concepts, and in particular one can simply forget about the peculiar physical situations and the specific historical problems which prompted the advent of the theories, e.g. the problem of the electromagnetism of moving bodies, the problem of the black body radiation, and the problem of the hydrogen atomic spectrum. As a matter of fact, at this level, the criteria of demarcation between Classical Mechanics (CM) and respectively a) SR and b) WM can be, and have been condensed in two simple prepositions. a) For SR: the (discursive) concept of (physical) time is structurally coded as a parameter of evolution in physical space by Newton, and as a coordinate of the space of the events by Einstein. b) For WM: the structure of WM can be, and has been ascribed to the duality between tangent and cotangent planes prevailing in any differentiable manifold (and not only in a Hilbert space, as e.g. Dirac's Bras and Kets), when due account is taken of the fact that physical grandeurs are not pure numbers (here called *c*-numbers after Dirac's terminology) but are μ-numbers, i.e. each endowed with a specific physical dimension (here called *megethos* in honour of Eudoxus) [6]. Thus the striking divergent characteristics of WM and CM, undeterminism and probability versus determinism, show to be in fact inherent properties of the differentiability structure.

But working only with structural concepts presents a serious inconvenient: the danger to be accused of an excessive formalism and of a lack of (physical) interpretation. This is the reason why the present paper aims at giving a more conventional analysis of the problem of derevolutionizing SR and WM, within a form more recognizable and understandable particularly by the physicists. It will be shown indeed that the simple substitution in SR of the 2nd postulate on the velocity of light, which is not of an universal character as referring only to the specific interaction of electromagnetism, by an universal postulate TEI about the structural coding of time, energy, and information yields a theory, the Special Quantum Kinetics (SQK), which could also properly be named after Newton-Einstein-de Broglie-

Schrödinger, since it not only recuperates SR in the same structural frame as Newton's CM but also encompasses de Broglie's and Schrödinger's WM (and even can be easily extended to Gibb's theory of canonical ensembles!)

It must be evident at this point that the present work proceeds necessarily from an essentially interdisciplinary research. Such a practice was even a necessary condition for the appearance of the derevolutions of SR and WM as a mere problem. First of course, there is the proper study of differentiable reversible motion and irreversible evolution in dynamical systems. The intentional restriction to the most elementary situation of the motion or evolution of a free material point system allows to treat the local and linear theories of SR and WM with an amazingly elementary mathematical apparatus: a *more graeco* calculus of proportions with differentials and derivatives. This is enough to exhibit the structural characteristics of the theory at the most intuitive level. For the same reason of intuitiveness, no recourse is made either to logic (which might be too crude: it is trivially true that two theories with different predictions are logically incompatible) or to axiomatics (which is too far away from intuition, especially regarding Quantum Mechanics). Still for the same reason important actual notions and problems have been deliberately disregarded: nothing will be said here about quantum fields and their interactions, about non-linearity and non-locality. That is not to say that they necessarily escape from the scope of the present theory. Only that this is matter for further intensive study.

Second, this scientific research is interwoven in an essential way with correlative studies both in the history and in the philosophy of the sciences. The proposed reconstruction of the present theories of motion and evolution necessitates tracing back the origins of the formation of their principal concepts (e.g. time itself) [7], in particular to inquire when, how and why those concepts were abstracted from practice and erected to their status. That means a serious (critical) knowledge of all the history of the various, first philosophical and later scientific, representations of motion, from Eudoxus' elementary kinematics to the present sophisticated theories of dynamical systems and of gauge fields. In one sense, this requires a deepening, an enlarging and a continuing of Mach's famous work about the critical history of mechanics.

Philosophy of science is called for at least in three of its branches: epistemology, ontology and methodology. For the first two, one can only refer to other papers. The author develops an original *epistemology of research*, called *SHCD* after the initial letters of its three main components: it is structural (à la Bourbaki, not à la Lévi-Strauss!), historical, critical, and dialectical (see above) [8]. Ontology, an appalling word to most hard empiricists, but an unavoidable field as any theory has to bear on definite entities (the "metaphysical" problem of the relation of these entities to the "out there" reality is something else, of semantic order), has to say about the nature of entities like time, energy, information, and so on [9]. For the third branch, there is less question of methodology as such than of a definite method, the procedure of double coding in discursive and structural concepts, which was already alluded to, but needs some more precision in the presentation.

§ 2. THE PROCEDURE OF DOUBLE CODING IN DISCURSIVE AND STRUCTURAL CONCEPTS

This is the most recent stage in the development of ideas held by the author in the early seventies. It is the occasion for him to apply his own conception of the slow emergence of the structuring reason as the characteristic feature of scientific rationality. He began by significantly opposing language to discourse in the physical theories. For instance, Ptolemaeus' and Copernicus' systems of the world are two different discourses in the same mathematical language (cycles and epicycles) referring to the same observed world (thus the deficiency of the theories of meaning of the scientific theories!), whilst the same discourse of Maxwell's electromagnetism can be displayed in different mathematical languages: quaternions, vectors, tensors or differential forms. In his 1979 essay [2] he made the opposition more precise: he spoke of the language of the (bourbachic) structures and of the regulative concepts in the discourse. Then in his 1982 papers [4] he conceived of the double coding of experimental and observational data into concepts and structures. Realizing that concepts are also present in the structures, and that the theories, at least of mathematical physics, in fact use *two* complete languages, each endowed with both a syntax and a semantic, one for the description of the physical world, thus with discursive concepts, and the other for the description of the mathematical structure, thus with structural concepts [10]. Typical of the first ones are time (absolute or relative), energy, physical space, etc., and of the second ones are parameter, coordinate, tangent vector, etc.

This procedure shows to be a magnificent methodological tool. It has been used for the discrimination of the three modalities of physical time that are motion, evolution and history [11]. It has been used previously and *de facto* in the determination of the demarcation criterium between Newton's and Einstein's dynamics by the structural coding of (discursive) time as a parameter, respectively a coordinate. It can also provide a possible explanation of this fantastic blindness of Schrödinger, who, at the very beginning of the first of his four famous 1926 papers [12], puts the relation $S = K \log \psi$ for the dèvelopment of his Wave Mechanics, without any recognition of the structural identity of this relation with the Boltzmann equation: from the very start, his theory was probabilistic! [13] Finally, for the time being, it plays an essential role in the setting up of SQK through the postulate TEI (see § 6).

§ 3. SQK IN A NUTSHELL

Consider the Newton-Einstein axiom: $p = mv$. Actually this is the first definition and the local version of the first law in the *Principia*. The extension of this axiom to Einstein is valid in so far as time is coded in a coordinate. Using the standard quadridimensional notation, then $p^4 = mv^4 = mc$, as $dx^4 = cdt$ due to the homogeneity of the megethos of the coordinates of an event (part T of Post TEI). By the same token, the two other laws of Newton's dynamics (which is really a kinematics) can be extended to the fourth component of the force: $F^4 = dp^4/dt = (dp^4/dp)$ $(dp/dt) = v \cdot F$, and to the conservation of energy (as the fourth component of linear momentum: $d(p_4{}^I + p_4{}^{II}) = 0$ (action = reaction).

Now inverse the *N-E* axiom: $1/p = 1/v \cdot 1/m = c^2/v \cdot 1/mc^2$. As in SR $E = c^2m$ (here obtained from the proposed PAM relation between passion, action and metrics, see § 5) and as $V = c^2/v$ by the Brillouin-de Broglie relation (here obtained from the symmetry TANTRA between time and trajectory, named after the French words: *temps-trajectoire*), one yields $E = Vp$ and $ET = p\lambda$ as $\lambda = VT$. It is interesting to note that all the ideas of de Broglie boil down to the photonization of the particle, where the phase velocity V is substituted to the light velocity c. Then $E = cp$ comes opposed to $E = Vp$, $\lambda = cT$, to $\lambda = VT$, and even $cdt = c/c\,dx$ to $cdt = V/c\,dx$, this last equality yielded directly by TANTRA symmetry.

Finally, Schrödinger's theory is recaptured from the part I of Post TEI (information is the parameter and is coded as action: $S = -h\xi$) and the fact that the unit tangent vector $d/d\xi$ obtained from the parameter ξ has the squared norm -1 for reversible motion and $+1$ for irreversible evolution. Hence, in the case of dynamics (reversible motion), the simple eigenvalue problem $\dfrac{d}{d\xi}\,\psi(\xi) = \pm\, i\psi(\xi)$ and the following sequence of solutions which transform the parameter as a trajectory into a progressive wave: for $-i$,

$$\psi \sim e^{-i\xi} = e^{i\frac{S(\xi)}{h}} = e^{i\frac{px-Et}{h}} = e^{-i\frac{\tau}{\tau_0}} = e^{-i\left(\frac{t}{T}-\frac{x}{\lambda}\right)}.$$

There the third term is given by embedding ξ into spacetime, the fourth one by reparametrizing action into proper time $\tau\,(S = -E_0\tau)$, and the last one by performing a Lorentz transformation on proper time τ, with a convenient definition of the Lagrangian time $T = \tau_0\sqrt{1-\beta^2}$. The justification of all those derivations follows, as well as the discussion of irreversible evolution in the canonical ensemble, when the natural temperature ϑ (defined as minus the inverse of absolute temperature) is conceived as an imaginary time (in conformity to Kubo-Martin-Schwinger's formalism), and information is encoded in entropy. This presentation in a nutshell should raise in the reader, the alluded impression of a fantastic triviality of all kinetics at the structural level (but only at this level!)

§ 4. SOME CRITICAL HISTORICAL REMARKS ABOUT MOTION, TIME, VELOCITY, AND DYNAMICS

It is still a lively tradition, surely in almost all philosophers and probably in most scientists, to conceive the natural development of the study of motion in the following order. First there is physical space, endowed with its Euclidean geometry. Then the introduction of time as a parameter allows to build kinematics, with its typical concepts of velocity and acceleration. Finally the introduction of mass opens the field of dynamics, with the typical concepts of momentum, force, and energy. This is surely the historical order which is respected scrupulously in all highschools of the world. But today such an order is subject to strong objections. It is no longer accepted plainly that geometry should be more elementary than kinematics, and that the latter be more elementary than dynamics. It is even questionable whether the three famous Newton's laws of dynamics are truly dynamical. In the same vein the careful examination of an apparently so intuitive concept as velocity can display many instructive, epistemological surprises. Due to lack of place only brief indications will be given here.

It is fascinating to follow the way that time itself has been deprived of its discursive richness in the course of history up to its reduction to the status of a real variable by Newton. Here Aristotle's modelization of all change after the pattern of local motion, and Descartes' decisive transformation of the status of motion from change to state have been both decisions of primary relevance. In spite of Heidegger, who sees in the works of Plato and Aristotle the majestic beginning of the decline of western thought, this extreme reductionism has been the price to pay in order to get the key to complexity. As in biology the reduction of the fantastic variety of living beings to the simplicity of the genetic code has been the necessary condition to the first steps into a real understanding of that complexity, so the reduction of common discursive time to mechanical time has opened the door to the comprehension of more and more complex systems. The richness of time is on its way to become recovered. In particular, it is now possible to distinguish clearly at least three modalities of physical time: the reversible time of mechanical motion, the irreversible orientable time of evolution, and the irreversible oriented time of history. This problem has been analyzed in a more philosophical paper [14] by means of the procedure of double coding.

Here we are concerned with another aspect of the structural coding of time, as a parameter *or* a coordinate [15]. A serious defect of CM is to code ambiguously the discursive concept time both as a parameter *and* a coordinate. This ambiguity is suppressed in SR, where the structural coding of time as a coordinate t entails the necessary introduction of another entity, the proper time τ, in the role of the parameter. At rest, t and τ can be totally identified, which is the source of the confusion in CM.

The feeling that time and space are of a very distinct nature (are different megethos' as it is said, here), that time is absolutely independent of space, has constituted a formidable obstacle in the kinematical problem of the composition of the velocities of two bodies A and B moving uniformly in the same direction relatively to a referential frame K. If A moves with velocity v_1 relatively to B, and B with velocity V relatively to K, what is the velocity v of A relatively to K? Before the advent of the differential calculus, Galileo had yet given his solution, very well argumented, to this problem: simply add the velocities: $v = v_1 + V$. On the other hand rather early in the development of the calculus, it was recognized that, in the case of a (numerical) function y (in the role of the ordinate) of the variable x (in the rôle of the abcissis), the ratio $dy/dx = tg\alpha$, where α is the (variable) angle of the direction of the tangent to the curve with the x-axis. But for the function position x of the parameter t, if it has been put quickly that $dx/dt = v$, with v along the geometric tangent to the trajectory, this velocity v was never conceived as the ratio of the differential of two coordinates, and then as a *trigonometric tangent*. Up to this century this conceptual obstacle prohibited the raising of the question: in the case of the composition of two tangents, what one must add first: the tangents themselves, as Galileo boldly did when such a problem did not yet exist in his conceptual horizon, or first add the arguments and then take the tangent of their sum, which Einstein really did, at least at the structural level if not in his historical work. (Naturally the hyperbolic Minkowski metrics transform the circular functions into the hyperbolic ones, whence the actual Einstein formula: $v = \dfrac{v_1 + V}{1 + v_1 V/c^2}$.)
It is interesting to recognize here another intuitive criterium of demarcation between CM and SR: the mere inversion of order in the sequence of two elementary operations: adding and taking a tangent!

The above mentioned conceptual obstacle can be traced back in Greek Antiquity, to the famous mathematician Eudoxus, who not only has conceived of the first model of the heavens by means of an ingeniously articulated set of spheres, but has developed the calculus of proportions (later on axiomatized by Euclides) and has explicited the concept of megethos itself. It is really fantastic to realize that his conceptions have been in use for about two millenia, but also that, by the same token, they have raised two formidable obstructions, which needed about the same duration to be finally overcome. The first obstruction bears on our rational numbers. For Eudoxus as already for Pythagoras, only the natural numbers are numbers (the zero is not yet conceived as a definite object). Thus the ratios (in Greek: logos) of two natural numbers are not numbers, even if it is practically possible to develop a certain kind of reckoning with them (addition, later on multiplication). In this matter Eudoxus is still right within our own standards: for instance, the rational number $2/1 = 4/2 = 6/3 = \cdots$ is not the natural number 2, even if we can make the identification without trouble. Effectively we define the rational numbers as equivalence classes of pairs of natural numbers. This first Eudoxus' obstruction has been finally overcome, especially by S. Stevin in the 16th century, with an extension of the concept of number itself.

The second obstruction concerns the impossibility to compose different megethos' in order to build a new megethos. After the scandal created by the recognition of the irrationality of $\sqrt{2}$, which stroke a lethal blow to the Pythagorean dream that all is number, an answer was brought, in the realm of geometry, with the creation of the concept of measure *(metron)*. But in geometry there are entities of different nature: lines, areas, volumes, angles. Precisely these are the megethos' (= magnitudes) together with time and weight. Velocity is consequently excluded from this list. Indeed the ratio of the measure of a certain distance to the measure of a certain duration in a definite motion cannot yield the measure of a new megethos velocity. Only ratios of the same megethos are definable, e.g. the ratio of two distances traveled during the same time or the ratio of two durations used in travelling the same distance. That was only in the 14th century, at Merton College in Oxford and at the University of Paris that scholastics finally overcame this second formidable obstruction, in their smart abstract study of typical motions as uniform (still our meaning) and difform (accelerated) motions, by means of that kind of ancestors of the coordinates that were *longitudines* and *latitudines formarum* [27].

Jumping boldly to the other end of history, the very recent mathematical theory of differentiable manifolds (DM) provides us with a new clue about the relation between velocity and megethos, with the encoding of the former into a tangent vector. This vector is no longer the well known entity found in the elementary textbooks on vector calculus, and usually written $\vec{v} = \sum_i v_i \vec{e}_i$. In a DM M, when a natural frame of coordinates x^i has been selected, the tangent plane, a vector space, admits the partial derivatives $\partial/\partial x^i$ for its basis vectors, and thus $\vec{v} = \sum_i v^i \partial/\partial x^i$ becomes a differential operator on the maps on M. For a parametrized curve, the simple application of Leibniz' chain rule yields the tangent vector

$$v = \frac{d}{dt} = \sum_i \frac{dx^i}{dt} \frac{\partial}{\partial x^i}$$

Two observations can relevantly be made: first, the tangent vector presents a double face: 1) a total derivative, linked by Einstein, in his famous 1905 paper on the light quanta, with the description of particles, and 2) a linear expansion of partial derivatives, linked by Einstein with the description of waves! The author is convinced that the famous duality wave-particle finds its source in this situation. More on this point will be said below.

The second observation opens the door to a direct perspective on Quantum Mechanics. Mathematicians call velocity the tangent vector v, because, applied on the coordinates v, it gives the components v_j of the velocity. But for them, t and x are c-numbers. This situation cannot be satisfactory to physicists: for them, t and x are μ-numbers: t is endowed with the megethos time, and x^i with the megethos length. Thus it becomes necessary to introduce a correcting μ-factor λ of megethos length in order to conserve to velocity its own megethos. Using in addition Newton's axiom $p = mv$, then

$$p = p^i\lambda \frac{\partial}{\partial x^i} = \frac{p^i}{p}(p\lambda)\frac{\partial}{\partial x^i}$$

p becomes P^j when an active projection is made of p to the axis x^j. Then $P^j = p\lambda \dfrac{\partial}{\partial x_j}$. Up to the imaginary factor i and the de Broglie relation $p\lambda = h$, one gets the operator $\hat{P}_j$ in the Schrödinger picture of elementary QM!! [18]

Finally, a remark about the shift of the borderline between kinematics and dynamics. It is now well known that momentum-energy and spin are the Casimir operators of the Poincaré group, the group of rotations and translations in spacetime. That means that Newton's couple momentum and mass (p, m) for a particle is no more properly dynamical than the couple length and period (λ, T) for a wave is properly kinematical. Thence the proposal of the term *kinetics* in order to design the study of motion-evolution by means of the differential calculus. Consequently the famous three Newtonian laws of dynamics are only kinetical, and dynamics enters into the picture only when Newton gives his law of gravitation. That is now the definite position of the theoretical physicists: dynamics is the field of study of the (elementary) interactions. And indeed, the actual size of Planck's constant of action h, which plays a role in kinetics, can only be determined by its ratio to the coupling constants of the various interactions: e^2, G, g_w and still indirectly g_s.

§ 5. PASSION, ACTION, METRICS: THE RELATION PAM

Consider the following identity in c-numbers:

$$\frac{d\xi}{X}\cdot g_{\xi\xi}X\,d\xi \equiv g_{\xi\xi}d\xi^2 \tag{1}$$

and call the first factor passion P, the second action A, and the right member metrics M. Symbolically it can be written: $P \cdot A = M$ [19].

Now take $\xi \in \mathbf{R}$ as a parameter; then $d/d\xi$ is the basis vector of the tangent space $T(\mathbf{R}) \sim \mathbf{R}$ and $d\xi$ the basis covector of the cotangent space $T^*(\mathbf{R}) \sim \mathbf{R}$.

There is a Riemannian metrics $g_{\xi\xi} = (d/d\xi, d/d\xi)$. $g_{\xi\xi} = \pm 1$ as a consequence of the Frobenius theorem in a *flat* space (possibility to find a orthonormed natural frame of coordinates, with $g_{\mu\nu} = (\partial_\mu, \partial_\nu)$, the scalar product). It will be shown that the choice $g_{\xi\xi} = -1$ makes of ξ the parameter of a *reversible motion*, while $g_{\xi\xi} = +1$ makes of ξ the parameter of an *irreversible evolution*. Finally X is the component of a vector $\hat{X}$, an operator of derivation on the derivable functions of $\xi: \hat{X} = X \dfrac{d}{d\xi}$.

One obtains the structural transcription of a physical discourse by a *μ-reparametrization*, that means by endowing ξ and X with convenient megethos. For instance, for a particule at rest, ξ becomes μ-reparametrized in proper time τ, X becomes the rest mass m_o and $g_{\xi\xi}$ is recalibrated as $-c^2$. Then (1) becomes:

$$\frac{d\tau}{m_0} \cdot (-m_0 c^2 d\tau) = -c^2 d\tau^2 \tag{2}$$

which entails the justification of the denominations passion, action and metrics.

The most interesting situation is given by passing from the rest frame to a mobile frame in spacetime. Then, by evident transpositions, one yields the general PAM relation:

$$\frac{d\tau}{m_0}(g_{\mu\nu} p^\mu dx^\nu) = g_{\mu\nu} dx^\mu dx^\nu, \quad \mu, \nu \in \{1, 2, 3, 4\} \tag{3}$$

Here, a first result. When time t is taken as a coordinate of the space of the events, along with the coordinates x^i of physical space, it is necessary to introduce a μ-factor of homogeneization between the megethos of time μ (time), and the megethos of length, μ (length). By history and tradition, this μ-homogenization has been made in the sense: time is an additional coordinate to space as to form the continuum spacetime while the reversal should have been more consistent: changes in time can happen to the same place, but up to now never changes in place at the same moment of time have been observed (at least for science: the magic ubiquity belongs to a total other realm!) Then, using a celebrated notation, $x^4 = ct$, where c is a constant of megethos velocity. For the moment, nothing has to be known about the existence of a phenomenon moving at such a velocity c: thus one comes to depart from Einstein's original treatment. In a moment his second postulate for SR will be abandoned. What could *a priori* happen is a dependance of c on the instant τ, that means $c(\tau)$. This is the situation of the variation of the natural constants first seriously contemplated by Dirac. For reasons of symmetry (conservation of the rest mass of a free particle) the homogeneity of proper time and then, for SQK, of spacetime will be assumed, making of c an universal constant. Consequently, the Newton-Einstein axiom and the celebrated equivalence of mass and energy follow from the PAM relation:

From (3), one sees immediately that

$$p^\mu = m_0 \frac{dx^\mu}{d\tau} \tag{4}$$

which can better be written under the form of a bicharacteristic (Monge's language and theory!)

$$\frac{d\tau}{m_0} = \frac{dx^\mu}{p^\mu} = \frac{dx}{p_x} = \frac{dy}{p_y} = \frac{dz}{p_z} = \frac{dx^4}{p^4} \equiv \frac{cdt}{p^4} \tag{5}$$

This is equivalent to the local version of Newton's first law, with integration of the first definition of the *Principia*:

$$p^i = m\frac{dx^i}{dt} = mv^i; \quad p^4 = m\frac{dx^4}{dt} = mv^4 = mc \tag{6}$$

(Come back to § 3 for the extension of the two other Newton's laws to x^4.) Still from (3), now one obtains, with the Minkowskian metrics

$$\frac{dt}{m}\left(\textstyle\sum_i p^i dx^i - p^4 dx^4\right) = \textstyle\sum_i (dx^i)^2 - (dx^4)^2$$

By identification with the well known Hamiltonian formalism, in the case of a free particle, $p^4 = E/c$, and then

$$p^4 = mc = E/c \Leftrightarrow E = c^2 m\,. \tag{7}$$

After these first results, one proceeds further by retaining the hyperbolic signature of the metrics for the time component, as in "dynamics" time is a μ-reparametrization of the c-parameter ξ for a *reversible motion*, but giving up the 4-dimensionality of the manifold.

Thus, one is freeing oneself from the too restrictive specificity of Einstein's original postulate (on the velocity of light). For historical reasons, SR issued from the passionate efforts of Einstein to set up a more symmetrical interpretation of Maxwell's equations and especially of Faraday's induction experiment. Anyway, the title of his celebrated 1905 paper is exactly chosen: "Zur Elecktrodynamik bewegter Körper." The reformulation of kinematics of space and time has come to him as a necessary precondition to solving his problem, which imposed its necessity to him unavoidably. On that score it is remarkable that Einstein did neither think to give the Lorentz transformations for the linear momentum p and for the energy E (that was done by Planck, who was eager to know whether his constant of action was or not a relativistic invariant!) nor conceive of the momentum of the light quantum $E = cp$ before 1917 (and further on: $p = E/c = h\nu/c = h/\lambda$!!!) It is remarkable too that he regarded the mathematical treatment of his SR by Minkowski as a superfluous scholarly piece of work ("überflüssige Gelehrsamkeit"), anyway before engaging himself into his researches for GR, and that nevertheless up to his last days he constantly held that the 4-dimensional treatment was not absolutely necessary in the case of SR (see his "Autobiographisches") [20].

The 4-dimensionality of spacetime is necessarily related to the electromagnetic interaction. If "visible" is taken in all the extension of the electromagnetic spectrum of radiation, it can be said that 4 is the dimension of the visible world. Maxwell's equations retain their form only for a 4-dimensional manifold. This has been shown by H. Weyl for the 4-vector potential A_μ [21], and for the differential 2-form $B_{\mu\nu}$ by the author on a totally different ground [22]. Thus the renouncement to the original Einstein's second postulate of SR about the independance of the light velocity c from the state of motion of its source permits to consider any n-dimensionality for the manifold into which the parameter is embedded as a parametrized curve. In particular the case $n = 2$ turns out to be of the highest interest. It was prohibited for the photon, whose spin gives way to the phenomenon of polarization (yet Newton asked whether light had not sides, a question which has inclined E. Mach to see in Newton the true discoverer of this phenomenon, one century before Malus!). Leaving aside all questions about spin, or equivalently considering only particles of spin 0, the *manifold time-trajectory*, duly endowed with its hyperbolic metrics, exhibits a remarkable symmetry, named here *TANTRA*, which up to now has escaped observation. In the usual derivations of the Lorentz transformation (only x and t are taken), it is said that the coordinates y and z can be discarded without loss of generality. It should be said rather that, because the isotropy of physical space, the orientation of the direction of a determined motion is not singularized in any way.

Moreover, the identity $\sum_i(\cos\alpha_i)^2 = 1$ regarding the principal cosinuses of the angles of projection α_i of a determinate direction to each axis of an orthonormed frame in a n-dimensional manifold plays an ubiquitous role. In its essence, it is the n-dimensional version of Pythagoras' theorem in the Euclidean plane. There the above identity becomes $\cos^2 + \sin^2 = 1$. In the time-trajectory manifold, it will be transformed into $ch^2 - sh^2 = 1$ [23].

But before examining the symmetry TANTRA any further, it is worthwile to look at the place of the latter identity in the PAM relation itself. Apply this identity to (2). One obtains when (7) is used:

$$\frac{d\tau}{m_0}(-E_0 d\tau)(ch^2 - sh^2) = -c^2 d\tau^2(ch^2 - sh^2) \tag{8}$$

The identifications $dt = d\tau ch$, $dx = cd\tau sh$ and correlatively

$$E = E_0 ch, \quad p = p_0 sh\,(p_0 = m_0 c)$$

give:

$$\frac{d\tau}{m_0}(pdx - Edt) = dx^2 - c^2 dt^2 \tag{9}$$

Introducing the Hamilton-Jacobi function of action $S(x, t)$ one sees that passion remains unaltered, as

$$\frac{d\tau}{m_0} = \frac{dx}{p} = \frac{dt}{m} \tag{10}$$

while action (its differential dS) bears the signature of the metrics, as it has to be from the identity $-E_0 d\tau = \frac{m_0}{d\tau}(-c^2 d\tau^2)$. This is not trivial: the minus sign of the metrics is assigned to the time component and is determined by the relation (TEI!) between action, time and energy, i.e. $\partial S/\partial t = -E$. That should eliminate the often used alternative choice: $c^2 d\tau^2 = c^2 dt^2 - dx^2 = dx_0^2 - dx^2$. This is a general fact. It is commonplace that a parametrized curve in a n-dimensional DM M, ($x \in M$, $x^i(x) \in \mathbf{R}$ the n coordinates functions, $\xi \in \mathbf{R}$ the parameter) can be given by the system (passion!)

$$\frac{dx^1}{X^1} = \frac{dx^2}{X^2} = \cdots = \frac{dx^n}{X^n} = d\xi \tag{11}$$

It is also well known that, in such a system, ξ can be discarded and a coordinate be elected as the new parameter, e.g. x^n, at the condition that $X^n \neq 0$. That is exactly what CM does when it elects time as the parameter. Then a first integral (solution) of this system is given by:

$$\frac{dx^1}{0} = \frac{dx^2}{0} = \cdots = \frac{dx^{n-1}}{0} = \frac{dx^n}{X^n} \tag{12}$$

which means that all the change is along x^n, while the other coordinates are kept constant.

That is exactly what Minkowski does in his reconstruction of SR with his *World Postulate*: For any substantial (i.e. $m_0 \neq 0$) system, it is always possible to find a referential frame where the system is at rest. Moreover, with the passion (11), it is possible to construct the PAM relation: action and metrics, both for the x^i's and the X^i's; by means of the calculus of proportions [24]:

$$\frac{dx^n}{X^n} = \frac{g_{ik} dx^i dx^k}{g_{ik} dx^i X^k} \quad \text{and} \quad \left(\frac{dx^n}{X^n}\right)^2 = \frac{g_{ik} dx^i dx^k}{g_{ik} X^i X^k} \tag{13}$$

i.e. symbolically: $P \cdot A = M_{(x)}$ and $M_{(x)}/P^2 = A/P = M_{(X)}$. The fact that the X^i generate a manifold of the same dimension n and with the same metrics as M means, in dynamics, that the space of the events (of coordinates x^i) has to be parallelized by another space (of coordinates $X^i = p^i$). We are back to the already emphazised defect by Einstein. He does not parallelize spacetime with the manifold momentum-energy. This observation can be made more acute. In his 1905 paper, Einstein is very clear about events: an event e is determined by a date and a place.

But he totally neglects to say what e is the event of. "Is" or "happens" has to be said of something, not necessarily material. Minkowski chose substantial. This word will be rejected here because its too "metaphysical" connotations. One could better speak of a what-space, or a quid-space, or a tí-space as parallel to the event-space. Finally the word *being* (Seiendes, on, ens, étant, etc.) will be retained here and one will speak of the being-space. In this choice one has been guided someway by Heidegger and his *Sein und Zeit*: in a rest frame, the event e is only dated by the proper time τ, and the being b is determined by its rest energy E_0. Those considerations are made precise when structurally coded into the postulate TEI.

§ 6. THE POSTULATE TEI

It is high time to come to this postulate TEI, which takes the place in SQK of Einstein's 2nd postulate in the original SR. As already amply emphasized, it must have an universal character. The precedings sections have given all the necessary material in order to understand now its statement:

Postulate TEI

T: Time is structurally coded into a coordinate of the event-space.

E: Energy is structurally coded into a coordinate of the being-space.

I: Information is structurally coded into the parameter of motion-evolution. Usually the latter is variously μ-reparametrized.

Here are some μ-reparametrizations of information: a) proper time in kinematics (old meaning); b) action in dynamics (old meaning); c) entropy in Gibb's theory of the canonical ensembles: then time is also μ-reparametrized into natural temperature; d) other choices are not excluded [25].

§ 7. SOME FIRST CONSEQUENCES OF THE POSTULATE TEI

Consider first the case of dynamics, where at rest $dS = - E_0 d\tau$. Thus the megethos action is the product of the megethos energy by the megethos time. It can be said, introducing the notion of μ-inversion, that energy and time have action-inverse megethos'. As information is a c-number, by the I Part of Post. TEI it would be sane to take action also as a c-number. That has been the position held by H. Weyl, for instance, but his motivation, at least in his book [6, 1921], is based upon a rather weak reason: in Quantum Theory, action appears always in integer numbers of quanta (that was indeed before the discovery of the half spin of the fermions!). With action as a c-number, the megethos' of energy and time would be really inverse: μ (energy) $= \mu^{-1}$ (time).

Anyway, by this part I of Post. TEI, there must exist a calibration factor between information and parameter $dS = -hd\xi$. In effect, in the case of the free particle, the rest mass E_0 is a constant, it is conserved during the motion, and then $S = -E_0\tau$ up to an additive constant (one has still $dS/d\tau = -E_0$). *A priori*, h could be dependant on ξ. But for the same reason of homogeneity as previously, in the simplest theory, h is taken as an universal constant.

As $dS = -hd\xi = -E_0 d\tau$, one gets the kinematical μ-reparametrization into the proper time as $d\tau = h/E_0 d\xi \equiv \tau_0 d\xi$. Here τ_0 is a natural standard of proper time, while it is arbitrary when the mass is taken out of the picture as that happens in the usual case. It can be said then that kinematics is a dynamics which forgets about mass (for instance, the dynamical principle of least action becomes simply converted into the kinematical principle of maximal proper time). It is clear that, in this case, the PAM relation is blurred trivially: (2) becomes $d\tau \cdot (-c^2 d\tau) = -c^2 d\tau^2$!

The constancy of h can be argued too on the following lines. In a 1-dimensional manifold each differentiable 1-form is a multiple of any other one. Then,

$$c^2\tau_0 d\tau = (m_0 c^2 \tau_0)\frac{d\tau}{m_0} = h\frac{d\tau}{m_0} \tag{14}$$

appears as a recalibration of the passion. In one dimension the argument seems rather weak, but in two dimensions it gives the most solid derivation of de Broglie's solution $p\lambda = ET = h$. Indeed, two expressions of the null 1-form must be proportional. On the one hand, (10) gives $\frac{dx}{p} - \frac{dt}{m} = 0$. With the same identifications of $dt = d\tau ch$ and $dx = cd\tau sh$, one must have $c\tau_0 sh^{-1}dx - c^2\tau_0 ch^{-1}dt = 0$. It is easy to show that $c\tau_0 sh^{-1} = \lambda$ and that $\tau_0 ch^{-1} = T$ (see below). Then $\lambda dx - c^2 T dt = 0$ and by (14), one yields

$$\lambda dx - c^2\tau dt = h\left(\frac{dt}{p} - \frac{dt}{m}\right) \Leftrightarrow \lambda = \frac{h}{p} \quad \text{and} \quad T = \frac{h}{E} \tag{15}$$

Einstein-de Broglie's relations

By their definitions,

$$\frac{\lambda}{T} = c\left(\frac{ch}{sh}\right) = \frac{c^2}{v} = \frac{c^2 dt}{dx} \overset{\text{det}}{=} V \tag{16}$$

V is really the phase velocity, but has been obtained without any reference to a wave! On the contrary it is associated to the inversion of the role of dx and dt obtained by their mutual exchange in the metrics: that is precisely the symmetry TANTRA (to come in § 9).

§ 8. VELOCITY AND THE COUPLES (x, t), (p, m) AND (λ, T)

1) First, always in the case of a free particle, with constant rest energy E_0, from the identifications $E = E_0 ch$ and $p = p_0 sh$, one derives

$$(17) \qquad \frac{dE}{dp} = c \cdot th = v \quad \text{and} \quad \frac{E}{p} = c \cdot th^{-1} = \frac{c^2}{v} = V$$

$$\text{and then} \quad \frac{dE}{dp} = \beta^2 \frac{E}{p}$$

which is nothing else than $c^2 pdp - EdE = 0 = d(E_0^2)$!

2) Second, one refers to the well known Hamilton-Lagrange (HL) formalism. It is well known for energy, but up to now it has not been clearly recognized that the same formalism applies to time: t is the Hamiltonian time, and T is the Lagrangian time. This is equivalent to the recognition by L. de Broglie of two different kinds of frequenties: $\nu_H = \nu_0/\sqrt{1-\beta^2}$ and $\nu_L = \nu_0\sqrt{1-\beta^2}$ (the author's indexes!). He has been right in his evaluation of the importance of these dual frequencies. But he has been totally wrong in his substantiating them with different kinds of clocks ("les petites horloges"), as in his substantiating the material waves: he was forced to involve an *ad hoc* principle of phase concordance, in a very mysterious way. In fact, they are only two dual expressions of time [26].

Consider the identities:

$$(18) \qquad \begin{aligned} E_0^2 &= E_0 ch \cdot E_0 ch^{-1} \equiv E(-L) \\ \tau^2 &= \tau ch \cdot \tau ch^{-1} \equiv t \cdot T \\ d\tau^2 &= d\tau ch \cdot d\tau ch^{-1} \equiv dt \cdot dT \end{aligned}$$

The first identification is made by means of the well known relation in the HL formalism $dS = Ldt$. Then

$$(19) \qquad dS = -hd\xi = Ldt = -E_0 d\tau = -HdT$$

where the last member is new. Similarly

$$-S = E_0\tau = ET = p\lambda = h\xi$$

For a free particle, as E_0 is constant, $T = \tau_0 ch^{-1}$ and $\lambda = c\tau_0 sh^{-1}$. It can be noticed once more that $T = h/E$ and $\lambda = h/p$. Their differentials are respectively:

$$dT = -\tau_0 shch^{-2} d\vartheta \quad \text{and} \quad d\lambda = -c\tau_0 chsh^{-2} d\vartheta,$$

whose ratio is:

$$(20)\qquad \frac{d\lambda}{dT} = c\cdot\frac{ch^3}{sh^3} = \frac{c^4}{v^3} = \frac{V^2}{v}$$

what makes v a group velocity, once more without any reference to a wave! (Naturally, one could also use $dT = -hdE/E^2$ and $d\lambda = -hdp/p^2$ for the same result). One notices also that

$$(21)\qquad \frac{d\lambda}{dT} = th^{-4}\frac{dx}{dt}$$

which makes complete the correspondences between the couples (dx, cdt), (p, mc) designed for a particle and (λ, cT) designed for a wave, for the expression of the velocity (or velocities) [27].

Still two remarks, useful for the symmetry TANTRA:

a) It is known that an hyperbolic metrics is indefinite and admits of three kinds of vectors: timelike (<0), spacelike (>0) and isotropic ($=0$). As (x, ct), the couple (p, cm) is timelike, while (λ, cT) is spacelike (Proof: $\lambda_0^2=\lambda^2-c^2T^2 = \dfrac{c\tau^2(ch^2-sh^2)}{sh^2ch^2}>0$). Thus:

$$(22)\qquad \lambda = \lambda_0 ch = c\tau sh^{-1}, \quad cT = \lambda_0 sh = c\tau ch^{-1}, \quad \text{then} \quad c\tau = \lambda_0 chsh$$

(Even if $c\tau=c\tau_0=$constant, λ_0 is not ($c\tau_0$ is then Compton's length)):

b) It is clear that if p_0 = constant, then $c\tau_0 = h/p_0$ is also constant. The inversion of each component of a vector changes its likeness:

$$(23)\qquad -p_0^2 = p^2-(mc)^2 < 0 \Leftrightarrow \frac{1}{p^2}-\frac{1}{mc^2} = \frac{p_0^2}{p_0^4 sh^2ch^2} = \frac{\lambda_0^2}{h^2} > 0$$

In summary, the exchange between timelike and spacelike can be obtained by two different ways: a) exchange the time and space components; b) inverse each component. This remark leads us finally to the presentation of TANTRA.

§ 9. THE TIME-TRAJECTORY SYMMETRY TANTRA

It is not so easy to reduce to the concatenation order of writing a structure partaking rather of a lattice. Nevertheless that has to be done here regarding the Lorentz transformation and the symmetry TANTRA.

The derivation of the Lorentz transformation for the time component presents no special trouble. It is sufficient to pass from the rest frame of the particle to a frame $K(x, t)$ where the latter moves with velocity $v = dx/dt$. In (8) a first application of the identity $ch^2 - sh^2 = 1$ to the metrics $-c^2d\tau^2$ has given the identification $dt = d\tau ch$ and $dx = cd\tau sh$. Apply it again to $cd\tau$ itself:

$$d\tau = (ch^2 - sh^2)d\tau = ch(chd\tau - thshd\tau) = \frac{dt - \frac{v}{c^2}dx}{\sqrt{1 - v^2/c^2}} \tag{24}$$

For the x component, the derivation is less direct. First, one can pass to the hyperbolic rotations, as Minkowski did. $ch^2 - sh^2 = 1$ is the determinant of the matrix in such a rotation in the plane $\{x, ct\}$

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} ch & -sh \\ -sh & ch \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} \tag{25}$$

Here TANTRA comes to show up its nose! (25) will be written again under the precise form given to it by Einstein in his *Grundlagen* (1916), where $a = ch$ and $b = sh$

$$\begin{aligned} x' &= ax - bct \\ ct' &= act - bx \end{aligned} \tag{26}$$

In this form, TANTRA is evident. There is only one unique formula when the permutation of x and ct is performed. This property is specific to hyperbolic rotations. In usual Euclidean rotations, this symmetry is lost because the difference of sign for the sinus in the matrix of rotation.

Thus the symmetry TANTRA was lying open to Einstein's eyes but remained a "hidden truth" to him, and for many others [28].

The exchange $x \leftrightarrow ct$ (equivalently $ch \leftrightarrow sh$) transforms a timelike vector into a spacelike vector, and vice-versa. In effect, defining:

$$TANTRA \quad \begin{cases} dx_T = cdt \\ cdt_T = dx \end{cases} \quad (c_T = c)! \tag{27}$$

one yields for metrics:

$$-c^2d\tau^2 = dx^2 - c2dt^2 = c^2dt_T^2 - dx_T^2 = +c^2d\tau_T^2$$

Now, applying the procedure (8) and (24) to $cd\tau_T$, one yields

$$c^2d\tau_T^2 = dx^2 - c^2dt^2 \Leftrightarrow \begin{aligned} dx &= chcd\tau_T = cdt_T \\ cdt &= shcd\tau_T = dx_T \end{aligned} \tag{28}$$

and

$$cd\tau_T = ch(chcd\tau_T - thshcd\tau_T) = \frac{dx - vdt}{\sqrt{1 - v^2/c^2}}. \tag{29}$$

The reason lying under TANTRA is the total equivalence, in the time-trajectory manifold, of the choice of the time component ct or of the space component x in order to reparametrized the proper time $c\tau$ (all three have the same megethos). If usually $dx = vdt$ is defined (and even $x = vt$ for uniform motion), that means if x is parametrized by t as $x(t)$, there is no reason to exclude the dual possibility to define $dt = v^{-1}dx$ (and $t = v^{-1}x$ for uniform motion) by the parametrization of t by x as $t(x)$. In that case, one must not forget that x plays the role of "time" integrally! Then, if

$$(30) \qquad \frac{dx}{cdt} = \frac{v}{c} = th\,, \qquad \text{then} \qquad \frac{dx_T}{cdt_T} = \frac{c}{v} = th^{-1} = \frac{v_T}{c} \overset{\text{det}}{=} \frac{V}{c}$$

Recall the last remark of section 7. These is a rotation between the exchange $x \leftrightarrow ct$ and the inversion of the components, here of their ratio [29].

Now one gets immediately the Brillouin-de Broglie kinematic relation

$$(31) \qquad vV = c^2$$

which, by μ-reparametrization, yields the Quantum Einstein and de Broglie relations (up to the constancy of the product, which has been derived previously (eq. 15))

$$(32) \qquad mvVT = p\lambda = ET = E_0\tau$$

One can close this section by the following list of equations now immediately understandable by its symmetry:

$$(33) \qquad \frac{E}{cp} = \frac{mc}{mv} = \frac{c}{v} = \frac{cdt}{dx} = \frac{dx_T}{cdt_T} = \frac{V}{c} = \frac{TV}{Tc} = \frac{\lambda}{cT}\,.$$

§ 10. THE RECOVERY OF EINSTEIN'S ORIGINAL SR

The essentials of de Broglie's theory have been recovered by SQK. Now it is an imperative necessity for the latter theory to recover all of SR, and especially Einstein's original second postulate, which has been destituted of this status in SQK.

First, the law of composition of velocities. This subject has been discussed in section 4. It may be difficult to hard empiricists to swallow the idea that a velocity, "naturally" associated to a translation, is in fact a rotation in a space and time plane. But as this rotation can act only along an unique spatial axis, its effect is similar to a translation. The same situation arises with the electric field, acting as a polar vector, while it is also an axial vector in space and time. For those people, they can refer to their standard textbooks, or to the note [30].

Second, time dilatation and length contraction. As for the derivation of the Lorentz transformations, the timelike part is immediate: $dt = d\tau ch$, while the spacelike part relies also on TANTRA: with the help of (22), $cdT = cd\tau ch^{-1}$. It should be clear that the time component of a spacelike vector corresponds to the space component of a timelike vector.

Third, light and its non dependance on the velocity of its source. A ray of light is an isotropic vector, i.e. $c^2 d\tau^2 = 0$, $d\tau = 0$, $m_0 = 0$ and $dS = 0$. In SQK, light is the phenomenon travelling in space with a velocity precisely equal to the constant μ-factor c. Then

$$x = ct \quad \text{and} \quad dx = cdt \quad \text{then} \quad v = V = c \tag{34}$$
$$dx^2 - c^2dt^2 = 0 \quad \text{and} \quad xdx - c^2tdt = 0 \quad \text{(Lorentz self orthogonality)}$$

Then

$$th\vartheta_l = th\vartheta_l^{-1} = 1 \Leftrightarrow \vartheta_l = \infty \tag{35}$$

Consequently, on can add to or substract from ϑ_l *any finite velocity* v, the argument (now called rapidity by some physicists) remains infinite and $th(\vartheta_l \pm v) = th\vartheta_l = 1$. Thus the velocity of light remains unaffected by the velocity of its material source, and Einstein's second postulate is recovered.

§ 11. REVERSIBLE MOTION AND IRREVERSIBLE EVOLUTION (SCHRÖDINGER AND GIBBS)

When Leibniz introduced the operator d (*differentia*), beside its inverted companion S (*summa*; become $\int$ in the hands of one of the Bernouilli's), he could not imagine what a proteiformous symbol he had created. According to the number of dimensions of the manifold where it operates, indeed, d takes the form of a vector with differential components or equivalently of a covector with vector components. Thus:

$$d = d\xi \frac{d}{d\xi} = \textstyle\sum_i dx^i \frac{\partial}{\partial x^i} = \sum_i \gamma_i dx^i \gamma^i \frac{\partial}{\partial x^i} = \ldots \tag{36}$$

In our case of the 1-dimensional parameter ξ, by Riemanian metrics one obtains the metrics by the norm of d:

$$(d, d) = \left(d\xi \frac{d}{d\xi}, d\xi \frac{d}{d\xi}\right) = d\xi^2 \left(\frac{d}{d\xi}, \frac{d}{d\xi}\right) = g_{\xi\xi} d\xi^2 \tag{37}$$

Further, one considers the double operation of d: $d^2 = d \circ d$. By construction of the identity

$$(d, d) g_{\xi\xi} \frac{d^2}{d\xi^2} = g_{\xi\xi}{}^2 d^2 \tag{38}$$

knowing that on a flat space it is always possible to find an orthonormed frame such that ${g_{\xi\xi}}^2 = +1$, one obtains the prototype of the *Klein-Gordon equation*, for c-numbers and one dimension:

(39) $$d^2 = ds^2 \,\Box \qquad \text{or} \qquad \frac{d^2}{ds^2} = \Box$$

where the usual notations are used: ds^2 for the metrics (here $ds^2 = g_{\xi\xi} d\xi^2$) and the Dalembertian $\Box$, the wave operator [31], $\left(\text{here } \Box = g_{\xi\xi} \dfrac{d^2}{d\xi^2}\right)$ in prevision of the extension to n-dimensional manifolds possibly endowed with a megethos (of length e.g.). Then, symbolically anyway, (36) can be written again as: [32]

(40) $$d = ds \,\Box\, {}^1/_2 \qquad \text{or} \qquad \frac{d}{ds} = \Box\, {}^1/_2$$

Now consider the following eigenvalue problem [33]:

(41) $$\frac{d}{d\xi}\frac{d}{\delta\xi}\varphi(\xi) = \left(\frac{d}{d\xi}, \frac{d}{d\xi}\right)\varphi(\xi) \qquad \text{or} \qquad \frac{d^2}{d\xi^2}\varphi(\xi) = g_{\xi\xi}\varphi(\xi), \quad \varphi(\xi) \in \mathbf{R}$$

As $g_{\xi\xi} = \pm 1$, it will boil down to the *eigenvalue problem for the unit tangent vector*. For the Euclidean choice $g_{\xi\xi} = +1$, the unit tangent vector will be the usual tangent to a curve in $\mathbf{R}^n$ when embedded into this space. Then:

(42) $$\left(\frac{d}{d\xi} + 1\right)\left(\frac{d}{d\xi} - 1\right)\varphi(\xi) = 0, \quad \varphi(\xi) \in \mathbf{R}$$

is associated with an *irreversible evolution*, with a real dissymmetry between a "past" direction and a "future" direction given by the contrast between a divergence in one sense and a convergence in the other sense of the solution $\varphi(\xi) = \varphi_0 e^{\pm\xi}$. A μ-realization of this pattern is given by the *Gibbs theory* of canonical ensembles, where the equation applies:

(43) $$\text{Gibbs:} \quad k\frac{d}{d\vartheta}W = EW$$

(k: Boltzmann's constant, here taken as the quantum of entropy; ϑ: the natural temperature $\vartheta = -T^{-1}$, T the absolute temperature; E: energy; W: the "complexion"). On the other side, the Minkowskian choice $g_{\xi\xi} = -1$ entails that $d/d\xi$ is the unit tangent to proper time or time, according to the μ-reparametrization of ξ. Then:

(44) $$\left(\frac{d}{d\xi} + i\right)\left(\frac{d}{d\xi} - i\right)\varphi(\xi) = 0 \qquad \varphi(\xi) \in \mathbf{R}.$$

A convenient choice of $\varphi(\xi)$ as

$$\varphi(\xi) = \overline{\psi}(\xi)\psi(\xi) \qquad \psi(\xi) \in \mathbf{C}$$

transforms (44) into

$$\left(\frac{d}{d\xi} + i\right)\psi \cdot \left(\frac{d}{d\xi} - i\right)\overline{\psi} = 0 \tag{45}$$

where each factor is the ξ-*reversal* of the other. Thus (45) is associated with a *reversible motion*, where no dissymmetry permits to discriminate between a "past" and a "future" directions. The circularity of the solution $\psi(\xi) = \psi_0 e^{\pm i\xi}$ is typical of a progressive wave. It should even suggest to call this reversible motion a *revolution*, in the first meaning of the word, as in the *De Revolutionibus Orbium Caelestium*. And the factor $e^{\pm i\xi}$ will entail the omnipresence of the Fourier transform in the theories built after this pattern!

Of course the paradigmatic case of those theories is the *WM of Schrödinger*, which is so recovered by SQK:

$$\text{Schrödinger:} \quad ih\frac{d}{d\tau}\psi = E_0\psi \quad \text{or} \quad ih\frac{\partial}{\partial t}\psi = E\psi \tag{46}$$

Yet again, the relation between action, energy and time will determine the positive direction of time (one cannot say the direction of the future): $i\dfrac{d}{d\tau}\psi = +\dfrac{d\xi}{d\tau}\psi$, i.e. $id\log\psi = d\xi$ and then, up to an additive constant, disclosing its character of information, the action S reads:

$$S = -ih\log\psi \tag{47}$$

the final form that Schrödinger should have given to its initial formula $S = K\log\psi$ [34]. Without further ado, the various appearances of $e^{-i\xi}$ can simply be recalled:

$$e^{-i\xi} = e^{\frac{iS}{h}} = e^{i\frac{(px-Et)}{h}} = e^{-i\frac{\tau}{\tau_0}} = e^{i\left(\frac{x}{\lambda}-\frac{t}{T}\right)} \tag{48}$$

§ 12. THE TANGENT VECTOR AND THE DUALITY WAVE-PARTICLE

We recall two formulas here:

(49)

$$\frac{d}{ds} = \frac{dx^\mu}{ds}\frac{\partial}{\partial x^\mu} \qquad \text{the tangent vector (particle)}$$

$$\frac{d^2}{ds^2} = \Box \qquad \text{the wave operator}$$

Thus the particle aspect is associated with the first derivative of the parameter, and the wave aspect with its second derivative. In the simple case of a free particle, according to Newton-Einstein axiom, $p^\mu = m_0 \frac{dx^\mu}{d\tau}$, and then the force can be expressed alternatively as $K^\mu = \frac{d}{d\tau}(p^\mu) = m_0 \frac{d^2}{d\tau^2}(x^\mu)$. Nevertheless, the wave aspect cannot appear, as in this case the p^i's do not depend on the x^j's. Then $\frac{\partial p^i}{\partial x^j} = 0$ in the tangent vector, and $\frac{\partial^2 x^i}{\partial x^{j2}} = 0$ in the Dalembertian. The result is essentially the Newtonian relation $F^i = \frac{dp^i(t)}{dt}$, up to a factor. But in hydrodynamics, this independance disappears, and then the wave aspect can develop.

A more interesting situation takes place when one recalibrates proper time by time itself, as CM does it (but at the price of a terrible ambiguïty, as will be shown presently).

In this paper, at least three different structural times have been determined: the proper time τ, the ordinary time t which is the Hamiltonian time, and the Lagrangian time T. They are related by the structural relation:

$$t = \tau ch = T ch^2 \quad \text{and consequently} \tag{50}$$
$$\frac{d}{dt} = \frac{d}{d\tau} ch^{-1} = \frac{d}{dT} ch^{-2}$$

Historically, in kinetics, t has been introduced the first. Relatively to it, the recalibration of the metrics reads, as now the x^i's are dependant on t:

$$- c^2 d\tau^2 = - c^2 dt^2 ch^{-2} = - c^2 dt^2 (1-\beta^2) \tag{51}$$

which entails for (49) the so called convective derivative:

$$\frac{d}{dt} = \frac{d}{d\tau ch} = \frac{dx^\mu}{dt}\frac{\partial}{\partial x^\mu} = v^i \frac{\partial}{\partial x^i} + \frac{\partial}{\partial t} \tag{52}$$

Here lies the ambiguïty. t enters twice in the formula: 1) in the total derivative as the parameter t_p, and 2) in the partial derivative as the coordinate t. An immediate flaw of this notation is that $\frac{\partial t_p}{\partial t_i} \neq 1$ while usually $\frac{\partial t}{\partial t} = 1$. Similarly, $\frac{\partial x^{\bar{i}}}{\partial x^i} = 1 - \frac{1}{v^i}\frac{\partial x^i}{\partial t}$! And the frame appears to be no more orthonormed, when the megethos enters into the picture.

For the physicists indeed the metrics $g_{\mu\nu}dx^\mu dx^\nu$ is endowed with megethos length square L^2 if the x^i's are endowed with length L. That means that the coefficients

$g_{\mu\nu}$ of the metrics tensor are taken as c-numbers. This is consistent with the use of the $g_{\mu\nu}$ to raise or lower the indexes of the components of the tensors and cotensors, without alteration of their megethos. For instance in the metrics $g_{\mu\nu}dx^{\nu} = dx_{\mu}$ keeps its megethos length.

But this practice is no longer consistent, structurally, with the canons of the Riemanian metrics, where the $g_{\mu\nu}$ are the scalar products of the basis vectors of a natural frame of coordinates x^i's: $g_{\mu\nu} = (\partial/\partial x^{\mu}, \partial/\partial x^{\nu})$ and should have megethos L^{-2}. Then metrics should be a pure c-number, as it has to be when used to transform a vector into a covector without alteration of its intrinsic (component + basis) megethos. On the contrary, in the standard practice of the physicists, for instance the covector momentum-energy $p_{\mu}dx^{\mu}$ is taken as the differential of action according to its megethos, MLT^{-1} and the same vector $p^{\mu}\partial/\partial x^{\mu}$ is usually discarded: it would have a megethos MT^{-1}.

From this example, the way out is to recalibrate the $g_{\mu\nu}$ and the x^{μ} by eliminating their megethos by a μ-factor of inverse megethos, in order to recover the mathematical purety of c-numbers. That has already been alluded to in section 4: introduce a (for the moment) arbitrary μ-factor λ of megethos length, and use systematically $\lambda^{-1}x^i$ as c-numbers. That means also that now the basis vectors and covectors are c-numbers again. Thus, for our examples, one gets:

$$(53) \qquad p^{\mu}\left(\lambda \frac{\partial}{\partial x^{\mu}}\right), \quad p_{\mu}(dx^{\mu}/\lambda) \quad \text{and} \quad g'_{\mu\nu} = \left(\lambda \frac{\partial}{\partial x^{\mu}}, \lambda \frac{\partial}{\partial x^{\nu}}\right)$$

Then the following recalibrations for the metrics coefficients:

$$(54) \qquad g_{\xi\xi} = \left(\frac{d}{d\xi}, \frac{d}{d\xi}\right) = \tau_0^2 g_{\tau\tau} = t_0^2 g_{t_p t_p} = T^2 g_{TT}$$

or equivalently, according to (50)

$$(54) \qquad g_{t_p t_p} = g_{\tau\tau} ch^{-2} = g_{TT} ch^{-4}$$

It will be shown now that the partial derivative $\partial/\partial t$ is in fact equal to the total derivative d/dT, which explains the above mentioned ambiguïties. Once more again the identity $ch^2 - sh^2$ will be on duty.

Reconsider (52) in the time-trajectory manifold:

$$(52.1) \qquad \frac{d}{dt}(ch^2 - sh^2) = v\frac{\partial}{\partial x} + \frac{\partial}{\partial t}$$

whence the identifications:

$$(52.2) \qquad \frac{\partial}{\partial t} = ch^2 \frac{d}{dt} = \frac{d}{dT} \quad \text{and} \quad \frac{\partial}{\partial x} = -\frac{1}{V}\frac{\partial}{\partial t}$$

Thus $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t}$ are no more the independent basis vectors of the Minkowskian frame. Their linear dependance keeps the trace of the 1-dimensional submanifold of the parameter. The identifications (52.2) are not arbitrary. Here is a very simple cross-checking:

Using (47): $$-E = \frac{\partial S}{\partial t} = -ih\frac{\partial}{\partial t}\log\psi \equiv -\hat{H}\psi$$

and $$+p = +\frac{E}{V} = \frac{\partial S}{\partial x} = -ih\frac{\partial}{\partial x}\log\psi = \frac{1}{V}\left(ih\frac{\partial}{\partial t}\right)\log\psi = \hat{P}\psi$$

which gives the same relation (52.2). This relation shows that $E = Vp$ is transmitted to the operators $\hat{H} = V\hat{P}$. Or course, the frame is no more orthonormed, as $\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial t}\right) = g_{TT} = ch^4 g_{t_p t_p} < 0$, $g_{xx} = \frac{1}{V^2}\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial t}\right) < 0$ and $g_{xt} = -\frac{1}{V}\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial t}\right) > 0$.

But the metrics is conserved

(55) $$g_{\tau\tau}c^2 d\tau^2 = g_{t_p t_p}c^2 dt^2 = c\check{h}4\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial t}\right)c^2 dt^2$$

Moreover, it is now possible to fix the recalibration. As the quantum commutator $\left[\frac{d}{dt}, t\right] = \mathbf{1} = (ch^2 - sh^2)\,\mathbf{1} = \left[\frac{\partial}{\partial t}, t\right] + \left[v\frac{\partial}{\partial x}, t\right]$, one easily gets:

(56) $$\left[\frac{\partial}{\partial t}, t\right] = ch^2\,\mathbf{1} \quad \text{and} \quad \left[\frac{\partial}{\partial x}, x\right] = -sh^2\,\mathbf{1}.$$

This is the price to be paid for the ambiguity of reparametrizing proper time as time. But there are also some advantages.

First (52.1) is consistent with the wave operator $\square$ (using again the Minkowskian metrics)

(57) $$\frac{d^2}{-c^2 d\tau^2}(ch^2 - sh^2) = \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{c^2\partial t^2} = \square$$

Second, for the eigenvalue problem of the unit tangent vector, the up to now arbitrary factor λ in $\lambda\frac{\partial}{\partial x^\mu}$ of (53) can now be determined in various ways:

(58) $$\mathbf{1} = i\frac{d}{d\xi} = i\tau_0\frac{d}{d\tau} = iT\frac{\partial}{\partial t} = -i\lambda\frac{\partial}{\partial x} = -ih\frac{d}{dS} = -it\frac{d}{dt}$$

Not only the well known eigenvalue problems for energy and impulse are recovered:

$$E\left(iT\frac{\partial}{\partial t}\right)\psi = ih\frac{\partial}{\partial t}\psi = \hat{H}\psi = E\psi$$

(59)

$$p\left(-i\lambda\frac{\partial}{\partial x}\right)\psi = -ih\frac{\partial}{\partial x}\psi = \hat{P}\psi = p\psi$$

but still other forms are provided, as

(60) $$(px-Et)\left(-ih\frac{d}{dS}\right)\psi = \hat{S}\psi = S\psi$$

giving directly the Schrödinger equation, and especially:

(61) $$(pv-E)\left(-it\frac{d}{dt}\right)\psi = -ih\frac{d}{dt}\psi = \hat{L\psi} = L\psi$$

With this last expression, the well known relation between Lagrangian and Hamiltonian pass to the operators:

(62) $$\hat{L} = v\hat{P} - \hat{H}\,.$$

An ubiquitous confusion in non relativistic Quantum Mechanics is done away with for good from now on. In that theory t is the parameter, without question to be a reparametrization of a non existing proper time τ. Hence the usual calibrations of the commutators $\left[\frac{\partial}{\partial t}, t\right] = \left[\frac{\partial}{\partial x}, x\right] = \mathbf{1}$ and the writing of $\hat{H} = ih\frac{\partial}{\partial t}$. That means that on each axis the partial derivative functions as the total derivative of a parameter, which is called here an active projection. That means also, and this is the most important, that any linear dependance due to the remembrance of the embedding of an external parameter ξ onto a parametrized curve in space-time is totally repudiated, i.e. there is *no more motion of a trajectory!*

By the same token, another confusion is cleared up. $\hat{H}$ is generally written $ih\frac{\partial}{\partial t}$, but sometimes also as $ih\frac{d}{dt}$, as it should be done for a parameter. Here both expressions receive their specific meaning:

(63) $$\hat{H} = ih\frac{\partial}{\partial t} \quad \text{and} \quad \hat{L} = -ih\frac{d}{dt}$$

The same precision has to be made for the Liouville operator in phase space!

One cannot resist the pleasure to close this section by a final touch, where all the presented notions come back to dance the last figure of a ballet. First enters Newton and his law

(64) $$p = mv$$

Next comes TANTRA, the magician who inverts the law and converts it into $h/p = Vh/E$, and lets appear Einstein and de Broglie with their relations:

$$E = Vp\,, \quad \lambda = VT \qquad \text{and} \qquad h = ET = p\lambda\,. \tag{65}$$

The first of them is transferred to the operators $\hat{H}$ and $\hat{P}$, and then passion *fait son entrée* slightly disguised under the form:

$$\frac{dx}{\partial/\partial x} = \frac{c^2 dt}{-\partial/\partial t} = \frac{-c^2 d\tau^2}{d}, \tag{66}$$

dropping a curtsey to Leibniz. Then the old Eudoxus comes along with his calculus of proportions and launches the trio PAM, where action is also slightly disguised:

$$\frac{dx}{\partial/\partial x} = \frac{dx\,dx - c^2 dt^2}{dx\dfrac{\partial}{\partial x} + dt\dfrac{\partial}{\partial t}} = \frac{ds^2}{d} \qquad (P = M_{(x)}/A) \tag{67}$$

and

$$\frac{dx}{\partial/\partial x} = \frac{dx\dfrac{\partial}{\partial x} + dt\dfrac{\partial}{\partial t}}{\partial x \partial x - \dfrac{1}{c^2}\dfrac{\partial}{\partial t}\dfrac{\partial}{\partial t}} = \frac{d}{\square} \qquad (P = A/M_{(x)})$$

who then introduce Klein and Gordon:

$$d^2 = ds^2\,\square\,. \tag{68}$$

Schrödinger is the ballet master. The critic expresses his admiration, and adds his own touch: this entire ballet has been composed only upon the structure of differentiability and megethos. Curtain!

§ 13. DETERMINISM, UNDETERMINISM, AND PROBABILITY AS ROOTED IN THE STRUCTURE OF DIFFERENTIABILITY AND MEGETHOS

It is well known that a DM supplemented with an algebraic structure of group transform into a Lie group structure. Then one can work with the Lie algebra of the Lie group and stay in the algebra. That is the way Dirac started the development of Quantum Mechanics. The anecdote is known: Dirac had quickly recognized that the essentials of the then recent "discovery" of Heisenberg, who worked it out without any knowledge that *de facto* he had used the matrix calculus, consisted in fact in the non commutativity of the p's and the q's, which could not be usual numbers (his c-numbers) but had to be special entities (his q-numbers).

During a Sunday walk, it came suddenly to Dirac that this situation was very similar to the Poisson brackets of CM. He had only a rather vague remembrance of this subject from previous Tolman's lectures. He wanted to check his intuition immediately in some treatise on CM, but had to refrain his eagerness up to the next morning's opening of the library. But it seems that Dirac failed to ask the question: why the Poisson brackets. These are related to *conjugated canonical variables.* But what does this concept of canonical precisely mean? It is true that, at Dirac's time, the mathematical correlative concepts of tangent and of cotangent bundles were not yet clearly exposed. And this duality tangent/cotangent that he could not perceive in the phase space for historical necessity, nevertheless he succeded to reproduce it at the level of the functional analysis, with his famous kets $| >$ and bras $< |$.

So QM was more and more engaged into more and more complex and sophisticated algebra. By contrast, the author's opinion is that the essentials of the characteristics differences between CM and QM, i.e. determinism versus undeterminism and probability can be traced back to the more elementary structure of a DM prepared for physical grandeurs, that means in the structure of differentiability and megethos.

Here follows a very condensed argumentation of those topics, which have given birth to thousands of pages, essentially at the level of the discursive concepts. Keeping the discussion only at the structural level permits a rather direct presentation, as the relevant relations are ultimately rather elementary, and can be found in any serious textbook on DM's.

Let M be a n-dimensional DM, of elements $x \in M$, and $\{x^i\}$ a frame of natural coordinates on an open subset $U \subset M$. Let $\xi \in I \subset R$ be a parameter (all is kept local), thus extraneous to M ("absolute" says Newton for his time!). Let $\hat{X} = X^i \dfrac{\partial}{\partial x^i}$ a vector field on U: $\hat{X} \in T(U)$ and $\varphi(x)$ an element of the module of differentiable functions (real or complex) defined on U. A parametrized curve in U is the mapping from I to U, $\xi \to x(\xi)$. Then $\varphi(x, \xi) \equiv \varphi_\xi(x)$ is defined on $I \times U$ given by:

$$(58) \qquad \frac{d\varphi(x, \xi)}{d\xi} = \hat{X}\varphi(x, \xi)$$

whose solution is the exponential mapping

$$(59) \qquad \varphi(x\xi) = \varphi_0(x)\, e\xi X$$

Applied to the functions coordinates $x^i(x, \xi) \equiv x^i(\xi)$ themselves, $\dfrac{dx^i}{d\xi} = X^i$ is nothing else than the system (11) of passion

$$(60) \qquad \frac{dx^i}{X^i} = d\xi \qquad i \in \quad \{1, 2, \dots n\}$$

The *determinism* is a necessary condition that the differential equation (58) can admit of the solution (59). This is the Einsteinian determinism only, differential and local, while the Laplacian determinism, analytic and global, meaningful only in the case of the irreversible evolution, is dead for good [35].

Let us consider now the eigen value problem, and first in the general case. For a fixed value X (recall that X is a c-number, as all the other entities!), the problem reads

$$\hat{X}\varphi = X\varphi \Leftrightarrow X = \frac{d}{d\xi}\log\varphi \Leftrightarrow \log\varphi = X\xi \tag{61}$$

up to an integration constant.

Introducing a constant $K(K \in \mathbf{R})$ as a standard unit, and putting

$$S = K\log\varphi = K\,X\,\xi \tag{62}$$

(almost Schrödinger's initial formula (36)), one makes more obvious the analogy of S (and of ξ) with information. By definition, the information $I = I_0 \log P$, where I_0 is the standard unit (e.g. a bit, or an octet, etc.) and P a probability, $0 \leqslant P \leqslant 1$. Here φ is not necessarily normalized: for $\varphi \in \mathbf{R}$, only the condition $0 \leqslant \varphi$ applies. In the physical applications, $S \in \mathbf{R}$, which entails $X \in \mathbf{C}$ if $\varphi \in \mathbf{C}$ (case of reversible motion).

Up to that point, there was only question of c-numbers and pure mathematics. In preparation for physics, (62) has to be endowed with convenient megethos, and so

$$\tilde{S} = \tilde{K}\,X\,\xi \tag{63}$$

where the tilda can be taken as a sort of deformed μ. $\tilde{K}$ is the quantum of $\tilde{S}$. The case of interest in motion and evolution is that $\tilde{K}$ *can be conceived as the product of two conjugated* μ*-factors*, as

$$\tilde{K} = \mathbf{1}_{\tilde{S}} = \tilde{X}_0\,\tilde{\xi}_0 \tag{64}$$

which transforms (63) into

$$\tilde{S} = (\tilde{X}_0\,X)\,(\tilde{\xi}_0\,\xi) \equiv \tilde{X}\,\tilde{\xi} \tag{65}$$

with the relation of $\tilde{S}$-inversion between $\tilde{X}$ and $\tilde{\xi}$:

$$\mu(\tilde{S}) = \mu(\tilde{X})\cdot\mu(\tilde{\xi}) \tag{66}$$

It is this conjugation which makes possible to find *mean values* and *mean quadratic deviations* for $\tilde{X}$ when $\tilde{\xi}$ is the parameter. That cannot be done in the case of classic kinematics where of course the omission of $\tilde{X}$ blocks this mechanism. Nevertheless it could be reintroduced when a grandeur of megethos velocity squared would be defined as $\tilde{S}$, with $\tilde{X} = V$, $\tilde{\xi} = v$ and $\tilde{K} = c^2$ (the related quantum!) [37].

As it has been shown in section 10, there is a simplification for motion and evolution: $X = \pm i$ for reversible motion, and $X = \pm 1$ for irreversible evolution. But $\tilde{X}_0$ still remains.

From now on, we will restrict ourselves to those two cases as they are realized in nature, i.e. $X = -1$ for an evolution which converges toward equilibrium in the future, and $X = -i$ for a motion toward the positive direction of time. It will be enough to set up the convenient translations of the abstract structural concepts into a table of isomorphisms.

Abstract:	*Irreversible evolution* ($X = -1$)	*reversible motion* ($X = -i$)
$\tilde{S}$	entropy $S > 0$	action $S < 0$
$\tilde{\xi}$	natural temperature $\vartheta < 0$	time $t > 0$
φ	complexion $W \in \mathbf{R}$	amplitude of probability $\psi \in \mathbf{C}$
$\tilde{K}$ (quantum)	Boltzman's constant k	Planck's constant h
(62), (65)	$S = -k \log W = -E_0\vartheta_0$	$S = -ih \log \psi = -E_0\tau$
	reparametrization by means of a coordinate:	
(61)	$-\dfrac{\partial S}{\partial \vartheta} = E = k\dfrac{\partial}{\partial \vartheta}\log W$	$-\dfrac{\partial S}{\partial t} = E = ih\dfrac{\partial}{\partial t}\log \psi$
(61) eigen value problem	$k\dfrac{\partial}{\partial \vartheta}W = EW$	$ih\dfrac{\partial}{\partial t}\psi = E\psi$

Now, the most astonishing object, and the idea of genius of Boltzmann, is the *Zustandssumme*, the partition function Z for evolution and the superposition of the states Ψ (the wave function) for motion, in the case that there exists a system of different fixed energies E_i at the same natural temperature ϑ or at the same time t. Then:

Z.S.	$Z = \sum_i e^{\dfrac{E_i\vartheta}{k}}$	$\Psi = \sum_{-\infty}^{+\infty} c_i\, e^{\dfrac{-iE_it}{h}}$
mean value	$<E> = k\dfrac{\partial}{\partial \vartheta}\log Z$	$<E> = \bar{\Psi}\left(ih\dfrac{\partial}{\partial t}\log \Psi\right)\bar{\Psi}^{-1} = \sum_i \dfrac{<\Psi E_i \Psi>}{\bar{\Psi}\Psi}$
mean quadratic deviation	$\Delta E^2 = k^2\dfrac{\partial^2}{\partial \vartheta^2}\log Z$	$\Delta E^2 = -h^2\dfrac{\partial^2}{\partial t^2}\log \Psi$

Thus the *probabilistic* character of Q.M. is rooted in the structure of differentiability itself.

Finally, the *undeterminism* is also rooted in the same structure, in the duality between tangent and cotangent planes prevailing on any DM.

commutator of the parameter	$\left[\frac{d}{d\vartheta}, \vartheta\right] = \mathbf{1}$	$\left[\frac{d}{dt}, t\right] = \mathbf{1}$
contraction of the basis vector and covector	$<\frac{d}{d\vartheta}, d\vartheta> = 1$	$<\frac{d}{dt}, dt> = 1$

In elementary QM, it is well known that the non commutativity of the commutator $[P, Q] = -ih\,\mathbf{1}$ entails the existence of Heisenberg's uncertainty relations between P and Q, which certainly is an essential feature of QM. The point here made is that such a feature is present in any DM when the vectors and covectors are treated intrinsicly, as elements of the tangent and cotangent spaces. Of course, it is prohibited to appear in CM by the mere fact that CM operates only with componants, i.e. functions, which necessarily do commute. In this precise sense, Einstein is wrong: if a theory is incomplete, it is not QM, but CM itself!

All this justifies, more than amply, the author's thesis of a scientific derevolution (remember: in fact an overrevolution): an unique structure (differentiability and megethos) lies under all the known forms of Kinetics.

§ 14. CONCLUSION

Functorially, one should be able to set up such similar translations for other theories, in the first place for the electromagnetism, where $e^2 = \alpha hc$, by making use of spinors. One should be able also to translate more "exotic" (from the present point of view) theories, like thermostatics where the parameter is energy: that could be done probably by a convenient duality. But there is enough for this paper.

Let it find an end with an opening to further progress.

1) A natural extension of SQK is GQK, by substitution of the general relativity postulate No. 1 to the special one used in SQK and SR. That means the consideration of the linear connexions, called gauges by the physicists.

2) The introduction of the interactions needs to pass to non linear relations.

3) A new way opens toward the possibility to set up this so long awaited General Relativistic Quantum Theory. Already in de Broglie's relation, it has to be observed that a "curvature" is proportional to the momentum, as

$$\lambda^{-1} = h^{-1} p \tag{67}$$

This is coherent with Einstein's celebrated 2-tensor equation

$$S^{\mu\nu} = K_E \, \theta^{\mu\nu} \tag{68}$$

The similarity at the level of differential geometry is obvious. Nevertheless there is a big trouble at the level of megethos. If h can be conceived as a c-number, and then e^2, that cannot be the case of K_E, which then would have a megethos time square. Here might lie the principal difficulty.

NOTES AND REFERENCES

[1] T. S. KUHN: *The Structure of Scientific Revolutions*, Chicago Univ. Press, Chicago 1969².

[2] P. SCHEURER: *Révolutions de la science et permanence du réel*, P.U.F., Paris 1979.

[3] *Ibidem.*

[4] P. SCHEURER: ref. (2) and espec. a) "Vues nouvelles sur la science du temps et l'histoire", in *Penser dans le temps, Mélanges Jeanne Hersch*, L'Age d'Homme, Lausanne 1977, p. 157-172; b) "Relations d'incertitude de Heisenberg et déterminisme géométrique", Actes du Séminaire Mathématiques et Philosophie, ed. M. Loi et R. Thom, Ecole Norm. Sup., Paris 1977; c) "Einsteins Relativiteitstheorie: Revolutie en Derevolutie", in *Albert Einstein: bijdragen over zijn wereld, zijn wetenschap, zijn denken*, ed. G. Debrock and P. Scheurer, Nijmegen Studies in the Philosophy of Nature and its Sciences No. 1, Kath. Univ. Nijmegen 1982, p. 95-118; d) "Leibnizian Quantum Structure, Irreversible Dynamics, Quantum Kinetics, and all that", *Archives des Sciences, 34*, No. 3, 1981, p. 383-388.

[5] P. SCHEURER: a) "Quantum Kinetics: a structural reinterpretation of the quantization in dynamical systems", and b) "Relativisation: une réinterprétation structurale de la relativité restreinte", *Archives des Sciences, 35*, No. 2, 1982, p. 197-216 and p. 217-226.

[6] This seems reminiscent of the concept of metrical space by H. Weyl, where a referential frame is given by a system of coordinates plus a calibration: cf. H. Weyl: a) "Gravitation und Elektricität", Sitz. Preuss. Akad. Wissenschaften, 1918; b) *Raum, Zeit, Materie*, 1921. Here one demands something more: not only to fix an unit standard, but also to specify what physical dimension (megethos) it is the standard of. Thus regarding those papers, it is significant that Weyl did neither perceive the PAM relation (see section 5) nor conceive that his demand that action be a pure number entails that energy's megethos, resp. linear momentum's megethos should be inverse to time's megethos, resp. to length's megethos (thus pointing to de Broglie's relations!)

[7] P. SCHEURER: "Mouvement, évolution, histoire: sur trois modalités du temps physique", *Revue Philosophique de Louvain, 83*, fév. 1985.

[8] P. SCHEURER: ref. (2) and more recently; a) "L'importance d'une analyse historique critique dans les sciences expérimentales: de la vitesse comme vecteur tangent", in *Quels types de recherches pour rénover l'éducation en sciences expérimentales?*, ed. A. Giordan et J. L. Martinand, Univ. Paris VII, 1983, p. 531-538; b) "Schoonheid

als een heuristisch princip bij theorievorming" in *Waarden voor-bij Wetenschap*, ed. G. Debrock and P. Scheurer, Nijmegen Studies in the Philosophy of Nature and its Sciences, No. 3, Kath. Univ. Nijmegen 1983, p. 177-199.

[9] P. SCHEURER: ref. (2), (7), "Temps, énergie, information", in *Teoria dell'informazione*, ed. J. Roger, Seminari interdisciplinari di Venezia, il Molino, Bologna 1974, p. 199-220. Here, see section 6.

[10] See ref. (7), (8a and b).

[11] Ref. (7).

[12] E. SCHRÖDINGER: "Quantisierung als Eigenwertproblem" I, II, III, IV, *Annalen der Physik 79* (1926) 361; 489; *80* (1926) 437; *81* (1926) 109.

[13] A paper is under preparation on this matter.

[14] Ref. (7).

[15] As even many physicists do not see the pertinence of the opposition of those two concepts, because both are numbers, it is not useless to remember that the difference lies in the relation of those numbers with the manifold. Parametrization embeds (ordered) numbers onto a curve in the manifold, while a coordinate is a numerical mapping defined on an open subset of the manifold. Then the relations between numbers and manifold appear as dual, what is not without significance for Quantum Mechanics. Ordinarily the parameter is taken as the set (or a subset) of the real numbers, while to each point of the manifold there are assigned so many real (sometimes complex) numbers as the dimensionality of the manifold.

[16] For more details, cf. Ref. (8a).

[17] *Ibidem.*

[18] This part of the communication (16) is not reproduced in (8a), for question of admissible length!

[19] As in dynamics the product of momentum p with the differential of coordinate x enters in the differential of action, it seems natural to call "passion" their quotient dx/p. Passion plays a crucial role, as able to construct from itself both action and metrics (see below formula (13)).

[20] Einstein's well known "Autobiographisches", in *Albert Einstein, Philosopher Scientist*, ed. P. Schilp.

[21] See Ref. (6, 1918).

[22] E.g. in Ref. (2) and Ref. (4a).

[23] Deliberately the argument will be omitted, when no confusion can arise. As usual the notation $v/c = th = \beta$ will be freely used.

[24] The angular momentum can be constructed in the same way: e.g.

$$\frac{dt}{m} = \frac{ydx - xdy}{yp_x - xp_y} = \frac{(x^2+y^2)d\vartheta_{xy}}{L_{xy}} \Leftrightarrow L_{xy} = I_{xy}\omega_{xy}$$

The same construction applies also for spinors:

$$\frac{d\tau}{m_0} = \frac{\gamma_1 dx^1}{\gamma_1 p^1} = \frac{\Sigma_\mu \gamma_\mu dx^\mu}{\Sigma_\mu \gamma_\mu p^\mu}$$

One does not go further on that way in this paper.

[25] For instance, the microcanonical ensemble, where the μ-parameter is energy. For the present time, those relatively deviant cases will be no further investigated (they can be tried as exercises!)

[26] For a free particle, there exists a narrow association between the Lorentz transformation, from the frame (x, t) to the rest frame, and the Legendre transform. For energy indeed the former is the latter (with the notations (18)):

$$\alpha)\ -L = E_0\sqrt{1-\beta^2} = E - pv = E - p\frac{dE}{dp}$$

while for time, one has only:

$$\beta)\ c^2T = c^2\tau\sqrt{1-\beta^2} = c^2t - xv = c^2t - x\frac{dx}{dt}$$

in accordance with Einstein's-de Broglie's relations

$$\frac{h}{E} = \frac{-h}{L} - \frac{h}{\lambda}\frac{x}{E} \Leftrightarrow L + E = -\frac{xL}{\lambda} = pv$$

and of course with the PAM relation: the mere multiplication of α) by the factor $\frac{\tau}{m_0} = \frac{-c^2\tau^2}{-m_0c^2\tau}\left(\frac{\text{metrics}}{\text{action}}\right)$ yields β) up to c^2!

The entity c^2T should deserve a special name, at least a special symbol (proposal: $\sqcup\!\sqcup$) by its rôle of an analogous dual of energy $E = c^2m$; cf.:

$$E = c^2m = pV = (mv)V$$

and $\quad \sqcup\!\sqcup = c^2T = \lambda v = (TV)v$

In the case of energy, it may be added that, for the free particle, as $d(H+L) = d(pv)$ and $dE = vdp$, then $dL = pdv$. One passes very easily to the usual case in CM of a particle moving in a potential $V_{pot}(q)$: $d(H+L) = (vdp+dV_{pot}) + (pdv-dV_{pot})$, i.e. the usual $H(p, q)$ and $L(v, q)$, at the condition of the non dependance of p and v on q.

[27] And more: as $v = dx/dt = dE/dp$, then

$$dpdx = dEdt$$

a relation very reminiscent of Heisenberg's uncertainty relations. Other correlated relations follow:

$$\text{a)}\ \frac{dp}{dt}\cdot\frac{dx}{dt} = \frac{dE}{dt} = v\cdot F \quad \text{thus} \quad dE = vdp = Fdx$$

Then, if E depends on x too, i.e. is the Hamiltonian $H(p, q) \equiv E(p, x)$ and if E is conserved (conservative systems), trivially

$$\text{b)}\ dE = \frac{\partial E}{\partial p}dp + \frac{\partial E}{\partial x}dx = vdp - Fdx = 0$$

that means Hamilton's equations on the one hand

$$\text{c)}\ dx/dt = \frac{\partial E}{\partial p}; \qquad dp/dt = -\frac{\partial E}{\partial x}$$

and on the other hand Monge's equation for the bicharacteristics:

$$\text{d)}\ \frac{dx}{\partial E/\partial p} = dt = \frac{-dp}{\partial E/\partial x}.$$

[28] A. EINSTEIN: *Die Grundlagen der Allgemeinen Relativitätstheorie*, Barth, Leipzig, 1916, Appendix. This is a paradigmatic case of blindness in sight, of the dialectics of the unseen in the seen (2). TANTRA escaped also the observation of Minkowski, too engaged in his discovery of the 4-dimensionality of space-time, even if he first proposed the interpretation of the Lorentz transformations as rotations in the plane $\{x, t\}$. The same misadventure has happend to Landau and Lifschitz, who use

rotations but with circular functions by means of Minkowski's (pedagogic?) *truc* to put $-dt^2 = +(idt)^2$. The author himself can take his place in this list: about ten years ago, still ignoring Einstein's Appendix, he gave the symmetric presentation of Lorentz transformations, using ct instead of the usual t, but failed to recognize the presence of TANTRA. Still again his main thesis of the slow emergence of concepts from practice!

[29] Remember section 3: de Broglie is Newton inverted by TANTRA!

[30] As usual, a free particle has a velocity v in a frame $K(x, t)$ and v' in a frame $K'(x', t')$. K' has velocity V relatively to K. Putting, $v = th$, $v' = th'$ and $V = Th$, one finds (using TANTRA's exchange $x \leftrightarrow ct$)

$$x = c\tau sh = ch' Ch c\tau (th' + Th)$$

$$ct = c\tau ch = ch' Ch c\tau (1 + th' Th)$$

whose ratio yields the required formula.

One additional remark: the here chosen exchange of the axis x and t transforms a right frame (x, t) into a left frame (x_T, t_T). That can have some consequences in cases where the orientation of the frame is relevant.

[31] Strictly, one should use the Laplacian Δ in all generality.

[32] And then, why not consider relations as: $d^1/_2 = ds^1/_2 \square\ ^1/_4$ and even $d^1/_3 = ds^1/_3 \square\ ^1/_6$ in a sense similar to what is attempted in the theories of supersymmetries?

[33] From now on, the text explicits the Appendix of Ref. 5,2. "Quantum Kinetics..."

[34] Ref. (12).

[35] Cf. Ref. (7).

[36] See end of section 2 and formula (47).

[37] To be checked as an exercise.

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