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# LEIBNIZIAN QUANTUM STRUCTURE, IRREVERSIBLE DYNAMICS, QUANTUM KINEMATICS, AND ALL THAT

# STRUCTTRE QUANTIQUE LEIBNIZIENNE, DYNAMIQUE IRRÉVERSIBLE, CINÉMATIQUE QUANTIQUE ET *TUTTI QUANTI*

BY

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ABSTRACT

For any differentiable dynamical system, when functions operate on the system state, Leibniz' rule of derivation yields a quantum structure. The physical calibration of the unit ($\hbar$, $c^2$, $k$, $e^2$, etc.) affords to the various phenomenologies (Quantum Mechanics, Kinematics, Thermics, Electromagnetics, etc.). Phy(sical)-dimension and change from passive to active projection transform geometric entities into physical ones, both classical and quantum. This is illustrated by the case of linear momentum.

Leibnizian Quantum Structure, or LQS, is a very simple and general mathematical structure, and supports Einstein's conviction that nature realizes the most simple mathematical structures. It displays a more coherent and more intuitive view of elementary Quantum Mechanics (Q.M.) and of Statistical Dynamics than the usual views inherited from the Knabenphysik of 1925-1928, and than Prigogine's new type of complementarity in irreversible mechanics [1]. Moreover Quantum Logics [2] attains a deep abstract coherence, but at the price of renunciation to intuitiveness and of some axiomatic arbitrariness (e.g., to ascribe "observables" only to Hermitian operators, what can be perceived as a dogma and critized as such [3].)

LQS finds its intuitiveness in the systematic use of the algebraicogeometrical properties of differentiable manifolds, or DM. Structurally, DM provide a very adequate and elegant language to reformulate coherently various dynamical theories:

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mathematically, with the theory of classic dynamical systems, as well as physically, for the diverse phenomenologies. [4]. For example, the ascription of a different geometrical nature to time — parameter or coordinate — enables us to specify both Newtonian and Einsteinian dynamics in the same mathematical structure (I call this kind of process a scientific derevolution in [5]).

My claim is that a similar process of derevolution of Q.M. is now permitted by consideration of LQS. Quantum structural properties are deeply rooted in the fundamental duality existing on a DM between the tangent bundle $T(M)$ (whose elements can be tangent vectors $\vec{v}$, and then linear momenta $\vec{p}$) and the cotangent bundle $T^*(M)$ (whose elements are numerical functions and differentials, and then positions $Q$) [4, Scheurer]. This duality is embedded implicity in LQS, in what may be the most elementary way [6].

As a mathematical structure, indeed, LQS is nothing else than Leibniz' rule of ordinary differential calculus on the real numbers, when functions are conceived as multiplicative operators on functions. For let $d$ be any linear differential operator, and $f$ and $g$ differentiable functions on $R$, then from Leibniz' original form (1677): $d(fg) = gdf + fdg$, through the modern definition of derivation: $d(fg) = (df)g + f(dg)$, one gets LQS form: $d(fg) - fdg = [d,f]g = (d(f))g$ when breaking the symmetry of $f$ and $g$ and singularizing $g$ for some non-mathematical reason. In dynamics the motivation to proceed like that is founded in the necessity, at last clearly recognized and stated here, to allow the physical grandeurs ($d$ and $f$) to act not directly on the system itself, but on its state function ($g$). For instance, if $d$ be the linear momentum $P_i$ proportional to $\partial/\partial x^i$, $f$ the position, i.e. the coordinate function $Q^j = x^j$, and $g$ the wave function $\Psi(x)$, the commutator $[P_i, Q^j]$ of elementary Q.M. obtains immediately, and hence the Heisenberg Uncertainty Relations, or HUR. Amazingly, almost every textbook on elementary Q.M. relies on the reversed process of *i*) postulating the wave function $\Psi(x)$, and *ii*) using Leibniz' rule to obtain the commutator, but nevertheless completely fails to recognize the very structure of LQS [7].

This structure becomes even more impressive by two extensions. First we can extend Leibniz' rule to the cases where the operators are both of the same geometrical nature, vector fields $v_1$ and $v_2$, or functions $f_1$ and $f_2$. We define $v_1(v_2) = [v_1, v_2] = \mathscr{L}_{v1}v_2$, (the Lie bracket and the Lie derivative of two vector fields), and $f_1(f_2) = [f_1, f_2] = 0$ by commutativity of ordinary product of functions. This way the algebra generated by the $P_i$'s, the $Q^j$'s and $I$ is fully recovered.

Second, we can define a priori a *fermiderivation* [8] by the following *Antileibniz' rule* on "fermifunctions": $\tilde{\partial}(\tilde{f}\tilde{g}) = (\tilde{\partial}\tilde{f})\tilde{g} - \tilde{f}\tilde{\partial}\tilde{g}$. Thus, operating on a convenient state "fermifunction" $\tilde{g}$, one gets the anticommutator $[\tilde{\partial}, \tilde{f}]_+ = \tilde{\partial}(\tilde{f})$. Similarly to Bergmann geometrical realization of the bosonic commutator (9, section 12.5), the AntiLQS structure can be realized in terms of a Clifford algebra (spinors) to represent the fermionic anticommutator. For that put $\tilde{\partial} = \gamma\partial$ (tangent operator) and $\tilde{f} = \gamma^+ f$ (cotangent operator) with $\gamma$ satisfying the conditions $\gamma = -\gamma^+$; $\gamma^2 = -1$ and $\gamma^+\gamma = -\gamma^+\gamma$. Thus $\tilde{\partial}(\tilde{f}) = [\gamma\partial, \gamma^+ f]_+ = \gamma\gamma^+ [\partial, f] = \partial(f)$ in full compatibility with Leibniz' rule for functions. A further extension completes the fermionic algebra, like for LQS proper.

On a DM, LQS applies for any *continuous parameter of evolution* $t \in R$ (usually time, but occasionally energy, or even temperature). Here are some realizations of LQS on various DM's. Mathematical precisions on the operator characteristics (domain, boundedness, spectrum, etc.), however necessary, will be dropped out here.

i) Given a DM $M^n$ and the parameter $t$. Then $d/dt = v^i\partial_i$, $i = 1, 2, \ldots n$, and $[d/dt, t] = I$. By active projection (see below for a precise meaning of this concept), of the parameter onto each axis $i$ successively, one gets $[\partial/\partial x^i, x^i] = I$. By physical calibration of the unit $I$, one reobtains the Schrödinger picture of the commutator $[-ih\partial_i, x^i] = -ihI$. As $-ih\partial_i$ is identified with $P^i$, so is $-ihd/dt$ with $L_{op}$, the Lagrangian operator. Indeed, when passing to the usual extension to $DM\ M^n xR$, then $d/dt = v^i\partial_i + \partial_t$ represents the substantial fluxion and $\partial/\partial t$ the local fluxion (11). Identifying $ih\partial_t$ with $H_{op}$, the Hamiltonian operator, one gets a Legendre transformation for operators: $L_{op} = v^i P_i - H_{op}$. In non relativistic cases, because $H_{op}$ is a positive operator but not necessarily $L_{op}$, the commutator $[L_{op}, t] = -ihI$ can occur, but not $[H_{op}, t] = ihI$. However in relativistic cases negative energies restaure this possibility for $H_{op}$, in full accordance with the fact that now time becomes a coordinate, and the same active projection of the parameter (proper time) onto it can apply too.

ii) Take now the phase space, $M^{2n}$, of element $(p^i, q^i)$ (resp. the state space $M^{2n}xR$, of element $(p^i, q^i, t)$). Then $id/dt = i\dot{q}^i\,\partial/\partial q^i + i\dot{q}^i\,\partial/\partial p^i\ (+\ i\partial_t)$ becomes Liouville operator $L$ in case of Hamiltonian systems. This is enough to explain the so claimed new complementarity $[L, T] = i$ by Prigogine [1]. This "second, fluctuant, historical time" $T$ is another way to introduce irreversibility in dynamics than by the usual way in thermodynamics of introducing a non geometrical (out of the DM) variable entropy $S$ [11]. $[L, T] = i$ runs in full parallelism with $[d/dt, S] = \dot{S} \geqq 0$. When $\dot{S} = 0$, then $S$ and $T$ become constant, and then usual time $t$ functions as a coordinate, or a reversible dynamical parameter (i.e. no more as a true parameter). This discussion will be more fully developed elsewhere.

iii) Besides Q.M., and even before it, there exists a Q.K., a Quantum Kinematics. As mass $m$ and period $T$ are related to parameter $t$, it is possible on the DM itself to introduce a Q.K. by defining the operators velocity $v^i := P^i/m$ and phase velocity $V^i{}_\phi$: $Q^i/T$. Then $[v^i, V^j{}_\phi] = -ic^2\,\delta^{ij}$, if due count is taken of the Brillouin-de Broglie relation $vV_\phi = c^2$ and $p\lambda = ET = h$. Then, in Q.K., HUR read now $\Delta v \Delta V_\phi \geqq c^2/2$, what is not microscopic at all! (Thus, kinematics would be nothing but a denenerated dynamics, where mass and period are forgotten or neutralized, $mT = 1$ ?) Various realizations of $v^i$ are possible: $v^i = -ih/m\partial_i = -ic^2T\partial_i = -ic^2\partial/\partial V_\phi$ = etc. (Note that the last form is to Brillouin relation what $-ih\partial_i$ is to de Broglie's!). Now a lot of mixed representations of the commutator can be conceived, as $[v^i, Q^j] = -ih/m\delta^{ij}$, the true classical limit with $h/m \rightarrow o$, or $[P^i, V_\phi{}^j]$, etc. with the associated various HUR. One can even play with a brand of QED, defining electric current to be $j^i := e/cv^i = -ieh/mc\partial_i = -2\mu_B\partial_i$, with $\mu_B$ the Bohr magneton: then $\Delta j \Delta Q \geqq \mu_B$! Correspondingly there exists $[j^i, J^j] = e^2/c^2\, [v^i, V_\phi{}^j]$. And so on.

iv) More interesting is the consideration of Thermodynamics and its LQS form: Statistical Mechanics. We don't come back to Prigogine complementarity discussed in ii). We proceed instead formally and a priori to the following transformation: $ihd/dt = kd/d(-it) = kd/d\theta$, identifying $h = k$ and $-it = \theta$. Chosing now the thermodynamic DM to be the generalized momentum space [11], $k$ becomes then Boltzmann quantum of entropy and $\theta = -1/T$ Stueckelberg's natural temperature (minus the inverse of absolute temperature $T$). Thus the statistical analogon of Schrödinger equation reads $kd/d\theta\phi = E\phi$ or $\phi = \exp(-S(\theta)/k) = \phi_o \exp(-E/kT)$. Here $\theta$ is the parameter and $E$ has fixed value. More usually, one works with systems in the dual representation (conjugation relatively to entropy!), with energy $H$ the parameter and fixed temperature $T$, i.e. $kd/dH\phi = \theta\phi$ or $\phi = \exp(-S(H)/k) = \phi_o \exp(-H/kT)$, i.e. the Gibbs canonical distribution. Here also mixed forms may occur. Note that HUR read now $\Delta E \Delta \theta \leqq k/2$ and limit in size the fluctuations in the neighborhood of a regular point (but say nothing for a critical point!).

The large variety in the possible physical phenomenologies shows that LQS, as such, is a pure mathematical structure, valid for any, "classic dynamical" system (measure preserving vector fields on DM's), endowed with physical signification only when a calibration (a gauge) of definite physical dimension (hereafter refered to as *phy-dimension*) is imparted to the geometric unit I. Hence the various constant gauges $h$, $c^2$, $e^2$, $k$, etc, which are interrelated by common reference to energy and time. Thus quantum structure is independant of the actual size of the gauge, and then is totally decoupled from micro- or macroscopicity!

This is a first important epistemic feature of LQS. A second one concerns the relation between classical and quantum systems. There is no break in character, inasmuch LQS can occur in any description of a dynamical system in term of a *continuous parameter of evolution*. In this sense HUR are included in a *deterministic* frame (not Laplacian determinism, of course, but Einsteinian determinism as well). I conjecture even that continuity could be dropped, and then some version of LQS could work for "abstract dynamical systems", where the notion of parameter still remains. As for the change between classical and quantum character, I ascribe it to the change in the mode of action of the physical entities on dynamical systems: classically, direct action on the *actual* system; LQSly, action on the *state* of the system, the physical entities being operators acting on the state function (wave function, Gibbs canonical distribution, which, as well known, are both related to probabilities; the formal relation with information is evident for Gibbs distribution: acting on it, the operator entropy $S$ gives $S \exp(-S/k)$, directly of the form $x \log x$). In his [1], Prigogine is right when remarking that operators are not necessarily linked with Q.M., but considerably less when bounding them with the abandon of the trajectories of classical mechanics, due to a condition of weak stability. This is a sufficient condition, but not a necessary one. For me, classical description obtains always when the field of all possible states is reduced to the unique actual state, blurring any difference between the state of the system and the system itself. The fact that the state appears to be more fundamental than the system itself, the latter being an actual "substantialization" of the former [11], is of considerable philosophical importance, for example in the discussion of non-localizability. This will be treated elsewhere.

Another reason of the superiority of LQS description of nature over the conventional one is to be found in the existence of the *phy-dimensionality*, which makes physics to be more than pure geometry. If on a DM the coordinates $x^i$ are endowed with length, then the geometrical linear momentum $\vec{p} = p^i\partial_i$ has no more the required phy-dimension of momentum. The restauration of the latter and the use of active projection instead of passive (geometric) projection lead easily to the usual quantum form of $\vec{p}$.

Let $\vec{p}$ have scalar magnitude $p$. Passive projection projects $\vec{p}$ geometrically into some previously chosen frame $\{x^i\}$, and so $\vec{p} = p^i\partial_i$, with $p^ip_i = p^2$, or equivalently $\vec{p} = v^ip/v\partial_i$. In active projection, the $i$-axis is physically put along the direction of $\vec{p}$, and so $P^i = p\partial_i$. (This is in complete analogy with the question of the velocity of a light ray, along or at angle with the ray).

Now the condition that phy-dimension is only defined for the *intrinsic* physical entity (11, app. A1) entails to working on the DM with basis vectors $\lambda\partial_i$ and basis covectors $dx^i/\lambda$, where $\lambda$ could be first any arbitrary length (and then to write $\vec{p} = p^i\lambda\partial_i$ and $P^i = p\lambda\partial_i$). This condition is reported upon the components when the basis elements are dimensionless (like in the Euclidian case $\vec{p} = p^i\vec{e}_i$). Then by the Broglie relation $p\lambda = h$, one can write $\vec{p} = (v^i/v)\, h\partial_i$ and $P^i = h\partial_i$ ($\vec{p}$ becomes a superposition of the quantum $P^{i'}s$). The final touch for $P^i$ is given by the condition of hermiticity. There remains only to justify the universality of the constant $h$. This is still pending.

Phy-dimensionality plays also a part in the covector formulation of both classical $\overleftarrow{p}$ and quantum $P_i$, which must equally present the phy-dimension of momentum. This condition imposes that the metric tensor be instrinsically phy-dimensionless: $g_{ij}/\lambda^2 dx^i dx^j = g'_{ij}dx^i dx^j$. (Thus eo ipso the $g'_{ij}$ are endowed with the phy-dimension required in the General Theory of Relativity of the gravitation potentials.) One gets then classically $\overleftarrow{p} = p_i dx^i/\lambda$ and quantumly

$$P_i = +ih/\lambda^2 dx^i \quad \text{or} \quad P_i = +ih/\lambda^2 x^i$$

as the $x^i$'s are as a good basis for $T^*(M)$ as the $dx^i$'s. The second form admits quasi-automatically of two noticeable interpretations: i) it gives a kind of Regge-trajectory condition: $P^2 = +ih/\lambda^2 P_i x^i$; and ii) it exhibits potentials of the form of quark potentials: $E = +ihc/\lambda^2 r$, with $ct = r$.

The (non universal) constant $h/\lambda^2 = p/\lambda = p^2/h$ deserves a proper name. As the product of $p$ and $\lambda$ is called *action*, their quotient may be called *passion*. In dynamics, passion is as fundamental as action. It appears in the characteristic equations [4] $dx/p = dt/m = cd\tau/p_o$. And effectively, when $\lambda$ is allowed to pass to the "differential limit" $vdt$ (resp. $\lambda_o$ to go to $cd\tau$) for parameter $t$ (resp. $\tau$), passion permits to recover both classical momentum and potentials in $1/r$, and then yields a definite relation between the forms of Newton Laws and of the physical interactions. In this limit, indeed, with parameter $\tau$, $P_i = p_o/\lambda_o dx^i$ goes to $p_o dx^i/cd\tau = m_o dx^i/d\tau$, and $P_i = \alpha h/\alpha\lambda^2{}_o dx^i = e^2/cr_o dx^i/\lambda_o$ goes to $e/cA^i_{self}$, where $r_o = \alpha\lambda_o$ is the classical electron radius, and $A_{self}$ a sort of self-potential. The same can be done for gravitation with $\beta = Gm^2/hc$.

In conclusion, I wish to thank my friends P. Kroes and Prof. A. Janner for helpful discussions.

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[1] PRIGOGNINE, I. *From Being to Becoming*, Freeman, San Francisco, 1980, esp. chap. 8 and App. A. He gives the commutators $[L, T] = i$ and $[L, M] = D \geqq 0$, which are rewritten here as $[d/dt, t] = I$ and $[d/dt, S] = dS/dt \geqq 0$.

[2] Cf. the works of J. VON NEUMANN, G. W. MACKEY, J. M. JAUCH and C. PIRON. For a recent survey, see H. PRIMAS, *Chemistry, Quantum Mechanics and Reductionism*, Springer, Berlin, 1981, §4.4.

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[6] Another expression of this duality reveals in the theories of homology and cohomology. Cf. the last two ref. of (4).

[7] For ex. A. MESSIAH, *Mécanique Quantique*, Dunod, Paris, 1959, vol. I, chap. 11.

[8] As an antiderivation presents an alternating sign, another appropriate name had to be coined!

[9] JAUCH, J. M. *Foundations of Quantum Mechanics*, Addision-Wesley, Reading, Mass., 1968.

[10] No distinction is made between $h$ and $\hbar$!

[11] STUECKELBERG E. C. G. and P. B. SCHEURER: *Thermocinétique phénoménologique galiléenne*, Birkhäuser, Basel, 1974.