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# DETAILS OF HYDROGEN-BURNING THERMONUCLEAR REACTIONS

BY

P. BOUVIER and L. WEIBEL

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## ABSTRACT

In connection with the triple  $pp$  chain, we study here the temperature dependence of the rate of energy production in the range where the  $PP\ II$  chain is predominant. Moreover, we examine the approach towards equilibrium of the complete  $CNO$  cycle, composed of the  $CN$  and the two  $NO$  cycles, showing that the presence of the second  $NO$  cycle entails a quasi-equilibrium for the  $CN$  cycle.

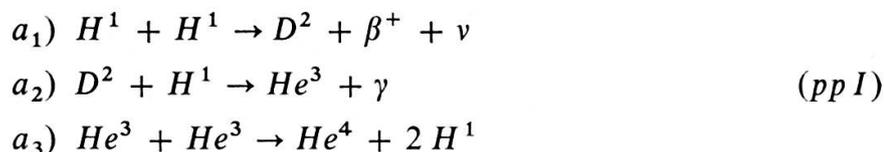
## RÉSUMÉ

Relativement à la triple chaîne  $pp$ , on examine ici la dépendance du débit d'énergie à l'égard de la température, dans l'intervalle où la chaîne  $PP\ II$  domine. Par ailleurs, nous abordons l'étude de l'approche à l'équilibre du cycle complet  $CNO$  formé du cycle  $CN$  et des deux cycles  $NO$ , en montrant que la présence du deuxième cycle  $NO$  entraîne un quasi-équilibre pour le cycle  $CN$ .

## 1. ENERGY PRODUCTION IN THE PROTON-PROTON CHAIN

The two most important ways by which hydrogen is converted into helium within stars are the so called proton proton chain and the  $CNO$  cycle.

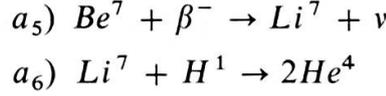
Now the  $pp$  chain operates over the temperature range  $8 \leq T_6 \leq 30$  (where  $T_6 = T \times 10^{-6} \text{ }^\circ K$ ), yielding  $26.21 \text{ MeV}$  in form of radiation energy; it consists in fact in a three-fold chain of reactions; the 1<sup>st</sup> being



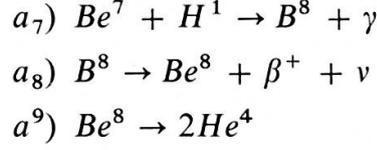
When  $He^4$  becomes sufficiently abundant, we may have instead of  $a_3$ ),



and whether  $Be^7$  will capture either a free electron or a proton, we will finally obtain  $He^4$  by the completion of the  $pp-II$  or the  $pp-III$  chain respectively, *ViZ.*



or



When the complete  $pp$  chain proceeds in equilibrium, the rate of helium formation is given by the expression (see Clayton, 1968)

$$\frac{d\text{He}^4}{dt} = \left( \frac{d\text{He}^4}{dt} \right)_I \varphi(\alpha_1)$$

where  $(d\text{He}^4/dt)_I$  is the rate of formation by the  $ppI$  chain only and

$$\varphi(\alpha_1) = 1 - \alpha_1 + \alpha_1 \left( 1 + \frac{2}{\alpha_1} \right)^{\frac{1}{2}}$$

where

$$\alpha_1 \left( T, \frac{\text{He}^4}{\text{H}} \right) = \frac{\lambda_{34}^2}{\lambda_{33} \lambda_{pp}} \left( \frac{\text{He}^4}{\text{H}} \right)^2$$

the  $\lambda$ 's being the reaction rates per pair of interacting nuclei. [ $\text{cm}^3 \text{sec}^{-1}$ ].

The rate of energy liberation per unit volume,  $\rho\epsilon$  [ $\text{erg sec}^{-1} \text{cm}^{-3}$ ], will not include the energy carried off by the neutrinos which amounts respectively, for each of the neutrinos appearing in  $a_1$ ,  $a_5$ ,  $a_8$ ), to 1.9, 4.0 and 28 per cent of the total energy connected to the mass defect  $4m_{\text{H}} - m_{\text{He}}$ .

Denoting by  $F_{ppi}$  the fraction of  $\alpha$  particles produced by the chain  $i$  ( $i = I, II, III$ ), we may write

$$\rho\epsilon = \frac{d\text{He}^4}{dt} (4m_{\text{H}} - m_{\text{He}}) c^2 (0.981 F_{ppI} + 0.960 F_{ppII} + 0.720 F_{ppIII})$$

whence, if  $\epsilon = \epsilon_I$  when  $F_{ppI} = 1$  ( $ppI$  acting alone),

$$\epsilon = \frac{\epsilon_I}{0.981} \varphi(\alpha_1) [0.981 F_{ppI} + 0.960 F_{ppII} + 0.720 F_{ppIII}]$$

There is no difficulty in showing that, under equilibrium conditions (Clayton, 1968)

$$\frac{F_{ppI}}{F_{ppII} + F_{ppIII}} \equiv \frac{F_{ppI}}{1 - F_{ppI}} = \frac{1}{4} \left[ \left( 1 + \frac{2}{\alpha_1} \right)^{\frac{1}{2}} - 1 \right]$$

whence

$$F_{ppI} = \left[ \left( 1 + \frac{2}{\alpha_1} \right)^{\frac{1}{2}} - 1 \right] \left[ \left( 1 + \frac{2}{\alpha_1} \right)^{\frac{1}{2}} + 3 \right]^{-1}$$

Furthermore, in terms of the branching ratio

$$\alpha_2 = \frac{F_{ppIII}}{F_{ppII} + F_{ppIII}}$$

of the  $ppIII$  chain at  $Be^7$ , we have

$$F_{ppIII} = \alpha_2 (1 - F_{ppI}) \quad F_{ppII} = (1 - \alpha_2) (1 - F_{ppI})$$

so that, after some easy algebraic manipulation, we can write  $\epsilon$  in the form

$$\epsilon = \epsilon_I \psi(\alpha_1, \alpha_2)$$

where

$$\psi(\alpha_1, \alpha_2) = 1 + (0.962 - 0.5 \alpha_2) [\varphi(\alpha_1) - 1]$$

which is (improved value) the expression given by Reeves (1965), obtained here in a slightly different way, through the explicit introduction of the  $F_{ppi}$ .

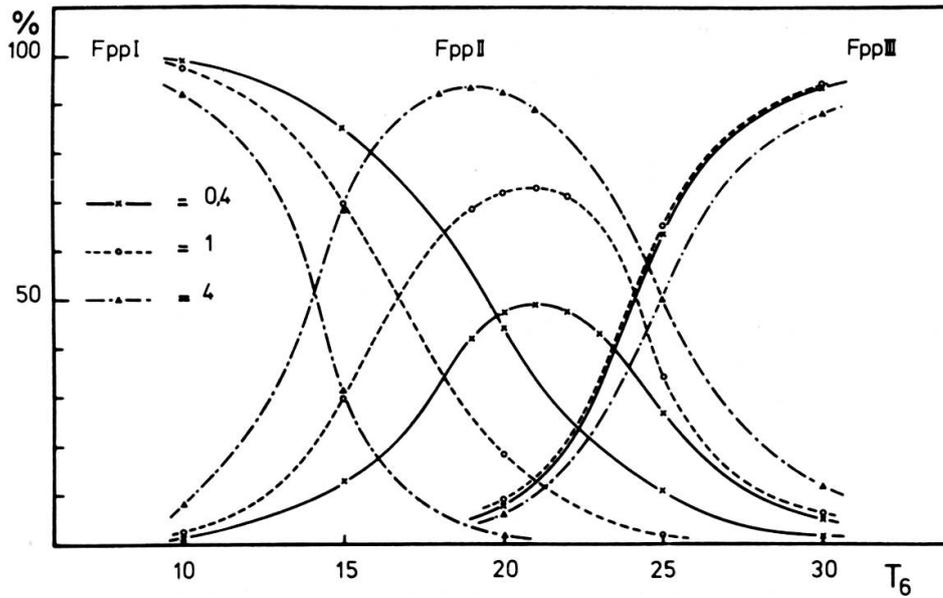


FIG. 1. — Illustrates the dependence of the fractions  $F_{ppi}$  on temperature, at given  $Y/X = 4 He^4/H$  ratio.

## 2. PREDOMINANCE OF $pp II$ CHAIN

Fig. 1 reveals that, when enough  $He^4$  is present, there is a temperature interval in which the  $pp II$  chain prevails distinctly over the 2 other chains ( $17 \lesssim T_6 \lesssim 22$ ); since  $pp III$  has not started significantly yet, we have  $\alpha_2 = 0$  and consequently,

$$\epsilon \approx \epsilon_I \varphi(\alpha_1)$$

$\epsilon_1$  is proportional to the function  $\tau^2 e^{-\tau}$  characteristic of a non resonant reaction, where  $\tau = BT_6^{-1/3}$ ,  $B$  being a constant depending on the chemical parameters ( $A, Z$ ) of the interacting nuclei.

On the other hand, it is customary to express the  $\epsilon(T)$  dependence as a power law, so that

$$\epsilon \div T^v \div \tau^{-3v} \div \tau^2 e^{-\tau} \varphi(\alpha_1)$$

By logarithmic derivation, we obtain

$$3v = \left(1 - \frac{\varphi'_\tau}{\varphi}\right) \tau - 2$$

The explicit form for  $\alpha_1$  is, to a high degree of accuracy,

$$\alpha_1 = A \exp(-100 T_6^{-1/3})$$

where  $A$  is practically independent of  $T$ ; consequently,

$$\varphi'_\tau = \varphi'_{\alpha_1} \frac{\partial \alpha_1}{\partial \tau} = \frac{100}{B} \alpha_1 \varphi'_{\alpha_1} \simeq -3\alpha_1 \varphi'_{\alpha_1}$$

since the value of  $B$  in the present  $pp$  reaction amounts to 33.7.

Therefore

$$v = \frac{\tau - 2}{3} + \alpha_1 \frac{\varphi'_{\alpha_1}}{\varphi} \tau$$

the first term pertains to the  $pp I$  chain only, the second term being the contribution of the  $pp II$  chain. Table 1 summarizes the values obtained for  $v$  at different  $Y/X$  and  $T_6$  values.

TABLE 1

$Y/X \backslash T_6$	18	20	22
0.4	3.62 + 1.03	3.50 + 1.32	3.34 + 1.32
0.63	3.62 + 1.32	3.50 + 1.37	3.34 + 1.08
1	3.62 + 1.42	3.50 + 1.21	3.34 + 0.73
2	3.62 + 1.23	3.50 + 0.65	3.34 + 0.27
4	3.62 + 0.65	3.50 + 0.23	3.34 + 0.08

$v$  is increased by the  $pp II$  influence, but not considerably;  $pp II$  is less sensitive to the temperature as the helium abundance is high ( $Y/X$  large).

3. THE *CNO* TRICYCLE

When  $T_6$  reaches 12, hydrogen may also burn by the *CNO* cycle, which around  $T_6 = 18$ , for standard compositions of population *I* stars, soon becomes the dominant reaction as  $T_6$  goes on rising. The *CNO* tricycle, sketched in fig. 2, is made up of the basic *CN* cycle (hexagon) and of two additional *NO* cycles, the first of which, connected to the *CN* cycle joins  $N^{15}$  to  $N^{14}$  by passing  $O^{16}$  whereas the second *NO* cycle, added to the first one, joins  $O^{17}$  to  $N^{15}$  through  $O^{18}$ .

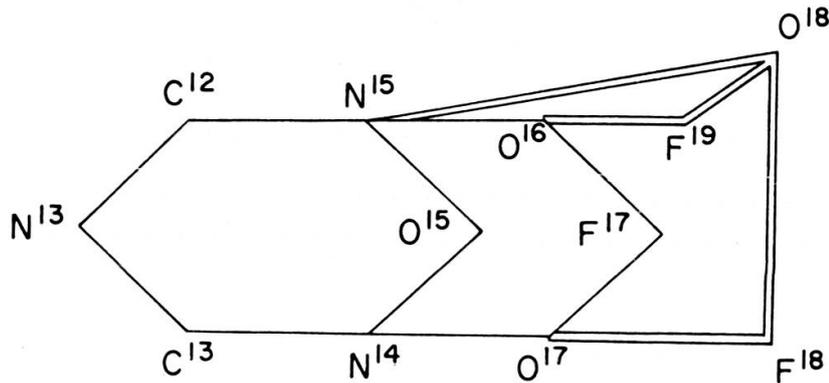


FIG. 2.

All reactions considered here are induced by protons; we neglect any  $\alpha$ -induced reaction, corresponding to simultaneous hydrogen and helium burning (Caughlan and Fowler, 1962).

The branching ratio of the  $N^{15} (p, \gamma) O^{16}$  reaction is approximately  $\gamma_1 = 8 \times 10^{-4}$  according to Fowler et al. (1975; quoted as *FCZ II*), so that it takes about  $10^3$  complete *CN* cycles to go through before a significant number of *CN* nuclei will have switched over to the *NO* cycles. In general, the lifetimes of the different nuclear species are fairly time-independent during a steady hydrogen burning phase, so that we may regard these lifetimes as constant at a given temperature; the branching ratios will then also be constant and moreover weakly dependent on temperature changes.

In the physical conditions  $T_6 = 25$ ,  $\rho X_H = 28$  which we had adopted in a preliminary study to test the assumption of constant nuclear lifetimes during a particular nuclear burning stage, the lifetimes of the relevant nuclei are, in years,

$$\begin{aligned} \tau_{12} &= 8.98 \times 10^2 & \tau_{16} &= 4.51 \times 10^6 \\ \tau_{13} &= 2.58 \times 10^2 & \tau_{19} &= 1.98 \times 10^5 \\ \tau_{14} &= 8.38 \times 10^4 & & \end{aligned}$$

Beyond  $F^{19}$ ,  $p$ -induced reactions could lead either to  $O^{16}$  or to  $Ne^{20}$ ; both such reactions are extremely slow, the second being still slower than the first which could close the chain at  $O^{16}$ , thus completing a *CNOF* quadricycle.

We further label  $\tau_{A,B}$  the lifetimes of nucleus  $A$  with respect to the transmutation  $A \rightarrow B$  through  $p$ -capture; always according to the *FCZ II* reaction rates, we have, in years,

$$\tau_{15,12} = 3.3 \quad \text{and} \quad \tau_{15,16} = 4.1 \times 10^3$$

For the isotopes  $O^{17}$ ,  $O^{18}$ , the values are sensitive to experimental uncertainties; according to whether a resonance in the compound nucleus contributes by a vanishing, partial or dominant amount to the  $p$ -capture cross-section, *FCZ II* give three possible values (denoted  $L$ ,  $I$ ,  $H$  respectively). Retaining here only the intermediate values, viz. in years,

$$\begin{aligned} \tau_{17,14} &= 2.78 \times 10^6 & \tau_{18,15} &= 12,4 \\ \tau_{17,18} &= 3.71 \times 10^6 & \tau_{18,19} &= 7.4 \times 10^5 \end{aligned}$$

whence

$$\gamma_2 = \frac{\tau_{17,14}}{\tau_{17,14} + \tau_{17,18}} = 0.43 \quad \gamma_3 = \frac{\tau_{18,15}}{\tau_{18,15} + \tau_{18,19}} = 1.68 \cdot 10^{-5}$$

In spite of the uncertainty in these values, they do show us that the frequency of occurrence of both *ON* cycles is about the same, and that of the complete quadricycle is comparatively much less.

#### 4. APPROACH TOWARD EQUILIBRIUM IN THE *NO* CYCLES

According to the  $\tau_{12}$  value given above, we see that the *CN* cycle achieves equilibrium in about  $10^3$  years and only later will the *NO* cycles become significantly operative. The stellar environment adopted here, namely  $\rho X_H = 28$ ,  $T_6 = 25$  corresponds closely to the central conditions of a  $3 M_\odot$  main sequence star (Iben, 1965); the fact that  $T_6$  lies well below 100 prevents us from having to consider  $\alpha$ -induced reactions and the complications of the rather extensive reaction network for the hot *CNO-Ne* cycle (Audouze et al., 1973).

The time variations of chemical abundances during the approach to equilibrium of the *NO* cycles are the following (see fig. 2):

$$\begin{aligned} \frac{dN^{14}}{dt} &= (1-\gamma_2) \frac{O^{17}}{\tau_{17}} + (1-\gamma_1) \frac{N^{15}}{\tau_{15}} - \frac{N^{14}}{\tau_{14}} \\ \frac{dN^{15}}{dt} &= \frac{N^{14}}{\tau_{14}} - \frac{N^{15}}{\tau_{15}} + (1-\gamma_3) \frac{O^{18}}{\tau_{18}} \\ \frac{dO^{16}}{dt} &= \gamma_1 \frac{N^{15}}{\tau_{15}} - \frac{O^{16}}{\tau_{16}} + \frac{F^{19}}{\tau_{19}} \\ \frac{dO^{17}}{dt} &= \frac{O^{16}}{\tau_{16}} - \frac{O^{17}}{\tau_{17}} \end{aligned} \tag{1}$$

where we have neglected the short-lived positron emitters. The running of the  $NO$  cycles alters somewhat the  $CN$  equilibrium abundances reached after  $10^3$  years; we now have

$$\frac{C_e^{12}}{\tau_{12}} = \frac{C_e^{13}}{\tau_{13}} = \frac{N_e^{15}}{\tau_{15}} = \frac{N^{14}}{\tau_{14}} + \gamma_2 \frac{O^{17}}{\tau_{17}}$$

where  $N^{14}$  and  $O^{17}$  will change on a longer time scale ( $\sim 10^5$  y); the  $CN$  cycle is therefore in a quasistatic equilibrium.

Dropping further the  $F^{19}$  channel ( $F^{19} = O, \gamma_3 = O$ ) and noticing that  $O^{18}$  reaches its equilibrium value in less than 20 years:

$$\frac{O_e^{18}}{\tau_{18}} = \frac{O_e^{18}}{\tau_{18,15}} = \gamma_2 \frac{O^{17}}{\tau_{17}}$$

we obtain

$$\frac{dN^{15}}{dt} = \frac{N^{14}}{\tau_{14}} - \frac{N^{15}}{\tau_{15}} + \gamma_2 \frac{O^{17}}{\tau_{17}}$$

Since  $\frac{1}{\tau_{15}} \approx \frac{1}{\tau_{15,12}}$ , and  $\tau_{15,12}$  is of order  $3\frac{1}{2}$  years, while  $\tau_{14}$  and  $\tau_{17}$  are much

larger, we may consider that in a few years time,  $N^{15}$  has attained its equilibrium value and we are left with the three equations

$$\begin{aligned} \frac{dN^{14}}{dt} &= (1 - \gamma_2 \gamma_1) \frac{O^{17}}{\tau_{17}} - \gamma_1 \frac{N^{14}}{\tau_{14}} \\ \frac{dO^{16}}{dt} &= \gamma_1 \frac{N^{14}}{\tau_{14}} + \gamma_1 \gamma_2 \frac{O^{17}}{\tau_{17}} - \frac{O^{16}}{\tau_{16}} \\ \frac{dO^{17}}{dt} &= \frac{O^{16}}{\tau_{16}} - \frac{O^{17}}{\tau_{17}} \end{aligned} \quad (2)$$

which entail  $N^{14} + O^{16} + O^{17} = N_{CNO}$  (const.). Letting  $\gamma_2 = O$  brings us back to the case of the  $CNO$ -bicycle.

## 5. TEMPORAL BEHAVIOUR OF THE CHEMICAL COMPOSITION

Labeling  $N_i$  ( $i=1, 2, 3$ ) the fractional abundances of the 3 isotopes involved in equations (2), it is easy to find, by suitable elimination, a  $2^d$ -order differential equation for each of the  $N_i$ , of the form

$$\frac{d^2 N_i}{dt^2} + A \frac{dN_i}{dt} + B N_i - C_i = 0 \quad (3)$$

where

$$N_1 = N^{14}, \quad N_2 = O^{16}, \quad N_3 = O^{17},$$

$$C_1 = \frac{1 - \gamma_1 \gamma_2}{\tau_{16} \tau_{17}} N_{CNO}, \quad C_2 = \frac{\gamma_1}{\tau_{14} \tau_{17}} N_{CNO}, \quad C_3 = \frac{\gamma_1}{\tau_{14} \tau_{16}} N_{CNO},$$

$$A = \frac{\gamma_1}{\tau_{14}} + \frac{1}{\tau_{16}} + \frac{1}{\tau_{17}} \quad N_{CNO} B = \sum_{i=1}^3 C_i$$

The general solution of (3) has the form

$$N_i(t) = K_i e^{r_1 t} + L_i e^{r_2 t} + \frac{C_i}{B} \quad (4)$$

where  $K_i, L_i$  are constants,  $\frac{C_i}{B}$  is the equilibrium value of  $N_i$  and

$$r_1 = -\frac{A}{2} + \frac{1}{2} \sqrt{A^2 - 4B}, \quad r_2 = -\frac{A}{2} - \frac{1}{2} \sqrt{A^2 - 4B}$$

The initial conditions to which  $K_i, L_i$  are connected are taken at a time  $t = 0$  chosen as the starting time for the  $NO$  cycles display, well after the  $CN$  cycle has attained equilibrium. In the particular stellar environment adopted here and also for other temperatures between 20 and 80 million degrees, it turns out that  $A^2 - 4B$  is always positive (sometimes only weakly); therefore  $r_1, r_2$  and consequently  $K_i, L_i$  are real and we get a superposition of exponential variations for  $N_i(t)$ . Some of the abundances will increase from  $N_i(0)$  to  $C_i/B$  while others shall decrease, since at all times we have

$$\sum_{i=1}^3 N_i(t) = N_{CNO}$$

But the accuracy of the  $\tau$ -values is often poor and, for other similar situations, we could possibly be faced with the case  $A^2 - 4B < 0$ ; this happens, for example, when the lifetimes  $\tau_{14}/\gamma, \tau_{16}, \tau_{17}$  become very close to each other and the approach to equilibrium would then have an oscillatory character.

Letting  $\frac{1}{2} \sqrt{4B - A^2} = \omega$ , the solution (4) becomes explicitly

$$N_i(t) = \left( N_i(0) - \frac{C_i}{B} \right) \exp\left(-\frac{A}{2}t\right) \left[ \cos \omega t + \frac{A}{2\omega} \sin \omega t \right] + \frac{C_i}{B}$$

having assumed  $\left(\frac{dN_i}{dt}\right)_{t=0} = 0$ .

The condition that  $N_i(t)$  must always remain positive is warranted by the fact that the period  $2\pi/\omega$  is distinctly larger than the damping time  $2/A$ . After a quarter of period, for instance, when  $t = \pi/2\omega$ , the amplitude of the oscillation is already reduced by  $e^{-\pi u/2}$  (where  $u = A/2\omega$ ) and this factor is much less than unity

because, from the definitions of  $A$  and  $B$ ,  $A^2 - 2B$  is always  $> 0$ , so

$$u = \frac{A}{\sqrt{4B - A^2}} > 1.$$

We conclude that  $N_i(t)$  is indeed always  $\geq 0$ , whatever the sign of  $N_i(0) - C_i/B$ .

We are indebted to Professor W. A. Fowler for having sent us in advance of publication, a preprint of his revised thermonuclear reaction rates.

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