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TRANSMISSION ELECTRON SPIN RESONANCE THE INTERPRETATION OF TWO EXPERIMENTS¹

BY

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ABSTRACT

A brief review is given of the principles of transmission electron spin resonance. Previous work by the author which 1) provides a basis for interpreting the ion-implantation experiments of Monod, Hurdequin, Janossy, Obert, and Chaumont (1972) and 2) gives an explanation of the anomalies observed by Schultz and Dunifer (1967) in their experiments on *Na* and *K*, is then reviewed.

INTRODUCTION

First of all, I would like to say that it is a great pleasure for me to be on the same program as the preceding speakers, Philippe Monod and Andrew Janossy. As you will see before long, much of my work in the area of transmission electron spin resonance has been stimulated by their publications.

In the next forty-five minutes or so, I would like to accomplish the following two things:

1. to review work (Walker 1973a, b) which gives a basis for interpreting the ion-implantation experiments of Monod, Hurdequin, Janossy, Obert and Chaumont (1972).
2. to review work (Walker, 1973c) which gives a qualitative explanation of the anomalies observed by Schultz and Dunifer (1967) in their experiments on *Na* and *K* (these are the experiments in which spin waves were observed in *Na* and *K*).

This talk will rely on qualitative arguments, and will refer to work already published, or to be published in the near future, for detailed calculations.

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Background material on transmission electron resonance (*TESR*) relevant to the following discussion can be found in the reviews by Walsh (1968) and Platzman and Wolff (1973).

A RECIPROCITY THEOREM FOR TRANSMISSION EXPERIMENTS

Before getting down to the details of the ion-implantation experiments, it is of interest to note that a reciprocity theorem can be proved which is applicable to transmission experiments in general. Consider the slab shown in Figure 1. In a

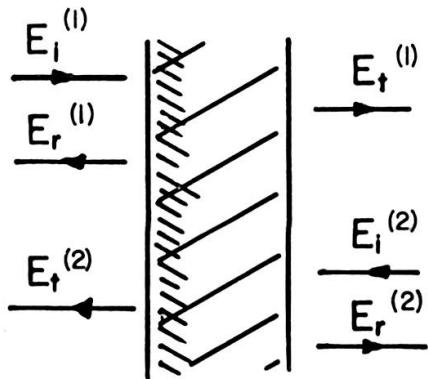


FIG. 1. — The slab shown may vary in composition as one goes from left to right, as indicated by the heavily shaded region. The slab can be excited either from the left (case 1) or right (case 2); the reciprocity theorem says that the transmission coefficient is the same in either case.

transmission experiment one can either excite from the left and observe the transmitted field on the right (call this case 1 and label the fields by a superscript 1 in Figure 1) and visa-versa (call this case 2 and label the fields by a superscript 2 in Figure 1). It was demonstrated experimentally in the *TESR* experiments of Monod *et al.* (1972) and of Janossy and Monod (1973) that the transmission coefficient was independent of which side one excited the slab from. A formal expression of this result is as follows. The transmitted field amplitudes (subscript t) in each case can be linearly related to the incident field amplitudes (subscript i) i.e.

$$E_{t\alpha}^{(1)} = T_{\alpha\beta}^{(1)}(\vec{H}_0) E_{i\beta}^{(1)}$$

$$E_{t\alpha}^{(2)} = T_{\alpha\beta}^{(2)}(-\vec{H}_0) E_{i\beta}^{(2)}$$

where α, β take the values x or y , the summation convention is used, and the external magnetic is \vec{H}_0 in case 1 and $-\vec{H}_0$ in case 2. It can then be proved (Walker, 1973b) that the relation

$$T_{\alpha\beta}^{(1)}(\vec{H}_0) = T_{\beta\alpha}^{(2)}(-\vec{H}_0)$$

holds.

REVIEW OF PRINCIPLES OF TRANSMISSION EXPERIMENTS

A brief review of the principles of transmission electron spin resonance (*TESR*) is perhaps appropriate. Consider the metallic slab shown in Figure 2. As a result of the skin effect, the incident microwave field penetrates into the metal only a

distance ζ , the skin depth. The metal slab has a thickness L much greater than the skin depth so that if the electrons did not have spins, there would be no transmission. If the magnitude of the static external field, H_0 , is now adjusted so that the electron spin resonance frequency coincides with the frequency of the microwave field, electron spins entering the skin depth will be induced to begin precessing about \vec{H}_0 as shown in Figure 2. These electrons diffuse across the slab, and if their spins have not relaxed before they get to the far side (e.g. point B of Figure 2) will radiate giving rise to the transmitted field H_t . The distance a spin diffuses in its spin relaxation

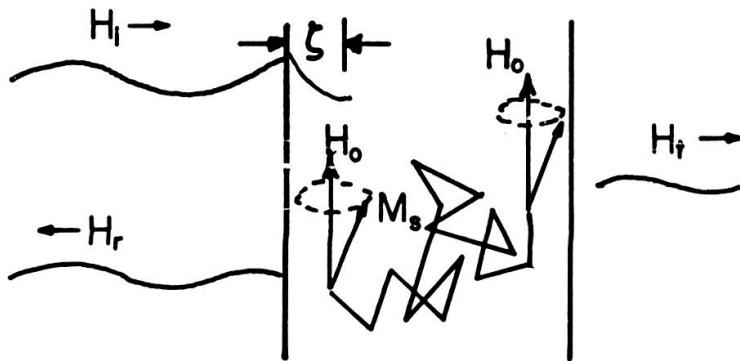


FIG. 2. — Illustrating the transmission of microwave power through a metallic slab by electron spins.

time, τ_s , is $(D\tau_s)^{1/2}$, where D is the diffusion constant; hence the condition that power is transmitted through the slab is $\sqrt{D\tau_s} > L$.

This phenomenon whereby the metallic slab becomes partially transparent at the spin resonance frequency was predicted theoretically by Azbel, Gerasimenko and Lifshitz (1957), seven years before it was first observed experimentally (Lewis and Carver (1964), Vander Ven and Schumacher (1964)).

Assuming that the incident microwave field has set up a precessing magnetization density, $M_-(z) = M_x(z) - iM_y(z)$, in the slab, the next problem is to calculate the amplitude of the transmitted field. That portion of the transmitted field radiated by the precessing moment in the heavily cross hatched volume element of Figure 3 is given by

$$dH_t = 4\pi i k_v M_-(z') dz' e^{ik_0(L-z')}$$

where $k_0 = (1+i)/\zeta$, ζ being the skin depth (classical skin effect conditions are assumed) and $k_v = \omega/c$ is the vacuum wave number of light. The factor $M_-(z') dz'$ is the magnetic moment of the volume element and the factor $\exp[ik_0(L-z')]$ gives the attenuation and phase shift suffered by the wave in travelling the distance $L - z'$ to the surface of the metal; the constant of proportionality, $4\pi i k_v$, follows from more detailed considerations.

Summing up contributions from all volume elements one finds (Walker 1973b)

$$H_t = 4\pi ik_r \int_0^L M_-(z) e^{ik_0(L-z)} dz$$

The value of this formula lies in the fact that no assumption has been made concerning the nature of the spatial variation of $M_-(z)$. This makes it very convenient for the study of the ion-implantation problem where the magnetization density near the surface varies rapidly as a result of the contribution of the implanted ions.

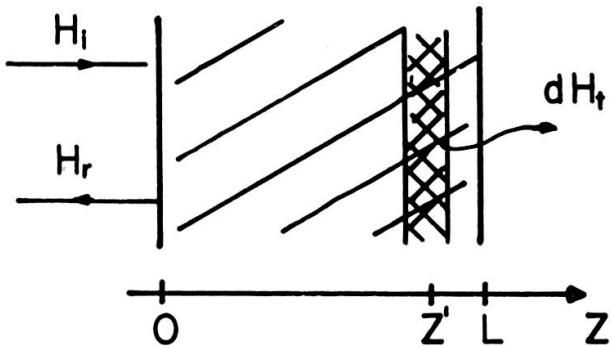


FIG. 3. — The transmitted field is a superposition of fields radiated by the precessing magnetic moments at different points in the metal.

If $M_-(z)$ varies slowly in the skin depth, one can write $M_-(z) \approx M_-(L)$ when evaluating the integral in the expression for H_t ; thus finding

$$H_t = -4\pi(k_r/k_0) M_-(L)$$

which is the classical skin effect limit of the well-known result of Lampe and Platzman (1966).

TRANSMISSION THROUGH ION-IMPLANTED FILMS

Monod *et al.* (1972) have found that by implanting one surface of a copper foil with manganese ions the intensity of the *TESR* signal was enhanced by a factor of as much as 50; in the case of two-sided implantation the enhancement factor was found to be $(50)^2 = 2500$. This improvement in sensitivity is potentially of great practical importance. The preceding talks by Monod and Janossy have described these experiments in detail, and also show that other mechanisms for enhancing the signal have much promise.

The original paper of Monod *et al.* listed a number of unusual features of the observations which any theory of the phenomenon would have to explain. Subsequently, a theory was proposed by myself (1973a, b) which gave at least a qualitative understanding of the phenomenon, and that theory will now be reviewed.

Let us assume that the concentration of implanted ions is zero for $z < L - \delta'$ and non-zero but uniform for $L - \delta' < z < L$ (see Fig. 4). For $z < L - \delta'$, the precessing magnetization density, M_{s-} , is entirely due to conduction electron spins; for $L - \delta' < z < L$, the precessing magnetization density, $M_{s-} + M_{d-}$, contains a contribution, M_{d-} , due to the transition metal local moments. Thus, in this case, the transmitted field will be given by

$$H_t = 4\pi ik_v \left[\int_0^{L-\delta'} M_{s-} e^{ik_0(L-z)} dz + \int_{L-\delta'}^L (M_{s-} + M_{d-}) e^{ik_0(L-z)} dz \right]$$

Assuming M_{s-} and M_{d-} are independent of z , one finds

$$H_t = -4\pi(k_v/k_0) [M_{s-} e^{ik_0 \delta'} + (M_{s-} + M_{d-})(1 - e^{ik_0 \delta'})]$$

To proceed further it is necessary to know something about the two normal modes of oscillation of the two degrees of freedom, M_{s-} and M_{d-} , in the implanted region. It will be assumed that M_{s-} and M_{d-} are strongly coupled together so that a description in the so-called bottleneck limit is applicable. The appropriate normal modes are shown in Figure 5. In mode 1, \vec{M}_s and \vec{M}_d remain parallel to each other

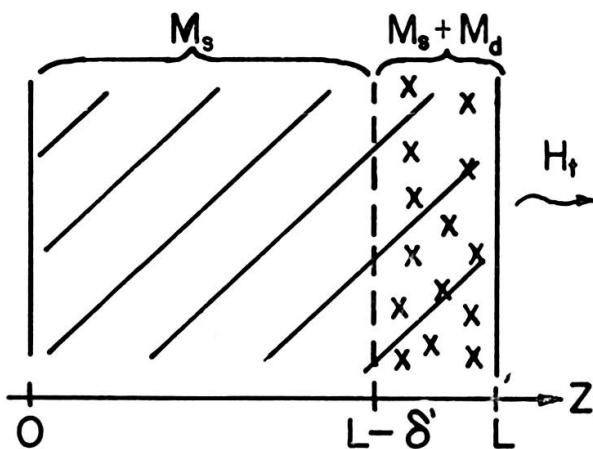


FIG. 4. — Idealized geometry describing ion-implantation experiments.

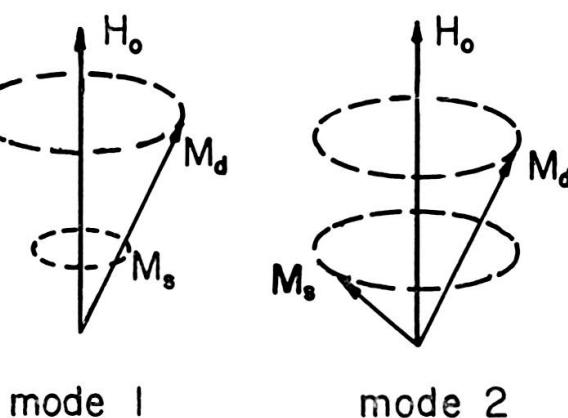


FIG. 5. — The normal modes of a homogeneous coupled local-moment conduction-electron system in the bottleneck limit.

as they precess, whereas in mode 2, the precessing components of M_s and M_d are equal in magnitude but have opposite directions. Since mode 2 has no net precessing moment, it can not couple to the microwave magnetic field and for our purposes can be ignored. A detailed study of $Cu : Mn$ using the transmission technique has been made by Schultz, Shanaberger, and Platzman (1967).

Thus, in mode 1, M_{s-} and M_{d-} precess in phase with each other with relative amplitudes given by $M_{d-}/M_{s-} = \chi_d/\chi_s$ where χ_d and χ_s are the local moment and conduction electron susceptibilities. Hence it follows that

$$H_t = -4\pi(k_v/k_0) M_{s-} E(\delta')$$

where

$$E(\delta') = e^{ik_0 \delta'} + [(\chi_s + \chi_d)/\chi_s] (1 - e^{ik_0 \delta'})$$

The factor $E(\delta')$ represents the enhancement of the transmitted field as a result of implantation. In the absence of implantation, $\delta' = 0$, and $E(\delta') = E(0) = 1$. If the implantation depth, δ' , is greater than the skin depth, ζ , the enhancement factor reduces to $E(\delta') \approx [(\chi_s + \chi_d) / \chi_s]$; this enhancement factor can be large at low temperatures as a result of achieving relatively large local moment susceptibilities, χ_d .

This mechanism of enhancement is illustrated figuratively in Figure 6. The incident microwave field excites a precessing magnetization density at the surface $z = 0$. The precessing electrons then diffuse across the metal slab into the implanted

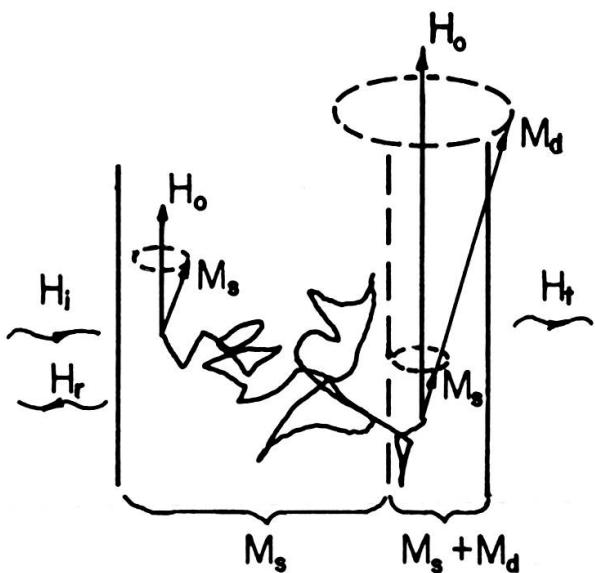


FIG. 6. — Illustrating the mechanism of signal enhancement due to ion implantation.

region. The appropriate normal mode of motion in the implanted region is however mode 1 of Figure 5; hence a very large M_{d-} will now accompany M_{s-} in its precessional motion. As a result of the presence of M_{d-} , the radiating moment is much larger, and the amplitude of the transmitted field is correspondingly enhanced.

The implantation of the slab (see Fig. 4) affects the observed resonance frequency and line width. If ω_P and ω_A are the resonance frequencies of the pure metal and of a homogeneous alloy similar to the implanted region, the observed resonance frequency, ω_R , is given as a weighted average of these two quantities,

$$\omega_R = \frac{L_P \omega_P + L_A \omega_A}{L_P + L_A}$$

where $L_p = L - \delta'$ and $L_A = \chi_r \delta'$ where $\chi_r = [(\chi_s + \chi_d) / \chi_s]$. Notice that the effective thickness of the alloy region, L_A , contains the factor χ_r , so that the properties of the alloy region become more heavily weighted as the temperature is lowered and the Curie law susceptibility of the local moments increases in magnitude. The above formula predicts a shift of the resonance frequency from that of the pure metal, and it is of interest to note that this has recently been observed (P. Monod—preceding talk).

Similarly, the observed line width, τ_R^{-1} , is given by the result

$$\frac{1}{\tau_R} = \frac{1}{L_p + L_A} \left[\frac{L_p}{\tau_p} + \frac{L_A}{\tau_A} \right]$$

The experimental results of Monod *et al.* display the unusual feature that the line width increases as the temperature is lowered. The reason for this is that at lower temperatures, the effective thickness of the alloy region, L_A , becomes greater, and hence the line width associated with the alloy region, τ_A^{-1} is weighted more heavily. Presumably $\tau_A^{-1} > \tau_p^{-1}$ and this leads to an increased τ_R^{-1} at lower temperatures. τ_A^{-1} itself will contain some temperature dependence, but this is difficult to predict for the relatively high concentration alloys used in the experiments of Monod *et al.*

As a result of the reciprocity theorem, discussed at the beginning of this talk, the above results also apply if the incident microwave field is incident from the right, in Figure 4, rather than from the left.

In the case of implantation of one side to a depth δ and the other to a depth δ' , the enhancement factor turns out to be $E(\delta)E(\delta')$, the product of the two enhancement factors, in agreement with the experimental results of Monod *et al.*

In concluding this section of the talk, we note that the predictions of the theory are in excellent qualitative agreement with the experiments on copper implanted with manganese described by Monod in the preceding talk (see also Monod *et al.* (1972)). Further details of the calculations described above can be found in Walker (1973a, b).

REVIEW OF THE EXPERIMENTS OF SCHULTZ AND DUNIFER (1967)

In 1958, Silin, on the basis of the Landau theory of Fermi liquids predicted the existence of weakly damped spin waves in a degenerate Fermi liquid in a magnetic field. The dispersion relation for these spin waves is shown in Figure 7. The infinite wavelength spin wave, in which all spins precess in phase with each other, is excited when the magnetic field satisfies the usual condition for conduction electron spin resonance, i.e. $g\mu_B H = \hbar\omega$.

The spin wave dispersion relation depends on the angle, ϑ , which the external magnetic field makes with the normal to the slab. The spin waves occur in fields

greater than $(\hbar\omega/g\mu_B)$ when Δ is greater than a certain angle, Δ_C , called the critical angle, and they occur in fields less than $(\hbar\omega/g\mu_B)$ when Δ is less than the critical angle (see Fig. 8).

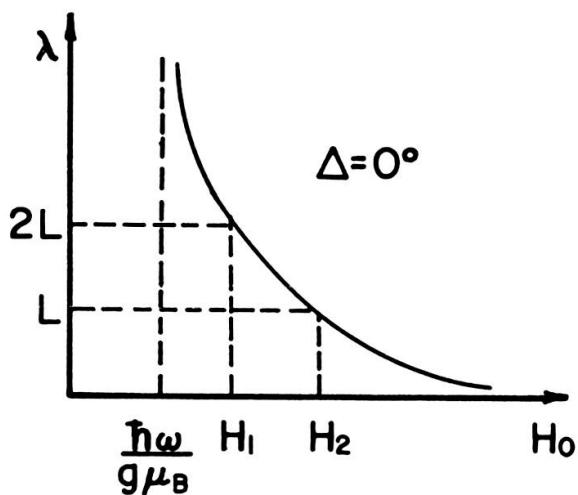


FIG. 7. — The dispersion relation for spin waves in sodium or potassium plotted assuming an infinitely long orbital collision time for electrons.

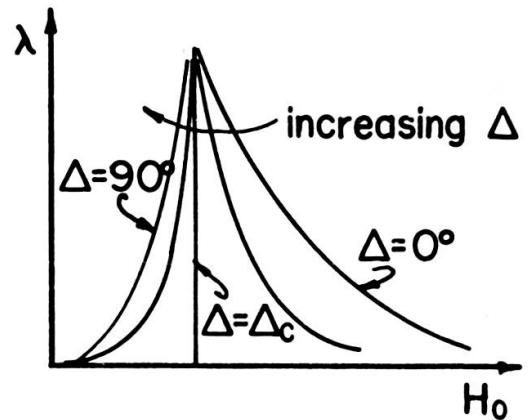


FIG. 8. — The angular dependence of the dispersion relation (Δ is the angle the external magnetic field makes with the normal to the slab)

The transmission experiments of Schultz and Dunifer were done on thin slabs of sodium and potassium metals. The slab acts like a resonant cavity in which normal modes of oscillation of spin density can be set up. The first few normal modes are shown in Figure 9; as one would expect, they are labelled by the number of nodes which they possess. The modes labelled 0, 1, 2, etc. have wavelengths $\lambda_0 = \infty$, $\lambda_1 = 2L$, $\lambda_2 = L$, etc. and occurs at fields $\hbar\omega/g\mu_B$, H_1 , H_2 , etc. (see Fig. 7). A qualitative sketch of the observations of Schultz and Dunifer is shown in Figure 10; the amplitude of the static external field H_0 is varied, and a peak in the amplitude of the transmitted microwave field is observed whenever the field H_0 takes the values $\hbar\omega/g\mu_B$, H_1 , H_2 , etc. setting up a normal mode of oscillation of the magnetization density.

A theory which fits the experimental results shown in Figure 10 was given by Platzman and Wolff (1967) and the agreement between their theory and the results as plotted in Figure 10 is impressive.

There were, however, certain anomalies in the experimental results which could not be explained by the theory of Platzman and Wolff (1967), and it is this anomalous behavior which I would like to explain in the last half of my talk.

The measured linewidth, ΔH , of the infinite wavelength mode (see Fig. 10 for a definition of ΔH) varies rapidly with the angle Δ (Dunifer, 1968, see Fig. 11). In fact, the linewidth is more than an order of magnitude greater at the critical angle than at $\Delta = 0$. Furthermore, the field for resonance of the infinite wavelength mode varies with angle as shown in Figure 12. These results were unexplained by the theory of Platzman and Wolff, which predicted no angular dependence of these

quantities. Subsequent theoretical papers by Wilson and Fredkin (1970) and Walker (1971) also failed to shed any light on the origins of these anomalies, although in retrospect, a more careful analysis of the results of the latter paper could have provided some clues.

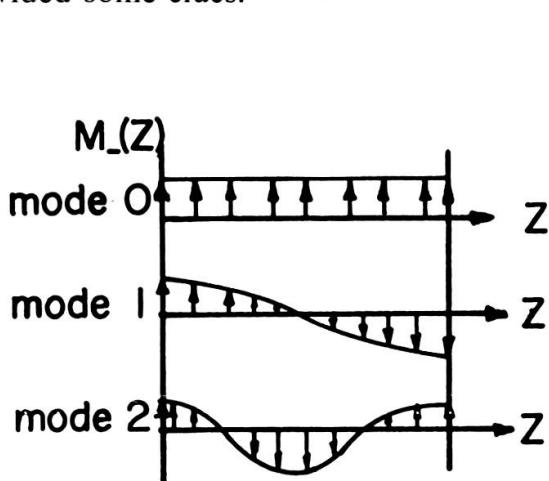


FIG. 9. — The normal modes of oscillation of the spin density in a metallic slab.

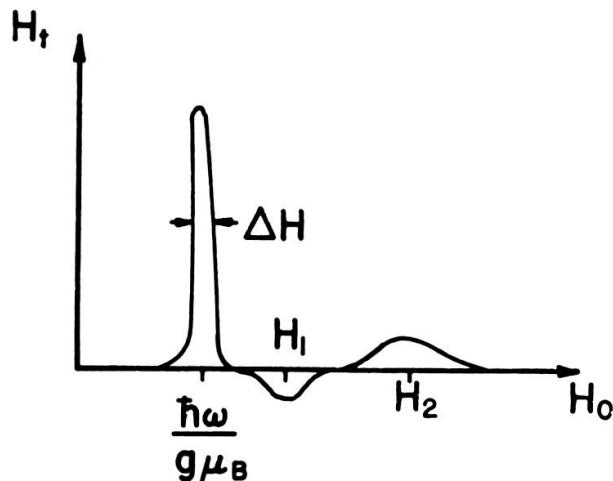


FIG. 10. — A qualitative sketch of the dependence of the transmitted field amplitude on H_0 as observed by Schultz and Dunifer (1967).

The experiments of Schultz and Dunifer are of great importance as a comparison of experimental results with theory provides a way of measuring a few of the Landau Fermi-liquid interaction parameters describing the spin-dependent interactions between quasi-particles. It was therefore one of the more interesting problems of recent years to find some explanation of the anomalous behavior just described.

To begin with, assume that as a result of collisions of the electrons with impurities or phonons, the motion of the electron spins can be described as a spin diffusion process with the dispersion relation

$$\omega = \frac{g\mu_B H_0}{\hbar} + i \left(\frac{1}{\tau_s} + Dk^2 \right)$$

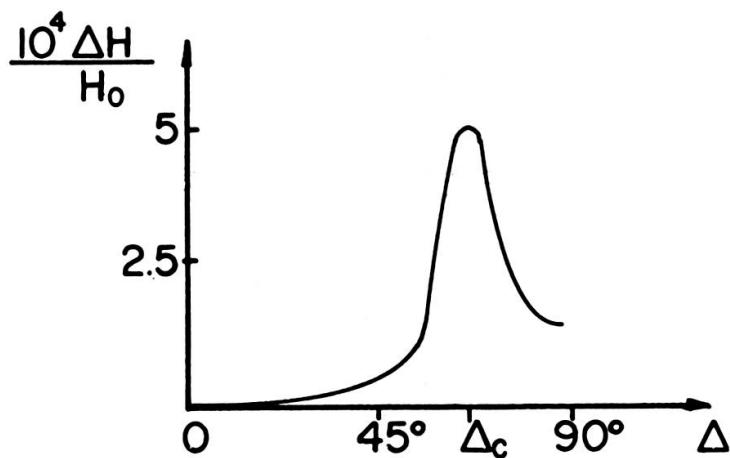


FIG. 11. — A sketch of the observed angular dependence of the line width of the infinite wavelength mode (Dunifer 1968).

where τ_s is the bulk spin relaxation time, D is the diffusion constant, and $k = (2\pi/\lambda)$ is the angular wave number of the disturbance.

The contribution Dk^2 to the relaxation rate can be understood in the following way. If an inhomogeneous disturbance in the spin density is characterized by a wavelength λ , particles have to diffuse a distant $\frac{1}{2}\lambda$ from regions of high spin density to regions of low spin density in order to relax the disturbance. By elementary diffusion theory, this takes a time $(\frac{1}{2}\lambda)^2/D$, and the associated relaxation rate is just the recip-

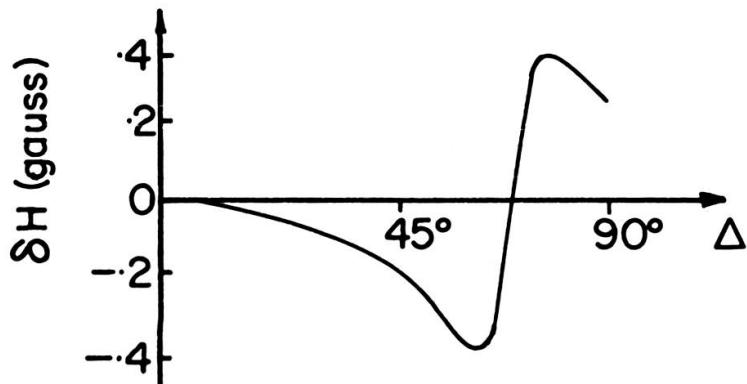


FIG. 12. — A sketch of the observed angular dependence of the shift of the field for resonance from its value at $\Delta = 0^\circ$.

rocal of this, i.e. approximately Dk^2 . Dk^2 is thus the rate at which an inhomogeneous disturbance relaxes due to diffusion. It is this additional relaxation rate due to diffusion which we shall see is the source of the anomalous behavior described above.

It is now necessary to study the effects of relaxation of the electron spins at the surface of the metal on the normal modes of oscillation of the magnetization density. If there is no surface relaxation, i.e. if electrons do not flip their spins on striking the surface, the number of spin up electrons leaving the surface is equal to the number of spin up electrons incident on the surface. The net spin current at the surface is therefore zero. In a macroscopic theory, the expression for the spin current is $j = -D(dM/dz)$ ¹, and the absence of surface relaxation implies that (dM/dz) is zero at the boundaries. This condition is satisfied by a homogeneous normal mode (Figure 13a) and since this mode has essentially an infinite wavelength, the Dk^2 contribution to the relaxation rate is zero.

If an electron has its spin flipped from up to down at the surface, however, (see Fig. 13b) there is a net loss of magnetic moment of two Bohr magnetons at the surface, one because of the loss of a spin up, and another because of the negative contribution of the spin down. Hence, the effect of surface relaxation is to cause a net loss of magnetization density at the surface, and the appropriate boundary condition becomes $j = -D(dM/dz) \neq 0$. That is, a gradient of magnetization

¹ The correct expression for the current density of M_- in a metal is $j = -D[d(M_- - \gamma H_-)/dz]$ (Walker 1971); however, since we are describing normal modes, we are looking at the oscillations of spin density in the absence of a driving field, and $H_- = 0$.

density must be maintained at the surface to provide a spin current flowing out of the surface. As a result, the normal mode with no nodes (also called the zeroth mode) must have a somewhat bowed shape, characterized by a non-zero wave-number k . Hence surface relaxation causes the zeroth mode to be inhomogeneous, and this inhomogeneity results in an additional contribution, Dk^2 , to the relaxation rate.

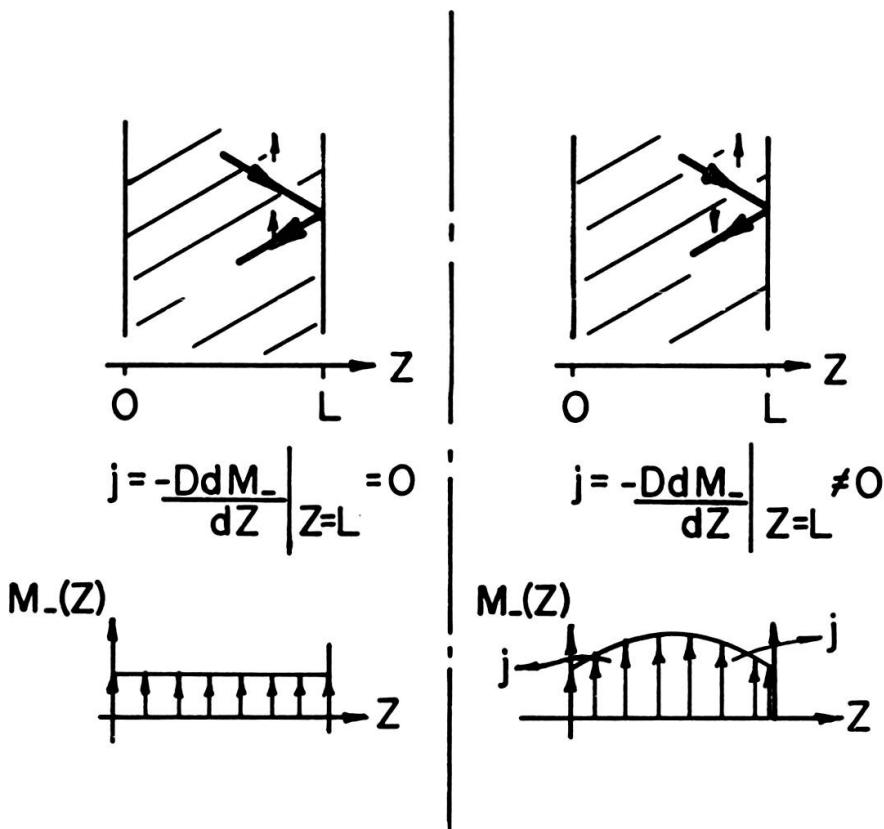


FIG. 13. — Illustrating the effects of surface relaxation.

The wavelength of the zeroth mode in the presence of surface relaxation can be calculated (Walker 1973c) and when the result for k is substituted into the dispersion relation given above it is found that the frequency of the mode is

$$\omega = \frac{g\mu_B H}{\hbar} + i \left[\frac{1}{\tau_s} + \frac{\varepsilon' v}{L} + \frac{\varepsilon'^2 v^2}{4D} \right]$$

where ε' is closely related to the probability that an electron flips its spin when striking the surface, and v is the Fermi velocity.

To proceed further it is necessary to have an accurate expression for the diffusion constant, D . At low temperatures, and in very pure metals, the electrons undergo relatively few collisions with phonons and impurities; the force exerted on a given electron by other electrons then becomes more important than the force on the given electron as a result of collisions. In this case the process of diffusion

is no longer a correct physical description of the electron spin motion; in spite of this it can be shown that the dispersion relation given above is correct provided the constant D is appropriately chosen. D can be calculated using the Landau theory of Fermi liquids and turns out to be a complex number which depends on frequency, magnetic field, the angle Δ , and the spin-dependent Landau Fermi liquid interaction parameters. It is in fact just this calculation of D which is necessary to determine the dispersion relation sketched in Figures 7 and 8. Writing the above expression for the frequency of the normal mode in the form $\omega = \omega_R + i\tau_R^{-1}$, and making use of the microscopic calculation of D , one finds that the resonance frequency and line width are given by (Walker 1973c)

$$\omega_R = \frac{g\mu_B H}{\hbar} - \frac{3}{4} \varepsilon'^2 (1+B_0)(1+B_1) \omega_B \frac{X}{X^2 + Y^2}$$

$$\frac{1}{\tau_R} = \frac{1}{\tau_s} + \frac{\varepsilon' v}{L} + \frac{3}{4} \varepsilon'^2 (1+B_0)(1+B_1) \frac{Y}{X^2 + Y^2}$$

where

$$\hbar\omega_B = [(B_0 - B_1)/(1+B_0)] g\mu_B H,$$

B_0 and B_1 are Landau Fermi-liquid interaction parameters, and, when the cyclotron frequency is greater than the orbital collision time of the electrons, τ_0 , X and Y are given by

$$X = \cos^2 \Delta - [B^2/(1-B^2)] \sin^2 \Delta$$

$$Y = [\omega_B \tau_0]^{-1} [\cos^2 \Delta + [B^2(1+B^2)/(1-B^2)^2] \sin^2 \Delta]$$

From experiment, $B = \cos \Delta_c \approx 1/3$.

It is clear that the angularly dependent contribution to the line width, considered as a function of Δ , has a resonance type shape, whereas the corrections to the resonance frequency have a dispersion type shape. Hence these results are qualitatively in agreement with the experimental results of Dunifer sketched in Figures 11 and 12. In fact accurate graphs of the above expression for ω_R and τ_R^{-1} appear to be in excellent agreement with the experiment results; furthermore, the very unusual temperature dependence of the line width is explained by the above expressions (Walker 1973c).

The value of ε'^2 necessary to produce the correct magnitude of the angularly dependent contribution to the line width is 8.8×10^{-3} . However, with this value of ε' , the angularly independent contribution to the line width, $(\varepsilon' v/L)$, is much too large if L is taken to be the thickness of the foil (i.e. $L \approx 10^{-2}$ cm.).

S. Schultz¹ has informed me that the experiments were done under conditions such that strong surface relaxation of the conduction electrons is more likely to

¹ S. Schultz, private communication.

take place at the edge of the sodium film, where it comes into contact with the metal of which microwave cavity is made, than at the surfaces $z = 0$ and $z = L$ as indicated in Figure 13b. In this case, the results quoted above might have some validity, but only if L is taken to be the size of the sodium window in the microwave cavity. This would reduce the magnitude of the angularly independent contribution to the line width by about two orders of magnitude, which is what is required. Thus it would appear that our results are not inconsistent with Schultz's suggestion that "edge relaxation" is responsible for the anomalies observed by himself and Dunifer, and in fact they give considerable support to this suggestion.

A final remark on the history of this problem is perhaps appropriate. The first person to recognize that surface relaxation might play a part in giving rise to anomalous contributions to the line width similar to those discussed above was Melle Colette de Botton who made this suggestion in her Master's thesis¹; this work was carried out under the supervision of P. Nozières and completed about six years ago but was never published due to the lack of stimulation from published experimental results at that time. My own contribution was to rediscover the above results, and to show by numerical comparison with experiment how accurately they accounted for the experimental results of Dunifer (1968). Finally, J. Dobson recently informed me that he had independently carried out a theoretical investigation of the finite slab problem; his results have recently appeared (Dobson 1973) and are of interest because he has explicitly solved a two-dimensional problem appropriate to the case of edge relaxation; his treatment also differs from ours in the use of different boundary conditions.

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¹ I am indebted to S. Schultz for having brought this work to my attention.

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DISCUSSION

MONOD: I would just like to make a comment that in fact was made to me by Abragam about the reciprocity of the transmission in fig. 1. The argument is that although the total transmission is exactly the same whether you do it from left to right or from right to left, the details of what is happening inside the metal are completely different. The RF field seen by the ions is totally different in each case.

PETER: I think the second part of your talk was of particular interest to those who do ion resonance. Since this part concerned other modes of the conduction electrons, it is to be hoped that somebody will again implant ions on something that is in this regime where the conduction electrons in the bulk show spin-wave dispersion. Then you have a tool to look at the resonance of the implanted ions whose *g* value is considerably different from 2, mainly at the frequency of the corresponding spin-wave mode of the conduction electrons. Of course, your analysis will then be very interesting because this type of relaxation will occur also on the surface where the ions are implanted.

WALKER: One thing that may inhibit it is that the spin-wave modes are very broad. The central resonance line is sharp but these spin-wave modes are almost as broad as their separation. They are not the sharp resonances with which you would like to do these coupling experiments.

PETER: But I would have thought you have a broad mode and now you have a sharp *Mn* resonance on the surface; at this resonance lots of power is coupled out. So the width of the phenomena you see will be given essentially by the *Mn* resonance, and the function of the mode is simply to make possible the transmission of microwaves by electrons at this frequency.

WALKER: Well, you could only get a coupling between the *Mn* and spin-waves if they had the same frequency. So the only mode the *Mn* could couple to would be the mode with no nodes.