A FITTING PROCEDURE FOR CONDUCTIVE OPACITY IN A STELLAR MEDIUM

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ABSTRACT

An expeditious although sufficiently accurate procedure is proposed here to include the conductive opacity into the overall opacity of the stellar medium, according to the most recent advances in the theory of electron conduction. The method has been successfully tested in a case where previously tabulated opacity values were available.

RÉSUMÉ

Un procédé rapide mais suffisamment exact est proposé pour inclure, dans l'opacité globale du milieu stellaire, l'opacité conductive telle qu'elle résulte des versions les plus récentes de la théorie de conduction électronique. Cette méthode a été correctement vérifiée dans un cas où l'on disposait de valeurs d'opacité préalablement tabulées.

1. CONDUCTION VERSUS RADIATION TRANSFER

The radiative energy transfer inside a star is connected to the opacity of the stellar medium by the well-known expression for the radial energy flux

$$F_{\text{rad}} = -\frac{4\,ac\,T^3}{3\,\rho\kappa_{\text{rad}}}\,\frac{dT}{dr} \tag{1}$$

where a, c, ρ, T have their usual meanings and κ_{rad} denotes the mean radiation absorption coefficient per unit mass. Consequently,

$$\lambda_{\text{rad}} = \frac{4\,ac\,T^3}{3\,\rho\kappa_{\text{rad}}} \tag{2}$$

may be called the radiation conductivity coefficient.

On the other hand, the radial flux of energy transported by electron conduction

$$F_{\text{cond}} = -\lambda_{\text{cond}} \frac{dT}{dr} \tag{3}$$

enables us to define the thermal electron conductivity coefficient λ_{cond} to which corresponds, in the sense of relation (2), a coefficient of conductive opacity

$$\kappa_{\text{cond}} = \frac{4\,a\,c\,T^3}{3\,\rho\,\lambda_{\text{cond}}} \tag{4}$$

When both these modes of energy transfer are simultaneously at work, the fluxes (1) and (3) should be added, therefore also the conductivities, so that the overall opacity κ will be given by

$$\frac{1}{\kappa} = \frac{1}{\kappa_{\text{rad}}} + \frac{1}{\kappa_{\text{cond}}} \tag{5}$$

Insofar as the mean free path of the photons is distinctly larger than that of the electrons, as it occurs in the interior of main sequence stars, the conductive transfer is quite inefficient when compared to radiative transfer. Now, if the density increases and the electron gas becomes degenerates, the lower momenta states will all be occupied, thereby hindering electron scattering. At high degeneracy, conductive transfer shall predominate so that, according to (5), $\kappa \approx \kappa_{\text{cond}}$ and the opacity is essentially conductive.

In the stellar medium, electrons will undergo collisions with ions (*e-i* collisions) and with other free electrons (*e-e* collisions); for high degeneracy of the electron gas, the electrons tend to build a continuous sea of uniform negative charge, unsuitable for scattering and moreover in a (*e-e*) scattering, the final quantum state of both electrons must be unoccupied, not only that of the single electron involved in a (*e-i*) scattering. For such reasons, only (*e-i*) interactions were considered in the early treatments of thermal electron conduction (MESTEL, 1950); nevertheless, the method is incorrect with a partially degenerate medium and LAMPE (1968a) has shown that (*e-e*) interactions should be retained even in highly degenerate plasmas.

2. ELECTRON CONDUCTION IN THE STELLAR MEDIUM

The stellar medium is usually regarded as a gaseous mixture of non degenerate ions and of free electrons which may present any degree of degeneracy. The gaseous character of the plasma implies weak coupling between the particles; the potential energy of any particle in the plasma remains much smaller than its average kinetic energy.

For the sake of clarity about the later developments, we shall recall concisely, in the present section and the following one, the main features of the conduction

theory in the light of the recent advances of LAMPE (1968a, 1968b) and HUBBARD-LAMPE (1969).

Owing to their large mass M, the ions do not contribute appreciably to the conductive energy transfer, so that they may be considered in local maxwellian equilibrium, described by the familiar distribution function of positions and momenta

$$f_i(\mathbf{x}, \mathbf{p}, t) = \frac{n_i}{(2\pi MkT)^{3/2}} \exp\left(-\frac{p^2}{2MkT}\right) \qquad (6)$$

where the ion concentration n_i and temperature T both depend on $\mathbf{x}$, t.

The electrons, of mass m and assumed non relativistic, will be distributed in phase space according to a nearby Fermi-distribution

$$f_e(\mathbf{x}, \mathbf{p}, t) = f_e^{(0)}(\mathbf{x}, p^2, t) + f_e^{(1)}(\mathbf{x}, \mathbf{p}, t) \qquad (7)$$

where

$$f_e^{(0)} = \frac{2}{h^3} \left[\exp\left(\frac{p^2}{2mkT} - \psi\right) + 1\right]^{-1} \qquad (8)$$

ψ being the degeneracy parameter depending, like T, on $\mathbf{x}$, t.

The perturbation term $f_e^{(1)}$ is due to the temperature gradient ∇T imposed on the medium by the energy flux, together with the electric field $\mathbf{E}$ required to ensure the average electric neutrality.

The distribution functions just introduced are normalized to the concentrations:

$$\int f_{i,e}(\mathbf{x}, \mathbf{p}, t) d^3p = n_{i,e}(\mathbf{x}, t) \qquad (9)$$

In the first-order perturbation theory, (7) will currently be written in the form

$$f_e(\mathbf{x}, \mathbf{p}, t) = f_e^{(0)} \left[1 + \left(1 - \frac{h^3}{2} f_e^{(0)}\right) \Phi(\mathbf{x}, \mathbf{p}, t)\right] \qquad (10)$$

where $1 - \frac{h^3}{2} f_e^{(0)}$ is the probability for the perturbed electron to find itself in a state unoccupied in the equilibrium distribution.

The evolution of $f_e(\mathbf{x}, \mathbf{p}, t)$ in time is governed by a Boltzmann-type of equation

$$\frac{Df_e}{Dt} = \left(\frac{\partial f_e}{\partial t}\right)_{\text{coll.}} \qquad (11)$$

the l.h.s. of which can be written,

$$\frac{\partial f_e}{\partial t} + (\mathbf{v} \cdot \nabla T) \frac{df_e^{(0)}}{dT} + \frac{e}{m} \mathbf{E}' \cdot \frac{\partial f_e^{(0)}}{\partial \mathbf{v}} \qquad (12)$$

where $e\mathbf{E}'$ is the total force acting on an electron, due to the electric field $\mathbf{E}$ and to the pressure gradient ∇P:

$$e\,\mathbf{E}' = e\,\mathbf{E} + \frac{1}{n_e}\,VP \qquad \text{(CHAPMAN and COWLING, 1961)}.$$

The r.h.s. of (11) describes the stochastic variation of f_e on account of $e\text{-}i$ and $e\text{-}e$ encounters; it includes the sum of two contributions, each of them represented by a multiple integral over the initial momenta and the parameters specifying the encounter. The cross-sections for both types of collision are calculated in the Born approximation with a shielded Coulomb field and, in view of the $e\text{-}e$ collisions, care is taken of energy as well as momentum exchange during the collision (dynamical shielding). Keeping also in mind the exclusion principle for the possible electron states, one finally gets for the r.h.s. of (11) the form proposed by Lenard and Balescu (LAMPE, 1968a).

Replacing f_e by its value (10) and neglecting terms quadratic in $f_e^{(1)}$, we may then write, symbolically,

$$\left(\frac{\partial f_e}{\partial t}\right)_{\text{coll}} = L(\Phi) \tag{13}$$

where L is a linear functional transformation of the unknown function Φ.

3. POLYNOMIAL SOLUTIONS

To solve this Boltzmann (or Lenard-Balescu) equation (11), linearized in Φ according to (10), (12), (13), it is sufficient (CHAPMAN-COWLING, 1961) to retain for Φ an expression linear both in $V \log T$ and $\mathbf{E}'$, viz.

$$\Phi\,(\mathbf{x}, \mathbf{p}, t) = A\,(p^2)\,(\mathbf{p}/m)\cdot(V\,T/\,T) + D\,(p^2)\,(\mathbf{p}/mkT)\cdot e\,\mathbf{E}' \tag{14}$$

A, D being two unknown scalar functions of the electron's momentum modulus.

Going back with (10), (12), (14) into (11), one obtains, after equating separately the coefficients of $V\,T/T$ and $\mathbf{E}'$, two linear integral equations, one for $A\,(p^2)$, the other for $D\,(p^2)$.

Following here the Champan-Enskog method of resolution, we expand A and D in polynomial series of the form

$$A\,(p^2) = \sum_{j=0}^{\infty} a_j P_j\,(p^2), \qquad D\,(p^2) = \sum_{j=0}^{\infty} d_j P_j\,(p^2) \tag{15}$$

where the $P_j\,(p^2)$ are expected to build a complete set of orthonormal functions. As a matter of fact, the eigenfunctions of the operator L are unknown in general; but in case of a nondegenerate electron gas, the orthogonality relations appearing in the problem lead us to choose the Sonine polynomials for the $P_j\,(p^2)$, in spite of the fact that these polynomials, multiplied by the spherical functions $Y_l^m\,(\theta, \varphi)$

only coincide with the eigenfunctions of L for a very restricted type of interaction between the plasma particles (JANCEL et KAHAN, 1963).

Other polynomials must be found for a degenerate electron gas, suggested by the orthogonality relations and LAMPE (1968a) gives the explicit form of such polynomials in the case of a highly degenerate electron gas.

Anyway, the integral equations for A, D will then yield a system of order $n \to \infty$ of linear algebraic equations for the a_j, d_j respectively:

$$\sum_{j=0}^{n} a_{jk} a_j = \alpha_k, \qquad \sum_{j=0}^{n} a_{jk} d_j = \delta_k \tag{16}$$

Approximate solutions of (16) will be found by truncating the sums to a finite number n of terms and with a suitable choice for the $P_j(p^2)$ we may expect the approximate solutions $a_j^{(n)}$, $d_j^{(n)}$ to converge rapidly towards the exact solutions a_j, d_j as $n \to \infty$.

4. THE FIRST TWO POLYNOMIAL APPROXIMATIONS FOR THERMAL CONDUCTIVITY

The energy flux carried by electron conduction is equal to

$$\mathbf{F} = \frac{1}{2m^2} \int f_e^{(1)} p^2 \, \mathbf{p} \, d^3 p \tag{17}$$

Substituting $f_e^{(1)}$ by its expression in (10) and eliminating further $\mathbf{E}'$ between (17) and the condition that the electric density current vanishes in the medium, it becomes possible to express $\mathbf{F}$ in the form (3), written as

$$\mathbf{F} = -\lambda \nabla T$$

where the value of the thermal conductivity $\lambda = \lambda^{(n)}$ depends of the order n to which the algebraic system (16) has been truncated.

With $n = 1$, one obtains (LAMPE, 1968a) in terms of some Fermi functions,

$$\lambda^{(1)} = \frac{128 \pi^2 m}{27} \left(\frac{k}{h}\right)^6 (21 F_{5/2} - 25 F_{3/2}^2/F_{1/2})^2 \, T^5/a_{11} \tag{18}$$

further, when $n = 2$;

$$\lambda^{(2)} = \lambda^{(1)} \left(1 - \frac{a_{12}^2}{a_{11} a_{22}}\right)^{-1} \tag{19}$$

the a_{jk} are collisional integrals containing the physics of the problem; the contribution of the e-i collisions can be expressed through Fermi functions $F_\nu(\psi)$ of integer

and half-integer order; it also contains a logarithmic factor θ_{ei} brought in the $(e\text{-}i)$ scattering cross-section by the shielding effect. As for the contribution to a_{jk} from the $e\text{-}e$ collisions it is much more difficult to evaluate and it also contains a similar collision logarithm θ_{ee}.

Relying on the numerical values given by LANDSHOFF (1949) for the matrix elements included in the a_{jk} pertaining to a non degenerate plasma made up of electrons and single ions of isotopic number Z, we verify that

$$\lambda^{(2)} = 2.4\, \lambda^{(1)} \text{ if } Z = 1 \text{ (hydrogen)}$$
$$\text{and} \quad \lambda^{(2)} = 3.1\, \lambda^{(1)} \text{ if } Z = 2 \text{ (helium)}$$

Furthermore, when Z becomes large, the $(e\text{-}e)$-collisions contribution decreases and if we neglect it, retaining only the $(e\text{-}i)$-interactions, then $\lambda^{(2)} \to 6.5\, \lambda^{(1)}$.

Moreover, the introduction of a third polynomial solution appears unnecessary, since it only results in a correction less than 8 per cent (HUBBARD and LAMPE, 1969).

On the other hand, if the electron gas is highly degenerate ($\psi \gg 1$), we may adopt part of the Lorentz approximation to evaluate the $(e\text{-}i)$ contribution to a_{jk}, namely that the maximum energy transferred in a $(e\text{-}i)$ collision remain much less than kT. Consequently the exact value of the thermal conductivity is then already given by the one polynomial solution to an accuracy of 1 per cent, independently of the $(e\text{-}e)$ encounters.

We now turn back to expression (18) for $\lambda^{(1)}$; the element a_{11} has the form

$$a_{11} = a_{11ei} + a_{11ee}$$

where a_{11ei}, itself sum of three terms (LAMPE, 1968b) is proportional to

$$Z\, \xi\,(\psi)\, \theta_{ei}\, T^{5/2}$$

with Z as a mean isotopic number for a mixture of different ions of species α:

$$n_e Z = \sum_\alpha n_\alpha Z_\alpha^2 \tag{20}$$

n_e, n_α being respectively the electron and ion concentrations.

Further

$$\xi\,(\psi) = \left[2\, F_1 F_{1/2} - \frac{10}{3} F_0 F_{3/2} + \frac{25}{9} \frac{F_{3/2}^2}{F_{1/2}} \,(1 + e^{-\psi})^{-1} \right] \tag{21}$$

while $a_{11}ee$ is proportional to

$$\eta\,(\psi)\, \theta_{ee}\, T^{5/2}$$

where $\eta\,(\psi)$ may be written, after an integration by parts,

$$\eta\,(\psi) = \int_0^\infty V d\, V \ln(1 + e^{\psi - V})^2 \int_0^V \left[e^{w^2 - \psi} + 1 \right]^{-1} dW \tag{22}$$

Putting all this into (18) allows us finally to write for the first polynomial approximation of the thermal electron conduction

$$\lambda^{(1)} = \frac{k^{7/2}}{54\pi\sqrt{2\,m}\,e^4}\;\frac{(21\,F_{5/2} - 25\,F_{3/2}^2/F_{1/2})^2\,T^{5/2}}{Z\,\xi(\psi)\,\theta_{ei} + 4\,\eta(\psi)\,\theta_{ee}} \qquad (23)$$

The calculation of $\lambda^{(2)}$ according to (19), requires still the evaluation of the elements a_{12}, a_{22}, which is quite difficult with regard to the e-e terms (LAMPE 1968b).

4. OPACITY TABLES

On the basis of the results recalled in the previous sections, HUBBARD and LAMPE (1969) managed to compute conduction opacity tables for hydrogen, helium, carbon and two special mixtures, one for a typical red-giant core ($X_{He} = 0.98$, $X_N = 0.015$, $X_{Ne} = 0.005$) and the other for a solar-type star.

The density range runs from $10^{-5.75}$ to 10^{+6} g cm^{-3}, the temperature-range from 10^3 to 10^9 deg K but the log κ values have been computed, in function of log ρ, log T, only within the domain of validity of the theories underlying the above results, in particular for weakly coupled and non-relativistic electrons.

In the stellar evolutionary calculations which we intend to carry out, at first during the core hydrogen-burning phase from the zero age main sequence to the red giant tip, we shall have to interpolate the opacity at each stage, between two of four opacity tables corresponding to decreasing hydrogen abundances. These four tables we have selected among those given by COX and STEWART (1969); they are related to the mixtures named MAS II, CS XII, CS XIII, CS XIV characterized respectively by the hydrogen abundances $X_H = 0.70, 0.50 . 0.20, 0.00$.

The Cox-Stewart tables contain, in function of log ρ and log T values of the radiative opacity (without and with the effect of lines) and of the effective opacity (log κ) by including the electron conduction according to the MESTEL (1950) and LEE (1950) theory where e-e interactions were ignored.

More recently, PACZYNSKI (1970) and DEMARQUE and HEASLEY (1971) have been using the radiative Cox-Stewart opacities together with the conductive Hubbard-Lampe opacities, but we cannot use their results because of the unknown procedure which they applied to combine opacities pertaining to different chemical compositions.

The direct calculation of λ from the expressions (19) and (23) is most unwieldy on account, partly of uncertainties in the evaluation of θ_{ee} and partly of the extreme sensitivity of the factor $(21\,F_{5/2} - 25\,F_{3/2}^2/F_{1/2})^2$ appearing in (23) to the accuracy adopted for the numerical values of the relevant Fermi functions.

By way of short cut, we propose here an approximate method to include systematically for any degree of degeneracy, the Hubbard-Lampe conductive opacities into the four afore mentioned Cox-Stewart tables which we need in our evolutionary computations.

5. A METHOD FOR INCLUDING CONDUCTIVE OPACITY AT ANY ψ

We retain the fact that we have at our disposal a conductive opacity table $\mathcal{T}_0$ established for a given chemical composition denoted symbolically by χ_0, and giving the values of log κ_{cond} in terms of log ρ, log T as usual.

$$\kappa_{\text{cond}} = \kappa\,(\rho,\,T;\,\chi_0)$$

and the corresponding degeneracy parameter ψ can always be calculated from the basic relation

$$\rho\,(\psi,\,T;\,\chi_0) = \mu_e\,m_H\,n_e$$
$$= \frac{4\pi}{h^3}\,m_H\,(2\,mk)^{3/2}\,F_{1/2}\,(\psi)\,T^{3/2}\left(\sum_\alpha \frac{X_\alpha Z_\alpha}{A_\alpha}\right)^{-1} \tag{24}$$

where, apart from the well known universal constants m, m_H, and k, n_e is the free electron concentration already present in (20), μ_e the mean mass number per free electron and X_α the relative abundance by mass of the element (Z_α, A_α).

Instead of deriving ψ from given ρ and T, let us from now on regard ψ and T as the independent variables; then we may obtain ρ for the given composition χ_0 and ψ, T values from the relation (24) and further, interpolating in table $\mathcal{T}_0$, the opacity value

$$\kappa\left(\rho\,(\psi,\,T;\,\chi_0),\,T,\,\chi_0\right) = \kappa_0 = \kappa\,(\psi,\,T;\,\chi_0)$$

Keeping first the same T-value, we may similarly obtain by interpolation in $\mathcal{T}_0$ other values for κ_0 and then start the same procedure again for another value of T, thus building gradually a limited table $\mathcal{T}_0'$ extrapolated from $\mathcal{T}_0$ in the ψ range of interest.

The problem is now to find out the conductive opacity $\kappa_1 = \kappa\,(\psi,\,T;\,\chi_1)$ for another chemical composition χ_1 not too widely different from χ_0, and for which we have no table $\mathcal{T}_1$. To attain this end, let us consider the opacity ratio

$$\frac{\kappa_1}{\kappa_0} = \frac{\mu_e(\chi_0)}{\mu_e(\chi_1)}\,\frac{\lambda\,(\psi,\,T;\,\chi_0)}{\lambda\,(\psi,\,T;\,\chi_1)} \tag{25}$$

deduced from (4) and (24) at given ψ, T. We now require that, in passing from χ_0 to χ_1, the mean isotopic number Z defined by (20) should not vary appreciably.

By examining the form of the coefficients a_{jk} (LAMPE, 1968b) occurring in expression (19), it is easy to see that they have the general form

$$(\alpha_{jk} Z + \beta_{jk}) T^{5/2}$$

where the chemical composition appears mainly in Z, although also in α_{jk} but there only through the collision logarithm θ_{ei}. Neglecting this last dependence, we will assume that $a_{12}^2/a_{11} a_{22}$ is a slowly varying function of Z and that, in first approximation, $\lambda^{(2)}/\lambda^{(1)}$ is practically independent of chemical composition. Then the λ ratio in the r.h.s. of (25) can be taken as that of the $\lambda^{(1)}$'s, equal, according to (23), to

$$\frac{\lambda(\psi, T; \chi_0)}{\lambda(\psi, T; \chi_1)} = \frac{Z_1 \xi(\psi) \theta_{ei}(\psi, T) + 4\eta(\psi) \theta_{ee}(\psi, T)}{Z_0 \xi(\psi) \theta_{ei}(\psi, T) + 4\eta(\psi) \theta_{ee}(\psi, T)} \tag{26}$$

where Z_1, Z_0 are the mean Z values for each of the chemical compositions, $\xi(\psi)$, $\eta(\psi)$ are given by (21) and (22) and the dependence of θ_{ei} on χ has been dropped.

In a wholly ionized medium, we recall that

$$\mu_e^{-1} = \sum_\alpha \frac{X_\alpha Z_\alpha}{A_\alpha} \tag{27a}$$

$$Z \mu_e^{-1} = \sum_\alpha \frac{X_\alpha Z_\alpha^2}{A_\alpha} \tag{27b}$$

Let the composition χ_1 be, for sake of definiteness, such that $\mu_e(\chi_1) > \mu_e(\chi_0)$, therefore also $Z_1 > Z_0$. Owing to the positive character of the quantities ξ, η, θ, it is apparent from (26) that

$$1 < \frac{\lambda(\psi, T; \chi_0)}{\lambda(\psi, T; \chi_1)} < \frac{Z_1}{Z_0}$$

so that, in consequence of (25),

$$\frac{\mu_e(\chi_0)}{\mu_e(\chi_1)} < \frac{\kappa_1}{\kappa_0} < \frac{Z_1 \mu_e(\chi_0)}{Z_0 \mu_e(\chi_1)}$$

the bounding values being all the more close to each other that the chemical compositions are more similar. Let us denote by κ_1' and κ_1'' the opacities obtained by equating the κ-ratio to its lower, respectively upper bound:

$$\frac{\kappa_1'}{\kappa_0} = \frac{\mu_e(\chi_0)}{\mu_e(\chi_1)} \tag{28a}$$

$$\frac{\kappa_1''}{\kappa_0} = \frac{Z_1 \mu_e(\chi_0)}{Z_0 \mu_e(\chi_1)} \tag{28b}$$

The λ-ratio could only attain its upper bound, according to (26), if the Z are large or if the e-e contribution becomes very much smaller than the $(e$-$i)$ contribution. In fact we are dealing here with Z's fairly close to unity and the e-e contribution is never negligible, even to high degeneracies (LAMPE, 1968a).

Therefore we shall try to approximate the conductive opacity κ_1 either by κ_1' or by the mean of $\log \kappa_1'$, $\log \kappa_1''$ preferentially weighted in favour of $\log \kappa_1'$.

6. TESTING THE METHOD

As a preliminary test, let us apply the procedure in a case where previously tabulated values exist for both chemical compositions envisaged; for example pure hydrogen (χ_0) and a solar composition (χ_1) as used by HUBBARD and LAMPE (1969) in connection with a paper of BAHCALL, BAHCALL, SHAVIV (1968) dealing with the solar-neutrino problem and involving several possible solar models. Retaining more specially model C, the composition χ_1 is defined by $X_H = 0.764$, $X_{He} = 0.221$, $\hat{X} = 0.015$ ($\hat{X}$, heavy element abundance).

Adopting the same distribution of heavy elements as the one determined by LAMBERT and WARNER for the solar photosphere (1968) we find, from the relations (27) applied to composition χ_1,

$$\mu_e^{-1} = 0.882 , \ Z\mu_e^{-1} = 1.073 \tag{29}$$

By the way, the MAS II mixture used in one of our interpolating tables for stellar evolutionary computations, is a little less hydrogenrich ($X_H = 0.70$) than the Bahcall-Shaviv models, but the heavy element distribution ($\hat{X} = 0.02$) appears to be very close to that of Lambert-Warner; the carbon is equally abundant and the oxygen is slightly more abundant in MAS II. We have, for the MAS II mixture,

$$\mu_e^{-1} = 0.850 \ Z\mu_e^{-1} = 1.068$$

Let us choose for $\log T$ the three values 6, 7, 8 and for $\log \psi$ the three values 0, 1, 2; the Hubbard-Lampe opacity table for hydrogen $\mathscr{T}_0$ yields, by linear interpolation, the $\log \kappa_0$ values for the densities $\rho_0 (\psi, T)$; whence table 1 containing the values of $\log \kappa_0$ written just under those of $\log \rho_0$

TABLE 1

$\log T$ \ ψ	0	1	2
6	0.791 3.328	1.105 2.844	1.358 2.401
7	2.291 2.511	2.605 2.046	2.858 1.629
8	3.791 1.654	4.105 1.216	4.358 0.809

values of $\log \rho_0$ (above) and $\log \kappa_0$ (below) in terms of $\log T$, ψ

Keeping the same ψ and T-values, we pass from ρ_0 to ρ_1 with the values (29) for chemical mixture χ_1, viz.

$$\log \rho_1 = \log \rho_0 + 0.055$$

the new densities ρ_1 leading, as one interpolates in the Hubbard-Lampe opacity table $\mathcal{T}_1$ for solar composition, to values of $\log \kappa_1$ and the table 2 contains, similarly to table 1, the $\log \rho_1$ and $\log \kappa_1$ values for composition χ_1.

TABLE 2

log T \ ψ	0	1	2
6	0.846 3.294	1.160 2.813	1.413 2.378
7	2.346 2.475	2.680 2.043	2.913 1.600
8	3.846 1.617	4.160 1.179	4.413 0.776

values of $\log \rho_1$ (above) and $\log \kappa_1$ (below) in terms of $\log T$, ψ

If $\mathcal{T}_1$ did not exist, we could always calculate κ_1' and κ_1'' by the relations (28), viz. in the present case,

$$\log \kappa_1' = \log \kappa_0 - 0.055$$

$$\log \kappa_1'' = \log \kappa_0 + 0.031$$

and the mean values $\bar{\kappa}_1$, $\tilde{\kappa}_1$ defined by

$$\log \bar{\kappa}_1 = \frac{1}{2}(\log \kappa_1' + \log \kappa_1'') \qquad (30a)$$

$$\log \tilde{\kappa}_1 = \frac{1}{2}(\log \kappa_1' + \log \bar{\kappa}_1) \qquad (30b)$$

become here

$$\log \bar{\kappa}_1 = \log \kappa_0 - 0.012$$

$$\log \tilde{\kappa}_1 = \log \kappa_0 - 0.034$$

In the following table 3, we have collected the values of $\log \bar{\kappa}_1$ and $\log \kappa_1'$ since it turns out that, among various approximations to $\log \kappa_1$, $\log \tilde{\kappa}_1$ leads to values in closest agreement with the interpolated values given in table 2.

TABLE 3

	0	1	2
6	3.294 3.273	2.810 2.789	2.367 2.346
7	2.477 2.456	2.012 1.991	1.595 1.574
8	1.620 1.600	1.182 1.161	0.775 0.754

values of $\log \tilde{\kappa}_1$ (above) and $\log \kappa_1'$ (below) in terms of $\log T$, ψ

Comparing tables 2 and 3, one verifies that the relative deviations $(\tilde{\kappa}_1 - \kappa_1)/\kappa_1$ all remain below 3 per cent; the approximation $\log \kappa_1'$ is still fairly good, $(\kappa_1' - \kappa_1)/\kappa_1$ never exceeding 7.5 per cent and if we content ourselves with an accuracy of 10%, $\log \kappa_1$ or $\log \kappa_0$ may also be used.

This preponderance in the choice of $\log \tilde{\kappa}_1$ is again apparent if one considers the Hubbard-Lampe table $\mathcal{T}_1$, as connected to a slightly different chemical composition, like those of the other Bahcall-Shaviv solar models.

7. CONCLUSION

In conclusion, as we pass from one particular chemical composition χ_0 for which we do have a table $\mathcal{T}_0$ of conductive opacities, to another χ_1, not too different from χ_0 but for which an opacity table $\mathcal{T}_1$ does not exist, we may approximate the conductive opacity κ_1 by the mean value $\tilde{\kappa}_1$ defined by (30b) or, in terms of (30a), (28), by

$$\log \tilde{\kappa}_1 = \log \kappa_0 + \frac{3}{4}\log \frac{\mu_e(\chi_0)}{\mu_e(\chi_1)} + \frac{1}{4}\log \frac{Z_1 \mu_e(\chi_0)}{Z_0 \mu_e(\chi_1)} \tag{31}$$

The procedure just described is easy to apply and avoids a considerable amount of calculations; it appears to us to be more expeditious than the limited fitting formulae of the type used by SWEIGART (1973) for the HUBBARD-LAMPE (1969) and the CANUTO (1970) data. According to the previous test, the method is quite accurate and we should recall that, in most cases which we are facing, conductive opacity is only part of the overall opacity (5).

In our stellar evolutionary computations, we have to interpolate, as mentioned before, over a 4-fold grid of opacity tables $\mathscr{T}_1$, $\mathscr{T}_2$, $\mathscr{T}_3$, $\mathscr{T}_4$, corresponding to chemical compositions χ_1, χ_2, χ_3, χ_4 with decreasing values of X_H (section 4).

These tables contain separately the radiative opacities and on the other hand, we have the Hubbard-Lampe conductive opacities for pure hydrogen ($\mathscr{T}_0$) and pure helium ($\mathscr{T}_5$). We apply the foregoing method in passing first from pure hydrogen to composition χ_1 to get the conductive opacities κ_1 which we combine, using (5), to the radiative opacities of table $\mathscr{T}_1$, thus obtaining a table $\mathscr{T}'_1$ of overall opacities where the electron conduction is taken care of according to the Hubbard and Lampe results.

Similarly, we pass from composition χ_1 with table $\mathscr{T}'_1$ to composition χ_2 (not too different from χ_1), thus constructing a table $\mathscr{T}'_1$ of overall opacity. It will not be necessary to distinghuish here between composition χ_4 and pure helium so that, starting from χ_4 and table $\mathscr{T}_5$ we first combine the radiative opacity from $\mathscr{T}_4$ with the conductive opacity from $\mathscr{T}_5$, getting the overall opacity table $\mathscr{T}'_4$ for composition χ_4. Then we pass on to χ_3, for which a table $\mathscr{T}'_3$ will be similarly established and as a matter of verification, we could pass from χ_3 to χ_2 in order to confirm table $\mathscr{T}'_2$.

Interpolation over the 4 fold grid $\mathscr{T}'_1$ $\mathscr{T}'_2$ $\mathscr{T}'_3$ $\mathscr{T}'_4$ will enable us to compute, at every stage of evolution, the overall opacity including electron conduction according to the latest improvements. Insofar as we dispose of appropriate radiative opacity tables, the procedure can readily be extended to the core helium burning stage.

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