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COMPUTING EXPERIMENTS ON STELLAR SYSTEMS

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Star gatherings

Among the various stellar systems observed in space, we may broadly distinguish between galaxies, galaxy clusters and star clusters. Since the preceding lecture was devoted to evolutionary computations of galactic disks, the aim of the present considerations will consist essentially in the study of the dynamical evolution of clusters by means of numerical methods. The richest star clusters are the so-called globular clusters, some of them containing up to a few tens of millions of stars; in our galaxy, they number about 120 and belong to the oldest objects known. Five billion years is a characteristic figure for their age. In connection with their name, the globular clusters present a circular or nearly circular image, which is not always the case of the open star clusters. The latter exhibit a large range of ages, from million to billion years, the smallest of these clusters reducing to multiple stars.

In a first approximation picture, an isolated star cluster is envisaged as a group of N point-like stars, held together by their mutual gravitational attraction so that, from the dynamical standpoint, we are facing straight-away the famous N-body problem. We know for a long time that this problem can analytically be solved only for $N = 2$ and investigated in very special cases when $N = 3$; for larger N, we have to rely entirely on numerical methods and the first attempts in this direction had to be delayed until the advent of powerful and fast computers.

Statistical description

Meanwhile, some progress was achieved for large stellar systems on statistical lines by introducing a continuous distribution function $f_m(\mathbf{x}, \mathbf{v}, t)$ of the positions $\mathbf{x}$, velocities $\mathbf{v}$ and masses m at any instant of time t. The time-variation of f_m is then followed with the help of a Boltzmann equation (1), the *r. h. s.* of which is given by the Fokker-Planck expression (2) appropriate to long-range forces.

$$\frac{\partial f_m}{\partial t} + \mathbf{v} \cdot \frac{\partial f_m}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{x}} \cdot \frac{\partial f_m}{\partial \mathbf{v}} = \sum_{m'} \left(\frac{\partial_c f_m}{\partial t} \right)_{m'} \tag{1}$$

where the stochastic variation of f_m per unit time, due to the encounters with stars of mass m', amounts to

$$\left(\frac{\partial_c f_m}{\partial t} \right)_{m'} = - \frac{\partial}{\partial \mathbf{v}} \cdot (f_m \langle \Delta \mathbf{v} \rangle_{m'}) + \frac{1}{2} \frac{\partial}{\partial \mathbf{v}} \frac{\partial}{\partial \mathbf{v}} (f_m \langle \Delta \mathbf{v} \, \Delta \mathbf{v} \rangle_{m'}) \tag{2}$$

$\langle \Delta \mathbf{v} \rangle_{m'}$ being the average velocity change per unit time of a star of mass m encountering stars of mass m'.

The self-gravitating property of the isolated system is ensured by requiring the smoothed-out potential $\Phi(\mathbf{x}, t)$ to obey Poisson's equation

$$\nabla^2 \Phi = 4 \pi G \sum_m m \int f_m \, d^3 v \tag{3}$$

The integro-differential system of equations (1), (2), (3) non-linear even without the encounter effect, is far too intricate to be solved in a general way.

The effect of binary encounters undergone by a star passing successively near other stars, tends to bring the system towards a state of statistical equilibrium; the associated time-scale is called relaxation time T_{rel} and shall vary from one place to another in the cluster. By comparing $\overline{T}_{rel}$ averaged over the whole cluster (or a limited part of it) to the age A of the system, usually obtained from the astrophysical properties of its brightest stars, one is led to an important distinction. When the ratio $\overline{T}_{rel}/A$ is much less than unity, the system considered (or part of it) is said to be relaxed; such is the case, for instance, of the dense central regions of a globular cluster. If conversely, the above ratio appears to be much larger than unity, the system is unrelaxed because the encounters have not had sufficient time to influence its dynamical state; such a situation is expected to prevail in the outer tenuous parts of any stellar system.

Another characteristic time which we shall evoke sometimes later on is the crossing-time T_{cr}, given by the time taken by a star to cross the whole system with the average (or r.m.s.) velocity of the stars in the cluster. For large systems, T_{cr} is shown to be much shorter than $\overline{T}_{rel}$.

The full complexity of the general problem embodied in equations (1), (2), (3) is avoided by laying forward a certain number of simplifying assumptions; this may lead to some restricted solutions of the problem. In spite of the somewhat crude nature of the assumptions and approximations made, it has been possible to obtain by this method a first overall picture of the dynamical evolution of a star cluster. This evolutionary picture, independent of the initial conditions and later confirmed by other methods, reveals the development of an increasing central concentration, together with the formation of an extended outer halo while some stars, having acquired a high energy through encounters, will escape from the cluster.

The instantaneous evaporation time T_{ev} is defined by

$$\frac{dN}{N} = -\frac{N(t)}{T_{ev}} \tag{4}$$

and if the escape rate dN/dt were constant, T_{ev} would be equal to the remaining lifetime of the cluster.

Numerical experiments

Twelve years ago, S. von HOERNER (1960), using a 2002 Siemens computer, was the first to investigate the dynamical evolution of a small star cluster by numerical simulation and similar attempts by several authors were soon to follow after.

The essential advantage of this method of numerical experiments, as it is also called, resides in its being free from any simplifying assumptions; given arbitrary initial conditions, one just has to solve numerically the first-order equations of (non-relativistic) motion for the N point-like stars composing the system ($i = 1, 2, ..., N$; $k = 1, 2, 3$)

$$\frac{dx_{ik}}{dt} = v_{ik} \tag{5}$$

$$\frac{dv_{ik}}{dt} = -G \sum_{j \neq i} m_j \frac{x_{ik} - x_{jk}}{|\mathbf{x}_i - \mathbf{x}_j|^3} \tag{6}$$

The evaluation of the r.h.s. of (6) requires an increasingly large computing time T_{cp} when N becomes large; estimated over one relaxation time of the system, T_{cp} appears roughly proportional to N^3, so that the method is limited to systems of a few hundred members, corresponding to small and medium star clusters of galaxy clusters. $N = 500$ represents an upper limit at the present time (AARSETH, 1971 (1971 b)).

The time-consuming computation of the acceleration (6) for large N leads us to prefer the use of multistep integration methods, implying high-order difference schemes. When two stars approach each other very closely, we must reduce the time-step in order to follow correctly the variation of the physical parameters connected with these two stars, but this reduction should not affect the motion of other stars, following smooth orbits. Consequently, each pair of stars must have its own individual time-step, adjustable in course of evolution.

The interpretation of the numerical results is a delicate matter; in a comparative study, carried out by eleven teams, each working with a different computer and using often different integration methods, the discussion of the results obtained for a 25-body problem under the same initial conditions, showed good agreement at first, followed by strong divergences as the effect of close encounters became predominant (LECAR, 1967). We therefore do not obtain a unique solution for the specified set of initial conditions and the numerical experiments appear to be reproducible only as long as the motion is dominated by the mean field — grad Φ. Alternatively, time reversal tests usually fail over durations exceeding one or two crossing times of the system (MILLER, 1964).

In such circumstances, it may be asked whether the numerical methods are doomed to failure.

In reality, this kind of method yields superabundant information; we are not really interested in the detailed motion of every individual member of the system, but only in the evolution of the system as a whole. The solution obtained corresponds to a trajectory in phase space which diverges, on account of the amplifying effect on the numerical errors due to the close encounters, from the trajectory of the true solution; it may therefore be looked at as a fluctuation of the true solution and any such fluctuation is equally acceptable as the true solution, so far as the trajectory remains inside the phase domain defined by the constants of the evolution. In other words, so long as the constancy of the total energy of the cluster, of its resultant angular momentum and of the parameters defining the centre-of-mass motion is verified with sufficient accuracy, we will be entitled to consider the statistical behaviour of the system to correspond to its true dynamical evolution.

The numerical experiments simulating the evolution of an isolated star cluster have all confirmed the main features already disclosed in the statistical calculations derived from the Boltzmann and Poisson equations: formation in a finite time of a high-density core, of an extended halo, and slow loss of stars by evaporation.

More precisely, a large fraction of high velocity stars go into the halo along elongated orbits, later returning towards the central region, but most of the time is being spent in the halo where the encounters are very rare.

Furthermore, close binary stars or tightly bound subsystems often appear in the central region of an evolving cluster, where the increasingly high density favours multiple encounters; such a region thus behaves as a sink of energy. The continual ejection of stars from the strongly bound central core eventually leads to the appearance of one close pair at the centre (VAN ALBADA, 1968). This might indeed be the final state of cluster evolution, since the 2-body system is the only known stable configuration; the other members would be removed at infinity (AARSETH, 1971 a). On account of mass segregation, the most massive stars generally tend to gather in the central core, enhancing binary activity therein.

In reality, a star cluster is never quite isolated; it moves in the gravitational field of the Galaxy and, when orbiting near the galacitc plane, may also interact with clouds of diffuse matter. The first of these external influences lowers the energy threshold beyond which a star may escape from the cluster; while escapers are generally identified by their positive total energy when the cluster is isolated, we have rather to consider their Jacobian integral per unit mass in the presence of the galactic field. Computing experiments by HAYLI (1971) and others have shown that the evaporation time (4) is noticeably reduced by the effect of the galactic field.

On the other hand, the effect of rapidly passing-by interstellar clouds tends to increase the total energy of the cluster as a result of energy exchange, thus favouring a gradual disruption of the cluster. This case has also been submitted lately to numerical simulation (BOUVIER and JANIN, 1970 b) for a small cluster of 25 stars. A movie film illustrates the evolution of that cluster, either isolated or non-isolated; in the first case, the formation of the core-halo structure is clearly evidenced, with rapid creation of a few binaries near the centre, some of which remain stable over several revolutions while others disrupt by subsequent encounters; in the second case, the energy transfer from the clouds to the cluster results in a marked inhibition to the formation of core and halo as well as in the evaporation rate.

The inclusion of both the preceding external influences presents many difficulties; a recent attempt (BOUVIER, 1971, 1972) leads to a lifetime of slightly over 10^8 years for a 25-star cluster against its internal dynamical evolution together with the tidal effects of the Galaxy and of passing clouds.

The most direct information on the time-scale of the dynamical evolution of open clusters is provided by the observed distribution of the cluster ages (WIELEN, 1971), since under the assumption that the rate of formation of clusters has not appreciably changed in time, the age distribution reflects the finite lifetimes of open clusters. The evaporation time T_{ev} defined in (4) depends mainly on the median radius (containing half the mass of the cluster), the total mass M and the total number of stars N; for actual open clusters, M and N are strongly correlated. The observed distribution of ages reveals a wide spread which can then be interpreted as due to the variety of total masses and radii of the clusters.

Simplified methods for large systems

In contrast to small systems which are subject to rather large fluctuations, the evolution of large stellar systems will remain closer to the statistical expectations derived from a continuous distribution function but, as mentioned before, we meet the difficulty of the computing time T_{cp} increasing sharply with N. In order to reduce

T_{cp}, one might be tempted to seek for a method simpler than the direct numerical integration of the equations of motion, which would keep at least part of the various simplifying assumptions put forward in the statistical calculations based on the Boltzmann and Poisson equations. Indeed, in a large system, the gravitational field acting on a star may be split in two parts: a) the mean smoothed-out field $-$ grad Φ due to the bulk of the other stars of the system and which, if acting alone, would determine the " regular " orbit of the star, b) a small " irregular " fluctuating field produced by encounters suffered by the star with closely passing-by stars and which shall gradually alter the parameters of the regular orbit.

Let us consider the motion of a star during a time long compared to the crossing time T_{cr} but short with respect to the relaxation time $\bar{T}_{rel}$ in the cluster; this motion is then governed by the mean field and the latter can be taken as time-independent; if it is, furthermore, spherically symmetric, the stellar orbit is then a plane rosette. Instead of computing the effect of all the stars on the one under consideration as is implicitly done in the numerical integration, we may proceed to a random selection, in the sense of the Monte-Carlo tactics (HENON, 1966), with the result that T_{cp} is now proportional to N at most.

More precisely, one first selects just one point of the regular orbit and computes the perturbation only at that point; secondly, one selects just the effect of one field star and computes the perturbation from the star; finally we multiply this perturbation by an appropriate factor in order to account for all the orbital points and field stars which have not been considered.

This will, of course, not give the exact perturbation in the stars motion, but on account of the random character of the perturbation, noted before while discussing the interpretation of the numerical results, we are only interested in the statistical properties such as the velocity moments of 1st and 2nd order. The Monte-Carlo scheme can also be visualized as a convenient algorithm for the numerical solution of equations (1) and (2), because of the same basic assumptions made here, namely that the system can be adequately described by a one-particle distribution function and that the evolution is due only to binary encounters. Multiple encounters are ignored, therefore also the formation and disruption of binaries; this omission is probably not serious when the system is large (HÉNON, 1971).

Another simplified method for computing the evolution of a large spherical stellar system has been proposed by LARSON (1970). It uses a fluid-dynamical approach based on the numerical solution of the " moment equations " derived from (1). In order to take into account the radial energy flow characterizing the cluster's evolution, the moment equations have to be carried to at least the fourth order before closing the chain of equations by assuming a relation between the fifth and lower-order moments. For this purpose, the velocity distribution function is expanded around a Maxwellian in Legendre polynomials and the relation sought for is obtained by retaining only the first three terms of the expansion, while the collision terms in the

Fokker-Planck expression (2) are evaluated on assuming a Maxwellian distribution for the field stars of mass m'.

Unrelaxed systems

We pointed out before the distinction met when the ratio of relaxation time T_{rel} vs. age A of the system is either much smaller or much larger than unity. In the first case, the system is said to be relaxed, in the sense that most of its dynamical evolution is governed by the encounter effect between member stars whereas, in the second case, viz. that of an unrelaxed system, encounters have not yet manifested appreciably their influence on the cluster's evolution. In particular, any recently formed stellar system will be unrelaxed to begin with, undergoing an initial phase of orbital mixing dominated by collective motions in a time-dependent potential. The time-scale T_{mix} for this initial phase is of the order of a few crossing times (or average stellar periods), which is generally short compared with T_{rel}, the time-scale for the " thermalization " due to encounters.

It can be shown that the ratio T_{rel}/T_{mix} is nearly proportional to $N/\log N$; it therefore increases with N, so that the initial phase is well separated for large systems. The orbital mixing phase is described, in terms of distribution functions, by the collisionless Boltzmann equation, namely (1) with vanishing r.h.s. (also called then Liouville or Vlasov equation), coupled with the Poisson equation (3). The problem being still non-linear, progress in analytical solutions is very limited; recourse to numerical simulation has been successful mostly for so-called stratified systems, which possess symmetries that make their properties depend only on a single space co-ordinate. For example, the system can be stratified in planes, all perpendicular to the x-axis; grouping all the stars of same x-co-ordinate and velocity u along x into a single object (superstar), we may write the equations of motion of the superstar labelled i ($i = 1, 2, ..., n$) in the usual form

$$\frac{dx_i}{dt} = u_i \qquad \frac{du_i}{dt} = a_i \tag{7}$$

The N actual stars of the system are thus distributed into n sub-groups or superstars; if μ_m is the number of stars having mass m (discrete spectrum), we may assume, without loss of generality, that each superstar carries the same mass $\mu = \sum_m m\mu_{m/n}$ uniformly distributed with a constant surface density σ.

The n parallel planes may freely cross each other and the resulting potential fluctuations are all the more negligible that n is larger, therefore the stellar system can indeed be regarded as unrelaxed. Owing to the particularly simple expression for the attraction between two parallel planes, the computation of the acceleration in (7) is an easy matter; we just have

$$a_i = 2\pi G \sigma (n - 2i + 1)$$

independently of the distance.

Such a plane-parallel system could eventually describe the collective motions perpendicular to the galactic plane in the vicinity of the Sun.

Starting from any initial distribution of the n superstars, the evolution is computed according to equations (7) and illustrated in the (x, u) phase plane. HOHL and FEIX (1967) treated the case of the water-bag model, where the superstars are uniformly distributed in a given domain of the (x, u) plane. As a consequence of the collisionless Boltzmann equation which expresses the conservation of phase-density, this domain changes its form but not its volume; the points on the boundary remain on the boundary and their motion is sufficient to determine the motion of the whole system. Consequently, if we retain the motion of the boundary only, we shall be led to a much faster computational method for studying the evolution of a water-bag model; this method has been successfully applied to the two-stream instability of a plasma (ROBERTS and BERK, 1967) and later to a plane-parallel stratified stellar system (JANIN, 1971).

That such a method be equally applicable to a fully ionized plasma and to a stellar system is not altogether surprising, since both these domains are basically connected to the same law of force, proportional to the inverse square of the distance; nevertheless, some essential differences do exist also, on account of the sign of the force, viz. repulsion between electrons, attraction between stars.

Similar considerations may be carried out for spherical systems, the superstars being now concentric shells of given radius, r, radial velocity u, on which the actual stars move in any direction with a given tangential velocity v; here too, the acceleration is fairly easy to calculate and such a model has been used to describe the initial evolution of a spherical star cluster (HÉNON, 1964). Furthermore, the spherical water-bag has been studied by BOUVIER and JANIN (1970 a) with the use of a suitable representation in two-dimensional varieties of the (r, u, v) space. As for the plane case, the system evolves towards a quasistationary state made up of a central core surrounded by some filamentary structure corresponding to high energy halo superstars.

Large relaxed spherical systems

Consider further a spherical stratified system; if it consists, for instance, of 2000 shells each of them loaded with 500 stars of equal mass, it will picture a spherical unrelaxed system containing a million stars. One could then think of adding a slight perturbation to the velocities of each shell (superstar) in order to obtain statistically the same results as would be anticipated from random encounters between neighbouring stars. This point of view, adopted lately by SPITZER and HART (1971 a), should then allow us to follow the dynamical evolution of a relaxed spherical cluster much larger than one could deal with by using the direct N-body problem integration (500); the irregular variation of the gravitational field, owed to stellar encounters, is taken care of by perturbing the velocity of each superstar over specified time-intervals.

This is achieved by means of a simplified Monte Carlo scheme designed to give, with the minimum of computations, results consistent with the average values per unit time of the velocity moments Δv, $(\Delta v)^2$. More precisely, while HÉNON (1967) selects all the variables characterizing a binary encounter at random, except the impact parameter p, chosen so that the mean values $\langle \Delta v \rangle$, $\langle (\Delta v)^2 \rangle$ per unit time, integrated over all p's, are correctly given, Spitzer only selects the velocity of a certain star randomly, the values of $\langle \Delta v \rangle$, $\langle (\Delta v)^2 \rangle$ being then averaged over all the other variables defining the encounters undergone by that star. The procedure requires that specific velocity distributions be assumed; in most situations, a Maxwellian distribution will be sufficient. The results obtained by this method, at least in the case of equal stellar masses and with a convenient choice of the time interval between successive velocity perturbations, may be taken as a general confirmation of Larson's approach discussed before, based on small deviations from the Maxwellian distribution, together with large velocity anisotropies in the halo. The present numerical models are probably more realistic than most previous ones, although it has not been possible to carry through the computations for systems with as many stars as are found in galactic nuclei or in many globular clusters.

Concluding remarks

All the different methods reviewed here contain many approximations and assumptions, some of which remain controversial; nevertheless, it is satisfying to note that they agree on general features characterizing the overall picture of the dynamical evolution of a stellar cluster, mainly the separation into a central contracting core and an outer halo, together with some evaporation. The contraction of the core towards some kind of singularity is more rapid in cases where the stars have unequal masses, because the relaxation effect or tendency towards energy equipartition slows down the more massive stars which form a rapidly contracting core at the centre (SPITZER and HART, 1971 b).

The core contraction accelerates in time, but the rate of this contraction appears to be sensitive to external perturbations such as passing clouds; the dynamical evolution of the 25-star cluster, alluded to previously, had shown a slowing down of the building of the core-halo structure in presence of clouds. Moreover, the mass loss of stars due to their internal evolution could also interfere, but presumably without changing the nature of the evolution (SPITZER and CHEVALIER, 1972).

The cause of this accelerated contraction may eventually be found in some instability of the system, similar to the gravo-thermal instability of an isothermal sphere (LYNDEN-BELL and WOOD, 1968); the inner part of the sphere behaves as a medium of negative specific heat; by contracting, it gets hotter, transferring heat to the outer region (halo), but nonetheless the temperature difference between centre and edge increases and accelerates the evolution. At some finite time, the central density becomes singular.

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