THEOREM 6. *If* $G = G_u \rtimes T$ *is a connected solvable affine group and* $S \subset G_s$ *is a commuting set then* $Z_G(S)$ *is connected and* $aSa^{-1} \subset T$ *for some* $a \in G$. *In particular, all maximal tori of* G *are conjugate.*

Proof. We use induction on $\dim G$. The assertions are obvious if $S \subset Z(G)$. Otherwise choose $s \in S \backslash Z(G)$. By Theorem 5 we may assume that $s \in T$. Then $Z_G(s)$ is a proper closed subgroup of G containing T and S. By Theorem 3, $Z_G(s)$ is connected. Since $\dim Z_G(s) < \dim G$, we can apply the induction hypothesis to conclude the proof. □

It is now easy to describe connected nilpotent affine groups.

THEOREM 7. *A connected solvable affine group* $G = G_u \rtimes T$ *is nilpotent if and only if* $G_s = T \subset Z(G)$. *In that case* $G = G_u \times T$.

Proof. Assume that G is nilpotent. We prove that $G_s = T \subset Z(G)$ by induction on $\dim G$. We may assume that G is not abelian. Let N be the last non-trivial term of the lower central series of G. Let f be as in Theorem 2 and $\bar{G} = f(G)$. Then $\bar{G} = f(G_u T) = (\bar{G})_u f(T)$. By induction hypothesis we have $f(T) = (\bar{G})_s \subset Z(\bar{G})$. Consequently if $t \in T$ and $x \in G$ then $u := txt^{-1}x^{-1} \in N$. Since $N \subset Z(G) \cap G_u$, and $xtx^{-1} = u^{-1}t = tu^{-1}$ we must have $u = 1$. Thus $T \subset Z(G)$ and, by Theorem 5, $G_s = T$. The converse is obvious. □

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