Proof. Since we consider the graded inverse lexicographic ordering, we have for all $P \in R[X] - \{0\}$: $\mathrm{in}(P) = \mathrm{in}(\tilde{P})$. Hence $<\mathrm{in}(\tilde{J})> = <\mathrm{in}(J)> = <\mathrm{in}(G)> = <\mathrm{in}(\tilde{G})>$.

5.2. *Example.* Let R be a field. Consider the "twisted cubic"

$$Z := \{(t, t^2, t^3) \mid t \in R\} \subseteq R^3 .$$

Then

$$J := <X_1^3 - X_3, X_1^2 - X_2> \leqslant R[X_1, X_2, X_3]$$

is the ideal of polynomials vanishing on Z.

Recall that the set of zeroes of $\tilde{J}$ in the projective space $\mathbf{P}_3(R)$ is the closure (with respect to the Zariski topology) of Z.

The polynomials $X_1^3 - X_3X_4^2$ and $X_1^2 - X_2X_4$ do *not* generate the ideal $\tilde{J} \leqslant R[X_1, X_2, X_3, X_4]$.

By 3.6. $G := \{X_1^2 - X_2, X_1X_2 - X_3, X_2^2 - X_1X_3\}$ is a Gröbner basis of J with respect to the graded inverse lexicographic ordering. Hence $\tilde{J}$ is generated by $\{X_1^2 - X_2X_4, X_1X_2 - X_3X_4, X_2^2 - X_1X_3X_4\}$.

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See also the literature cited in these articles.

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