

§4. Proof of Theorems 1 and 2

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **34 (1988)**

Heft 1-2: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **18.04.2024**

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.

Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.

Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

3.3. LEMMA. Let R be a local ring and F be the associated field. Let $f: C_1 \rightarrow C_0$ be a R -homomorphism of finitely generated free R -modules and let $\bar{f}: F \otimes_R C_1 \rightarrow F \otimes_R C_0$ be the induced F -homomorphism. If $\text{rk } f = \text{rk } \bar{f}$ then with respect to some bases in C_1, C_0 the homomorphism f is presented by the matrix $\begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}$ where E is the unit matrix of order $\text{rk } f$.

Proof. Since F is a field we can choose bases d_0, d_1 respectively in $F \otimes_R C_0, F \otimes_R C_1$ so that the matrix of $\bar{f}$ regarding these bases has the form $\begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}$. Let $\mathcal{D}_i$ be a lifting of d_i to $C_i, i = 1, 2$. Here $\mathcal{D}_i$ is a sequence of $\text{rg } C_i$ elements of C_i . In view of Nakayama's lemma $\mathcal{D}_i$ generate C_i . This implies that $\mathcal{D}_i$ generates the $(\text{rg } C_i)$ -dimensional vector space $Q(R) \otimes_R C_i$ over the field $Q(R)$. Therefore, the elements of the sequence $\mathcal{D}_i$ are linearly independent over $Q(R)$ and, hence, over R . Thus $\mathcal{D}_i$ is a basis of C_i for $i = 0, 1$. The matrix of f with respect to bases $\mathcal{D}_0, \mathcal{D}_1$ has the form $\begin{bmatrix} E+U & Z \\ X & Y \end{bmatrix}$ where U, X, Y, Z are matrices over the maximal ideal u of R . Note that $\det(E+U) = 1 \pmod{u}$. Since all elements of $R \setminus u$ are invertible in R the square matrix $E+U$ is invertible over R . Therefore we can choose bases in C_0, C_1 so that the corresponding matrix of f equals $\begin{bmatrix} E & 0 \\ 0 & Y' \end{bmatrix}$. Since $\text{rk } f = \text{rk } \bar{f} = \text{rk } E, Y' = 0$.

3.4. LEMMA. Let R be a local ring and F be the associated field. Let $C = (\cdots \rightarrow C_{i+1} \rightarrow C_i \rightarrow \cdots)$ be a finitely generated free chain complex over R . Let C' be the chain F -complex $F \otimes_R C$. Let ∂_i, ∂'_i be the boundary homomorphisms $C_{i+1} \rightarrow C_i, C'_{i+1} \rightarrow C'_i$. If $\text{rk}_R H_i(C) = \text{rk}_F H_i(C')$ for some i then: $H_i(C), \text{Im } \partial_{i+1}, \text{Im } \partial_i$ are free R -modules and $C_i = \text{Im } \partial_{i+1} \oplus H_i(C) \oplus \text{Im } \partial_i$; the projection $C \rightarrow C'$ induces F -isomorphisms $F \otimes_R H_i(C) \rightarrow H_i(C'), F \otimes_R \text{Im } \partial_j \rightarrow \text{Im } \partial'_j$ with $j = i, i+1$.

This Lemma directly follows from Lemmas 3.2 (ii) and 3.3.

§ 4. PROOF OF THEOREMS 1 AND 2

4.1. PROOF OF THEOREM 1. Denote by Q_n the fraction field of the ring $\Lambda_n = \mathbf{Z}[t_1, t_1^{-1}, \dots, t_n, t_n^{-1}]$. Denote by Q_n^0 the subring of Q_n which consists of rational functions fg^{-1} with $f, g \in \Lambda_n$ and $g \notin (t_n - 1)\Lambda_n$ (so that

$g(t_1, \dots, t_{n-1}, 1) \neq 0$). The homomorphism $f \mapsto f(t_1, \dots, t_{n-1}, 1): \Lambda_n \rightarrow \Lambda_{n-1}$ uniquely extends to a ring homomorphism $Q_n^0 \rightarrow Q_{n-1}$ which is denoted by φ .

Denote by X the exterior of K and by Y the exterior of L .

We shall prove the following two statements.

(4.1.1). $\varphi(\Delta(K)) = \Delta(K)(t_1, \dots, t_{n-1}, 1)$ divides $\Delta(L)$ in Λ_{n-1} .

(4.1.2). There exists a representative ω of the torsion $\omega(X) \subset Q_n$ such that $(t_n - 1)\omega \in Q_n^0$ and $\varphi((t_n - 1)\omega)$ represents $\omega(Y) \subset Q_{n-1}$.

Let us show first that these two statements imply the Theorem. Let ω be the element of Q_n produced by (4.1.2). Put $\pi = \prod_{i=1}^{n-1} (t_i - 1)$. According to the results formulated in Sec. 2.2 the product $(t_n - 1)\pi \cdot \Delta(K)$ represents $\omega(X)$. Thus

$$\omega \doteq \frac{f\bar{f}}{g\bar{g}}(t_n - 1)\pi\Delta(K)$$

where $f, g \in \Lambda_n \setminus 0$. We may assume that $f\bar{f}$ and $g\bar{g}$ are relatively prime. If $t_n - 1$ does not divide g then $\omega \in Q_n^0$ and $\varphi((t_n - 1)\omega) = 0$ which contradicts to the inclusion $\varphi((t_n - 1)\omega) \in \omega(Y)$. Thus $g = (t_n - 1)h$ with $h \in \Lambda_n$. In view of (4.1.1), $\varphi(\Delta(K)) \neq 0$, i.e. $t_n - 1$ does not divide $\Delta(K)$. If $\varphi(h) = 0$ then $(t_n - 1)^2$ divides g which obviously contradicts the inclusion $(t_n - 1)\omega \in Q_n^0$. Thus $\varphi(h) \neq 0$. We have

$$h\bar{h}(t_n - 1)\omega \doteq f\bar{f}\pi\Delta(K).$$

Since $\varphi(h\bar{h}(t_n - 1)\omega) \neq 0$ we have $\varphi(f) \neq 0$. This implies that $\pi \cdot \varphi(\Delta(K)) \doteq q\bar{q}\varphi((t_n - 1)\omega)$ where $q = \varphi(h)/\varphi(f)$. Thus $\pi\varphi(\Delta(K))$ represents $\omega(Y)$. Since $\pi\Delta(L) \in \omega(Y)$ we have

$$\varphi(\Delta(K))\lambda\bar{\lambda} = \Delta(L)\mu\bar{\mu}$$

with non-zero $\lambda, \mu \in \Lambda_{n-1}$. We may assume that $\lambda\bar{\lambda}$ and $\mu\bar{\mu}$ are relatively prime. Since $\varphi(\Delta(K))$ divides $\Delta(L)$ we immediately obtain $\mu\bar{\mu} = 1$. Thus, $\Delta(L) = \varphi(\Delta(K))\lambda\bar{\lambda}$.

Let us prove (4.1.1) and (4.1.2). We may assume that $X \subset Y$ and that $Y \setminus X$ is the interior of the regular neighborhood $U \subset Y$ of the n -th component of K in Y . Let $p: \tilde{X} \rightarrow X$ and $q: \tilde{Y} \rightarrow Y$ be the maximal abelian coverings with the groups of covering transformations respectively $H_1(X) \approx \mathbf{Z}^n$ (generators $t_1, \dots, t_n$) and $H_1(Y) \approx \mathbf{Z}^{n-1}$ (generators $t_1, \dots, t_{n-1}$). It is clear that p is the composition of an infinite cyclic covering $\tilde{X} \rightarrow q^{-1}(X)$ and the covering $q: q^{-1}(X) \rightarrow X$.

Fix a C^1 -triangulation of Y so that X and U are simplicial subcomplexes of Y . Fix also the induced equivariant triangulations in $\tilde{X}$ and $\tilde{Y}$.

The ring Λ_{n-1} determines via the natural homomorphism $\mathbf{Z}[\pi_1(Y)] \rightarrow \mathbf{Z}[H_1 Y] = \Lambda_{n-1}$ a system of local coefficients on Y which we denote by the same symbol Λ_{n-1} . According to definitions, for any simplicial subsets $A \supset B$ of Y the Λ_{n-1} -module $H_*(A, B; \Lambda_{n-1})$ equals $H_*(C(q^{-1}(A), q^{-1}(B); \mathbf{Z}))$. Here the simplicial chain complex $C_*(q^{-1}(A), q^{-1}(B); \mathbf{Z})$ is a finitely generated free Λ_{n-1} -complex. Analogously Λ_n defines a system of local coefficients on X and for simplicial subsets $A \supset B$ of X the Λ_n -module $H_*(A, B; \Lambda_n)$ equals $H_*(C(p^{-1}(A), p^{-1}(B); \mathbf{Z}))$. Note that

$$\Lambda_{n-1} \otimes_{\Lambda_n} C_*(p^{-1}(A), p^{-1}(B); \mathbf{Z}) = C_*(q^{-1}(A), q^{-1}(B); \mathbf{Z})$$

where Λ_n acts on Λ_{n-1} via ϕ .

Claim 1. For $i \neq 1, m-1$,

$$\mathrm{rk}_{\Lambda_n} H_i(X; \Lambda_n) = \mathrm{rk}_{\Lambda_{n-1}} H_i(X; \Lambda_{n-1}) = \mathrm{rk}_{\Lambda_{n-1}} H_i(Y; \Lambda_{n-1}) = 0.$$

For $i = 1, m-1$,

$$\mathrm{rk}_{\Lambda_n} H_i(X; \Lambda_n) = \mathrm{rk}_{\Lambda_{n-1}} H_i(X; \Lambda_{n-1}) = n-1; \quad \mathrm{rk}_{\Lambda_{n-1}} H_i(Y; \Lambda_{n-1}) = n-2.$$

Proof of Claim 1. We shall compute the rank of $H_i(X; \Lambda_n)$; modules $H_i(X; \Lambda_{n-1})$ and $H_i(Y; \Lambda_{n-1})$ can be treated similarly.

Denote by V a wedge of n circles in X such that the inclusion homomorphism $H_1(V; \mathbf{Z}) \rightarrow H_1(X; \mathbf{Z}) = \mathbf{Z}^n$ is bijective. Then $H_i(X, V; \mathbf{Z}) = 0$ for $i \leq m-2$. Therefore an application of Lemma 3.2(i) to complexes $C_*(\tilde{X}, p^{-1}(V); \mathbf{Z})$ and $C_*(X, V; \mathbf{Z})$ gives that $\mathrm{rk}_{\Lambda_n} H_i(X, V; \Lambda_n) = 0$ for $i \leq m-2$. This implies that $\mathrm{rk} H_i(X; \Lambda_n) = \mathrm{rk} H_i(V; \Lambda_n)$ for $i \leq m-3$ and that $\mathrm{rk} H_{m-2}(X; \Lambda_n) \leq \mathrm{rk} H_{m-2}(V; \Lambda_n)$. The rank of $H_i(V; \Lambda_n)$ can be computed directly: It is equal to 0 if $i \neq 1$ and to $n-1$ if $i = 1$. Thus the rank of $H_i(X; \Lambda_n)$ equals 0 if $i \neq 1, m-1$ and equals $n-1$ if $i = 1$. The equality $\mathrm{rk} H_{m-1}(X; \Lambda_n) = n-1$ follows from duality or from the equalities

$$\sum_{i=0}^m (-1)^i \mathrm{rk} H_i(X; \Lambda_n) = \chi(X) = 0.$$

Claim 2. The exact homology sequence of (Y, X) with coefficients in Λ_{n-1} splits into short exact sequences

$$\begin{aligned}
0 &\rightarrow H_m(Y, X; \Lambda_{n-1}) \rightarrow H_{m-1}(X; \Lambda_{n-1}) \rightarrow H_{m-1}(Y; \Lambda_{n-1}) \rightarrow 0, \\
0 &\rightarrow H_i(X; \Lambda_{n-1}) \xrightarrow{\approx} H_i(Y; \Lambda_{n-1}) \rightarrow 0, \quad (i \neq 1, m-1) \\
0 &\rightarrow H_2(Y, X; \Lambda_{n-1}) \xrightarrow{\partial_1} H_1(X; \Lambda_{n-1}) \rightarrow H_1(Y; \Lambda_{n-1}) \rightarrow 0.
\end{aligned}$$

Proof of Claim 2. Clearly, $H_i(Y, X; \Lambda_{n-1}) = H_i(U, \partial U; \Lambda_{n-1}) = 0$ for $i \neq 2, m$. Therefore the only thing to prove is the injectivity of ∂_1 . According to Claim 1 $\text{rk } H_1(X; \Lambda_{n-1}) = n - 1$ and $\text{rk } H_1(Y; \Lambda_{n-1}) = n - 2$. Since $H_2(Y, X; \Lambda_{n-1}) = \Lambda_{n-1}$ we see that ∂_1 is injective.

Proof of (4.1.1). In view of the equalities $\text{rg } H_i(X; \Lambda_n) = \text{rg } H_i(X; \Lambda_{n-1})$, $i = 0, 1, \dots$ we may apply Lemma 3.2 (iii) to the chain complexes $C_*(\tilde{X}; \mathbf{Z})$ and $C_*(q^{-1}(X); \mathbf{Z})$ respectively over Λ_n and Λ_{n-1} . Since $m - 1 > r > 1$ Claims 1, 2 show that $H_r(X; \Lambda_n)$ and $H_r(X; \Lambda_{n-1})$ are torsion modules respectively over Λ_n and Λ_{n-1} and $H_r(X, \Lambda_{n-1}) = H_r(Y; \Lambda_{n-1})$. By definition $\Delta(K) = \text{ord } H_r(X; \Lambda_n)$ and $\Delta(L) = \text{ord } H_r(Y; \Lambda_{n-1}) = \text{ord } H_r(X; \Lambda_{n-1})$. Lemma 3.2 (iii) directly implies that $\phi(\Delta(K))$ divides $\Delta(L)$.

It remains to prove Statement (4.1.2) which is, of course, the core of Theorem 1. For simplicial subsets $A \supset B$ of Y we shall denote by $C(A, B)$ the (simplicial) chain Q_{n-1} -complex $Q_{n-1} \otimes_{\Lambda_{n-1}} C_*(q^{-1}(A), q^{-1}(B); \mathbf{Z})$. Clearly

$$H_i(A, B; Q_{n-1}) = H_i(C(A, B)) = Q_{n-1} \otimes_{\Lambda_{n-1}} H_i(A, B; \Lambda_{n-1}).$$

Consider the short exact sequence of chain Q_{n-1} -complexes

$$(5) \quad 0 \rightarrow C(X) \rightarrow C(Y) \rightarrow C(Y, X) \rightarrow 0.$$

Provide the homology modules of complexes $C(X)$, $C(Y)$, $C(Y, X)$ with bases as follows. It is evident that $H_i(C(Y, X)) = 0$ for $i \neq 2, m$ and

$$H_i(C(Y, X)) = H_i(C(U, \partial U)) = H_i(U, \partial U; Q_{n-1}) = Q_{n-1}$$

for $i = 2, m$. Fix a lifting $\tilde{U} \subset \tilde{Y}$ of $U \approx S^{m-2} \times D^2$. Fix in $H_m(C(Y, X))$ the generator $[\tilde{U}, \partial \tilde{U}]$. Fix in $H_2(C(Y, X))$ the generator $[\Delta, \partial \Delta]$ where Δ is the meridional disk of $\tilde{U}$.

It follows from Claim 1 that $H_i(C(X)) = H_i(C(Y)) = 0$ for $i \neq 1, m - 1$. Fix an arbitrary basis f in the $(n-2)$ -dimensional vector Q_{n-1} -space $H_1(Y; Q_{n-1})$. Fix the dual basis g in $H_{m-1}(Y; Q_{n-1})$. It follows from Claim 2 that inclusion homomorphisms $H_i(C(X)) \rightarrow H_i(C(Y))$ are surjective for all i . Let F and G be sequences of $n - 2$ vectors in $H_1(C(X))$ and in $H_{m-1}(C(X))$ whose images under these inclusion homomorphisms are equal respectively to f and g . Claim 2 implies that $[\partial \tilde{U}], G$ is a basis in $H_{m-1}(C(X))$ and

$[\partial\Delta]$, F is a basis in $H_1(C(X))$. Now all homology modules of complexes $C(X)$, $C(Y)$, $C(Y, X)$ are provided with bases.

Provide the modules of $C(X)$, $C(Y)$, $C(Y, X)$ with natural bases (see Sec. 2.2). We may choose these bases to be compatible in the sense of Lemma 2.1.1. According to this Lemma

$$\tau(C(Y)) = \pm \tau(C(X))\tau(C(Y, X))\tau(\mathcal{H})$$

where $\mathcal{H}$ is the homology sequence associated with the exact sequence (5). It is evident that $\tau(\mathcal{H}) = \pm 1$. It is easy to verify that $\tau(C(Y, X)) = \tau(C(U, \partial U)) = \pm 1$. (Indeed, the pair $(U, \partial U)$ has a cell structure such that $\text{Int } U$ contains 2 open cells; the meridional disc and its complement; for such cell structure the equality $\tau(C(U, \partial U)) = \pm 1$ is evident. The case of an arbitrary cell structure (or triangulation) follows from the invariance of torsion under cell subdivision). Thus $\tau(C(Y)) = \pm \tau(C(X))$. Note that $\tau(C(Y))$ represents $\omega(Y)$. Therefore $\tau(C(X))$ also represents $\omega(Y)$.

Consider the chain complex

$$C = Q_n^0 \otimes_{\Lambda_n} C_*(\tilde{X}; \mathbf{Z}).$$

Note that Q_n^0 is a local ring with the maximal ideal $(t_n - 1)Q_n^0$ and associated field Q_{n-1} . Clearly, $Q_{n-1} \otimes_{Q_n^0} C = C(X)$. The natural bases in chain modules of $C(X)$ lift to natural bases in chain modules of C . Claim 1 implies that for all $i \geq 0$

$$\text{rk}_{Q_n^0} H_i(C) = \text{rk}_{\Lambda_n} H_i(X; \Lambda_n) = \text{rk}_{Q_{n-1}} H_i(C(X)).$$

Therefore we may apply Lemma 3.4 to complexes C , $C(X)$. This lemma shows that: $H_i(C) = H_i(C(X)) = 0$ for $i \neq 1, m - 1$; the basis $[\partial\Delta]$, F in $H_1(C(X))$ lifts to a basis, say, $f_0, f_1, \dots, f_{n-2}$ in $H_1(C)$; the basis $[\partial\tilde{U}]$, G in $H_{m-1}(C(X))$ lifts to a basis, say, $g_0, g_1, \dots, g_{n-2}$ in $H_{m-1}(C)$; the submodules of cycles and boundaries of C are free in all dimensions. Thus we may apply the constructions of Sec. 2.1 to C which gives rise to a torsion $\tau(C) \in Q_n^0$. It follows directly from the formula (3) that $\phi(\tau(C)) = \tau(C(X))$. Thus $\phi(\tau(C))$ represents $\omega(Y)$.

Let v be the matrix of the semi-linear intersection pairing

$$\langle , \rangle : H_1(X; Q_n^0) \times H_{m-1}(X; Q_n^0) \rightarrow Q_n^0$$

with respect to bases $f_0, f_1, \dots, f_{n-2}$ and $g_0, g_1, \dots, g_{n-2}$. (Here $H_i(X; Q_n^0) = H_i(C)$). It is clear that $\tau(C) (\det v)^{-1}$ represents $\omega(X)$. Put $\omega = \tau(C) (\det v)^{-1}$. We shall prove that

$$(6) \quad \det v = \pm (t_n - 1) + (t_n - 1)^2 a$$

where $a \in Q_n^0$. Then $(t_n - 1)\omega \in Q_n^0$ and

$$\varphi((t_n - 1)\omega) = \varphi(\tau(C)[\pm 1 + (t_n - 1)a]^{-1}) = \pm \varphi(\tau(C)) \in \omega(Y).$$

This would complete the proof of (4.1.2).

It is obvious that

$$v = \begin{bmatrix} \langle f_0, g_0 \rangle & (t_n - 1)\alpha \\ (t_n - 1)\beta & E + (t_n - 1)\gamma \end{bmatrix}$$

where α, β, γ are respectively a $(n-2)$ -row, $(n-2)$ -column and $(n-2) \times (n-2)$ -matrix over Q_n^0 . It turns out that

$$(7) \quad \langle f_0, g_0 \rangle = \pm (t_n - 1) + (t_n - 1)^2 b$$

with $b \in Q_n^0$. This immediately implies (6).

I shall prove (7) for a special choice of f_0 which is sufficient for our aims. Let $\theta: [0, 1] \rightarrow \partial\tilde{X}$ be a path whose projection to $\tilde{Y}$ is a loop parametrizing $\partial\Delta \subset \partial\tilde{U}$. Let $\eta: [0, 1] \rightarrow \tilde{X}$ be a path such that $\eta(0) = \theta(0)$ and $\eta(1) = t_1 \cdot \theta(0)$. Consider the singular chain $\mathfrak{g} = \theta - t_1\theta + t_n\eta - \eta$. It is easy to check up that $\mathfrak{g}$ is a cycle in $\tilde{X}$ and that its homology class $[\mathfrak{g}] \in H_1(C)$ projects to $(1 - t_1)[\partial\Delta] \in H_1(C(X))$. Put $f_0 = (1 - t_1)^{-1}[\mathfrak{g}]$. Then $\langle f_0, g_0 \rangle = (1 - t_1)^{-1}\langle [\mathfrak{g}], g_0 \rangle = (1 - t_1)^{-1}(t_n - 1)\langle \eta, g_0 \rangle$ where in the right part the brackets $\langle \ , \ \rangle$ denote the intersection pairing

$$H_1(X, \partial X; Q_n^0) \times H_{m-1}(X; Q_n^0) \rightarrow Q_n^0.$$

The image of $\langle \eta, g_0 \rangle$ under $\varphi: Q_n^0 \rightarrow Q_{n-1}$ can be computed using the analogous pairing

$$H_1(X, \partial X; Q_{n-1}) \times H_{m-1}(X; Q_{n-1}) \rightarrow Q_{n-1}.$$

Namely, $\varphi(\langle \eta, g_0 \rangle) = \pm (t_1 - 1)$. Thus $\langle \eta, g_0 \rangle = \pm (t_1 - 1) + (t_n - 1)c$ with $c \in Q_n^0$. Therefore $\langle f_0, g_0 \rangle = \pm (t_n - 1) + (t_n - 1)^2 b$ where $b = (1 - t_1)^{-1}c$. This implies (7).

4.2. Proof of Theorem 2. We may assume that $\Delta_{u-1}(L) \neq 0$ and $l_1 = l_2 = \dots = l_{n-1} = 0$. Then the n -th component of K lifts to the maximal abelian covering of the exterior Y of L . The remaining part of the proof is analogous to the proof of Theorem 1. Note, however, the necessary changes. In Claim 1 for $i = 1, 2$

$$\text{rk}_{\Lambda_n} H_i(X; \Delta_n) = \text{rk}_{\Lambda_{n-1}} H_i(X; \Lambda_{n-1}) = u - 1; \text{rk}_{\Lambda_{n-1}} H_i(Y; \Lambda_{n-1}) = u - 2.$$

In the proof of (4.1.1) one should take into account that $\text{Tors}_{\Lambda_{n-1}} H_1(X; \Lambda_{n-1})$ injects into $\text{Tors}_{\Lambda_{n-1}} H_1(Y; \Lambda_{n-1})$ and thus the order of the first of these 2 modules divides the order of the second one.

REFERENCES

- [1] HILLMAN, J. A. *Alexander Ideals of Links*. Lecture Notes in Math. 895, Springer-Verlag, New York, 1981.
- [2] LEVINE, J. P. Links with Alexander Polynomial zero. *Indiana Univ. Math. J.* 36 (1987), 91-108.
- [3] MILNOR, J. W. Whitehead Torsion. *Bull. Amer. Math. Soc.* 72 (1966), 358-426.
- [4] TURAEV, V. G. Reidemeister Torsion in the Knot Theory. *Uspechi Matem. Nauk* 41 (1986), 97-147 (Russian); English translation: *Russian Math. Surveys* 41 (1986), 119-182.
- [5] TORRES, G. On the Alexander polynomial. *Annals of Math.* 57 (1953), 57-89.

(Reçu le 1^{er} juillet 1987)

V. G. Turaev

University of Geneva

Permanent address:

Leningrad Branch of Steklov Math. Inst.
Fontanka 27
Leningrad 191011 (USSR)