Computation of river suspended-sediment discharge: revisited

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Computation of River Suspended-Sediment Discharge—Revisited

Practically every nation excels in one field or another. Among the remarkably many records of outstanding achievement Switzerland has compiled in diverse areas, none is more striking than its accomplishments in scientific hydraulic engineering (the melding of the formalism of fluid mechanics and the art of engineering). The numerous contributions of Professor Th. Dracos to both the science and to the engineering application of hydraulics have earned him a well deserved place in the roster of great Swiss hydraulicians. Indeed, his career achievements add luster to the already brilliant history of sustained accomplishment by Swiss engineering researchers and practicioners. It is a pleasure and a privilege to participate in the recognition being accorded Professor Dracos, on the occasion of his sixtieth birthday, in this dedicated issue of «Schweizer Ingenieur und Architekt».

Sediment suspension and Discharge

One of the few bright spots in analytical river mechanics is the diffusion theory of sediment suspension, developed by Rouse and Ippen. (Short reviews of the history of their ana-

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lyses are given by Vanoni [1, p. 76], and Kennedy [2, pp. 1257–8].) In the form derived by Rouse [1, p. 76], the distribution of suspended-sediment concentration is given by

$$(1) \qquad \frac{C}{C_a} = \left[\frac{d-y}{y} \frac{a}{d-a} \right]^z$$

where

$$(2) \qquad z = \frac{w}{\kappa \beta_{u_*}}$$

These equations are so well known as not to require definition of their notation. The turbulent diffusion coefficient, $\varepsilon_s(y)$, incorporated into (1) is based on the logarithmic velocity distribution,

$$(3) \qquad \frac{u}{u_*} = \frac{1}{\kappa} \ln \zeta + \text{const.}$$

and is given by

$$(4) \qquad \frac{\varepsilon_s(y)}{u_* d} = \beta \kappa \zeta (1-\zeta)$$

in which $\zeta = y/d$ and the other notation is as universally recognized.

The suspension formulation given by (1) was widely recognized and extensively verified shortly after its publication, but remained more of an academic triumph than an engineering tool until the Swiss-educated engineer Hans Albert Einstein [3] combined it with a form of the logarithmic velocity distribution given by (3) to compute the suspended-sediment discharge per unit width of stream, q_s, from the integral

$$(5) \qquad q_s = \int_{y_0}^{d} Cu\, dy$$

in which y_0 is the lower limit of integration, specification of which proves to be a major problem.

Application of (5) to the calculation of suspended-sediment discharge was beset by three major stumbling blocks: estimation of the reference concentration C_a; the bed-level (y = 0) singularities in the equations for C and u, (1) and (3), respectively, and the attendent problem of specifying the lower limit of integration, y_0; and the effect of the moving sediment on κ. Values of q_s computed from (5) incorporating (1) and (3) proved to be especially sensitive to y_0, because of their bed-level singularities. This point was considered at length by Brooks [4]. Einstein [3] adopted the clear-water value of Karman's constant, κ = 0.4, and presented methods for estimating C_a and y_0. His formulation for computing suspended and bed-load discharges of streams, albeit based on several tenuous assumptions, proved to be a classic of sediment-transport theory and, for over three decades, one of the linchpins of river engineering.

Most of the problems that arise in the computation of q_s from (1) and (3) arise from the degenerate behavior of these equations as $y \to 0$. These problems are more than just nuisances, because of the high sediment concentrations that occur near the bed, and the correspondingly large contribution the near-bed region makes to q_s. The difficulties can be circumvented if, instead of the logarithmic distribution (1), one adopts the power-law velocity distribution,

$$(6) \qquad \frac{u}{U} = \frac{n+1}{n} (\zeta)^{1/n}$$

where, in addition to the notation introduced above, U is the mean velocity and n is the inverse of the exponent. The dependence of n on the flow and sediment-transport characteristics of the flow is considered below.

Proceeding exactly as Rouse did in the calculation of (1), except for utilization of the power-law velocity relation, (6), instead of the logarithmic, (3), yields

$$(7) \qquad \frac{\varepsilon_s}{u_* d} = \beta_n \frac{n^2}{n+1} \frac{u_*}{u} \zeta^{\frac{n-1}{n}} (1-\zeta)$$

and

$$(8) \qquad \ln C = \ln C_b - z_k \zeta^{1/n} \left[n - \ln(1-\zeta) - \sum_{i=1}^{\infty} \frac{\zeta^i}{1+n, i^2} \right]$$

where

$$(9) \qquad z_k = \frac{n+1}{n} \frac{\sqrt{8}\, w}{\beta_n K_s u_*}$$

and C_b is the reference concentration at the bed level, $\zeta = 0$; β_n is the ratio of turbulent sediment diffusion coefficient to the momentum-diffusion coefficient given by (7); and

$$(10) \quad K_s = n\sqrt{f}$$

in which f is the Darcy-Weisbach friction factor and K_s, as derived by Karim and Kennedy [5], is given by

$$(11) \quad K_s = 0.4\sqrt{8}\left[1 + 1.5\,(s-1)\,C_b\,\frac{u_b y_b}{UdS}\left(1 + \frac{u_*^2}{w^2}\right)\tan 24°\right]^{-1}$$

In (11),

$$(12) \quad u_b = U\left(\frac{y_b}{d}\right)^{1/n} = U(\zeta_b)^{1/n}$$

is the mean velocity of the particles in the moving bed layer, whose thickness is given by

$$(13) \quad y_b = D_{50}\,\frac{u_*}{u_{*c}}$$

where D_{50} is the median bed-particle size; u_{*c} is the Shields value of u_* for initiation of motion; C_b is the concentration at $y = y_b$ (i.e., $\zeta = \zeta_b$); and s is the specific gravity of the bed particles. The effect of the moving sediment on the velocity distribution is included in (10) and (11).

The series term in (8) usually is very small compared to the other terms, and therefore the concentration relation may be reduced to

$$(14) \quad \frac{C}{C_b} = \exp\{-z_k\zeta^{1/n}[n - \ln(1-\zeta)]\}$$

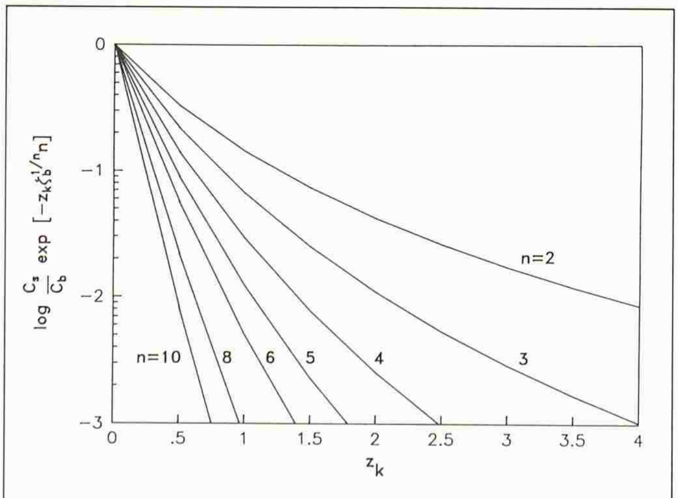
The sediment concentration at the bed, C_0 at $\zeta = 0$, is related by means of (14) to the sediment concentration C_b at $\zeta = \zeta_b$;

$$(15) \quad C_0 = C_b \exp\{-z_k\zeta_b^{1/n} n\}$$

in which ζ_b has been treated as negligibly small in the logarithmic term. The suspended-load discharge then may be computed as

$$(16) \quad \frac{q_s}{UdC_b} = \frac{n+1}{n}\exp\{z_k\zeta_b^{1/n} n\} \int_0^1 \zeta^{1/n}\exp\{-z_k\zeta^{1/n}[n - \ln(1-\zeta)]\}\,d\zeta$$

Figure 1. Results of Numerical Integration of (16)



Note that because of the orderly behavior of (6) and (14) as $\zeta \to 0$, the lower limit of the integral now is taken as zero.

Figure 1 shows (16) for the usually encountered ranges of n and z_k. The bed-level concentration is computed from the regression relation presented by Karim and Kennedy [5]:

$$(17)$$
$$\log C_b = -4.44 + 3.57\,V_1 + 1.23\,V_2 - 2.87\,V_3 + 1.58\,V_5V_6 - 1.40\,V_1V_4 + 0.41\,V_4V_6 - 0.85\,V_2V_3V_4 + 0.65\,V_2V_5V_6 - 5.43\,V_1V_3V_5 + 2.19\,V_3V_5V_6 - 1.27\,V_1V_5V_6$$

in which $V_1 = \log\{u/\sqrt{g(s-1)D_{50}}\}$; $V_2 = \log(S \cdot 10^3)$; $V_3 = \log(u^*/w)$; $V_4 = \log(w \cdot D_{50}/\nu)$; $V_5 = \log\{(u_* - u_{*c})/\sqrt{g(s-1)D_{50}}\}$; and $V_6 = \log(d/D_{50})$. (For other applications, in which the mean, total-load sediment-discharge concentration, $\bar{C}$, is known, the following predictor for C_b, developed from regression analysis of the data base used in the derivation of (17), may be used:

$$(18) \quad \log\left(\frac{C_b}{\bar{C}}\right) = 1.16 + 1.09\,V_1 - 0.74\,V_2 - 0.62\,V_3 - 0.45\,V_3V_6 + 0.15\,V_2V_3V_6)$$

Both (17) and (18) were found by Karim and Kennedy [5] to yield very good predictions of C_b.

Validation

Table 1 presents a comparison of experimental and computed values of q_s, based on data reported by Karim and Kennedy [5] for the four experiments of Vanoni and Brooks [6] in which concentration profiles were measured in flows over unconsolidated beds. Note that the mean concentrations, $\bar{c}$, in Table 1, are based on the measured total-load discharges, and therefore include also the bed-load discharges. In the computations of q_s reported in Table 1, C_b was determined from (17), and incorporated: n from (10) and (11) using the reported bed friction factors for f; u_b given by (12); and y_b given by (13). The calculated suspended-load mean concentrations, C_s, are, as expected, consistently lower than the measured total-load mean concentrations, C. The ratios of these two concentrations seem reasonable and consistent, especially in view of the fact that two of the flows (Nos. 1 and 3) had dune beds, and the other two flat beds.

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Table 1. Computed Suspended-Load Concentrations C_s, and Measured Total-Load Concentrations, $\bar{C}$, for Experiments of Vanoni and Brooks [6]

Run No.	d	U	S	D_{50}	$\frac{y_b}{D_{50}}$	C_b a)	K_s b)	n c)	z_k d)	C_s (comp'd)	$\bar{C}$ (meas'd)
	cm	cm/s		mm		g/l				g/l	g/l
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	8.7	37.5	0.0025	0.091	6.31	6.97	1.07	3.28	0.76	1.13	3.64
3	7.4	61.6	0.0020	0.091	4.92	48.2	0.77	5.31	1.33	1.93	4.60
5	7.4	68.9	0.0021	0.091	5.38	72.4	0.66	5.06	1.53	2.40	6.92
7	7.7	69.5	0.0026	0.148	4.89	107.2	0.80	5.35	2.14	2.51	3.61

a) Computed from (17)
b) Computed from (11)
c) Computed from (10), using K_s from col. 8
d) Computed from (9) using K_s from col. 8, n from col. 9, and $\beta_n = 1$

Summary

The algebraic simplicity of the power-law velocity profile, and the accuracy with which it predicts velocity and concentration distributions [5] and suspended-sediment discharge would appear to make it an attractive alternative to the river-flow formulations based on the logarithmic velocity distribution.

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Sicherheit gegen Schadstoff-einleitungen in Flüsse

Einleitung

Eine moderne Industriegesellschaft muss Industriebetriebe zulassen, in denen gefährliche Stoffe gelagert und verarbeitet werden. Auch wenn die Handhabung solcher Stoffe mit grösster Sorgfalt geschieht, lässt sich ein Unfall, durch den solche gefährliche Stoffe in die Vorfluter gelangen können, nicht mit absoluter Sicherheit vermeiden.

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Eine verantwortliche Firmenleitung wird daher mit den Aufsichtsbehörden zusammenarbeiten und gemeinsam Vorkehrungen treffen, um gegebenenfalls auftretende Schäden zu begrenzen. Hierfür müssen Szenarien erstellt werden, in denen der Unfall als eingetroffen angenommen wird. Aus der Analyse der Folgen eines solchen gedachten Unfalls können dann Bereiche identifiziert werden, in denen die Konsequenzen eines Schadens besonders hoch sind, und für solche Punkte können dann Vorsorgemassnahmen gegen einen allfälligen Unfall getroffen werden.

Es erscheint aber wenig sinnvoll, bei Vorsorgemassnahmen von einer gedachten allerungünstigsten Schadenssituation auszugehen, insbesondere wenn eine solche Betrachtung zu sehr hohen Investitionen führen würde. Statt dessen sollte die Wahrscheinlichkeit für das Auftreten möglicher Kausalketten bei der Betrachtung der Konsequenzen berücksichtigt werden. Dadurch können solche Situationen, deren (bedingte) Auftretenswahrscheinlichkeit bei gegebenem Unfall sehr klein ist, ausgeschlossen werden. Eine solche Betrachtung erfordert eine gemischt deterministisch-statistische Analyse der Kausalkette der Unfallfolgen. Eine solche Analyse soll an einem Beispiel durchgeführt werden.

Ausgangspunkt ist die Lage eines Betriebes an einem Punkt A eines Flusses, analog zu der in Bild 1 gezeigten Lage. In diesem Betrieb werden bis zu 500 kg eines toxischen Stoffes in wässriger Lösung gelagert und für einen wichtigen Herstellungsprozess des Betriebes derart verwendet, dass immer, wenn der Stoff aufgebraucht ist, eine neue Menge von 500 kg hergestellt und langsam abgearbeitet wird. Ein Unfall könnte also mit gleicher Wahrscheinlichkeit bei irgendeinem der Füllungszustände zwischen $M = 0$ und $M_{max} = 500$ kg auftreten, d. h. die Wahrscheinlichkeitsdichte für das Vorhandensein des Stoffes ist:

$$(1) \qquad f_M(M) = \frac{1}{M_{max}} \qquad 0 \leqq M \leqq M_{max}$$

Bei einem Betriebsunfall muss damit gerechnet werden, dass ein Teil oder die ganze gelagerte Menge in den Vorfluter gelangt. Dadurch können Anrainer geschädigt werden, solange die Konzentration des Stoffes eine vorgegebene Toxizitätsgrenze von $c_{zul} = 0{,}1$ g/m^3 überschreitet. Entlang der gefährdeten Strecke des Flusses sollen vorsorgliche Massnahmen getroffen werden, um die Anrainer vor den Unfallfolgen zu schützen. Im Sinne der in der Einleitung formulierten Sicherheitsbetrachtung soll festgestellt werden, wie gross die bedingte Wahrscheinlichkeit für das Auftreten einer über der Toxizitätsgrenze liegenden Konzentration ist.

Der Vorfluter sei ein Fluss mit Rechteckquerschnitt und kon-

Bild 1. Lageskizze und Definitionen