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Some Applications of Infinite Dimensional Analysis in Mathematical Physics

By Sergio Albeverio

Mathematisches Institut, Ruhr-Universität, D-44780 Bochum (Germany)
BiBoS Research Centre, SFB 237 (Essen-Bochum-Düsseldorf)
CERFIM (Locarno), Acc. Arch., USI (Mendrisio)

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Abstract. We give a survey of some applications of infinite dimensional analysis in mathematical physics, which have occurred in the last few years. In particular we cover applications of both types of infinite dimensional integrals, the "Wiener type" (probabilistic functional integrals) and "Feynman type" (oscillatory functional integrals). The applications involve in particular the theory of infinite dimensional processes (stochastic analysis, Dirichlet forms theory), as applied to systems of classical and quantum statistical mechanics, polymer models and quantum fields.
Gauge fields and a recent application of infinite dimensional oscillatory integrals to the proof of the Atiyah-Witten conjecture (topological invariants from Chern-Simons quantum fields) are also discussed.

Dedicated to Klaus Hepp and Walter Hunziker, for their 60th birthday

1 Introduction

We shall briefly discuss some recent developments in infinite dimensional analysis, as they arose mainly in connection with problems of physics.
The choice of topics is largely based on the author's personal experience, taste and interests and should of course not be considered as exhaustive in any sense.

Infinite dimensional analysis is here understood as analysis in its full extent, including, in particular, integration, differentiation, as well as the study of differential and integral equations on infinite dimensional (flat or curved) spaces.
This theory has its roots in many areas, here are some:

1) Calculus of variations (as developed ca 1880–1920 by, e. g., Volterra, Tonelli, Fréchet, Gâteaux)

2) The theory of integration on infinite dimensional spaces (as it developed from work by Borel, Lebesgue, via Daniell, Fréchet, and especially Wiener; this development was stronly influenced by foundational work in classical statistical mechanics, through Maxwell, Boltzmann, Gibbs, Einstein and others, with Wiener measure on the path space of Brownian motion emerging as a prototype of interesting measures in infinite dimensions).

3) Quantum theory (in its relations with operator theory, especially through J. v. Neumann's and H. Weyl's work, and with infinite dimensional oscillatory integrals, first introduced as a heuristic tool by Feynman, in its "space-time visualization approach" to quantum theory. The influence of these developments made itself felt in the theory of stochastic processes, particularly through the work of Kac).

4) Potential theory and its relations with the theory of stochastic processes (first developed in the 40's for finite dimensional stochastic processes).

Advances in infinite dimensional analysis have thus been closely connected with developments in the study of stochastic processes. In applications to quantum theory and the study of the heat equation the stochastic processes involved have finite dimensional state space. However there is a great need to develop also the theory of infinite dimensional stochastic processes, in connection with the natural problem of introducing random terms in such important physical equations as those of hydrodynamics and the theory of wave propagation—for the latter the problem is also closely connected with that of stochastic quantization of quantum fields. In fact $\Box\varphi + v'(\varphi) = \eta$, with η a space-time white noise, $\Box$ the d'Alembert (wave-) operator and v a non quadratic function (giving the interaction), can be looked upon both as a non-linear stochastic wave equation as well as the stochastic quantization equation for a Euclidean quantum field. $t \to \varphi(t, \cdot)$ can be seen as an infinite dimensional process (or, alternatively, $\varphi(t, x)$ as a generalized random field). For quantum fields, the t-variable is the "auxiliary time", the other variable x being taken in Euclidean space-time.

An important tool for the study of infinite dimensional analysis as connected with measure and integration theory over infinite dimensional spaces is the theory of Dirichlet forms and associated processes, and therefore we shall start our exposition by recalling briefly the basic elements of that theory.

2 A general setting for infinite dimensional stochastic analysis

We shall discuss briefly the approach to infinite dimensional analysis with associated measure theory given by the theory of Dirichlet forms. This seems to be the most natural and fruitful approach permitting to extend to the case of infinite dimensional state spaces the fruitful relations between classical potential theory and the theory of associated stochastic processes, see e. g. [Alb 1–6]. Let us start with the "analysis part".

2.1 Analysis

Let (E, B, μ) be a measure space (which is going to be the "state space" on which our process will run). Let $L^2(\mu)$ denote the corresponding L^2-space. Let $\mathcal{E}$ be a symmetric positive bilinear closed form, densely defined, with domain $D(\mathcal{E})$, dense in $L^2(\mu) \times L^2(\mu)$ (for basic functional analytic concepts of this type see, e. g., [MR] and references therein).

By the representation theorem to $\mathcal{E}$ there corresponds uniquely a self-adjoint, positive operator H acting on a dense domain $D(H) \subset L^2(\mu)$, the correspondence being given by $\mathcal{E}(u, v) = (H^{1/2}u, H^{1/2}v)_{L^2(\mu)}\, \forall u, v \in D(\mathcal{E}) = D(H^{1/2})$.

In turn, to H there corresponds uniquely a (self-adjoint strongly continuous contraction) semigroup $T_t \equiv e^{-tH}, t \geq 0$, generated by H, in $L^2(\mu)$.

$\mathcal{E}$ with the additional contraction property $\mathcal{E}(u^\#, u^\#) \leq \mathcal{E}(u, u)$, with $u^\# = (u \vee o) \wedge 1$, $\forall u \in D(\mathcal{E})$, is called a *Dirichlet form* (the name being justified by the fact that a form like $\frac{1}{2} \int \nabla u.\ \nabla u \rho(x)\, dx$, with ρ smooth with support say a compact set occurs in the classical Dirichlet principle).

What are the semigroups which correspond to Dirichlet forms? It is not difficult to see that these are the *Markov semigroups*, i. e. the semigroups (in above sense) which are in addition s. t. $0 \leq u \leq 1 \longrightarrow 0 \leq T_t u \leq 1 \forall u \in L^2(\mu)$. One has a well known bijective correspondence between Dirichlet forms and Markov semigroups (given, e. g., in terms of H, since H enters both $\mathcal{E}$ and P_t). It is also easy to see that any Markov semigroup (on $L^2(\mu)$) automatically is a strongly continuous contraction semigroup in all $L^p(\mu)$, $\forall 1 \leq p < \infty$, see e. g. [FOT], [MR].

2.2 Probability

We want to associate to $\mathcal{E} \leftrightarrow H \leftrightarrow T_t = e^{-tH}$ a "nice" stochastic (Markov) process $(X_t)_{t \geq 0}$ with state space E. Because of our desire to get a "nice process", we assume that (E, μ) is a Hausdorff topological space with a σ- finite "reference measure".

To get the associated "nice stochastic Markov process" X it turns out that the basic condition on $\mathcal{E}$ is that $\mathcal{E}$ be "quasi regular" (we shall then also say for simplicity that $\mathcal{E}$ resp. H are "good").

To $\mathcal{E}$ is then associated a Markov process X in a proper way i. e. such that $(T_t u)(x) = E^x(u(x_t))$ (for any $u \in C_b \cap L^2(\mu)$), $x \in E$, and s. t. the right hand side is a "quasi- Feller modification" of the left hand side, see [FOT], [MR]. X, started with distribution μ at "time" $t = 0$, is then a symmetric (i. e. time reversible) process, a conservative one if T_t is conservative, i. e., s. t. $T_t 1 = 1$. μ is then the invariant measure ("equilibrium distribution") for X. X is a "nice process" in a technical, probabilistic-potential theoretic sense, e. g., it is strong Markov and has "càdlàg paths" (running in the state space E).

If the state space E has the structure of a manifold (finite or infinite dimensional) then one can show that $\mathcal{E}$ (and consequently H) splits additively into two parts $H = H_{\text{loc}} + H_{\text{nonloc}}$, the local part H_{loc} being of the form of a 2nd order differential operator, to which there is associated a diffusion process (i. e. a "nice process" with continuous paths), and the non local part H_{nonloc}, being of the form of an integral operator, to which there is properly associated a jump process. For these results see [AlbMR 1–5], [AlbM], [AlbRZha 1, 2] and the books [FOT], [Sil], [BouH], [MR].

Remark "Symmetry" can be relaxed, see [AlbMR1], [MR], but is important in many applications in physics, especially in quantum theory; work in this sense goes back to Schrödinger and Kolmogorov, for newer developments see e. g. [Ne], [Fö 1, 2], [Na], on "stochastic mechanics", and [Zam] [CrZ], on "Euclidean quantum mechanics" (and references therein).

We shall now concentrate on the case on symmetric diffusions, which arise normally in connection with quantum theory.

2.3 Diffusions

In this case we shall assume a little more on the state space E, in fact we assume E to be a Souslin space (i. e. the continuous image of a polish space), i. e., e. g., a separable Banach space, or a distributional space like $\mathcal{S}'(\mathbb{R}^d)$. E can also be a nonlinear infinite dimensional space like the loop space $C(S^1, M)$. In applications to classical statistical mechanics E has yet a different form, a suitable subspace of the space of maps from a lattice, say $\mathbb{Z}^d$, into $\mathbb{R}^n$ or a manifold M. Let us assume there is a separable Hilbert space $\mathcal{H}$ closely and continuously contained in E ($\mathcal{H}$ should be thought as a "tangent space" to E). Let μ be a probability measure on E. Let FC_b^∞ be the subspace, dense in $L^2(\mu)$, of smooth cylinder functions on E.

Let us consider the "pre-Dirichlet form" $\mathring{\mathcal{E}}_\mu(u,v) = \frac{1}{2}\int \langle \nabla u, \nabla v\rangle \, d\mu$, $u, v \in FC_b^\infty$, ∇ being the natural gradient defined in FC_b^∞, $\langle , \rangle$ being the scalar product in $\mathcal{H}$. $\mathring{\mathcal{E}}_\mu$ is only a pre-form, inasmuch as it is not yet closed. In fact, closability of $\overset{\circ}{\mathcal{E}}_\mu$ requires (a very weak form of) regularity of μ.

If μ is s. t. $\overset{\circ}{\mathcal{E}}_\mu$ is closable (and sufficient conditions for this are known—which are even necessary if all associated "components forms" should also be closeable, see [AlbR2]) then we can take the closure $\mathcal{E}_\mu$ of $\overset{\circ}{\mathcal{E}}_\mu$. $\mathcal{E}_\mu$ is called the classical Dirichlet form given by μ. It is a quasiregular local form, and to it there is a properly associated diffusion process. X is the sum of a martingale part and a zero energy part ("Fukushima composition", see [FOT] for the finite dimensional case, [AlbR 1, 2, 3] for the infinite dimensional case).

Only in the case where μ satisfies the additional condition of possessing a "logarithmic derivative" we can write more explicitely the generator H_μ associated with $\mathcal{E}_\mu$ and a stochastic differential equation for X. For μ to have a logarithmic derivative β^μ means that there is a measurable vector field $\beta^\mu \in L^2(\mu) \otimes E$, s. t. the integration by parts formula

$$\int \frac{\partial u}{\partial k}\, d\mu = -\int u\beta_k^\mu \, d\mu$$

holds, $\forall u \in FC_b^\infty$, where $\frac{\partial}{\partial k}$ means directional derivative with respect to an arbitrary element k in an orthonormal basis $\{k\}$ in $\mathcal{H}$, and $\beta_k^\mu = \langle k, \beta^\mu \rangle$.

Remark In the finite dimensional case $E = \mathcal{H} = I\!R^n$ the existence of β^μ is equivalent with $\mu(dx) = \rho(x)\, dx$ with $\rho \in H^{1,1}$. In this case $\beta_k^\mu(x) = \frac{\partial}{\partial x_k} \ln \rho(x)$ with $x = (x_k)_{k=1,\ldots,d}$ (see [MR]). Logarithmic derivatives in infinite dimensions have been discussed in the 70's by Daletskii, Fomin and Albeverio-Høegh-Krohn [AlbHK2], [AlbHK6], see also the references in [ABR].

In the case where μ is absolutely continuous with respect to a Gaussian measure, integration by parts is well known in quantum field theory, by work of Dimock and Glimm, Fröhlich and Albeverio and Høegh-Krohn, see e. g. [GliJ], [Frö], [AlbHK2], [AlbHK6], [Sim]. In stochastic analysis integration by parts has been made into an essential tool and used for obtaining striking results (also in finite dimensional hypoelliptic problems) by Malliavin, see e. g. [Mall 1–2].

In the case, where β^μ exists, the Dirichlet operator H_μ associated uniquely to the classical Dirichlet form $\mathcal{E}_\mu$ takes the form

$$\begin{aligned} H_\mu &= -\frac{1}{2}\Delta - \frac{1}{2}\langle \beta^\mu, \nabla \rangle \\ &= \frac{1}{2}\sum_k \left(-\frac{\partial^2}{\partial k^2} - \beta_k^\mu(\cdot)\frac{\partial}{\partial k}\right), \end{aligned}$$

on FC_b^∞ (Δ, ∇ can be seen as natural Laplacian resp. gradient, in the sense of Gross, associated with $\mathcal{H} \subset E$).

The diffusion process X properly associated with $\mathcal{E}_\mu$ satisfies the infinite dimensional stochastic differential equation

$$dx_t = \beta^\mu(x_t)\, dt + dw_t \tag{1}$$

w_t being a Brownian motion on E with covariance given by the scalar product in $\mathcal{H}$. β^μ appears here as a measurable (in general not at all continuous!) drift coefficient.

Remark This formula is written for E being linear, similar formulae hold in the case of nonlinear state spaces E, e. g. in the case of loop spaces, see [DrR], [AlbLR].

The proof of the above results has a long history, begun by work of Albeverio and Høegh-Krohn in 75–77 [AlbHK2], [AlbHK6], continued by essential contributions by S. Kusuoka, in the generality described above the results are due to recent work by Albeverio, Röckner [AlbR 1–3] and Schmuland [Schm] (for more detailed references see e. g. [AlbKR5]).

X and $T_t^{(\mu)} \equiv e^{-tH_\mu}$ give a "stochastic dynamics" (or "Glauber dynamics" or "stochastic quantisation dynamics") associated with μ (or β^μ), in the sense that (1) can be looked upon as a stochastical differential equation whose equilibrium measure (stationary measure) is precisely μ (for the interplay between stochastic dynamics and equilibrium measure see e. g. [AlbKR3], [AlbKT] for statistical mechanics, [Ne], [AlbHK 1, 2, 6] for stochastic mechanics, and, [JLM], [AKR] for quantum field theory).

3 A simple class of examples

Let us consider the case of a lattice $\mathbb{Z}^d$ with a spin variable $x = (x_k)$ attached to each point $k \in \mathbb{Z}^d$ (the spin variable takes values in M, where M can be a finite set, like in the case of the Ising model, or $\mathbb{R}^\nu$ or a compact manifold).

Let μ be a Gibbs measure, describing the distribution of the spin variables. Here E is a suitable negative index Sobolev space (manifold) of mappings from $\mathbb{Z}^d$ into M and $\mathcal{H}$ is a suitable tangent space (e. g. $\mathcal{H} = \ell^2(\mathbb{Z}^d, \mathbb{R}^\nu)$ for $M = \mathbb{R}^\nu$). μ is heuristically of the form $\mu =$"$Z^{-1}e^{-W(x)} \prod_{k\in\mathbb{Z}^d} dx_k$", $x \in E$, with Z a normalization constant and W the sum of 1, 2,..., n-body "potentials" over $\mathbb{Z}^d$ (satisfying some conditions of the "superstable regular" type; e. g., the case of $P(\varphi)_d$-lattice models is included). Here β^μ exists and $\beta_k^\mu = \beta_k$ is independent of μ (and equal to $\beta_k^\mu(x) = -\frac{\partial}{\partial x_k} W(x)$).

By the general theory then there exists a "stochastic dynamics" diffusion $X(t) = (X_k(t))_{k\in\mathbb{Z}^d}$ with equilibrium measure μ.

For these results see [Alb 1–5], [AlbAAK], [AlbKR 1–5], AlbKT]. Natural questions which arise are: does H_μ have a ground state? Does there exist a mass gap? These are relevant questions since the asymptotics of e^{-tH_μ} for $t \to \infty$, and hence of the process X_t for $t \to \infty$, and thus the question of the "approach to the equilibrium measure", are intimately connected with the spectral properties of H_μ. The discussion of these questions is a particular case of an ergodic theory of Dirichlet forms, to which we come to speak in Section 5. Before however we need to discuss briefly uniqueness properties for Dirichlet forms.

4 Uniqueness properties for Dirichlet forms

There are two basic concepts of uniqueness: Markov uniqueness and essential self-adjointness. One has Markov uniqueness if e^{-tH_μ} is the only Markov semigroup whose generator coincides with H_μ on FC_b^∞. Sufficient conditions for this to happen are known and applied in many examples, see [MR], [AlbR5], [AlbRZha2]. Essential-self-adjointness of H_μ on FC_b^∞ is stronger than Markov uniqueness. In [AlbKR1] a useful sufficient condition for the former in terms of an "approximation criterion" (with the "bad" β^μ being approximated by good "β^{μ_n}") is given. This result has applications to the case of the lattice spin models described in section 3 (including e. g. the case of $P(\varphi)_d$-lattice models).

5 Ergodic theory for Dirichlet forms

We state a particular result of a general theory contained in [AlbKR3], (to which we refer for more details) (the roots of such results can be traced back to [AlbHK2]): if Markov uniqueness holds for a classical Dirichlet form $\mathcal{E}_\mu$ then the following holds: X time ergodic $\longleftrightarrow$ $\mathcal{E}_\mu$ irreducible $\longleftrightarrow$ T_t^μ irreducible $\longleftrightarrow$ $\{0\}$ unique ground state eigenvalues of H_μ $\longleftrightarrow$ μ is extremal (in the set of probability measures satisfying the integration by parts formula with a given drift coefficient; in the case where μ is a Gibbs state, as in Section 3, this corresponds to μ being an extreme Gibbs state). One has then in particular the ergodic theorem in the form:

$$T_t^\mu u(x) \longrightarrow \int u\, d\mu \quad , t \rightarrow \infty, \tag{2}$$

in $L^2(\mu)$ and $\mathcal{E}$-quasi everywhere.

Moreover in this case, μ is $\mathcal{H}$-ergodic (in the sense of ergodicity with respect to translations by vectors in $\mathcal{H} \subset E$). In case μ is "strictly positive" one has also the converse, namely that μ-$\mathcal{H}$-ergodic implies T_t^μ irreducible.

Remark These results extend previous results for finite spin-systems by Spitzer, Dobrushin ('71), Holley, Stroock, Deuschel, Zegarliński,...; they also have applications to quantum field theory, extending results by Fröhlich [Frö] and Albeverio/Høegh-Krohn [AlbHK2], see below.

The above results imply a characterization of Gibbs states by Glauber dynamics. For the case of the lattice models of Section 3 one has essential self-adjointness under some conditions on the interaction [AlbKR 1–3]. In this case for μ extremal one has that T_t^{μ} is the extension to $L^2(\mu)$ of a Feller semigroup T_t on $C_b(E)$, *independent* of μ. In this case the convergence in the ergodic theorem (2) holds, in some cases, even in sup-norm, which permits then to determine μ from the Glauber dynamics. One can also show for these models that μ is the *unique* invariant reversible measure for the Glauber dynamics [KoRZ].

The question of when one has an exponential rate of convergence in the statement (2) of the ergodic theorem is answered by the proof of *logarithmic Sobolev inequalities* for the Dirichlet form $\mathcal{E}_\mu$ (this is only a sufficient condition, hypercontractivity would also do). We recall that a probability measure μ, with associated classical Dirichlet form $\mathcal{E}_\mu$, is said to satisfy a *logarithmic-Sobolev inequality* with coefficient $\frac{1}{\lambda}$ ($\lambda > 0$) if

$$\int |u|^2 \log |u| \, d\mu \leq \frac{1}{\lambda}\mathcal{E}_\mu(u,u) + \|u\|_2^2 \log \|u\|_2$$

(where $\| \ \|_2$ means the $L^2(\mu)$-norm). This kind of inequalities were studied systematically by L. Gross (for the history of the subject see e. g. Davies, Gross and Simon's article in [AlbFeHL 1, 2]). In the case where H_μ is essentially self-adjoint (on FC_b^∞) and μ is uniformly log-concave, it was shown in [AlbKR2] that μ satisfies a $\frac{1}{\lambda}$-Log-Sobolev- inequality (with parameter λ related to the concavity property of μ). A corollary to the $\frac{1}{\lambda}$-Log-Sobolev inequality is the hypercontractivity of T_t^{μ} implying that H_μ has mass-gap at least $\frac{\lambda}{4}$ i. e. $H_\mu \geq \frac{\lambda}{4}$ on the orthogonal complement in $L^2(\mu)$ to the subspace spanned by the constants. In this case in (2) we have exponential rate of convergence.

Remark The log-Sobolev inequality has been proven for extremal Gibbs states in classical lattice spin models (under assumptions on the potential W which permit the inclusion of the the case of weak coupling $P(\varphi)_d$-lattice models), see the work by Stroock and Zegarlinski for bounded spins mentioned in [AlbKR3], and [AlbKR 2–4], [LiFY] for continuous spins.

6 Other examples

6.1 Quantum statistical mechanical lattice models

In this case one has a quantum Hamiltonian H_{QM} in any bounded subset Λ of $\mathbb{Z}^d$ given by

$$H_{QM}^{(\Lambda)} = -\frac{1}{2} \sum_{k \in \mathbb{Z}^d} \Delta_{x_k} + W(x)$$

acting in $\bigotimes_{k\in\Lambda} L^2_{(k)}(\mathbb{R}^\nu)$. W is of the same type as in Section 3. The Gibbs state ω^Λ in Λ is given by

$$\omega^\Lambda(\cdot) \equiv \operatorname{Tr}\left(e^{-\frac{1}{kT}H_{QM}^{(\Lambda)}} \cdot\right) / \operatorname{Tr}\left(e^{-\frac{1}{kT}H_{QM}^{(\Lambda)}}\right)$$

with k being Boltzmann's constants, T the temperature. By work in [AlbHK3] (related to previous work by Høegh-Krohn in the case of relativistic fields, of great importance in cosmology, see [AlbFHL 1, 2]) one can express $\omega^\Lambda(\cdot)$ by an expectation with respect to the "restriction to Λ", μ^Λ, of a Gibbs measure μ on a space of loops variables over $\mathbb{Z}^d$: heuristically $d\mu(x) = Z^{-1}e^{-S(x)}\,dx$, with $S(x) = \int_{S^1} |\dot{x}(\tau)|^2\,d\tau + \int_{S^1} v(x(\tau))\,d\tau$, $x = (x_k)_{k\in\mathbb{Z}^d}$, $\tau \in S^1$

$$\beta_k^\mu(x) = \beta_k(x) = \frac{\partial^2}{\partial\tau^2}x_k(\tau) + \tilde{\beta}_k(x(\tau))$$

with $\tilde{\beta}_k$ equal to the drift of the corresponding classical spin-system (described in 2.4).

In [AlbAAK], [AlbKRT] the classical Dirichlet form associated with μ is constructed, as well as the corresponding "stochastic dynamics for interacting loops". For results on exponential ergodicity, essential self-adjointness, logarithmic Sobolev inequalities associated with this system see [AlbKT], [AlbKRT], [DaPZ], [LiPY], [BarK], [GloK].

In particular in [AlbKRT] uniqueness results for Gibbs states have been given. See also [AlbKK] for results on absence of abnormal fluctuations.

6.2 Diffusions on loop spaces

To construct diffusions on the loop space $C(S^1; M)$, M a manifold, one can consider a natural measure (Wiener type measure) μ on $C(S^1; M)$. One constructs then a classical Dirichlet form given by μ and an associated diffusion, see [DrR], [AlbLR]. There are only partial results on essential self-adjointness, Log-Sobolev inequalities and on mass gap, see e. g. [Ac], [Ai], [Fa], [Gr 1, 2]. The study of (stochastic) analysis on loop spaces is interesting as a prototype of infinite dimensional geometry, with important relations to the representation theory of infinite dimensional "current groups", see [AlbHK1], [AHKMTT] and infinite dimensional Hodge-de Rham theory (see [Alb 1, 3, 5, 6]: the kernel of the analogue of H_μ on k-forms should be related to cohomology groups on loop spaces).

Remark

a) For a supersymmetric Dirichlet forms setting see [AlbKo] (and references therein).

b) For further work on loop spaces, index theory and infinite dimensional geometry see also [Mall 1, 2], [Lé], [LéR], [JoL].

6.3 Stochastic quantization of polymer measures

Edwards model of polymers in $\mathbb{R}^d$ is described by a heuristic measure of the form

$$\mu_\lambda(d\omega) = \text{“}Z^{-1}e^{-\lambda\int_0^1\int_0^1 \delta(\omega_s-\omega_t)\,ds\,dt}\mu_0(d\omega)\text{”},$$

with μ_0 standard Wiener measure on

$$C([0,1];\mathbb{R}^d),\ \ \lambda \in \mathbb{R}.$$

For $d=2$ and 3 there are constructions of μ_λ (for $\lambda \geq \lambda_0 > -\infty$ for $d=2$ and for $\lambda \geq 0$ for $d=3$): see [AlbFeHKL], [AlbHRZ] and [AlbRZ] and references therein. In [AlbHRZ] resp. [AlbRZ] the classical Dirichlet form associated with μ_λ has been constructed (despite the fact that β_μ here does not exist as a measureable function!) and shown to be irreducible for $d=2$, so that the associated diffusion X is ergodic in this case.

Remark Also finite dimensional cases where β^μ does not exist as a measureable function have been discussed, with applications to very singular interactions in quantum mechanics, see e. g. [AGHKH], [AlbBlM] and references therein.

6.4 Quantum fields and stochastic partial differential equations

6.4.1 Hamiltonian

If we take $E=\mathcal{S}(\mathbb{R}), H=L^2(\mathbb{R}), \mu$ the probability measure which gives the ground state for a "time zero" $v(\varphi)_2$–model (v standing for a polynomial, an exponential or a trigonometric function), a Dirichlet operator associated with μ describes the Hamiltonian of this $v(\varphi)_2$–model (thanks to the global Markov property, proven in [AlbHK1], [AlbHZ] see also [AZe],[AGieRu] and references therein) (in a finite space volume, this is identified with the corresponding classical Dirichlet operator, in the infinite volume case the identification is open except for $v=0$; see [AlbR5]). The corresponding process as infinite dimensional process with state space $\mathcal{S}'(\mathbb{R})$ satisfies a stochastic differential equation of the form (1), for a certain β^μ (which depends on μ): see [AlbHK 1–2], [AlbR 1–3]. The distribution of X as a space–time random field is the Euclidean (Markov) $v(\varphi)_2$–field.

6.4.2 Stochastic quantization

If we take $E=\mathcal{S}(\mathbb{R}^2), H=L^2(\mathbb{R}^2), \mu$ the Euclidean $v(\varphi)_2$–measure, to the classical Dirichlet form $\mathcal{E}_\mu$ given by μ there is properly associated a diffusion process X satisfying the stochastic partial differential equation (1) with drift.

$$\beta^\mu(X)=(\Delta_x-m^2)\varphi(X(x)) - {:}v'{:}(X(x)), x\in\mathbb{R}^2.$$

In this case dw_t is a space–time white noise. For these results see [AlbR 1–3], [AlbHaRu 1–3] (for stochastic pseudo–differential equations yielding the same invariant measure see [JLM]).

Similar methods and results can be used for handling the corresponding stochastic partial differential equations of hyperbolic type (stochastic wave equations) mentioned in Sect. 1., see [AlbHaRu 1–3]. Markov uniqueness is proven for these models only in a bounded volume (a result by Röckner and Zhang, see [Alb1]). Also log–Sobolev inequalities are open. However recently a strong result on ergodicity (extending previous work of [Frö]) has been proven [AKR]: μ–extremal (in the class of Guerra–Rosen–Simon states) implies T_t^{μ} irreducible and equivalently ergodicity of the stochastic quantization process.

Remark For higher dimensional space time there are only partial results: e. g. stochastic pseudodifferential equations with other types of Euclidean white noise have been discussed, and yield models which satisfies all axioms of indefinite metric local relativistic quantum field theory [AlbHKI],[AlbGWu1,2]. The stochastic quantization of $(\varphi^4)_3$ is still open, despite the fact that for the case of the "zero-component" $(\varphi^4)_3$-model (polymer measure) the stochastic dynamics has recently been constructed, see Section 6.

7 Some remarks on infinite dimensional differential geometry and related quantum fields

Except for the case of paths and loop spaces "natural measures" do not live on smooth geometrical objects, as we know from the case of Euclidean field measures for space–time dimension $d > 1$and for the case of Euclidean gauge field measures i.e. measures on space of connections, see e.g. [AlbHKHK1,2],[AlbKu],[Frö2]. This comes from the coexistence of the covariance/invariance properties and markovian properties required by the physics inbuilt in the model (expressions of the basic causality and relativistic invariance). These gives the well known difficulties (divergences) arising in the construction of higher dimensional quantum fields, Yang–Mills fields, as well as in models of (relativistic) strings [AlbJPS]. The basic problem can be seen as that of giving a meaning to the underlying functional integrals (of the Wiener resp. Feynman type, see [Alb5]). Sometimes this is however possible, by a suitable exploitation of the underlying geometry. This is the case of the Yang–Mills fields over a 2– dimensional manifold M, heuristically given by a probability measure μ of the type

$$d\mu(A) = Z^{-1}e^{-S_{YM}(A)}dA,$$

with A a connection 1–form ("gauge field") (of a principal fibre bundle over M, with compact structure Lie groups), S_{YM} being the Yang–Mills action $S_{YM}(A) = \frac{1}{2}\int_M F_A^* \wedge F_A$, with F_A the 2–form to A.

In this case expectation of products of (stochastic) holonomy operators $m_C(A)$ associated with loops C can be rigorously defined and computed ($m_C(A) = exp[\int_C A]$ in the abelian case), using a stochastic parallel transport (see [AlbHKHK2], [FrK], [GKS], [Se]). King and Sengupta [KiS] recently proven the concentration of μ on flat connections in the "semiclassical limit", where S_{YM} is replaced by $\frac{1}{h}S_{YM}$ and $h \to 0$.
For probabilistic results on the $(Higgs)_2$ model, using stochastic holonomy along Brownian

loops, see [AKu]. For (partial) results on a related 4–space–time dimensional model see [AlbT].

8 A remark on infinite dimensional oscillatory integrals and topological quantum fields

There is a rigorous mathematical theory of Feynman path integrals which works very well for non relativistic quantum theory, covering e.g.

1) representation of solutions of the Schrödinger equation, Green's functions, scattering and spectral quantities in terms of oscillatory integrals, see e.g. [AlbHK4], [ABrz 1, 2], [AlbBBr], [ABrB], [Alb 1–6]

2) detailed approach to the semiclassical limit (in forms of asymptotic expansion in power of Planck's constant, with strict control on remainders), see e.g. [AlbHK5], in particular trace formula for Schrödinger operators have been proven (which extend those of Selberg's type to the case of Schrödinger operators), see [ABlHK], [AlbBBr]

3) representation of "mean values" of products of holonomy operators ("Wilson loops") with respect to the Chern–Simons functional

$$I_{C_1,\dots,C_n} \equiv \int \prod_{l=1}^{n} m_{C_l}(A) d\mu(A),$$

with

$$d\mu(A) = Z^{-1} e^{iS_{CS}(A)} dA$$

$$S_{CS}(A) = \frac{k}{4\pi} \int_M A \wedge dA + \frac{2}{3} A \wedge A \wedge A,$$

with A a connection on a principal fibre bundle over a 3– dimensional manifold M; C_l are loops in $M, k \in \mathbb{Z}$ E.g. for $M = \mathbb{R}^3$ a rigorous representation of $I_{C_1,\dots,C_n}$ as infinite dimensional oscillatory integrals has been found [AlbS], [LS], [AlbSe] and a proof of the Atiyah–Witten conjecture has been fully provided in the abelian case ([AlbS], [LS]) resp. reduced to the computation of $I_{C_1,\dots,C_n}$ using stochastic parallel transport, following [FrK], in the non abelian case [AlbSe]. In the abelian case, replacement of $\int_C A$ by the corresponding De Rham currents and of $e^{i\int_C A}$ by $:e^{i\int_C A}:$ yields

$$I_{\alpha_1 C_1,\dots,\alpha_n C_n} = e^{-i\frac{\pi}{k}\sum\limits_{l\neq l'} \alpha_l \alpha_{l'} LK(C_l, C_{l'})}, \alpha_i \in \mathbb{Z}$$

LK being the Gaussian linking number.

9 Conclusion

We have presented some methods of infinite dimensional analysis intensively developed in the last 25 years concentrating particularly on those having applications in statistical mechanics and quantum theory. Related methods have been developed for other areas, see e.g. [Alb4], [Mall1,2], [EM], [DaPZ2], [AlbKol]. In the area of infinite dimensional analyis combined with geometry we have presently only the beginnings of a theory, whose full development constitutes a great challenge for the next century. Most probably mathematical physics will continue to provide a great stimulus for these exciting developments.

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Note added

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