Symmetries of impulsive gravitational waves

Autor(en): **Aichelburg, Peter C. / Balasin, Herbert**

Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **69 (1996)**

Heft 3

PDF erstellt am: **21.09.2024**

Persistenter Link: https://doi.org/10.5169/seals-116951

Helv Phys Acta
Vol. 69 (1996)

0018-0238/96/030337-04$1.50+0.20/0

Symmetries of Impulsive Gravitational Waves

By Peter C. AICHELBURG * §

Institut für Theoretische Physik, Universität Wien, Boltzmanngasse 5,
A - 1090 Wien, AUSTRIA

and Herbert BALASIN † ‡

Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstraße 8–10,
A - 1040 Wien, AUSTRIA

Abstract. We give a classification of pp-waves, where the curvature is concentrated on a null plane, in terms of their Killing symmetries. Our approach is based on the analysis of the group of normal-form-preserving diffeomorphisms. In order to obtain a complete classification it is necessary to extend the symmetry-concept to allow for discontinuous Killing fields.

PACS numbers: 9760L, 0250

1 Motivation

Consider the ultrarelativistic limit of the Schwarzschild geometry, the so-called AS–geometry [1], which describes the gravitational field of a massless particle.

$$ds^2 = -dudv + dx^i dx^i + \delta(u) \log\rho \, du^2 \qquad \rho^2 = x^i x^i \qquad i = 1, 2 \tag{1.1}$$

*e-mail: pcaich @ pap.univie.ac.at
§supported in part by the FUNDACION FEDERICO
†e-mail: hbalasin @ email.tuwien.ac.at
‡supported by the APART-program of the Austrian Academy of Sciences

This geometry has the standard form of a pp-wave as described in [3], namely

$$ds^2 = -dudv + dx^i dx^i + f(u, x^i)du^2, \qquad (1.2)$$

where f denotes the wave-profile. Following the classification given in [3], one would expect that (1.1) has only a two-dimensional Killing-algebra, since its profile is of the form

$$f(u, x^i) = F(u) \log \rho. \qquad (1.3)$$

However, a direct calculation [2] shows that the Killing–algebra of (1.1) is actually four-dimensional, as might have been expected from the fact that the AS-geometry is the limit of Schwarzschild. The reason for this discrepancy is that the $F(u)$ appearing in (1.3) is implicitly assumed to be a function and not a (singular) distribution. Therefore one is lead to an extension of the classification given in [3] allowing for distributional profiles. despite the fact that the metric is distributional the Lie derivative of the metric is well-defined for arbitrary C^∞ vectorfields.

2 The method

The difficulty for classifying pp-waves in terms of their symmetries comes from the fact that the metric (1.2) characterizes a pp-wave only up to "normal form preserving" (nfp) diffeomorphisms given by

$$\begin{aligned} \tilde{u} &= au + b, \\ \tilde{v} &= \frac{1}{a}\left(v + 2(\Omega x \cdot d'(u) + n(u))\right), \\ \tilde{x}^i &= \Omega^i{}_j x^j + d^i(u), \end{aligned} \qquad (2.1)$$

which act on the wave profile according to

$$gf(u, x) = a^2 f(\Omega x + d(u)) + d'^2(u) - 2(n'(u) + \Omega x \cdot d''(u)). \qquad (2.2)$$

These nfp diffeomorphisms form an infinite-dimensional group G, whose general element is conveniently parametrized by $g = (a, b, \Omega^i{}_j; d^i(u), n(u))$. Isometries of a given profile are contained in G as finite-dimensional subgroups (which leave the profile invariant). The change of frame does not only affect the metric (i. e. the profile) but also the Killing vectors. This is reflected by the adjoint action of the G on its Lie algebra, whose general element is pararmetrized by $X = (\alpha, \beta, \omega^i{}_j; D^i(u), N(u))$ in accordance with the parametrization of G. We may use the above freedom to choose a simple representative from every orbit of the adjoint action and impose it as Killing-vector thereby determining the profile. Higher symmetries are obtained by the combination of orbits. For a detailed discussion we refer the reader to [4].

3 Impulsive pp-waves

Impulsive waves are characterized by a profile of the form

$$f(u, x^i) = \delta(u)\tilde{f}(x^i). \tag{3.1}$$

Restriction of the nfp group G to those elements that preserve (3.1) leaves us with a finite dimensional subgroup $\tilde{G}$ of restricted nfp diffeomorphisms (rnfp) with general element

$$g = (a, 0, \Omega^i{}_j; d_0^i + ud_1^i, n_0 + u\frac{1}{2}d_1^2). \tag{3.2}$$

The action of $\tilde{G}$ on $\tilde{f}$

$$g\tilde{f}(x^i) = a\tilde{f}(\Omega^i{}_j x^j + d_0^i) \tag{3.3}$$

immediately identifies n_0 and d_1^i as isometry parameters. Stated differently, impulsive profiles admit at least a three-dimensional Killing algebra. Following the systematic approach outlined in the previous section produces the higher symmetry-classes [4]. The AS-geometry emerges now naturally as symmetry class with a four-dimensional Killing algebra.
However, the profile that describes a plane impulsive wave $\tilde{f} = x^i M_{ij} x^j$ appears to have only three Killing vectors, which is in contrast to its non-impulsive analog, whose Killing algebra is five-dimensional.

4 Generalized symmetries

Since impulsive profiles may be regarded as idealized objects in the class of non-impulsive ones the above situation is physically unsatisfactory. More specifically, there is no physical reason for the apparent symmetry reduction. Relaxing the C^∞–condition on the Killing vectors in fact produces the "missing" symmetries, which now appear as discontinuous transformations. Mathematically the possibility of such an extension is due to the well–defined multiplication of $\delta(u)$ by continuous functions. Going back to (2.1) we find that the $\tilde{G}$ may be extended to transformations parametrized by

$$\begin{aligned} &g = (a, 0, \Omega^i{}_j; d_0^i + ud_1^i + \theta(u)ud_1^{+i}, n_0 + \frac{u}{2}d_1^2 + \theta(u)\left(n_0^+ + \frac{u}{2}({d_1^+}^2 + 2d_1 \cdot d_1^+)\right)) \\ &g\tilde{f}(x^i) = a\tilde{f}(\Omega x + d_0) - 2(d_1^+ \cdot \omega x + n_0^+). \end{aligned} \tag{4.1}$$

The results [5] of the classification process following the method outlined in section 2 are summarized in the following table, which gives the correct result for both the AS and the plane-wave geometry.

profile	Killing vectors	r	type
$f = f(x)$	$\xi_1 = \partial_v$, $\xi_2 = 2x\partial_v + u\partial_x$, $\xi_3 = 2y\partial_v + u\partial_y$	3	abelian
$f = h(\rho)e^{\frac{1}{\gamma_1}\phi}$	ξ_1, ξ_2, ξ_3, $\xi_4 = u\partial_u - v\partial_v - \gamma_1\partial_\phi$	4	$[\xi_4, \xi_1] = \xi_1$, $[\xi_4, \xi_2] = \xi_2 + \gamma_1\xi_3$, $[\xi_4, \xi_3] = \xi_3 - \gamma_1\xi_2$
$f = h(\rho) - \frac{2}{\gamma_1}\phi$	ξ_1, ξ_2, ξ_3, $\xi_4 = -\gamma_1\partial_\phi + 2\theta(u)\partial_v$	4	$[\xi_4, \xi_2] = \gamma_1\xi_3$, $[\xi_4, \xi_3] = -\gamma_1\xi_2$
$f = h(\rho)$	ξ_1, ξ_2, ξ_3, $\xi_4 = \partial_\phi$	4	$[\xi_4, \xi_2] = -\xi_3$, $[\xi_4, \xi_3] = \xi_2$
$f = h(y)e^{-\frac{1}{\lambda}x}$	ξ_1, ξ_2, ξ_3, $\xi_4 = u\partial_u - v\partial_v + \lambda\partial_x$	4	$[\xi_4, \xi_1] = \xi_1$, $[\xi_4, \xi_2] = \xi_2 + 2\lambda\xi_1$, $[\xi_4, \xi_3] = \xi_3$
$f = h(y) + \left(\lambda x^2 + 2\bar{\lambda}xy\right)$	ξ_1, ξ_2, ξ_3, $\xi_4 = \partial_x + \theta(u)(\lambda\xi_2 + \bar{\lambda}\xi_3)$	4	$[\xi_4, \xi_2] = 2\xi_1$
$f = h(y)$	ξ_1, ξ_2, ξ_3, $\xi_4 = \partial_x$	4	$[\xi_4, \xi_2] = 2\xi_1$
$f = h_0 + \lambda\rho^2$	ξ_1, ξ_2, ξ_3, $\xi_4 = \partial_x + \lambda\theta(u)\xi_2$, $\xi_5 = \partial_y + \lambda\theta(u)\xi_3$ $\xi_6 = \partial_\phi$,	6	$[\xi_4, \xi_2] = 2\xi_1$, $[\xi_4, \xi_6] = \lambda\theta(u)\xi_3$, $[\xi_5, \xi_3] = 2\xi_1$, $[\xi_5, \xi_6] = -\lambda\theta(u)\xi_2$ $[\xi_6, \xi_2] = -\xi_3$ $[\xi_6, \xi_3] = \xi_2$
$f = h_0e^{-\frac{1}{\lambda}x}$	ξ_1, ξ_2, ξ_3, $\xi_4 = u\partial_u - v\partial_v + \lambda\partial_x$, $\xi_5 = \partial_y$	5	$[\xi_4, \xi_1] = \xi_1$, $[\xi_4, \xi_2] = \xi_2 + 2\lambda\xi_1$, $[\xi_4, \xi_3] = \xi_3$, $[\xi_5, \xi_3] = 2\xi_1$
$f = h_0 + \left(\lambda x^2 + \bar{\lambda}y^2\right)$	ξ_1, ξ_2, ξ_3, $\xi_4 = \partial_x + \lambda\theta(u)\xi_2$, $\xi_5 = \partial_y + \bar{\lambda}\theta(u)\xi_3$	5	$[\xi_4, \xi_2] = 2\xi_1$ $[\xi_5, \xi_3] = 2\xi_1$

References

[1] Aichelburg P and Sexl R, *Gen. Rel. Grav.* **2** (1971) 303.

[2] Aichelburg P and Balasin H, *Class. Quantum Grav.* **11**, L121 (1994).

[3] Jordan P Ehlers J and Kundt W, *Akad. Wiss. Lit. (Mainz) Abhandl. Math.-Nat. Kl.* **2**, 21 (1960).

[4] Aichelburg P and Balasin H, *Class. Quantum Grav.* **13**, 723 (1996).

[5] Aichelburg P and Balasin H, *Generalized Symmetries of Impulsive Gravitational Waves*, UWThPh-1996-28,TUW-96-07,gr-qc/9607045.