# Lattice calculations of f_D AND f_B

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# LATTICE CALCULATIONS OF $f_D$ AND $f_B$

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**Abstract**

The status of lattice calculations of the axial coupling of D and B mesons is reviewed.

## 1. Foreword

Before starting my talk, I would like to thank Raoul Gatto, for having guided my first steps in theoretical Particle Physics. This leads me, naturally, to recall the exciting days of the Florence school.

A small group of former students of the University of Roma, Altarelli, Buccella and Gallavotti, had followed Gatto to Florence, when he had moved from Frascati, in 1963, and joined to the people already there, Ademollo, Chiuderi, Celeghini, Giusti and others.

I myself had not been a student of Gatto. I had done an experimental thesis and had little or no background in theoretical physics. However, when I asked him to join the group, supported by a fellowship of the Istituto Superiore di Sanita', he accepted me, told me what to study first (the book of Bogolioubov) and, after a few months, gave me the first problem. This was the beginning of a collaboration, which later involved Giuliano Preparata, which has deeply marked my way of understanding Physics.

In Florence, we, the romans, had no families, no teaching or academic duties. So, in the beautiful surrounding of Arcetri, we studied and discussed physics all the time. Physics was great -

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$SU_6$, the Cabibbo angle, the quark model, current algebra - and Gatto was there, at the center of all the activity, commenting papers, assigning problems, discussing results. Above all, he was showing us the perseverance, intuition and integrity, typical of his way of doing research.

These years have been a really exceptional introduction to Particle Physics, and I am glad to have here the opportunity to express in public my deep gratitude to Raoul Gatto.

## 2. How heavy is heavy

The interaction of quarks and gluons is described, in QCD, by a dimensionless, bare, coupling constant, $g_0$:

$$L = -\frac{1}{8}\,\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + \sum_{\text{light}} \bar{q}(-i\not{D} + m_{\text{light}})q + \sum_{\text{heavy}} Q(-i\not{D} + M_{\text{heavy}})Q$$

$$D_\mu = \partial_\mu + ig_0 A_\mu \tag{2.1}$$

Dimensional transmutation, however, makes so that renormalized quantities depend upon a finite energy scale, $\Lambda_{QCD}$. For example, the running coupling constant is given by:

$$\frac{g(Q^2)^2}{4\pi} = \frac{12\pi}{(33-2F)\ln\left(\frac{Q^2}{\Lambda_{QCD}{}^2}\right)} = \frac{1}{b\ln\left(\frac{Q^2}{\Lambda_{QCD}{}^2}\right)}$$

with F the number of light quark flavours.

The dimensional parameter $\Lambda_{QCD}$, in turn, makes it possible to define in absolute terms what a heavy quark is. A quark is heavy when its mass is larger than some critical value, of the order of $\Lambda_{QCD}$:

$$M >> M_{crit} \sim \text{const.}\ \Lambda_{QCD} \tag{2.2}$$

$M_{crit}$ could range from 200 MeV, the value of $\Lambda_{QCD}$ itself, to 1 GeV, the proton mass, which is also "of the order of" $\Lambda_{QCD}$, or be even larger. We have no way to guess its value "a priori", only a non-perturbative calculation of pre-asymptotic effects can tell.

A possible criterion, to judge if a given quark flavour has reached asymptopia in mass, is offered by the inclusive weak decay-rates of the corresponding mesons. For a $Q\bar{q}$ meson (Q the heavy and q a light quark) and for Q heavy enough, quark weak decay dominates, and we obtain:

$$\Gamma_{NL}(Q\bar{u}) = \Gamma_{NL}(Q\bar{d}) \tag{2.3}$$

or, equivalently, equal semileptonic branching ratios.

Eq.(2.3) is grossly violated for K mesons and, to a lesser extent, for D mesons. The b-quark seems to be heavy enough so that eq.(2.3) is obeyed, within 20% or so [1]. Thus, the critical mass, $M_{crit}$, seems to be closer to the proton mass rather than to $\Lambda_{QCD}$. Accordingly, the known quark flavours can be classified as: massless or light (up, down, strange), border-case (charm), heavy (beauty) definitely heavy (top).

To study $Q\bar{q}$ systems, one can perform a systematic expansion in powers of 1/M, see ref.[2]. The exact $M \to \infty$ limit exists. It corresponds to the heavy quark being a static color source, which sits in a definite point in space and propagates only in time. As we shall see, this approximation allows for a fully non-perturbative study of beauty mesons, even with the presently available computer power.

## 3. Heavy quarks on the lattice

The radius of a $Q\bar{q}$ meson is much the same as that of a light hadron, of the order of 1 Fermi or so, while its propagation in time involves wave-lengths of the order of the Compton wave-lenght of the meson. Therefore, we need both:

$$a \ll \frac{1}{M} \tag{3.1}$$

to avoid lattice artifacts, and:

$$L \gg R \sim 1\,\mathrm{Fermi} \tag{3.2}$$

to reduce finite-volume effcts.

These two conditions are very demanding, and set a significant limit to the mass of the quarks we can put today on a lattice, as we shall see presently.

In lattice simulations, one starts with a fixed value of the bare coupling constant, $g_0$, or, equivalently, with a fixed value of $\beta$:

$$\beta = \frac{6}{g_0^2}$$

We may regard $g_0$ as being approximately equal to the running coupling at $Q^2 \sim \Lambda^2 \sim a^{-2}$:]

$$g_0^2 \sim \frac{4\pi}{b\ln(\frac{C^2}{a^2\Lambda^2_{QCD}})}$$

or:

$$a^{-1} = \frac{\Lambda_{QCD}}{C} \exp(\frac{2\pi}{bg_0{}^2}) \qquad (3.3)$$

with C a (non-perturbative, lattice dependent) constant and b derived from the perturbative calculation:

$$b = \frac{(33-2F)}{12\pi} \qquad (3.4)$$

where F is the number of flavours.

The result (3.3), a manifestation of the asymptotic freedom, shows that the lattice spacing in physical units vanishes for $g_0 \to 0$, or $\beta \to \infty$.

Denoting by $m_{LATT}(g_0)$ any hadronic mass computed on the lattice, we may write:

$$m_{LATT}(g_0) = m_{PHYS}\, a \qquad (3.5)$$

where $m_{PHYS}$ is the physical mass. Using eq.(3.5), we can determine the lattice spacing from the ratio of the lattice to the physical mass of any given hadron (any other dimensionful quantity, e.g. $f_\pi$, can be used to this purpose).

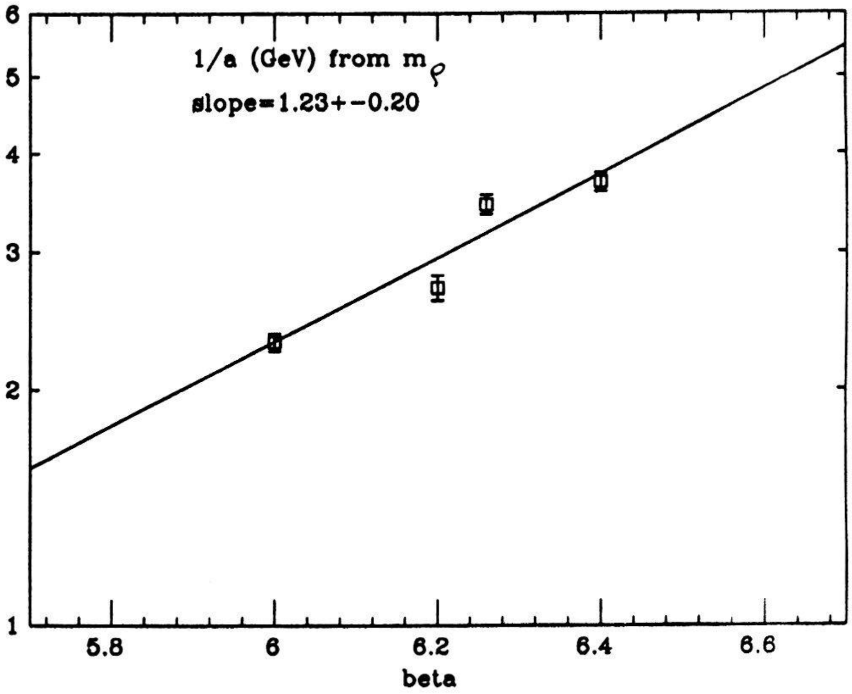
For sufficiently small bare coupling, eqs.(3.3), inserted in (3.5), gives the $g_0$ dependence of any lattice mass. If this scaling law holds, it means that we are close to the continuum limit.

The numerical $a^{-1}$-$\beta$ relation is illustrated in Tab 3.1 and Fig.3.1, where the values of $a^{-1}$ obtained from lattice determinations of the $\rho$-meson mass or, alternatively, from $f_\pi$ are reported (data have been taken from reff.[3,4a,4b,5b,6b], the error on $a^{-1}$ at $\beta$=6.26 is simply guessed to be equal to that estimated for the calculation at $\beta$=6.4, which has a similar statistics).

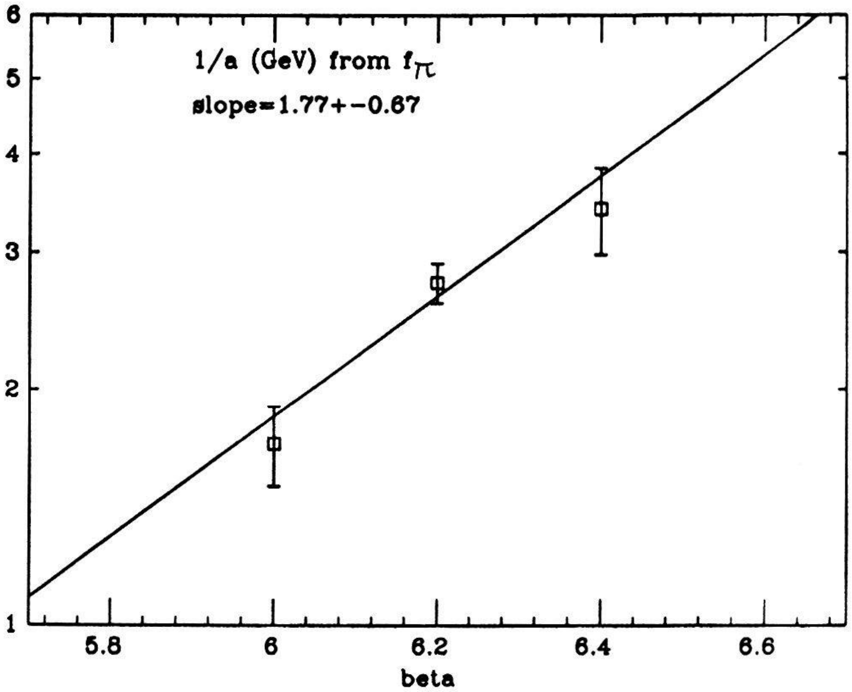
The scaling law (3.3) implies the linear relation:

$$\ln(a^{-1}) = \text{const} + S\,\beta = \text{const} + \frac{4\pi^2}{33-2F}\,\beta \qquad (3.6)$$

For quenched calculations, F=0, and the slope S in (3.6) is about 1.20, in quite good agreement with the slope of the best-fit line to the data, shown in Fig.3.1.




(a)




(b)

Fig. 3.1
The inverse lattice spacing, in GeV, v.s. $\beta$ , obtained from the $\rho$-meson mass (a) or from $f_\pi$ (b), in the quenched approximation. Data are those given in Tab.3.1. The best-fit lines have the slope: S= $1.23 \pm 0.20$ (a) and S= $1.77 \pm 0.67$ (b).

**Tab.3.1.**
The ultra-violet cut-off, $a^{-1}$, in GeV, vs $\beta$, determined from the $\rho$-meson mass (a) or $f_\pi$ (b), in the quenched approximation.

| $\beta$ | $a^{-1}$ (GeV) a | $a^{-1}$ (GeV) b | source |
|---|---|---|---|
| 6.00 | 2.30±0.06 | | APE coll.[3] |
| 6.00 | 2.2±0.1 | 1.7±0.2 | Gavela et al.[4a] |
| 6.20 | 2.7±0.1 | 2.74±0.16 | Gavela et al.[4] and Allton et al.[6a], average |
| 6.26 | 3.45±0.1 (error guessed) | | Alexandrou et al.[5b] |
| 6.40 | 3.7±0.1 | 3.41±0.43 | Boucaud et al.[6b] |

Present memory capacities and computing speeds, imply lattice dimensions in the order of $20^4$ sites. With this limitation, if we try to decrease the lattice spacing so as to satisfy (3.1), the lattice size decreases rapidly below 1Fermi, and we meet more and more severe finite-volume effects. The range of $\beta$ shown in Tab.3.1 represents the compromise which is possible today. Simulation of B mesons is out of question, but realistic charmed meson simulation is possible, already at $\beta$=6.0. A somewhat heavier quark can be simulated, at $\beta$=6.2 or 6.4.

## 4. The axial coupling of charmed mesons

The axial coupling of D or $D_s$ mesons is obtained from the $t\to\infty$ limit (in Euclidean time) of the two-point correlation function:

$$\sum_{\mathbf{x}} <0|A_0^{Loc}(\mathbf{x},t)A_0^{Loc}(\mathbf{0},0)|0> \underset{t\to\infty}{=} \frac{1}{2M_D} M_D{}^2 f_D{}^2 e^{-M_D t} + ... \qquad (4.1)$$

where $A_\mu^{Loc}$ is the local axial current:

$$A_\mu^{Loc}(x)= \bar{Q}(x)\gamma_\mu\gamma_5 q(x) \qquad (4.2)$$

Dots denote contributions from heavier, excited, states. For large times, these contributions are exponentially suppressed, with respect to the dominant term, by a factor:

$e^{-[M_{D'}-M_D]t}$

The values of $f_D$ and of $f_{Ds}$ obtained thus far by us and by other groups are reported in Tab. 4.1. To minimize the systematic errors, we compute $f_\pi$ and define $f_D$ as:

$$f_D = \frac{(f_D)_{LATT}}{(f_\pi)_{LATT}} (f_\pi)_{expt} \tag{4.3}$$

The procedure is consistent because the value of $f_\pi$ we obtain[4a] when the lattice spacing is calibrated on hadron masses, is compatible with the experimental result:

$$f_\pi = 140 \pm 20 \text{ MeV} \quad ((f_\pi)_{expt} = 132 \text{ MeV}) \tag{4.4}$$

The value of $f_K$ is also reported.

**Tab.4.1.**

Values of $f_K$, $f_D$ and $f_{Ds}$ computed in quenched lattice QCD (in parenthesis the value of $\beta$). Axial couplings are normalized to $f_\pi$. Third column: results from QCD sum rule calculations. Last column: experimental data.

| Meson | $f_M$(MeV) Lattice | $f_M$(MeV) QCD sum rules | expt. |
|---|---|---|---|
| K | 158±13 (6.0[4a]) | | 165 |
| | 173±13 (6.0[7]) | | |
| | $161 \pm 7^{+37}_{-16}$ (6.1[8]) | | |
| D | 180±30 (6.2[4b]) | 172±15[10] | <290[12] |
| | 215±60 (6.0[7]) | 224±26[11] | |
| | 174±26±46 (6.1[8]) | | |
| | 198±17 (6.0[5a]) | | |
| $D_s$ | 218±30 (6.2[4b]) | | |
| | ~220 (6.0[7]) | ~220[10] | |
| | 234±46±55 (6.1[8]) | 277±13[11] | |
| | 209±18 (6.0[5a]) | | |

Lattice calculations agree with each other and with the results of QCD sum rules. An experimental measure of $f_D$ is very important, it would provide a quite significant test of lattice QCD with heavy flavours.

Asymptotically, for very large quark mass, one can prove the QCD scaling law (see Sect. 5):

$$f(M) = \frac{\text{const}}{\sqrt{M}} \tag{4.5}$$

If scaling would apply already at the charmed meson mass, we could estimate $f_B$ without further effort, using eq.(4.5). From $f_D \sim 180$ MeV, one gets:

$$(f_B)_{\text{scaling}} = \sqrt{\frac{M_D}{M_B}}\, 180\ \text{MeV} \sim 110\ \text{MeV} \tag{4.6}$$

We have seen in Sect.2 that the charm mass is not asymptotic for weak lifetimes. It would be surprising if the deviations from parton model did not reflect in a similar violation of the scaling law (4.5).

Different groups have done or are doing calculations with a heavy quark mass above the charmed quark mass, with the aim of extrapolating to beauty **from below**.

A compilation of present results at different values of M is illustrated in Fig.4.1, where $f(M)\sqrt{M}$ is plotted against 1/M, in physical units[13]. We have taken the results of reff.[4,5,6a] in lattice units and translated them into physical units using the $\ln(a^{-1})$-$\beta$ linear relation which gives the best fit to the data of Figs.3.1a or 3.1b.

Data from different sources are remarkably consistent. Scaling is definitely **not** observed at the charm mass.

Data at very large quark mass, 5 and 10 GeV, have been reported[5b]. However, with $\beta$=6.26 and $a^{-1}$= 3.45 GeV, the ultraviolet cut-off is smaller than the quark mass, so that the results at these masses and the corresponding flattening of $f(M)\sqrt{M}$ are quite suspicious.

## 5. Beauty as a static source

The propagator of a massive quark in a given gauge field configuration can be found exactly in the $M \to \infty$ limit, that is when the typical wave-lengths of the gauge field are much longer than the quark Compton wave-length, see e.g.ref.[14]. Because of asymptotic freedom, which decouples wave-lenghts shorter than $\Lambda_{QCD}^{-1}$, we can use this approximation even in the full quantum theory, where we have to make a functional integral over all gauge configurations.

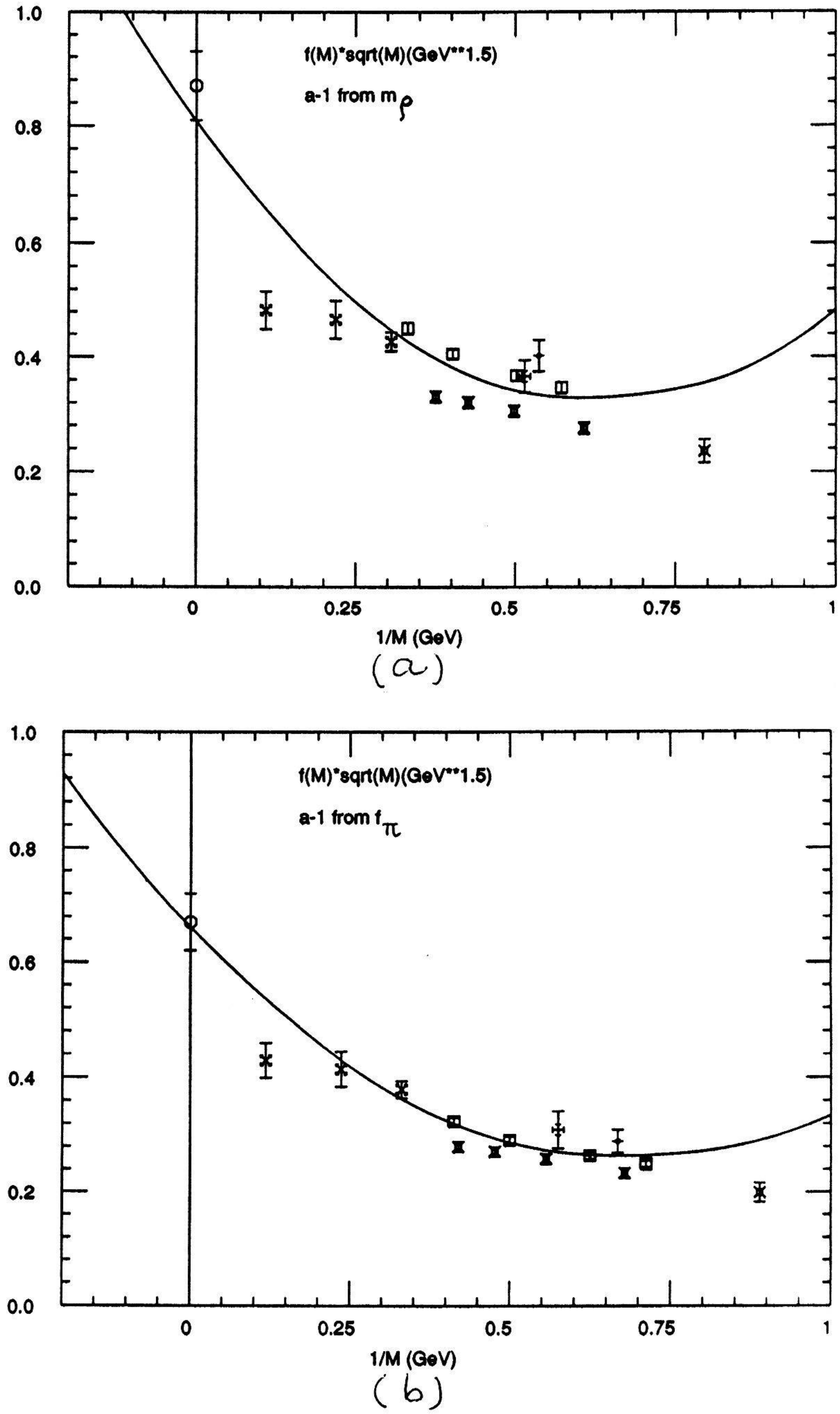



Fig. 4.1

$f(M)\sqrt{M}$ ($GeV^{3/2}$)vs 1/M (GeV). To transform from lattice to physical units, we have used the best-fit to the $a^{-1}$-$\beta$ relation obtained from the ρ-meson mass (a) or from $f_\pi$ (b). Data: full circle, $\beta$=6.0, ref.[4a]; vertical cross, $\beta$=6.2, ref.[4b]; open square, $\beta$=6.0, ref.[5a]; diagonal cross, $\beta$=6.26, ref.[5b]; open circle with cross, $\beta$=6.2, ref.[6a]. The static result, average of reff.[18, 5a, 5b] is also indicated. The solid line is the quadratic best-fit to the data, see eq.(7.1), excluding the two highest-mass points of ref.[5b].

In Euclidean space, the quark propagator is the solution[1] of:

$$(\not{D} + M)S(x; y) = (D_0\gamma^0 + \mathbf{D}\cdot\gamma + M)S(x; y) = \delta^{(4)}(x-y) \tag{5.1}$$

The static approximation corresponds to neglecting $\mathbf{D}\cdot\gamma$. For $t>0$, the solution is:

$$S_Q(\mathbf{x},t; \mathbf{0},0) \equiv S^{(0)}(\mathbf{x},t; \mathbf{0},0) =$$

$$= \delta^{(3)}(\mathbf{x})\, e^{-Mt}\, P(t, 0; \mathbf{0})\, \left(\frac{1+\gamma^0}{2}\right) \qquad (t>0) \tag{5.2}$$

$$P(t, 0; \mathbf{0}) = T\{\, \exp[-ig_0 \int_0^t dt' A_0(\mathbf{0},t')]\}$$

where T denotes time-ordering of the QCD, non-abelian, gauge fields.The quark sits at the space origin, $\mathbf{x}=\mathbf{0}$, and propagates in time.

We can use the static approximation to compute correlation functions involving heavy and light quarks. For example, the correlation function (4.1) is obtained, at vanishing space 3-momentum, as:

$$\int d^3x <0|A^{Loc}_0(\mathbf{x},t)A^{Loc}_0(\mathbf{0},0)|0> =$$

$$= e^{-Mt} \int d^3x \int D[\mu] e^{-S_{gauge}} \{\, Tr[\gamma_0\gamma_5 \left(\frac{1+\gamma^0}{2}\right) P(t)\gamma_0\gamma_5\, S_q(0; \mathbf{x},t)]\} \tag{5.3}$$

$S_q$ is the light quark propagator, in the same gauge field configuration, $D[\mu]$ the functional measure of the gauge fields. The vacuum expectation value in the l.h.s. of eq.(5.3) is obtained by functional integration with this measure, weighted with the exponent of the gauge action, $S_{gauge}$.

For large t, the l.h.side can be approximated, as in eq.(4.1), with the lowest-lying B meson state. Putting together esponentials with large masses, we obtain:

$$\frac{1}{2M_B} M_B^2 f_B^2 e^{-[M_B-M]t} + \ldots =$$

$$= \int d^3x \int D[\mu] \{\, Tr[\gamma_0\gamma_5 \left(\frac{1+\gamma^0}{2}\right) P(t)\gamma_0\gamma_5\, S_q(0; \mathbf{0},t)]\} \tag{5.4}$$

[1]S is not a function of the difference x-y only, as translation invariance does not apply in a given gauge configuration.

The important fact is that all quantities in eq.(5.4) tend to a finite limit, for $M_B \to \infty$. This is the case, for instance, of the difference $M_B$-M in the exponent, the binding energy of the B-meson, which is a pure QCD quantity of order $\Lambda_{QCD}$, and is finite in the above limit.

A calculation of the r.h.side of eq.(5.4), which involves only quantities pertaining to the light degrees of freedom, gluons and the light quark q, gives then a determination of:

$$\phi_\infty = \lim_{M \to \infty} f(M)\sqrt{M} \tag{5.5}$$

By the way, eq.(5.4) proves that indeed the scaling law eq.(4.5) holds in QCD.

As proposed originally by Eichten[15], eq.(5.4) can be used to compute $f_B$ with a lattice which would not allow the propagation of a heavy B-quark. The path-ordered integral is translated into the product of the gauge matrices associated to the links at $\mathbf{x}=\mathbf{0}$:

$$P_{LATT}(t) = \left[U(t,t-1)U(t-1,t-2)....U(1,0)\right]_{\mathbf{x}=\mathbf{0}} \tag{5.6}$$

and $S_q$ into the lattice light quark propagator. Functional integration corresponds to the average over a set of configurations, MonteCarlo generated with probability $\exp(-S_{gauge})$.

Besides these obvious translations, there are a few technical problems, by now well understood, and which I simply mention. One is that we have to multiply the r.h.side by an appropriate conversion factor, Z, which makes so that the lattice value of $f_B$ is correctly normalized to the continuum-limit axial current. Because of asymptotic freedom, a perturbative calculation of Z is sufficient, and has been done[16]. Also, on the lattice, the binding energy in the exponent is replaced by an unphysical quantity, linearly divergent with $a^{-1}$, related to the Coulomb self-energy of the string (5.6).

In conclusion, the lattice translation of eq.(5.4) reads:

$$\begin{aligned} &\frac{1}{2M_B} M_B^2 f_B^2 e^{-E_B t} + ... = \\ &= Z\left\{\frac{1}{N_{CONF}} \sum_{CONF} \mathrm{Tr}\left[\gamma_0\gamma_5 \left(\frac{1+\gamma^0}{2}\right) P_{LATT}(t) \gamma_0\gamma_5\, S_q(0;\, \mathbf{0},t)\right]\right\} \end{aligned} \tag{5.7}$$

The r.h.side is computed as function of the Euclidean time t and fitted to the exponential form in the l.h.side, for large t .

## 6. Calculation of $f_B$ in the static limit

The first attempts to determine $f_B$ in the static limit[17] have been rather inconclusive, because the correlation function did not show a pure exponential behaviour, in the available region of times.

Attributing this problem to the imperfect decoupling of the excited states, we decided to have another try[18] using a "smeared" axial current, instead of the local one given in eq.(5.4). This is a method developed for light hadron spectroscopy by the APE collaboration[19], to obtain a fast-in-time decoupling of the excited states. In our case, it amounts to use the smeared axial current:

$$A^{Sme}_{\mu}(\mathbf{x}, t) = \sum_{ij} \bar{Q}(\mathbf{x}+\mathbf{r}_i, t)\gamma_\mu\gamma_5 q(\mathbf{x}+\mathbf{r}_j, t) \tag{6.1}$$

where the vectors $\mathbf{r}_i$ run over the lattice points inside a cube of a given side around the origin. The smeared, mixed and local correlation functions, defined according to:

$$\begin{aligned} G^{SS} &= \langle A^{Sme}_0 A^{Sme}_0 \rangle \sim F_{SS} e^{-E_B t} \\ G^{LS} &= \langle A^{Loc}_0 A^{Sme}_0 \rangle \sim F_{LS}\, e^{-E_B t} \\ G^{LL} &= \langle A^{Loc}_0 A^{Loc}_0 \rangle \sim F_{LL} e^{-E_B t} \end{aligned} \tag{6.2}$$

are computed in a fixed gauge, the Coulomb gauge.

Of course, we are interested only to find $F_{LL}$, which then gives $f_B{}^2$ via eq.(5.7). However, if the lowest state dominates, each coefficient factorizes:

$$F_{ab} = F_a F_b \tag{6.3}$$

and we may get $F_L$ from $G^{LS}$ and $G^{SS}$:

$$(F_L)^2 = F_{LL} = \frac{(F_{LS})^2}{F_{SS}} = \left(\frac{F_{LS}}{F_{SS}}\right)^2 F_{SS} \tag{6.4}$$

The advantage is that **both** $G^{LS}$ and $G^{SS}$ should exhibit a precocious exponential behaviour, if the smeared current is efficient in decoupling the excited states, unlike $G^{LL}$.

Remarkably, all this works, as illustrated by Fig.6.1, a and b. We have reported the effective exponent as function of time:

$$E_{eff}(t) = \ln\frac{G(t+1)}{G(t)} \tag{6.5}$$

for the local, LL, and the smeared, SS, correlation functions. The latter exhibits a clear plateau in $E_{eff}$, while in the former the plateau, if any, appears only for large times, where the signal is almost non-existent. Fig. 6.2 shows the ratio $G^{LS}/G^{SS}$. Also in this case a clear plateau is seen, for t=5-10.

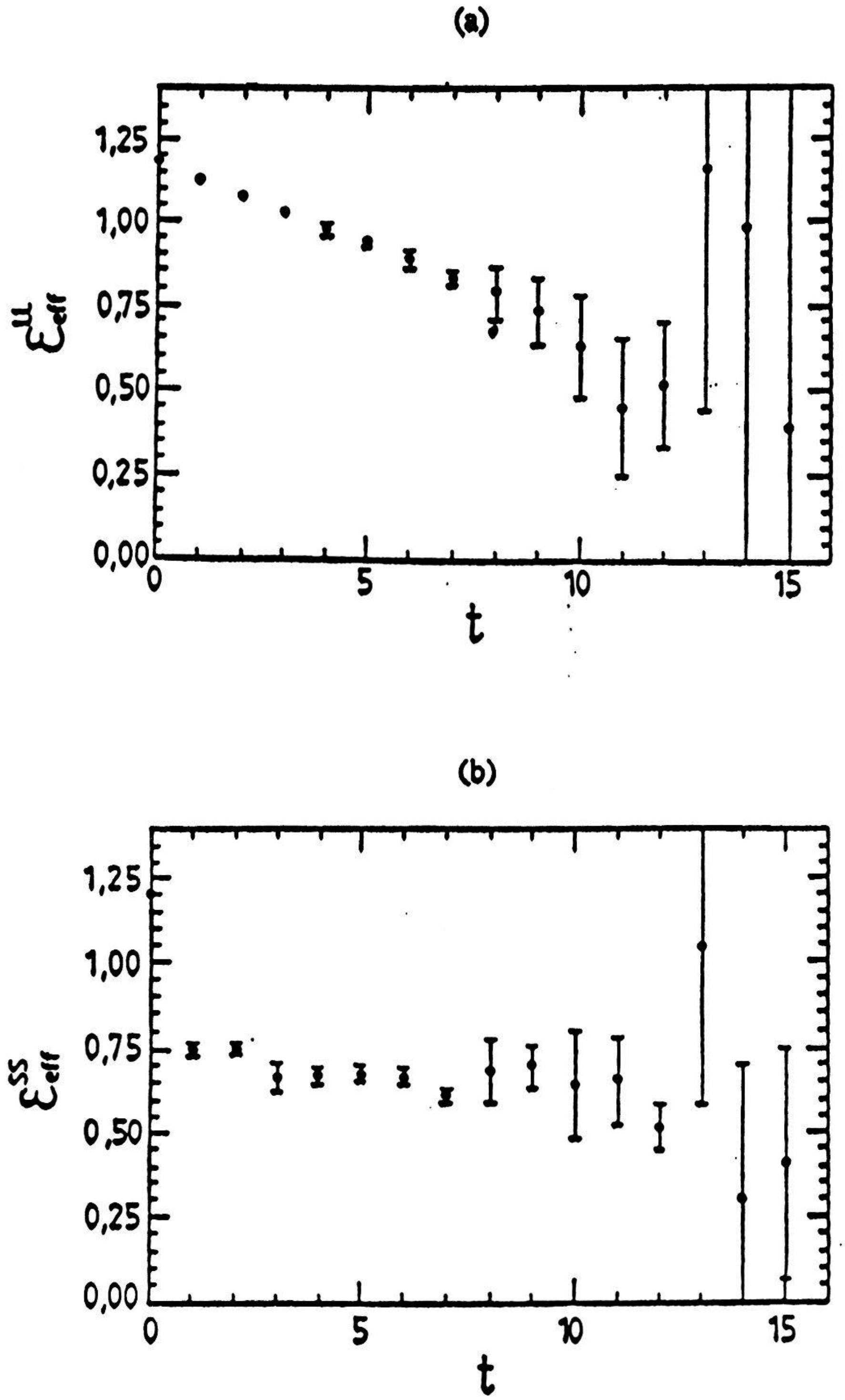



Fig. 6.1
Effective mass as function of time: (a) from the correlation functions of the local currents, $G^{LL}$; (b) from the correlation functions of the smeared currents, $G^{SS}$, figure from ref.[18].

In conclusion, we obtain the ratio:

$$\frac{F_{LS}}{F_{SS}} = \left[\frac{G^{LS}(t)}{G^{SS}(t)}\right]_{t=5-10} \tag{6.6}$$

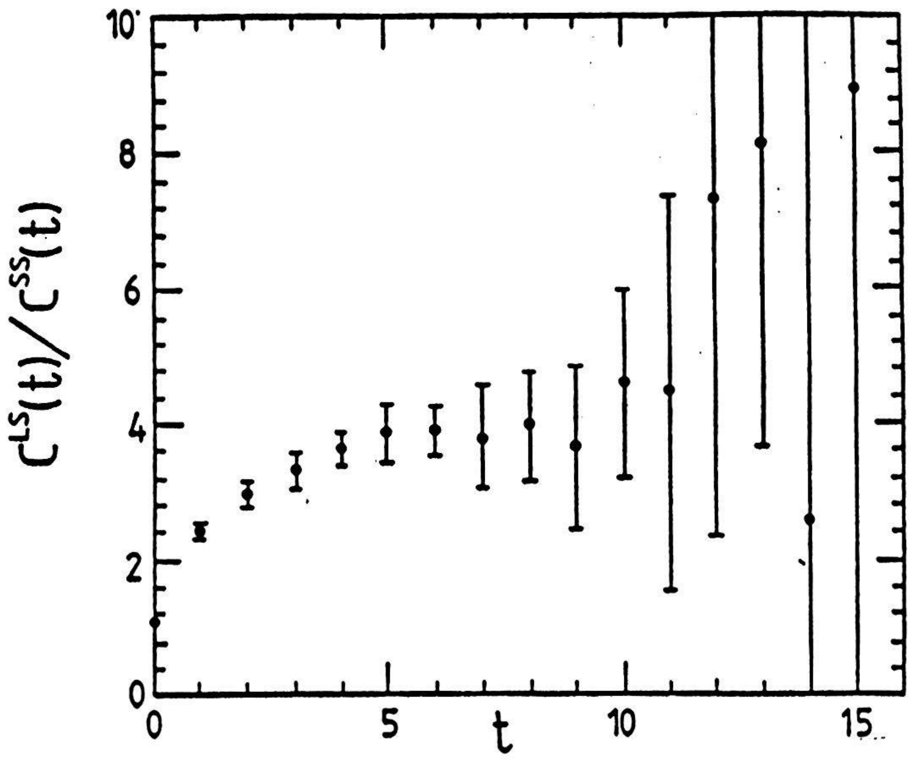



Fig. 6.2
Ratio of the correlation functions $G^{LS}$ and GSS vs. time, figure from ref.[18].

directly from Fig. 6.2, $F_{SS}$ from an exponential fit to $G^{SS}$, and $F_L$ from eq.(6.4). Finally:

$$\phi_\infty = \lim_{M\to\infty} f(M)\sqrt{M} = \sqrt{2Z}\,F_L$$

Our final result is[18]:

$$\phi_\infty = 0.71 \pm 0.14 \text{ GeV}^{3/2} \tag{6.7}$$

($\beta$=6.0, $a^{-1}$=2.0±0.2GeV; 30 configurations)

which yields:

$$(f_B)_{stat} = \frac{1}{\sqrt{M_B}}\,\phi_\infty = 310 \pm 25 \pm 50 \text{ MeV} \tag{6.8}$$

The statistical error is quite small, reflecting good identification of the signal with the smearing method. An earlier calculation of Eichten et al.[17] had given a much more uncertain (although compatible) result:

$$(f_B)_{stat} = 780 \pm 400 \text{ MeV} \tag{6.9}$$

The result (6.8) is much larger than that deduced from the charm result, in eq.(4.6), and it indicates substantial scaling violation at M~2GeV, later confirmed by the direct calculations illustrated in Sect.4.

By now, the result (6.8) has been confirmed by independent calculations of the Wuppertal group (Alexandrou et al.), $\beta$=6.0 and large statististics and $\beta$=6.26, as seen in Tab. 6.1. A similar result is found also by the UCLA-Stony Brook collaboration, ref.[9], (but no definite number is quoted).

**Tab. 6.1**
Lattice calculations of $f_B$ in the static limit.

| Authors | $\beta$ | n. of confs. | $(f_B)_{stat}$(MeV) |
|---|---|---|---|
| Allton et al.[18] | 6.0 | 30 | 310±25±50 |
| Alexandrou et al.[5a] | 6.0 | 100 | 366±22±55 |
| Alexandrou et al.[5b] | 6.26 | 15 | 416±35 |

## 7. An overall view

There is a good consistency between the static results of the different groups, even more than indicated by Tab.6.1. For $\beta$=6.0, in fact, the results in lattice units are closer and the difference between us and ref.[6a] is mostly due to the different assumed value of the lattice spacing.

The static result, averaged over the values of reff.[18,5a,5b], is reported in Figs.4.1 a and b. The results are translated from lattice to physical units with the same procedure used for the points at $1/M \neq 0$ (Sect.4).

To check for the overall consistency of the data, we have tried a linear and quadratic fit[13]:

$$\phi(M) = f(M)\sqrt{M} = \phi_\infty + \frac{A}{M} + \frac{B}{M^2} + \dots \tag{7.1}$$

leaving out the two points at the largest values of M, for the reasons explained at the end of Sect.4.

A linear fit gives a rather poor representation of the data, but the quadratic fit is very good, see Figs.4.1. For the two cases, we obtain:

$$\phi_\infty = 0.80 \text{ GeV}^{3/2}; \quad A = -1.6 \text{ GeV}^{5/2}; \quad B = 1.3 \text{ GeV}^{7/2} \quad [\, a^{-1} \text{ from } \rho \text{ mass}] \tag{7.2a}$$

$$\phi_\infty = 0.66 \text{ GeV}^{3/2}; \quad A = -1.2 \text{ GeV}^{5/2}; \quad B = 0.86 \text{ GeV}^{7/2} \quad [\, a^{-1} \text{ from } f_\pi \,] \tag{7.2b}$$

The linear term at the B-meson mass is about -34% for case (a) (-37% for case (b)) and the quadratic term is 5% (6%). The rather flat behaviour of $\phi(M)$ around the charmed meson mass is explained, in the fit, by the rather large quadratic corrections in this region. Notice that the fit is systematically larger than the value at the beauty mass ($1/M \sim 0.2$ GeV$^{-1}$) determined in ref.[5b], but almost consistent with it, in case a, and even better in case b, although that point was not used in the fit itself. The point at the largest mass, instead, is definitely off the fit.

The overall picture seems to be reasonable. The large linear and quadratic corrections at the charm scale are in line with the considerations made in Sect.2, about scaling in weak decays.

**Interpolating** with the best-fit curve, one finds:

$$f_B = 240 \text{ MeV} \; ; \; f_D = 235 \text{ MeV} \qquad [\, a^{-1} \text{ from } \rho \text{ mass}] \tag{7.3a}$$

$$f_B = 203 \text{ MeV} \; ; \; f_D = 201 \text{ MeV} \qquad [\, a^{-1} \text{ from } f_\pi \,] \tag{7.3b}$$

This, I think, is the best one can say about $f_{B,D}$ from lattice calculations, at present. We do not attempt to give an error to each determination. The difference between the two set of results in (7.2) and (7.3) gives a probably optimistic, but not too unrealistic idea of the systematic errors involved .

## 8. Outlooks

The calculation of the axial couplings of heavy-flavoured pseudoscalars may provide a significant testing ground for lattice QCD, for many reasons.

Axial couplings are relatively simple quantities, but still require a fully non-perturbative and relativistic (in the light quark) calculation. Present calculations are done within the quenched approximation but the axial couplings will keep their meaning also when internal quark loops will be introduced, unlike most hadron spectroscopy.

Finally, there are no clear-cut quark model estimates and no experimental determinations, which makes lattice predictions quite interesting. The more so, since, as discussed in ref.[20], a large value of $f_B$, in conjunction with a large mass of the t-quark ($f_B > 200$ MeV; $m_t > 130$ GeV) would lead to an interestingly large CP violation in the weak decay of a B-$\bar{B}$ pair.

The lattice prediction of $f_D$ is well established, by now. The value of $f_B$ has been the object of considerable discussion. As I have tried to illustrate here, the results obtained by extrapolating from below are consistent with the static limit and point to a large value of $f_B$, if we accept that sizeable violations of the scalig law are present at the charm scale.

A crucial consistency check would be to calculate the coefficient A, the 1/M correction in eq.(7.1), on the lattice, from a development around the static limit itself, so as to extrapolate to beauty from above. Unfortunately, as we have found quite recently [21], the proper definition of the 1/M corrrections seems to require non-perturbative subtractions, which would make rather problematic a lattice determination of A. The problem is still under study, but I am, today, much less optimistic than I was when I gave this talk. If this is so, the refinement of the lattice prediction (7.3a,b) will have to go by the hard way, namely the achievement of a computing power such as to be able to get closer to beauty, with really propagating quarks.

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