On the problem of non-locality in quantum mechanics

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On the problem of non-locality in quantum mechanics

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Abstract : Many experimental results and theoretical reasonings on "gedanken" experiments suggest that quantum entities behave in a "non-local" way. In order to make the discussion more precise, we propose an operational definition of "non-locality". We discuss two examples of spin superposition experiments which illustrate the use of this definition. They show that quantum entities like a neutron or an atom are non-local. We make an analysis of the difficulty to imagine such non-local entities and to conciliate this non-locality with the locality of classical entities.

1. Introduction.

From the very beginning of the quantum theory it was clear that the so-called "quantum world" is very different from our familiar or "classical" world. For a long time however the study of the quantum properties remained limited to the domain of the stationary states of microscopical physics and this has certainly greatly contributed to a comfortable attitude of "sleeping on a gentle pillow" which is still so general among the physicists [1]. Recent experiments on spin correlation measurements in EPR situations have shown that some of the paradoxical aspects of the quantum formalis put forward in the original EPR paper can be realized (although not easily) in the laboratory. Many different analysis have been made of these EPR experiments and the idea of

"non-locality" has been put forward to characterize the following strange assertion : the results of measurements performed on distant parts of a system with distant local apparatuses are statistically correlated in a way that indicates (or strongly suggests ?) the existence of a link between the measurements events, and this correlation persists even when these distant measurement events are separated by a minkowskian space-like distance. However, an "exact" analysis of real EPR experiments with two photons in a singlet spin state is very difficult because one has to estimate the influence of a lot of "technical details" and "theoretical implicit hypotheses" on the interpretation of the experimental results. This has been the source of an abundant controversial literature [2]. Therefore, it is perhaps wise not to base the discussion of a possible "quantum non-locality" on the EPR experiments and to look for some more direct evidence.

It is well known that even in the case of one single particle, one meets experimental situations wherein the paradoxical aspects of the quantum formalism become apparent. A good account of some of these paradoxical aspects can be found in the standard analysis of the double slit "gedanken" experiment [3]. The interpretation of this double slit experiment within the conceptual framework of wave-particle duality is widely accepted and offers all that can be said about such a situation. However, we think that recent "real" experiments, which are indeed more sophisticated than the simple double slit experiment, offer more information on what actually happens at the quantum level, so that the old analysis should be refined and completed.

In section 3 of this paper we want to present an analysis of a specific experiment, one of the many neutron interferometric experiments performed by the group of Rauch [4], that can be considered as a version of the double slit experiment. We try to point out which kind of new experimental information it gives beyond the well known wave-particle duality. Roughly speaking it becomes an experimental fact, that even if a quantum entity is in a wave-like situation it can be influenced without destroying its wave properties (interference), in a sense that one could only imagine for a particle-like entity. This influence is not a measurement of localization, but entails an aspect of "locus" in a very specific, and non-local way. Namely, the apparatus defines several widely separated regions of space with the following properties : on the one hand, a localization measurement would find the neutron in one and only one (although only statistically known) of these regions ; on the other hand, the neutron can be influenced as a whole by a local device that acts at a given instant of time in any one of these regions, the effect being independent on the chosen region.

The analysis of this experimental result has led us to believe that non-local aspects of quantum entities can be, and should be investigated apart from other quantum peculiarities, i.e. that they should not be considered as side aspects of the wave-particle duality, as is usually done when one discusses one single quantum entity.

It has also led us to propose a new, operational definition of "non-locality" which can be used to analyse classical and quantum situations. This definition is operational in the sense that it

only refers to operational concepts that one can realize in laboratory situations, and it does not make use of the concept of space in relation with the quantum entity, i.e. the space concept is only related to the classical aspects of the macroscopical apparatuses used in the experiment.

We state this definition in section 2, before the presentation of the Rauch experiment, which was the source of our inspiration, so that we can analyse this Rauch experiment in the light of this definition, with the conclusion that the neutron is a non-local entity.

Our conclusion can of course be criticised on several grounds. Firstly, it is reached on basis of a definition of non-locality directly inspired by this experiment itself and therefore it has a little taste of a circular reasoning. Secondly, a change in an interference pattern is essentially a statistical change in the distribution of an ensemble of particle detections, and it is always a delicate methodological problem to infer from such a change a conclusion on individual cases. Therefore, we look for another kind of experiment where the same conclusion can be reached on individual cases, or equivalently, with a statistical weight equivalent to certainty.

We turn to the old Stern-Gerlach experiment (section 4) for which we have given elsewhere a complete quantum mechanical analysis. By means of a "gedanken" experiment combining several Stern-Gerlach apparatuses we show in section 5 that also an atom is a non-local entity (again according to our definition). As explained in detail in section 5 this non-locality can be exhibited on each individual atom. Roughly speaking a commutator which switches on or off a homogeneous magnetic field in any one of two macroscopically separated regions and not elsewhere can send each atom (and therefore all atoms) on the left or alternatively on the right of a detecting screen.

In section 6 we analyse more deeply our definition of non-locality. We show that according to this definition all typical classical entities are local. Even a wave in three dimensional euclidean space is a local entity for our definition. This underlines the deep difference between the quantum wave function and a classical wave. Every physicist knows that for a system of several particles the quantum wave function is not a wave in three dimensional euclidean space, but many physicists still consider that for one single entity the quantum wave function is some kind of classical wave propagating in euclidean space. The quantum non-locality that we want to grasp with our definition is different from the very fact that waves are extended in space. In the light of the fact that according to our definition classical entities are local, the difference between quantum entities and classical entities with respect to locality and non-locality becomes more evident. We point out again this difference in section 6, and state more carefully how some aspects of this difference are not accounted for by the standard interpretation of quantum mechanics.

In section 7 we propose a set of ideas, finding their roots in an analysis of the construction of reality originated by H. Poincaré, that leads to a new interpretation of non-locality. In short, one can say that the environment where we live (and in particular the common presence of rigid bodies) has deeply influenced our way of thinking. Our model of the physical entities and their

evolution is directly inspired from this macroscopical environment. We believe that this is the main reason for our reaction against some particular aspects of quantum mechanics that we perceive as paradoxical. We propose to reverse the trend and not consider these facts as paradoxical, but rather to look for an explanation of their disappearance in our macroscopical world.

Many physicists and philosophers think that the crisis of "understanding" that has come to humanity when quantum theory was born, can only be solved by the introduction of new concepts, and new ways of thinking about these concepts. The proposal in section 7 must be understood as an attempt in this direction.

We believe that the problem of the non-locality of individual quantum entities is so strange and so complex that only a detailed presentation of its empirical aspects makes it understandable. Therefore we have written the paper in a self-contained way, so that it becomes readable by any physicist, working or not in the foundations of quantum mechanics. We hope that by doing so, this paper also has gained some pedagogical value.

2. An operational definition for non-locality.

We propose the following definition for a non-local entity :

Definition : An entity is "non-local" if it can be prepared in a state which introduces several macroscopically separated regions of space with the following alternative physical operations at some definite instant of time:
- on the one hand, the detection of the whole entity would occur with certainty in one and only one of these regions (this one being possibly unknown before the detection occurs) if a measurement of detection would be performed at this instant of time.
- on the other hand, the entity can be influenced at this instant of time from at least two of these regions by means of a local apparatus whose range of action, in its classical description, is limited to one region only.

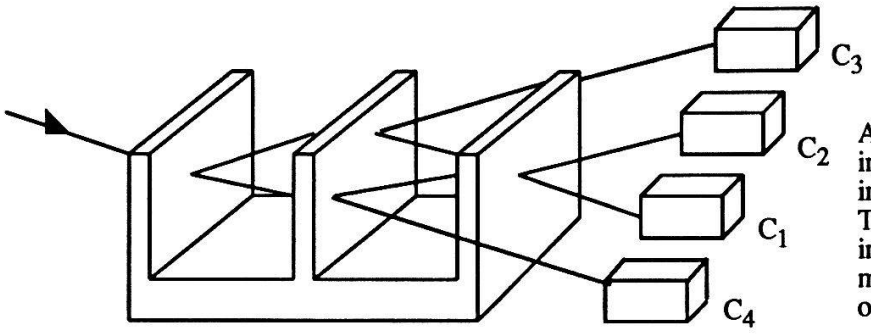
We shall show that according to this definition, entities like one single neutron or one single atom are non-local entities. This definition is not meant to analyse non-locality in the way it has been done in the EPR situation. Although we are convinced that the correlations in the EPR situation are somehow related to the same type of non-locality as the one that we try to grasp in our definition, the analysis of non-locality by means of the violation of the Bell inequalities in the EPR situation is different. It has been shown that the violation of the Bell inequalities is directly related to the "non-separation" of the two entities under consideration, which is not the same as the "non-locality" that we study here. For example, Bell inequalities can be violated by mechanistic classical

entities that are non-separated [5] which shows that the violation of the Bell inequalities does not distinguish between classical entities and quantum entities, and hence this violation cannot be considered as a sufficient condition for the presence of quantum non-locality in our sense.

3. The experiment of the Rauch's group.

Beautiful interference experiments with thermal or ultra-cold neutrons have been carried out at the Laue-Langevin Institute in Grenoble by the group of H. Rauch [4]. All these neutron interferometric experiments pertain to the case of self-interference, where during a certain time interval, only one neutron is inside the interferometer, if at all. We are interested in the experiment about the so called 4π-symmetry of spinors, where it is shown that if one introduces a local constant magnetic field on one of the two paths inside the interferometer with the aim of causing a Larmor precession of the spin of a neutron that "occasionally" follows this path, one has to create a 4π rotation in order to recover the original interference pattern. (Notice that we use the word "path" as it is usually done in interferometry to indicate the region of space that is defined by the geometrical approximation to wave mechanics, i.e. the mean path defined by the Ehrenfest theorem).

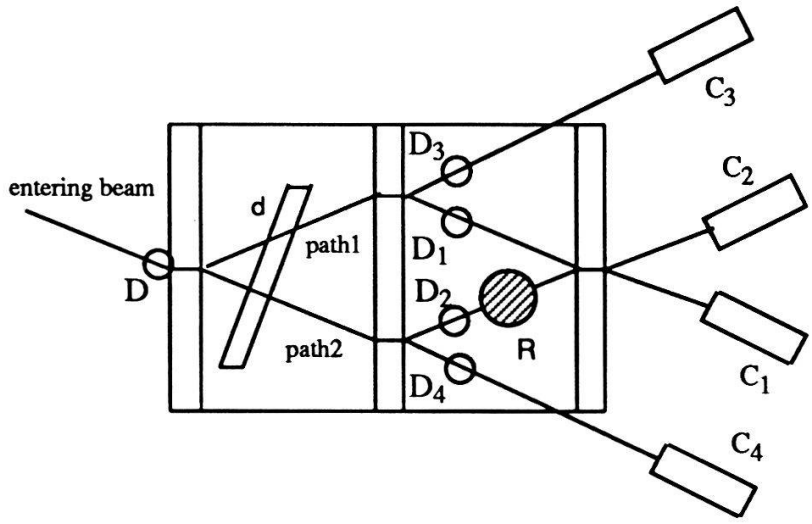
Let us briefly describe this experiment in order to make our point [6]. A perfect crystal of silicon has been cut to give an interferometer of the Laue type. The thermal neutrons are dynamically Bragg-diffracted at the first crystal plate, then reflected at the middle plate, and finally superposed at the third plate (fig.1).



A perfect silicon crystal is cut in such a way to form an interferometer of the Laue type. The beam of neutrons comes in at the left, and counters measure the intensities of the outgoing beams (fig 1).

Introducing a crystal plate of variable orientation (d on fig.2) one can change the relative phase of the two beams, and create an interference pattern in both of the detecting counters C_1 and C_2 as a function of the orientation χ of the plate, as predicted by wave mechanics. The interference patterns observed in C_1 and C_2 as functions of χ are complementary : maxima in C_1 correspond to minima in C_2 and vice-versa. A controller counter C_3 which detects the neutrons that escape on the left-side of the crystal after crossing the second plate and therefore don't participate to the interference, shows a flat distribution in χ. On purpose, we add a fourth counter C_4 which detects the neutrons escaping on the right-side of the crystal after crossing the second plate. We assume

that these counters are perfect so that any neutron which enters into the crystal will eventually be detected in one of these four counters.



A drawing of a sight from above of the experiment with the neutron interferometer. On path 2 a magnetic field is applied confined to a small region R, such that an 'occasional' neutron flying on this path has to pass through the magnetic field (fig 2)

One can modify this interference pattern by acting locally with a constant magnetic field **B** confined to a small region R on path 2, such that an "occasional" neutron flying on this path would pass for a short period of time Δt through **B**, and have its spin turned over an angle $\beta = \gamma \cdot B \cdot \Delta t$ around the direction of this field by the Larmor precession, γ being the gyromagnetic factor of the neutron. If **B** is taken to define the x_3-direction, a spin in direction $\mathbf{n}(\theta, \phi) = (\sin\theta \cdot \cos\phi, \sin\theta \cdot \sin\phi, \cos\theta)$ will be transformed into a spin in direction $\mathbf{n}(\theta, \phi+\beta)$. The complex vector (spinor) $S(\theta, \phi) = (\exp(-i\phi/2) \cdot \cos\theta/2, \exp(i\phi/2) \cdot \sin\theta/2)$ of C^2 representing the spin in the quantum formalism will be changed into $S(\theta, \phi+\beta)$. We consider the case $\beta = 2\pi$; then $\mathbf{n}(\theta, \phi+2\pi) = \mathbf{n}(\theta, \phi)$; a Larmor precession of 2π does not change the spin direction, and this is logical since if we turn any space direction over an angle of 2π, we get the same space direction. However, since the vectors representing the spin states are vectors of C^2, and the rotation group of C^2 is SU(2), the action of such a 2π rotation is different for these vectors, in the sense that $S(\theta, \phi+2\pi) = -S(\theta, \phi)$. There is nothing mysterious in this fact, because in quantum mechanics the state is represented by the normalized vector, or by the ray of the Hilbert space. Therefore $S(\theta, \phi)$ and $-S(\theta, \phi)$ represent the same spin state. This point is sometimes misunderstood although it is mathematically very simple and deeply rooted in the quantum formalism, and due to the fact that SO(3), the rotation group in three dimensional Euclidean space, is represented by SU(2) in the group of symmetries of C^2. This is so because one has to consider "projective representations" of SO(3) [7], i.e. to take into account explicitly that the state of the quantum entity is represented by the ray of the Hilbert space, and not by the vector.

Rauch et al. found that if $\beta = 2\pi$, the observed interference patterns are changed completely: maxima became minima and vice versa just as if one would interchange the patterns observed in

C_1 and C_2. The original interference pattern is only recovered if $\beta = 4\pi$ (or a multiple of 4π), and this is the reason why they speak of the 4π- symmetry of spinors.

This fascinating result shows very convincingly that the picture of a "localized" neutron following a definite path is wrong. Such a "localized" neutron would only explore a narrow neighbourhood of one of the two paths, with the following alternatives: either it travels on path 1 and is not affected by the magnetic field, or, it travels on path 2 and its spin is turned over an angle 2π. Hence, in both cases, nothing happens. So, the behaviour of the neutron remains somehow dependent on the two paths. But it is interesting to remark that this dependence goes beyond the ordinary statement on the wave-particle duality. The usual application of the wave-particle duality to the double slit experiments offers the following alternatives [3]:

- either one doesn't observe on which path the quantum entity travels and then it behaves like a wave and gives rise to an interference pattern;
- or one does make this observation and then the quantum entity behaves as a particle and no interference pattern exists.

In the present experiment, one doesn't try to determine on which path the neutron travels but one influences this neutron as a whole on one of the two paths. We say "as a whole" because the magnetic field is such that the "full" gyromagnetic moment of the neutron must be "present" in the small region R where the field is acting in order to obtain the desired rotation of the spin orientation (i.e. 2π, or 4π). This is a typical particle like behaviour. Nevertheless the interference pattern still exists, although it is completely changed in the case of the 2π rotation because the part of the wave function associated to this path has changed its sign.

Let us now consider a quantum mechanical description of this situation, using "localized" wave-packets to represent the individual neutron, travelling alone in the apparatus. Let us firstly define what we mean by an approximately localized particle state: *a normalized wave-packet $\Psi(x_1, x_2, x_3)$ is ε-localized in the space region D if the corresponding probability integral calculated on D is larger than 1-ε,*

$$\int_D dx_1 dx_2 dx_3 \, |\Psi(x_1, x_2, x_3)|^2 \; > \; 1 - \varepsilon \qquad (1)$$

which physically means that the probability of detecting the quantum entity in region D is larger than 1 - ε. Such a state will be represented by the symbol Ψ_D or alternatively by $\Psi_{D\,t}$ if we like to emphasize that this happens at time t. When it enters the interferometer, at time t=0, the neutron is ε-localized in a small region D just in front of the interferometer (fig.2), with a definite spin in direction $\mathbf{n}(\theta, \phi)$:

$$\Psi_{D\,0} = \Psi_{D\,0}(x_1,x_2,x_3) \otimes S(\theta,\phi) \qquad (2)$$

The experimental situation is a bit too complicated to allow an exact treatment of the Schrödinger equation, but this is not really necessary in order to understand what is going on. The initial wave packet evolves into two separated wave packets the centres of which follow approximately the paths previously described. The state of the neutron at a certain time t, when it is inside the first part of the apparatus, is described by a wave function

$$\Psi_t = \Psi_{D_1 t} + \Psi_{D_2 t}$$

$$\text{where } \Psi_{D_1 t} = \Psi_{D_1}(x_1,x_2,x_3)\otimes S^1(\theta,\phi)$$

$$\text{and} \quad \Psi_{D_2 t} = \Psi_{D_2}(x_1,x_2,x_3)\otimes S^2(\theta,\phi) \qquad (3)$$

$\Psi_{D1\,t}$ is a product state, the space part of which is represented by the wave packet Ψ_{D1}, ε–localized in a region D_1 on path 1(analogous $\Psi_{D2\,t}$). The precise form of the wave packets is rather irrelevant for our purpose, and therefore we forget about the deformation of the wave-packets ε-localized in regions D_1 and D_2, and consider instead these regions as evolving along the paths 1 and 2, respectively, when the time elapses (fig.2). In the second part of the apparatus, the wave function splits further into four such product states

$$\Psi_t = \Psi_{D_1 t} + \Psi_{D_2 t} + \Psi_{D_3 t} + \Psi_{D_4 t} \qquad (4)$$

where D_3 and D_4 leave the apparatus and do not participate to the interference. Rauch applies a magnetic field in a region R which is crossed by the region D_2 during some time interval (t_0, t_0 + Δt) causing a Larmor precession of the spinor S^2, so that only one of the four parts of the wave function of the neutron is affected by this magnetic field. If the experimental arrangement is such that $\beta = 2\pi$, then the spinor S^2 changes its sign, which means that, comparing two experimental situations, the first one without the magnetic field (or B inactive) and the second one with the magnetic field as here above described (B active), one gets after the crossing of the region R:

$$\{\Psi_{D2}\}_{\text{B active}} = -\{\Psi_{D2}\}_{\text{B inactive}} \qquad (5)$$

But then the complete wave function Ψ_t representing the state of the neutron undergoes the following changes. Before the time t_0 where the movable region D_2 reaches the region R, the wave function remains the same in both cases (i.e. B active or inactive). After the time $t_0 + \Delta t$ where the movable region D_2 has fully crossed the region R, the wave function becomes when B is active :

$$\{\Psi\}_{\text{B active}} = \{\Psi_{D_1} + \Psi_{D_2} + \Psi_{D_3} + \Psi_{D_4}\}_{\text{B active}}$$

$$= \{\Psi_{D_1} - \Psi_{D_2} + \Psi_{D_3} + \Psi_{D_4}\}_{\text{B inactive}} \qquad (6)$$

which is of course deeply different from $\{\Psi\}_{B\ inactive}$ at the same time. The change in the interference patterns observed by the experimenters is nothing else than the statistical realization of this elementary change, and it is indeed explained in that way in their publications.

We want to insist, on purpose, on the local character of the action which changes the state of one single neutron (even if the experimental observation of this fact is statistical). It is clear that one can make the same reasoning about an experiment placing the region R on the path of the wave-packet number 1. One could just as well place the region R on the paths of D_3 and D_4 and this would also change the state of the neutron but would not change the interference. Let us now apply our definition of non-locality to this situation. The state Ψ introduces behind the second plate four macroscopically separated regions of space, and for sure, one would detect the neutron as a whole in one of these four regions if one would perform a local detection experiment. On the other hand, the interference patterns observed in C_1 and C_2 are interchanged, which means that, at least statistically, a neutron which would be detected by C_1 when B is inactive is detected by C_2 when B is active and vice versa. This means that the entity has been influenced by the activation of the local field in the region R placed either on path 1 (i.e. when R and D_1 intersect) or on path 2 (i.e. when R and D_2 intersect). This reasoning explains exactly what, in our opinion, can be added to the analysis of a "double slit" experiment, using the results of the Rauch experiment. It is our operational criterion of non-locality : the neutron is a non-local entity.

To emphasize the amazing character of the effect of "de-localization" of one neutron realized in the experiment, we like to make a rough comparison based on a scaling which respects the relative orders of magnitude. The longitudinal coherence length of these neutrons, experimentally shown to be of the order of 20 Å, is a good measure of the size of a wave packet and hence in a certain sense an upper bound for the "size" of the neutron. Since the regions D, D_1, and D_2 are represented on the figures by means of spots of more or less 0.5 cm diameter, there is a scaling factor of $0.25 \cdot 10^7$. On the other hand, the incoming neutrons follow each other at an average distance of some 300 m, the real distance between the two paths is of the order of 5 cm, and the region R is a square with sides equal to 0.15 cm. Hence, in an "on scale" drawing, neutrons like little balls of 0.5 cm diameter would be shot having an average distance of 750.000 km between them (i.e. nearly 2 times the distance moon earth), the two paths would be separated by 125 km and the region R would be 4 km squared. At this scale, the experiment literally means that we can de-localize one single object for which we normally would imagine it to be enclosed in a sphere of 0.5 cm, over a distance of 125 km, in the sense that this "one" object can be influenced from two space regions that are like small villages of a few km^2 some 125 km apart. It is clear that this "non local Rauch effect", is not some kind of peculiar property of neutrons or magnetic fields and the conclusion about the non-locality should be accepted also for other quantum-entities, and also for other force fields. Notice however, that a weak point of the reasoning is that interference patterns are always the result of a statistical sampling of individual cases. Hence, to be able to draw more

direct conclusions about single entities, it would be better to discuss experiments where the effect of a local perturbation can be seen on each individual entity or equivalently, experiments where the statistical prediction turns out to be a certainty. We want to consider now such an experiment, the principle of which is quite analogue to the one of Rauch, in which exactly the same effect of delocalization of one quantum entity happens and can in principle be observed on each single entity.

4. The Stern-Gerlach experiment.

We now want to discuss the old Stern-Gerlach experiment, which in fact can be considered as the true discovery of the electron spin [8]. Stern and Gerlach prepared a beam of silver atoms travelling with a rather well defined velocity, in a certain space direction **b** (beam). The beam enters a strong inhomogeneous magnetic field, with a strong constant part along a space direction **a**, orthogonal to **b**, and also having a strong gradient. When the atoms of the beam leave the magnet, they are detected on a screen, placed orthogonal to **b**. Stern and Gerlach found that the individual atoms are detected in two spots on the screen: one spot, "upwards", in the **a** direction, and another spot "downwards", in the **-a** direction. In a time when modern quantum mechanics did not yet exist, but the idea of quantization was already very common, they interpreted their result as an indication that the silver atoms have a "quantized magnetic moment". They had in mind the following classical picture of the mechanism taking place during the experiment . The silver atom is imagined to be a classical particle of mass M with a magnetic moment μ due to its orbiting valence electron. When it enters the region of the magnetic field, it interacts with this magnetic field in essentially a classical way: the magnetic moment vector μ precesses rapidly around the **a** direction and the whole atom undergoes a force due to the coupling of the magnetic moment to the gradient of the field. The classical calculations show that the field gradient causes a deflection of the impact of the beam on the screen which extends on a full interval along **a** whose extremities correspond to the spots detected in the experiment. The quantization of the orientation of the angular momentum restricts the possible impacts to the two spots. This quantized orientation of the angular momentum remains up to our days the basic concept that many physicists use when they try to imagine the spin of a quantum entity.

Let us now consider the modern quantum mechanical description of this situation. Unlike the case of the Rauch experiment, one can calculate in detail the quantum mechanical evolution of one single atom in the Stern-Gerlach experiment, by solving the time dependent Schrödinger-Pauli equation with an interaction potential energy equal to $-\mu\cdot(\sigma\cdot\mathbf{B})$, where μ is the magnetic moment of the atom, $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices, and **B** is the magnetic field (limited in practice to a small region along the magnet), with an appropriate initial condition:

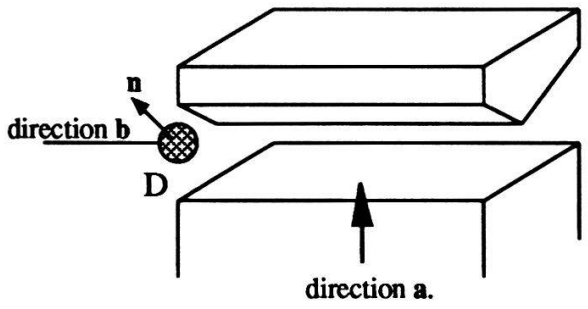
$$i\bar{h}\frac{\partial}{\partial t}\Psi(t) = -\frac{\bar{h}^2}{2M}\Delta\Psi(t) - \mu\cdot(\sigma\cdot\mathbf{B})\cdot\Psi(t)$$
$$\Psi(0) = \Psi_0 \qquad (7)$$

We consider the situation where an atom with a well defined spin direction enters the Stern Gerlach magnet, at time t = 0. Its state is then represented by a normalized vector Ψ_0, of the form $\Psi_D(\mathbf{x}) \otimes (\alpha, \beta)$, where $\Psi_D(\mathbf{x})$ is a wave-packet ε-localized in region D, localized at the entrance of the Stern-Gerlach apparatus (fig.3), and $(\alpha, \beta) = S(\theta, \phi) = (\cos(\theta/2)\exp(-i\phi/2), \sin(\theta/2)\exp i(\phi/2))$ representing the spin direction $\mathbf{n}(\theta, \phi)$ defined with respect to the directions $\mathbf{a} = (0,0,1)$ and $\mathbf{b} = (1,0,0)$.

It is remarkable, taking into account the importance of the Stern-Gerlach experiment and the apparent simplicity of the problem, that nowhere in the literature a sound quantum mechanical treatment exists. Most treatments consider incorrect magnetic fields (not satisfying the Maxwell equations), or they do not give any satisfactory explanation for the approximations used to construct a solution of the equation. Therefore we have investigated again the quantum mechanical treatment of the Stern-Gerlach experiment, trying to avoid the weakness of the earlier attempts [9]. This work will be published elsewhere and we only give here a short account of this analysis. We consider the case of an incoming atom represented by a small three dimensional gaussian wave packet with a mean velocity v_0 in the $\mathbf{b} = (1, 0, 0)$ direction. We follow the evolution of the wave packet, when it travels through the magnet and analyze in which way we can construct a solution, making the appropriate physically significant approximations.This allows us to compute the wave function at a time τ when the atom leaves the magnet and we find that it is then clearly separated into two wave packets. We then follow the further evolution of these free wave packets until they reach the screen. The result is rather complex to write down explicitly but for the present discussion a detailed expression is not necessary. Enough is to say that the wave function that leaves the magnetic field is of the form

$$\Psi_{out} = \alpha\cdot\Psi_A \otimes (1,0) + \beta\cdot\Psi_B \otimes (0,1) \qquad (8)$$

where A and B are two separated regions in space (fig.3), and where we keep the same notations as previously to indicate the ε-localization of a wave packet in a region of space. Remember that in all the fig. of this paper, we do not make a drawing of the quantum entities but only of the regions where they are ε-localized ; the arrows attached to these regions are mathematical directions which only occasionally indicate the spin direction of the atom (this is the case for region D, this is not the case for regions A and B).



The region D is the space region where the incoming atom is ε–localized, and the vector **n** is the space direction of the spin of the incoming atom. The regions A and B are the space-regions where the component wave-functions of the superposition wave function representing the state of the outgoing atom, are ε–localized. These component wave functions appear in the superposition with spin-vectors corresponding to respectively space direction a and -a. The complete wave function representing the state of the outgoing atom does not have a spin corresponding to a fixed space direction (fig 3).

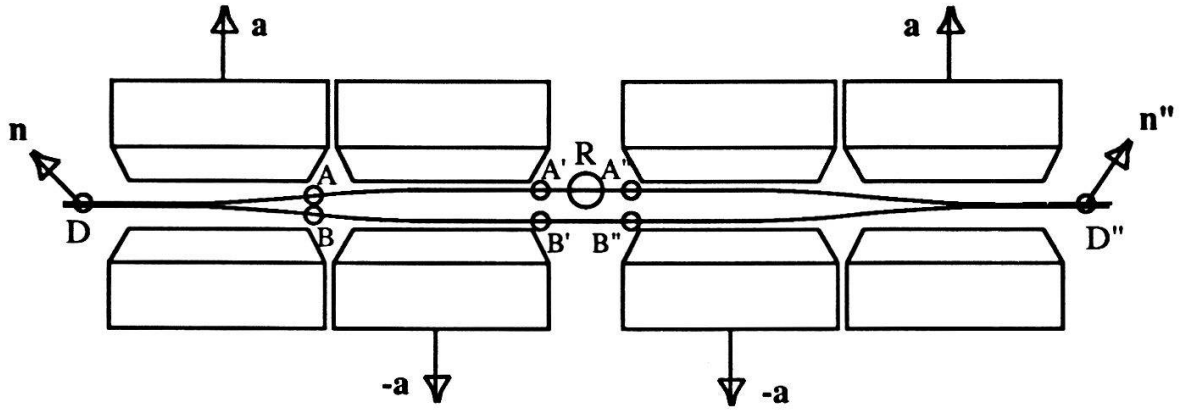
Let us now analyse the physical meaning of the state represented by this vector Ψ_{out}, the state of the single atom when it leaves the magnet before it is detected. Most physicists agree on the fact that the purely classical model of a particle having a quantized rapidly precessing magnetic moment, is not a correct model for the spin of a quantum-entity. Through the years some kind of "semi-classical" picture has developed. The final state vector Ψ_{out} is often implicitly interpreted in the following way . When an atom enters the Stern-Gerlach apparatus, it either really "flips up", and flies on the upper beam, and it is then *de facto* in a state represented by the vector $\Psi_A \otimes (1, 0)$, or it "flips down" and flies on the lower beam, and is then *de facto* in a state represented by the vector $\Psi_B \otimes (0,1)$. The very fact that, according to the calculations of quantum mechanics, the final state is represented by the vector Ψ_{out} is interpreted as reflecting our lack of knowledge about which of the two possibilities the atom chooses. If this semi-classical interpretation would be correct, then the atoms that fly through a Stern-Gerlach magnet have always a well defined spin direction, and are always ε-localized in one of the separated regions A or B. Of course, this is not true from first principles of quantum mechanics and also from the results of the Rauch experiment that one can translate *mutatis-mutandis* to the Stern-Gerlach case. But, if this semi-classical interpretation is wrong, what else can we propose ? Let us try to describe a possible experiment of the Rauch type which sustains the thesis that atoms coming out the magnet are in a non-local state, i.e. that atoms also are to be considered as non-local entities.

5. The non-locality of an atom leaving the Stern Gerlach apparatus.

Experiments that consist in combining different Stern-Gerlach apparatuses to illustrate the amazing properties of the superposition principle has been considered in the past by many authors and are described in various text-books [3]. We consider such an experiment on purpose of showing that an atom in the state Ψ_{out} is non-local in the sense of our definition. It is true that the experiment we shall describe is probably impossible to realize now and perhaps for ever because one

cannot control the inhomogeneous magnetic fields in the different magnets of the set-up with a sufficient accuracy [10]. But we are confident in the reasoning because of the great similarity with the work of Rauch et al. with neutrons. And this "gedanken" experiment has the advantage of working directly on one single atom or equivalently with the statistical weight of certainty.

One sends a beam of atoms flying in direction (1, 0, 0), with a definite spin direction along $\mathbf{n}(\theta, \phi)$ into a first Stern-Gerlach magnet and a magnetic field in direction $\mathbf{a} = (0, 0, 1)$. Then one puts right behind it a second Stern-Gerlach magnet identical to the first one except for the field direction which is opposite: $-\mathbf{a} = (0, 0, -1)$. This second magnet will converge the two beams (to describe the experiment we use the classical language) in such a way that they become parallel (fig4). Then one puts a third Stern-Gerlach magnet, identical to the second one, which will incline



A representation of the experiment, combining four Stern-Gerlach magnets, such that the original beam is recombined. In region R one can apply a constant magnetic field, and turn the spin over an angle of 2π. In this case the spin of an outgoing atom will be turned over an angle of π around the **a** direction (fig 4).

the beams towards each other. Finally one puts a fourth Stern-Gerlach magnet, identical to the first one, which will have the effect of reconstituting the two beams into a single one which is in principle identical to the original incoming beam. In a region R one applies a constant magnetic field, such that the spin can be turned by the Larmor precession, over different angles. One makes the experimental arrangement in such a way that the spin will turn over an angle of 2π. Notice that because such a rotation is equivalent to the identity, the axis of rotation (i.e. the direction of the magnetic field) is irrelevant for the results of the experiment.

Let us now analyze the quantum mechanical description, to see what will happen. As before, the state at time $t = 0$ is represented by a normalized vector Ψ_0 of the form $\Psi_D(x) \otimes (\alpha, \beta)$ (see section 4). Again we have to solve the Schrödinger-Pauli equation (7) for the evolution of this vector through the first Stern-Gerlach magnet, such that the state of the atom coming out of the first magnet is given by Ψ_{out} (see (8)), where A and B are separated regions (see section 4 and fig. 4). The second Stern-Gerlach magnet will not change anything to the spin components, since they

are (1, 0) and (0, 1), but will influence the direction of flight of the two wave packets Ψ_A and Ψ_B, so that they become parallel at its end, so that the atom coming out of the second magnet will be in a state $\alpha\cdot\Psi_{A'} \otimes (1,0) + \beta\cdot\Psi_{B'} \otimes (0,1) = \Psi'_{out}$ (see fig. 4). If we now apply a constant magnetic field in the region R localized on the upper beam (fig.4), then only the spin of the partial vector $\alpha\cdot\Psi_{A'} \otimes (1,0)$ will be influenced. If it is turned over an angle of 2π, then the spinor (1, 0) will be changed into the spinor (-1,0), no matter what the direction of the magnetic field applied in the region R can be. Of course, as we recall it in our reasonings about the Rauch experiment, these two spinors represent the same state. So by the effect of the Larmor rotation we find at the entry of the third Stern-Gerlach magnet the atom in a state represented by a vector $\alpha\cdot\Psi_{A''} \otimes (-1, 0) + \beta\cdot\Psi_{B''} \otimes (0, 1) = (-\alpha)\cdot\Psi_{A''} \otimes (1, 0) + \beta\cdot\Psi_{B''} \otimes (0, 1) = \Psi''_{out}$, where A" and B" are again separated regions in space (fig 4). To proceed and find the quantum mechanical effect of the two last Stern-Gerlach magnets, we notice that the Schrödinger dynamics is invariant for time reversal. So instead of trying to calculate the evolution up to the end of the fourth magnet of a state which is of the form Ψ''_{out} at the entrance of the third one, we can inverse the problem and consider the question : which state of the form $\Psi_{D''}(x) \otimes (\gamma, \delta)$ at the end of the fourth magnet will give rise to the state Ψ''_{out} at the entrance of the third magnet by a time reversed Schrödinger evolution ? The answer to this question is immediately given by the first part of the calculation, and is of course $\Psi_{D''}(x) \otimes (-\alpha, \beta)$. Hence if the spin of the incoming atom is in direction $\mathbf{n}(\theta, \phi)$, then using in the region R a constant magnetic field of appropriate magnitude but <u>arbitrary</u> direction, the spin state $\mathbf{n}''(\theta, \phi)$ of the outgoing atom has turned over an angle π around the direction **a** defined by the Stern-Gerlach magnets. In the classical picture of spinning particles this magnetic field in region R would have turned the spin over 2π, which means that each atom would leave the apparatus exactly in the same state as it enters. The same result would be found in the "semi-classical" picture discussed in section 3. If we complete the experiment by a Stern-Gerlach measurement of the orientation of the spin of the final atom, there is no doubt that it will fully confirm the standard quantum mechanics prediction of a rotation of π ! For instance, a preparation of the incoming atoms with spin in the direction $(\mathbf{a} \times \mathbf{b}) = (0,1,0)$, i.e. $S_{in}(\theta, \phi) = \sqrt{2}/2\cdot(1 - i, 1 + i)$ will give outgoing atoms with the same spin if the magnetic field in region R is inactive and will give outgoing atoms with spin in the opposite direction $-(\mathbf{a} \times \mathbf{b}) = (0,-1,0)$, i.e. $S_{out}(\theta, \phi) = \sqrt{2}/2\cdot(1 + i, 1 - i)$, if the magnetic field in region R is active. An analysis of the spin of the outgoing atoms with an appropriate Stern-Gerlach magnet (i.e. placed along the beam **b**, with a field orientation along $(\mathbf{a} \times \mathbf{b})$, with therefore a left-right alternative for the result), will show that **all** the outgoing atoms are left deflected if the magnetic field in region R is inactive, and **all** the outgoing atoms are right deflected if the magnetic field in region R is active. Remember that *the orientation of the magnetic field in region R is arbitrary*, and also that region R can be *situated on any of the two "paths"* inside the combined Stern-Gerlach apparatus we described. Also, *if the region R be placed elsewhere, in particular between the two paths*, then *no difference* can be seen whether the

magnetic field in R be active or inactive. In this sense it is clear that the experiment shows a realization of our definition of non-locality for quantum entities.

6. Imagining non-locality.

Let us now consider again the question of the physical meaning of a state like $\alpha \cdot \Psi_A \otimes (1,0) + \beta \cdot \Psi_B \otimes (0,1)$ for a single quantum entity . As was shown in the previous sections, such a state can easily be prepared for a particle like a neutron, or a more complex entity like an atom.

The standard interpretation of quantum mechanics allows the following statement. This state represents a quantum entity which is neither localized in region A with spin in the direction **a**, nor localized in region B with spin in direction - **a**. These two possible physical situations would be represented by different state vectors, i.e. by $\Psi_A \otimes (1,0)$ or $\Psi_B \otimes (0,1)$, respectively. In the framework of this standard interpretation, in our case, the physical properties "position" and "spin orientation" are only *potentialities* which can be *actualized* when proceeding to a measurement. Let us consider such a measurement which consists in installing in region A a perfect detector of the presence of the entity. Then, **either** the detector registrates the entity and we know for sure that immediately after this measurement the entity is localized in region A and has spin orientation **a** (reduction of the state vector $\alpha \cdot \Psi_A \otimes (1,0) + \beta \cdot \Psi_B \otimes (0,1)$ to $\Psi_A \otimes (1,0)$), **or** the detector doesn't registrate the entity and we know for sure that immediately after the measurement the entity is localized in region B and has spin orientation **-a** (reduction of the state vector $\alpha \cdot \Psi_A \otimes (1,0) + \beta \cdot \Psi_B \otimes (0,1)$ to $\Psi_B \otimes (0,1)$). The repetition of this experiment will give a statistical distribution of the alternative results in the proportion $|\alpha|^2 : |\beta|^2$. According to this standard interpretation, it does not make sense to speak about the position and about the spin orientation of the entity before the measurement, i.e. as long as these properties remain potentialities which are not yet actualized.

We think however that this interpretation of the state vector $\alpha \cdot \Psi_A \otimes (1,0) + \beta \cdot \Psi_B \otimes (0,1)$ can now be refined, because *some physical property related to the concept of position exists before the detection*. On the one hand, the very fact that the entity can only be found and would certainly be found in the union of the two separated regions, is in itself a property of the entity related to the concept of position. On the other hand, according to our discussion, one can influence the entity as a whole from either one of the two separated regions A and B by means of a local (macroscopically local) apparatus, which is not a detector of localization, acting only in one of the two regions at one time, and also, that such an action on the entity is not possible if this local apparatus is placed elsewhere (f.i. between A and B). And this very fact has been demonstrated by the experimental results of the Rauch experiment.

This particularity would easily be understood if we knew of classical examples of such a situation. But no such classical model can be found and this is of course the reason why the fact

looks so strange and why one tries to describe it by the negation of a classical property: the entity is non-local.

Let us briefly review how this property of locality manifests itself in classical physics. In order not to have to repeat several times the same explanation let us make some conventions about the words and the symbols we use. The regions where the classical entity can be found *as a whole*, i.e. with all of its invariant properties (mass, electric charge, energy for a conservative evolution, etc...) will always be refereed to as A, B ... regions. Locality contains the idea that these regions have a finite extension, i.e. that there is a finite largest linear dimension l_A, l_B, ... Because of this, we generally consider that the regions A, B, ... are moving when the time ellapses : they accompany the entity in its motion. The region where one tries to influence the entity, i.e. to change at least one of its properties will be refereed to as the R region. R has a finite extension, i.e. there is a largest linear dimension r. We assume that the apparatus which is build inside R in order to influence the entity has no action outside R, at least for all known fields of force (i.e. it is a macroscopical local apparatus). All the regions A, B..., R are simply connected regions. Now, A and B are separated if the distance $d_{AB} = \text{Min } \{|x_\alpha - x_\beta|, \alpha \in A, \beta \in B\}$ is larger than r ; in practical applications (see for example the cases discussed in the previous sections), the ratio d/r reaches several orders of magnitude, but this is only important in that it makes the conclusions more sure and more spectacular.

- A classical particle or a classical rigid body has a priori some definite spatial extension and it can at every moment be thought to lie in some region A. One can influence it, at some instant of time with an apparatus installed in R if and only if A and R intersect. Therefore, it is clearly impossible to do so for two separated regions A and B at one time. This is the reason why we agree that such a classical system is a local entity.

-The same is true of a classical entity composed of several particles or rigid bodies which we consider as a model of a deformable body, having permanent properties (total mass, total energy, total angular momentum, ...). The previous argument is easily repeated with of course a large enough region A but this is not an objection. Therefore, this entity also can be considered, in this sense, as a local entity.

- The same is true of any piece of continuous material like a plastic body, a liquid or a gas. If we insist on finding the totality of any invariant property of the entity inside a region A (which actually may expand with time), it can only be influenced from a region R if A and R intersect. And again, this cannot happen for two separated regions A and B at one time.

- The same is even true of a classical wave. Of course, we loose some invariant properties like the mass or the electric charge but there remain invariant properties like the total energy or the total momentum associated to the wave. A wave packet can then be localized in a region A which generally will expand. We speak of the wave as a whole in region A if and only if one can find the totality of its invariant properties inside of A. A system of wave packets obtained after the separation of one single wave packet by an interferometer can be considered as one or as several entities. If we insist on finding the totality of its energy in one region A (i.e. if we consider it as one single entity), then one has to take this region large enough in order to contain all the fragmentary wave packets. In this sense, it remains a local entity which cannot be influenced from a region R which does not intersect with A. One can of course modify the properties of the wave by a more local action on any one of the fragmentary wave packets (situated in A_1, A_2, ...). But one does not find the totality of any of the invariants associated to the wave in only one of these subregions. Therefore, although the wave remains a whole, it is not at all a whole like in the quantum case, and in order to find the totality of the wave we must consider the larger region A and not its subregions A_1, A_2, ...

We see that no one of these classical entities does fully reflect the quantum dilemma :

- on the one hand, the full entity (represented by its invariant properties) can be found in one region, either A or B.

- on the other hand the entity can be influenced (without detection) either from A or from B and not from other regions, at the same time.

Of course, the essence of this dilemma is not new, and it has been abundantly discussed in the literature on the foundations of quantum physics. One can find a lot of ingenious proposals aiming to solve the dilemma but we dare say that no one is satisfactory. Among them, we distinguish the following two which seem to be the most popular and which are at any rate the most "successful" ones.

- We already mentioned the standard interpretation "à la Bohr" which denies the actualisation of a property like the localisation before a suitable measurement. We recall that the action we described to take place in the region R is not a measurement of position. Therefore, as we said before, this interpretation, although correct, should now be completed taking into account the results of the new experiments described in this paper. It lacks to make reference to the very existence of some physical property related to the concept of position that exists, as we show, aside of a detection.

- Another ingenious proposal is that the entity consists of a wave and a particle (Bohm-De Broglie-Vigier theory). The particle is really localized somewhere, either in A or in B, while the wave exists in both regions. The invariant properties of the entity (mass, charge, total energy, spin) are

localized with the particle and the wave has no mechanical or electrical properties. Although the model can successfully explain the experiment, one can make the following remarks :

1) such a wave, which doesn't transport any mechanical or electrical property seems to be a very strange wave with no classical analogy. Its effects amounts only to create some de-localization of the entity, while keeping all its invariant properties localized, although statistically distributed.

2) such a particle seems to be a very strange object : it is truly localized in A or in B (with a statistically determined distribution) but it can be influenced, independently of this localization, and just in the same way by our local apparatus R working only in A (say).
For these reasons, we consider that this model also doesn't provide a satisfactory interpretation.

7. Proposal for a new interpretation of non-locality.

So what ? Confronted with this dilemma, we propose to go back to an old discussion of H. Poincaré [11] on the construction of the concept of euclidean space in the human mind. H. Poincaré emphasizes the important role of the human senses, in particular of the touch, in stimulating our feeling and our consciousness of the existence of space. He also emphasizes the capital role of the very physical existence of rigid bodies in the construction, by the human mind, of a concept of euclidean space. Now, let us for a moment suppose that another type of intelligence has developed, not like the human intelligence in contact with the classical entities, but with the quantum entities as primary object of knowledge. This is purely speculative but, we guess that such an intelligence would not develop the concept of euclidean space as an essential frame for the study of the environment. The surprise would probably be that an exploration of the world at large scale (by which we mean of course, at our classical scale), leads to the discovery of a property of localization and of an euclidian structure. For this intelligence, it would be a common place remark that one can influence an entity from two separated regions at the same time, and it would be a surprise that this property disappears at large scale. We do not want to push further this analysis because it probably meets profound questions on the very possibility of the function of intelligence at a quantum scale, but we only want to mention that it presents the dilemma with another enlightening. If we forget about our prejudices about an a priori euclidean structure of something we call space, an a priori which results from millions of years of formation of the human mind in contact with rigid bodies, there is no logical paradox of locality for the quantum entities. The euclidean space is a convenient theatre to describe the motions and the interactions of classical entities ; it is not convenient any more for the quantum entities. Although we **feel** such a situation as very paradoxical (presumably because our feelings cannot undo themselves from the just mentioned a priori), the reasoning we just made shows that our ratio can in principle imagine and

understand what the origin could be of this strange quantum situation. If this hypothesis is true, then the problem is elsewhere: one has to explain why the strongly correlated quantum entities, get local when organized into more complex entities. A possible explanation for this fact could be that the quantum correlations are very fragile and very sensitive to perturbations which are common in complex systems. As soon as these quantum correlations are broken, there are generally not any more states like our $\alpha \cdot \Psi_A \otimes (1,0) + \beta \cdot \Psi_B \otimes (0,1)$ but only states of the first kind, either $\Psi_A \otimes (1,0)$ or $\Psi_B \otimes (0,1)$, i.e. localized states.

Our conclusion is therefore that :

1) quantum entities are non local entities, and there is no logical paradox in this fact ; the paradox is rooted in the structure of the human mind which cannot easily imagine a world where the continuum we call space would not be a convenient tool any more for describing the entities.

2) locality is a common property for classical entities, not as an unavoidable a priori, but as a consequence of the organization of quantum entities into complex systems which generally causes a destruction of the fragile quantum correlations.

Let us finally remark that our interrogation on the role of space in quantum mechanics doesn't proceed along the lines sometimes suggested. It has been proposed by several physicists [12,13] that something in the intrinsic nature of the physical space, represented in classical non-relativistic theory by an euclidean space and in classical relativistic theory by some continuous manifold in a flat or curved space-time, should be changed in order to construct a unified theory covering the quantum phenomena and gravitation. These proposals aim generally to change the structure of space which becomes fluctuating at small scale [13]. We think that such a change which can be useful in order to handle features like indeterminism, uncertainty relations, state vector reduction, etc... does not help much for the problem of the macroscopical non-locality of quantum entities. In our approach, the change doesn't concern the structure of space but the role of space. A structured space is not considered as a necessary a priori theatre for the motion and interactions of the physical entities. Such a structured space contains a part of physical reality and a part of human mind construction. This last one can be misguiding when we meet new physical phenomena which can then appear as paradoxical. To use a kind of analogy, the structured space is like coloured spectacles through which we observe, study and partly understand the world. In order to discover, to study and to understand the scenes that happen to be of the complementary colour, it would be better to put the spectacles aside, or at least, if we cannot, to be conscious of the fact that we have them on.

There do exist approaches to the foundations of quantum physics which do not use space as a primary concept [14]. This type of approach could perhaps lend itself to a more explicit exploration of the general ideas that we presented.

On the other hand it seems also useful to proceed along the general lines of our paper in order to invent new experiments where the proposed ideas about the non-local character of quantum entities could be put in evidence.

8. References.

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3.See for instance *"The Feynman Lectures on Physics"*, Vol. III, Ch. 5,6.

4. These experiments are described in great details in several publications of members of this group. One finds a sufficient list of references in Rauch, H.,*"Neutron interferometric tests of quantum mechanics"* , Helv. Phys. Acta, 61, 589 (1988).

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11. About this question of the human part of the construction of space and other similar questions it is still very interesting to read again the famous trilogy of H. Poincaré *"La science et l'hypothèse"*, *"La valeur de la science"*, and*"Science et méthode"* , published by Flammarion, Paris, at the beginning of the century.

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13. See for example § 43.4 in , Misner, C. W., Thorne, K. S., and Wheeler, J. A., *"Gravitation"*, W.H.Freeman and Company, San Francisco (1973) ; Wheeler, J.A., *"Information, Physics, Quantum: the Search for Links"*, Proceed. 3rd Int. Symp. Foundations of Quantum Mechanics, Tokyo (1989), 354-358.

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