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Electromagnetic properties of the pion in the dressed quark model

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Abstract. The dressed quark model is extended to the pion. The form-factor and the structure function are calculated in the covariant approach with a Hulthén-type wave function for the pion. The Q^2-dependence of the structure function is presented.

1. Introduction

The application of quantum chromodynamics to processes with hadrons in initial and final states requires the knowledge of hadron structure in terms of quarks and gluons. An interesting approach in this direction has been proposed in [1, 2], where a 'two-level' structure of hadrons is assumed: mesons and baryons are supposed to be constructed of 'dressed' (or constituent) quarks and antiquarks which are built out of pointlike (or current) quarks and gluons. In this model the strong interaction is characterized by two different radii of confinement. One of them characterizes the dimension of hadrons, while the other is related to a dressed quark structure. Estimates [2] show that the dressed quark radius is 3–5 times smaller than the hadron one. As a consequence, the structure of dressed quarks is not seen at small momentum transfers. This fact is confirmed by successes of a simple constituent quark model in explaining the relations between hadronic total and elastic cross-sections [2] and an analysis of the hadron–nucleus scattering [2]. In these papers, however, a detailed structure of hadrons in terms of dressed quarks was not considered.

In this paper we extend the dressed quark model to the pion. In this case a hadronic structure reveals itself in much more detail with both the composition of the pion in terms of dressed quarks and the structure of dressed quarks being involved. Assuming that characteristics of a dressed quark inside a hadron do not depend on the type of the hadron, we use to this end the results [3] obtained from an analysis of nucleon structure functions. Then it remains to determine a pionic wave function in terms of dressed quarks. We choose its form and parameters from the behaviour of the pionic form-factor at moderate momentum transfers in the space-like region. As a consequence, pion structure functions are defined for the whole range of q^2, where q is the virtual photon momentum, because, as it is known [4, 5] the q^2-dependence of structure functions at large q^2 follows from the asymptotic properties of quantum chromodynamics.

The comparison of our calculations with future experimental data can give a valuable information concerning an applicability of the dressed quark model to the pionic structure. Existing experimental data on pionic structure functions at various q^2 are of preliminary and inconclusive character. On the whole, they speak in favour of the dressed quark hypothesis for the pion.

2. Pion form-factor

In our calculation we follow the technique developed by Landshoff and Polkinghorne [6] in the framework of the covariant parton model. Consider diagrams for the pion form-factor shown on Fig. 1. Internal lines correspond to

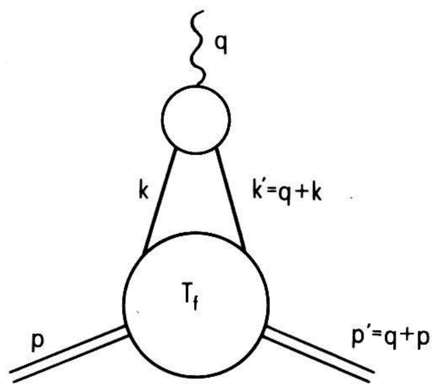

Figure 1.
Electromagnetic form-factor of pion in a dressed quark model.

dressed quarks (antiquarks). The momentum transfer q is supposed to be not large so that one can neglect the form-factor of dressed quarks. Then the pion form-factor is given by

$$2p^*_\alpha F(q^2) = -i(2\pi)^{-4} \sum_f \int d^4k \; \mathrm{Sp}\,\{e_f \gamma_\alpha T_f(p, k, q)\} \tag{1}$$

where we sum over all flavours f with quark charges e_f and the quark–pion interaction amplitude T_f; $p^* = (p+p')/2$. The vector in the integrand can be decomposed into vectors p^* and $k^* = (k+k')/2$ with scalar coefficient functions of $s = (p+k)^2$, q^2, k^2 and k'^2.

We calculate (1) in the infinite momentum frame assuming that p^* is a large longitudinal vector while q is a transversal one. Then the appropriate integration variables are s, $x = k^*_z/p^*_z$ and the transverse momentum $\mathbf{k}_\perp$. The Feynman contour of integration over s can be deformed to encircle right hand singularities of the amplitude $T(s)$. The nearest singularity is a pole corresponding to the one-particle quark intermediate state with a momentum $k_1 = p - k$ (Fig. 2). After the integration over s this intermediate quark will be on the 'mass shell' $k_1^2 = M_q^2$. As usual, we assume that within the confinement region quarks behave like ordinary particles with a mass M_q. Thus, the integral over x and $\mathbf{k}_\perp$ in (1) contains a pole term of the amplitude, and, therefore, two vertex parts determine its value. The vertex parts correspond to an interaction of a pion with a real quark and with a virtual one. We parametrize them by two scalar functions of virtuality k^2

$$\Gamma(p, k) = \gamma_5[a_1(k^2) + a_2(k^2)(M_q + k)^{-1}] \tag{2}$$

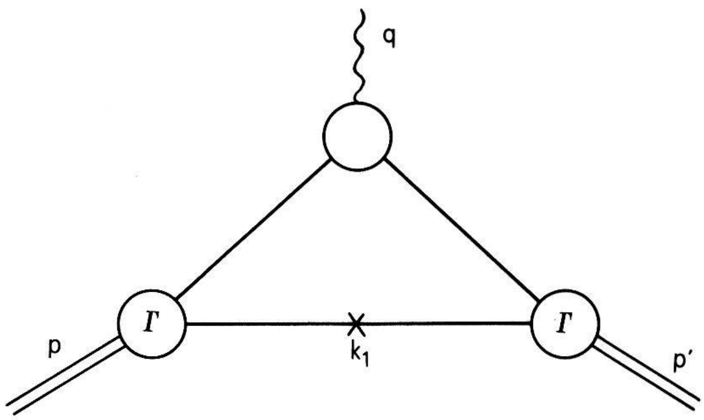

Figure 2.
One-quark pole approximation.

We introduce now two pion wave functions

$$\psi_i(k^2) = a_i(k^2)(M_q^2 - k^2)^{-1}, \qquad i = 1, 2. \tag{3}$$

Then the pion form-factor is

$$F(q^2) = (2\pi)^{-3}\int_0^1 dx(1-x)^{-1}\int dk_\perp \psi(k'^2)B(q)\psi(k^2) \tag{4}$$

Here ψ denotes the set ψ_i, $i = 1, 2$; 2×2 matrix B has matrix elements

$$\begin{aligned} B_{11} &= x(m_\pi^2 - q^2/2) + M_q^2 - (k^2 + k'^2)/2 + q^2/2 \\ B_{22} &= 1 - x \qquad B_{12} = B_{21} = M_q \end{aligned} \tag{5}$$

Virtualities are expressed in terms of integration variables

$$\begin{aligned} k^2 &= xm_\pi^2 - M_q^2 x(1-x)^{-1} - \mathbf{k}_\perp^2(1-x)^{-1} \\ k'^2 &= xm_\pi^2 - M_q^2 x(1-x)^{-1} - (\mathbf{k}_\perp - \mathbf{q}(1-x))^2(1-x)^{-1} \end{aligned} \tag{6}$$

The functions $\psi_i(k^2)$ are normalized by the condition $F(0) = 1$ at $q^2 = 0$.

Equation (4) expresses the pion form-factor through wave functions $\psi_i(k^2)$ which describe the pion structure in terms of dressed quarks. The relativistic invariance of these functions is a consequence of a covariant approach: it was assumed that at small momentum transfers one can use standard Feynman diagrams with dressed quark propagators having fixed singularities.

In a more general approach only the existence of wave functions $\varphi(x, \mathbf{k}_\perp)$ in an infinite momentum frame may be postulated. In this case, the form-factor is given by

$$F(q^2) = (2\pi)^{-3}\int dx[x(1-x)]^{-1}\int d^2k_\perp \varphi(x, \mathbf{k}_\perp)\varphi(x, \mathbf{k}_\perp + (1-x)\mathbf{q}) \tag{7}$$

It is easily seen that (4) and (7) are closely related. In the covariant approach both variables x and $\mathbf{k}_\perp$ are taken together in a combination (6). In principle, wave functions may not be relativistically invariant, because the condition of relativistic invariance applies only to the form-factor $F(q^2)$. There is a great variety of admissible noncovariant wave functions $\varphi(x, \mathbf{k}_\perp)$. In some models described by the light-cone Hamiltonian with fixed number of particles this arbitrariness can be eliminated by an additional condition that the spin of the pion is zero [8]. But no additional condition has been found up to now to exclude completely the freedom in the choice of φ. Therefore, we shall use both types of functions $\varphi(x, \mathbf{k}_\perp)$ and $\psi(k^2)$. This point will be discussed in §4.

3. Pion structure function

The calculation of the pion structure function in the covariant approach is based on diagrams of Fig. 3 for the virtual forward Compton scattering.

$$A_{\alpha\beta} = -i(2\pi)^{-4} \sum_f \int d^4k \, \mathrm{Sp}\,\{a^f_{\alpha\beta}(q, k) T_f(p, k, q)\} \tag{8}$$

where we sum over all flavors f, while $a^f_{\alpha\beta}$ is an amplitude for the virtual Compton scattering by a quark of flavor f. The integral (8) is evaluated in the infinite

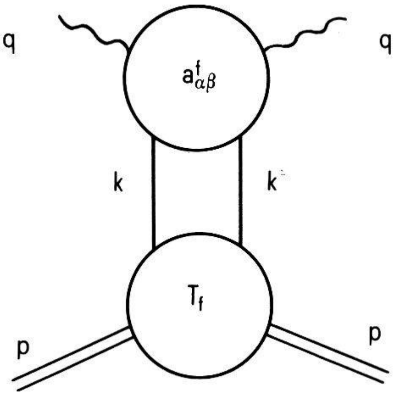

Figure 3.
Amplitude for the virtual forward Compton scattering.

momentum frame of the pion. As in the case of the pion, we introduce variables $s = (p+q)^2$, $x = k^*_z/p^*_z$ and $\mathbf{k}_\perp$, and replace the contour integral over s by a contribution from the one-quark state (Fig. 4). Then a quark of momentum k_1 will be on the mass shell and the remaining integral will contain the same vertex parts Γ which entered in the pion form-factor. Next we assume that one can disregard the k^2-dependence of the amplitude $a^f_{\alpha\beta}$, so that its discontinuity can be

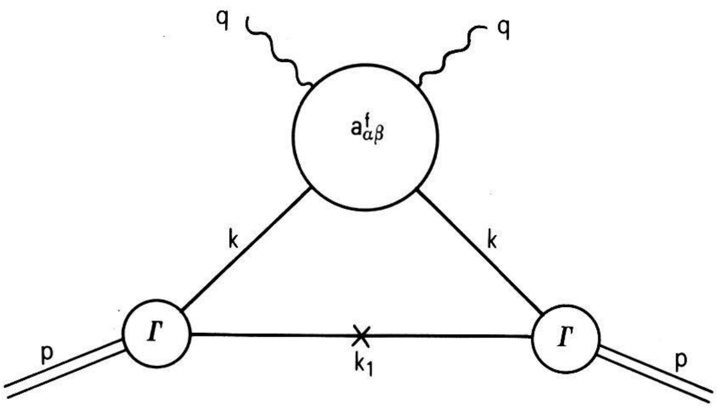

Figure 4.
One quark pole term in the amplitude for the forward Compton scattering.

related to quark structure functions. After all these approximations one comes to the following expression for the structure function F_2^π

$$F_2^\pi(\xi, q^2)=(2\pi)^{-3}\int_\xi^1 dx(1-x)^{-1}\int d^2k_\perp \psi(k^2)B(0)\psi(k^2)\sum_f F_2^f(\xi/x, q^2)$$

Here $F_2^f(y, q^2)$ is the structure function of the dressed quark of flavor f; $\psi(k^2)$ is a set of pion wave functions given by (3); k^2 is given by (6); the matrix $B(0)$ is defined by (5) with $q^2=0$.

Let us introduce a positive function $p(x)$

$$(1-x)p(x)=(2\pi)^{-3}\int d^2k_\perp \psi(k^2)B(0)\psi(k^2) \tag{10}$$

for which the condition $F(0)=1$ is equivalent to

$$\int_0^1 p(x)\,dx=1 \tag{11}$$

One considers $p(x)$ as a probability that a dressed quark inside a pion has a part x of the pion momentum in the infinite momentum frame. The pion structure function may be now rewritten in a simple form

$$F_2^\pi(\xi, q^2)=\int_\xi^1 dxp(x)\sum_f F_2^f(\xi/x, q^2) \tag{12}$$

The quark structure function F_2^f may be expressed in terms of probabilities $D_{f'}^f$ to find inside a dressed quark a point quark f' or gluon ($f'=g$)

$$F_2^f(x, q^2)=x\sum_{f'} e_{f'}^2 D_{f'}^f(x, q^2) \tag{13}$$

$D_{f'}^f(x, q^2)$ as functions of q^2 can be found from chromodynamics for sufficiently large q^2 [4, 5]. The chromodynamical expression for $D_{f'}^f$ may be presented in the form

$$D_{f'}^f(x, q^2)=\int_0^1 dx_1\,dx_2\,\delta(x-x_1x_2)E(x, t)D^f(x_2, 0) \tag{14}$$

where the evolution matrix E acts on the column of valence quark, sea and gluon probabilities, and

$$t=(2\pi)^{-1}\int_{-q_0^2}^{-q^2} dp^2\alpha(p^2)p^{-2} \tag{15}$$

$\alpha(p^2)$ is the running quark–gluon constant. Numerical values for the matrix E are given in [9] for $0\leqslant t\leqslant 0.5$.

Note that the structure function (12) contains only the total probability $p(x)$ to find a dressed quark with a given x inside the pion, so that the dependence on the transverse momenta $k_\perp$ is not essential in our approximation. Therefore, structure functions will not change drastically, if instead of a function $\psi(k^2)$ one uses a function $\varphi(x, k^2)$ introduced at the end of §3.

4. The choice of pion wave functions

As we have seen, in the covariant approach the pion is characterized by two wave functions (3) describing its quark structure. As it follows from (2) and (3), the function $\psi_2(k^2)$ has no pole singularity at $k^2 = M_q^2$. According to (4), at relatively large q^2 (but within the range of the dressed quark model) the form-factor behaves like $q^2\psi_1(q^2)$ or $\psi_2(q^2)$. It is natural to conjecture that at large q^2 one has $\psi_1(q^2) \sim (q^2)^{-2}$ and $\psi_2(q^2) \sim (q^2)^{-1}$. The simplest choice compatible with such behaviour is the Hulthén type double pole form

$$\begin{aligned} \psi_1(k^2) &= \xi_1(M_q^2 - k^2)^{-1} - (m_2^2 - k^2)^{-1}) \\ \psi_2(k^2) &= \xi_2(m_2^2 - k^2)^{-1} \end{aligned} \tag{16}$$

Constants ξ_1, ξ_2, quark mass M_q and a mass m_2 are the parameters to be determined. One of them is fixed by the normalization condition (11). Other parameters should follow from the comparison with an experiment.

The quark mass is not a completely free parameter. The dressed quark model assumes that dressed quarks are the same in all hadrons. Therefore, their mass should be close to 0.3 Gev. Numerical results in the next paragraph confirm that an agreement with an experiment may be obtained with M_q in a rather narrow region around 0.3 Gev. The pion appears here as a system with a very large binding energy. As a consequence, the probability $p(x)$ has no maximum at $x = \frac{1}{2}$; contrary to all expectations it is monotonously decreasing function of x.

We have studied also a non-covariant description of the pion by the wave function $\varphi(x, \mathbf{k}_\perp)$ [10] (see (7)). As in the case of nucleons [1], we suppose that $\varphi(x, \mathbf{k}_\perp)$ is a Lorentz scalar, and we establish the relation between x and a momentum in the center-of-mass system k_z through the equality of volumes, or

$$x(1-x)\, dk_z = m\, dx \tag{17}$$

where m is a mass scale. Then $k_z = m \ln x(1-x)^{-1}$.

Suppose that in the center-of-mass system $\varphi(x, \mathbf{k}_\perp)$ depends only on one argument $k^2 = k_z^2 + k_\perp^2$. Then $\varphi(k^2)$ is symmetrical under $x \to 1-x$ and has a maximum at $x = \frac{1}{2}$. The simplest choice for $\varphi(k^2)$ in this case is an oscillator-type wave function with a new parameter β

$$\varphi(k^2) = \exp(-\beta k^2/m^2) \tag{18}$$

Such a non-covariant approach can be easily criticized: the limiting procedure transformation from the infinite momentum frame to the center-of-mass system involves some arbitrariness, zero value of the pion spin is not taken into account. Nevertheless, we use this function in our calculations in order to study the dependence of structure function on the shape of $p(x)$ and, in particular, to study the role of the peak in $p(x)$-distribution. As we have seen, in the covariant approach such a peak may exist only for small binding energies.

5. Numerical calculations

Starting from (4) and (7) we have calculated the pion form-factor with both types of wave functions (16) and (18) for negative q^2 in the range $-6\ \text{Gev} \leq q^2 \leq 0$. The parameters ξ_1 and ξ_2 in (16) and m in (18) were fixed by the normalization

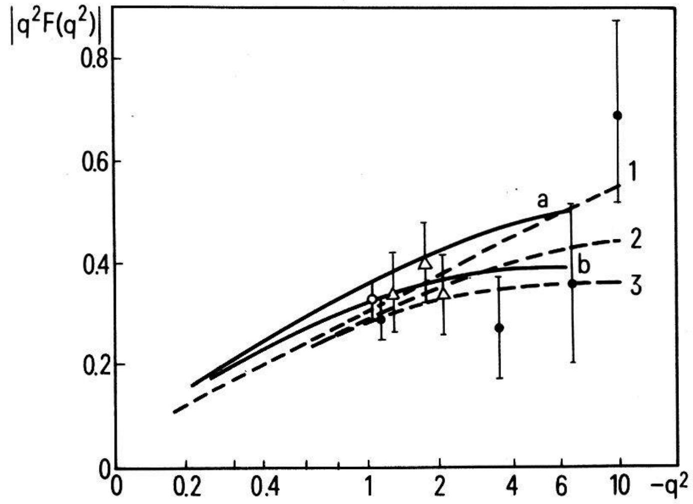

Figure 5.
$|q^2F(q^2)|$ for different wave functions. Data from [14]. Wave function (16) (solid line): (a) $M_q^2 = 4.9\,m_\pi^2$, $m_2^2 = 18.8\,m_\pi^2$ (b) $M_q^2 = 4.0\,m_\pi^2$, $m_2^2 = 25.2\,m_\pi^2$. Wave function (18) (dashed line): 1. $\beta = 0.30$, 2. $\beta = 0.35$, 3. $\beta = 0.40$.

condition (11) and by the requirement that the form-factor slope at $q^2 = 0$ corresponds to the experimental value $0.46f^2$ for the pion radius. The parameters M_q and m_2 in (16) and β in (18) were chosen to fit the data for $|q^2F(q^2)|$ at $q^2 = -4$ Gev^2. It was found that this condition fixes rather strictly both parameters M_q and m_2. Their optimal combination is $M_q^2 = 4.9m_\pi^2$ and $m_2^2 = 18.8m_\pi^2$. Our fits for $|q^2F(q^2)|$ are shown at Fig. 5 together with experimental points. Any deviation from the optimal combination leads to a sharp increase of this function at large q^2. In the case of wave function (18) the optimal points is $\beta = 0.35$.

Thus, experimental data for the pion form-factor in the space-like region enable us to determine pion wave functions practically without ambiguity in both cases (16) and (18). The distribution $p(x)$ of dressed quarks inside the pion calculated with wave functions (16) and (18) is shown on Fig. 6.

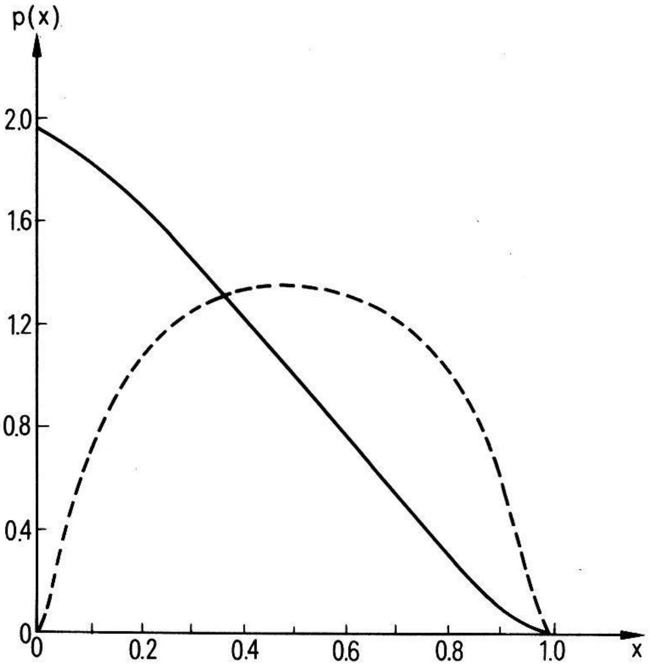

Figure 6.
Probability distribution $p(x)$ for wave functions (16) (solid line) and (18) (dashed line).

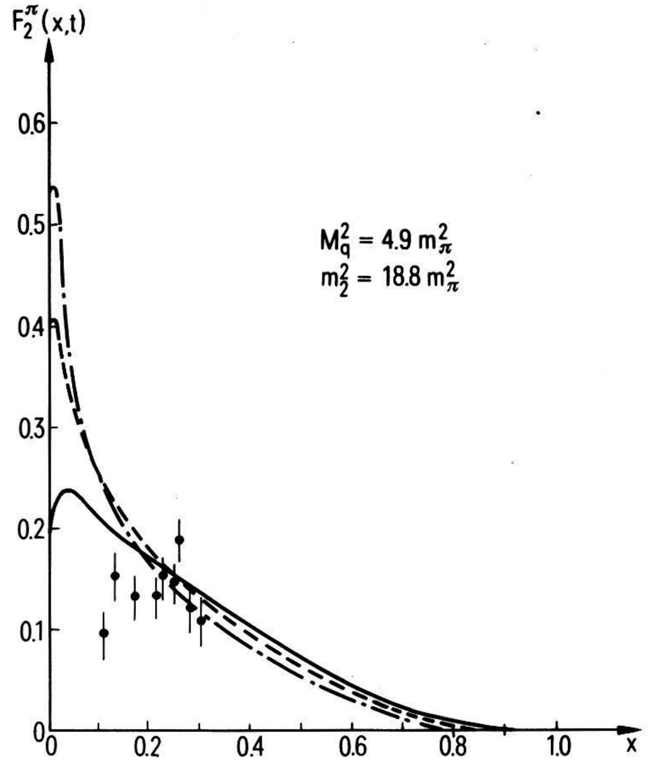

Figure 7.
Pion structure function with the wave function (16) for $t=0$ (solid line), $t=0.05$ (dashed line) and $t=0.10$ (dash-and dotted line.

As soon as pion wave functions are known one can go over to the determination of structure functions. To this end it is necessary to know an initial distribution of bare quarks and gluons in dressed quarks. In the dressed quark model this distribution is universal for hadrons, so that we can use the distribution found for nucleon at $-q^2=9\,\text{Gev}^2$ [3]. The bare quark-gluon content D^f of a

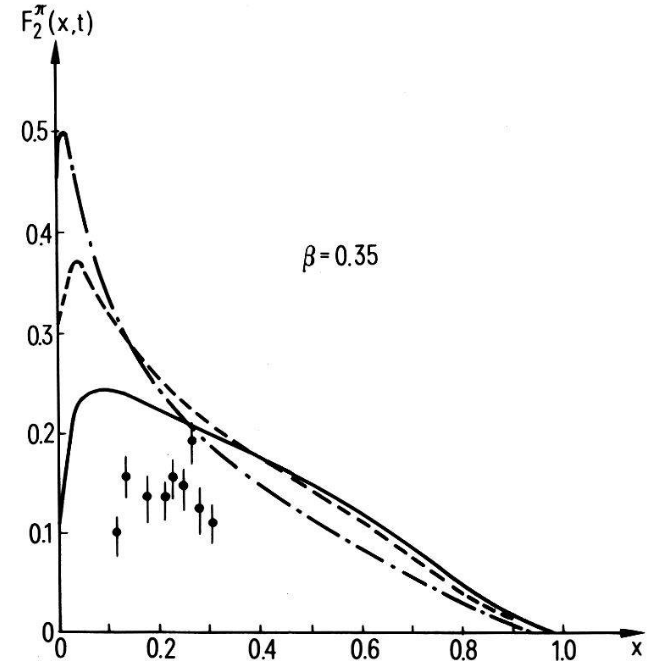

Figure 8.
Pion structure function with function (18) for $t=0$ (solid line), $t=0.05$ (dashed line) and $t=0.10$ (dash-and-dotted line). Data from [12].

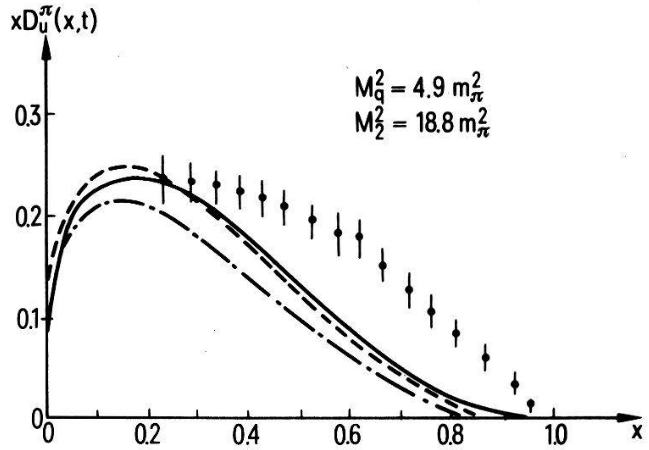

Figure 9.
Valence quark distribution in the pion with wave function (16) for $t=0$ (solid line), $t=0.05$ (dashed line), $t=0.10$ (dashed-and-dotted line); points from [13].

dressed quark f is usually subdivided into the contributions coming from valence quarks $V(x)$, sea quarks $S(x)$ and gluons $G(x)$. For $-q^2=9\ \mathrm{Gev}^2$ one has

$$V(x)=\frac{\Gamma(A+\frac{1}{2})}{\Gamma(A)\Gamma(\frac{1}{2})}\frac{(1-x)^{A-1}}{x^{1/2}} \tag{19}$$

$$S(x)=Cx^{-1}(1-x)^{D-1} \qquad G(x)=Gx^{-1}(1-x)^{E-1}$$

where $A=0.847$, $D=3.29$, $C=0.082$, $E=2.4$ and $G=E(1-6C/D-(2A+1)^{-1})$. With $D^f(x)$ given by (19) one can find from (12) and (13) a structure function at $-q^2=9\ \mathrm{Gev}^2$, while at large q^2 one should also use (14). Pion structure functions calculated for different t with the wave functions (16) and (18) are represented in Figs. 7 and 8.

Experimentally the pion structure function can be determined from the data on deep inelastic lepton-nucleon scattering with registration of a slow nucleon in the final state [11]; the structure function data [12] are also shown in Figs. 7 and 8. As far as our phenomenological structure functions are concerned, their behavior depends weakly on the choice of wave functions. The position of the maximum in the covariant approach is shifted towards small x. With increasing q^2 this shift is diminishing.

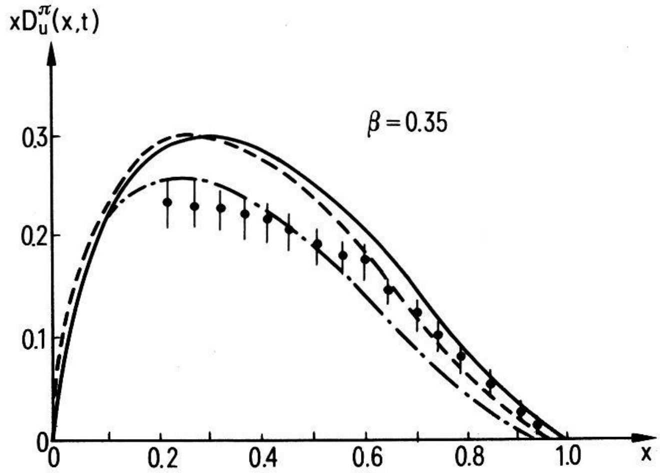

Figure 10.
Valence quark distribution in the pion with the wave function (18) for $t=0$ (solid line), $t=0.05$ (dashed line) and $t=0.10$ (dash-and-dotted line); points from [13].

The valence quark distribution in the pion $D^{\pi}(x, t)$ calculated with different wave functions are represented in Figs. 9 and 10 together with experimental data [13] obtained from the cross-section of the Drell-Yan process. We see that the calculations with the non-covariant wave function (18) are in better agreement with the experiment. In the case of the covariant function (16) the maximum of valence quark distribution lies considerably to the left from the experimental one. This discrepancy can be attributed to the too high binding energy of the pion in the covariant approach which leads to the absence of a maximum, as well as to the lack of symmetry under $x \to (1-x)$ for the probability $p(x)$. We would like to stress that, within the covariant approach, it is very difficult to displace the maximum of structure function into the region of large x. Such displacement would require a considerable decrease in the binding energy and, correspondingly, an increased value of the pion radius-in contradiction with the experiment.

We conclude by stating that although the existing experimental data about pion structure function are of rather preliminary character, because of large experimental errors and complexity of data extraction, the comparison of our results with data favors clearly a model of the pion as a system of dressed quark and antiquark.

REFERENCES

[1] G. Altarelli, N. Cabbibo, L. Maiani and R. Petronzio, Nucl. Phys. *B69*, 531 (1974).
[2] V. Anisovich, Proc. of the XIV Winter School, Leningrad, LINP, 1979.
[3] N. Cabbibo and R. Petronzio, Nucl. Phys. *B137*, 395 (1978).
[4] Yu. Dokshitser, D. Dyakonov and S. Troyan, Phys. Reports. *58*, 269 (1980).
[5] G. Altarelli and G. Parisi, Nucl. Phys. *B126*, 298 (1977).
[6] P. Landshoff and J. Polkinghorne, Nucl. Phys. *B53*, 473 (1973).
[7] S. Drell and T. Yan, Phys. Rev. Lett. *24*, 181 (1970).
[8] M. V. Terentiev, Jadern. Fizika. *24*, 207 (1976) (Sov. J. Nucl. Phys. *24*, 106 (1976)).
[9] W. Furmansky and S. Pokorski, CERN Report TH. 2685 (1979).
[10] M. Braun and V. Yu. Novozhilov, Vestnik of Leningrad University, *22*, 5 (1979).
[11] N. Volkonsky, V. Yu. Novozhilov, L. V. Prokhorov and V. Soloviev, Jadern. Fizika. *25*, 145 (1977) (Sov. J. Nucl. Phys. *25*, 79 (1977)).
[12] V. Yu. Novozhilov, Jadern. Fizika. *28*, 1656 (1978) (Sov. J. Nucl. Phys. *28*, 853 (1978)).
[13] J. Badier et al., CERN Report EP. 80–148 (1980).
[14] C. Bebek et al., Phys. Rev. *D17*, 1693 (1978).
[15] S. Brodsky and G. Lepage, Phys. Rev. *D23*, 1152 (1981).