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On the Theory of Particles of Spin 1

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Zusammenfassung. Verschiedene Formulierungen der Theorie des Vektormesons werden diskutiert. Es wird gezeigt, dass die Quellenterme, mit denen ein Vektormeson gekoppelt werden kann, im allgemeinen verschieden sind von den Termen, die man naiverweise als Quellen identifizieren würde. Gewöhnlich enthalten die wahren Quellen Ableitungen der Feldoperatoren, aber in Sonderfällen, zum Beispiel in der Theorie von YANG und MILLS, fallen die Ableitungen fort. Ein solches Verhalten ist für die Renormierung der Theorie günstig.

1. Introduction

The quantum field theory of particles of spin one originated around 1937 and for a time it was widely used in attempts to construct a meson theory of nuclear forces. The interest in spin one particles naturally diminished when the pseudoscalar nature of the pion was established. It also became clear that spin 1 theories, unlike those of spins 0 and 1/2, presented grave difficulties to renormalization programmes. It then seemed at least a fortunate accident, if not part of a deeper design that, with the exception of the photon, particles of spin 1 seemed to form no part of nature's scheme.

Recently, however, attractive developments of the theory of weak interactions have been put forward which are based on the idea of a heavy boson providing an intermediate link in β-decay processes[1][2]. The symmetries of these processes indicate that if such intermediates exist they should have spin 1. (See, however, FRONSDAL and GLASHOW[3].)

If these ideas are correct, we can no longer shirk facing the difficulties presented by spin 1 theory. Already it has been pointed out that for certain special interactions renormalization difficulties are less grave than was originally thought[4]. However, spin 1 theory is notorious for the variety of the equivalent, or nearly equivalent ways in which it is possible to present it, and there appears to the writer to be a certain lack of clarity, if not confusion, in what is being and has been said about these particles. It is the aim of this article to present an account of spin 1 theory which may illuminate some of the questions recently discussed and which in particular subjects the problem of renormalization to a rescrutiny. This study leads to the establishment of certain fairly simple general criteria of renormalizability. The theory of YANG and MILLS is used as the one known concrete example of a theory, for which such criteria hold.

2. Different Formulations of the Theory of Spin 1 Particles

At a very early stage it was pointed out that a formulation of the theory of free particles of mass m and spin 1 was provided by Proca's equations*) **)

$$\left.\begin{aligned} \partial_\mu A_\nu - \partial_\nu A_\mu - m\, F_{\mu\nu} &= 0 \\ \partial_\nu F_{\mu\nu} - m A_\mu &= 0 \end{aligned}\right\} \tag{1}$$

In this formulation the field associated with the particle is described by ten components, the A_μ which form a four-vector and the $F_{\mu\nu}$, an antisymmetric tensor, the two quantities appearing on a completely equal footing. By analogy with the electromagnetic field most authors have preferred to use the quantities

$$a_\mu = m^{-\frac{1}{2}} A_\mu \text{ and } f_{\mu\nu} = m^{\frac{1}{2}} F_{\mu\nu},$$

so as to give the equations the form

$$\left.\begin{aligned} \partial_\nu f_{\mu\nu} - m^2 a_\mu &= 0 \\ f_{\mu\nu} &= \partial_\mu a_\nu - \partial_\nu a_\mu \end{aligned}\right\} \tag{1'}$$

but, although (for $m \neq 0$) this transformation is trivial, the original form seems slightly preferable for the following analysis.

The free particle equations (1) have as immediate consequences the relations

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0 \tag{2a}$$

$$\partial_\mu A_\mu = 0 \tag{2b}$$

The first line of (2) represents four component equations of which, however, only three are essentially independent, so that altogether (2) cuts down the 10 field components to six independent ones, just the number necessary to describe a particle capable of three states of polarization.

In the more fashionable formalism the same situation is described somewhat differently. There the variables are in the first place usually normalised as in equations (1), but in addition a more substantial difference in emphasis is made by replacing (1) by the single second-order equation

$$\partial_\nu (\partial_\mu A_\nu - \partial_\nu A_\mu) - m^2 A_\mu = 0 \tag{3}$$

*) A completely equivalent formulation of this theory is provided by the equation

$$(\partial_\mu \beta_\mu + m)\, \psi = 0$$

where the β_μ are 10×10 matrices satisfying the Petiau-Duffin commutation rules

$$\beta_\lambda \beta_\mu \beta_\nu + \beta_\nu \beta_\mu \beta_\lambda = \delta_{\lambda\mu} \beta_\nu + \delta_{\mu\nu} \beta_\lambda,$$

but we shall not make use of this formulation here.

**) We use the metric in which

$$a^2 = a_\nu\, a_\nu = a_0^{\,2} - \boldsymbol{a} \cdot \boldsymbol{a}.$$

with the field $F_{\mu\nu}$ eliminated by means of the second set of equations of (1). If the field $F_{\mu\nu}$ is used at all, it is thought of as the four-dimensional curl of A_μ *by definition*:

$$F_{\mu\nu} \equiv \frac{1}{m} (\partial_\mu A_\nu - \partial_\nu A_\mu). \tag{4}$$

While for the free Proca field the change from (1) to (3) and (4) is a trifling matter, we shall see below that an interesting difference in the approach to the problem of interacting fields results from it.

It should also be noted that a corresponding change of emphasis in the direction opposite to the conventional one is just as legitimate. We may, if we wish, look on the Proca scheme as given by the equations

$$\partial_\mu(\partial_\rho F_{\nu\rho}) - \partial_\nu(\partial_\rho F_{\mu\rho}) - m^2 F_{\mu\nu} = 0 \tag{5}$$

which are obtained by eliminating the vector A_μ from (1). On this view A_μ would be defined by

$$A_\mu \equiv \frac{1}{m} \partial_\nu F_{\mu\nu}. \tag{6}$$

When scheme (1) is used, equation (3) is often thought of as composed of the equations

$$(\partial^2 + m^2) A_\mu = 0 \tag{7}$$

and

$$\partial_\mu A_\mu = 0 \tag{8}$$

the latter of which is a consequence of (4), but not of (7). Together, equations (7) and (8) are equivalent to (4) and they are particularly convenient for one familiar version of the quantization of spin 1 equations, in which $\partial_\mu A_\mu$ is treated as a subsidiary condition restricting the independence of the four components of A_μ allowed by (7).

This attitude is equally possible if one chooses the $F_{\mu\nu}$ field as independent variables, for equation (5) may be replaced by

$$(\partial^2 + m^2) F_{\mu\nu} = 0 \tag{9}$$

and

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0 \tag{10}$$

and the relation between (5), (9) and (10) is precisely the same as that between (4), (7) and (8). One may, if one so chooses, regard (9) as the equation of motion and (10) as a subsidiary condition.

The reason these variations of approach deserve some attention becomes apparent when one goes over to the study of the Proca field in interaction with external sources or other fields. According to which approach is chosen, such sources will be defined differently and as a result confusion may arise. Let us discuss the cases one by one.

In a theory based on equations (1), one would naturally introduce interactions with other fields by writing

$$\left.\begin{aligned} \partial_\mu A_\nu - \partial_\nu A_\mu - mF_{\mu\nu} &= m_{\mu\nu} \\ \partial_\nu F_{\mu\nu} - mA_\mu \qquad\quad &= j_\mu \end{aligned}\right\} \tag{11}$$

and the two types of 'sources' j_μ and $m_{\mu\nu}$ appear on equal footing. For instance, in discussing renormalization, as we shall do below, there is no reason to expect more divergent behaviour from $m_{\mu\nu}$ sources than from j_μ of similar structure; on the other hand, the same theory written in the form (5) would take on the form

$$(\partial^2 + m^2)\, A_\mu = s_\mu \tag{12}$$

where

$$- s_\mu = m j_\mu + \partial_\nu m_{\mu\nu}$$

and it would then *appear* that the part of the source involving $m_{\mu\nu}$ must necessarily have the characteristics of a derivative coupling. That in general this appearance has little physical meaning can be seen, for instance, by writing the theory instead in the form (9) which becomes

$$(\partial^2 + m^2)\, F_{\mu\nu} = q_{\mu\nu} \tag{13}$$

where

$$- q_{\mu\nu} = m m_{\mu\nu} + \partial_\mu j_\nu - \partial_\nu j_\mu$$

so that now the j_μ coupling appears to be of the derivative kind. These general considerations will form the background of the following discussion of two separate, but related problems.

3. The Independent Degrees of Freedom of the Spin 1 Field

We have seen that the equations of non-interacting spin 1 particles include, besides the equation of motion proper, the equations (2), which according to the particular formulation of the theory may appear either all as consequences of the equations of motion, or partly as such consequences and partly as subsidiary conditions. What happens to these relations in the presence of sources? Clearly they will in general no longer hold. In the linear (Proca) formulation the first set of equations (2) will only hold if

$$\partial_\lambda m_{\mu\nu} + \partial_\mu m_{\nu\lambda} + \partial_\nu m_{\lambda\mu} = 0 \tag{14}$$

and the second only if

$$\partial_\mu j_\mu = 0 \tag{15}$$

In the conventional second order formulation, (2b) will be violated unless $\partial_\mu s_\mu = 0$, which is the same as (15), and in the 'inverted' second order scheme (2a) is violated unless $\partial_\lambda q_{\mu\nu} + \partial_\mu q_{\nu\lambda} + \partial_\nu q_{\lambda\mu} = 0$, which evidently reduces to (14).

Now it was already mentioned that the conditions (2) are necessary to guarantee that the field should have only the degrees of freedom characteristic of a spin 1 particle. Does this mean that in interactions violating either (14) of (15) additional degrees of freedom of the field, latent for a free field, are excited? This point of view has been expressed occasionally[5]), at least in respect of (15), though due to the particular choice of formulation the equivalent role of (14) has apparently not been noted. This apparent difference in the conclusions with regard to two precisely similar situations shows that a somewhat deeper analysis is needed.

One is here concerned with the total time development of the fields—in quantal terms with the Heisenberg picture—and therefore one should state the field equations in a canonical form. In the most familiar presentations such a canonical form is usually given only in momentum space and to state the corresponding equations in coordinate space is a little troublesome and unless one employs Schwinger's[6]) general scheme, cannot be done simply in terms of covariant quantities only, although the theory is of course covariant as a whole. In these circumstances it is appropriate to make the otherwise retrograde step of writing Proca's equation in a three-dimensional vector notation. Using symbols that stress the formal similarity to electromagnetism, we put

$$F_{0i} = E_i,\ \frac{1}{2}\varepsilon_{ijk} F_{jk} = B_i,\ A_0 = \phi,\ m_{0i} = P_i,\ \frac{1}{2}\varepsilon_{ijk} m_{jk} = M_i,\ j_0 = \varrho$$

Then equations (1) become

$$\left.\begin{aligned} -\nabla\phi - \dot{\boldsymbol{A}} - m\boldsymbol{E} &= \boldsymbol{P} \\ \nabla_\wedge \boldsymbol{A} - m\boldsymbol{B} &= \boldsymbol{M} \\ \nabla_\wedge \boldsymbol{B} + \dot{\boldsymbol{E}} - m\boldsymbol{A} &= \boldsymbol{j} \\ -\nabla.\boldsymbol{E} - m\phi &= \varrho \end{aligned}\right\} \tag{16}$$

while the consequent equations (2) appear as

$$\left.\begin{aligned} \nabla_\wedge \boldsymbol{E} + \dot{\boldsymbol{B}} &= 0 \\ \nabla.\boldsymbol{B} &= 0 \\ \nabla.\boldsymbol{A} + \dot{\phi} &= 0 \end{aligned}\right\} \tag{17}$$

The scheme (17) may be derived by variation of the Lagrangian*)

$$\left.\begin{aligned} L = {} & \frac{m}{2}(\boldsymbol{E}^2 - \boldsymbol{B}^2) + \frac{m}{2}(\boldsymbol{A}^2 - \phi^2) \\ & + \frac{1}{2}\dot{\boldsymbol{A}}.\boldsymbol{E} - \frac{1}{2}\boldsymbol{A}.\dot{\boldsymbol{E}} + \nabla\phi.\boldsymbol{E} + \nabla_\wedge \boldsymbol{B}.\boldsymbol{A} \\ & + \boldsymbol{P}.\boldsymbol{E} - \boldsymbol{M}.\boldsymbol{B} + \boldsymbol{j}.\boldsymbol{A} - \varrho\phi. \end{aligned}\right\} \tag{18}$$

*) We assume for simplicity that the sources are not themselves functions of the field variables, but only slight modification of the argument is needed for the more general case.

but in this form L is not directly suitable for bringing into canonical form, in the first place because the variables ϕ and $\boldsymbol{B}$ enter without their time derivatives. However, these variables may be eliminated by using the second and fourth equations of (16).

We then get

$$\left.\begin{aligned} L = \frac{m}{2}(\boldsymbol{E}^2 + \boldsymbol{A}^2) + \frac{1}{2m}((\nabla.\boldsymbol{E})^2 + (\nabla_{\wedge}\boldsymbol{A}^2) + \frac{1}{2}\dot{\boldsymbol{A}}.\boldsymbol{E} - \frac{1}{2}\boldsymbol{A}.\dot{\boldsymbol{E}} \\ + \boldsymbol{j}.\boldsymbol{A} + \boldsymbol{P}.\boldsymbol{E} - \frac{1}{m}\boldsymbol{M}.\nabla_{\wedge}\boldsymbol{A} + \frac{1}{m}\varrho\nabla.\boldsymbol{E} + \frac{1}{2m}\boldsymbol{M}^2 + \frac{1}{2m}\varrho^2. \end{aligned}\right\} \tag{19}$$

Here we have dropped a spatial divergence; the last two terms may also be disregarded in this discussion as they only influence the equations of motion of the sources**).

From this Lagrangian, equations (16) are easily derivable, in the sense that the symbols $\boldsymbol{B}$ and ϕ are considered as given by their values from (16) by *definition*.

Even in this form L is not yet ready for a canonical formalism. The next step is to split the two fields $\boldsymbol{A}$ and $\boldsymbol{E}$ into (spatially) transverse and longitudinal parts.

For any 3-dimensional vector field $\boldsymbol{v}$ we make the (spatially***)) non-ocal transformation

$$\boldsymbol{v} = \boldsymbol{v}^{(1)} + \boldsymbol{v}^{(2)}; \ \nabla.\boldsymbol{v}^{(1)} = 0 \qquad \nabla_{\wedge}\boldsymbol{v}^{(2)} = 0 \tag{20}$$

In this notation we find from (16) that

$$\left.\begin{aligned} -\dot{\boldsymbol{A}}^{(1)} - m\boldsymbol{E}^{(1)} &= \boldsymbol{P}^{(1)} \\ \dot{\boldsymbol{E}}^{(2)} - m\boldsymbol{A}^{(2)} &= \boldsymbol{j}^{(2)} \end{aligned}\right\} \tag{21}$$

and these relations may be used to eliminate $\boldsymbol{E}^{(1)}$ and $\boldsymbol{A}^{(2)}$ from (19). In this way we find

$$\left.\begin{aligned} L = &-\frac{1}{2m}(\dot{\boldsymbol{A}}^{(1)2} + \dot{\boldsymbol{E}}^{(2)2}) + \frac{1}{2m}((\nabla_{\wedge}\boldsymbol{A}^{(1)})^2 + (\nabla.\boldsymbol{E}^{(2)})^2) \\ &+ \frac{m}{2}(\boldsymbol{A}^{(1)2} + \boldsymbol{E}^{(2)2})) \\ &+ \boldsymbol{A}^{(1)}.(\boldsymbol{j}^{(1)} + \frac{1}{m}\dot{\boldsymbol{P}}^{(1)} - \frac{1}{m}\nabla_{\wedge}\boldsymbol{M}) \\ &+ \boldsymbol{E}^{(2)}.(\boldsymbol{P}^{(2)} - \frac{1}{m}\dot{\boldsymbol{j}}^{(2)} - \frac{1}{m}\nabla\varrho) \\ &- \frac{1}{2m}(\boldsymbol{P}^{(1)2} - \boldsymbol{M}^2 + \boldsymbol{j}^{(2)2} - \varrho^2) \end{aligned}\right\} \tag{22}$$

**) As FRONSDAL and GLASHOW[3]) point out, the form of these terms is important for calculating the form of the interaction between the source fields.

***) This splitting must be distinguished from the four-dimensional split discussed, e. g. by GLASHOW[4]).

Here again we have discarded a spatial divergence, and now also a total time derivative. The last term is again of no importance to the equations of motion we are considering. From this Lagrangian the required equations of motion follow again, but now clearly in a form which shows that the variables canonically conjugate to $\boldsymbol{A}^{(1)}$ and $\boldsymbol{E}^{(2)}$ respectively are $-\dot{\boldsymbol{A}}^{(1)}/m$ and $-\dot{\boldsymbol{E}}^{(2)}/m$, so that the transition to Hamiltonian form is immediate.

We are particularly interested in the commutation rules for the canonical variables. In writing them down we must ensure that they are compatible with the conditions (20) on $\boldsymbol{A}^{(1)}$ and $\boldsymbol{E}^{(2)}$. To ensure this one may put

$$\left.\begin{aligned} [A_m^{(1)}(\boldsymbol{x}),\ A_n^{(1)}(\boldsymbol{y})] &= \frac{m}{i}\left[(\delta_{mn} - \frac{\partial_m \partial_n}{\nabla^2}\,\delta(\boldsymbol{x}-\boldsymbol{y})\right] \\ [E_m^{(2)}(\boldsymbol{x}),\ E_n^{(2)}(\boldsymbol{y})] &= \frac{m}{i}\,\frac{\partial_m \partial_n}{\nabla^2}\,\delta(\boldsymbol{x}-\boldsymbol{y}) \end{aligned}\right\} \tag{23}$$

(all other pairs commuting).
To eliminate the spatially non-local operators on the right hand sides, we may go back to the total fields A_m and E_n by using (21). This gives

$$\left[A_m(\boldsymbol{x}) + \frac{1}{m}\,j_m^{(2)}(\boldsymbol{x}),\ E_n(\boldsymbol{y}) + \frac{1}{m}\,P_n^{(1)}(\boldsymbol{y})\right] = \frac{1}{i}\,\delta(\boldsymbol{x}-\boldsymbol{y})\,\delta_{mn} \tag{24}$$

The quantities

$$\hat{\boldsymbol{A}} \equiv \boldsymbol{A} + \frac{1}{m}\,\boldsymbol{j}^{(2)} \text{ and } \hat{\boldsymbol{E}} \equiv \boldsymbol{E} + \frac{1}{m}\,\boldsymbol{P}^{(1)}$$

are thus the canonical variables in terms of which the field equations in the Heisenberg representation should be discussed. Going right back to the form (16) of Proca's equations, with the use of the further convenient definitions $m\hat{\boldsymbol{B}} \equiv \nabla_\wedge \hat{\boldsymbol{A}}$, $m\hat{\phi} \equiv -\nabla . \hat{\boldsymbol{E}}$, we see that their canonical form is

$$\left.\begin{aligned} -\nabla\hat{\phi} - \dot{\hat{\boldsymbol{A}}} - m\hat{\boldsymbol{E}} &= \boldsymbol{P}^{(2)} - \frac{1}{m}\,\boldsymbol{j}^{(2)} - \frac{1}{m}\,\nabla\varrho \\ \nabla_\wedge \hat{\boldsymbol{A}} - m\hat{\boldsymbol{B}} &= 0 \\ -\nabla_\wedge \hat{\boldsymbol{B}} + \dot{\hat{\boldsymbol{E}}} - m\hat{\boldsymbol{A}} &= \boldsymbol{j}^{(1)} + \frac{1}{m}\,\boldsymbol{P}^{(1)} - \frac{1}{m}\,\nabla_\wedge \boldsymbol{M} \\ -\nabla . \hat{\boldsymbol{E}} - m\hat{\phi} &= 0 \end{aligned}\right\} \tag{25}$$

It is now apparent that the sources entering the *canonical* equations automatically satisfy conditions (14) and (15). There is therefore now no question of spurious degrees of freedom being present in the field, whatever the coupling. What is not made clear in the usual treatment is that the sources to which the Proca field can couple are not those one would expect from naively writing down the equations.

The appearance of the equations (25) is of course not covariant; nevertheless they are equivalent to the original equations. This is evidently so because the definitions of the quantities $\hat{\boldsymbol{A}}$, $\hat{\boldsymbol{E}}$ etc. are themselves not covariant. It should perhaps be re-emphasized that the transformation to the new variables is non-local, but only in a three-dimensional spatial sense, so that it presents no obstacles to this discussion based on the Heisenberg representation.

3. Renormalization of Spin 1 Theories

To discuss the renormalization of the theory, let us return to equations (11) as our starting point. We obtain by simple calculation the second order equations

$$\left.\begin{aligned} -(\partial^2+m^2)\,A_\mu &= m j_\mu + \partial_\nu m_{\mu\nu} + \frac{1}{m}\,\partial_\mu\partial_\nu j_\nu \\ -(\partial^2+m^2)\,F_{\mu\nu} &= m m_{\mu\nu} + \partial_\mu j_\nu - \partial_\nu j_\mu + \frac{1}{m}\,\partial_\rho(\partial_\mu m_{\nu\rho} + \partial_\nu m_{\rho\mu} + \partial_\rho m_{\mu\nu}) \end{aligned}\right\} \tag{26}$$

These equations may evidently also be symbolically written as

$$\begin{aligned} A_\mu &= -\frac{m}{\partial^2+m^2}\left(\delta_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{m^2}\right)\left(j_\nu + \frac{1}{m}\,\partial_\rho m_{\nu\rho}\right) \\ F_{\mu\nu} &= -\frac{m}{\partial^2+m^2}\left[\delta_{\mu\rho}\,\delta_{\nu\sigma} + \frac{1}{m^2}\,(\partial_\mu\partial_\sigma\,\delta_{\nu\rho} + \partial_\nu\partial_\rho\delta_{\mu\sigma} + \partial^2\delta_{\mu\rho}\delta_{\nu\sigma})\right] \times \\ &\qquad \left[m_{\rho\sigma} + \frac{1}{m}\,(\partial_\rho\, j_\sigma - \partial_\sigma\, j_\rho)\right]. \end{aligned} \tag{27}$$

and hence we see that a theory in which the $F_{\mu\nu}$ are treated as an independent set of variables involves as Green's functions the well known expression

$$\frac{\mathfrak{P}_{\mu\nu}}{(\partial^2+m^2)} \equiv \frac{m}{(\partial^2+m^2)}\left(\delta_{\mu\nu} + \frac{\partial_\mu\partial_\nu}{m^2}\right)$$

for the A_μ field and the less familiar Green's function

$$\frac{\mathfrak{P}_{\mu\nu,\,\varrho\sigma}}{(\partial^2+m^2)} \equiv -\frac{m}{(\partial^2+m^2)}\left[\delta_{\mu\rho}\,\delta_{\nu\sigma} + \frac{1}{m^2}\,(\partial_\mu\partial_\sigma\delta_{\nu\rho} + \partial_\nu\partial_\rho\delta_{\mu\sigma} + \partial^2\delta_{\mu\rho}\delta_{\nu\sigma})\right] \tag{28}$$

For the $F_{\mu\nu}$ field*). In an approach that does not treat the $F_{\mu\nu}$ as independent variables, which therefore does not include $m_{\mu\nu}$ sources explicitly, the term involving $1/m^2$ in $\mathfrak{P}_{\mu\nu,\rho\sigma}$ drops out. In our approach, however, it must clearly play as important a part as the corresponding term involving $1/m^2$ in $\mathfrak{P}_{\mu\nu}$.

In the usual procedure of discussing renormalization in terms of a series expansion of the S-matrix these Green functions reappear as the

*) We omit here the antisymmetrization of the Green's function with respect to μ and ν and with respect to ϱ and σ.

propagators associated with the internal lines, and in them the terms in $1/m^2$ are responsible for the fact that spin 1 theories are in general not renormalizable.

It has recently been pointed out[4]) that in the special case when the term $\partial_\mu \partial_\nu/m^2$ in $\mathfrak{P}_{\mu\nu}$ has no effect, i.e. if in terms of the current s_μ of equation (12) we have $\partial_\mu s_\mu = 0$ (a conserved current), the theory is renormalizable provided only s_μ satisfies certain conditions of simplicity. If, for the sake of brevity, we confine ourselves to source functions constructed from boson fields the renormalizability conditions on a theory with conserved current are that any term in s_μ should involve (i) the products of no more than three boson field operators, (ii) if it involves the derivative of one field operator this may be multiplied by at most one non-differentiated field operator, and (iii) not more than one derivative of a field shall occur in any term.

In this context one should note that the condition of current conservation is necessary simply because the offending term in $\mathfrak{P}_{\mu\nu}$ contains two derivatives; the quantity that determines renormalizability is in fact not s_μ but $\mathfrak{P}_{\mu\nu} s_\nu$. It is therefore clear that the condition of current conservation may be *relaxed,* provided that, by virtue of the equations of motion $\mathfrak{P}_{\mu\nu} s_\nu$ rather than s_μ itself satisfies conditions (i), (ii) and (iii). This is precisely the situation in the case of 'partial current conservation' as defined by GLASHOW.**)

The above discussion is but a restatement of known results. It is based on the conventional statement of spin 1 theory. If however we choose the linearized form of the theory the statement of renormalizability conditions appears in a somewhat different form. We have then to consider the sources j_μ and $m_{\mu\nu}$ separately. Apparently this is a complication, but it brings us the gain that now the appearance of the $F_{\mu\nu}$ field in the sources need not be classed as an appearance of derivatives. We then find that a theory of Proca particles is renormalizable if

$$\partial_\mu j_\mu = 0 \quad \text{and} \quad \partial_\lambda m_{\mu\nu} + \partial_\mu m_{\nu\lambda} + \partial_\nu m_{\lambda\mu} = 0,$$

or if at least the quantities $\mathfrak{P}_{\mu\nu}\, j_\nu$ and $\mathfrak{P}_{\mu\nu,\rho\sigma}\, m_{\rho\sigma}$ involve:
(i)′ no terms with more than three boson field operators (among which both the A_μ and the $F_{\mu\nu}$ may be included),
(ii)′ only one derivative term multiplied by no more than one non-differentiated field operator (defined as above), and
(iii)′ not more than one derivative of a field operator which is not re-expressible in terms of the A_μ and $F_{\mu\nu}$.

It is also clear from our previous discussion that the 'inverted' scheme may equally well be used for the discussion of renormalizability. With it

**) Recently SALAM has stated objections to GLASHOW's treabment.

conditions similar to our first three must clearly be imposed on $\mathfrak{P}_{\mu\nu,\rho\sigma} q_{\rho\sigma}$ with $q_{\rho\sigma}$ defined by equation (13) and with the $F_{\mu\nu}$ now functioning as the sole non-derivative field operators.

4. The Yang Mills Theory

We shall now discuss briefly the renormalizability of the Yang-Mills[7]) theory in terms of our criteria for the linearized approach. This theory involves an isobaric triplet of fields A^i_μ and $F^i_{\mu\nu}$ $(i = 1, 2, 3)$ and a corresponding isobaric triplet of currents

$$j^i_\mu = g\,(A^j_\nu F^k_{\mu\nu} - A^k_\nu F^j_{\mu\nu})$$

and

$$m^i_{\mu\nu} = g\,(A^j_\mu A^k_\nu - A^k_\mu A^\nu_j),$$

$(i, j, k$ cyclic). We take all three fields to have the same *non-vanishing* mass m. It is then readily calculated from the equations of motion that $\partial_\mu j^i_\mu = 0$. On the other hand we find

$$\left.\begin{aligned}\partial_\lambda m^i_{\mu\nu} + \partial_\mu m^i_{\nu\lambda} + \partial_\nu m^i_{\lambda\mu} = gm(&F^j_{\lambda\mu} A^k_\nu + F^j_{\mu\nu} A^k_\lambda + F^j_{\nu\lambda} A^k_\mu \\ &- F^k_{\lambda\mu} A^j_\nu - F^k_{\mu\nu} A^i_\mu - F^k_{\nu\lambda} A^j_\mu)\end{aligned}\right\} \tag{29}$$

This is not zero, but nonetheless $\mathfrak{P}_{\mu\nu,\rho\sigma} m_{\rho\sigma}$ satisfies criteria (i)', (ii)', (iii)'. Hence the above criteria are satisfied.

SALAM and WARD[8]) have pointed out that the situation remains unchanged if the third component of the Yang-Mills field is replaced by the electromagnetic field. This can be checked by noting that our criteria are still satisfied if in the *second*, but not the first set of the Proca equations for the three fields the mass terms (even all three) are taken to be different.

The question whether it may be possible to construct other schemes satisfying our conditions, will not be examined here. Except for the neutral vector meson no other renormalizable scheme has so far been suggested. It may well be that the analysis presented here can do no more than restate known results in what is perhaps a more direct and simple form.

The writer is sincerely grateful to be able to join in this tribute to the memory of Wolfgang Pauli, one of the greatest physicists of this century, who was also so good a friend and wise a counsellor.

References

1) R. P. FEYNMAN and M. GELL-MANN, Phys. Rev. *109*, 193 (1958).
2) J. SCHWINGER, Ann. Phys. *2*, 407 (1957).
3) C. FRONSDAL and S. L. GLASHOW, Phys. Rev. Lett. *3*, 570 (1959).
4) S. L. GLASHOW, Nucl. Phys. *10*, 107 (1959).
5) N. BYERS and R. E. PEIERLS, Nuov. Cim. *10*, 520 (1958).
6) J. SCHWINGER, Phys. Rev. *91*, 713 (1953).
7) C. N. YANG and R. MILLS, Phys. Rev. *96*, 190 (1954).
8) A. SALAM and J. C. WARD, Nuov. Cim. *11*, 568 (1959).