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Primordial Inhomogeneities Spectra in Mixed Dark Matter Models with Non-zero Cosmological Constant

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Abstract. The transfer functions of power spectra for mixed (cold + hot) dark matter models with a cosmological constant are calculated. Under the assumption of the post-inflationary Harrison-Zeldovich spectrum some characteristics of the large scale structure of the Universe are calculated and compared with observable ones.

1 Introduction

Results of measurements of temperature fluctuations of cosmic microwave background radiation (CMBR) in COBE experiment [18] give no chance for the surviving of the most simplest and explored standard cold dark matter (SCDM) model [8, 16, 15]. The more complicated two-parameter model with mixed (cold + hot) dark matter (MDM) has not difficulties like

SCDM and its predictions marginally match observable characteristics of the large scale structure of the Universe (LSS). It was discussed in a dozen papers, e.g. [19, 17, 14] and references cited therein. However, all models with $\Omega_{matter} = 1$ have an essential deficiency, namely that the age of Universe is below present estimates [5]. Low density models with $\Omega_{tot} < 0.5$ can remove this problem but at the cost of having difficulties with observable upper limits of CMBR temperature fluctuations on small angular scale. Because of this problem and in order to keep the inflation paradigm, to solve "age problem" and to overcome the difficulties like those in low density models the cosmological constant $\Lambda > 0$ must be introduced [4, 12], [11], [10]. In the last reference it is shown that models with $\Omega h \geq 0.25 - 0.3$ match observations only marginally.

The goal of this work is to analyze the possibility of explanation of LSS characteristics in the framework of MDM models with cosmological constant ($MDM + \Lambda$). For this we shall first calculate the transfer function of $MDM + \Lambda$ models with different values of free parameters (Ω_{HDM}, Ω_Λ, number of species of massless and massive collisionless particles and dimensionless Hubble parameter $h \equiv H/100\frac{km}{s\,Mpc}$). Then assuming the post-inflationary Harrison-Zeldovich spectrum and normalizing it to COBE data on the temperature fluctuations of CMBR we shall calculate some characteristics of the LSS and compare them with observable ones.

2 Transfer function and power spectrum: equations and method

We investigate small potential (scalar) perturbations in four component medium (Cold DM, Hot DM, radiation, massless collisionless ν-particles). The baryonic content of the Universe ($\Omega_b \sim 0.03$) is neglected. (Baryons slightly change the state equation of photons $p_{rad} = \rho_{rad}/3$, just before the hydrogen recombination.) The total background density (including density corresponding to the cosmological constant) is assumed to be equal to the critical density.

The numbers of species (independent spin-degrees of freedom) of collisionless particles (ν) and (h) are described by parameters β_ν and β_h respectively for massless and massive particles. The mass of h-particles is connected with Hot component part in total density Ω_h in a following way: $m_h = 70.5\Omega_h h^2/({\alpha_h}^3\beta_h)eV$, where $\alpha_h = T_h/T_{rad} = (4/11)^{1/3}$ is the ratio of temperature of h-particles and radiation, h is Hubble constant in the unit $100kms^{-1}Mpc^{-1}$. The scale factor $a(t)$ is normalized to be unity at the moment when h-particles become nonrelativistic that gives the present value $a_0 = 3{\cdot}10^5\Omega_h h^2{\beta_h}^{-1}{\alpha_h}^{-4}$. The time and length units (τ, l) are chosen to be: $\tau = (\frac{3}{8\pi G}\rho_{rad}a^4)^{1/2} = 6.88{\cdot}10^8{\alpha_h}^8{\beta_h}^2(\Omega_h h^2)^{-2}s$, $l = a_0\tau c = 2.03{\cdot}{\alpha_h}^2\beta_h(\Omega_h h^2)^{-1}Mpc$ correspondingly.

Cold component can be described as a hydrodynamical pressureless fluid (dust - like medium) with equation of state $p_c = 0$. Baryons and radiation are assumed to be a single ideal fluid, with the sound velocity $c_s = c/\sqrt{3}$, that is fairly reasonable if $\Omega_B \leq 0.03$. To describe this fluid it is enough to investigate the equations for density contrast and perturbed velocity. Because in collisionless media the anisotropy of pressure is different from zero, the equations for density contrast and velocity are not sufficient for description of perturbations in h and ν particles. Collisionless components must be described by perturbed distribution functions, which satisfy Boltzmann-Liouville equations. All components are interacting via the gravitational field. Expanding all perturbations into plane waves finally we have obtained the self-consistent system of equations. The method of numerical solution of this system is described in [19]. Our numerical calculation is done in the range from $z_{init} = 10^9$ to $z_{end} = 20$, for mass of perturbation from $M = 10^{22}M_\odot$ to $M = 10^{12}M_\odot$ and primordial (post-inflationary) power spectrum $P_{pi}(k) \equiv < \delta_k\delta_k^* >= Ak^n$ with n=0, where A is normalization constant. As a result, we obtain the density perturbation of each component as a function of time and scale of perturbations (wave-number k). It is easy to obtain the relation for total density perturbations:

$$\delta_m = \frac{\delta\rho_c + \delta\rho_h}{\rho_c + \rho_h} = \frac{1}{\Omega_m}(\Omega_c\delta_c + \Omega_h\delta_h)$$

Transfer function $T(k)$ is calculated as ratio of amplitude of Fourier mode $\delta_m(z_{end}, k)$ to one of minimal k, which corresponds to wave number of maximal mass perturbation $M = 10^{22}M_\odot$.

The final power spectrum at $z = 0$ will be used to compute radiation anisotropies, to be compared with the temperature fluctuations of CMBR detected by COBE DMR observations [18]. Non-zero cosmological constant affects only the expansion rate (scale factor $a(t)$) and changes the growth of perturbations after the moment of equality of the cosmological constant density to the matter one [11, 12, 10]. Taking into account the reduction of the amplitude of the density perturbations due to the cosmological constant the power spectrum at present epoch will be

$$P(k;\Omega_\Lambda,\Omega_{HDM},\beta_\nu,\beta_h) = AkK_\delta^2(\Omega_\Lambda)T^2(k;\Omega_\Lambda,\Omega_{HDM},\beta_\nu,\beta_h),$$

where K_δ is coefficient of such reduction and is determined as ratio δ_m at z=0 for $\Omega_m = 1-\Omega_\Lambda$ to one for $\Omega_m = 1$.

3 Models and normalization

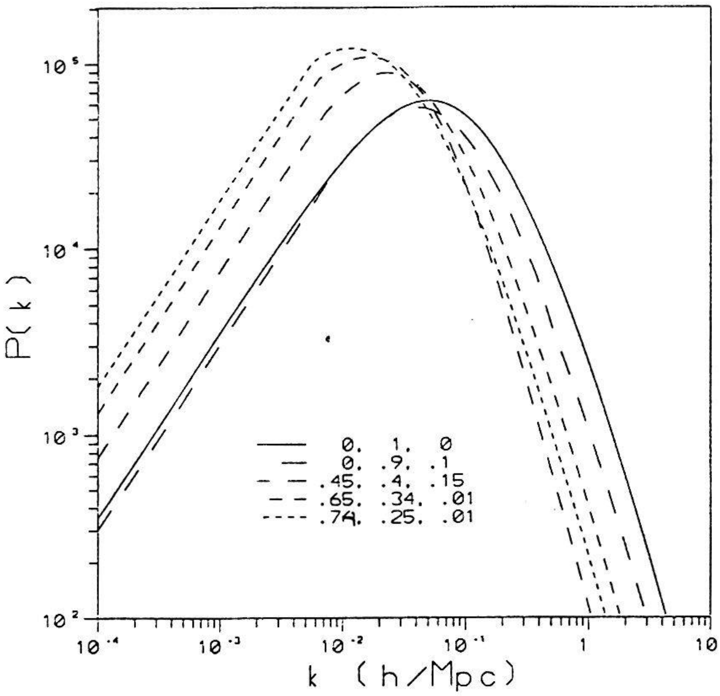
For analyzing the influence of cosmological constant on the predicted characteristics of LSS by $MDM+\Lambda$ models we set Λ=0, 0.45, 0.65 and 0.74. The reduction coefficients K_δ for them are equal to 1, 0.89, 0.81, 0.75 accordingly. The age of Universe for $\Lambda \neq 0$-models is larger than 18 Gyr (when $h = 0.5$), that removes the "age problem". For each Λ-model the ratio Ω_{HDM}/Ω_m is defined within the range 0.03-0.65. For analyzing the influence of ν-particle species number on transfer function the parameter β_h is set 2, 4 and 6, the parameter β_ν - 0, 2, 4 and 6. The fifth free parameter h was equal 0.5 and 0.75. On the whole forty models were calculated.

We have normalized the spectra of all models to produce the COBE data on CMBR temperature anisotropy at angular scales 10^o: $< (\frac{\Delta T}{T})^2 >^{\frac{1}{2}}= 1.1\ 10^{-5}$. We calculate $< (\frac{\Delta T}{T})^2 >= C(0;10^o)$ using the approximation formula from [20] for correlation function of

CMBR temperature anisotropy measured by a receiver with Gaussian response function and exact formula from [13] for correlation function $C(\alpha) \equiv < \frac{\Delta T}{T}(0) \ \frac{\Delta T}{T}(\alpha) >$. Both Sachs-Wolfe and Doppler effects are taken into account. The effect of amplification of $\Delta T/T$ in the model with cosmological constant is taken into account by coefficient $K_{\Delta T/T} = exp[\Omega_\Lambda^{3.6}(1 + 0.008(\Omega_\Lambda/\Omega_m)^{4/3})]$ [10], by which the postrecombination spectrum is multiplied when $C(\alpha)$ is calculated. The tensor mode is taken into account like [10]. It gives about 10% of contribution to $\Delta T/T$. The tensor mode and effect of amplification reduce the normalization constant A of power spectrum of $\delta\rho_m/\rho_{cr}$.

4 Results and Conclusions

The power spectra of δ_m normalized to COBE $\Delta T/T$ at 10^o angular scale and recalculated to z=0 in the framework of linear theory for Λ-models are shown in Fig.1. $P(k)$ has maximum at k_{eq} which is approximately equal to inverse horizon size at the moment when $\rho_r + \rho_\nu \approx \rho_m$ and decreases $\sim \Omega_m$ when Ω_Λ increases. The influence of massless collisionless particles on the power spectrum $P(k)$ is shown in Fig.2. Analyzing the results of calculations (see Table) we can conclude that the models with small number of massless particles are in better accordance with observations, than the models with large ones.



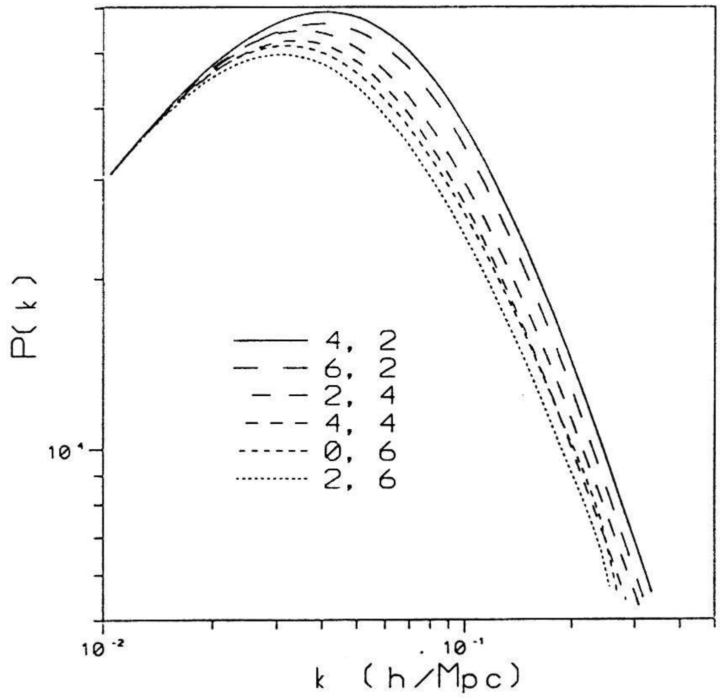


Fig.1. The initial power spectra for $MDM + \Lambda$ models with β_ν=4, β_h=2 and different values Ω_Λ, Ω_{CDM} and Ω_{HDM} ($h = 0.5$).

Fig.2. The initial power spectra for MDM models with $\Omega_\Lambda = 0$, $\Omega_{CDM} = 0.8$, $\Omega_{HDM} = 0.2$ and different values of β_ν and β_h ($h = 0.5$).

To test the basic assumption about Gaussian statistics let us calculate for a given spectrum all characteristics of the large scale structure of the Universe to confront them with observations. The most important among them are: anisotropy of the relic background radiation, correlation functions of galaxies and clusters, large-scale bulk motions and phenomenological biasing relating the distributions of galaxies and dynamical matter. The results of calculation of predictions for ten models with $h = 0.5$ are presented in Table. The galaxy biasing is calculated as the inverse mass fluctuations in a top-hat sphere with radius $8h^{-1}Mpc$ $b_g = 1/\sigma_8$, the rich cluster one b_{cl} according to [1]. All linear correlation functions and bulk velocities can be easily derived if the power spectrum $P(k)$ is known: $\xi(r) = b^2/2\pi^2 \int_0^\infty dk\ k^2\ P(k)\ W^2(kR_f) sin(kr)/kr$, $V_R^2 = H^2 K_v^2 K_\delta^{-2}/2\pi^2 \int_0^\infty P(k)\ W^2(kR)\ dk$, where the filtering function $W(kR_f)$ singles out the object scales in the spectrum $P(k)$, reduction coefficients K_v and K_δ are calculated like in [11]. Here we took for bright galaxies $R_f = 0.35h^{-1}Mpc$ and for rich clusters $5h^{-1}Mpc$. The correlation radius defined as $\xi(r^c) = 1$ for them as well as $\sigma \equiv \sqrt{\xi(0; R_f)}$ are presented in Table. The bulk motion is calculated for radius 5 (Gaussian filtering) and for radii 40 and $60h^{-1}Mpc$ of top-hat sphere with preliminary Gaussian filtering in scale $R_f = 12h^{-1}Mpc$. On calculating the quadrupole $Q_2 = (5a_2^2/4\pi)^{1/2} f_2(\sigma)$, $a_2^2 = \frac{8}{\pi}\frac{K_2^2(\Omega_m)}{R_h^4}\int_0^\infty P(k)k^{-2}J_2(kR_h)dk$ we have taken into account only the Sachs-Wolfe effect. Here $R_h = 2c/H_o$ is the contemporary horizon, $K_2(\Omega_m)$ is the amplification coefficient due to the cosmological constant [11] and $J_2(x)$ the spherical Bessel function. The factor f_2 is the expansion coefficient of an angular smoothing function with respect to the Legendre polynomials. In the case of the COBE experiment it is approximately equal to $f_2(\sigma) \simeq exp(-6\sigma^2) \simeq 0.964$ for double Gaussian smoothing with 7 deg FWHM, which was carried out in this experiment.

Table. Observable and theoretical characteristics of the large-scale structure of the Universe in MDM models with different values of Ω_Λ and numbers of massive and massless collisionless particle species β_h and β_ν for $\Lambda = 0$ model ($h = 0.5$).

	Observ.\ Ω_Λ				0				0.45	0.65	0.74
Ω_{HDM}		0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.15	0.01	0.01
β_ν		4	4	6	2	4	0	2	4	4	4
β_h		2	2	2	4	4	6	6	2	2	2
b_g	....	1.2	1.3	1.4	1.5	1.55	1.6	1.7	2.0	1.5	1.8
σ_g	....	3.8	2.7	2.4	2.5	2.3	2.5	2.3	1.3	2.4	1.7
r_g^c	5 – 7 [1)]	6.4	6.7	6.7	6.8	6.8	6.5	6.5	7.0	6.7	6.8
b_{cl}	....	3.4	3.5	3.7	3.9	4.0	4.0	4.2	5.1	3.8	4.8
σ_{cl}	....	0.67	0.60	0.58	0.54	0.52	0.51	0.48	0.44	0.55	0.45
r_{cl}^c	15 – 25 [2)]	15.4	17.0	16.8	16.6	16.9	16.8	17.0	24.7	18.3	21.2
V_5	$\simeq 580 \pm 60$ [3)]	629	613	594	570	552	544	528	392	309	230
V_{40}	$\simeq 390 \pm 70$ [3)]	323	325	319	316	311	309	304	263	205	165
V_{60}	$\simeq 330 \pm 80$ [3)]	259	260	257	256	253	252	249	223	177	146
Q_2	$\simeq 0.5 \pm 0.2$ [4)]	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.58	0.61

[1)] [6, 7], [2)] [2, 9], [3)] [3], [4)] [18], velocities are expressed in terms of km/s, linear scales in $h^{-1}Mpc$, Q_2 in 10^{-5}.

The more crucial test for Λ-models is the bulk motions. In models with $\Omega_\Lambda \geq 0.5$ the observable large scale peculiar velocities of galaxies in bulk of 5, 40 and $60h^{-1}Mpc$ are out of 95% confidential level. Increasing h to 0.75 does not change the situation essentially. The comparison of predictions with observable data constrains the parameters of MDM models in $\Lambda \neq 0$ cosmology so that $\Omega_\Lambda \leq 0.5$ and $\Omega_{HDM}/\Omega_m \leq 0.2$ when $h \cong 0.5$.

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