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Vavilov-Cerenkov Radiation in a Finite Region of Space

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Abstract. Exact expressions are found for the electromagnetic field arising from the instant acceleration of a charged particle, its subsequent motion with the velocity exceeding the light velocity in the medium and the instant transition into the state of rest. It turns out that these expressions have definite advantages over the usual ones grounded on the use of the Fourier transform. In particular, they clearly show when and where the Cerenkov radiation should be observed in order to discriminate it from the bremsstrahlung.

1 Introduction

The Vavilov-Cerenkov effect (VCE) is a well established phenomenon widely used in physics and technology. Its nice exposition may be found in the Frank book [1]. In most text-books and scientific papers the VCE is considered in terms of the Fourier components. To get an answer in the physical space, the inverse Fourier transform should be performed. The occurring divergent integrals obscure the physical picture. As far as we know, there are only few attempts in which the VCE is treated without making the Fourier transform. At first, we should mention Sommerfeld's paper [2] in which the hypothetical motion of the extended charged particle in the vacuum with the velocity $v > c$ has been considered. Although the relativity principle prohibits such a motion in the vacuum, all the equations of [2] are valid in the medium if we identify c with the light velocity in the medium. Unfortunately, due to the finite dimensions of the charge, equations describing the field strengths are so complicated that are not suitable for the physical analysis. The other reference treating the VCE without recourse to the Fourier transform is the Heaviside book [3] where the superluminal motions of the point charge both in the vacuum and infinitely extended medium were considered. Yet Heaviside was not aware of Sommerfeld's paper [2] as well as Tamm and Frank [4,5] did not know about the Heaviside's investigations. It should be noted that Frank and Tamm formulated their results in terms of Fourier components. The results of Heaviside (without referring to them) were translated into the modern physical language in ref. [6]. A similar motion of the charge with finite dimensions has been considered in ref. [7]. The charge had zero dimensions in the direction normal to the velocity and the Gauss distribution along the velocity. It was shown there that the singular Cerenkov shock wave did not arise in this case. Instead, the field strengths had a finite maximum at the Cerenkov angle. It is the goal of present investigation to investigate electromagnetic effects arising from the point-like charged particle motion in the finite medium.

2 Mathematical Preliminaries

Let a charged point particle moves inside the medium with the polarizabilities ϵ and μ along the given trajectory $\vec{\xi}(t)$. Then, its electromagnetic field (EMF) at the observation point $(\vec{r}, t)$ is given by the Lienard-Wiechert potentials

$$\Phi(\vec{r},t) = \frac{e}{\epsilon}\sum \frac{1}{Z_i}, \quad \vec{A}(\vec{r},t) = \frac{e\mu}{c}\sum \vec{v}_i/Z_i, \quad (div\vec{A} + \frac{\epsilon\mu}{c}\dot{\Phi} = 0) \tag{2.1}$$

Here $\vec{v}_i = (d\vec{\xi}/dt)|_{t=t_i}$, $Z_i = |\vec{r} - \vec{\xi}(t_i)| - \vec{v}_i(\vec{r} - \vec{\xi}(t_i))/c_n$ and c_n is the light velocity inside the medium ($c_n = c/\sqrt{\epsilon\mu}$). The summing in (2.1) is performed over all physical roots of the equation

$$c_n(t - t') = |\vec{r} - \vec{\xi}(t')| \tag{2.2}$$

To preserve the causality, the time of radiation t' should be smaller than the observation time t. Obviously, t' depends on the coordinates $\vec{r}, t$ of the point P at which the EMF is observed. Let a particle move with the constant velocity v along the z axis: $\xi = vt - z_0$ Then, Eq.(2.2) has two roots

$$c_n t' = \frac{c_n t - \beta_n(z + z_0)}{1 - \beta_n^2} \mp \frac{r_m}{|1 - \beta_n^2|} \tag{2.3}$$

Here $r_m = \sqrt{(z + z_0 - vt)^2 + \rho^2(1 - \beta_n^2)}$, $\rho^2 = x^2 + y^2$, $\beta_n = v/c_n$ In what follows we need also $c_n(t - t')$ which is given by

$$c_n(t - t') = \beta_n \frac{vt - z - z_0}{\beta_n^2 - 1} \pm \frac{r_m}{|\beta_n^2 - 1|} \tag{2.4}$$

We shall denote t' corresponding to the upper and lower signs in (2.3) and (2.4) as t'_1 and t'_2, resp. It is easy to check that

$$c_n^2(t-t'_1)(t-t'_2) = \frac{1}{\beta_n^2-1}r, \quad r = [(z+z_0-vt)^2+\rho^2]^{1/2} \tag{2.5}$$

Consider a few particular cases.

3 Particular Cases

3.1 The Uniformly Moving Charge With the Velocity $v < c_n$

It follows from (2.5) that $t - t'_1$ and $t - t'_2$ have different signs for $\beta_n < 1$. As only positive $t - t'$ correspond to the physical situation, one should choose the plus sign in (2.4) that corresponds to the upper signs both in (2.3) and (2.4). For the electromagnetic potentials one obtains the well-known expressions

$$\epsilon\Phi = \frac{e}{r_m}, \quad A_z = \frac{e\beta\mu}{r_m}, \quad (\beta = v/c) \tag{3.1}$$

It follows from this that the uniformly moving charge carries the EMF with itself.

3.2 The Uniformly Moving Charge With the Velocity $v > c_n$

This section briefly reproduces the contents of ref. [6]. It follows from (2.5) that for the treated case $(t - t'_1)$ and $(t - t'_2)$ are of the same sign which coincides with the sign of the first term in (2.4). It is positive if

$$t > (z+z_0)/v \tag{3.2}$$

The two physical roots are

$$c_n t'_{1,2} = -\frac{c_n t - \beta_n(z+z_0) \pm r_m}{\beta_n^2 - 1}$$

The positivity of the expression staying under the square root in r_m requires

$$\mathcal{M} = vt - z - z_0 - \rho/\gamma_n > 0 \quad (\gamma_n = \sqrt{\beta_n^2 - 1}) \tag{3.3}$$

. As this inequality is stronger than (3.2), one may use only (3.3) which shows that the EMF is enclosed inside the Mach cone given by (3.3). For the electromagnetic potentials one gets

$$\epsilon\Phi = \frac{2e}{r_m}\Theta(\mathcal{M}), \quad A_z = \frac{2e\mu\beta}{r_m}\Theta(\mathcal{M}) \tag{3.4}$$

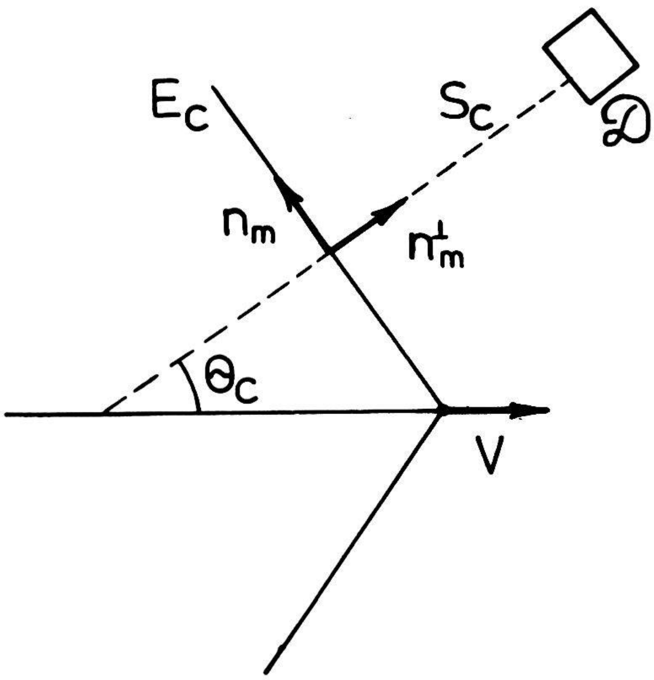

Figure 1: Cerenkov shock wave propagating in the infinite medium. There is no EMF in front of the Mach cone. Behind it there is EMF of the moving charge. At the Mach cone itself there are singular electric E and magnetic H fields. The latter having only the ϕ component is perpendicular to the plane of figure.

(the factor 2 appears because there are two physical roots meeting (2.2)). The electromagnetic strengths ($\vec{D} = \epsilon\vec{E}$, $\vec{E} = -grad\Phi - \dot{\vec{A}}/c$, $\vec{B} = \mu\vec{H} = curl\vec{A}$) are given by

$$H_\phi = -\frac{2e\rho\beta}{\gamma_n^2 r_m^3}\Theta(\mathcal{M}) + \frac{2e\beta}{\gamma_n r_m}\delta(\mathcal{M}),$$

$$\epsilon\vec{E} = -\frac{2er}{\gamma_n^2 r_m^3}\vec{n}_r \cdot \Theta(\mathcal{M}) + \frac{2e\beta}{\gamma_n r_m} \cdot \delta(\mathcal{M})\vec{n}_m \tag{3.5}$$

Here $n_r = (\rho\vec{n}_\rho + (z+z_0-vt)\vec{n}_z)/r$ is the unit radial vector directed from the current position of the charge inside the Mach cone and $\vec{n}_m = \vec{n}_\rho/\beta_n - \vec{n}_z/\beta_n\gamma_n$ is the unit vector lying on the surface of the Mach cone (fig.1). The δ-function terms in these Eqs. correspond to the Vavilov-Cerenkov radiation. They are different from zero only on the surface of the Mach cone.

We observe that both terms in $\vec{E}$ and $\vec{H}$ are singular on the Mach cone (as r_m vanishes there). On the other hand, according to the Gauss theorem, the integral from E taken over the sphere surrounding the charge should be equal to $4\pi e$. The integrals from each of the terms entering into E are divergent. Only their sum is finite (take into account their different signs). This was explicitly shown in ref.[6].

The observer at the (ρ, z) point will see the following picture. There is no EMF for $c_n t < R_m$ $(R_m = (z + z_0 + \rho/\gamma_n)/\beta_n)$. At the time $c_n t = R_m$ the Cerenkov shock wave reaches the observer. At this moment, the actual and retarded positions of the charged particle are: $z_a = z + \rho/\gamma_n$ and $z_r = z - \rho\gamma_n$, resp. For $c_n t > R_m$ the observer sees the EMF of the charged particle originating from the retarded positions of the particle lying to the left and right from z_r.

At large distances the terms with the Θ functions die out, and for the Cerenkov radiation field one gets

$$\epsilon\vec{E} = \frac{2e\beta_n}{\gamma_n r_m}\delta(\mathcal{M}) \cdot \vec{n}_m, \quad \vec{H} = \frac{2e\beta}{\gamma_n r_m}\delta(\mathcal{M}) \cdot \vec{n}_\phi$$

The Poynting vector is equal to

$$\vec{S}_c = \frac{c}{4\pi}\vec{E} \times \vec{H} = \frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}} \cdot [\frac{2e\beta}{r_m\gamma}\delta(\mathcal{M})]^2 \cdot \vec{n}_m^\perp$$

Here $\vec{n}_m^\perp = n_\rho/\beta_n\gamma_n + n_z/\beta_n$ is the unit vector normal to the surface of the Mach cone (fig.1). The observer being placed at the ρ, z point will detect the Cerenkov photon at the moment $t = (z + z_0 + \rho/\gamma_n)/v$. The beam of charged particles propagating along the z axis with the velocity $v > c_n$ produces the continuous Cerenkov radiation in the $\vec{n}_m^\perp$ direction with the electric vector in the $\vec{n}_m$ direction.

3.3 The Uniform Motion With $v < c_n$ On a Finite Space Interval

Let the charged particle rest at the point $z = -z_0$ for time $t < 0$. During the time interval $0 < t < 2z_0/v$ the particle moves with the constant velocity $v < c_n$. For $t > 2z_0/v$ the particle again rests at the point $z = z_0$. The electromagnetic potentials are equal to

$$\epsilon\Phi = \frac{e}{r_1}\Theta(r_1 - c_n t) + \frac{e}{r_2}\Theta(c_n t - \frac{2z_0}{\beta_n} - r_2) + \frac{e}{r_m}\Theta(c_n t - r_1)\Theta(\frac{2z_0}{\beta_n} + r_2 - c_n t),$$

$$A_z = \frac{e\beta\mu}{r_m}\Theta(c_n t - r_1)\Theta(\frac{2z_0}{\beta_n} + r_2 - c_n t) \tag{3.6}$$

where we have put for brevity $r_1 = [\rho^2 + (z+z_0)^2]^{1/2}$, $r_2 = [\rho^2 + (z-z_0)^2]^{1/2}$. The particular terms of Eq.(3.6) have a simple interpretation. The information on the beginning of the particle motion has not reached the points for which $c_n t < r_1$. At these points there is a field of the charge resting at $z = -z_0$ (first term in Φ). The information on the motion ending has passed through the points for which $c_n t > 2z_0/\beta_n + r_2$. At those space-time points there is a field of the charged particle resting at $z = z_0$ (second term in Φ). Finally, at the space-time points for which $r_1 < c_n t < 2z_0/\beta_n + r_2$ there is a field of the uniformly moving charge (last term in Φ). The electromagnetic field strengths are equal to

$$H_\phi = \frac{e\beta(1-\beta_n^2)\rho}{{r_m}^3}\Theta(c_n t - r_1)\Theta(\frac{2z_0}{\beta_n} + r_2 - c_n t) + \frac{e\beta\rho}{r_1}\frac{\delta(c_n t - r_1)}{r_1 - \beta_n(z+z_0)} - \frac{e\beta\rho}{r_2}\frac{\delta(c_n t - r_2 - \frac{2z_0}{\beta_n})}{r_2 - \beta_n(z - z_0)},$$

$$\epsilon\vec{E} = \frac{e}{{r_1}^2}\vec{n}_r^1\Theta(r_1 - c_n t) + \frac{er(1-\beta_n^2)}{{r_m}^3}\vec{n}_r\Theta(c_n t - r_1)\Theta(\frac{2z_0}{\beta_n} + r_2 - c_n t) + \frac{e}{{r_2}^2}\vec{n}_r^2\Theta(c_n t - \frac{2z_0}{\beta_n} - r_2) +$$

$$+\frac{e\rho\delta(c_nt-r_1)}{r_1}\frac{\beta_n}{r_1-\beta_n(z+z_0)}\vec{n}_\theta^1-\frac{e\rho\delta(c_nt-r_2-\frac{2z_0}{\beta_n})}{r_2}\frac{\beta_n}{r_2-\beta_n(z-z_0)}\vec{n}_\theta^2, \qquad (3.7)$$

Here $\vec{n}_r^1$, $\vec{n}_\theta^1$, $\vec{n}_r^2$ and $\vec{n}_\theta^2$ are the radial and polar unit vectors lying on the spheres S_1 and S_2 with their centers at the points $z=-z_0$ and $z=z_0$, resp.:

$$\vec{n}_r^1=(\rho\vec{n}_\rho+(z+z_0)\vec{n}_z)/r_1, \quad \vec{n}_\theta^1=(\vec{n}_\rho(z+z_0)-\vec{n}_z\rho)/r_1$$

$$\vec{n}_r^2=(\rho\vec{n}_\rho+(z-z_0)\vec{n}_z)/r_2, \quad \vec{n}_\theta^2=(\vec{n}_\rho(z-z_0)-\vec{n}_z\rho)/r_2$$

The observer at the (ρ,z) point will detect the radiation arising from the particle's instant acceleration and deceleration at the moments $c_nt=r_1$ and $c_nt=r_2+2z_0/\beta_n$, resp. For the distant observer the radiation field is given by

$$\epsilon\vec{E}=\frac{e\beta_n\rho}{r_1(r_1-\beta_n(z+z_0))}\delta(c_nt-r_1)\cdot\vec{n}_\theta^1-\frac{e\beta_n\rho}{r_2(r_2-\beta_n(z-z_0))}\delta(c_nt-r_2-2z_0/\beta_n)\cdot\vec{n}_\theta^2$$

$$\vec{H}=\vec{n}_\phi e\beta\rho[\frac{\delta(c_nt-r_1)}{r_1(r_1-\beta_n(z+z_0))}-\frac{\delta(c_nt-r_2-2z_0/\beta_n)}{r_2(r_2-\beta_n(z-z_0))}]$$

The total Poynting vector is equal to the sum of energy fluxes emitted at the$z=\pm z_0$ points:

$$\vec{S}=\vec{S}_1+\vec{S}_2,$$

$$\vec{S}_1=\frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{e\beta\rho\delta(c_nt-r_1)}{r_1(r_1-\beta_n(z+z_0))}]^2\cdot\vec{n}_r^1, \quad \vec{S}_2=\frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{e\beta\rho\delta(ct-r_2-2z_0/\beta)}{r_2(r_2-\beta(z-z_0))}]^2\cdot\vec{n}_r^2$$

Here $\vec{n}_r^1=(\rho\vec{n}_\rho+(z+z_0)\vec{n}_z)/r_1$ and $\vec{n}_r^2=(\rho\vec{n}_\rho+(z-z_0)\vec{n}_z)/r_2$ are the unit vectors normal to S_1 and S_2. It turns out that the vector $\vec{S}$ describes divergent spherical waves emitted at the $z=z_0$ and $z=-z_0$ points.

3.4 The Uniform Motion With $v>c_n$ on a Semifinite Space Interval

a) The charge particle motion begins from the state of rest (fig.2).

Let the particle rest at the point $z=-z_0$ up to the moment $t'=0$. For $t'>0$ it moves with the velocity $v>c_n$. For the observer being placed at the point (ρ,z) the condition for the particle to be at rest is $c_nt<r_1$. The condition $t'>0$ for the particle motion to start is different for upper and lower signs in (2.3) (see the Appendix). The solution corresponding to the upper sign exists only if $z>\rho\gamma_n-z_0$ and $R_m<c_nt<r_1$ $(R_m=(z+z_0+\rho/\gamma_n)/\beta_n)$. The solution corresponding to the lower sign exists both for $z<\rho\gamma_n-z_0$ and $z>\rho\gamma_n-z_0$:

$$c_nt>r_1 \quad \text{for} \quad z<\rho\gamma_n-z_0 \quad \text{and} \quad c_nt>R_m \quad \text{for} \quad z>\rho\gamma_n-z_0$$

The electric scalar and magnetic vector potentials are given by

$$\epsilon\Phi=\frac{e}{r_1}\Theta(r_1-c_nt)+\frac{e}{r_m}\Theta(z+z_0-\rho\gamma_n)\Theta(r_1-c_nt)\Theta(c_nt-R_m)$$

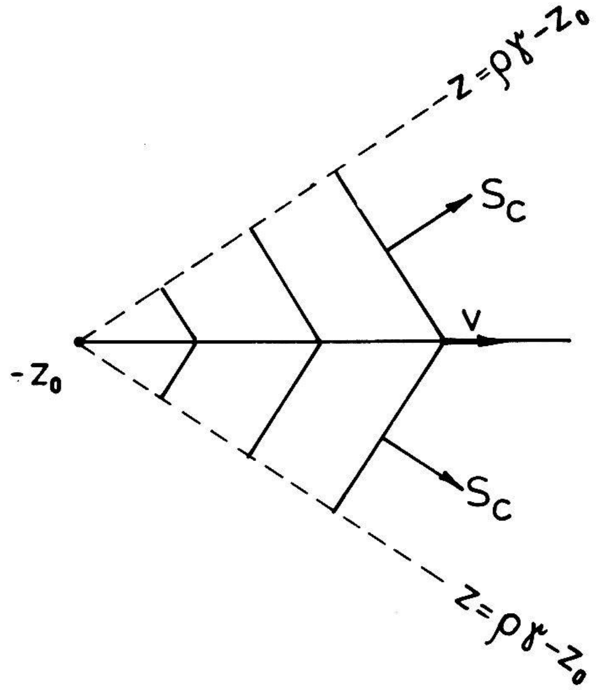

Figure 2: The superluminal motion of the charge begins from the rest state at $z = -z_0$. In the $z < \rho\gamma_n - z_0$ region the observer sees (consecutively in time) the EMF of the resting charge, the bremsstrahlung shock wave and the EMF of the moving charge. There is no Cerenkov radiation in this space region. In the $z > \rho\gamma_n - z_0$ region the observer consecutively sees the EMF of the resting charge, the Cerenkov shock wave, the EMF from two retarded positions of the charge, the bremsstrahlung and the EMF from the moving away retarded position of the charge.

$$+\frac{e}{r_m}\Theta(\rho\gamma_n - z_0 - z)\Theta(c_n t - r_1) + \frac{e}{r_m}\Theta(z + z_0 - \rho\gamma_n)\Theta(c_n t - R_m),$$

$$\frac{1}{\mu}A_z = \frac{e\beta}{r_m}[\Theta(z + z_0 - \rho\gamma_n)\Theta(r_1 - c_n t)\Theta(c_n t - R_m)$$

$$+\Theta(\rho\gamma_n - z_0 - z)\Theta(c_n t - r_1) + \Theta(z + z_0 - \rho\gamma_n)\Theta(c_n t - R_m)]$$

As a result, the observer being placed at the point (ρ, z) will see the following picture.

Let $z < \rho\gamma_n - z_0$. Then, for $c_n t < r_1$ the observer sees the EMF of the charge resting at $z = -z_0$. For $c_n t = r_1$ he will observe the bremsstrahlung originating from the $z = -z_0$ point. For $c_n t > r_1$ the observer detects the EMF of the charge moving with the velocity $v > c_n$. There is no Cerenkov radiation in this space region.

Let the observer be in the space region where $z > \rho\gamma_n - z_0$. In this case, for $c_n t < R_m$ the observer sees the EMF of the charge resting at $z = -z_0$. At the time $c_n t = R_m$ the shock Cerenkov wave reaches him. At this moment the retarded position of the charge particle is $z' = z - \rho\gamma_n$. In the time interval $R_m < c_n t < r_1$ the solution corresponding to the upper sign gives EMF from the retarded positions of the particle in the interval $-z_0 < z' < z - \rho\gamma_n$. At the moment $c_n t = r_1$ the bremsstrahlung from the $z = -z_0$ point reaches the observer. On the other hand, the solution corresponding to the lower sign for $c_n t > R_m$ describes the electromagnetic field from the retarded positions of the charged particle lying on the right of the $z' = z - \rho\gamma_n$. As time goes the distance between the observation point and the particle retarded position increases. Correspondingly, the EMF diminishes at the observation point.

For the distant observer only the singular parts of the field strengths survive

$$\epsilon\vec{E} = -\frac{e\beta_n\rho\delta(c_n t - r_1)}{(\beta_n(z + z_0) - r_1)r_1}\cdot\vec{n}_\theta^1 + \frac{2e}{\gamma_n r_m}\Theta(z + z_0 - \rho\gamma_n)\delta(c_n t - R_m)\cdot\vec{n}_m,$$

$$\vec{H} = [-\frac{e\beta\rho\delta(c_n t - r_1)}{(\beta_n(z + z_0) - r_1)r_1} + \frac{2e}{\gamma_n r_m\sqrt{\epsilon\mu}}\Theta(z + z_0 - \rho\gamma_n)\delta(c_n t - R_m)]\cdot\vec{n}_\phi$$

The Poynting vector is equal to $\vec{S} = \vec{S}_1 + \vec{S}_c$, where

$$\vec{S}_1 = \frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{e\beta\rho\delta(c_n t - r_1)}{(\beta_n(z + z_0) - r_1)r_1}]^2\cdot\vec{n}_r^1, \quad \vec{S}_2 = \frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{2e\beta}{\gamma_n r_m}\Theta(z + z_0 - \rho\gamma_n)\delta(\mathcal{M})]^2\cdot\vec{n}_m^{\perp}$$

For the space region $z < \rho\gamma_n - z_0$ the distant observer detects the bremsstrahlung at the moment $c_n t = r_1$. There is no Cerenkov radiation in this region of space. For $z > \rho\gamma_n - z_0$ the distant observer detects the Cerenkov radiation at the moment $c_n t = R_m$ and the bremsstrahlung at the moment $c_n t = r_1$.

b) The charge particle motion ends by the state of rest (fig.3).

Let a particle moves with the velocity $v > c_n$ up to a point $z = z_0$. After that it rests there. The condition for the particle to be at rest is $c_n t > 2z_0/\beta_n + r_2$. The solution corresponding to the lower sign exists only for $z < z_0 + \rho\gamma_n$ and $R_m < c_n t < 2z_0/\beta_n + r_2$ (see the Appendix). The solution corresponding to the upper sign exists both for $z > z_0 + \rho\gamma_n$

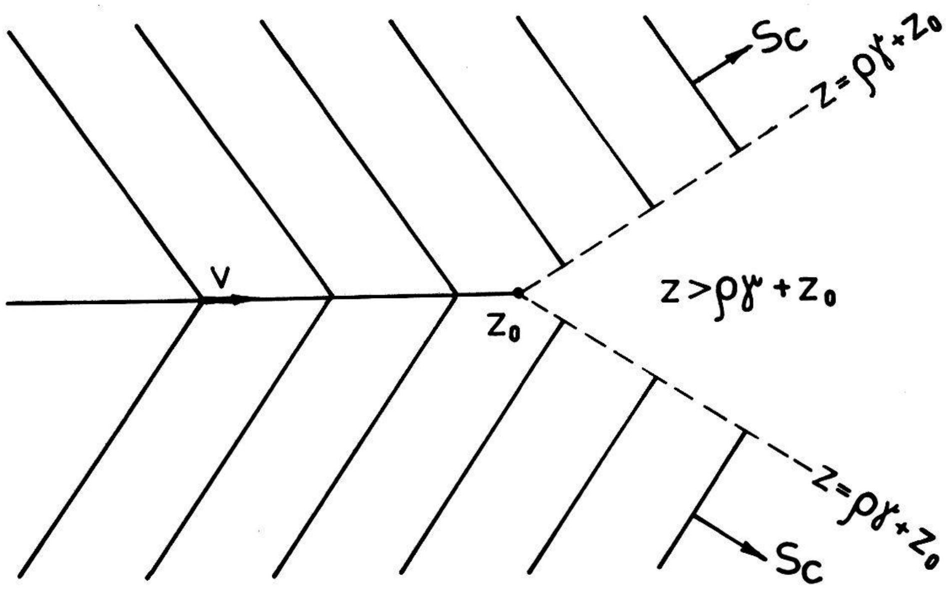

Figure 3: The superluminal motion ends by the rest state at $z = z_0$. In the region $z > \rho\gamma_n + z_0$ the observer sees no field up to some moment, when the shock bremsstrahlung wave reaches him. Later he sees the EMF of the resting charge and the EMF from one retarded position of the charge. In the region $z < \rho\gamma_n + z_0$ the EMF equals zero up to some moment when the Cerenkov shock wave reaches the observer. After that he sees EMF from two retarded positions of the charge up to the moment when the bremsstrahlung shock wave reaches him. Later, the observer sees simultaneously the field of the resting charge and that of the retarded positions of the charge.

if $c_n t > 2z_0/\beta_n + r_2$ and for $z < z_0 + \rho\gamma_n$ if $c_n t > R_m$. The electromagnetic potentials are equal to :

$$\epsilon\Phi = \frac{e}{r_2}\Theta(c_n t - 2z_0/\beta_n - r_2) + \frac{e}{r_m}\Theta(z - z_0 - \rho\gamma_n)\Theta(c_n t - 2z_0/\beta_n - r_2)$$

$$+\frac{e}{r_m}\Theta(c_n t - R_m)\Theta(z_0 + \rho\gamma_n - z)[1 + \Theta(2z_0/\beta_n + r_2 - c_n t)],$$

$$\frac{1}{\mu}A_z = \beta\frac{e}{r_m}\Theta(z - z_0 - \rho\gamma_n)\Theta(c_n t - 2z_0/\beta_n - r_2)$$

$$+\beta\frac{e}{r_m}\Theta(c_n t - R_m)\Theta(z_0 + \rho\gamma_n - z)[1 + \Theta(2z_0/\beta_n + r_2 - c_n t)]$$

For the observer in the $z > \rho\gamma_n + z_0$ region there is no EMF for $c_n t < 2z_0/\beta_n + r_2$. At the moment $c_n t = 2z_0/\beta_n + r_2$ he detects the bremsstrahlung shock wave. For $c_n t > 2z_0 + r_2$ the observer sees the EMF of the charge resting at the $z = z_0$ point and the EMF of the retarded positions of the particle trajectory lying on the left of the $z = z_0$ point. There is no Cerenkov radiation in this space region. For the observer in the $z < \rho\gamma_n + z_0$ region the EMF equals zero for $c_n t < R_m$. At the moment $c_n t = R_m$ the Cerenkov shock wave reaches the observation point. At this moment the retarded position of the particle is $z' = z - \rho\gamma_n$. For $R_m < c_n t < 2z_0/\beta_n + r_2$ the solution corresponding to the lower sign gives the EMF emitted from the points of the charge trajectory lying in the interval $(z - \rho\gamma_n < z' < z_0)$. For the moment $c_n t = 2z_0/\beta_n + r_2$ the bremsstrahlung from the $z = z_0$ point reaches the observer. After that the lower sign solution gives the EMF of the charge resting at the $z = z_0$ point. On the other hand, the solution corresponding to the upper sign for $c_n t > R_m$ gives EMF from the retarded points lying on the left of $z - \rho\gamma_n$ point. The EMF at the observation point diminishes as the radiation arrives from more remote points. The field strengths and Poynting vector in the wave zone are:

$$\epsilon\vec{E} = e\frac{\delta(c_n t - r_2 - 2z_0/\beta_n)}{\beta_n(z - z_0) - r_2}\frac{\rho\beta_n}{r_2}\cdot\vec{n}_\theta^2 + e\delta(c_n t - R_m)\frac{2}{r_m\gamma_n}\Theta(\rho\gamma_n + z_0 - z)\cdot\vec{n}_m,$$

$$\vec{H} = e[\frac{\delta(c_n t - r_2 - 2z_0/\beta_n)}{\beta_n(z - z_0) - r_2}\frac{\beta}{r_2} + \frac{2}{r_m\gamma_n\sqrt{\epsilon\mu}}\delta(c_n t - R_m)]\cdot\vec{n}_\phi,$$

$$S = S_2 + S_c,\quad S_2 = \frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{\delta(c_n t - r_2 - 2z_0/\beta_n)}{\beta_n(z - z_0) - r_2}\frac{\rho\beta}{r_2}]^2\cdot\vec{n}_r^2,$$

$$S_c = \frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{2\beta}{r_m\gamma_n}\delta(\mathcal{M})\Theta(z_0 + \rho\gamma_n - z)]^2\cdot\vec{n}_m^\perp$$

In the space region $z > \rho\gamma_n + z_0$ the distant observer detects the bremsstrahlung at the moment $c_n t = 2z_0/\beta_n + r_2$. There is no Cerenkov radiation there. For $z < \rho\gamma_n + z_0$ the observer sees the Cerenkov radiation at the moment $c_n t = R_m$ and the bremsstrahlung at the time $c_n t = 2z_0/\beta_n + r_2$.

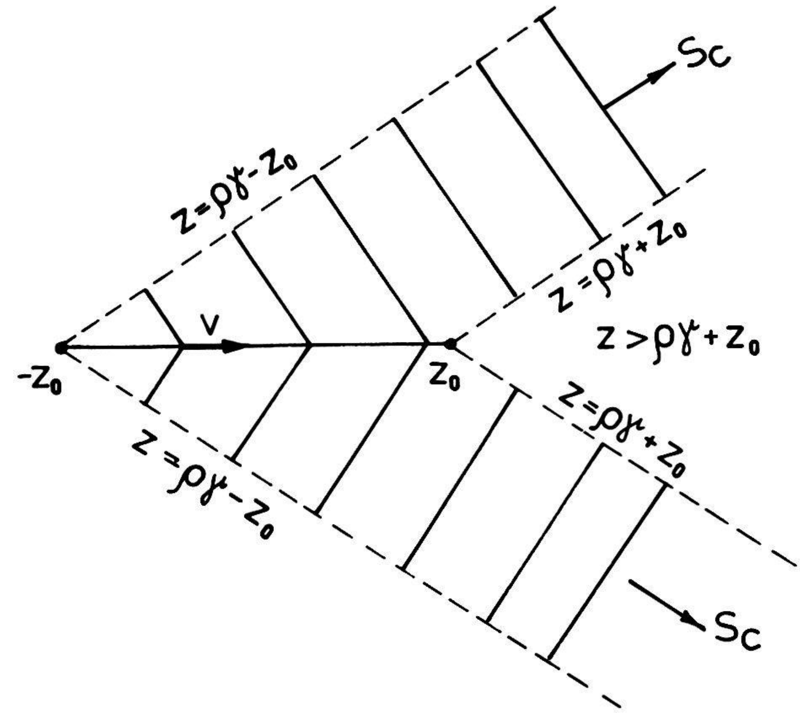

Figure 4: The superluminal motion begins from the rest state at the point $z = -z_0$ and ends by the rest state at the point $z = z_0$. For the finite distances the space-time distribution of EMF is rather complicated (see the text). The distant observer will see the following space-time picture. In the region $z < \rho\gamma_n - z_0$ he detects the bremsstrahlung shock wave from the $z = -z_0$ point first and from the $z = z_0$ point later. In the $z > \rho\gamma_n + z_0$ region these waves arrive in the reverse order. In the $\rho\gamma - z_0 < z < (\rho^2\gamma_n^2 + z_0^2/\beta_n^2)^{1/2}$ region the observer consecutively detects the Cerenkov shock wave, bremsstrahlung from the $z = -z_0$ point and bremsstrahlung from the $z = z_0$ point. In the region $(\rho^2\gamma_n^2 + z_0^2/\beta_n^2)^{1/2} < z < \rho\gamma_n + z_0$ the latter two waves arrive in the reverse order.

3.5 The Uniform Motion With $v > c_n$ on a Finite Time Interval

Let the charged particle rest at the point $z = -z_0$ for time $t < 0$. For the time interval $0 < t < 2z_0/v$ the particle moves with the constant velocity $v > c_n$. For $t > 2z_0/v$ the particle again rests at the point $z = z_0$ (fig.4). According to refs.[1,8] the physical realization of this model is, e.g.,a β decay followed by the nuclear capture.

The observer being placed into the different space-time regions will detect the following physical situation (see the Appendix).

i) $z < \rho\gamma_n - z_0$.

Then, for $c_n t < r_1$ the observer sees the EMF of the charge resting at $z = -z_0$. At the moment $c_n t = r_1$ the bremsstrahlung shock wave originating from $z = -z_0$ reaches him. For $r_1 < c_n t < 2z_0/\beta_n + r_2$ the observer sees the EMF of the charge moving with the superluminal velocity (lower sign in (2.3)). At the moment $c_n t = 2z_0/\beta_n + r_2$ the bremsstrahlung shock wave originating from the $z = z_0$ point reaches him. Finally, for $c_n t > 2z_0/\beta_n + r_2$ the observer sees the EMF of the charge resting at $z = z_0$. There is no Cerenkov radiation in this space region despite the observation of superluminal motion.

ii) $\rho\gamma_n - z_0 < z < (\rho^2\gamma_n^2 + z_0^2/\beta_n^2)^{1/2}$.

For $c_n t < R_m$ the observer sees the EMF of the charge resting at $z = -z_0$. At the moment $c_n t = R_m$ the Cerenkov shock wave reaches him. For $R_m < c_n t < r_1$ the observer simultaneously sees the EMF of the charge resting at $z = -z_0$ and the EMF of the moving charge (both signs give contribution). At the moment $c_n t = r_1$ the bremsstrahlung originating from the $z = -z_0$ point reaches him. For $r_1 < c_n t < 2z_0/\beta_n + r_2$ the observer will see the EMF of the moving charge (lower sign in (2.3)). At the moment $c_n t = 2z_0/\beta_n + r_2$ the bremsstrahlung shock wave originating from the $z = z_0$ point reaches him. At last, for $c_n t > 2z_0/\beta_n + r_2$ the observer sees the EMF of the charge resting at $z = z_0$.

iii) $[\rho^2\gamma_n^2 + z_0^2/\beta_n^2]^{1/2} < z < z_0 + \rho\gamma_n$.

For $c_n t < R_m$ the observer sees the EMF of the charge resting at the $z = -z_0$ point. At the moment $c_n t = R_m$ the Cerenkov shock wave reaches him. For $R_m < c_n t < 2z_0/\beta_n + r_2$ the observer sees the EMF of the charge resting at $z = -z_0$ and the EMF of the moving charge (both signs of Eq.(2.3) give contribution). At the moment $2z_0/\beta_n + r_2$ the bremsstrahlung shock wave originating from the $z = z_0$ point reaches the observation point. For $2z_0/\beta_n + r_2 < c_n t < r_1$ the observer simultaneously sees the EMF of the charge resting at $z = -z_0$, the EMF of the charge resting at $z = z_0$ and the EMF of the moving charge (upper sign in (2.3)). At the moment $c_n t = r_1$ the bremsstrahlung from the $z = -z_0$ point reaches him. Finally, for $c_n t > r_1$ the observer sees the EMF of the charge resting at $z = z_0$.

iiii) $z > z_0 + \rho\gamma_n$.

For $c_n t < 2z_0/\beta_n + r_2$ the observer will see the EMF of the charge resting at the $z = -z_0$ point. At the moment $c_n t = 2z_0/\beta_n + r_2$ the bremsstrahlung shock wave originating from $z = z_0$ point reaches him. For $2z_0/\beta_n + r_2 < c_n t < r_1$ he sees the EMF of the charge resting at the $z = \pm z_0$ points and the EMF of the moving charge (upper sign in (2.3)). At the moment $c_n t = r_1$ the bremsstrahlung shock wave originating from the $z = -z_0$ point reaches him. At last, for $c_n t > r_1$ the observer sees the EMF of the charge resting at $z = z_0$. There is no Cerenkov radiation in this space region.

The electromagnetic potentials are equal to

$$\epsilon\Phi = \Phi_1 + \Phi_2 + \Phi_m, \quad \frac{1}{\mu}A_z = \beta\Phi_m.$$

Here

$$\Phi_1 = \frac{e}{r_1}\Theta(r_1 - c_n t), \quad \Phi_2 = \frac{e}{r_2}\Theta(c_n t - r_2 - 2z_0/\beta_n), \quad \Phi_m = \frac{1}{r_m}\Theta(r_1 - c_n t).$$

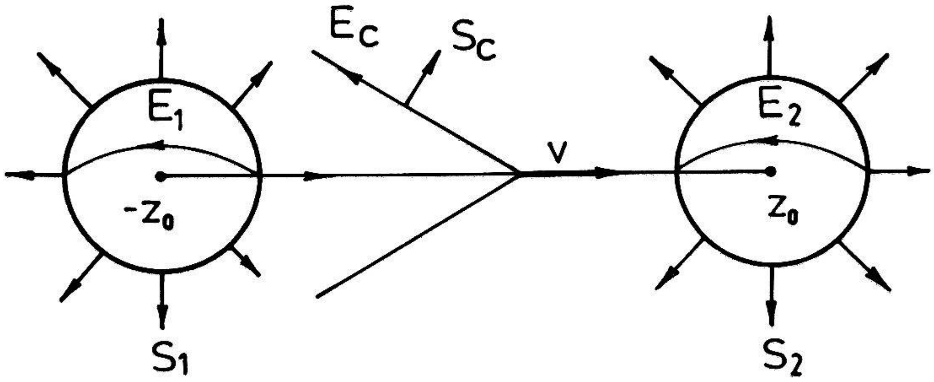

Figure 5: The schematic presentation of the EMF for the superluminal motion on the finite space interval. The magnetic field of the bremsstrahlung and of the moving charge has only the ϕ component. The electric field of the bremsstrahlungs has only the θ component. The electric field of the moving charge has singular and nonsingular parts. The singular part $\vec{E}_c$ lies on the Mach cone. The nonsingular part lies on the radius directed from the particle actual position towards the Mach cone.

$$\cdot[\Theta(z_0+\rho\gamma_n-z)\Theta(z+z_0-\rho\gamma_n)\Theta(c_nt-R_m)+\Theta(z-z_0-\rho\gamma_n)\Theta(c_nt-\frac{2z_0}{\beta_n}-r_2)]+$$

$$+\frac{1}{r_m}\Theta(\frac{2z_0}{\beta_n}+r_2-c_nt)[\Theta(z_0+\rho\gamma_n-z)\Theta(z+z_0-\rho\gamma_n)\Theta(c_nt-R_m)+\Theta(\rho\gamma_n-z-z_0)\Theta(c_nt-r_1)]$$

At large distances the field strengths are (fig.5)

$$\epsilon\vec{E}=-\frac{\delta(c_nt-r_1))}{\beta_n(z+z_0)-r_1}\frac{\rho\beta_n}{r_1}\cdot\vec{n}_\theta^1+\frac{\delta(c_nt-r_2-2z_0/\beta_n)}{\beta_n(z-z_0)-r_2}\frac{\rho\beta_n}{r_2}\cdot\vec{n}_\theta^2+$$

$$+\delta(c_nt-R_m)\frac{2}{r_m\gamma_n}\Theta(\rho\gamma_n+z_0-z)\Theta(z+z_0-\rho\gamma_n)\cdot\vec{n}_m,$$

$$\vec{H}=[-\frac{\delta(c_nt-r_1)}{\beta_n(z+z_0)-r_1}\frac{\rho\beta}{r_1}+\frac{\delta(c_nt-r_2)}{\beta_n(z-z_0)-r_2}\frac{\rho\beta}{r_2}+\frac{2}{r_m\gamma_n\sqrt{\epsilon\mu}}\delta(c_nt-R_m)]\cdot\vec{n}_\phi$$

The total Poynting vector reduces to the sum of energy fluxes radiated at the $z=\pm z_0$ points and to the Cerenkov one:

$$\vec{S}=\vec{S}_1+\vec{S}_c+\vec{S}_2,$$

$$\vec{S}_1=\frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{\delta(c_nt-r_1)}{\beta_n(z+z_0)-r_1}\frac{\rho\beta}{r_1}]^2\cdot\vec{n}_r^1,\quad \vec{S}_2=\frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{\delta(c_nt-r_2-2z_0/\beta_n)}{\beta_n(z-z_0)-r_2}\frac{\rho\beta}{r_2}]^2\cdot\vec{n}_r^2,$$

$$\vec{S}_c=\frac{c}{4\pi}\sqrt{\frac{\mu}{\epsilon}}\cdot[\frac{2}{r_m\gamma_n}\delta(\mathcal{M})\Theta(z+z_0-\rho\gamma_n)\Theta(z_0+\rho\gamma_n-z)]^2\cdot\vec{n}_m^\perp$$

For the distant observer the radiation field looks differently in various space regions.

i) $z < \rho\gamma_n - z_0$

At the moment $c_n t = r_1$ the observer detects the bremsstrahlung from the $z = -z_0$ point. At the later time $c_n t = 2z_0/\beta_n + r_2$ he detects the bremsstrahlung from the $z = z_0$ point. There is no Cerenkov radiation in this space region.

ii) $\rho\gamma_n - z_0 < z < (\rho^2\gamma_n^2 + z_0^2/\beta_n^2)^{1/2}$

The observer detects (consecutively in time) the Cerenkov shock wave at $c_n t = R_m$, the bremsstrahlung from the $z = -z_0$ point at the moment $c_n t = r_1$ and the bremsstrahlung from the $z = z_0$ point at the moment $c_n t = 2z_0/\beta_n + r_2$.

iii) $(\rho^2\gamma_n^2 + z_0^2/\beta_n^2)^{1/2} < z < \rho\gamma_n + z_0$

The observer sees the Cerenkov shock wave at the moment $c_n t = R_m$, the bremsstrahlung from the $z = z_0$ point at the moment $c_n t = 2z_0/\beta_n + r_2$ and the bremsstrahlung from the $z = -z_0$ point at the moment $c_n t = r_1$.

iiii) $z > \rho\gamma_n + z_0$.

At the moment $c_n t = 2z_0/\beta_n + r_2$ the observer fixes the bremsstrahlung from the $z = z_0$ point. At the later moment $c_n t = r_1$ he detects the bremsstrahlung from the $z = -z_0$ point. As in case i) there is no Cerenkov radiation in this space region.

4 Discussion

The bremsstrahlungs from the $z = \pm z_0$ points have maxima at the angles θ_1 and θ_2 numerically coinciding with the Cerenkov angle θ_c. One should bear in mind that these angles are ones between the z axis and the radius-vectors originating from different points. Indeed, θ_1 is the angle between the z axis and the radius-vector originating from the $z = -z_0$ point, θ_2 is the angle between the z axis and the radius-vector originating from the $z = z_0$ point, while θ_c is an angle between the z axis and the radius vector originating from the retarded position of the charged particle (fig.6). If the distance from the observation point is comparable with the motion distance $2z_0$, the inclination angles of the radius-vectors (i.e., angles between the radius-vectors and the z axis) directed from the $z = \pm z_0$ points toward the observer are certainly different from θ_c. This means that this observer will detect the bremsstrahlungs under the angles different from θ_c and for him the Cerenkov radiation will be clearly separated from the bremsstrahlungs. On the other hand, if the observer is at the distance much larger than $2z_0$, the bremsstrahlungs from the $z = \pm z_0$ points and the Cerenkov radiation reach the observation point almost at the same inclination angle θ_c. In this case, the angular separation of the Cerenkov radiation and bremsstrahlung is hardly possible. However, if the intensity of the charged particles is so low that inside the interval $(-z_0, z_0)$ there is only one charged particle at each instant of time, the time resolution between the Cerenkov photons and the bremsstrahlung ones is still possible. We conclude: the description of the Cerenkov

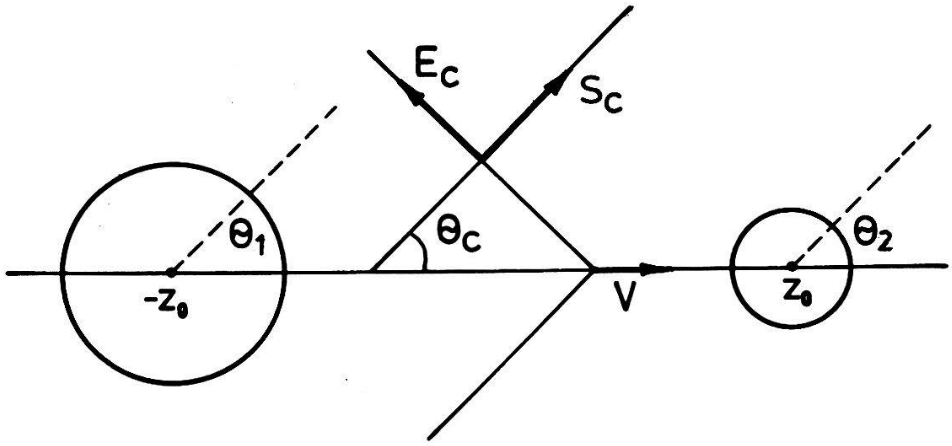

Figure 6: The observer not very far from the z axis sees the bremsstrahlung at the angle different from the angle of its (i.e., bremsstrahlung) maximal intensity coinciding with the Cerenkov angle θ_c. Thus, the angular resolution is possible for him. For the distant observer the time resolution between the Cerenkov radiation and bremsstrahlung is still possible.

radiation by direct solving of the Maxwell equations greatly simplifies the consideration. In particular, the prescriptions are easily obtained when and where the Cerenkov radiation should be observed in order to discriminate it from the bremsstrahlung. This is contrasted with the consideration in terms of the Fourier components where the discrimination of the Cerenkov radiation from the bremsstrahlung presents a problem (see, e.g., [1,8-10]). On the other hand, if the dependence of penetrabilities ϵ and μ of the photon frequency is essential, an analysis via the Fourier method seems to be more appropriate. In this sense, these two methods complement each other.

Appendix

We consider conditions arising from the inequality $0 < c_n t' < 2z_0/\beta_n$ for the upper and lower signs in (2.3) separately.

Upper sign.

For the upper sign in (2.3) one has

$$c_n t' = \gamma_n^2\{\beta_n(z+z_0) - c_n t - r_m\} \tag{A.1}$$

Now we impose the condition $t' > 0$. As the sign at r_m in (A.1) is negative, the first term should be positive. This means that the following two conditions should be satisfied simultaneously

$$c_n t < \beta_n(z+z_0) \quad \text{and} \tag{A.2}$$

$$\beta_n(z+z_0) - c_n t > r_m \tag{A.3}$$

The latter inequality being resolved WRT $c_n t$, gives:

$$c_n t < r_1 \tag{A.4}$$

As a result, one has

$$c_n t \le min\{\beta_n(z+z_0), \quad r_1\}$$

It is easy to check that $\beta_n(z+z_0)$ is greater or smaller than r_1 when z is greater or smaller than $\rho\gamma_n - z_0$, respectively. Thus, $c_n t < \beta_n(z+z_0)$ for $z < \rho\gamma_n - z_0$ and $c_n t < r_1$ for $z > \rho\gamma_n - z_0$. With the account of (3.3) one gets

$$R_m < c_n t < \beta_n(z+z_0) \quad \text{for} \quad z < \rho\gamma_n - z_0 \quad \text{and} \tag{A.5}$$

$$R_m < c_n t < r_1 \quad \text{for} \quad z > \rho\gamma_n - z_0 \tag{A.6}$$

Eq. (A.5) is satisfied if $R_m < \beta_n(z+z_0)$. This is possible only if $z > \rho\gamma_n - z_0$. This disagrees with the condition $z < \rho\gamma_n - z_0$ under which Eq.(A.5) was obtained. Thus, only Eq.(A.6) survives and the condition $c_n t' > 0$ reduces to

$$R_m < c_n t < r_1 \quad \text{for} \quad z > \rho\gamma_n - z_0 \tag{A.7}$$

Now we turn to the condition $c_n t' < 2z_0/\beta_n$. From (A.1) one gets

$$\beta_n(z-z_0) + \frac{2z_0}{\beta_n} - c_n t < r_m \tag{A.8}$$

This inequality is wittingly satisfied if $c_n t > \beta_n(z-z_0) + 2z_0/\beta_n$. Taking into account (3.3) one gets in this case

$$c_n t > max\{\beta_n(z-z_0) + 2z_0/\beta_n, \quad R_m\}$$

As $\beta_n(z-z_0) + 2z_0/\beta_n$ is greater or smaller than R_m when z is greater or smaller than $z_0 + \rho\gamma_n$, one obtains

$$c_n t > \beta_n(z-z_0) + 2z_0/\beta_n \quad \text{for} \quad z > z_0 + \rho\gamma_n \quad \text{and} \quad c_n t > R_m \quad \text{for} \quad z < z_0 + \rho\gamma_n \tag{A.9}$$

Now let

$$c_n t < \beta_n(z-z_0) + 2z_0/\beta_n \tag{A.10}$$

The following two inequalities stem from (A.8):

$$c_n t > \frac{2z_0}{\beta_n} + r_2 \quad \text{and} \quad c_n t < \frac{2z_0}{\beta_n} - r_2 \tag{A.11}$$

The second of these inequalities is incompatible with Eq.(3.3) for any z. Then, combining (3.3), (A.10) with the first of (A.11) one gets

$$max\{R_m, \quad \frac{2z_0}{\beta_n} + r_2\} < c_n t < \beta_n(z-z_0) + 2z_0/\beta_n \tag{A.12}$$

As $2z_0/\beta_n + r_2$ always exceeds R_m, inequality (A.12) reduces to

$$\frac{2z_0}{\beta_n} + r_2 < \beta_n(z - z_0) + 2z_0/\beta_n \tag{A.13}$$

The RHS of this inequality exceeds its LHS only if $z > z_0 + \rho\gamma_n$. From (A.13) and (A.9) one obtains that condition $c_n t' < 2z_0/\beta_n$ reduces to

$$c_n t > \frac{2z_0}{\beta_n} + r_2 \quad \text{for} \quad z > z_0 + \rho\gamma_n \quad \text{and} \quad c_n t > R_m \quad \text{for} \quad z < z_0 + \rho\gamma_n \tag{A.14}$$

Combining (A.7) and (A.14) one obtains equations guaranteeing the fulfillment of the inequality $0 < c_n t' < 2z_0/\beta_n$ for the upper sign in (2.3)

$$R_m < c_n t < r_1 \quad \text{for} \quad \rho\gamma_n - z_0 < z < \rho\gamma_n + z_0 \quad \text{and}$$

$$\tag{A.15}$$

$$\frac{2z_0}{\beta_n} + r_2 < c_n t < r_1 \quad \text{for} \quad z > z_0 + \rho\gamma_n$$

Lower sign.

For the lower sign one has

$$c_n t' = \gamma_n^2\{\beta_n(z + z_0) - c_n t + r_m\} \tag{A.16}$$

Consider at first the inequality $c_n t' > 0$. It is wittingly satisfied if $c_n t < \beta_n(z + z_0)$. Taking into account (3.3) one gets

$$R_m < c_n t < \beta_n(z + z_0) \tag{A.17}$$

It takes place when $z > \rho\gamma_n - z_0$. On the other hand, if $c_n t > \beta_n(z + z_0)$, there should be

$$c_n t - \beta_n(z - z_0) < r_m$$

Being resolved, this gives two solutions

$$c_n t < -r_1 \quad \text{and} \tag{A.18}$$

$$c_n t > r_1 \tag{A.19}$$

Consider at first (A.18). If it is satisfied, the following inequality should take place

$$max\{\beta_n(z + z_0), \quad R_m\} < c_n t < -r_1 \tag{A.20}$$

It turns out that $\beta_n(z + z_0)$ is greater or smaller than R_m when z is greater or smaller than $\rho\gamma_n - z_0$, resp. Then, for $z > \rho\gamma_n - z_0$ one gets $\beta_n(z + z_0) < c_n t < -r_1$. But for this range of z the RHS of this inequality is smaller than its LHS. Thus, (A.18) cannot be satisfied for $z > \rho\gamma_n - z_0$. For $z < \rho\gamma_n - z_0$ one obtains

$$R_m < c_n t < -r_1$$

The RHS of this inequality exceeds its LHS when the following two Eqs. are satisfied simultaneously

$$z + z_0 + \rho\gamma_n < 0 \quad \text{and} \quad r_1 < -R_m$$

The second of these Eqs. and, as a consequence, inequality (A.18) cannot be satisfied for any z. Thus, only Eq.(A.19) survives. It is valid only if

$$c_n t > max\{\beta_n(z + z_0), \quad R_m, \quad r_1\}$$

As always $r_1 > R_m$, so it remains

$$c_n t > max\{\beta_n(z + z_0), \quad r_1\}$$

Since $\beta_n(z + z_0) > r_1$ for $z > \rho\gamma_n - z_0$ and $\beta_n(z + z_0) < r_1$ for $z < \rho\gamma_n - z_0$ one gets $c_n t > \beta_n(z + z_0)$ for $z > \rho\gamma_n - z_0$ and $c_n t > r_1$ for $z < \rho\gamma_n - z_0$. Combining these Eqs. with (A.17) one gets the following Eqs. realizing the condition $c_n t' > 0$ for the lower sign in (2.3):

$$c_n t > R_m \quad \text{for} \quad z > \rho\gamma_n - z_0 \quad \text{and} \quad c_n t > r_1 \quad \text{for} \quad z < \rho\gamma_n - z_0 \tag{A.22}$$

Now we analyze what the condition $c_n t' < 2z_0/\beta_n$ means for the lower sign. It follows from (A.16) that

$$r_m < c_n t - \beta_n(z - z_0) - 2z_0/\beta_n \tag{A.23}$$

Obviously, this inequality can be satisfied only if

$$c_n t > \beta_n(z - z_0) + 2z_0/\beta_n \tag{A.24}$$

Resolving (A.23) gives

$$\frac{2z_0}{\beta_n} - r_2 < c_n t < \frac{2z_0}{\beta_n} + r_2 \tag{A.25}$$

Combining this Eq. with (3.3) and (A.24) one arrives at

$$max\{\frac{2z_0}{\beta_n} - r_2, \quad \beta_n(z - z_0) + \frac{2z_0}{\beta_n}, \quad R_m\} < c_n t < \frac{2z_0}{\beta_n} + r_2$$

As always $R_m > 2z_0/\beta_n - r_2$, so it remains

$$max\{\beta(z_n - z_0) + \frac{2z_0}{\beta_n}, \quad R_m\} < ct_n < \frac{2z_0}{\beta_n} + r_2 \tag{A.25}$$

Noticing that the sign of $\beta_n(z - z_0) + 2z_0/\beta_n - R_m$ coincides with that of $(z - \rho\gamma_n - z_0)$ one gets:

$$R_m < c_n t < \frac{2z_0}{\beta_n} + r_2 \quad \text{for} \quad z < \rho\gamma_n + z_0 \quad \text{and} \tag{A.26}$$

$$\beta_n(z - z_0) + 2z_0/\beta_n < c_n t < \frac{2z_0}{\beta_n} + r_2 \quad \text{for} \quad z > \rho\gamma_n + z_0 \tag{A.27}$$

The LHS of (A.26) is always smaller than its RHS. On the other hand, the RHS of (A.27) exceeds its LHS only if $z < z_0 + \rho\gamma_n$. This disagrees with the condition $(z > z_0 + \rho\gamma_n)$ under which the inequality (A.27) was obtained. As a result, only inequality (A.26) survives. It realizes the condition $c_n t' < 2z_0/\beta_n$ for the lower sign. Combining (A.26) with (A.22) one gets

$$max\{R_m, \quad r_1\} < c_n t < \frac{2z_0}{\beta_n} + r_2 \quad \text{for} \quad z < \rho\gamma_n - z_0 \quad \text{and}$$

$$R_m < ct < \frac{2z_0}{\beta_n} + r_2 \quad \text{for} \quad \rho\gamma_n - z_0 < z < \rho\gamma_n + z_0.$$

As r_1 always exceeds R_m, one finds that for the lower sign the condition $0 < c_n t' < 2z_0/\beta_n$ reduces to

$$r_1 < c_n t < \frac{2z_0}{\beta_n} + r_2 \quad \text{for} \quad z < \rho\gamma_n - z_0 \quad \text{and}$$

$$(A.28)$$

$$R_m < c_n t < \frac{2z_0}{\beta_n} + r_2 \quad \text{for} \quad \rho\gamma_n - z_0 < z < \rho\gamma_n + z_0$$

This completes the analysis of the boundary conditions influence on the space-time distribution of the Cerenkov radiation.

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