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The dependence of the QED short-distance enhancement factor on the top quark mass

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We point out that the QED short-distance enhancement factor $S(m_p, m_Z)$, which contributes to the radiative corrections to superallowed $0^+ \to 0^+$ nuclear β^+ decays, is independent of the mass of the top quark provided that this mass is greater than the Z^0 mass. This contradicts a paper published recently. We give the correct expression for $S(m_p, m_Z)$ and its numerical value.

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Two independent analyses of data on the ft values for superallowed nuclear β^+ decays within 0^+ isospin triplets have appeared recently [1, 2]. Both lead to values of $|V_{ud}|$ which, when combined with values of $|V_{us}|$ and $|V_{ub}|$ obtained from data on the semileptonic decays of hadrons containing strange and bottom quarks respectively, enable a test to be made of the unitarity of the quark mixing matrix. If one accepts the standard model of the electroweak interaction in its usual form, this matrix must be unitary.

The QED short-distance enhancement factor $S(m_p, m_Z)$ contributes to the radiative correction to superallowed decays and therefore affects the value of $|V_{ud}|$. Both analyses take the value of $S(m_p, m_Z)$ from Marciano and Sirlin [3], who used a top quark mass $m_t = 40$ GeV . Wilkinson [4] claimed that, after making what he considered to be a reasonable adjustment to the nuclear mismatch correction for superallowed decays, the dependence of $S(m_p, m_Z)$ on m_t is such that use of the more recent value $m_t = 130 \pm 30$ GeV leads to values of $|V_{ud}|$ consistent with unitarity. The formula that Wilkinson used for $S(m_p, m_Z)$, which was taken from ref. [3], is valid, however, only for $m_b < m_t < m_W$. Different formulae are required if $m_W < m_t < m_Z$, or if $m_t > m_Z$. It is now practically certain that $m_t > m_Z$ [5, 6], in which case $S(m_p, m_Z)$ no longer depends explicitly on m_t. We here give these modified formulae for $S(m_p, m_Z)$, and the value of $S(m_p, m_Z)$ for the presently accepted values of particle masses.

The radiative correction of $O(\alpha)$ to the ft values for superallowed decays was given by Sirlin [7] as

$$\left\{1 + \frac{\alpha}{2\pi}\, \bar{g}\,(E,E_0)\right\} \left\{1 + \frac{\alpha}{2\pi}\left[4 \ln\left(\frac{m_Z}{m_p}\right) + \ln\left(\frac{m_p}{m_A}\right) + 2C + \mathcal{A}_g\right]\right\} \qquad (1)$$

(for notation see ref. [2]). Marciano and Sirlin [3] incorporated the effect of leading logarithmic corrections of order $(\alpha \ln m_Z)^n$ by means of a re-normalisation group analysis, with the result that the expression (1) should be replaced by

$$\left\{1+\frac{\alpha}{2\pi}\left[\ln\left(\frac{m_p}{m_A}\right)+2C\right]+\frac{\alpha(m_p)}{2\pi}\left[\bar{g}\,(E, E_0)+\mathcal{A}_g\right]\right\} S(m_p, m_Z)\ . \qquad (2)$$

The factor $S(m_p, m_Z)$ effectively replaces $1+\frac{2\alpha}{\pi}\ln\left(\frac{m_Z}{m_p}\right)$.

The expression given by Marciano and Sirlin [3] for $S(m_p, m_Z)$ involves the running QED fine structure constant $\alpha(\mu)$, which is a function of the mass scale μ. In the modified minimal subtraction scheme, it evolves according to

$$\alpha^{-1}(\mu_2) = \alpha^{-1}(\mu_1) - b_0 \ln(\mu_2/\mu_1) \qquad (\mu_2 > \mu_1)\ , \qquad (3)$$

provided b_0 is constant in the interval (μ_1, μ_2). Also one has [8]

$$S(\mu_1, \mu_2) = \left[\alpha(\mu_2)/\alpha(\mu_1)\right]^{2/\pi b_0}. \qquad (4)$$

The value of b_0 changes each time one passes through the mass of an elementary fermion (lepton or quark) or of the $W^{\pm}$:

$$b_0 = \frac{2}{3\pi}\sum_f Q_f^2\,\theta(\mu - m_f) - \frac{7}{2\pi}\theta(\mu - m_W)\ , \qquad (5)$$

where at each quark mass one has to take account of the three colour degrees of freedom. Thus one has

$$\alpha^{-1}(\mu) = \alpha^{-1}(0) - \frac{2}{3\pi}\sum_f Q_f^2 \ln\left(\frac{\mu}{m_f}\right), \qquad (6)$$

where $\alpha^{-1}(0) = \alpha^{-1} + 1/6\pi = 137.089$ [3], and an extra term $(7/2\pi)\ln(\mu/m_W)$ is added to the right side of eq. (6) if $\mu > m_W$. The sum in eq. (6) is over fermions f for which $m_f < \mu$. Also $S(m_p, m_Z)$ is obtained by multiplying factors as in eq. (4) for each subinterval in which b_0 is constant.

Marciano and Sirlin [3] in their eq. (11) give the expression for

$S(m_p, m_Z)$ when $m_b < m_t < m_W$. For the case when $m_t > m_W$, the first three factors remain the same. When $m_W < m_t < m_Z$, we have

$$\begin{aligned} b_0 &= 40/9\pi , && m_b < \mu < m_W , \\ b_0 &= 17/18\pi , && m_W < \mu < m_t , \\ b_0 &= 11/6\pi , && m_t < \mu < m_Z , \end{aligned} \tag{7}$$

giving, instead of the last three factors of eq. (11) of ref.[3], the three factors

$$\left[\frac{\alpha(m_W)}{\alpha(m_b)}\right]^{9/20} \left[\frac{\alpha(m_t)}{\alpha(m_W)}\right]^{36/17} \left[\frac{\alpha(m_Z)}{\alpha(m_t)}\right]^{12/11} . \tag{8}$$

When $m_t > m_Z$, b_0 remains at $17/18\pi$ up to m_Z , so that there are just two additional factors and the complete expression becomes

$$S(m_p, m_Z) = \left[\frac{\alpha(m_c)}{\alpha(m_p)}\right]^{3/4} \left[\frac{\alpha(m_\tau)}{\alpha(m_c)}\right]^{9/16} \left[\frac{\alpha(m_b)}{\alpha(m_\tau)}\right]^{9/19} \left[\frac{\alpha(m_W)}{\alpha(m_b)}\right]^{9/20} \left[\frac{\alpha(m_Z)}{\alpha(m_W)}\right]^{36/17} . \tag{9}$$

We calculate the values of $\alpha(\mu)$ using the present mass values for the charmed and bottom quarks and for the W and Z bosons : $m_c = 1.35$ GeV, $m_b = 5.0$ GeV, $m_W = 80.19$ GeV and $m_Z = 91.18$ GeV . The relative values of the $\alpha^{-1}(\mu)$ can then be obtained from eq. (6). In order to calculate absolute values of $\alpha^{-1}(\mu)$ from eq. (6), one may use effective masses of the up, down and strange quarks. The values of $\alpha^{-1}(\mu)$ given by Marciano and Sirlin [3] are obtained by taking the masses of these three light quarks as all equal to 0.07 GeV, as stated by Wilkinson [4]. This gives, in particular,

$$\alpha^{-1}(m_p) = 133.93 , \tag{10}$$

which was the value used in the analyses of refs.[1, 2]. Also eq. (9) gives

$$S(m_p, m_Z) = 1.02249 , \tag{11}$$

which is close to the value 1.02256 given in ref. [3] for $m_t = 40$ GeV.

The quark contribution to the last term in eq. (6) can also be obtained directly by using dispersion theory fits to experimental data on $\sigma(e\bar{e} \to \text{hadrons})$. In this way, Degrassi *et al.* [9] obtain $\alpha^{-1}(m_Z) = 127.8\,(1)$ using their eq. (A.2). A more recent calculation by Jegerlehner [10] on the basis of [11] gives for the 5 quark contribution at m_Z the value 0.0282 (8) instead of the number 0.02875 (90) in eq. (A.2). The uncertainty in this number reflects the latitude in fitting the data. With Jegerlehner's new value, eq. (A.2) of ref. [9] gives

$$\alpha^{-1}(m_Z) = 127.85\,(11)\ . \tag{12}$$

From eq. (6),† the corresponding values of the other $\alpha^{-1}(\mu)$ are then

$$\alpha^{-1}(m_p) = 133.82\,(11) \tag{13}$$

and $\alpha^{-1}(m_c) = 133.51$, $\alpha^{-1}(m_\tau) = 133.20$, $\alpha^{-1}(m_b) = 131.81$, $\alpha^{-1}(m_W) = 127.89$ (all with an uncertainty 0.11), leading to

$$S(m_p\,,\, m_Z) = 1.02251\,(2)\ . \tag{14}$$

Since this second approach avoids the use of uncertain effective masses for the light quarks, it seems preferable to the first approach. Replacement of the values of $\alpha^{-1}(m_p)$ and $S(m_p\,,\, m_Z)$ used in the analyses of refs. [1,2] by the values (13) and (14) increases the values of $|V_{ud}|^2$ by only 3.10^{-5} and therefore has no influence on the test of unitarity.

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† It would not be consistent to calculate the values of all the $\alpha(\mu)$ occurring in eq. (9) from Jegerlehner's values of the quark contribution for each value of μ, since such values would not satisfy eqs. (3) and (5), which are the basis for eqs. (4) and (9).

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