Chaos in mechanical engineering devices

Autor(en):   **Hongler, Max-Olivier**

Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr):   **62 (1989)**

Heft 5

CHAOS IN MECHANICAL ENGINEERING DEVICES.

Max-Olivier HONGLER

Institut de Microtechnique

Département de Mécanique

Ecole Polytechnique Fédérale de Lausanne

CH-1015 LAUSANNE.

Abstract : Vibro-impact devices commonly used in mechanical engineering are discussed in the light of recent developments of non-linear dynamics. More specifically, the existence of typically non-linear phenomena such as cascade of bifurcations and chaotic solutions are examined. The illustrations presented here, namely vibratory transporters and models of gearboxes have, besides their own engineering interest, the merit to exhibit a dynamics described by well known 2-dimensional, dissipative mappings.

1. INTRODUCTION.

In 1961, the eminent Professor of Mechanical Engineering, R. M. Rosenberg concluded an article devoted to non-linear oscillations in the following terms [1] : "*The outlook regarding progress in non-linear oscillations is bright for those who like to do research, and bleak for those who like to see results. (....). At the present time, no hope exists for a unified theory or body of knowledge regarding the solutions of non-linear problems. The basic reason behind this statement is the failure to define the field .*" Since this remark was written, great progresses has been achieved in the field of non-linear dynamics. Recently, the engineering community has started to explore more systematically the implications of these new mathematical developments [2,3]. Illustrations ranging from magnetically levitated vehicles, chaos in elastic continua, impact print head, non-linear electric circuits, etc...are reported ; (see further references in [3]). The aim of the present paper is to bring a contribution to this exploration in the domain of mechanical engineering.

We shall discuss dissipative, non-linear systems driven by external impulsive forces and (or) also subject to impusive constraints. The Dynamics of these systems is described by discrete mappings which now stand as the corner-stone of the studies in non-linear science. Let us here emphasize that the Dynamics of the impulsively driven systems is exactly described by non-linear mappings ; whereas, in the study of differential equations, mappings often result either from approximations or modelizations of the original equations of the motion. Besides their own engineering interest, the devices discussed in this paper, present the advantage to admit dynamical equations already encountered in the mathematical literature.

Before, we introduce our particular devices, let us first formally exhibit the type of equations of motion we have to deal with. These have the recurrent form :

$$\tau_{n+1} = f_1(\tau_n, \psi_n) \tag{1a}$$

$$\psi_{n+1} = f_2(\tau_n, \psi_n), \tag{1b}$$

where f_1 and (or) f_2 are non-linear functions . The mappings to be derived are dissipative i.e. the Jacobian of Eqs. (1a,b) is less than one.. Among the infinitely rich variety of choices for the function f_1 and f_2 two situations have been largely studied, namely :

$$f_1 = \tau_n + \alpha \psi_n \tag{2a}$$

$$f_2 = \varepsilon \psi_n + (1+\varepsilon)\cos(\tau_{n+1}), \tag{2b}$$

and

$$f_1 = \tau_n + \alpha/\psi_n \tag{3a}$$

$$f_2 = -\varepsilon \psi_n + \beta\cos(\tau_n), \tag{3b}$$

where α, β are external parameters and $0 < \varepsilon < 1$ relates the dissipation ; the Jacobian of these transformations equals ε. Eqs. (2a, b) describe the so-called dissipative standard mapping which is discussed in [3]. Eqs. (3a, b) are known under the name of the dissipative Fermi map and are also considered in [3, 4].

Our paper is organised as follows : In section 2, we introduce the problem of vibro-transportation. It is observed that the dynamics of this system is governed by the mapping Eqs. (2a, b). In section 3, we discuss, as a second example, the modelization of gearboxes. In this last situation, the Fermi mapping Eqs. (3a, b) is found to be embedded in the dynamical equations of motion. Finally, section 4 is devoted to conclusions and remarks..

2. VIBRO-TRANSPORTATION.

One of the difficulties in the realization of automatic assembly lines is to convey parts to the ad-hoc locations in the chain. A commonly adopted solution is the use of vibratory transporters (also called vibratory feeders). Basically, a vibratory feeder is constituted by an oscillating track on which the parts to be conveyed are disposed. When the track is set into motion, the mobile lying on it is itself set in movement. Since the pioneering work A.H. Redford & G. Boothroyd [6], theoretical and experimental aspects vibro-transportation have been abundantly studied, (a selection of articles is given in [7]). This important activity of research clearly reflects the difficulties which the constructors of feeders have to deal with.

Schematically, the device is represented in Fig.1 and 2 where the notations to be used are introduced. The reference frame xOy is mobile and attached to the track.

In actual applications, the vibratory transporter is either a bowl or a linear track. Hére, we shall restrict our discussion to the linear case for which the centripetal and Coriolis accelerations are absent- (the dynamics for the bowl shape case presents, in its essence, identical features as locally it reduces to the case Fig. 2).
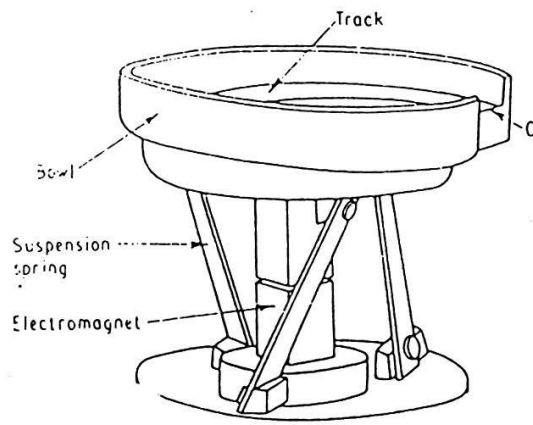
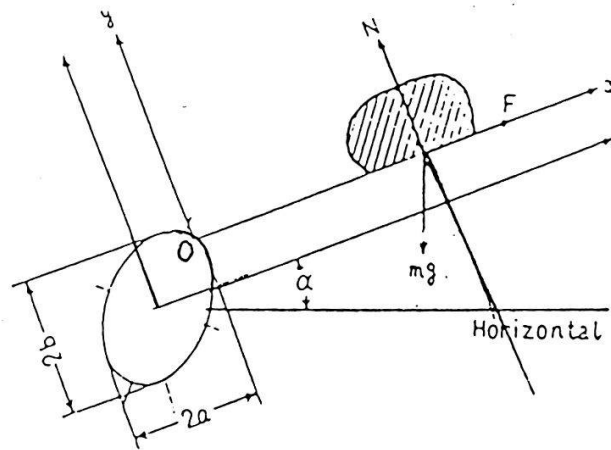*Vibratory bowl feeder*

Fig. 1

Fig. 2

In view of Fig. 2, the general equations of the motion have the form :

$$m\ddot{x}(t) = ma\omega^2 \sin(\omega t) - mg\sin(\alpha) + F \qquad (4a)$$

$$m\ddot{y}(t) = mb\omega^2 \sin(\omega t + \gamma) - mg\cos(\alpha) + N, \qquad (4b)$$

where dots denote the derivatives with respect to the time, F and N stand respectively for the friction and the constraints forces, α is the slope of the track, g the gravitational acceleration and γ the phase shift between the parallel and perpendicular components of the excitation force.

Depending on the external parameters, various type of motion exist and a detailed analysis of the possible periodic motions is given in [8]. Here, we shall confine our attention to the pure jumping regimes (i.e. sticking to the track is neglected). In these regimes, the dynamics between the impacts with the feeder, simply reduces to free flight equations, namely :

$$\ddot{u}(\tau) = \sin(\tau) - k \qquad (5a)$$

$$\ddot{v}(\tau) = \eta\sin(\tau + \gamma) - k\,tg(\alpha) \qquad (5b)$$

where τ, $u(\tau)$, $v(\tau)$, k, η are dimensionless quantities defined :

$$k = g\cos(\alpha)/b\omega^2 \quad ; \quad \eta = a/b \quad ; \quad \tau = \omega t$$

$$u = y/b \quad ; \quad v = x/a . \tag{6}$$

The dynamics at the nth impact time $\tau = \tau_n$ is specified in the form :

$$\frac{\partial}{\partial \tau} u(\tau) \Big|_{\tau = \tau_n + \varepsilon} = -R_\perp \frac{\partial}{\partial \tau} u(\tau) \Big|_{\tau = \tau_n - \varepsilon} \tag{7a}$$

$$\frac{\partial}{\partial \tau} v(\tau) \Big|_{\tau = \tau_n + \varepsilon} = -R_\perp \frac{\partial}{\partial \tau} v(\tau) \Big|_{\tau = \tau_n - \varepsilon} \tag{7b}$$

where the coefficients of the perpendicular, (parallel) restitution are denoted respectively by $R_\perp$ and $R_{//}$ and ε is a infinitesimal quantity which relates times just before and after the impact time $\tau = \tau_n$. Obviously we have :

$$0 < R_\perp < 1 \quad \text{and} \quad 0 < R_{//} < 1. \tag{8}$$

Now , let us introduce the notations :

$$\frac{\partial}{\partial t} u(t) \Big|_{t = \tau_n} = \Psi_n \quad \text{and} \quad \frac{\partial}{\partial t} v(t) \Big|_{\tau = \tau_n} = \Phi_n \tag{9}$$

Using Eqs. (7-9), we can reactualize the initial conditions each time an impact has occured. Hence the direct integration of the free flight Eqs. (5a, b) yields the set of non-linear mappings :

$$-k/2 \ (\tau_{n+1}-\tau_n)^2 + (\cos(\tau_n)+\Psi_n)(\tau_{n+1}-\tau_n) =$$

$$= \sin(\tau_{n+1})-\sin(\tau_n) \qquad (10)$$

$$\Psi_{n+1} = R_\perp(\ k(\tau_{n+1}-\tau_n)+\cos(\tau_{n+1})-\cos(\tau_n)-\Psi_n) \qquad (11)$$

$$\Phi_{n+1} = R_{/\!/}(-\eta \ (\cos(\tau_{n+1}+ \gamma) - \cos(\tau_n+\gamma)) -$$

$$-k tg(\alpha)(\tau_{n+1}-\tau_n) + \Phi_n). \quad (12)$$

The transport rate itself can be calculated with the mean velocity W_n (in the parallel direction) attained between successive impacts ; thus we obtain :

$$W_n = \frac{1}{\tau_{n+1}-\tau_n} \int_{\tau_n}^{\tau_{n+1}} \left(\frac{\partial}{\partial \tau'} v(\tau')\right) d\tau' = \qquad (13)$$

$$W_n = \left\{ \eta\cos(\tau_n+\gamma)+\Phi_n \right\} - \frac{k}{2} tg(\alpha)\left(\tau_{n+1}- \tau_n\right) - \left(\frac{\eta}{\tau_{n+1}-\tau_n} \left(\sin(\tau_{n+1}+\gamma)-\sin(\tau_n+\gamma)\right) \right)$$

The dynamics of the model is now completely characterized by Eqs. (10-13).

Let us discuss the solutions. First of all, one has to remark that Eqs. (10) and (11) can be discussed independently of Eq. (12). Eqs. (10) and (11) are precisely the mapping recently studied in [9,10]. Let us briefly recall the results obtained in [4,5]. The mappings Eqs. (10) and (11) exhibits the cascade of bifurcations (here, the control parameter is k), discovered by Myrberg and Feigenbaum [11]. The period one solutions (i.e. $\tau_n = \tau_0 + (2\pi n)r$, $\mathbb{N} \ni r$) are immediately found in the form:

$$\Psi_{n+1} = \Psi_n = \Psi = \frac{(2\pi r k)\, R_\perp}{1 + R_\perp} \tag{14a}$$

$$\tau_0 = A\cos\left\{ \pi r k \left(\frac{1 - R_\perp}{1 + R_\perp} \right) \right\} \tag{14b}$$

Once $R_\perp$ is fixed and hence the mobile to transport selected, the unique control parameter of the problem is k. The stability intervals for the period one solutions are obtained by a linearization procedure. The result reads [4,9,12] :

$$k_{1,r} > k > k_{2,r} \tag{15}$$

where $k_{1,r}$ and $k_{2,r}$ read :

$$(k_{1,r})^{-2} = \left\{ \pi^2 r^2 \left(\frac{1-R_\perp}{1+R_\perp} \right)^2 + 4 \frac{\left(1+R_\perp^2 \right)^2}{\left(1+R_\perp \right)^4} \right\}$$

and

$$k_{0,r} = \frac{1}{\pi r} \left(\frac{1+R_\perp}{1-R_\perp} \right)$$

When k is decreased below $k_{1,r}$ a stable period two orbit is found. This behaviour is observed until a new critical value, say $k_{2,r}$, is reached, where a new period doubling occurs.... and so on until $k_{\infty,r}$, where the chaotic regime is attained [4,5,11,13,14]. The succession of the critical values approach the accumulation point $K_{\infty,r}$ according to the the equation [4,5,11,13,14] :

$$\lim \frac{k_{\eta+1,r} - k_{\eta,r}}{k_{\eta+2,r} - k_{\eta+1,r}} = 0.46992... \qquad \eta = 1, 2, 3, \text{.......} \qquad (16)$$

Using Eq. (13), the transport rate takes the form :

$$W_n = W = \pi r k \left\{ \eta \cos(\gamma) \left(\frac{1 - R_\perp}{1 + R_\perp} \right) - tg(\alpha) \left(\frac{1 + R}{1 - R} \right) \right\} -$$

$$-\eta \sin(\gamma) \left(1 - \pi^2 k^2 r^2 \left(\frac{1 - R_\perp}{1 + R_\perp} \right)^2 \right)^{\frac{1}{2}} \qquad (17)$$

While it is relatively obvious to obtain Eq. (17), the estimation of the transport is far less trivial in the case of chaotic regimes. To simplify the expressions without lost of generality, let us confine ourselves now to the case $\gamma = 0$. With the use of Eq. (10), Eq. (13) can be written in the form :

$$W_n = \left(\Phi_n - \eta \Psi_n \right) + \left(\tau_{n+1} - \tau_n \right) \left(\frac{\eta k}{2} - \frac{k}{2} tg(\alpha) \right) \qquad (18)$$

In the chaotic regime, the quantities Φ_n, Ψ_n, and τ_n form pseudo-stochastic sequences which statistical properties are unknow. To calculate the average transport rate, one would in fact need the probability densities governing these quantities. Analytical results which give such invariant measures are not yet available. Hence, one has to resort to numerical exploration. Presently, such studies are performed [15].

For relatively large restitution parameter, namely $0.8 < R_\perp < 1$, the mapping Eqs. (10 - 11) can be approximated [4,16]. This is achieved by observing that the quantity :

$$(\sin(\tau_{n+1})-\sin(\tau_n))/(\tau_{n+1}-\tau_n)$$

is a vanishingly small quantity for appropriately choosen initial conditions [4,16]. Using this approximation, the mapping Eqs. (10-11) can be revritten in the form :

$$\tau_{n+1} = \tau_n + \frac{2}{k}\left(\cos(\tau_n) + \Psi_n\right) \qquad (19a)$$

$$V_{n+1} = R_\perp V_n + \left(1 + R_\perp\right)\cos\left(\tau_{n+1}\right) \qquad (19b)$$

where we have introduced the notation :

$$V_{n+1} = \cos(\tau_n) + \Psi_n \qquad (20)$$

The mapping Eqs. (19a,b) exhibits precisely the form of Eqs. (2a,b).

In actual feeders, the reliability of the mean transport rate is one of the crucial property. Indeed, the set of external control parameters (i.e. amplitude of the excitations, frequency etc....) are always subject to variations due industrial environment. Hence, the problem is to determine transport regimes which are not too sensitive to variations of these external parameters. It seems intuitively clear, that beside the simplest periodic solution (which are obtained for k in relatively large bands), the chaotic regime is also likely to favour a mean transport rate relatively insensitive to external parameters. In the subharmonic perodic regimes, very tiny changes in the operating conditions are sufficient to induce a change of period in the Feigenbaum cascade, (see Eq. (16)). On the other hand, we have to stress that external noise is always present in the system. This in turn has the effect of truncating the original cascade of bifurcations [9,10]. A discussion concerning the reliability of the chaotic regime is reported in [17] and we are presently performing further investigations [15]. From the conceptual point of view, the role played by the chaotic solutions is interesting. Indeed, here the chaos would appear as a useful behaviour in contrary to most situations where random solutions are considered as a nuisance.

3. GEARBOXES MODELS.

As a second illustration, we introduce now the discussion of gearboxes. Here, the problem is to determine the sequence of impacts between the cog-wheels of a schematic device represented in Fig. 3. In Fig. 4, we reproduce the results of a numerical computation of the times of impact between two cog-wheels as sketched in Fig.3 [18,19,20]. Fig. 4 clearly exhibit a random motion , a stochastic nature which, as we shall see, is here again solely due to the non-linearity of the equations of the motion.

The impulsive dynamics of this device has been considered in [18,19] . In view of Fig. 3, the model consists of a   wheel whose mass, radius and inertial moments respectively are M, $\mathcal{R}$ and Θ. This wheel is externally driven by another cog-wheel which radius is $\mathcal{R}_{ex}$. The   model includes three dynamical variables, namely φ, x and r which describe the dynamics of the driven cog-wheel. For these three quantities, we have to consider three intervals of tolerance (two of which are explicitely sketched in Fig. 3); the interval of tolerance I_z lies itself between the teeth of the cog-wheel and is therefore time dependent. Between the impacts the free dynamics of the system is obviously governed by the following equations [18,19]:
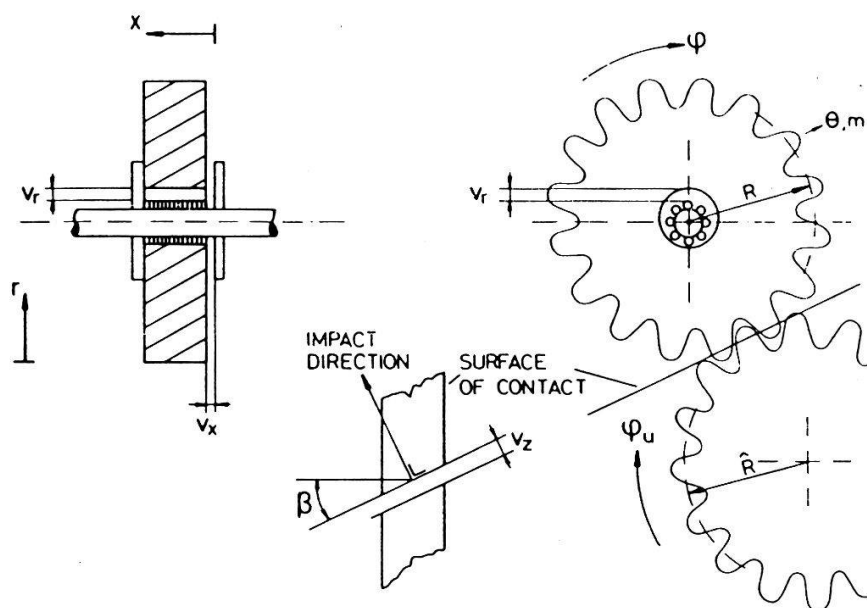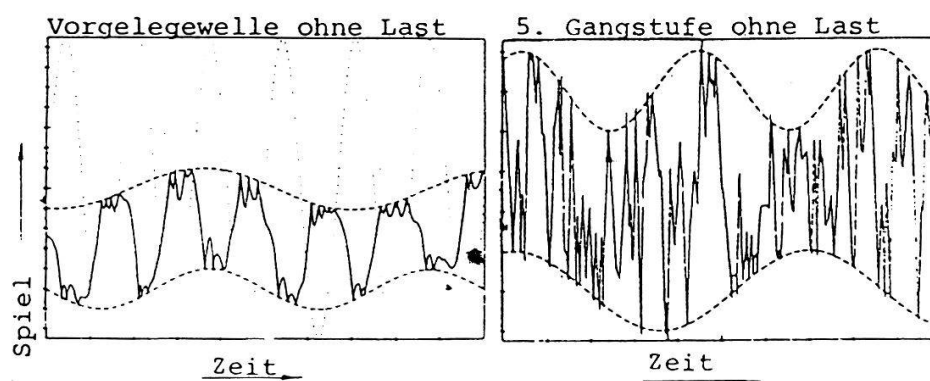
Fig. 3   The one-step gearbox model.

Fig. 4

$$\Theta\ddot{\varphi} = T_\varphi - d_\varphi\dot{\varphi} \qquad\qquad I_z \ni S_z \qquad (21)$$

$$m\ddot{x} = T_x - d_x\dot{x} \qquad\qquad I_x \ni x \qquad (22)$$

$$m\ddot{r} = T_r - d_r\dot{r} \qquad\qquad I_r \ni r \qquad (23)$$

where dots denote time derivatives and the quantity S_z which follows from geometric considerations, reads [18] :

$$S_z = e(t) - (R\cos(\beta)\varphi(t) + \sin(\beta)x(t) + \cos(\beta)r(t)) \qquad (24)$$

where :

$$e(t) = \mathcal{R}_{\theta x}\sin(\Omega t) \qquad\qquad (24a)$$

is the excitation force and $d_{\varphi,x,r}$ are friction coefficients.

Corresponding to the restitution Eq. (8), we have here:

$$S_z\big|_{\tau=\tau_\eta+\varepsilon} = -R_z S_z\big|_{\tau=\tau_n-\varepsilon} \quad ; \quad x\big|_{\tau_n+\varepsilon} = -R_x x\big|_{\tau=\tau_n-\varepsilon}$$

$$r\big|_{\tau=\tau_n+\varepsilon} = -R_r r\big|_{\tau=\tau_n-\varepsilon} \qquad (25)$$

where the coefficients $0 < R_{x,r,z} < 1$ express the dissipation of the system during the impacts.

Once again the dynamics of the system is found by integrating Eqs. (21)-(23) between the impacts and by using Eq. (25) to reactualize the initial conditions which are to be specified to integrate. In the simplest possible geometry ($\beta = 0$, $\mathcal{R} = m = 1$, $\Theta = 1$, $d_r = d_\varphi \ll 1$, $T = T_r + T_\varphi \ll 1$,

$R_z < 1$ and $\Omega(\tau_{n+1} - \tau_n) << 1$), we can derive an approximate mapping [20] in the form :

$$\Psi_{n+1} = -R_z\Psi_n + \mathcal{R}_{ex}\Omega\cos\left(\Omega\tau_n\right) \qquad (26a)$$

$$\tau_{n+1} = \tau_n + \left|\frac{v_z}{\Psi_{n+1}}\right| \qquad (26b)$$

where the quantity v_z is directly related to the tolerance interval between the cog-wheels and Ψ_n denotes :

$$\Psi_n = \frac{\partial}{\partial t}\left[\, r + \varphi\,\right]\Big|_{t = \tau_n}.$$

Eqs. (26a,b) are known as the Fermi dissipative mapping [4,5]. They are usually derived for the problem of a ball bouncing between two heavy walls, one of which being in sustained oscillations with the time. Hence, in view of our basic cog-wheel problem, it is not surprising that the Fermi map is embedded in this context. The map Eqs. (26a, b) does also present the phenomena of cascade of bifurcations and chaotic solutions [4,5,11,13,14] do then exist. Therefore, the behaviour observed in Fig. 4 can be understood from the equations of motion themselves and not from the presence of, for instance, round-off errors in the integration procedure.

In the search to reduce the clattering (accoustic) noise in gearboxes, the question whether the periodic or chaotic regime is more favorable or not is important. It might well happen that the chaotic regime, due to its wide emission spectrum, presents less annoyance to listeners than a purely periodic regime which is likely to induce resonnances. Then we will have another situation in which chaotic regimes constitute delibarately an advantage.

4. CONCLUSIONS AND PERSPECTIVES.

We have obtained the dynamical equation of motions of two common mechanical devices in the form of non-linear, dissipative sets of mappings Eqs. (2a,b) and (3a,b). These mappings are among the simplest models discussed in non-linear dynamics. In particular, cascade of bifurcations and chaotic solutions are present. In the devices presented here, we point out that not only the periodic behaviour is interesting for actual applications. Indeed , it might well happen that the chaotic regime is precisely the one to be tuned for the requested task. This situation can be intuitively expected when, for instance, the behaviour of a non-linear dynamical system has to be relatively insensitive to variations of the external parameters which govern the equations of the motions. Indeed, once in the chaotic regime, the details of the motion become almost irrelevant ; only the invariant distributions

of the pseudo-stochastic variables contain the relevant informations.

Although its ubiquous presence, the influence of external noise has been omitted in this paper. Its presence has a tendancy to smooth the invariant measures and hence, to favour the property of chaotic regimes to be less sensitive to small variations external parameters.

ACKNOWLEDGEMENTS.

REFERENCES.

1. R. M. ROSENBERG. "Nonlinear oscillations". App. Mech. Rev. 14 ,(1961), 837.

2. L.O. CHUA. "Special issue on chaotic systems". Proc. of IEEE. 75,

3. F.C. MOON. "Chaotic vibrations. An introduction for applied scientists". (1987, John Wiley.

4. J. GUCKENHEIMER & P.J. HOLMES. "Nonlinear oscillations, dynamical systems and bifurcations of vector fields" . App. Math. Sc. **42** , (1983), Springer Verlag.

5. A.J. LICHTENBERG & M.A. LIEBERMAN. "Regular and stochastic motion". App. Math. Sc. **38**, (1983), Springer Verlag.

6. A. H. REDFORD & G. BOOTHROYD. "Vibratory feeding". Proc Instn. Mech. Engrs. **182,** (1967-68), 135.

7. M.-O. HONGLER & J. FIGOUR. "Periodic versus chaotic motion in vibratory feeders". Helv. Phys. Acta. to appear.

8. O. TANIGUCHI, M. SAKATA, Y. SUZUKI & Y. OSANAI. "Studies on vibratory feeders". Bull. of the JSME. **6**, (1963), 37.

9. N. B. TUFILLARO, T.M. MELLO, Y.M. CHOI & A.M. ASLBANO. "Period doubling boundaries of a bouncing ball". J. Physique **47**, (1986), 173.

10. N.B. TUFILLARO, & A.M. ALBANO. " Chaotic dynamics of a bouncing ball". Am J. of Phys. **54**, (1986), 939.

11. C. MIRA . "Chaotic dynamics". (1987),World Scientific, Singapoore, NewJersey & Hong-Kong.

12. J. INOUE, S. MIYAURA & A. NISHIYAMA. " On the vibrotansportation and vibroseparation". Bull. of the JSME. **11**, (1968), 167.

13. M.J. FEIGENBAUM. "Qualitative universality for a class of transformations". J. Stat. Phys. **19**, (1978), 25.

14. P. Collet & J.-P. ECKMANN. "Iterated map on the interval as dynamical systems". (1980), Birckhaüser, Basel.

15. M.-O. HONGLER, P. CARTIER & P. FLURY. " Numerical study of a non-linear mapping describing vibrotransporation", (in preparation),

16. C. N. BAPAT,S. SANKAR & N. POPPLEWELL. " Repeated impacts on a sinusoidally vibrating table, reappraised". J. Sound & Vib. **108**, (1986), 1477.

17. Ya. F. VAYNKOF & S.V. INOSOV. "Non-periodic motion in vibratory conveyors". Mechanical Sc. Maschinovedeniye. **5**, (1976), 1.

18. F. PFEIFER & F. KUCUKAY. "Eine erweiterte Theorie mechanische Stosstheorie und uhre Anwendung in der Getriebdynamik". VDI-Zeitschr. Bd. **127**, (1985), 341.

19. F. KUCUKAY & F. PFEIFER. Uber Rasselschwingungen in KFZ-Schallgetrieben". Ing. Archiv. **56**, (1986), 25.

20. M.-O. HONGLER & L. STREIT. "on the origin of chaos in gearbox models". Physica **29D**, (1988), 402.