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RESONANT STIMULATION OF COMPLEX SYSTEMS

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Abstract: The dynamics of a huge variety of complex systems can be estimated from a low dimensional system of ordinary differential equations, although the number of degrees of freedom is large. Nearly all variables are slaved by a few order parameters. If the complex system is perturbed by an external force, slaved variables can be stimulated and a prediction of the response from the low dimensional system of differential equations is impossible. We show that it is generally possible to predict the response of the complex system and to control the complex system if the external forces are resonant perturbations.

1. Introduction

Sinusoidally driven nonlinear oscillators have a complex, in many cases chaotic dynamics. The response to externally applied forces is relatively small and does not match any well defined resonance condition. E.g. when a nonlinear mechanical pendulum is perturbed by a sinusoidal force, the response is comparatively small in amplitude/1/, is in many cases chaotic/2/ and does not satisfy any well defined resonance condition, even when the frequency of the driving force coincides with a peak (resonance) in the power spectrum of the dynamics of the unperturbed system/3/. In order to obtain a large response, the frequency of the driving force has to be shifted in such a way, that it coincides at all amplitudes with the characteristic frequency of the oscillator/1/. But even when the frequency of the driving force coincides at all amplitudes with the basic resonance of the unperturbed system, the resonace condition is not exactly satisfied, since all the other peaks/3/ of the power spectrum of the unperturbed system have to be taken into account as well. Recently a method has been presented to calculate driving forces which satisfy the resonance condition exactly/1/. A generalisation of this method/4/ provides us with the possibility to calculate perturbations which satisfy a certain condition or goal, but are of small amplitude. Perturbations of small amplitude are mostly important for the stimulation of complex systems. Its has been shown/5/ that due to the slaving principle a large variety of complex systems can be modeled by low dimensional systems of differential equations. If one uses this system of differential equations in order to calculate driving forces which force the experimental system to perform a special dynamics (goal dynamics dynamics) and applies these forces to the real system, generally some slaved varibales which are not included in the model will be excited too. However if the driving force is small, the stimulation of the slaved varibles generally will remain small. Therefore perturbations of very small amplitude have beside of their small amplitude a second advantage: one can predict the response using a low dimensional system of differential equations since excitations destinating away from the inertial manifold generally remain small. The aim of this paper is to show that it is possible to control complex systems by perturbations which force the experimental system to perform a goal dynamics but are of very small amplitude. These perturbation are defined by a variation principle and are called resonant perturbations. Usually these resonant perturbations are aperiodic.

In the second chapter of the paper we show that the response of a nonlinear oscillator is generally small for sinusoidal perturbations. In the third chapter we introduce a new definition of resonant perturbations using a variation principle. In the chapter 4 we present two experimental applications. First we show that a nonlinear mechanical pendulum can be stimulated resonantly by appropriate driving forces. Then we present experimental results which indicate that hydrodynamic systems can be stimulated resonantly too. Finally chapter 5 gives the conclusion.

2. Stimulation of nonlinear oscillators at resonance

A general feature of nonlinear oscillators is the amplitude frequency coupling/6/. The stimulation of a nonlinear oscillator is *at resonance* if the perturbation is sinusoidal and if the frequency of the perturbation coincides with the basic frequency of the oscillator. In order to illustrate that the response *at resonance* is small compared to the response to special nonsinusoidal perturbations which will be presented in chapter 3 we investigate Hamiltionians of the form

$$H = V_1(x_1) + \frac{1}{2}m_1\dot{x}_1^2 + V_2(x_2) + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}k(x_1 - x_2) \qquad (1)$$

where x_1, and x_2 are amplitudes, m_1 and m_2 are inertial constants, V_1 is an anharmonic and V_2 an harmonic potential and k is a small coupling constant which quantifies the magnitude of the coupling between the harmonic driving system and the nonlinear oscillator. In order to keep the feed back from the nonlinear oscillator to the dynamics of the driving system small we assume that the energy of the driving system is essentially larger than the energy of the nonlinear oscillator. In order to quantify the magnitude of the response the maximal energy transfer to the nonlinear oscillator is used. Fig. 1 illustrates a typical resonance curve.

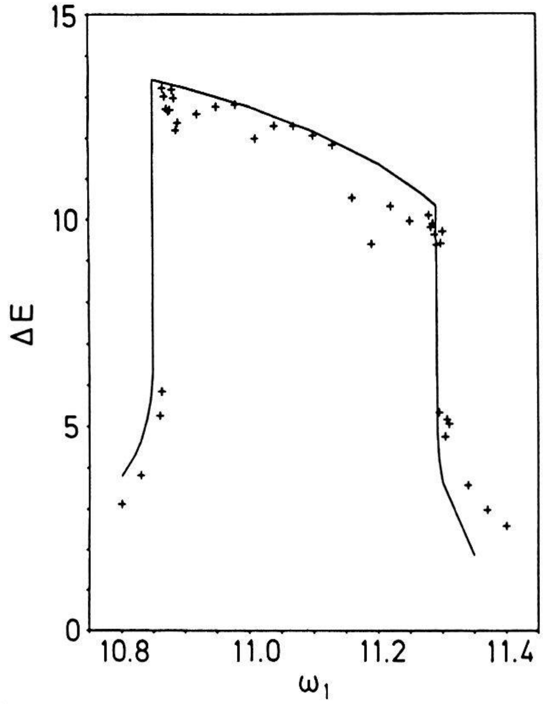

Fig.1 *Resonance curve of a nonlinear oscillator, where* $V_1 = x_1^6, m_1 = m_2 = 1, k = 0.1$, *and the energy of the harmonic oscillator* $E_2 \approx 100$ *is essentially larger than the energy of the nonlinear oscillator* $E_1 \approx 10$. *Plotted is the maximal energy transfer versus the frequency of the perturbation where the initial energy of the nonlinear oscillator was kept fixed. The continuous line represents the results of secular perturbation theory /8/ and (+) represents numerical results.*

The area of the resonance peak estimated by $A = \Delta\omega\Delta E_{max}$ is independent of the magnitude of the amplitude frequency coupling if $\Delta\omega$ and ΔE_{max} are estimated by a first order secular perturbation theory. Using the same approximation one can show/8/ that there is a simple relation between the quality factor of the resonance estimated by $Q = \frac{\Delta E_{max}}{\Delta\omega}$ and the magnitude of the amplitude frequency coupling

$$Q = \frac{1}{\frac{\partial \omega_1}{\partial E_1}} \tag{2}$$

where ω_1 is the basic frequency of the nonlinear oscillator for $k = 0$ and which depends on the energy E_1 of this oscillator. Since the area of the resonance peak is independent of the amplitude frequency coupling, we conclude from Eq. (2) that the resonanace curve becomes broad and small if the driven oscillator is essentially nonlinear, i.e. has a large amplitude frequency coupling.

3. Resonant stimulation of complex systems

In order to calculate driving forces $F(t)$ with a large response for nonlinear oscillators of type

$$\ddot{y} + \eta_1 \dot{y} + \frac{\partial V_1(y)}{\partial y} = F(t) \tag{3}$$

where η_1 is a friction coefficient and where F is a driving force, we use a goal equation/9/ of type

$$\ddot{x} + \eta_2 \dot{x} + \frac{\partial V_1(x)}{\partial y} = 0. \tag{4}$$

The driving force is calculated by $F(t) = (\eta_1 - \eta_2)\dot{x}$. Eq.(4) can be deduced by a variation of F. The variation is done by looking for a small driving force, i.g. $\int_0^T F^2 dt = small$ for a fixed energy transfer i.g. $\int_0^T F(t)\dot{y}dt = \Delta E$ and fixed boudary conditions $x(0) = y(0) = x_0$, $x(T) = y(T) = x_T$, i.e. the integral

$$I = \int_0^T \left(F^2 - \lambda F \dot{y}\right) dt \tag{5}$$

becomes minimal, where λ is a Lagrange parameter. If one considers the following set of goal equations

$$\ddot{x} + \eta_2 \dot{x} + \tilde{K}(x) = 0 \tag{6}$$

where $\tilde{K}$ is an unspecified driving force, and η_2 is an unspecified parameter, then Eq.(4) is a special solution of Euler's equations resulting from the variation of I. Since the amplitude of the driving force resulting from Eq.(4) is as small as possible in order to get a certain energy tranfer, we call those driving forces resonant. As indicated in chapter 1, driving forces of small amplitude have a certain relevance for the stimulation of complex systems. Therefore we generalise this definition of resonance to systems which are not just simple oscillators. We assume that the dynamics of the order parameters of a complex system can be modeled by a differential equation of type

$$\dot{\mathbf{y}}\left(t, r_1, r_2, \ldots\right) = \mathbf{f}\left(y_1, y_2, \ldots, t, r_1, r_2, \ldots, \frac{\partial}{\partial r_1}, \frac{\partial}{\partial r_2}, \ldots\right) + \mathbf{F}\left(t, r_1, r_2, \ldots\right) \tag{7}$$

where $\mathbf{y}$ is an n-dimensional vector of order parameters, and $\mathbf{f}$ is an n-dimensional flow vector field, which depends on the order parameters $y_1, y_2, \ldots$, spatial coordinates $r_1, r_2, \ldots$, time t, and spatial derivatives. $\mathbf{F}$ is a driving force. This driving force is called resonant perturbation if it forces the experimental system to reach a certain goal and is extraordinary small. The corresponding dynamics is called goal dynamics $\mathbf{x}\left(t, r_1, r_2, \ldots\right)$. If the goal dynamics can be modeled by a differential equation, this equation is called goal equation. In general the goal equation is equivalent to the equation of motion resulting from the variation which is used to define resonant perturbations, as indicated above.

4. Experimental applications

As a real physical oscillator we used a damped wheel with an excentric mass distribution/10/. The dynamics of the pendulum can be modeled by

$$\Theta \ddot{y} + \eta_1 \dot{y} + c_1 y + c_2 sin\left(y\right) = F\left(t\right) \tag{8}$$

where y is the angular displacement, $\Theta = 1.65 \cdot 10^{-3}\, kg\, m^2, c_1 = 1.62 \cdot 10^{-2}\, kg\, m^2 s^{-2}$, $c_2 = 0.03563\, kg\, m^2 s^{-2}$ and η_1 is a friction constant which can be varied in the range $5 \cdot 10^{-5}\, kg\, m^2 s^{-1} < \eta_1 < 6 \cdot 10^{-4}\, kg\, m^2 s^{-1}$. The time dependent driving force $F\left(t\right)$ is transmitted by a digital to force converter from a computer to the experimental pendulum.

Fig.2a shows a resonant stimulation of the pendulum, where the driving force was calculated by a Poincare map based on the dynamics of Eq.(4)/11/. Due to the strong amplitude frequency coupling there is a sensitive dependence of the basic frequency of the oscillator from the amplitude of the oscillation. Fig.2b illustrates that the phase relation between the driving force and y remains 90 degree, despite of the large shift of the frequency. The phase shift of 90 degree indicates that the reaction power is zero and that the driving force is resonant/1/. If the driving force is sinusoidal there is no 90 degree phase difference between y and F. This experiment illustrates that a resonant stimulation of a low dimensional nonlinear mechanical oscillator is possible. Even this mechanical pendulum has in principle a large number of different vibrational states because it is a macroscopic object. But in general these mechanical pendula are constructed in such a way that the basic frequencies of all those vibrational modes are essentially higher than the basic frequency of the oscillation. The stimulation of vibrational modes can be kept small by an appropriate construction of the pendulum. Completely different is the situation if one tries to control the dynamics of a hydrodynamic system.

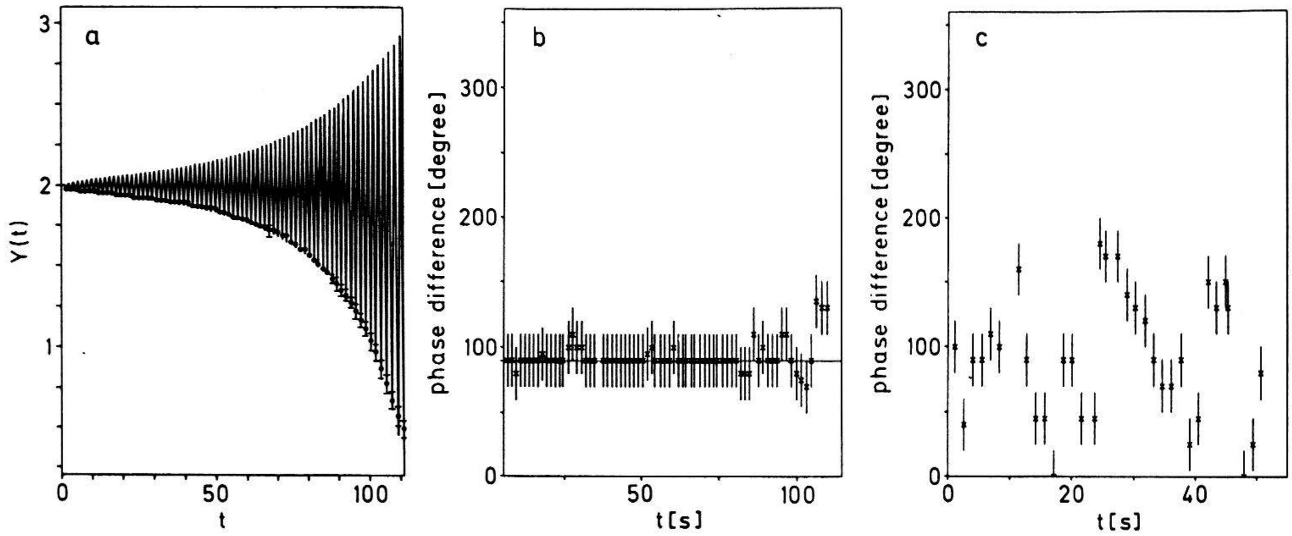

Fig.2 *The extrema of the amplitude of the experimental pendulum (a) and the phase difference between amplitude and driving force (b) versus time for a resonant perturbation where* $\eta_1 = 6 \cdot 10^{-4}\, kg\, m^2 s^{-1}$, $\eta_2 = -0.17\eta_1$. *If the driving force is sinusoidal, i.e.* $F = 4sin(\omega_0 t)$, *where* ω_0 *is the basic frequency at the minimum of the potential, the phase difference does not keep close to 90 degree (c).*

In hydrodynamic systems the basic frequency of slaved variables cannot be shifted. If the dynamics of some order parameters of the system is reconstructed from an experiment/12-14/, the dynamics of the slaved varibles is not known. However a general feature of the response of huge variety of oscillators is, that the excitation is small, if the perturbation is small. Therefore resonant driving forces seem to be mostly appropriate in order to control hydrodynamic systems if just an order parameter equation is known. Recently it has been shown, that the periodic dynamics of a velocity signal in the vortex street behind an circular cylinder can be modeled by a special low dimensional differential equation/15/. If the goal of a perturbation is to shift the basic frequency of the vortex street, stimulations with square waves are resonant perturbations of these special differential equations/16/. Fig.3 illustrates that the region of entrainment of the vortex street can really be enlarged by using square waves. This indicates that the low dimensional model can be used to predict the response of the high dimensional complex system, if the driving force is resonant.

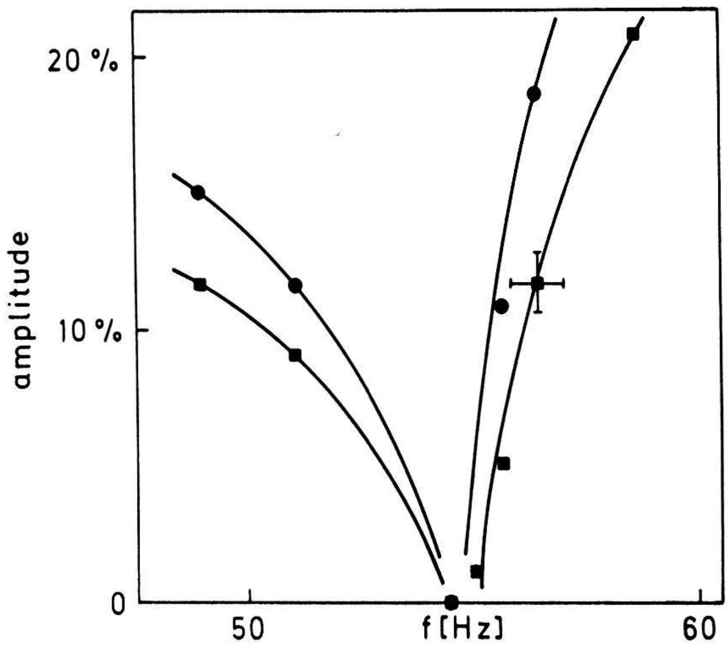

Fig.3 *The boundaries of the region of entrainment for sinusoidal (•) and for square wave perturbations (⊔). The position of the hot wire probe is 6 mm behind the cylinder (⌀ = 2mm) in the center of the vortex street (Reynold number = 70). The amplitude (distance between the extrema) of the acoustic perturbation is normalised to the amplitude of the variation of the velocity of the unperturbed system at this position.*

5. Conclusion

The state space of hydrodynamic systems and other complex systems has an large number of degrees of freedom. The dynamics of the order parameters is in a subspace, which is called inertial manifold. Generally nonlinear control theory provides us to calculate perturbations which satisfy a certain condition or goal, but are of small amplitude. Usually these resonant perturbations are aperiodic but the amplitude of excitations destinating away from the subspace of the model remains small. Therefore it seems to be possible to estimate the response of the complex system from the differential equation of the order parameters. Maybe resonant perturbations can be used to control chaotic flows.

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