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Helvetica Physica Acta, Vol. **61** (1988) 447–450
0018–0238/88/040447–04\$1.50 + 0.20/0

The lower critical field in high-T_c superconductors

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(26. XI. 1987)

In honor of Martin Peter's 60th birthday.

Abstract. Using a two-dimensional Fermi surface model for a high temperature superconductor we calculated the temperature dependence of the lower critical field H_{c1}. The new temperature dependence in H_{c1} is given by the temperature dependence of the London penetration depth $\lambda(T)$, but an additional contribution appears and this is contained in the number of superconducting electrons n_s and the gap which are also temperature dependent.

1. Introduction

The high-temperature superconductivity recently discovered by Berdnorz and Müler [1] in the La-Ba-Cu-O system has enjoyed an immense experimental [2] and theoretical interest [3–5] in the last period. The important problem which has been intensively studied by Chu et al. [6], Wu et al. [7] and Cava et al. [8] was to obtain a great number of materials which present superconductivity at high temperature. In order to predict the critical temperature and to propose a mechanism for the explanation of the superconductivity at high temperatures Jalborg, Junod and Peter [9] accurately calculated the critical temperature of the A15 compounds on the basis of the band structures.

In these new compounds the role of the electrons of O and of Cu (which seems to have the valence only approximatively fixed as Cu^{+2} (d^9)) have to be taken into consideration together.

Recently the systematic band calculations [10] showed that systems $La_{2-x}(Sr, Ba, Ca)_xCuO_4$ present a two dimensional Fermi surface with an imperfect nesting on the direction of the wave function $\vec{Q} = [\pi, \pi]$. Using these results Machida and Kato [11] considered a two dimensional model for the Fermi surface and calculated the critical temperature for a high temperature superconductor.

The purpose of this paper is to calculate the temperature dependence of the lower critical field H_{c1} a high temperature superconductor with two dimensional Fermi surface.

2. The Model

In order to calculate the lower critical field H_{c1} we start with the following mean field Hamiltonian

$$H = H_0 + H_{SC} + H_{SDW} \tag{1}$$

where

$$H_0 = \sum_{\vec{u},\alpha} \varepsilon(\vec{k}) C^+_{\vec{u}\alpha} C_{\vec{u}\alpha} \tag{2}$$

$$H_{SC} = -\sum_{\vec{u},\alpha} (\Delta(\vec{u}) C^+_{\vec{k}\alpha} C^+_{-\vec{k},-\alpha} + \text{h.c.}) \tag{3}$$

$$H_{SDW} = -\sum_{\vec{k}} M(C^+_{\vec{k}+\vec{Q}\uparrow} C_{\vec{u}\downarrow} + \text{h.c}) \tag{4}$$

The first term is the kinetic energy of two dimensional square lattice (the lattice constant $a = 1$), namely

$$\varepsilon(k) = -t(\cos k_x + \cos k_y) \tag{5}$$

where t is the nearest neighbour transfer integral. We assume that the band is half-filled to ensure the Spin-Density-Waves (SDW) nesting with the wave vector $\vec{Q}$. The SDW order parameter M is defined self-consistently by

$$M = U \sum_{\vec{k},\omega} \langle C^+_{\vec{k}+Q\uparrow} C_{\vec{u}\downarrow} \rangle \tag{6}$$

where U is the repulsive Coulomb interaction. We will consider the d-pairing state (which is compatible with the squate symmetry and in this case the superconducting order parameter is

$$\Delta_d(\vec{k}) = \sum_{\substack{\vec{k}',\alpha \\ w}} g_d(\vec{u}, \vec{k}') \langle C_{\vec{k}',\alpha} C_{-\vec{k}',-\alpha} \rangle \tag{7}$$

where

$$\Delta_d(\vec{k}) = \Delta \tau_d(\vec{k}) = \frac{\Delta}{2} (\cos k_x - \cos k_y)$$

and the coupling constant

$$g_\alpha(\vec{u}, \vec{u}') = g \tau_\alpha(\vec{u}) \tau_d(\vec{k}')$$

The equations for M and Δ have been obtained by Kato and Machida [11] as

$$1 = \frac{U}{\pi t} T \sum_\omega \frac{1}{\sqrt{\omega^2 + \Delta^2 + M^2}} \ln \frac{2\pi t}{\sqrt{\omega^2 + M^2}} \tag{8}$$

$$1 = \frac{g}{\pi t} T \sum_\omega \left\{ \frac{1}{\sqrt{\omega^2 + \Delta^2 + M^2}} \ln \frac{2\pi t}{\sqrt{\omega^2 + \Delta^2}} - \frac{1}{2\Delta} \ln \left| \frac{\sqrt{\omega^2 + \Delta^2 + M^2} + \Delta}{\sqrt{\omega^2 + \Delta^2 + M^2} - \Delta} \right| \right\} \tag{9}$$

These equations can be solved and one obtains three domains

(a) the superconducting domain $\Delta \neq 0$, $M = 0$

(b) the coexistence domain $\Delta \neq 0$, $M \neq 0$

(c) the magnetic domain $\Delta = 0$, $M \neq 0$.

In the following we concern ourselves with the superconducting domain and in this case the equation (9) becomes

$$1 = \frac{g}{\pi t} T \sum_{\omega} \left\{ \frac{1}{\sqrt{\omega^2 + \Delta^2}} \ln \frac{2\pi t}{|\omega|} - \frac{1}{2\Delta} \ln \left| \frac{\Delta + \sqrt{\omega^2 + \Delta^2}}{\Delta - \sqrt{\omega^2 + \Delta^2}} \right| \right\} \tag{10}$$

3. The Lower Critical Field

The lower critical field H_{c1} is expressed [12] by the energy of a single vortex E_v as

$$H_{c1} = 2|e| E_v \tag{11}$$

where E_v is given by the general expression [13]

$$E_v = 2\pi \int_0^{\infty} r \, d\pi \int_0^{g} d\left(\frac{1}{g}\right) [|\Delta(r)|^2 - |\Delta(o)|^2] \tag{12}$$

where $\Delta(r)$ is the superconducting order parameter in the presence of the magnetic field, and $\Delta(o)$ the order parameter for a homogeneous superconductor. The order parameter $\Delta(\vec{r})$ will be expanded as

$$\Delta(\vec{r}) = \Delta(o) + \Delta_1(\vec{r}) + \dots \tag{13}$$

where $\Delta_1(\vec{r})$ is the first order correction due to the presence of the magnetic field. Using now the equation (10) we can calculate $d(1/g)$ and (12) becomes

$$E_v = 2\pi \int_0^{\infty} r \, d\pi \int_0^{\infty} C(\Delta) \Delta_1(r) \frac{d\Delta}{\Delta} \tag{14}$$

where $C(\Delta)$ is

$$C(\Delta) = -\frac{1}{\pi t} \left[2\pi T \sum_{\omega}^{\omega_v} \left(\frac{\Delta^2}{E^3} \ln \frac{\pi t}{\omega} - \frac{1}{2\Delta} \ln \left| \frac{\Delta + E}{\Delta - E} \right| + \frac{1}{E} \right) \right] \tag{15}$$

where $E^2 = \omega^2 + \Delta^2$, $\omega = \pi T(m + \frac{1}{2})$. The first order correction $\Delta_1(r)$ has been calculated [13] as

$$\Delta_1(r) = -\frac{(ev_0 A(r))^2}{3} \left[\sum_{\omega} \frac{1}{\eta E^3} \right] \left[\sum_{\omega} \frac{1}{E^3} \right]^{-1} \tag{16}$$

where

$$\eta = 1 + \frac{1}{\tau_r E}$$

being the transport scattering time and $\vec{A}(\vec{r})$ the potential vector for a vortex. In the following we will take $\vec{A}(\vec{r})$ as [13]

$$A(r) = \frac{1}{2\,|e|\,r} K_1\left(\frac{r}{\lambda}\right) \tag{17}$$

where $K_1(x)$ is the Bessel function of the imaginary argument. For a London superconductor the integral over 'r' in the equation (14) gives the main contribution for $\lambda < r < \xi$ (ξ is the London penetration depth and λ the coherence length) and $\vec{A}(\vec{r})$ can be approximated as

$$A(r) \cong \frac{1}{2\,|e|\,r} \tag{18}$$

With these results we get from (11), (15) and (16) the critical field H_{c1} as

$$H_{c1} = \frac{\Phi}{u\pi\lambda^2}\left[\frac{1}{mt}\left(\ln\frac{\pi t}{\omega_D} + \frac{n}{n_s(T)}\ln\Delta(T)\right) - \frac{n}{n_s(T)} 2T \frac{\ln\cosh\Delta(T)/2T}{\Delta(T)\,\mathrm{tangh}\,\Delta(T)/2T}\right]\ln\frac{\lambda}{\xi} \tag{19}$$

where ω_D is the Debye frequency and $n_S(T)$ is the number of electrons in the superconducting state. For a BCS superconductor H_{c1} has been calculated [12–13] as

$$H_{c1} = \frac{\Phi}{4\pi\lambda^2}\ln\frac{\lambda}{\xi} \tag{20}$$

and from (19, 20) we see that in the two dimensional superconductor, as it is supposed to be a high temperature superconductor, we expect an important enhancement in the temperature dependence of the lower critical field.

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