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Autor(en): **Fivaz, R.**

Objekttyp: **Article**

Zeitschrift: **Helvetica Physica Acta**

Band (Jahr): **58 (1985)**

Heft 2-3

PDF erstellt am: **11.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-115616

Helvetica Physica Acta, Vol. **58** (1985) 497–505
0018–0238/85/020497–09$1.50 + 0.20/0

Symmetry breaking by random fields

By R. Fivaz, Institut de Physique Appliquée, Ecole Polytechnique Fédérale de Lausanne, PHB-Ecublens, CH-1015 LAUSANNE

(13. IX. 1984)

In honor of Emanuel Mooser's 60th Birthday

Abstract. In physical systems in which order results from coupling by long-ranged forces, a random field may of itself lead to instability and symmetry breaking. The reason is that random perturbations create simultaneously entropy and potential energy, the latter in large amounts since the forces are long-ranged. The resulting stability properties are worked out for the ferromagnetic conductor, a bistable two-dimensional system in which the impurity field raises the order of the singularity from three to five.

1. Introduction

Random fields are known to interfere with the onset of order in self-ordering physical systems. For instance, random magnetic fields raise the critical dimensionality of second order phase transitions in the Ising model [1]; the physical processes invoked are formation of small random domains and roughening of the domain walls [2]. In either case, a number of new states are made available to the system and the resulting increase of entropy favors disordered states. Hence a higher dimensionality is required to obtain order.

However, this result is contingent upon the type of forces responsible for self-ordering, which are assumed short-ranged in the Ising model. If forces are instead long-ranged, interaction between domain walls and the random field creates simultaneously large amounts of potential energy; these are volume contributions to free energy while entropy increases are restricted to the interfaces. Hence in some region of the parameter space, competition between these two contributions results in a new instability and symmetry breaking occurs as a specific effect of the random field. Thus in systems exhibiting metastability, it is found that the order of the singularity is raised by 2, so that the random field appears to favour order rather than disorder.

A first example is offered by the hysteresis effect observed in iron whiskers, which are long, practically perfect single crystals. In these simple ferromagnets, a single Bloch wall separates two domains of up- and down moments in weak magnetic fields. Net magnetization would then be expected to increase linearly with an external field as the Bloch wall shifts continuously and reversibly from the center to the boundary of the sample; such a linear response represents a singularity of first order. Yet experiment shows a definite hysteresis effect, even though whiskers have practically no extended defects which might explain irreversibility [3]. Therefore point defects such as impurities must be considered as responsible for the effect. This conclusion is actually borne out by calculations if the Bloch wall is assumed to warp randomly under the influence of these defects [4]. Thereby, two third order singularities of the cusp type are introduced. In the first one, symmetry breaking gives rise to spontaneous magnetization, while the second one, which is reached at large defect concentrations, connects partially ordered states to completely disordered states.

In this paper, another self-ordering system, the bistable ferromagnetic conductor, is investigated in order to work out the effect of a random field. As bistability originates in electrical forces which are long-ranged, two free energy terms are found to compete and an additional instability is introduced, raising the order of the singularity from three to five. Dimensionality makes a difference, however, since no completely disordered state is found to be stable in this two-dimensional system.

2. The perfect ferromagnetic conductor

At low temperatures, iron whiskers exhibit a magnetoresistance bistability, which connects states where the current flows in the whole crystal to states where it is restricted to a thin inner core around the crystal axis. The instability stems from domains rotations driven by the solenoidal field of the current itself: a pinch effect takes place as the field turns peripheral domains of low (longitudinal) magnetoresistance into domains of large (transverse) magnetoresistance; these form a current-depleted sheath while current density is increased in the core; thereby the self-field reinforces the pinch effect and the conductive core collapses to a thin filament [3].

The long-ranged forces leading to positive feedback and instability are electrical forces which maintain neutrality; this property is expressed by macroscopic current conservation along the crystal axis Oz. In addition magnetic forces are also long-ranged, a property expressed by Gauss' law as magnetic flux conservation along the axis [4]. Hence the core-sheath partition is submitted to a pair of macroscopic constraints:

$$\int_A \vec{j} \cdot d\vec{A} = I, \qquad \int_A \vec{B} \cdot d\vec{A} = \text{const.}, \tag{2.1}$$

with A the sample cross-section, which is assumed constant at all z, and I the total electrical current. These conditions are satisfied over the length of the crystal if the Bloch walls separating longitudinal and transverse domains are perfectly flat and parallel to the crystal axis, since then all derivatives with respect to z vanish. Thus $\partial j_z/\partial z = 0$ and $\partial B_z/\partial z = 0$ lead to uniform current densities in the ratio $\kappa = \rho_c/\rho_s$ of magnetoresistivities in core and sheath, while $\vec{B}$ has a purely axial symmetry.

As shown in Ref. 4, the total magnetic energy in a longitudinal field H is multivalued. In the calculation, the square-based cylindrical geometry was replaced with an equivalent circular basis geometry of external radius $\omega_c = 2\omega/\sqrt{\pi}$, with 2ω the width of the whisker; then the magnetic energy contained a contribution from the sheath and from the core:

$$e_m(J) = e_s(J) + e_c(J), \tag{2.2}$$

both dependent on the relative cross-section of the core, $J = A_c/A$. In energy units of $M_0 I/\omega_c$, with M_0 the saturation magnetization, these energies are given by:

$$e_s = (\kappa/3 + (1-\kappa)J - (1 - 2\kappa/3)J^{3/2})/(\kappa + (1-\kappa)J), \tag{2.3}$$

$$e_c = h(1-J)/2 \tag{2.4}$$

with $h = 2\pi H\omega_c/I$.

Minimization of e_m with respect to J, which here plays the role of an order parameter, leads to two branches of metastable states above critical values $h_c = 1$ and $\kappa_c = 1/2$; the distribution of magnetization may then switch between branches as H is varied, leading to order of magnitude changes of the apparent magnetoresistance of the sample, which is related to the order parameter by:

$$\rho = \rho_c(\kappa + (1-\kappa)J)^{-1}. \tag{2.5}$$

3. The real ferromagnetic conductor

In the presence of defects, walls are assumed warped, or roughened, in response to the random field surrounding the defects. Specifically walls are assumed to be broken into portions positioned randomly on each side of the ideal plane, so that areas of core and sheath fluctuate along the axis. In the square-based cylindrical geometry shown in Fig. 1, the mean position of each of the four sides of the core is given by its ditance ξ to the axis; the random coordinates x of the wall portions are then described with respect to the yz-plane at ξ by the plane wave expansion:

$$x = \xi + \int d\vec{k} a \sin(\vec{k}\cdot\vec{r}), \tag{3.1}$$

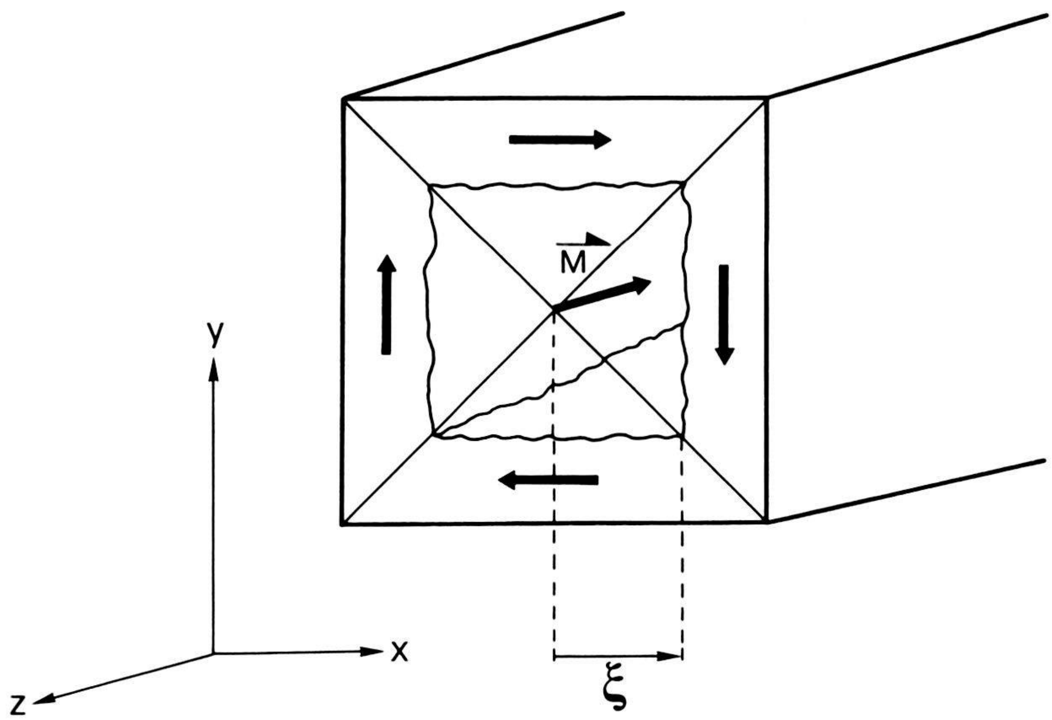

Figure 1
Core-sheath configurations in a real Fe-whisker where current flows along the z-axis. The magnetization remains longitudinal in the core and turns transverse in the sheath; the domains are separated by roughened Bloch walls of mean position ξ.

with $\vec{r} = {}_j y + {}_k z$ the vector position in the yz-plane, $\vec{k}$ its reciprocal vector and a the function of $\vec{k}$ measuring the kth mode amplitude. In the long wave length limit of interest with long-range forces, the z-dependence of the core cross-section is given for each mode by:

$$A_c(z) = 4\int_0^{\omega} dx \int_{-x}^{x} dyb(x-\xi-a\sin(k_z z+k_y y))$$
$$\cong 8\int_0^{\omega} dxxb(x-\xi-a\sin k_z z), \tag{3.2}$$

with $b(x)$ the moment distribution in the perfect Bloch wall, a soliton function for instance. Of course the sheath cross-section is $A_s(z) = A - A_c(z)$. Then the conservation conditions of equation (2.1) are expressed as:

$$\int d\vec{k}[j_c A_c(z) + j_s A_s(z)] = I,$$
$$\int d\vec{k}[M_c A_c(z) + M_s A_s(z)] = \text{const.}, \tag{3.3}$$

with j_c and M_c the longitudinal components of current density and magnetization in the core and j_s and M_s the longitudinal components in the sheath.

Since derivatives with respect to z no longer vanish, compensating space charge q and magnetic poles p form locally in order to maintain conditions (3.3) and 'dispersion' fields are set up according to:

$$\text{div } \vec{E} = \varepsilon_0^{-1} q, \qquad \text{div } \vec{H} = \mu_0^{-1} p, \tag{3.4}$$

with μ_0 the vacuum permeability.

To obtain these fields, their divergence is to serve as the source term in a Poisson Equation; since forces are long-ranged and act in a very long sample, the source is adequately represented by its zero-order Fourier component over the cross-section. Exchanging operations, one may first average the source over the cross-section and then take its divergence. Thus averaged fields over the sample cross-section are:

$$E_c = E_s = \varepsilon_0^{-1} \rho_c j_c A_c / A, \qquad H_c = \mu_0^{-1} M_c A_c / A. \tag{3.5}$$

The distributions of random charge and poles can then be calculated by taking the divergence of equations (3.5) after substitution by equations (3.2). Solution of the Poisson Equations then is straightforward and the stored potential energy is completely determined by

$$e_D = \langle E^2 \rangle / 2\varepsilon_0 + \langle B^2 \rangle / 2\mu_0 \mu, \tag{3.6}$$

where the brackets indicate averaging over all modes and μ is the relative permeability; this 'dispersion energy' will be evaluated in Section 4.

On the other hand, the expansion (3.1) leads to a distribution of random wall positions given by:

$$f(x) = \int d\vec{k} \cos^{-1}[(x-\xi)/a]. \tag{3.7}$$

This function also represents the magnetic moment density distribution, with which is associated an entropy density given by:

$$S = 2(M_0 k_B/\beta\omega^2)\int_0^{\omega} dx\, x[f \ln f + (1-f)\ln(1-f)], \tag{3.8}$$

with β the effective Bohr magneton and k_B the Boltzmann constant. The two terms in the integrand refer to the two different orientations which moments have on each side of the Bloch wall.

Finally, the free energy associated with the random field is the linear superposition:

$$e_R = e_D - T^* S, \tag{3.9}$$

with T^* a phenomenological pseudo-temperature scaling the energy cost of entropy and proportional to the random source intensity, i.e. the point defect concentration.

4. Dispersion energy

This quantity is first evaluated in terms of the moment distribution function by the method detailed in Ref. 4; here only the modifications needed to accommodate the two-dimensional geometry will be mentioned. The energy of the magnetization in its own demagnetizing field is by far the largest part in equation (3.6). To obtain this field, the procedure outlined below equation (3.4) is used. The averaged core magnetization is:

$$\bar{M}_c = M_0 A_c/A = (M_o/\omega^2)(\xi + a \sin k_z z)^2. \tag{4.1}$$

Since the perfect wall is very thin compared to the sample width, $b(x)$ in equation (3.2) has been approximated by a simple step function. The divergence is:

$$\begin{aligned}\frac{d\bar{M}_c}{dz} &= \frac{2M_0}{\omega^2}\int_0^{\omega} dx\, x \frac{db}{dx}\frac{d}{dz}[a \sin(k_z z)] \\ &= \frac{2M_0}{\omega^2}(\xi + a\sin(k_z z))(-ak_z \cos(k_z z)).\end{aligned} \tag{4.2}$$

The scalar potential then is the solution of a Poisson Equation:

$$\begin{aligned}\frac{d^2V}{dz^2} &= -\frac{1}{\mu_0}\frac{d\bar{M}_c}{dz}, \\ V &= -\frac{2M_0 k_z^{-1}}{\mu_0\omega^2}\left[a\xi \cos(k_z z) + \frac{a^2}{4}\sin(k_z z)\cos(k_z z)\right];\end{aligned} \tag{4.3}$$

the demagnetizing field is obtained as the gradient:

$$H_D = -\frac{dV}{dz} = -\frac{2M_0}{\mu_0\omega^2}\left[a\xi \sin(k_z z) - \frac{a^2}{4}\cos 2k_z z)\right]. \tag{4.4}$$

The dispersion energy in the kth mode is

$$e_D = -\tfrac{1}{2} H_D \bar{M}_c. \tag{4.5}$$

Averaging over all $\vec{k}$ finally gives the dispersion energy:

$$e_D = \frac{2M_0^2}{\mu_0\omega^4}\int\frac{d\vec{k}}{\pi}\int_{\sin^{-1}(-\xi/a)}^{\sin^{-1}[(\omega-\xi)/a]} dz'\left[a^2\xi^2\sin^2 z' + \frac{a^4}{16}\cos^2 2z'\right]. \tag{4.6}$$

It is understood that limits are $\pm\pi/2$ when the argument of $\sin^{-1}$ is larger than unity in absolute value. The variable is now changed to $u = a\sin z'$,

$$e_D = \frac{2M_0^2}{\mu_0\omega^4}\int\frac{d\vec{k}}{\pi}\int_{-\xi}^{\omega-\xi} du\left\{\frac{\xi^2u^2}{(a^2-u^2)^{1/2}} + \frac{1}{16}\left[(a^2-u^2)^{3/2} - 2u^2(a^2-u^2)^{1/2} + \frac{u^4}{(a^2-u^2)^{1/2}}\right]\right\}. \tag{4.7}$$

Here the energy is expressed in terms of the unknown amplitude function a, which, from equation (3.9), is related to the moment distribution function $f(x)$ by:

$$f(x) = \frac{1}{\pi}\int_0^\infty d\alpha\, n(a)\cos[(x-\xi)/a], \tag{4.8}$$

with $n(a)$ the mode density per unit amplitude. Then its derivative with respect to x is:

$$f'(x) = \frac{1}{\pi}\int_0^\infty d\alpha\, n(a)/(a^2-u^2)^{1/2}. \tag{4.9}$$

Now it is possible to exchange integrations in equation (4.7) while keeping the domain of integration invariant. As a result, the function a is eliminated in favour of the distribution $f(x)$:

$$e_D = \frac{2M_0^2}{\mu_0\omega^4}\int_u^\infty du[\xi^2u^2f'(u) + \tfrac{1}{16}G(u)], \tag{4.10}$$

with

$$G(u) = G_2(u) - 2u^2G_1(u) + u^4f'(u), \tag{4.11}$$

where

$$G_1(u) = \int_u^\infty (a^2-u^2)^{1/2}n(a)\,da = -\pi\int_u^\infty uf'(u)\,du, \tag{4.12}$$

and

$$G_2(u) = \int_u^\infty (a^2-u^2)^{3/2}n(a)\,da = -3\int_u^\infty uG_1(u)\,du. \tag{4.13}$$

Equations (4.10) to (4.13) complete the evaluation of the dispersion energy in terms of the moment distribution.

5. Stability properties

Addition of magnetic and random field contributions gives the total free energy as follows:

$$e = e_c(J) + e_s(J) + e_D(J), \tag{5.1}$$

since J is related to f through equation (3.4), or equivalently by:

$$J = \frac{1}{\omega^2} \int_0^{\omega} f(x)x\,dx. \tag{5.2}$$

To obtain a dimensionless form, a characteristic dispersion width σ is introduced by the variable change $y = u/\sigma$ in equation (4.10):

$$e_D = C_1 \cdot v, \qquad v = \frac{\xi^2\sigma^2}{\omega^4} \int_{-\xi/\sigma}^{(\omega-\xi)/\sigma} dyy^2 f'(y) + \left(\frac{\sigma}{2\omega}\right)^4 \int_{-\xi/\sigma}^{(\omega-\xi)/\sigma} G(y)\,dy, \tag{5.3}$$

with $C_1 = 2M_0/\mu_0$, and in equation (3.8):

$$S = C_2 \cdot s, \qquad s = \frac{\sigma^2}{\omega^2} \int_0^{\omega/\sigma} dyy[f \ln f + (1-f)\ln(1-f)], \tag{5.4}$$

with $C_2 = 2M_0 k_B/\beta$. Then the free energy in units of $M_0 I/\omega_c$ is given by:

$$e = e_s(J) + h(1-J)/2 + m(v - \tau s), \tag{5.5}$$

with $m = 4M_0\omega/\sqrt{\pi}\mu_0 I$ the relative potential energy in the random field and $\tau = \mu_0 k_B T^*/\beta M_0$ the relative weight of the random source.

Extrema of the free energy are now searched by the variational method. It will be shown first that e_D has a minimum as a functional of f for small τ. The dispersion width then is small too and e_D is essentially given by:

$$e_D = \text{const.} \cdot \sigma^2 \int_0^{\omega} dy\{[(y-\xi)/\sigma]^2 f'(y) - 2\tau y[f \ln f + (1-f)\ln(1-f)]\}. \tag{5.6}$$

The minimum of the integral is obtained from the Euler Equation:

$$y - \xi/\sigma + \tau y \ln[f/(1-f)] = 0. \tag{5.7}$$

Since all functions of f or f' in equation (5.6) are peaked at $y = \xi/\sigma$, equation (5.7) is solved by the Fermi function

$$f(y) = 1/[\exp(y - \xi/\sigma) + 1], \tag{5.8}$$

and the dispersion width at the minimum has then the value

$$\sigma = \tau/\xi. \tag{5.9}$$

It is concluded that for small impurity concentration, the Bloch walls have a negative and small self-energy in the random field; therefore it will not affect noticeably their stability properties, their equilibrium position being shifted only in second order.

For very large σ, no minimum exists. Indeed, the two-dimensional effects on the potential energy are all contained in the last term of equation (5.3); since it is weighted by σ^4, it grows always faster than the entropy weighted by σ^2 in equation (5.4); therefore no completely disordered state is found to be stable. This property is related to dimensionality and contrasts with the one-dimensional case where such states are stable at large defect concentrations.

For intermediate values of τ, the Fermi function may be used as a trial function with ξ and σ considered as variational parameters; thus extrema of the total energy $e(\xi, \sigma)$ are now sought with parameters h, m and τ fixed.

With $e' = de_s/dJ$ and partial derivatives represented by indices, extrema satisfy the two conditions:

$$(e'-h/2)J_\xi + m(v_\xi - \tau s_\xi) = 0, \tag{5.10}$$

$$(e'-h/2)J_\sigma + m(v_\sigma - \tau s_\sigma) = 0, \tag{5.11}$$

which are equations for J compatible only if

$$(v_\xi - \tau s_\xi)/J_\xi = (v_\sigma - \tau s_\sigma)/J_\sigma = \chi \tag{5.12}$$

with χ a constant. The various derivatives can be computed from equations (5.2)–(5.4) and are related to the external fields by:

$$\tau = (v_\xi J_\sigma - v_\sigma J_\xi)/(s_\xi J_\sigma - s_\sigma J_\xi), \tag{5.13}$$

$$h = 2(e' + m\chi). \tag{5.14}$$

The interesting situations arise when m is large enough for the constant χ to dominate in equation (5.14): in this case, the value of the external field H required for stability is determined solely by the random field and independently of the original mechanism producing bistability.

Thus χ values were calculated at constant τ and results are shown on Fig. 2. It is seen that χ is indeed a multivalued function of J, represented on the figure by means of its square root Y, i.e. the equivalent half-side of the core. Thus Y plays the role of the order parameter in an equilibrium diagram where stable states are localized on the negative slope segments of pseudo-isotherms τ = constant.

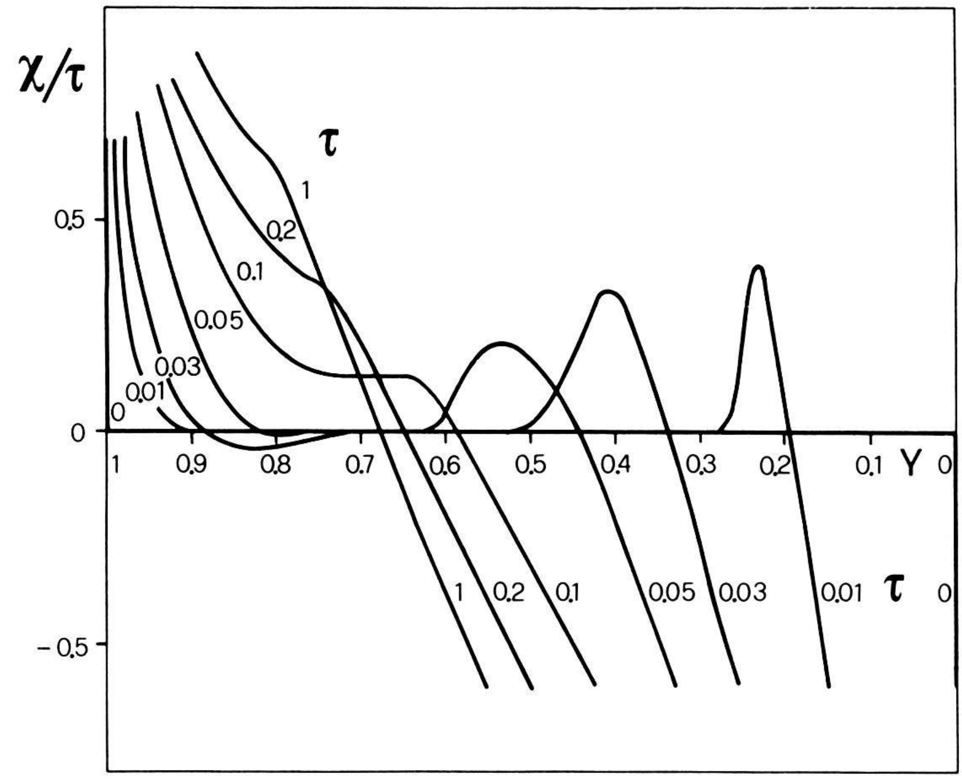

Figure 2
Equilibrium diagram for the real ferromagnetic conductor of large dispersion energy. Core-sheath configurations with mean core width $2Y$ are stable in the external field $2m\chi$ along the negative slope segments of the pseudo-isotherms τ, which characterize the defect concentration.

In particular, a critical pseudo-isotherm exists at $\tau \cong 0.1$, above which χ is a monotonous function of Y; e_F has a single minimum in this region, so that no discontinuity appears as a consequence of the random field. As the slope increases with τ, the original bistability due to the current pinch effect will disappear for large concentration of defects and the roughened Bloch wall will then shift continuously as the external field is varied.

Below the critical isotherm, however, the energy e_F has two branches of minima. For large m, the metastable states are determined by the random field but still depend on the magnetic field through the parameter χ. Hence transitions may take place between the branches as the relative height of the minima varies with the applied field; these first order transitions are presumbly of the spinodal type, since contact between different phases, i.e. regions of different determination of Y, would require large demagnetizing energies. Bistability would then be associated with symmetry breaking by the random field.

6. Conclusions

The presence of a random field is a cause for symmetry breaking when a system can order itself through long-ranged forces. This result was obtained for one- and two-dimensional systems where order is determined by magnetostatics or electrical current conservation.

More generally, when long-ranged forces are present, stationary systems acquire the properties of a continuous fluid, i.e. continuity equations apply stating that some field, say $\vec{A}$, must remain divergence-free everywhere; then macroscopic conservation laws follow to satisfy boundary conditions. As the random field introduces fluctuations $\delta\vec{A}$, a conjugate dispersion field $\vec{F}$ is set up such that $\operatorname{div}(\vec{F}+\delta\vec{A})=0$. Thereby conservation laws are preserved at an energy cost $e_D=-\vec{A}\cdot\vec{F}/2$. While this energy is a bulk property, entropy is generated only in low-dimensional inhomogeneities such as interfaces. Hence the free energy may be a multivalued function of their position and instability is obtained. Therefore symmetry breaking by random forces is likely to be found in other physical systems governed by macroscopic conservation laws, such as fluids in convective flow or solids undergoing plastic deformation.

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