Zeitschrift: Helvetica Physica Acta

Band: 56 (1983)

Heft: 5

Artikel: Triton bound state calculation with energy-dependent separable potentials

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DOI: https://doi.org/10.5169/seals-115433

Helvetica Physica Acta, Vol. 56 (1983) 1053–1063

0018-0238/83/051053-11$1.50 + 0.20/0

Triton bound state calculation with energy-dependent separable potentials*†

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Abstract. Energy-dependent *N–N* separable potentials proposed recently by Garcilazo are examined in trinucleon systems. For the special energy-dependence of the two-body potentials chosen there the Faddeev equations are well defined, unique, and preserve three-body unitarity. It is found that Garcilazo potentials overbind the triton by more than 14 MeV. A modified Garcilazo potential is proposed, which takes special care of the analytic continuation of the corresponding *t*-matrices to negative energies. The modified potentials describe not only the deuteron bound state and *N–N* phase shifts in the range of 0–450 MeV correctly, but give a correct triton binding energy of 8.59 MeV or 8.33 MeV depending on the uncertainty in the experimental data analysis of the energy position at which *N–N* phase shifts change the sign. The importance of correct description of phase shifts at higher energies for the low energy properties of a three-body systems versus two-body off-shell effects is found and discussed.

I. Introduction

Separable potentials are very useful in the numerical solution of the three-body problem, where they reduce the integral equations from two variables to one variable. Most work on the three-nucleon problem with separable potentials has treated the case of two neutrons and a proton, calculating neutron-deuteron scattering and the triton binding energy. The first calculations with spin effect included were performed by Mitra and his group [1] by Sitenko and Karchenko [2], and Aaron, Amado and Yam [3]. These authors used a simple rank one separable potential of Yamaguchi form [4] who was the first to introduce separable potentials in nuclear physics. It was found [1, 2, 3, 5] that this potential form which fit the *N–N* low energy behavior overbinds the triton by 3–4 MeV (the experimental binding energy for triton is 8.49 MeV [6]). On the other hand so called realistic potentials, which include repulsion at short distances underbind the triton by more than 1 MeV [5]. There is some hope that this gap between experiment and theory may be narrowed or closed by the Graz II separable potentials [8]. In the last decade more complex separable potentials were a proposed, for example by Doleshall [7] and the Graz group [8], which achieved better description of the *N–N* data then the Yamaguchi potential, leading with the exception of Graz II potentials nevertheless to overbinding of the triton [9] similar to the one obtained with Yamaguchi forces. Other separable *N–N*

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*) Supported in part by the Department of Energy under Contract No. DE-AC03-765F00515.
†) Supported in part by the U.S. National Science Foundation.
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potential by Pieper [10] were not applied to three-nucleon system because of its complexity, making the three-body equation difficult to solve. The failure of the extant separable potential with managable rank (complete calculation for Graz II force are under way) is usually attributed to the 'improper' off-energy-shell behavior of these forces. In this paper we indicate that this conclusion in the use of the Yamaguchi separable potential is by no means imperative. It seems that such effects as overbinding can be attributed to the proper description of phase shifts at high energies and to some properties of the potential which determine the analytical continuation of the corresponding t-matrix to negative energies, while leaving the off-shell properties unchanged.

In this paper we consider an energy-dependent rank one separable potential for N–N interaction which was proposed recently by Garcilazo [11]. The Garcilazo potential is of Yamaguchi type with energy-dependent potential strength. It acts as an attractive potential at low energies and as repulsive one at high energies. This leads to a proper description of N–N phase shifts, in the range of 0–450 MeV including the change of sign. The phase shifts derived from the Yamaguchi separable potentials are positive at all energies and therefore fail to reproduce the N–N phase shifts at high energies, where the phase shifts become negative. As is well known, this change of sign is due to a strong repulsion in the N–N interaction at short distances. In order to simulate this change of sign one needs at least a two-term separable potential with one term representing the attraction and the other the repulsion. It was found that in some cases the additional repulsive separable potential has almost no influence on the trinucleon binding energy [9].

It is therefore tempting to investigate whether an energy-dependent separable potential can cure the problem. As one knows a microscopic derivation of a force between particles with internal degrees of freedom will lead in general to an energy-dependent potential. This suggests that we can simulate the compositeness of the nucleon by a convenient phenomenological energy-parametrization of the N–N force.

In Section II we review the basic formulae for energy-dependent separable potentials. The study of the analytic continuation of the t–matrix generated by the Garcilazo potential suggests a modification of the Garcilazo potential which has almost no impact on the phase shift behavior, but which cures the unreasonable behavior of the t-matrix at negative energies. We discuss this modification in more detail in Section IV. In Section III several trinucleon calculations are presented using the original Garcilazo, the modified, and Yamaguchi potentials. It is shown that Garcilazo potential overbinds the trinucleon systems stronger than the Yamaguchin force, while the modified potential gives less binding and in case of a triton reproduces the experimental triton binding energy.

The results are discussed in Section IV with special emphasis to the relation between N-N phase shifts and the behavior of the N–N t-matrix at negative energies, and the influence of N–N phase-shifts in different energy intervals on the triton binding energy.

II. Energy-dependent two-body separable potentials

As mentioned already in the introduction the mathematical description of a force between particles with internal degrees of freedom leads in general to a nonlocal energy-dependent potential. As an example one might think of optical model [12] and the potentials of the resonanting group theory [13]. The use of energy-dependent (i.e. non-hermitian) potentials in Schrodinger equation has been discussed by many authors (see, for example, Ref. [12, 13] and specifically in Faddeev-like calculations of few-body systems was considered recently by Schmid [14] and Kim et al. [15].

Here we are considering a phenomenological energy-dependent two-body potential which might be regarded as an approximation of an energy-dependent theoretical potential derived from a microscopic theory in which the constituents of the particles under investigation are considered explicitly. In the context of a few-body system calculation the most convenient form of a two-body interaction for a numerical evaluation is the separable form used extensively in the literature [5] in the case of three-body calculation. Recently, an energy-dependent separable potential of Yamaguchi form for the N–N interaction was proposed by Garcilazo [11]:

$$V_G(p, p'; E) = g(p)g(p'), \tag{1}$$

in which

$$g(p) = (p^2 + \beta^2)^{-1} \tag{2}$$

and

$$\lambda_G \equiv \lambda_G(E) = \lambda_0 \tanh(1 - E/E_c) \tag{3}$$

E_c denotes here the energy at which the N–N phase shift changes sign. The modification of the original Yamaguchi potential is the energy dependence of its potential strength λ expressed in equation (3) by $\tanh(1 - E/E_c)$. For the scattering amplitude $f(k) = e^{i\delta} \sin \delta/k$ we obtain from potential (1) the expression:

$$f(k) = 1 \Bigg/ \left[-ik + \left(\frac{k^2 - \beta^2}{2\beta} + \frac{(\beta^2 + k^2)^2}{2\pi^2\lambda_G} \right) \right] \tag{4}$$

which in its structure is independent of the fact whether λ_G is energy dependent or not. From equation (4) it can be easily shown that $f(k)$ fulfills two-body unitary relation. This is important in regard to the Faddeev equations for the potential (1). In general as discussed by Kim et al. [15] an energy-dependent two-body potential due to its non-hermicity leads to nonunique Faddeev equations. Closely related to this shortcoming is the difficulty of a unique and proper continuation of the corresponding two-body t-matrix to negative energies. Here we can avoid these problems due to the special construction of the energy-dependence. Note that the energy-dependence of the potential (1) does not affect the off-shell properties because it is restricted to on shell energies. Therefore the cut structure of the corresponding t-matrix for positive and negative energies is uniquely defined. With this property and the fact that our two-particle t-matrix fulfills the off-shell-unitarity, it is straightforward to prove – following the procedure by Freedman, Lovelace, and Namyslowski [16] and discovered independently by Kowalski [17] – that the corresponding Faddeev-like equations are unique and the three-body unitarity is preserved. A similar discussion of these aspects based on Lippman–Schwinger equations is given by Garcilazo and Wilde in Ref. [18]).

From equation (4) we can determine the scattering length a

$$\frac{1}{a} = \frac{\beta}{2} \left(1 - \frac{\beta^3}{\pi^2 \lambda_G(E=0)} \right) \tag{5}$$

and the effective range r_0,

$$r_0=\frac{1}{\beta}\left(1+\frac{2\beta^3}{\pi^2\lambda_G}+\frac{\beta^5}{2\pi^2}\frac{d}{dE}\left(\frac{1}{\lambda_G(E)}\bigg|_{E=0}\right)\right). \quad (6)$$

One observes that equation (5) and equation (6) are identical with Yamaguchi formulae with the exception that in the expression for effective range the first derivative of the inverse potential strength enters the formula (6). In case of the parametrization (3) the corresponding term in equation (6) does no vanish.

In order to discuss some properties of potential (1) in the context of Faddeev equations it is convenient to introduce the operator τ. In case of separable potential (1) the t-matrix is of separable form

$$t(p, p'; E)=g(p)\tau_G(E)g(p') \quad (7)$$

where the quantity $\tau_G(E)$, we are interested in, is given by

$$\tau_G(E)=1\bigg/\left(\frac{1}{\lambda_G(E)}+2\pi^2\left[\frac{k^2-\beta^2}{2\beta(k^2+\beta^2)^2}-i\frac{k}{(k^2+\beta^2)^2}\right]\right), \quad (8)$$

with $k^2/M=E$, and M is the mass of the nucleon. In the case of Yamaguchi $\lambda_G(E)$ in formula (8) has to be replaced by a constant. Thus the t-matrices have the same form factors and differ only in the $\tau(E)$-matrices. This means, as we will see from the corresponding equations in Section III, that the Faddeev integral kernels are identical for all potentials we are using in this paper, and the only difference will come from the analytic continuation of $\tau(E)$ to negative energies: $\tau(E)\to\tau(-K^2-(3/4)p^2)$, where K^2/M denotes the binding energy of the trinucleon system, and p is the momentum of the spectator particle.

As already pointed out the energy-parametrization of the potential strength (3) assures that the potential (1) is attractive at low energies with decreasing strength (as energy decreases but is still beyond E_c) and is repulsive at high energies for $E>E_c$, where E_c is the energy where the phase shifts changes the sign. Note also that the potential strength remains finite in the whole energy-range $-\infty<E<+\infty$ and is bounded by

$$|\lambda(E)|\leq\lambda_0. \quad (9)$$

The separable potential (1) reproduces the S-wave N–N phase shifts for $^3S_1$ and $^1S_0$ satisfactorily through the energy range 0–450 MeV (E_{lab}) including the change of sign. Note that Yamaguchi phase shifts are positive at all energies and the discrepancies with empirical phase shifts are as big as 21°. The phase shifts for $^3S_1$ and $^1S_0$ for potentials used in this paper are given in Table II. In Table I parameters of all potentials used in the trinucleon calculations are given.

To conclude this section we discuss the behavior of the analytic continuation of the $\tau(E)$ matrix to negative energies. This behavior will be responsible for differences in the properties of trinucleon system (see Section III). It is easy to observe that for the continuation of $\tau(E)$ to negative energies the potential strength λ_G will remain positive and will increase with increasing momentum of the spectator particle p:

$$\lambda_G(-K^2-\tfrac{3}{4}p^2)=\lambda_0\tanh\left(1+\frac{K^2+\frac{3}{4}p^2}{ME_c}\right). \quad (10)$$

Table I.
List of separable forces used for trinucleon bound state calculations corresponding parameters.

Potential type	Partial wave	Notation used in the text	$\beta(fm^{-1})$	$\lambda_0(fm^{-3})$	$E_c(fm^{-1})$
Yamaguchi[1])	$^3S_1\,(n-p)$	YTNP*	1.450	0.54491	—
	$^3S_1\,(n-p)$	YTNP*	1.450	1.180	—
	$^1S_0\,(n-p)$	YSNP	1.304	0.2771	—
	$^1S_0\,(n-n)$[7])	YSNN	1.1295	0.17371	—
	$^1S_0\,(n-p)$	YSNP*	1.304	0.3149	—
Garcilazo parametrization (3)	$^3S_1\,(n-p)$	GTNP	1.745	0.8984178	0.816
	$^1S_0\,(n-p)$	GSNP	1.244	0.2398431	0.767
Parametrization (12)	$^3S_1\,(n-p)$	MTNP1	1.450	0.54491	0.816
	$^1S_0\,(n-p)$	MSNP1	1.304	0.2771	0.767
	$^3S_1\,(n-p)$	MTNP2	1.450	0.54491	0.897
	$^1S_0\,(n-p)$	MSNP2	1.304	0.2771	0.608
	$^1S_0\,(n-n)$	MSNN1	1.1295	0.17371	0.767
	$^1S_0\,(n-n)$	MSNN2	1.1295	0.17371	0.608

[1]) The Yamaguchi energy-independent potential strength is given by: $\lambda_{\text{Yamaguchi}} = \lambda_0 \tanh(1) = \lambda_0 \cdot 0.761594$.

Table II.
Triplet and singlet phase shifts for different forces.

	$\sin\delta_{\text{triplet}}$			$\sin\delta_{\text{singlet}}$		
	Force			Force		
K_{cm} in fm^{-1}	YTNP	GTNP	MTNP1	YSNP	GSNP	MSNP1
0.1	0.492	0.491	0.492	0.879	0.873	0.878
0.2	0.798	0.796	0.798	0.910	0.899	0.910
0.3	0.941	0.940	0.941	0.892	0.872	0.890
0.4	0.994	0.993	0.994	0.857	0.827	0.855
0.5	0.997	0.997	0.997	0.814	0.772	0.810
0.6	0.974	0.974	0.973	0.765	0.710	0.761
0.7	0.936	0.933	0.932	0.713	0.645	0.707
0.8	0.888	0.880	0.882	0.661	0.579	0.651
0.9	0.836	0.810	0.825	0.609	0.513	0.595
1.0	0.783	0.752	0.765	0.559	0.449	0.538
1.1	0.729	0.681	0.703	0.512	0.388	0.483
1.2	0.677	0.606	0.639	0.468	0.329	0.429
1.3	0.627	0.530	0.574	0.427	0.274	0.376
1.4	0.580	0.451	0.507	0.389	0.221	0.322
1.5	0.536	0.371	0.438	0.354	0.172	0.268
1.6	0.495	0.291	0.363	0.323	0.125	0.211
1.7	0.456	0.211	0.281	0.295	0.081	0.149
1.8	0.421	0.131	0.188	0.269	0.041	0.082
1.9	0.389	0.053	0.082	0.246	0.004	0.007
2.0	0.360	−0.022	−0.036	0.224	−0.030	−0.140
2.1	0.332	−0.0916	−0.157	0.206	−0.059	−0.140
2.2	0.307	−0.155	−0.267	0.188	−0.084	−0.194
2.3	0.284	−0.211	−0.347	0.173	−0.104	−0.229

This means that the t-matrix at negative energies does not know that the potential becomes repulsive at higher energies. On the contrary equation (10) lets the potential become even more attractive than the attractive Yamaguchi potential with energy-independent strength which is already more attractive than the potential (1). Since the Yamaguchi potentials already overbind the trinucleon system, potential (1) with the analytic continuation given in equation (10), will overbind the trinucleon system even more strongly. The upper bound for this overbinding is given by the binding energy of the trinucleon system calculated with Yamaguchi potential with the maximum strength λ_0 (see equation (9)). These expectations will be confirmed and discussed in Section III.

In order to prevent the shortcomings of the parametrization (3) we propose a modification of the energy-parametrization of λ without changing the phase shifts significantly:

$$\lambda_M(E) = \lambda_0 \tanh\left(1 - (E/E_c)^2\right) \tag{11}$$

for which

$$\lambda_M(-K^2 - \tfrac{3}{4}p^2) = \lambda_0 \tanh\left(1 - \left(\frac{K^2 + \frac{3}{4}p^2}{ME_c}\right)^2\right). \tag{12}$$

Now with increasing spectator momentum the potential strength decreases and changes the sign as soon as $K^2 + \frac{3}{4}p^2 = ME_c$ reflecting the fact that the potential becomes repulsive at higher energies. For the parametrization (11) we obtain exactly the Yamaguchi formulae for the scattering length and effective range, for in this case

$$\frac{d}{dE}\left(\frac{1}{\lambda_M(E)}\right)\bigg|_{E=0} = 0.$$

This means that we can use exactly the same parameters as in Yamaguchi case. Due to the parametrization (11), however, our phase shift decreases more strongly than the Yamaguchi phase shift and changes the sign at E_c. Since the coupling strength now also decreases for negative energies, as it should, we expect the binding of the trinucleon system with the parametrization (12) will not only be smaller than that obtained with Garcilazo potentials but also smaller than that obtained with Yamaguchi potentials. Indeed the results obtained in Section III confirm this prediction and even give (almost) the correct triton binding energy.

III. Trinucleon bound state calculation

We are solving a Faddeev bound state equation of the following type [2]

$$A_i(p) = 2\pi\tau_i(-K^2 - \tfrac{3}{4}p^2) \sum_j \chi_{ij} \mathcal{T}_{ij}(K; p, p') A_j(p') p'^2 \, dp', \tag{13}$$

where the kernel $\mathcal{T}_{ij}$ is given by

$$\mathcal{T}_{ij}(K; p, p') = \int_{-1}^{+1} \frac{g_i(\sqrt{p^2 + \frac{1}{4}p'^2 + pp'y})\, g_j(\sqrt{\frac{1}{4}p^2 + p'^2 + pp'y})}{K^2 + p^2 + p'^2 + p'py} \, dy$$

and $\tau_i(-K^2-\frac{3}{4}p^2)$ is easily obtained from equation (8). The summation in equation (13) runs over the spin coefficients

$$\chi^{(S=1/2,I=3/2)}=\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & -1 \\ -\frac{3}{2} & -\frac{1}{2} & 1 \\ -\frac{3}{2} & \frac{1}{2} & 0 \end{pmatrix} \tag{14a}$$

in the case of triton for charge-dependent N–N forces,

$$\chi^{(S=1/2)}=\begin{pmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{1}{2} \end{pmatrix} \tag{14b}$$

in the case of charge-independence, and $\chi=2$ for three indentical zero-spin particles, there S denotes total spin, and I total isospin of the trinucleon system. As already discussed in Section III equation (13) are unique and preserve three-body unitarity. The binding energy is found as the energy for which the corresponding Fredholm determinant vanishes. In Table III results for the binding energy for a system of three-identica zero-spin particles [21] for forces as listed in Table II are given. The results display in Table III confirm the expectations discussed in Section II. The energy-parametrization (3) of the potential strength leads for the force GTPN to very strong overbinding: 61.50 MeV compared with 25.40 MeV for the Yamaguchi triplet force (YTPN). The force YTPN* corresponds to the upper bound for the Garcilazo potential strength λ_G (i.e. $\lambda_{\text{Yamaguchi}}=\lambda_0$). The latter force leads to a binding energy of 75.40 MeV which is not too far from 61.50 MeV. This is due to the rapid convergence of tanh to one already for relatively small arguments. We can define an effective energy-independent potential strength λ_G^{eff} by noting that with this strength Yamaguchi potential reproduces the binding energy of 61.50 MeV. It holds then

$$\lambda_{\text{Yamaguchi}}=\lambda_0 \tanh(1)<\lambda_0 \tanh\left(1+\frac{61.50}{E_c}\right)<\lambda_G^{\text{eff}}<\lambda_0. \tag{15}$$

The lowest binding is obtained, as expected, for the parametrization (11) giving 17.35 MeV.

Corresponding results for the singlet force show the same qualitative features. Here the difference between the binding energy for Garcilazo potential and the binding energy for Yamaguchi potential is smaller than in the case of the

Table III.
Trinucleon binding energy for three identical zero-spin particles calculated by different forces.

Force	Partial wave	Binding energy (MeV)
YTPN	Triplet n–p	25.40
GTPN		61.50
YTPN*		75.40
MTPN1		17.35
YSPN	Singlet n–p	2.49
GSPN		3.87
YSPN*		13.00
MSPN1		1.94

Table IV.
Triton binding energy for different sets of charge-independent forces.

Force $^3S_1$	Force $^1S_0$	Binding energy in MeV
YTPN	YSPN	11.98
GTPN	GSPN	22.80
MTPN	MSPN	9.15

triplet force. On the other hand the energy of 13 MeV due to the force YSPN*, which corresponds to the upper bound for Garcilazo potential strength, is large compared with the binding energy of 3.87 MeV, obtained for GSPN. This peculiarity can be easily understood in terms of quantities of the inequality (15). For the triplet case λ_G^{eff} is closer to λ_0 than to $\lambda_{\text{Yamaguchi}} = \lambda_0 \tanh(1)$ due to the already large binding energy, whereas in the singlet case λ_G^{eff} is closer to $\lambda_0 \tanh(1)$ than to λ_0. The latter results from small binding energy in the case of a singlet force. Another way to express this behavior is to say that for large enough λ_0 ($\lambda_0 > 2.0$) $\lambda_G^{\text{eff}} \approx \lambda_0$. This means that we obtain the same binding for the energy-dependent and energy-independent potential, which is the upper bound for the energy-dependent potential regarding their potential strengths.

In Table IV we present binding energies for triton in a charge-independent treatment.

In Table V triton binding energies are given assuming charge-dependent nucleon–nucleon forces. The difference between the two versions of the separable force with the energy-parametrization given in equation (11) is due to the uncertainty in the experimental data in the vicinity of the energy position E_c at which N–N phase shifts change the sign. In the first version we have adopted the values of Ref. [19] following Garcilazo [11]. In the second version we exploit the recent data analysis by Arndt and VerWest [19]. Since there are no experimental phase shifts available for the neutron–neutron scattering we have assumed $E_c(n-n) = E_c(n-p)$ (see Table I). The results in Table V show that the binding energy is sensitive to the slight shift in E_c. This effect explains also the failure of the Yamaguchi potentials. In order to see this more clearly we use the following explanation. The separable potential with the parametrization (11) can reproduce the original Yamaguchi phase shifts even at higher energies if we let E_c go to infinity. In that case the energy dependence $E/(E_c \to \infty)$ is spurious and we end up with the original Yamaguchi potential

$$\lambda_M(E) \xrightarrow[E_c \to \infty]{} \lambda_{\text{Yamaguchi}} = \lambda_0 \tanh(1). \tag{16}$$

Table V.
Triton binding energy for different sets of charge-dependent forces.

Force $^3S_1\,(n-p)$	Force $^1S_0\,(n-p)$	Force $^1S_0\,(n-n)$	Binding energy in MeV
YTPN	YSPN	YSNN	10.80
MTPN1	MSPN1	MSNN1	8.59
MTPN2	MSPN2	MSNN2	8.33

More important than this is the possibility that using the energy-dependent separable potential with parametrization (11) we can gradually approach the Yamaguchi potential by continuously increasing E_c. This means that pushing E_c to higher and higher energies we overbind the triton more and more. The upper bound for this increment of the binding energy is just given by the limit (16), namely the original Yamaguchi potential. Since varying E_c we are not changing either the off-shell behavior nor the low energy on-shell behavior, the change in the binding energy is determined by the phase shifts at higher energies. We also discuss this point in Section IV. If we use the values of E_c given in Ref. [19] we obtain for the triton binding energy 8.59 MeV. If we use the recent data [20] for E_c (for triplet $E_c^{[19]} > E_c^{[20]}$ and for singlet $E_c^{[19]} < E_c^{[20]}$; see Table I) we obtain 8.33 MeV.

IV. Conclusions

We have shown that a simple, energy-dependent, separable potential of Yamaguchi form, which allows physically reasonable analytic continuation of the corresponding t-matrix to negative energies, describes correctly not only nucleon–nucleon observables, including high energy nucleon–nucleon phase shifts, but also provides a correct value for the triton binding energy. Moreover due to the construction of its energy-dependence this potential leads to unique Faddeev equations and preserves three-body unitarity.

Due to the fact that the potential proposed here (parameterization (11)) goes continuously in the limit $E_c \to \infty$ into the Yamaguchi potential, however without changing the off-shell behavior, we can attribute the overbinding obtained with Yamaguchi potential to its incorrect phase shifts at higher energies. Owing to the on-shell energy-dependence of our potential we can clearly decide in which energy range the phase shifts are important for the triton binding energy. One has to recall that the energy-dependent potential strength $\lambda_M(E)$, which determines the phase shift $\delta(E)$ at the same energy E, appears in the τ-matrix and must be continued to negative energies $\lambda_M(-K^2 - \frac{3}{4}p^2)$ in the context of Faddeev bound state equations. Since, due to formula (11), $\lambda_M(-E) = \lambda_M(E)$, and K^2 is a constant for given forces, we observe that in the context of Faddeev bound state equations $\lambda_M(E)$ is effective only in the range $E \in [K^2/M, \infty)$. This means that only the phase shifts in the same range are of immediate importance. (The phase shifts in the range [0, 17] MeV (E_{lab}) are only important in that sense that they are generated by the same potential and should for physical reasons approach the empirical phase shifts.) This finding has a general significance which can be expressed in the following way: In constructing a two-body potential for use in a triton bound state calculation one should pay more attention to a proper reproduction of the phase shifts above 17 MeV than below 17 MeV. Usually the Yamaguchi potential (and also other potentials) are chosen so that to reproduce first the two-body low energy behaviour. It is not straightforward to generalize this statement to other potentials. The reason is that in our case all potentials we have used have the same off-shell behavior and it is therefore easy to ascribe the differences in three-body magnitudes to the two-body on-shell variations. In general, a change of on-shell properties of a two-body force leads automatically to different off-shell behavior, which in turn has different implications on the three-body system and makes the above analysis more difficult. As far as the off-shell behavior is concerned, our result of obtaining an almost correct triton binding energy suggests that the off-shell behavior of our potential is not unreasonable.

Finally we like to emphasize that there is no reason not to consider energy-dependent two-body potentials within a few-body calculation, as long as the corresponding matrices fulfill the two-body unitarity condition. It is well known that as a basic input for Faddeev equations a two-body t-matrix may be considered instead of a potential. As discussed in Section II the two-body t-matrix must fulfill the two-body unitarity condition in order to make the Faddeev equations unique and guarantee their unitarity. A hermitian potential is a suitable generating operator of a two-body t-matrix which guarantees these properties automatically. However, as demonstrated here, the hermitian potential is not the exclusive mathematical tool for a proper generation of well behaved two-body t-matrices. Since the t-matrix is more closely related to observable data than the potential, the problem of its generation should be regarded of secondary importance. However, a potential, whether energy-dependent or energy-independent, can help to explain some features of the two-body t-matrix which are not accessible in the experiment as for example the analytic continuation of the two-body t-matrix to negative energies as required by the Faddeev equations. An obvious requirement, which motivated the parameterization (11), is that an energy-independent separable potential which would reproduce a t-matrix generated at a particular energy generated by an energy-dependent potential should not have a strength exceeding the maximum strength (attraction in our case) of the considered energy-dependent separable potential. The Garcilazo potential does not fulfill this requirement and leads to strong overbinding, since the improper choice of analytic continuation of the t-matrix corresponds to more attractive energy-independent potential than the maximum strength of the original energy-dependent potential. If we base our discussion on the t-matrix level without considering its generating potentials, we may say in view of the small differences between the phase shifts obtained with Garcilazo and our potential that given the empirical phase shifts a problem of a proper choice of continuation of the t-matrix to negative energies is an open question. (In our case this problem is solved by a specific construction of a generating potential.) This problem is of great importance in the context of the Zero-Range-Theory by Noyes [17], which is based only on the on-shell information of the two-body t-matrix. An analytic continuation of t-matrices constructed by means of dispersion theoretic equation and empirical phase shifts to negative energies predicts a triton binding energy of 2.5–3.0 MeV [21]. A similar results (2.8 MeV) was found recently by Kuzmichev and Kharchenko [22] in the approximation of two-particle correlations which corresponds to the zero range or pure on-shell limit. Thus it may be concluded that the two-nucleon off-shell effects account for approximately 5.5 MeV triton binding energy.

In view of the satisfactory results obtained with parameterization (11) it seems promising to use our potential in calculation of N–d reaction and ^3H and ^3He form factors [23]. We shall report on these calculations in a future publication.

Acknowledgements

The author would like to thank Professors H. Pierre Noyes, Yeong E. Kim, and Hossein Partovi, for helpful discussions.

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