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Diagonal QCD_2 with massless quarks: gauge transformations and mass perturbation[1])

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Abstract. The gauge transformations and the mass perturbation of massless $DQCD_2$, a gauge theory in two-dimensional space time with diagonal $SU(N)$ symmetry and massless quarks are discussed in detail. The implementable symmetry transformations are identified and the corresponding unitary operators are constructed. They determine the vacuum structure of strictly massless $DQCD_2$ with unbroken $U(1)$ chiral symmetry. The particle spectrum contains $(N-1)$ massive bosons and a massless quark. The addition of the mass term to the field algebra requires the inclusion of states with $U(1)$ charges localized at infinity into the vacuum space. The physical sectors become θ-sectors and chiral symmetry is broken. The energy associated to the mass perturbation is finite only on a subspace of a θ-sector. The physical quarks become unstable in most sectors.

1. Introduction

Diagonal quantum chromodynamics (DQCD) is a simplified abelian version of quantum chromodynamics (QCD) where the non-abelian gauge group is replaced by its maximal abelian subgroup. In two dimensional space-time, $DQCD_2$ is similar to the Schwinger model (QED_2) and in the case of massless quarks one has an explicit exact operator solution. This solution and approximate operator solutions of massive $DQCD_2$ have been discussed by Belvedere et al. [1] and by P. Mitra and P. Roy [2] ($DQCD_2$ is interpreted by the latter authors as a solution of QCD_2 with broken symmetry). The light spectrum of massive $DQCD_2$ in the strong coupling limit has been investigated by Steinhardt [3]. Gamboa Savari et al. [4] obtained some properties of massless $DQCD_2$ using path integral methods.

The purpose of the present work is two-fold. Motivated by our previous study of massless QED_2 [5, 6] we want first to elucidate the role of implementable symmetry transformations on the structure of massless $DQCD_2$. Our second goal is to get insight into the transition from massless to massive $DQCD_2$ from a detailed study of its response to a mass perturbation.

We have shown in [5] that there is an intimate relation between singular implementable local gauge transformations of massless QED_2 and its vacuum structure. The vacuum degeneracy is a consequence of the fact that the unitary operators implementing the singular gauge transformations act non trivially on the physical Hilbert space $\mathscr{H}_{\text{phys}}$.

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It is natural to ask if similar circumstances prevail in $DQCD_2$. If $DSU(N)$, the maximal abelian subgroup (torus) of $SU(N)$ is used as the gauge group of $DQCD_2$, there is an obvious difference with QED_2. The number N of fermion fields is no longer equal to the number $(N-1)$ of independent functions specifying a c-number local gauge transformation. Consequently, there is still a vacuum degeneracy but the fermionic degrees of freedom do not disappear completely from the physical spectrum. There is a massless colorless free quark.

This is related to a difference in the symmetries of massless QED_2 and $DQCD_2$. In addition to a global $DSU_V(N)\otimes DSU_A(N)$ symmetry associated to the local $DSU(N)$ symmetry, there is a global $U_V(1)\otimes U_A(1)$ symmetry which has no counterpart in QED_2. The $U(1)$ symmetry is unbroken and the colorless physical quark is the carrier of the corresponding charges. We may notice that the existence of this quark clearly indicates that the mechanism prohibiting physical states with non vanishing color is 'charge shielding' due to vacuum polarization rather than 'confinement' as an effect of long range forces.

As already mentioned, the first goal of the present work is the precise identification of the subgroup of implementable local and global gauge transformations of massless $DQCD_2$ and the explicit construction of the corresponding unitary operators (Section 4). Once this is done, we proceed to the decomposition of $\mathscr{H}_{\text{phys}}$ into its physically distinct sectors, each of them containing a unique vacuum. This is not as simple as in QED_2. A physical sector has to define an irreducible representation of some algebra $\mathscr{F}$. Due to the physical colorless quark each sector is a sum of $U(1)$ charge sectors and $\mathscr{F}$ has to contain charge creating operators interpolating them. The algebra $\mathscr{F}$ cannot coincide with the algebra $\mathscr{A}$ of gauge invariant observables; it is a field algebra in the sense of [7].

The algebra $\mathscr{F}$ is not uniquely defined and we discuss the implications of two distinct choices. In the first one (Section 6), $\mathscr{A}$ is the algebra of the observables which are invariant under the full symmetry group of massless $DQCD_2$ (including the chiral symmetries) and $\mathscr{F}$ is the minimal field algebra containing $\mathscr{A}$. We find that $\mathscr{H}_{\text{phys}}$ gets decomposed into sectors which are all equivalent. There is only one physically distinct sector; it is isomorphic to the product of the Fock spaces of $(N-1)$ massive bosons and the massless physical quark. Whereas the global $DSU_V(N)\otimes DSU_A(N)$ symmetry is broken, the global $U_V(1)\otimes U_A(1)$ is unbroken.

The purpose of our second choice for $\mathscr{F}$ (Section 7) is the announced study of the way massless $DQCD_2$ responds to a mass perturbation. The first choice does not allow this because the mass term $\bar{\psi}\psi$ is not in the corresponding $\mathscr{F}$. It provides an appropriate frame only for strictly massless $DQCD_2$. If $\mathscr{F}$ is changed into the minimal field algebra containing $\bar{\psi}\psi$, we have a drastic modification of the physical sectors. As they have to be invariant under the action of the $U_A(1)$ chirality changing $\bar{\psi}\psi$, the $U_A(1)$ charge is no longer defined on them. The new physical sectors are θ-sectors, labelled by a set of N chiral angles and all chiral symmetries are broken.

The mass perturbation is by no means a weak perturbation of massless $DQCD_2$ leading to a small explicit breaking of chiral symmetry. In order to define the mass perturbation on sectors with unique vacua, we are forced to break the $U(1)$ chiral symmetry by resorting to θ-sectors. Moreover, it turns out that the resulting interaction hamiltonian is finite only on a subspace of each θ-sector. For non-exceptional values of the chiral angles, this subspace is spanned by eigenstates of the $U_V(1)$ charge whose charge is an integer multiple of N. In particular, the colorless physical quark acquires an infinite energy and is totally unstable. There is no particle in the spectrum of massive $DQCD_2$ which turns into the physical quark of massless $DQCD_2$ in the limit of vanishing quark mass. The analysis of the states exhibiting a finite mass perturbation shows that a well defined parton picture applies to $DQCD_2$.

Our work is based on the covariant Lowenstein–Swieca type operator solution of $DQCD_2$ [8]. Most treatments of two-dimensional models with fermions bosonize the fermion fields. Whereas massless QED_2 can be treated without bosonization [5], a discussion of $DQCD_2$ avoiding this technique would be extremely intricate. The bosonization we use is not based on formal substitution rules transforming bilinears in the fermion fields into boson fields [9, 3]. We adopt the formalism originated by Becher [10] in which the fermion fields are rewritten in terms of regularized current potentials and operators creating localized charges with given charge distributions. The control of the various state spaces of the model is never lost. Limiting procedures which are hidden in more formal methods cannot be overlooked. In particular, one clearly sees at what point one is forced to construct states with charges localized at infinity. The θ-vacua are such states whereas the vacuum of strictly massless $DQCD_2$ has no charges at infinity.

Before we outline the plan of this article we specify our notations. The Lagrangian of massless $DQCD_2$ is:

$$\mathscr{L} = -\tfrac{1}{4}F^{i_D}_{\mu\nu}F^{\mu\nu,i_D} + \tfrac{1}{2}\bar{\psi}\gamma^\mu(i\partial_\mu + g\tfrac{1}{2}\lambda^{i_D}A^{i_D}_\mu)\psi. \tag{1.1}$$

The $N\times N$ matrices λ^i are the generators of the $SU(N)$ Lie algebra. The index i_D goes through the $(N-1)$ values of i labelling the diagonal λ's. If not otherwise stated, summation over repeated indices will be assumed. The fields $F^{i_D}_{\mu\nu}$ are given by:

$$F^{i_D}_{\mu\nu} = \partial_\mu A^{i_D}_\nu - \partial_\nu A^{i_D}_\mu. \tag{1.2}$$

We choose:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad g^{00} = -g^{11} = 1, \qquad \varepsilon_{01} = \varepsilon^{10} = 1, \tag{1.3}$$

and use the following notations:

$$x^\pm = x^0 \pm x^1, \qquad \gamma^5 = \gamma^0\gamma^1, \qquad \lambda^{i_D}_{a,b} = \delta_{ab}\lambda^{i_D}_a. \tag{1.4}$$

The first and second components of a spinor will be labelled '+' and '−'. This is justified because the first component of a free massless spinor is a left-goer depending on x^+ alone, the second component being a right-goer depending on x^-. An arbitrary component of ψ is noted $\psi_{\alpha,a}$, α = spinor index ($\alpha = +$ or $-$), a = color index ($a = 1, \ldots, N$). The fact that the current of a massless free spinor is a massless free field will be used repeatedly; its lightcone components $j_\pm = j_0 \pm j_1$ are functions of $x^\pm$.

This article is organized as follows. In Sections 2 and 3 we describe the covariant solution of $DQCD_2$ defined on an indefinite metric space $\mathscr{H}$ and review the construction of the positive metric space $\mathscr{H}_{\text{phys}}$. The so-called bleached fields are introduced as useful tools. In Section 4 we determine the implementable symmetry transformations and construct the corresponding unitary operators. We proceed to the bosonization of the bleached fields in Section 5 and obtain a simple characterization of $\mathcal{H}_{\text{phys}}$. The decomposition of $\mathcal{H}_{\text{phys}}$ into physical sectors in the case of a minimal field algebra is discussed in Section 6. The implications of the inclusion of the mass term into the field algebra and the construction of the appropriate θ-sectors are the subjects of Section 7.

2. The covariant solution

Our model being abelian, it has an operator solution in the covariant Landau gauge ($\partial_\mu A^\mu = 0$) which is a direct generalization of the Lowenstein–Swieca solution of massless QED_2 [8]:

$$\psi(x) = :\exp[i\sqrt{\pi/2}\,\gamma^5\lambda^{i_D}(\tilde{\Sigma}^{i_D} + \tilde{\eta}^{i_D})(x)]:\,\chi(x), \tag{2.1a}$$

$$A_\mu^{i_D}(x) = -(\sqrt{2\pi}/g)\varepsilon_{\mu\nu}\partial^\nu(\tilde{\Sigma}^{i_D} + \tilde{\eta}^{i_D})(x). \tag{2.1b}$$

The Wick ordered exponential $:\exp C:$ of a free field C is equal to $\exp C^{(+)}\exp C^{(-)}$, $C^{(\pm)}$ being the positive and negative frequency parts of C. The building blocks of (2.1) are: an N-plet of free massless Dirac fields χ_a, an $(N-1)$-plet of free massive pseudoscalar fields $\tilde{\Sigma}^{i_D}$ and $(N-1)$ free massless pseudoscalar fields $\tilde{\eta}^{i_D}$:

$$(\gamma\partial)\chi_a = 0, \qquad (\Box + (g^2/4\pi))\tilde{\Sigma}^{i_D} = 0, \qquad \Box\tilde{\eta}^{i_D} = 0. \tag{2.2}$$

The field $\tilde{\eta}^{i_D}$ has a Fock space $\mathcal{H}_{i_D}^{(\eta)}$ with indefinite metric defined by:

$$(\Omega, \tilde{\eta}^{i_D}(x)\tilde{\eta}^{i_D}(y)\Omega) = (1/4\pi)\ln[\mu^2(-x^2 + i\varepsilon x^0)], \tag{2.3}$$

the mass μ being arbitrary. The quanta of the χ-fields will be referred to as bare quarks.

The solution (2.1) is defined on the product space:

$$\mathcal{H} = \mathcal{H}^{(\Sigma)} \otimes \bar{\mathcal{H}}, \tag{2.4}$$

where $\mathcal{H}^{(\Sigma)}$ is the product of the Σ-fields Fock spaces $\mathcal{H}_{i_D}^{(\Sigma)}$ and $\bar{\mathcal{H}}$ is the product of the χ-fields Fock spaces $\mathcal{H}_a^{(x)}$ and the $\mathcal{H}^{(\eta)}$'s:

$$\mathcal{H}^{(\Sigma)} = \bigotimes_{i_D} \mathcal{H}_{i_D}^{(\Sigma)}; \qquad \bar{\mathcal{H}} = \left(\bigotimes_a \mathcal{H}_a^{(x)}\right) \otimes \left(\bigotimes_{i_D} \mathcal{H}_{i_D}^{(\eta)}\right). \tag{2.5}$$

The quark fields (2.1a) and the electric fields $E^{i_D} = F_{01}^{i_D} = -(g\sqrt{8\pi})\tilde{\Sigma}^{i_D}$ obtained from (2.1b) verify canonical equal time commutation relations.

The solution (2.1) satisfies a regularized form of Dirac's equation:

$$(\gamma\partial)\psi(x) = i(g/4)\lambda^{i_D}\lim_{\varepsilon\to 0}[\gamma^\mu A_\mu^{i_D}(x+\varepsilon)\psi(x) + \gamma^\mu A_\mu(x)\psi(x-\varepsilon)]. \tag{2.6}$$

The gauge invariant color currents $j_\mu^{i_D}$ appearing in Maxwell's equations:

$$\partial^\mu F_{\mu\nu}^{i_D} = -g j_\nu^{i_D} \tag{2.7}$$

are defined as point-split forms of $\bar{\psi}\gamma_\mu\frac{1}{2}\lambda^{i_D}\psi$ [8]; one gets:

$$j_\mu^{i_D} = j_{0,\mu}^{i_D} - (1/\sqrt{2\pi})\varepsilon_{\mu\nu}\partial^\nu(\tilde{\Sigma}^{i_D} + \tilde{\eta}^{i_D}), \tag{2.8}$$

where $j_{0,\mu}^{i_D} = :\bar{\chi}\gamma_\mu\frac{1}{2}\lambda^{i_D}\chi:$ is a bare quark current. The equations (2.7) are fulfilled in the mean in a subspace $\mathscr{H}'$ of $\mathscr{H}$:

$$\mathscr{H}' = \mathscr{H}^{(\Sigma)} \otimes \bar{\mathscr{H}}', \tag{2.9}$$

where $\bar{\mathscr{H}}'$ is a subspace of $\bar{\mathscr{H}}$ characterized by a set of subsidiary conditions:

$$\bar{\mathscr{H}}' = \{\Phi \in \bar{\mathscr{H}} \mid j^{i_D(-)}_{l,\mu}(x)\Phi = 0, \forall i_D, \mu, x\}. \tag{2.10}$$

The massless current $j^{i_D}_{l,\mu}$ is the longitudinal part of $j^{i_D}_\mu$ (equation (2.8)):

$$j^{i_D}_{l,\mu} = j^{i_D}_{0,\mu} - (1/\sqrt{2\pi})\varepsilon_{\mu\nu}\partial^\nu \tilde{\eta}^{i_D}. \tag{2.11}$$

The color currents which have been defined are not the only currents of $DQCD_2$. There is a current j_μ associated to the global $U_V(1)$ symmetry; it is defined through point splitting:

$$j_\mu(x) = Z_j \lim_{\varepsilon\to 0} [\bar{\psi}(x)\gamma_\mu(1 - i(g/2)\lambda^{i_D}A^{i_D}_\nu(x)\varepsilon^\nu)\psi(x+\varepsilon) + (\varepsilon \to -\varepsilon)]. \tag{2.12}$$

The contributions of the Σ- and η-fields cancel and what remains is a sum of bare quark currents:

$$j_\mu = \sum_a j_{a,\mu}, \qquad j_{a,\mu} = :\bar{\chi}_a\gamma_\mu\chi_a:. \tag{2.13}$$

The $U_A(1)$ current $\tilde{j}_\mu$ is related to j_μ by $\tilde{j}_\mu = \varepsilon_{\mu\nu}j^\nu$. The currents $j^{i_D}_\mu$, j_μ and $\tilde{j}_\mu$ are conserved. The gauge invariant axial color currents $\tilde{j}^{i_D}_\mu = \varepsilon_{\mu\nu}j^{i_D,\nu}$ have an anomaly:

$$\partial^\mu \tilde{j}^{i_D}_\mu = -(g/2\pi)E^{i_D}. \tag{2.14}$$

3. The bleached field and the physical Hilbert space

Equations (2.4), (2.9) and (2.10) show that the Σ-space $\mathscr{H}^{(\Sigma)}$ is not affected by the subsidiary condition; it plays the role of an unproblematic spectator. Fortunately it is also easy to characterize the factor $\bar{\mathscr{H}}'$ of $\mathscr{H}'$ in (2.9) because it is generated from the vacuum state by the action of an N-plet of bleached fields $\phi_a(x)$ describing screened bare quarks:

$$\phi_a(x) = :\exp[i\sqrt{\pi/2}(\eta_a + \gamma^5\tilde{\eta}_a)(x)]: \chi_a(x), \tag{3.1}$$

(no summation over a!). The N fields $\tilde{\eta}_a$ are linear combinations of the $(N-1)$ independent $\tilde{\eta}^{i_D}$:

$$\tilde{\eta}_a = \sum_{i_D} \tilde{\eta}^{i_D}\lambda^{i_D}_a, \tag{3.2}$$

and η_a is the scalar associated to the pseudoscalar $\tilde{\eta}_a : \partial_\mu\eta_a = \varepsilon_{\mu\nu}\partial^\nu\tilde{\eta}_a$. the bleached fields (3.1) satisfy the free massless Dirac equation and, by construction, commute with the various components of the longitudinal currents (2.11):

$$[j^{i_D(\pm)}_{l,\mu}(x), \phi_{\alpha,a}(y)] = 0 \tag{3.3}$$

According to the Definition (2.10), this implies that the ϕ's map $\bar{\mathscr{H}}'$ onto itself. As in massless QED_2 [5], where one has only one bleached field, $\bar{\mathscr{H}}'$ is actually spanned by the vectors obtained by applying monomials in the bleached fields on the canonical vacuum $\bar{\Omega}$ of $\bar{\mathcal{H}}$ ($\bar{\Omega}$ = product of the Fock vacua of all the factors in the Definition (2.5) of $\bar{\mathcal{H}}$).

One observes that the positive and negative frequency parts of $j^{i_D}_{l,\mu}$ commute among themselves and with j_μ. Consequently $\bar{\mathcal{H}}'$ is invariant under the action of these currents and the preceding statement implies that they are expressible in terms of the bleached fields. Indeed, one finds:

$$\begin{aligned} j^{i_D}_{l,\pm}(x) &= (j^{i_D}_{l,0} \pm j^{i_D}_{l,1})(x) \\ &= \lim_{\varepsilon\pm\to 0} \left[Z(\varepsilon^\pm)\phi^\dagger_\pm(x^\pm)\tfrac{1}{2}\lambda^{i_D}\phi_\pm(x^\pm+\varepsilon^\pm)+(\varepsilon^\pm\to-\varepsilon^\pm)\right], \end{aligned} \tag{3.4}$$

$j_\mu(x)$ having a similar expression with $\frac{1}{2}\lambda^{i_D}$ replaced by the unit matrix.

Whereas the bleached field ϕ_a creates one negative unit of bare charge Q_a, whose current is $j_{a,\mu}$ (2.13), it commutes with the full color current $j^{i_D}_\mu$ (2.10). This means that all vectors of $\bar{\mathcal{H}}'$ have vanishing color charges. Furthermore, the Σ-term in $j^{i_D}_\mu$ is a topological current and:

$$[\tilde{\Sigma}^{i_D}(x), j^{i_D'}_0(y)]_{\mathrm{E.T.}} = 0. \tag{3.5}$$

The factor $\mathcal{H}^{(\Sigma)}$ of $\mathcal{H}'$ in (2.9) being generated by the Σ-fields, we find that $\mathcal{H}'$ as a whole is a zero color space.

The commutation properties of the bleached fields (3.1) disclose an important difference between massless DQCD$_2$ and massless QED$_2$. In QED$_2$, the values of the bleached field at two points x and y commute or anticommute for all pairs (x, y). The x-dependence of this field is spurious, it is not related to any translational degree of freedom and $\mathcal{H}'$ is, in fact, a space of vacua. This is no longer the case here. The fields (3.1) have x-dependent commutation properties and describe physical massless bleached quarks. This will become clear after our analysis in Section 5. Here we write down the equations specifying the commutation properties of ϕ_a for space-like separations:

$$\phi_{\pm,a}(x)\phi_{\pm,a}(y) = \exp\left[\pm i(\pi/N)\varepsilon(x^1-y^1)\right]\phi_{\pm,a}(y)\phi_{\pm,a}(x), \tag{3.6}$$

for $(x-y)^2<0$. We see that the ϕ's are not local fields in the usual sense. However, there is no conflict with microcausality or Lorentz invariance. The exterior of the light is not simply connected in two-dimensional space-time and the phase factor in (3.6) cannot be excluded.

The structure (3.1) of the bleached fields and the selection rule resulting from the fact that ϕ_a creates a negative unit of charge Q_a imply that the metric of the space $\mathcal{H}'$ is semidefinite positive [11]. Let $\mathcal{H}'_0$ be the subspace of its zero norm vectors. The quotient

$$\mathcal{H}_{\mathrm{phys}} = \mathcal{H}'/\mathcal{H}'_0 \tag{3.7}$$

is a positive metric Hilbert space containing the physical states of our model. The main result of Section 4 will be the existence of gauge transformations which are implemented by unitary operators acting non trivially on $\mathcal{H}_{\mathrm{phys}}$. This means that $\mathcal{H}_{\mathrm{phys}}$ contains sets of vectors, in particular vacua, which are gauge equivalent. Therefore, $\mathcal{H}_{\mathrm{phys}}$ is not an acceptable state space with unique vacuum. It is a sum of physical sectors which will be determined in Sections 6 and 7.

4. The implementable symmetry transformations

In this section we discuss the implementable symmetry transformations of DQCD_2. We start with the local gauge transformations, which require a careful investigation, and defer the much simpler global transformations to the end of the Section.

The Lagrangian (1.1) is invariant under an abelian group of local c-number gauge transformations:

$$\psi_a \to \hat{\psi}_a = \exp(i\Lambda^a)\psi_a, \tag{4.1a}$$

$$A_\mu^{i_D} \to \hat{A}_\mu^{i_D} = A_\mu^{i_D} + (1/g)\sum_a \lambda_a^{i_D}\Lambda^a. \tag{4.1b}$$

There is no summation over a in (4.1a). The functions Λ^a are constrained by:

$$\sum_a \Lambda^a = 0. \tag{4.2}$$

In the Landau gauge the Λ's are solutions of d'Alembert's equation; they have the decomposition $\Lambda^a(x) = \Lambda_+^a(x^+) + \Lambda_-^a(x^-)$. We assume $\Lambda_\pm^a \in \mathscr{C}^\infty(\mathbb{R})$ and define $\tilde{\Lambda}^a = -\Lambda_+^a + \Lambda_-^a$.

The new fields $(\hat{\psi}, \hat{A}_\mu)$ have the form (3.1) if the building blocks χ and $\tilde{\eta}$ are replaced by:

$$\hat{\chi}_a = \exp[i(\Lambda^a + \gamma^5\tilde{\Lambda}^a)]\chi_a, \tag{4.3a}$$

$$\hat{\tilde{\eta}}^{i_D} = \tilde{\eta}^{i_D} - (1/\sqrt{2\pi})\sum_a \lambda_a^{i_D}\tilde{\Lambda}^a. \tag{4.3b}$$

and the $\tilde{\Sigma}$'s are kept unchanged.

We are looking for those automorphisms (4.3) which are implemented by a unitary operator U, i.e. for any operator B we must have

$$B \to \hat{B} = UBU^\dagger. \tag{4.4}$$

It is sufficient to consider transformations which affect only two quark fields, for instance $\psi_1(\chi_1)$ and $\psi_a(\chi_a)$, $a \neq 1$. They are defined by a single function Λ:

$$\Lambda^1 = \Lambda, \qquad \Lambda^a = -\Lambda, \qquad \Lambda^b = 0, \qquad b \neq 1, a. \tag{4.5}$$

A general local transformation is a product of such transformations.

The problem of identifying the implementable transformation (4.3) and constructing the operators U is the same as in massless QED_2. Therefore we state only the results, outlines of the proofs can be found in [5]. The structure of the transformation law (4.3) implies that if $U(\Lambda)$ exists, it is a product:

$$U(\Lambda) = U^{(\eta)}U^{(\chi)}, \tag{4.6}$$

where $U^{(\chi)}(\Lambda)$ implements (4.3a) and $U^{(\eta)}(\Lambda)$ implements (4.3b). The necessary and sufficient conditions for the existence of $U^{(\chi)}(\Lambda)$ are that $\Lambda_\pm(x^\pm)$ tend rapidly enough to finite limits as $|x^\pm| \to \infty$ and that the total increase of these functions is an integer multiple of π:

$$\Lambda_\pm(\infty) - \Lambda_\pm(-\infty) = -n_\pm \pi, \qquad n_\pm \in Z. \tag{4.7}$$

The existence of $U^{(\eta)}(\Lambda)$ is secured if $\Lambda_\pm(\infty) = -\Lambda_\pm(-\infty)$.

Condition (4.7) decomposes the group of implementable transformations (4.5) into disjoint classes $C(n_+, n_-)$. In the terminology of [5], $C(0, 0)$ is the class of weak transformations; all other transformation are strong. It is convenient to choose two representative strong transformations, one in $C(1, 0)$, the other in $C(0, 1)$, as standard strong transformations. Call $\Lambda^{(\pm)}$ the functions describing these transformations and $U^{(\pm)}$ the operators implementing them. An arbitrary function Λ belonging to $C(n_+, n_-)$ has the following decomposition:

$$\Lambda = n_+\Lambda^{(+)} + n_-\Lambda^{(-)} + \Lambda_{\text{weak}}, \tag{4.8}$$

where $\Lambda_{\text{weak}} \in C(0, 0)$. $U(\Lambda)$ is given by:

$$U(\Lambda) = [U^{(+)}]^{n_+}[U^{(-)}]^{n_-}U(\Lambda_{\text{weak}}), \tag{4.9}$$

it is determined once $U^{(\pm)}$ are constructed and if we know how to implement any weak transformation. It turns out that the generators of the weak transformations are the longitudinal currents (2.11):

$$U(\Lambda) = \exp\left[-i\int dx^1 \lambda_a^{i_D}(\Lambda^a j_{l,0}^{i_D} - \tilde{\Lambda}^a j_{l,1}^{i_D})\right], \tag{4.10}$$

if $\Lambda \in C(0, 0)$.

The bleached fields (3.1) are invariant under the local transformations (4.3). The space $\mathcal{H}'$ is therefore invariant under the action of the U's and the restriction of the standard operators $U^{(\pm)}$ to this space is expressible in terms of the bleached fields. One gets especially simple expressions if:

$$\Lambda^{(\pm)}(x) = -2 \text{ Arctg}\,[\kappa_\pm(x^\pm - c^\pm)], \qquad \kappa_\pm > 0. \tag{4.11}$$

Whereas the operators $U^{(\pm)}$ of massless QED_2 are linear in the bleached field, we have now bilinear expressions. They can be written in terms of local fields $\rho_{\pm,a}(x)$ which annihilate a unit of charge Q_1 and create a unit of charge Q_a and are formally proportional to $(\phi^\dagger_{\pm,a}\phi_{\pm,1})(x)$:

$$\begin{aligned}\rho_{\pm,a}(x^\pm) &= \lim_{\varepsilon \to 0} Z_\rho(\varepsilon)\phi^\dagger_{\pm,a}(x^\pm)\phi_{\pm,1}(x^\pm + \varepsilon) \\ &= \frac{2\pi}{\mu} :\exp\,[i\sqrt{2\pi}(\eta_{\pm,1} - \eta_{\pm,a})(x^\pm)]:\, \chi^\dagger_{\pm,a}(x^\pm)\chi_{\pm,1}(x^\pm).\end{aligned} \tag{4.12}$$

$a = 2, 3, \ldots, N$ and:

$$\eta_{\pm,a}(x^\pm) = \tfrac{1}{2}(\eta_a(x) \mp \tilde{\eta}_a(x)). \tag{4.13}$$

The standard operators $U^{(\pm)}$ are obtained from these $\rho_{\pm,a}$ according to the following recipe. Choose two test functions $h_\pm$ belonging to $\mathcal{S}(\mathbb{R})$ and whose Fourier transforms have prescribed values on $\mathbb{R}_+$:

$$\tilde{h}_\pm(k) = \int_{\mathbb{R}} dx h_\pm(x)e^{ikx} = e^{-(k/\kappa_\pm)}e^{-ikc^\pm} \tag{4.14}$$

for $k > 0$. Construct the smeared operators:

$$\rho_{\pm,a}[h_\pm] = \int_{\mathbb{R}} dx h_\pm(x)\rho_{\pm,a}(x). \tag{4.15}$$

The restrictions to $\mathcal{H}'$ of the operators $U_a^{(\pm)}$ implementing the standard strong give gauge transformations defined by (4.5) and (4.11) are given by:

$$U_a^{(\pm)} = e^{i\pi(Q_{\pm,1}-Q_{\pm,a})}\rho_{\pm,a}[h_\pm]. \tag{4.16}$$

The index a in $U_a^{(\pm)}$ recalls that the transformations (4.5) are specified by the value of a; $Q_{\pm,a}$ are the left- and right-going bare charges.

It follows from (4.12) that the fields $\rho_{\alpha,a}$ either commute or anticommute:

$$[\rho_{\alpha,a}(x), \rho_{\beta,b}(y)] = [\rho^\dagger_{\alpha,a}(x), \rho_{\beta,b}(y)] = 0 \quad \text{if} \quad \alpha \neq \beta \text{ or } \alpha = \beta \text{ and } a = b, \tag{4.17}$$
$$\{\rho_{\alpha,a}(x), \rho_{\alpha,b}(y)\} = \{\rho^\dagger_{\alpha,a}(x), \rho_{\alpha,b}(y)\} = 0 \quad \text{if} \quad a \neq b.$$

The fact that these relations hold for all values of x and y announces that the x-dependence of the ρ-fields is a spurious artifact of the semidefinite metric of $\mathcal{H}'$. We shall see in Section 5 that the ρ-fields become indeed x-independent at the level of $\mathcal{H}_{\text{phys}}$.

Definition (4.12) and equation (4.16) show that two bleached fields, ϕ_a and ϕ_1, are related by operators implementing gauge transformations. This means that the objects described by the different bleached fields are not physically independent. As we shall see in the next section, there is only one physically distinct bleached quark.

As announced, we close this section with the global symmetries of DQCD_2. We have a group $(U_V(1))^N \otimes (U_A(1))^N$ of global transformations:

$$\psi_a(x) \to e^{i\alpha_a}\psi_a(x), \qquad \psi_a(x) \to e^{i\tilde{\alpha}_a\gamma^5}\psi_a(x), \tag{4.18}$$

the angles α_a and $\tilde{\alpha}_a$ being independent. If we write these transformation laws in terms of the building blocks of the solution (3.1) we find that they affect only the bare quark fields. We just have to replace ψ_a by χ_a in (4.18). The generators of the global transformations are therefore the bare charges. The transformations (4.18) are implemented by:

$$U(\boldsymbol{\alpha}) = \exp\left[-i\sum_a \alpha_a Q_a\right], \qquad \tilde{U}(\tilde{\boldsymbol{\alpha}}) = \exp\left[-i\sum_a \tilde{\alpha}_a \tilde{Q}_a\right], \tag{4.19}$$

with $Q_a = Q_{-,a} + Q_{+,a}$, $\tilde{Q}_a = Q_{-,a} - Q_{+a}$, $\boldsymbol{\alpha}$ stands for the set $(\alpha_1, \ldots, \alpha_N)$.

In order to have a correspondence between the local transformations (4.1) and the global symmetries, we split the latter into global $DSU(N)$ and global $U(1)$ symmetries. Global $DSU_V(N)$ and $DSU_A(N)$ transformations are defined by sets $\boldsymbol{\alpha}$ and $\tilde{\boldsymbol{\alpha}}$ constrained by $\sum_a \alpha_a = \sum_a \tilde{\alpha}_a = 0$, whereas $U_V(1)$ and $U_A(1)$ transformations are specified by single angles α and $\tilde{\alpha}$, $\alpha_a = \alpha$, $\tilde{\alpha}_a = \tilde{\alpha}$ in (4.19).

Comparing (4.16) and (4.19) we see that $\rho_{\pm,a}[h_\pm]$ implements the product of a strong local gauge transformation with a special $DSU(N)$ global transformation. Thus, we may eliminate $U_a^{(\pm)}$ and generate the group of unitary operators implementing local and global transformations by the set $U(\Lambda_{\text{weak}})$, $\rho_{\pm,a}[h_\pm]$, $U(\boldsymbol{\alpha})$ and $\tilde{U}(\tilde{\boldsymbol{\alpha}})$. We notice that the $\rho_{\pm,a}$ do not all commute with each other (e.g. (4.17)), they neither commute with $U(\boldsymbol{\alpha})$ and $U(\tilde{\boldsymbol{\alpha}})$. We have a multiplier representation of the abelian group of implementable symmetry transformations of DQCD_2.

5. Bosonization and the structure of the physical Hilbert space

We investigate now the structure of the quotient space $\mathcal{H}_{\text{phys}}$ defined in (3.7). As it is the factor $\bar{\mathcal{H}}'$ of $\mathcal{H}'$ in (2.9) whose metric is semidefinite, (3.7) can be rewritten as follows:

$$\mathscr{H}_{\text{phys}} = \mathscr{H}^{(\Sigma)} \otimes \bar{\mathscr{H}}_{\text{phys}}, \qquad \bar{\mathscr{H}}_{\text{phys}} = \bar{\mathscr{H}}'/\bar{\mathscr{H}}_0, \tag{5.1}$$

where $\bar{\mathscr{H}}_0'$ is the norm zero subspace of $\bar{\mathscr{H}}'$. It is the content of $\bar{\mathscr{H}}_{\text{phys}}$ we have to find out.

We are interested in the operators which are defined on $\bar{\mathscr{H}}_{\text{phys}}$. Each operator which maps $\bar{\mathscr{H}}'$ onto itself and has an adjoint is defined on $\bar{\mathscr{H}}_{\text{phys}}$. To see why this is so, we remember that a zero norm vector Ψ of a space H with semidefinite metric is orthogonal to each vector of H. Therefore $(\Phi, A\Psi) = (A^\dagger\Phi, \Psi) = 0 \;\forall \Phi \in H$ if $\|\Psi\| = 0$. This implies $\|A\Psi\| = 0$; A leaves the zero norm subspace H_0 invariant. Consequently A is defined on the quotient H/H_0. There are, of course, equivalence classes of operators which reduce to the same operator on H/H_0: A and B are equivalent if $\|(A-B)\Psi\| = 0 \;\forall \Psi \in H$.

In massless QED_2, the identification of the classes of equivalent operators is easy. All smeared bleached fields $\phi_\alpha[g]$ with fixed α and conveniently normalized test functions g reduce to the same operator on $\mathscr{H}_{\text{phys}}$ [5].

In massless $DQCD_2$ the situation is more complicated and the expression (3.1) of the bleached fields does not allow an easy recognition of equivalent operators. As we shall see, this changes if we bosonize the bleached fields [9, 10]. Bosonization is therefore an extremely helpful tool of our discussion and greatly clarifies the physical content of our model.

The left- and right-goers being decoupled, we restrict ourselves to the right-going components $\phi_{-,a}$, $\rho_{-,a}$, $\eta_{-,a}, \ldots$. We drop momentarily the index '$-$', x stands for x^- and a current j is the light-cone component $j_- = j_0 - j_1$.

We use the bosonization technique described by Becher [10] and perform the following substitution for the bare quark fields:

$$\chi_a(x) \to \frac{1}{\sqrt{2\pi Z}} \exp(-2i\pi j_a^{(+)}[\theta_x - J])\sigma_a[J] \exp(-2i\pi j_a^{(-)}[\theta_x - J]), \tag{5.2}$$

where $j_a = :\chi_a^\dagger \chi_a:$, J is a standard $\mathscr{C}^\infty(\mathbb{R})$ kink function such that $J(-\infty) = 1$, $J(+\infty) = 0$. The function $\theta_x(y)$ is a $\mathscr{C}^\infty$ approximation of the step function $\theta(x-y)$. The unitary operator $\sigma_a[J]$ creates a negative unit of charge Q_a whose current density is $J'(x) = (d/dx)J(x)$:

$$[j_a(x), \sigma_b[J]] = \delta_{ab} J'(x)\sigma_a[J]. \tag{5.3}$$

The exponentials in (5.2) shift this charge to the neighbourhood of the point x. the σ's anticommute:

$$\{\sigma_a[J], \sigma_b[J]\} = \{\sigma_a^\dagger[J], \sigma_b[J]\} = 0 \tag{5.4}$$

The value of the constant Z in (5.2) is:

$$Z = \exp\left[\int dx \int dy\, J'(x) \ln(x - y - i\varepsilon) J'(y)\right]. \tag{5.5}$$

To bosonize the bleached fields we have also to transform the exponentials in (3.1). We write:

$$\eta_a(x) \to -\eta_a[\theta_x'] = \eta_a'[\theta_x - J] - \eta_a[J']. \tag{5.6}$$

A partial integration has been performed: it is licit because $(\theta_x - J)(y) \to 0$ for $|y| \to \infty$. Collecting (5.2) and (5.6) we obtain, after some manipulations, the bosonized form of the bleached fields:

$$\phi_a(x) = \frac{1}{\sqrt{2\pi}} \mu^{1/2} (\mu Z)^{-1/2N} \exp\left\{-2i\pi\left(j_a^{(+)} - \frac{1}{\sqrt{2\pi}} \eta_a'^{(+)}\right)[\theta_x - J]\right\}$$
$$V_a[J] \exp\left\{-2i\pi\left(j_a^{(-)} - \frac{1}{\sqrt{2\pi}} \eta_a'^{(-)}\right)[\theta_x - J]\right\}. \quad (5.7)$$

where $V_a[J]$ are anticommuting unitary operators creating a negative unit of bare charge Q_a:

$$V_a[J] = \exp(-i\sqrt{2p}\, \eta_a[J'])\sigma_a[J]. \quad (5.8)$$

If the function θ_x tends to a sharp step function, the right-hand side of (5.7) converges to a limit which is equivalent to the field defined in (3.1).

In the more usual versions of bosonization the charge distribution described here by $J'(x)$ is localized at infinity. There is no $J(x)$ appearing explicitly and one needs the pseudopotentials of the currents [1, 3]. In Becher's formalism, the operators $j_a[\theta_x - J]$ are regularized pseudo-potentials. It is instructive to start with standard charges localized at finite distance. This will lead to a clear understanding of why and when a shift to infinity becomes unavoidable.

The following transformation of (5.7) clarifies the physics of the bleached fields in a decisive way. Remembering the Definition (2.11) of the longitudinal currents we find that:

$$j_a - (1/\sqrt{2\pi})\eta_a' = (1/N)j + j_{l,a}, \quad (5.9)$$

Where j is the light-cone component of the $U(1)$ current, $j = \sum_a j_a$, and, in analogy with (3.2):

$$j_{l,a} = \sum_{i_D} j_l^{i_D} \lambda_a^{i_D}. \quad (5.10)$$

Inserting (5.9) into (5.7) we get:

$$\phi_a(x) = \exp(-2i\pi j_{l,a}^{(+)}[\theta_x - J])\bar{\phi}_a(x) \exp(-2i\pi j_{l,a}^{(-)}[\theta_x - J]), \quad (5.11)$$

with:

$$\bar{\phi}_a(x) = \left(\frac{\mu}{2\pi}\right)^{1/2} (\mu Z)^{-1/2N} \exp\left\{-\frac{2i\pi}{N} j^{(+)}[\theta_x - J]\right\}.$$
$$\cdot V_a[J] \exp\left\{-\frac{2i\pi}{N} j^{(-)}[\theta_x - J]\right\}. \quad (5.12)$$

The importance of the form (5.11) comes from the fact that the positive and negative frequence components of the longitudinal currents are equivalent to zero. This results from the Definition (2.10) of $\bar{\mathscr{H}}'$: $j_{l,a}^{(-)}$ annihilates every vector of $\bar{\mathscr{H}}'$ and $j_{l,a}^{(+)}$ transforms each vector into a zero norm vector. The expression (5.11) tells therefore that $\phi_a(x)$ is equivalent to $\bar{\phi}_a(x)$. The striking feature of these last fields is that they all have the same x-dependence determined by the $U(1)$ current j.

It is not obvious that the field $\bar{\phi}_a$ defined in (5.12) is really independent of J. One finds in fact that two choices, J_1 and J_2, for J lead to $\bar{\phi}_a$'s which are not strictly equal but equivalent: $\bar{\phi}_a[J_1] \sim \bar{\phi}_a[J_2]$. This is due to the transformation law of $V_a[J]$; (5.8) implies the equivalence:

$$V_a[J_2] \sim (Z_2/Z_1)^{1/2N} \exp\left\{-\frac{2i\pi}{N} j^{(+)}[J_2 - J_1]\right\} \cdot V_a[J_1] \exp\left\{-\frac{2i\pi}{N} j^{(-)}[J_2 - J_1]\right\}. \quad (5.13)$$

Our results lead to a very simple characterization of $\bar{\mathscr{H}}_{\text{phys}}$; in the same way as $\bar{\mathscr{H}}'$ is generated from its vacuum $\bar{\Omega}$ by the bleached fields $\phi_a(x)$ defined in (3.1), $\bar{\mathscr{H}}_{\text{phys}}$ is generated from its vacuum $\bar{\Omega}_{\text{phys}}$ (equivalence class of $\bar{\Omega}$) by the reduced bleached fields $\bar{\phi}_a(x)$ defined in (5.12). Equivalently, we may characterize $\bar{\mathscr{H}}_{\text{phys}}$ as a space carrying an irreducible representation of the algebra generated by the current $j(x)$, the unitary operators $V_a[J]$ and the bare charges Q_a. Notice that, contrary to the charges Q_a, the individual bare currents j_a do not map $\bar{\mathscr{H}}'$ onto itself and are not defined on $\bar{\mathscr{H}}_{\text{phys}}$. According to (2.8), the color currents j^{i_D} reduce to their Σ-term on $\mathscr{H}_{\text{phys}}$ and are equivalent to zero on $\bar{\mathscr{H}}_{\text{phys}}$.

The space $\bar{\mathscr{H}}_{\text{phys}}$ is a direct sum of charge sectors. If $\boldsymbol{n} = (n_1, \ldots, n_N)$, $n_a \in \mathbb{Z}$ we may write:

$$\bar{\mathscr{H}}_{\text{phys}} = \bigotimes_{\boldsymbol{n}} \mathscr{H}_{\boldsymbol{n}}, \quad (5.14)$$

$$Q_a \mathscr{H}_{\boldsymbol{n}} = n_a \mathscr{H}_{\boldsymbol{n}}. \quad (5.15)$$

The operators $V_a[J]$ interpolate the charge sectors $\boldsymbol{n}$:

$$V_a[J]\mathscr{H}_{\boldsymbol{n}} = \mathscr{H}_{(n_1,\ldots,n_a-1,\ldots,n_N)}. \quad (5.16)$$

The sector $\mathscr{H}_{\mathbf{0}}$ is the canonical Fock space of the current $j(x)$. The other sectors are copies of this space on which $j(x)$ acts in the displaced from $j_{\text{can}}(x) + (\sum_a n_a) J'(x)$.

The exponentials in the Definition (5.12) of $\bar{\phi}_a$ shift the charge created by $V_a[J]$ to the vicinity of x. The factor $1/N$ in the exponent is related to a factor N in the Schwinger term giving the value of the current commutator:

$$[j(x), j(y)] = i(N/2\pi)\delta'(x - y). \quad (5.17)$$

This factor N in turn comes from the fact that j is a sum of N free bare currents, $j = \sum_a j_a$. A $U(1)$ charge can be obtained in N ways. The factor $1/N$ in the exponents of (5.12) and the factor N in (5.17) are both responsible for the unconventional commutation properties of the bleached fields, for instance (3.6).

In the last part of this Section we discuss the fate of the symmetry transformations on $\bar{\mathscr{H}}_{\text{phys}}$. According to (4.16) we have to know how the operators ρ_a act on $\bar{\mathscr{H}}_{\text{phys}}$. After bosonization one finds the equivalence:

$$\rho_a(x) \sim \bar{\rho}_a = V_a^{\dagger}[J] V_1[J]. \quad (5.18)$$

The remarkable fact is that there is no x-dependence in $\bar{\rho}_a$; its J-dependence is spurious because, as in the case of $\bar{\phi}_a$, (5.13) implies $\bar{\rho}_a[J_1] \sim \bar{\rho}_a[J_2]$. Equation (5.18) tells us that the operators $\bar{\rho}_a$ which implement strong local gauge transformations act non trivially on $\bar{\mathscr{H}}_{\text{phys}}$. This is in contrast with the operators implementing the weak local transformations. The expression (4.10) and the fact that the $\bar{j}_l$'s are equivalent to zero imply that $U(\Lambda_{\text{weak}})$ reduces to the identity on $\bar{\mathscr{H}}_{\text{phys}}$. All charges Q_a being defined on $\bar{\mathscr{H}}_{\text{phys}}$, the operators implementing the global transformations, equation (4.19), are defined without modifications on this space.

As already mentioned in Section 4, the fact that ρ_a describes a gauge transformation reduces the number of physically distinct bleached quarks. Equations (5.18) and (5.12) imply

$$\bar{\phi}_a(x) = \bar{\rho}_b^{\dagger}\bar{\rho}_a\bar{\phi}_b(x), \tag{5.19}$$

with the convention $\bar{\rho}_1 = 1$. The component $\bar{\phi}_a$ is not a gauge transformed $\bar{\phi}_b$: this cannot be because each component is invariant under all local gauge transformations. Equation (5.19) tells us that $\bar{\phi}_a$ and $\bar{\phi}_b$ acting on the same vector of $\bar{\mathscr{H}}_{\text{phys}}$ produce two vectors which are related by a gauge transformation. All components of $\bar{\phi}_a$ are physically equivalent; we may use (5.19) and eliminate all $\bar{\phi}_a$'s $(a = 2, \ldots, N)$ in favour of $\bar{\phi}_1$. There is only on independent physical screened quark.

The equivalence of the $\bar{\phi}_a$'s is related to an equivalence of the charge sectors $\mathscr{H}_{\boldsymbol{n}}$ carrying the same total charge $\sum_a n_a$. They are interpolated by unitary operators implementing strong local gauge transformations. In particular, each sector with zero total charge contains a state which is gauge equivalent to the vacuum $\bar{\Omega}_{\text{phys}}$. Consequently, $\mathscr{H}_{\text{phys}}$ is not an admissible physical state space; it has to be decomposed in physical sectors. This is accomplished in the next Sections.

In this Section we considered only right-goers; all we have done has to be duplicated in order to include the left-goers as well.

6. The physical sectors of massless DQCD$_2$

We have to decompose $\mathscr{H}_{\text{phys}}$ defined in (3.7) into a direct sum of physical sectors $\mathscr{H}_\gamma$, γ standing for a set of yet unknown labels:

$$\mathscr{H}_{\text{phys}} = \bigoplus_\gamma \mathscr{H}_\gamma. \tag{6.1}$$

First we have to make clear what conditions an acceptable physical sector has to fulfil. We shall adopt the following requirements:

(i) two distinct vectors of $\mathscr{H}_\gamma$ represent distinct physical states,
(ii) $\mathscr{H}_\gamma$ contains a unique vacuum
(iii) $\mathscr{H}_\gamma$ defines an irreducible representation of an algebra $\mathscr{F}$, the so-called field algebra [7].

The following comments clarify and specify these requirements.

1. Condition (i) implies that $\mathscr{H}_\gamma$ contains no pairs of gauge equivalent vectors.

2. As pointed out at the end of Section 4, there are pairs of U's implementing symmetry transformations and acting non trivally on $\mathscr{H}_{\text{phys}}$ which do not commute. Consequently a sector $\mathscr{H}_\gamma$ cannot be a simultaneous eigenspace of all U's. What (i) really implies is the existence of equivalent sectors which are interpolated by the U's.

3. There is some freedom in the choice of the field algebra $\mathcal{F}$. In this Section, we shall require that it has to contain the algebra $\mathcal{A}$ of the observables which are invariant under the full group of symmetry transformations of massless $DQCD_2$. This algebra is generated by the Σ-fields, the $U(1)$ current $j_\pm(x^\pm)=(j_0\pm j_1)(x)$ and the left- and right-going $U(1)$ charges $Q_\pm$. Each sector $\mathcal{H}_\gamma$ being a sum of $U(1)$ charge sectors, $\mathcal{F}$ has to contain operators which interpolate these charge sectors. Our $\mathcal{F}$ will be a minimal algebra fulfilling these conditions.

4. Such a minimal algebra is entirely appropriate to massless QED_2. However, it does not contain the mass term $\bar\psi\psi$ and we cannot define a mass perturbation. As it will be shown in the next section, the inclusion of the mass term has dramatic effects on the physical sectors; it forces the breaking of chiral symmetry and the physical sectors become θ-sectors.

We start now the explicit construction of the physical sectors. The first step is the identification of the possible vacua, i.e. the charge zero Poincaré invariant vectors of $\mathcal{H}_{\text{phys}}$. The effect of a Poincaré transformation on an operator like $V_{\pm,a}[J]$ is to change its standard kink J. As the combinations $\bar\rho_{\pm,a}$ defined in (5.18) are J-independent, they are Poincaré invariant and transform a Poincaré invariant vector into another one. Their action on $\bar\Omega_{\text{phys}}$ produces a vacuum subspace $\bar{\mathcal{H}}_\Omega$ of $\bar{\mathcal{H}}_{\text{phys}}$. The vacuum of a physical sector $\mathcal{H}_\gamma$ will have the form:

$$\Omega_\gamma=\Omega^{(\Sigma)}\otimes\bar\Omega_\gamma \tag{6.2}$$

where $\Omega^{(\Sigma)}$ is the vacuum of $\mathcal{H}^{(\Sigma)}$ and $\bar\Omega_\gamma\in\bar{\mathcal{H}}_\Omega$. Using (5.18) we see that the vectors:

$$|\boldsymbol{n}^+,\boldsymbol{n}^-\rangle=(V_{+,1})^{n_1^+}\cdots(V_{+,N})^{n_N^+}(V_{-,1})^{n_1^-}\cdots(V_{-,N})^{n_N^-}\bar\Omega_{\text{phys}} \tag{6.3}$$

are in $\bar{\mathcal{H}}_\Omega$ if the sets $\boldsymbol{n}^+$ and $\boldsymbol{n}^-$ are such that:

$$\sum_a n_a^\pm=0. \tag{6.4}$$

The vectors (6.3) fulfilling this constraint form an orthonormal basis of $\bar{\mathcal{H}}_\Omega$.

Our second step is the complete specification of the field algebra $\mathcal{F}$ through the choice of its $U(1)$ charge creating elements. Each $V_{\pm,a}$ is such an operator. Whereas $\mathcal{F}$ has to contain at least one pair $V_{\pm,a}$ and its adjoint, the fact that the $\bar\rho_{\pm,a}$ are Poincaré invariant implies that it cannot contain more than one of them. For suppose that $V_{+,1}$ and $V_{+,a}$ $(a\neq 1)$ and their adjoints belong to $\mathcal{F}$. Then the product $\bar\rho_{+,a}=V_{+,a}^\dagger V_{+,1}$ belongs to $\mathcal{F}$ too and $\Psi=\bar\rho_{+,a}\Omega_\gamma$ is a vector of $\mathcal{H}_\gamma$. On the other hand, Ψ belongs to the vacuum space $\Omega^{(\Sigma)}\otimes\bar{\mathcal{H}}_\Omega$. Now Ω_γ is or is not an eigenvector of $\bar\rho_{+,a}$. In the first case, $V_{+,1}$ and $V_{+,a}$ have the same effect on the vectors of $\mathcal{H}_\gamma$, up to phase factors, and one of them is redundant. In the second case, $\mathcal{H}_\gamma$ contains more than one vacuum, in contradiction with condition (ii). Consequently, the determination of $\mathcal{F}$ involves the choice of one pair of operators $W_\pm$ creating units of $Q_\pm$ charges. For later convenience our choice is:

$$W_\pm[J]=V_{\pm,1}[J]e^{i\pi[Q_{-,1}\pm(Q_\pm/N)]}. \tag{6.5}$$

We have now a precise definition of the field algebra $\mathcal{F}$; it is generated by:

(a) the $(N-1)$ massive boson fields $\tilde\Sigma^{i_D}(x)$,
(b) the components $j_\pm(x^\pm)$ of the massless $U(1)$ current
(c) the total charges $Q_\pm$
(d) the charge creating operators $W_\pm$ and their adjoints.

The requirements (i)–(iii) don't restrict the choice of the admissible vacua in the space $\Omega^{(\Sigma)} \otimes \bar{\mathscr{H}}_\Omega$. Any vector in $\bar{\mathscr{H}}_\Omega$ is acceptable and we may define a set of orthogonal sectors $\mathscr{H}(\boldsymbol{n}^+; \boldsymbol{n}^-)$ obtained by acting on the vacuum $\Omega^{(\Sigma)} \otimes |\boldsymbol{n}^+; \boldsymbol{n}^-\rangle$ with the elements (a), (b) and (d) of $\mathscr{F}$. As long as we maintain our minimal algebra $\mathscr{F}$, the introduction of θ-sectors is unnecessary. A similar situation prevails in massless QED_2, as emphasized by Capri and Ferrari [12].

The representation of $\mathscr{F}$ on $\mathscr{H}(\boldsymbol{n}^+; \boldsymbol{n}^-)$ is irreducible by construction. It is immediately seen that the representations defined by the different sectors are all equivalent. The unitary operators $\bar{\rho}_{\pm,a}$ interpolate pairs of sectors; for instance:

$$\bar{\rho}_{+,a} \mathscr{H}(\boldsymbol{n}^+; \boldsymbol{n}^-) = \exp\left[i\pi\left(1 + \sum_{b=1}^{a-1} n_b^+\right)\right] \mathscr{H}(n_1^+ - 1, \ldots, n_a^+ + 1, \ldots,; \boldsymbol{n}^-). \tag{6.6}$$

On the other hand, we know that $\bar{\rho}_{\pm,a}$ implements a gauge transformation. The equivalence of the sectors is just a gauge equivalence. The existence of distinct sectors carrying equivalent representations is a necessary consequence of the existence of gauge transformations implemented by operators acting non trivially on $\bar{\mathscr{H}}_{\text{phys}}$.

The sectors $\mathscr{H}(\boldsymbol{n}^+; \boldsymbol{n}^-)$ being all gauge equivalent, they can be identified to a single physical sector $\hat{\mathscr{H}}$. It is clear that any other way of defining the initial physical sectors or any other choice for $W_\pm$ in (6.5) would lead to the same final $\hat{\mathscr{H}}$. The particle spectrum of $\hat{\mathscr{H}}$ contains:

(A) $(N-1)$ free massive bosons described by the Σ-fields. These particles carry the toplogical color current $j_\mu^{i_D} = -(1/\sqrt{2\pi})\varepsilon_{\mu\nu}\partial^\nu \tilde{\Sigma}^{i_D}$; their total color charge and their $U(1)$ charges are zero.

(B) One free massless bleached (colorless) quark described by the set $\{j_\pm(x^\pm), W_\pm[J]\}$. It carries a negative unit of $U(1)$ charge and its space-time behaviour is described by the fields:

$$\bar{\phi}_\pm(x^\pm) = \sqrt{\mu/2\pi}(\mu Z)^{-(1/2N)} \exp\left\{-\frac{2i\pi}{N} j_\pm^{(+)}[\theta_{x^\pm} - J]\right\}$$
$$W_\pm[J] \exp\left\{-\frac{2i\pi}{N} j_\pm^{(-)}[\theta_{x^\pm} - J]\right\}. \tag{6.7}$$

The existence of this quark, which distinguishes massless $DQCD_2$ from massless QED_2, is related to the fact that $DQCD_2$ has an additional unbroken $U_V(1) \times U_A(1)$ symmetry. The $U(1)$ charges need a carrier which the bleached quark turns out to be. This quark being massless, it generates a plethora of free massless multiparticle bound states which are a pecularity of two-dimensional space-time [13]. For example, the light-cone components $j_\pm$ describe left- and right-going quark-antiquark bound states. More will have to be said on these massless bound states in the next Section.

A final comment on the global $DSU(N)$ symmetries. The individual charges $Q_{\pm,a}$ are defined on the sectors $\mathscr{H}(\boldsymbol{n}^+, \boldsymbol{n}^-)$ and label their vacua. This leads to an apparent paradox. On the one hand, the generators of the global $DSU(N)$ symmetries map $\mathscr{H}(\boldsymbol{n}^+, \boldsymbol{n}^-)$ onto itself and this is a feature of an unbroken symmetry. On the other hand these generators don't annihilate the vacuum of $\mathscr{H}(\boldsymbol{n}^+, \boldsymbol{n}^-)$. There is in fact no paradox because of the equivalence of the different sectors $\mathscr{H}(\boldsymbol{n}^+, \boldsymbol{n}^-)$. The $DSU(N)$ symmetries are broken because the individual

$Q_{\pm,a}$ are not defined on $\hat{\mathscr{H}}$. Only the total charges $Q_{\pm}$ are defined on this sector, leading to unbroken $U(1)$ symmetries.

7. Mass perturbation and θ-sectors

In this last section we examine how the construction of the physical sectors has to be modified if we want to describe the response of massless $DQCD_2$ to a mass perturbation. To this end we have to include a regularized version $M(x)$ of the mass term $(\bar{\psi}\psi)(x)$ into the field algebra. This extension changes drastically the admissible physical sectors if one maintains the general requirements (i)–(iii) of Section 6.

To start, we need an explicit expression of $M(x)$. This operator is defined with the help of the limit of a bilocal string operator $S(x, y)$. Formally:

$$S(x, y) = \psi_-^\dagger(x) \exp\left[ig\tfrac{1}{2}\lambda^{i_D}\int_y^x dz^\mu A_\mu^{i_D}(z)\right]\psi_+(y). \tag{7.1}$$

Using equations (2.1), (3.1), (5.11) and (5.12) this formal definition is naturally transformed into an expression valid on $\mathscr{H}_{\text{phys}}$. The result is similar to the one obtained in massless QED_2 (equation (5.12) in [5]); it leads to:

$$S(x) = \lim_{y\to x} S(x, y) = \sum_a :\exp\left[i\sqrt{2\pi}\,\lambda_a^{i_D}\tilde{\Sigma}^{i_D}(x)\right]: K_a(x), \tag{7.2}$$

where

$$K_a(x) = \frac{1}{2\pi}\,\mu(\mu\,|Z|)^{-1/N} \exp\left\{\frac{2i\pi}{N}\left(j_-^{(+)}[\theta_{x^-} - J] - j_+^{(+)}[\theta_{x^+} - J]\right)\right\}$$
$$\cdot\, T_a[J] \exp\left\{\frac{2i\pi}{N}\left(j_-^{(-)}[\theta_{x^-} - J] - j_+^{(-)}[\theta_{x^+} - J]\right)\right\} e^{-i(\pi/N)Q} \tag{7.3}$$

and $T_a[J]$ is the following chirality changing operator:

$$T_a[J] = V_{-,a}^\dagger[J]V_{+,a}[J]e^{i(\pi/N)Q}. \tag{7.4}$$

The factors $\exp[\mp i(\pi/N)Q]$ have been introduced into (7.3) and (7.4) for later convenience. The mass term $M(x)$ is given by

$$M(x) = m_0(S(x) + S^\dagger(x)), \tag{7.5}$$

where m_0 is an unrenormalized mass. As $M(x)$ belongs now to the field algebra, a physical sector has to be invariant under its action. This excludes our previous sectors $\mathscr{H}(\boldsymbol{n}^+, \boldsymbol{n}^-)$. To see this, we observe that the relation (5.18) allows the elimination of $V_{\pm,a}$, $a \neq 1$, in favour of $\bar{\rho}_{\pm,a}$ and $V_{\pm,1}$. This leads to an expression of $T_a[J]$ containing the product $\bar{\rho}_{-,a}\bar{\rho}_{+,a}^\dagger$. According to (6.6), this product maps $\mathscr{H}(\boldsymbol{n}^+, \boldsymbol{n}^-)$ onto another sector and $M(x)$ does not map $\mathscr{H}(\boldsymbol{n}^+, \boldsymbol{n}^-)$ onto itself.

Another problem arises from the fact that we want to define a perturbation hamiltonian which is equal to the space integral of $M(x)$. This integral has to converge. Now, consider the matrix element $(\Psi, M(x)\Omega)$ where $\Omega = \Omega^{(\Sigma)} \otimes |\boldsymbol{n}^+, \boldsymbol{n}^-\rangle$ and $\Psi = W_-^\dagger W_+\Omega$. A calculation sketched in the Appendix shows that this matrix element behaves as $|x^1|^{-2/N}$ for large x^1, its integral diverges if $N \geqq 2$. This divergence is due to the chirality changing term T_a in (7.3) and the value of its commutator with the $U(1)$ current obtained from:

$$[j_\pm(x^\pm), V_{\pm,a}[J]] = J'(x^\pm)V_{\pm,a}[J]. \tag{7.6}$$

As it is the integral of nondiagonal matrix elements which diverges, it is of no help to redefine $M(x)$ by subtracting a constant multiple of the identity.

On the other hand, it is clear that the only way of making $M(x)$ integrable without altering its significance of a mass perturbation goes through the subtraction of a constant. What we have to admit is that this constant may not be the same in all physical sectors. In other words, the new physical sectors have to be such that in each of them an appropriate subtraction renders $M(x)$ integrable. A similar but simpler situation is encountered in massless QED_2. There we have only one K, which happens to be an x- and J-independent unitary operator. The appropriate physical sectors are eigenspaces of K (θ-sectors with eigenvalue $e^{i\theta}$) and $M(x)$ becomes integrable after the subtraction of a constant proportional to $\cos\theta$. This procedure works because the J-independence of K guarantees the Poincaré invariance of the θ-vacua.

The example of QED_2 suggests that in the case of DQCD_2 we may have to diagonalize a set of chirality changing Poincaré invariant unitary operators T_a, $a = 1, \dots, N$:

$$[T_a, \tilde{Q}_b] = -2\delta_{ab}T_a \tag{7.7}$$

We cannot identify T_a with $T_a[J]$; this operator being J-dependent, it is not Poincaré invariant. In fact, this choice wouldn't work at all. As $T_a[J]$ does not commute with the exponentials in (7.3), its diagonalization does not produce an $M(x)$ becoming integrable through subtraction.

The only operators defined on $\bar{\mathcal{H}}_{\text{phys}}$ which are explicitly J-independent are combinations of the operators $\bar{\rho}_{\pm,a}$ (equation (5.18)). It is impossible to construct chirality changing operators verifying (7.7) out of them. This does not mean that the T_a's we are looking for do not exist. It is known that J-dependent operators may have Poincaré invariant limits if their kink is shifted to infinity. Consider for instance the operators $V_{\pm,a}[R]$ where $R(x)$ is a kink function introduced by Becher [10]:

$$R(z) = \tfrac{1}{2}(\theta_L^\delta(z) + \theta_{-L}^\delta(z)). \tag{7.8}$$

It describes two charge clouds of charge $-\frac{1}{2}$ localized around $+L$ and $-L$ in intervals of width 2δ. Using Becher's techniques, one may show that the limit of $V_{\pm,a}[R]$ as $L \to \infty$ is a Poincaré invariant operator creating charges localized at infinity. This leads us to try the identification:

$$T_a = \lim_{L\to\infty} T_a[R]. \tag{7.9}$$

If we want to express $M(x)$ in terms of T_a we have to perform the substitution $J \to R$ in (7.3) and take the limit $L \to \infty$. We see that this requires not only the limit of $T_a[R]$ but also the limit of $j_\pm[\theta_{x^\pm} - R]$. As $L \to \infty$, $j_+[\theta_{x^+} - R]$ becomes the difference $v_+(x^+)$ of the left-going component of the current potential at x^+ and its average value at $\pm\infty$.

As already mentioned, we intend to define the new physical sectors as eigenspaces of T_a. Then, it should be possible to replace T_a by its eigenvalue in the restriction of $M(x)$ to such a sector. This cannot be done directly in the expression we have derived and a further transformation is needed. The operator T_a enters into $M(x)$ through $K_a(x)$ and after the substitution $J \to R$ has been performed in (7.3), $T_a[R]$ is sandwiched between two exponentials. Before we are allowed to replace $T_a[R]$, or its limit (7.9), by its eigenvalue we have to shift this operator to the right of the exponentials. It follows from (7.6) that $T_a[R]$ does not commute with $j_\alpha^{(-)}[\theta_{x^\alpha} - R]$, even in the $L \to \infty$ limit. Consequently, shifting $T_a[R]$ to the right introduces a multiplicative factor which, fortunately, turns out to become x-independent for finite x and $L \to \infty$:

$$K_a(x) = \tfrac{1}{2} Z_1 \mu \ {:}\exp\left\{\frac{2i\pi}{N}(j_-[\theta_{x^-} - R] - j_+[\theta_{x^+} - R])\right\}{:}$$
$$T_a[R]\left(1 + 0\left(\frac{x^+}{L}\right) + 0\left(\frac{x^-}{L}\right)\right), \qquad (7.10)$$

$\frac{1}{2}Z_1$ differs from the constant factor in (7.3). If $(d/dz)\theta_L^\delta(z)$ has a rectangular profile, one finds:

$$Z_1 = \text{const.}\ (\delta/\mu^2 L^3)^{1/2N}. \qquad (7.11)$$

Whereas $T_a[R]$ does not commute with the negative frequency part $j_\alpha^{(-)}[\theta_{x^\alpha} - R]$, it is easily established that it commutes with the full current $j_\alpha[\theta_{x^\alpha} - R]$ for large L. Therefore, one could also write (7.10) with $T_a[R]$ at the left of the Wick ordered exponential. With (7.10) we have now an expression for $K_a(x)$ in which T_a can be replaced by its eigenvalue if we are dealing with the restriction to one of its eigenspaces.

In what follows we do not attempt a technically precise definition of the limit $L \to \infty$ at the level of the operators of massless $DQCD_2$. We shall use L as a cutoff and perform our construction with a large but finite L. Our results will show how the hamiltonian of massive $DQCD_2$ has to be defined.

To get our new physical sectors we have to modify the field algebra $\mathscr{F}$ of Section 6. The chirality changing operators $T_a[R]$ have to be included into the new algebra $\hat{\mathscr{F}}$. As we want to diagonalize these operators, the chiral charge $\tilde{Q} = Q_- - Q_+$ has to be excluded. The operators $T_a[R]$ providing a link between left- and right-goers we need only one independent charge creating operator, for instance $W[J] = W_+[J]$ defined in (6.5). The right-going charges are then generated by:

$$W_-[J] = e^{-i\pi(N+1)/N} \exp\left\{-\frac{2i\pi}{N}(j_-[J-R] - j_+[J-R])\right\} \cdot W[J] T_1^\dagger[R] \qquad (7.12)$$

As we work with a finite L, consistency requires that the local fields of $\hat{\mathscr{F}}$ are localized in a finite region. The double cone $C_l = \{x \mid |x^\pm| < l, l < L - \delta\}$ is convenient if $(d/dz)\theta_L^\delta(z)$ has a compact support $[L-\delta, L+\delta]$. We conclude that $\hat{\mathscr{F}}$ is generated by:

(â) the $(N-1)$ boson fields $\tilde{\Sigma}^{i_D}(x)$,

($\hat{\text{b}}$) the current $j_\pm(x^\pm)$ for $x \in C_l$ and the ordered exponentials ${:}\exp\{(2i\pi/N) j_\pm[F-R]\}{:}$, the derivative F' of the kink function F having a compact support contained in $[-L+\delta, L-\delta]$,

(ĉ) the charge Q

(d̂) the charge creating operator W and its adjoint,
(ê) the chirality changing operators $T_a[R]$ and their adjoints.

We observe that $T_a[R]$ commutes with all elements of $\hat{\mathscr{F}}$; the sectors of this algebra are simultaneous eigenspaces of all T_a's. Their vacua have components in $\bar{\mathscr{H}}_{\text{phys}}$ which belong to an extended vacuum space $\hat{\mathscr{H}}_\Omega$ obtained by acting on $\bar{\mathscr{H}}_\Omega$ (defined in (6.3)) with the operators $T_a[R]$. Using (6.3) and the definition of $T_a[R]$ it is readily seen that $\hat{\mathscr{H}}_\Omega$ has the following basis:

$$|\mathbf{m};\boldsymbol{n}\rangle=(T_1[R])^{m_1}\cdots(T_N[R])^{m_N}(V_{+,1}[R])^{n_1}\cdots(V_{+,N}[R])^{n_N}\bar{\Omega}_{\text{phys}}. \tag{7.13}$$

The set $\boldsymbol{n}$ is constrained by $\sum_a n_a=0$ and the set $\boldsymbol{m}$ is unconstrained. The chiral charge of $|\boldsymbol{m};\boldsymbol{n}\rangle$ is $-2\sum_a m_a$. There is an arbitrariness in the definition of the basis (7.14). For example one would obtain an equivalent basis by replacing $(V_{+,a})^{n_a}$ by $(V_{-,a})^{n_a}$. The states (7.13) become Poincaré invariant in the limit $L\to\infty$.

The admissible vacua are simultaneous eigenvectors of the T_a's. The eigenvectors with prescribed eigenvalues $e^{i\theta_a}$ form a space of improper vectors spanned by the basis:

$$|\boldsymbol{\theta};\boldsymbol{n}\rangle=\frac{1}{\sqrt{2\pi}}\sum_{\boldsymbol{m}}\exp\left(i\sum_a m_a\theta_a\right)|\boldsymbol{m};\boldsymbol{n}\rangle. \tag{7.14}$$

We obtain orthogonal $\boldsymbol{\theta}$-sectors $\mathscr{H}(\boldsymbol{\theta};\boldsymbol{n})$ by applying the elements (â), (b̂) and (d̂) of $\hat{\mathscr{F}}$ on $\Omega^{(\Sigma)}\otimes|\boldsymbol{\theta};\boldsymbol{n}\rangle$. Two sectors with the same chiral angles $\boldsymbol{\theta}$ and different $\boldsymbol{n}$ define equivalent representations of $\hat{\mathscr{F}}$. As in Section 6, this is related to the fact that every element of $\hat{\mathscr{F}}$ is invariant under gauge transformations described by unitary operators interpolating sectors with different $\boldsymbol{n}$. In the present case these are the strong local gauge transformations implemented by $U_a^{(+)}$ (equation (4.16)):

$$[U_a^{(+)},A]=0,\qquad \forall A\in\hat{\mathscr{F}}$$
$$U_a^{(+)}\mathscr{H}(\boldsymbol{\theta};\boldsymbol{n})=\mathscr{H}(\boldsymbol{\theta};n_1+1,\ldots,n_a-1,\ldots,n_N). \tag{7.15}$$

The operators $\tilde{U}_a(\tilde{\alpha}_a)$ implementing the chiral transformations (4.18) and defined in (4.19) interpolate sectors carrying inequivalent representations of $\hat{\mathscr{F}}$:

$$\tilde{U}(\tilde{\alpha})\mathscr{H}(\boldsymbol{\theta};\boldsymbol{n})=\mathscr{H}(\theta_1+2\tilde{\alpha}_1,\ldots,\theta_N+2\tilde{\alpha}_N;n). \tag{7.16}$$

This is a manifestation of chiral symmetry breaking.

In order to keep only physically distinct sectors we identify all sectors $\mathscr{H}(\boldsymbol{\theta};\boldsymbol{n})$ with the same $\boldsymbol{\theta}$ to a single sector $\mathscr{H}(\boldsymbol{\theta})$. Clearly, all these sectors have the same particle spectrum as that found in Section 6. What distinguishes them is the value of the mass term. Collecting (7.5), (7.2), (7.3) and (7.10), we see that on $\mathscr{H}(\boldsymbol{\theta})$, $M(x)$ reduces to:

$$M(x)=m_0\mu Z_1\sum_a\Big\{:\cos\Big[\theta_a+\sqrt{2\pi}\,\lambda_a^{i_D}\tilde{\Sigma}^{i_D}(x)$$
$$+\frac{2\pi}{N}(j_-[\theta_{x^-}-R]-j_+[\theta_{x^+}-R])-\frac{\pi}{N}Q\Big]:-\cos\theta_a\Big\} \tag{7.17}$$

We have dropped the $0(x^\pm/L)$ terms appearing in (7.10); they have a vanishing limit if $x^\pm$ are kept finite and $L\to\infty$. Furthermore $M(x)$ has been normalized to a vanishing vacuum expectation value by subtraction of a constant proportional to $\sum_a\cos\theta_a$.

The value (7.11) of Z_1 shows that $M(x)$ has a finite limit for $L \to \infty$ only if m_0 is made cut-off dependent. We define a renormalized mass parameter through:

$$m^2 = \lim_{L \to \infty} m_0 \mu Z_1 \tag{7.18}$$

The mass term (7.17) coincides with the form obtained through the formal bosonization techniques [3]; the quantity $(j_-[\theta_{x^-} - R] - j_+[\theta_{x^+} - R] - \frac{1}{2}Q)$ has to be identified with the pseudopotential of the $U(1)$ current. In order to understand the content of (7.17) in our context, we evaluate the mean value of $M(x)$ in the state:

$$\Psi = \prod_{i=1}^{n_+} W_+^\dagger[F_i^+] \prod_{j=1}^{m_+} W_+[G_j^+] \prod_{k=1}^{n_-} W_-^\dagger[F_k^-] \prod_{l=1}^{m_-} W_-[G_l^-] \Omega(\boldsymbol{\theta}) \tag{7.19}$$

The $F_i^\pm$ and $G_j^\pm$ form a collection of kink functions whose derivatives have a compact support contained in the interval $[-l, +l]$. Notice that $W_+[F_i^+]$, for instance, belongs to the algebra $\hat{\mathscr{F}}$ because $W_+[F_i^+] = \exp\{-(2i\pi/N) j_+[F_i^+ - J]\} W$. Using the commutation relations (7.6), one finds for $|x^\pm| < l$:

$$\begin{aligned}\langle \Psi, M(x)\Psi\rangle_{\boldsymbol{\theta}} = m^2 \sum_a \Bigg\{ \cos\Bigg[\theta_a + \frac{2\pi}{N}\Bigg(\sum_{i=1}^{n_+} (F_i^+(x^+) - 1) \\ - \sum_{j=1}^{m_+} (G_j^+(x^+) - 1) - \sum_{k=1}^{n_-} F_k^-(x^-) + \sum_{l=1}^{m_-} G_l^-(x^-)\Bigg)\Bigg] - \cos\theta_a \Bigg\}\end{aligned} \tag{7.20}$$

The $\boldsymbol{\theta}$-sector matrix elements $\langle\ \rangle_{\boldsymbol{\theta}}$ are obtained from $(\Phi(\boldsymbol{\theta}), A\Psi(\boldsymbol{\theta}')) = \delta(\boldsymbol{\theta} - \boldsymbol{\theta}')$ $\langle \Phi, A\Psi\rangle_{\boldsymbol{\theta}}$. The right-hand side of (7.20) is a rigorous consequence of (7.17) and (7.18); it yields the exact $L \to \infty$ limit of $\langle \Psi, M(x)\Psi\rangle$. This means that we may define the matrix element of the interaction hamiltonian H_{int} through:

$$\langle \Psi, H_{\text{int}}\Psi\rangle_{\boldsymbol{\theta}} = m^2 \lim_{l \to \infty} \int_{-l}^{+l} dx^1 \langle \Psi, M(x)\Psi\rangle_{\boldsymbol{\theta}} \tag{7.21}$$

The argument of the first cosine in the curly bracket of (7.20) tends to θ_a for $x^1 \to -\infty$ and to $(\theta_a + (2\pi/N)(m_+ + m_- - n_+ - n_-))$ for $x^1 \to +\infty$. For arbitrary values of the angles θ_a, this implies that the integral in (7.21) converges only if $(m_+ + m_- - n_+ - n_-)$ is an integer multiple of N. In the case of a generic $\boldsymbol{\theta}$, we reach the important conclusion that the mass term produces an interaction hamiltonian which is finite only on the $U(1)$ charge sectors with charge $Q = pN$, $p \in \mathbb{Z}$. In particular the $Q = -1$ massless bleached quark gets an infinite energy and is totally unstable under the mass perturbation. This explains why the light particle spectrum of massive $DQCD_2$ contains no particle corresponding to the bleached quark in the limit of small m [3]. This spectrum contains mesons $(Q = 0)$, baryons and antibaryons $(Q = \mp N)$ and their bound states. A similar light spectrum has been found in the small quark mass limit of QCD_2 [14].

Among the states of massless $DQCD_2$ which have a finite mass perturbation we find some of the massless bound states of the bleached quark mentioned at the end of Section 6. These bound states exhibit a very peculiar property if they are described by means of the bleached fields in their original form (3.1). The right moving baryon, for instance, is created in the vacuum by a regularized version of

$$B_-(x^-) = \prod_{a=1}^{N} \phi_{-,a}(x^-) \tag{7.22}$$

The matrices λ^{i_D} being traceless, $\sum_a \lambda_a^{i_D} = 0$ and (3.1) and (3.2) imply that the η-fields disappear from this product. One is left with:

$$B_-(x^-) = \prod_{a=1}^{N} \chi_{-,a}(x^-) \tag{7.23}$$

The χ-fields being independent, no further regularization is required. Similarly, there are meson states created by the $U(1)$ current, which is also a function of the bare quark fields alone. We conclude that the zero mass bound states which survive the mass perturbation are those physical states which can be generated by color singlet combinations of the bare quark fields, without any admixture of η-fields [15]. These states correspond to the light particle spectrum of massive $DQCD_2$ [14]. Light particles having the same structure as the states generated by $B_\pm(x^\pm)$ and $j_\pm(x^\pm)$ have been found in the chiral limit of QCD_2 [16]. We see that the bare quarks are in fact the partons of $DQCD_2$ in a mathematically precise sense. Due to the peculiarities of two dimensional space-time, we have an idealized parton picture [17] in the massless case. The physical particles are colorless bound states of free massless partons; these bound states are of purely kinematical origin, no confining force is needed to keep the partons together.

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Appendix

According to (7.5), the matrix element $(\Psi, M(x)\Omega)$ is obtained from $s(x) = (\Psi, S(x)\Omega)$ and its complex conjugate. Using (7.2), (7.3) and (7.4) one finds:

$$s(x) = \text{const.} \left(\Omega,\ V^\dagger_{+,1} V_{-,1} \exp\left\{\frac{2i\pi}{N}(j_-^{(+)}[\theta_x - J] - j_+^{(+)}[\theta_{x^+} - J])\right\} \cdot V^\dagger_{-,1} V_{+,1}\Omega\right). \tag{A.1}$$

The fact that the $\tilde{\Sigma}$-fields and the current $j_\pm$ have negative frequency parts which annihilate Ω has been taken into account. The commutation relations (7.6) imply:

$$s(x) = \text{const.}\ e^{q(x)}, \tag{A.2}$$

where

$$q(x) = -\frac{2i\pi}{N}\int_{-\infty}^{+\infty} J'^{(+)}(y)(\theta_{x^+} + \theta_{x^-} - 2J)(y). \tag{A.3}$$

To estimate this integral, we insert the following expression for the positive frequency part of J':

$$J'^{(+)}(y) = -\frac{1}{2i\pi}\int_{-\infty}^{+\infty} dz \frac{1}{y - z + i\varepsilon} J'(z). \tag{A.4}$$

Integrating by parts over y we get:

$$q(x) = -\frac{1}{N}\int_{-\infty}^{\infty} dy\, dz(\theta'_{x^+} + \theta'_{x^-} - 2J'](y) \ln (y - z + i\varepsilon)J'(z). \tag{A.5}$$

If J' has a compact support the limit to sharp step functions can be performed in the integrand provided $x^{\pm}$ are outside this support. At fixed x^0 this happens if x^1 is large enough:

$$q(x) = \frac{1}{N}\int_{-\infty}^{+\infty} dz(\ln (x^0 + x^1 - z + i\varepsilon) + \ln (x^0 - x^1 - z + i\varepsilon))J'(z) + \frac{2}{N} Z \tag{A.6}$$

the constant Z is defined in (5.5). As $J(+\infty) - J(-\infty) = -1$, we see that $\operatorname{Re} q(x)$ behaves as $-(2/N) \log |x^1|$ for large x^1 and this implies $|s(x)| = 0(|x^1|^{-2/N})$.

REFERENCES

[1] L. V. Belvedere, J. Swieca, K. Rothe, and B. Schroer, Nucl. Phys. *B153* (1979), 112.
[2] P. Mitra and P. Roy, Phys. Rev. *D21* (1980), 511, 521 and 2926.
[3] P. J. Steinhardt, Ann. Phys. (N.Y.) *132* (1981), 18.
[4] R. E. Gamboa Saravi, F. A. Schaposnik and J. E. Solomin, Nucl. Phys. *B185* (1981), 239.
[5] A. K. Raina and G. Wanders, Ann. Phys. (N.Y.) *132* (1981), 404.
[6] A. K. Raina and G. Wanders, Helv. Phys. Acta *54* (1981), 419.
[7] S. Dopplicher, R. Haag and J. E. Roberts, Commun. Math. Phys. *13* (1969), 1 and *15* (1969), 173.
[8] J. H. Lowenstein and J. A. Swieca, Ann. Phys. (N.Y.) *68* (1971), 172.
[9] S. Coleman, Phys. Rev. *D11* (1975), 2088.
[10] P. Becher, Nuovo Cimento *47A* (1978), 151.
[11] J. A. Swieca, Fortschritte der Physik *25* (1977), 303.
[12] A. Z. Capri and F. Ferrari, Nuovo Cimento *62A* (1981), 273.
[13] R. Seiler, D. A. Uhlenbrok and D. N. Williams, Nucl. Phys. *B116* (1976), 133.
[14] P. J. Steinhardt, Nucl. Phys. *B176* (1980), 100.
[15] G. Wanders, in preparation.
[16] W. Buchmüller, S. T. Love and R. D. Peccei, Phys. Lett. *108B* (1982), 426; D. Amati, K. C. Chou and S. Yankielowicz, Phys. Lett. *110B* (1982), 309.
[17] R. P. Feynman, Phys. Rev. Lett. *23* (1969), 1415; J. D. Bjorken and E. A. Paschos, Phys. Rev. *158* (1969), 1975.