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PATHS TO TURBULENCE

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Sometimes, a physicist must step back from his everyday problem and try to understand, or at least to feel, the possibilities offered to him beyond those tools he is accustomed to. The aim of this talk is to provide such a perspective and to encourage you to consider its applicability in your future research.

Such an enterprise cannot provide readymade answers to the major open problems of theoretical physics, and in particular, this talk does not answer in any definitive, or even provisional way, the quest for an understanding of the beauties of turbulence, in fact, the title has been chosen by lack of better alternatives.

My main aim is thus to transmit a set of ideas which seem most helpful in a class of problems of classical physics, although there are some interesting connections with quantum-mechanical problems.

The starting point of the subject of "dynamical systems" are classical evolution equations, which are supposed to model the time evolution of a given physical system. While the general ideas probably apply to partial differential equations, detailed results are very scarce in this respect, so we shall deal with ordinary differential equations, only. Such equations can always be brought to the form $\dot{\vec{x}}(t) = \vec{F}(\vec{x}(t),t)$, or in components

$$\frac{d}{dt}x_j(t) = F_j(x_1(t),\ldots,x_n(t),t) \ , \quad j = 1,2,\ldots,n \ . \tag{1}$$

If F does not depend explicitly on t, the system is called autonomous. Our desire is to analyze such equations. Now, as soon as F is not linear, the task of solving such an equation explicitly becomes almost surely impossible, and I would like to illustrate this with the example of the (nonlinear) parametric pendulum :

$$\ddot{\varphi} = -\omega(1 + \mu \cos 2t)\sin \varphi - k\dot{\varphi} \ . \tag{2}$$

This equation can, obviously, been brought to the form (1). When $k = 0$ the system is conservative, if $k > 0$ one calls it dissipative. We shall concentrate on this latter possibility. Note that the pendulum is truly nonlinear, it is a "swing", going over the top.

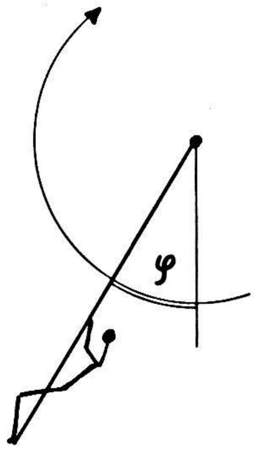


Figure 1

Of course, everybody is familiar with the stability analysis for small oscillations, as they have been done by Lord Rayleigh, and it is well known that the instability regions are to be found in the shaded parts of the following diagram, and that a periodic motion can be obtained by forcing the pendulum at this frequency, when $k = 0$.

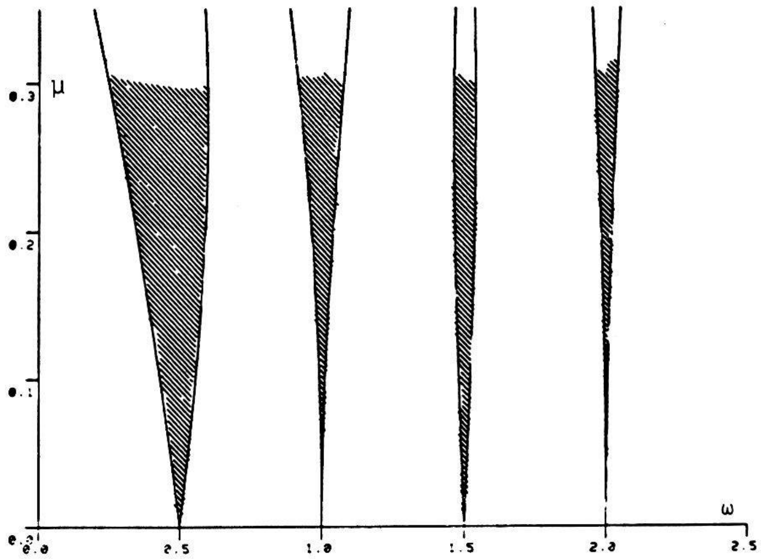


Figure 2

We shall show how the pendulum evolves, for $k = 0.1$, $\omega = 1$ and four values of μ, $\mu = 0.5, 0.88, 1.02, 1.5$. The initial conditions are $\varphi(0) = 2$, $\dot{\varphi}(0) = -1.5$.

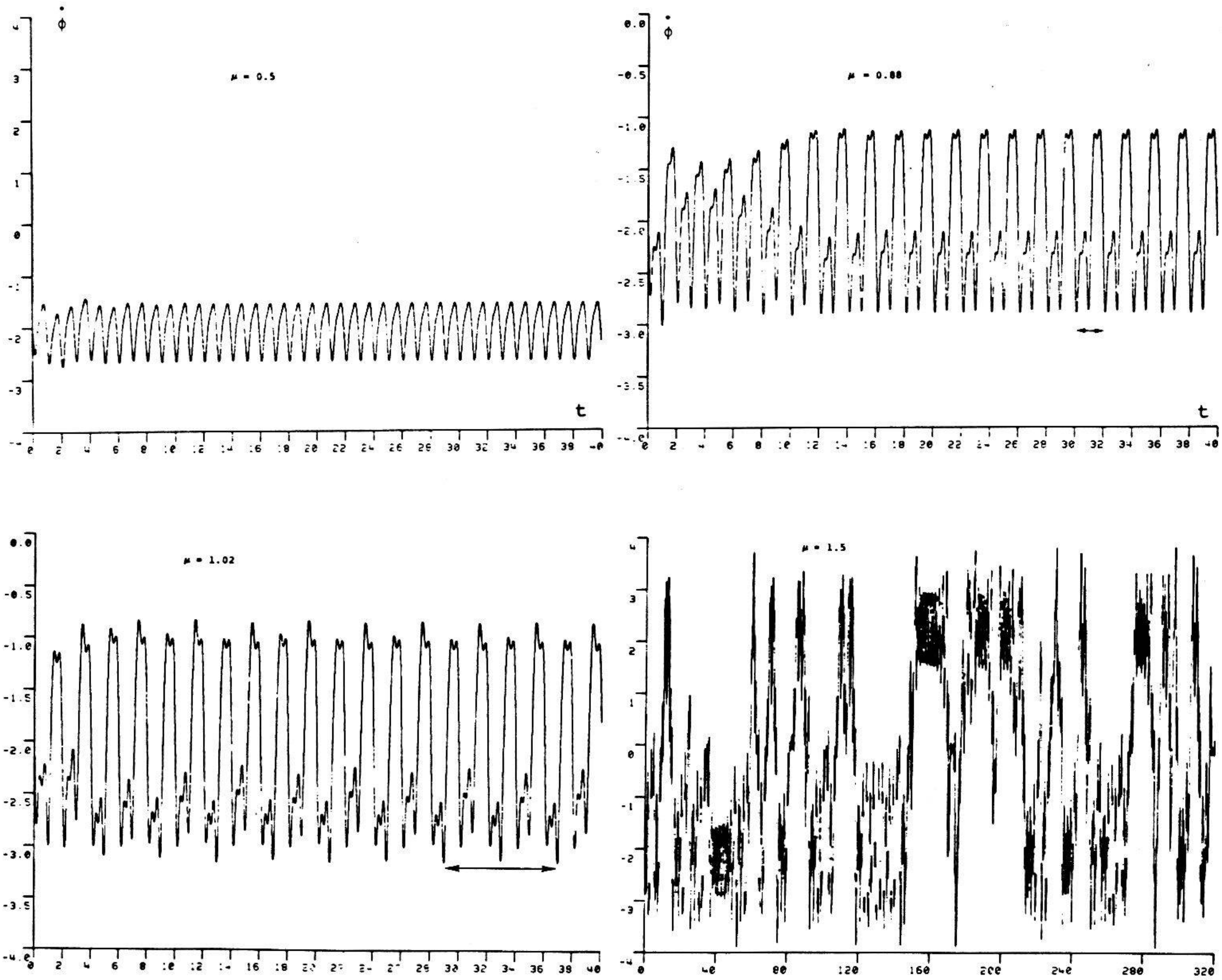


Figure 3

In all cases but the last we observe a transient regime, followed by a manifestly periodic solution, while in the last figure the transient state seems to last forever (it can be shown that there are initial conditions for which this is the case, and for which the number of rotations in a given sense before reversal follows (essentially) any given sequence [Melnikov]). The preceding observations teach us :

- One should neglect transients
- Even if one neglects transients (and considers what is usually called an attractor) one faces apparently unsurmontable problems of classification.

This is easily seen for the following examples from the literature

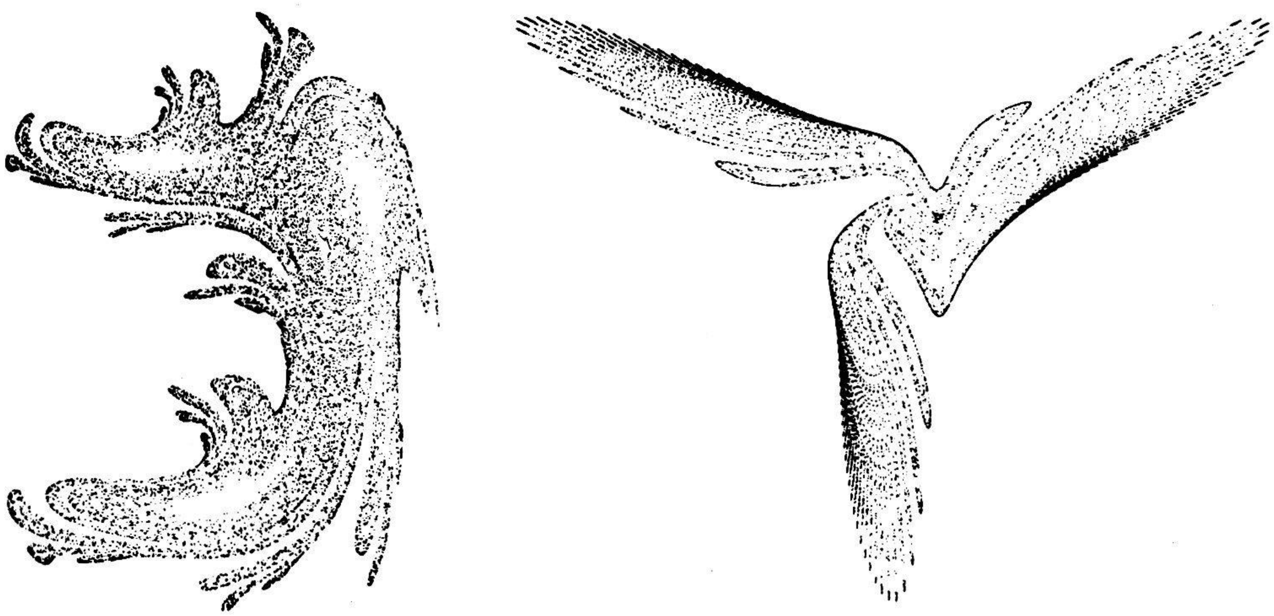
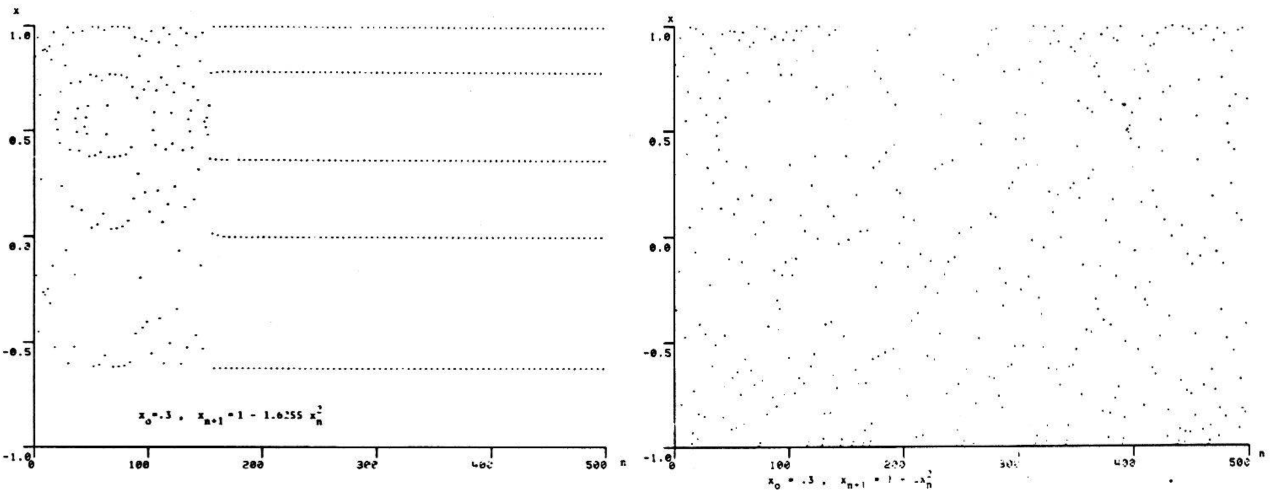


Figure 4 : Examples of attractors for simple systems [Ueda] left, [Mira, Gumowski] right.

The idea which this talk then tries to illustrate is to be more modest and to classify some nontrivial attractors which have the additional feature that they arise as modifications of trivial attractors as an external parameter μ is modified, as in the examples above. Such changes and their associated attractors are called scenarios. In this talk, due to lack of time, I will outline just one such scenario, with which I have been involved most. In my review article [Eckmann], I had outlined two others. Various reactions have shown that, although a disclaimer was made, some people have been lead to believe that there are only three possible scenarios. This is not true, and potentially there are many others of the same importance. In fact, a very appealing one has been found recently [Feigenbaum et al., Rand et al.] which applies to dissipative and conservative systems alike and which deals with invariant circles with irrational rotation number, and their destruction. Aubry has insisted for years on the importance of these circles for incommensurate crystals, and conduction in them.

The scenario I wish to outline has been discovered, based on Feigenbaum's analysis of 1-parameter families of maps of the interval. A typical such one parameter family is $F_\mu(x) = 1 - \mu x^2$. The analogue of equation 1 is $x_{n+1} = F_\mu(x_n)$, $n = 0,1,2,\ldots$; the analogue of Fig. 3 is given in Fig. 5, for $\mu = 1.625$ and $\mu = 2$



Again, we observe a period after a transient (in the first case, the period is 5) and is the second case, there is no periodicity, the transient lasts forever. Feigenbaum went on to observe a striking fact about such one-parameter families. If I plot the final (periodic or aperiodic) regime as a function of μ, I observe a "bifurcation diagram" as in Figure 6 below. This bifurcation diagram shows, from left to right, a stable fixed point, a period 2, then 4, then 8, 16, 32,... . (All these are trivial attractors.) If we denote by $\mu_1, \mu_2, \ldots, \mu_j$ the parameter values for which the periods $2, 4, \ldots, 2^j$ end, then we find that $\lim_{j\to\infty} \frac{\mu_j - \mu_{j+1}}{\mu_{j+1} - \mu_{j+2}} = 4.6692\ldots$. The striking fact is that this number is universal and hence independent of the one-parameter family in question. Feigenbaum proposed a renormalization-group explanation of the phenomenon (the analogue of, say, decimation being the map $f \to f \circ f$ (composition)). This proposal was subsequently put on a sound mathematical basis, and was considerably extended in collaboration with P. Collet, H. Koch and O. Lanford. It leads to the following sort of prediction, typical for a scenario :

Whenever a system undergoes a few period-doubling bifurcations it is "probable" (in the sense of universality of renormalization-group theory) that this doubling will occur at parameter values accumulating again with ratios 4.9962... .

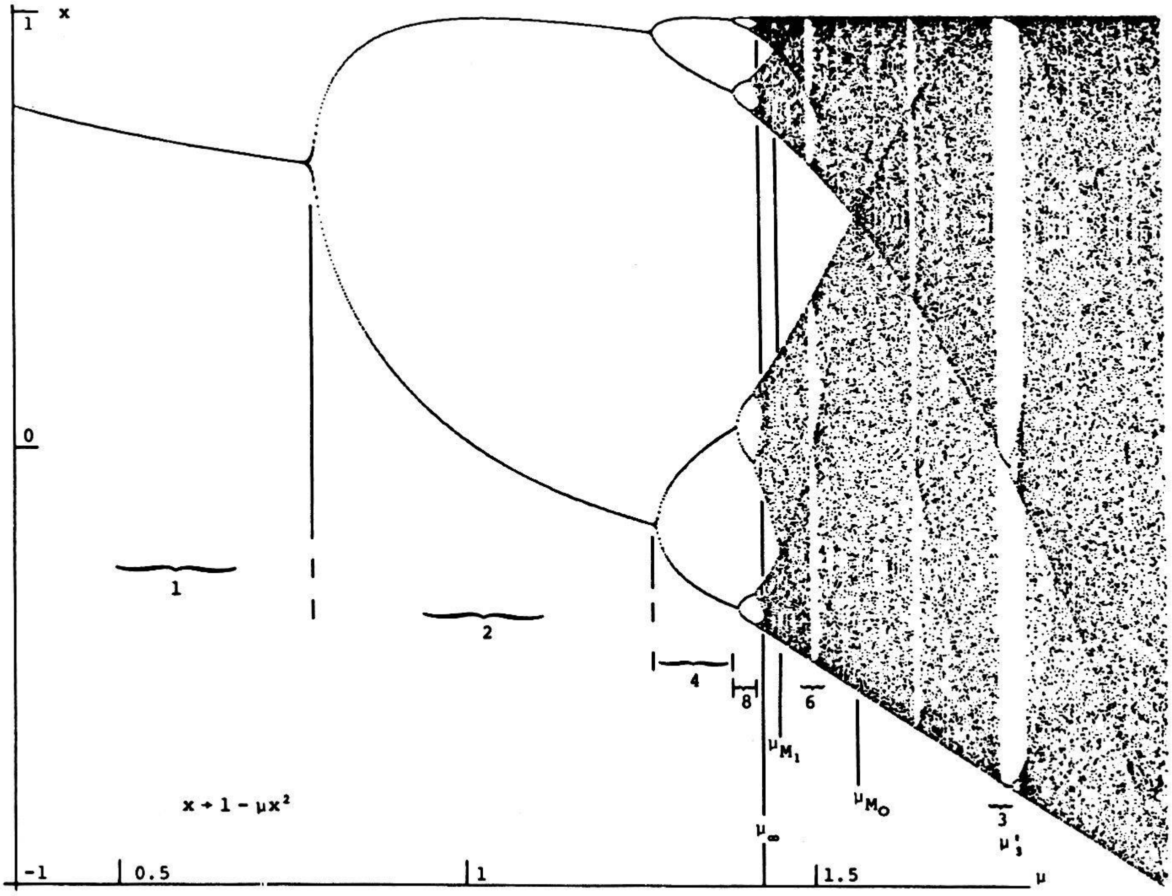


Figure 6

In addition, the "other side" of μ_∞ (see Fig. 6) shows the same scaling behavior, and allows thus to bring some order into a (albeit small) parameter region of chaotic behavior. These predictions have been beautifully confirmed (among others) in the experiments of Libchaber and Maurer.

This example illustrates several features of what I believe to be a successful approach in studying some complicated system : To find a simple - but universal - mechanism, to generalize it adequately, and to devise experiments which concentrate on its predictions. This method of controlled (hydrodynamic) and modest experiments seems a helpful tool in the further study of those chaotic phenomena which have so far defied any systematic analysis.

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